

Information Asymmetries and an Endogenous Productivity Reversion Mechanism

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Abstract

Several studies among recent empirical work have suggested that the systematic behavior of lending standards over the business cycle, with laxer standards applied during expansions and tighter standards applied during recessions, may be responsible for driving economic fluctuations. We build a dynamic screening model with informational asymmetries in credit markets that rationalizes these findings and generates endogenous fluctuations of total output and productivity. When the capital stock is high, which evolves endogenously, liquidity is high for all types of producers, allowing even the unproductive type to meet the early payments on the loan, and thus making signals inferred from such payments less informative. The cost that accomplishes successful screening thus rises, resulting in the emergence of pooling contracts which allow financing of low productivity entrepreneurs. The composition among capital producers then sets off a recession. The opposite happens at troughs.

JEL Codes: E32, E44, D24

I. Introduction

Several studies among recent empirical work have suggested that the systematic behavior of lending standards over the business cycles may be responsible for the reversion of trends in aggregate productivity. Laxer standards during economic booms allow for unproductive firms to enter the market, reducing aggregate productivity and leading to defaults in subsequent periods. On the contrary, tight credit standards during economic downturns allow the exclusion of bad projects, thus sowing the seeds of an economic recovery.

Asea and Blomberg (1998) use a panel data set of two million commercial and industrial loans to find that laxer lending standards occur during expansions and tighter standards occur during recessions, and

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that such behavior of considerably influences the dynamics of aggregate fluctuations. Lown and Morgan (2004) use a survey of loan officers and find a similar behavior of lending standards. Moreover, they point out that loan standards are a more important force than loan rates for explaining variation in business loans and output in the time series.¹ Finally, Berger and Udell (2004) document a similar behavior of lending standards, and validate the hypothesis of loan officers' suboptimal behavior.

Consistent with these empirical findings, the data reveals that delinquency rates and loan charge-off rates,² which we use as a measure of default rates, lag behind the business cycle, since they are high following expansions and low following recessions (Figure 1). It is also observed from the data (Figure 2) that the value of outstanding commercial and industrial loans is highly procyclical.³ It is indeed possible that the cyclical behavior of lending standards is one contributing factor.

Such behavior by banks and entrepreneurs is bound to have an impact on the economy, due to the high proportion of business value that is financed through banks. In fact, financing through financial institutions is widespread in the U.S. Non-corporate business, which represent roughly 1/3 of total business net worth since 1952, relies entirely on bank loans for financing of production. Corporate business, representing roughly 2/3 of the net worth, although it has considerably reduced its dependence on bank financing, still holds over 25% of its debt in bank loans and mortgages and, as recently as in the mid-1970s, held around 45% of their debt in these instruments.⁴

The extent of bank financing, and the seeming imperfections arising from credit standards that are either too lax or too tight, have generated much concern among the policy makers. Alan Greenspan, speaking at the Chicago Bank Structure Conference in 2001, alarmingly stated that "the worst loans are made at the top of the business cycle...[and at the bottom] the problem is not making bad loans, it is not making any loans, whether good or bad...".

It is useful to summarize the empirical facts as follows.

- Loan standards are relaxed at the top of the cycle and tightened at the bottom.
- Delinquency rates lag after the cycle.
- Financing of production through financial institutions is widespread.

¹It is important to make a distinction between the loan interest rate and lending standards, such as the size of the credit line extended, balance sheet variables of the borrower, time to loan maturity, etc. Both, standards and rates, can be used by the bank to screen borrowers.

²Charge-offs, which are the value of loans removed from the books and charged against loss reserves, are measured net of recoveries as a percentage of average loans and annualized. Delinquent loans are those past due thirty days or more and still accruing interest as well as those in nonaccrual status. They are measured as a percentage of end-of-period loans. Source: The Board of Governors, downloaded from <http://www.federalreserve.gov/releases/chargeoff/default.htm>. GDP and GDP deflator data are taken from the Bureau of Economic Analysis.

³Data on C&I loans is taken from the Federal Reserve Board, <http://www.federalreserve.gov/releases/>.

⁴The source for the aggregate balanced sheet data for corporate and non-corporate business is The Board of Governors, downloaded from <http://www.federalreserve.gov/releases/z1/Current/data.htm>.

Motivated by these findings, we investigate the role of credit market frictions in generating an endogenous reversion of productivity and, as a consequence, a reversion on total output. Specifically, we examine the role of information asymmetry in credit markets and endogenous screening costs as the cause for such a reversion. We thus depart from explanations associated with non fully rational expectations on the part of bank loan officers (as short memory, in Berger and Udell 2004), or explanations associated with an exogenous reversal of aggregate productivity (as is standard in the real business cycles literature).

We build an overlapping generations model with two types of entrepreneurs who differ in their productivity and obtain financing through a competitive banking sector. We assume that entrepreneurs and bankers have rational expectations and take decisions accordingly. Entrepreneurs produce next period's capital in two stages, for both of which they require external financing. They can be screened by the bank, which can require a partial loan repayment upon completion of the first stage of production. Entrepreneurs can always default after the second stage of production has taken place, and because of limited liability the bank recovers just a fraction of total output in that case. In the equilibrium path we analyze in this paper, both entrepreneurs seek financing whenever the size of the early payment allow it, but only good entrepreneurs repay in full, while bad entrepreneurs default after financing their second stage production. Banks can offer basically two types of contracts: separating contracts, with an early repayment not payable by bad entrepreneurs and a risk free interest rate (since there is no default), and pooling contracts, with no early repayment but a higher interest rate (since bad entrepreneurs will obtain financing and default afterwards).

The early payment can always be set high enough so that unproductive entrepreneurs cannot afford it. However, this payment hurts productive entrepreneurs as it lowers their reinvestment into the second stage of production. There are two effects that along with competition in the banking sector deliver a pooling contract, with both types of entrepreneurs obtaining financing, at the top of the cycle and a separating contract, with unproductive entrepreneurs unfinanced, at the bottom. At peaks, when all entrepreneurs enjoy higher liquidity (in the form of labor income), it takes a higher payment to screen the bad entrepreneurs out, and hence the productive entrepreneurs must be hurt more for the separation to be viable. Moreover, pooling the unproductive and productive producers into the same contract is less costly for the bank (since they recover more money after default) and hence a lower interest rate is needed to ensure bank participation. This implies that good entrepreneurs are more willing to accept a pooling contract (which does not imply an early repayment and hence allows for higher reinvestment) at the peak than at the trough.

Hence, competition in the banking sector, which leads to the selection of the best contract for good entrepreneurs subject to a zero profit condition, ensures that unproductive entrepreneurs are financed at the peak, thus decreasing the aggregate productivity in the capital good sector and sending the economy to a state with low levels of capital. This situation, in turn, generates the conditions for separating contracts to be selected, thereby increasing productivity in the capital good sector, and leading to an economic recovery.

The basic fact captured by our model is that a certain signal about an entrepreneur's productivity (a particular amount required as early payment) which is informative at the trough is not informative at

the top. This is so because bad entrepreneurs can "fake" their type more easily, using their higher labor income to do so. Consequently, screening out the bad projects becomes more costly, separating contracts become less attractive for the good entrepreneur and are beaten in the market by pooling contracts, which imply a higher interest rate but no early repayment, allowing for high reinvestment levels. These pooling contracts then imply that a mix of good and bad entrepreneurs are financed, lowering productivity and sending the economy into a recession. The opposite happens at a trough, where only a small early repayment is needed to signal a good type, so separating contracts are attractive for good entrepreneurs, due to the small cost of repayment and the big benefits generated by a lower interest rate.

Our model can thus give rise to endogenous fluctuations of capital, total output and productivity through the lending standards channel. This is due to an endogenous screening cost that is cyclical: the higher the availability of extra sources of income for entrepreneurs, the more difficult is to tell them apart. We see the early repayment requirement as a stylized version of lending standards⁵, and we are able to capture the fact that at the peak of the economic cycle there are laxer standards than at the bottom.

Our model does not require exogenous technological shocks to set off a recession after a period of high output, a peak after a period of low output. Therefore it is not an amplification mechanism (like the ones in Bernanke and Gertler (1994), Kiyotaki and Moore(1997) or Rampini(2004)), and captures the stylized fact of lagging delinquency rates without any need for additional explanations that relate the failure of projects to changes in macroeconomic conditions. In our model, it is not the case that ex-ante good projects are financed, become ex-post bad investments because a shock (which was deemed a low probability event) and then become bankrupt. What we have is *ex-ante bad projects*, which are still financed because screening is costly, and even good entrepreneurs prefer pooling contracts where they can reinvest more, even if at the cost of higher interest rates. This endogenous selection of ex-ante bad projects is probably best exemplified with the dot com craze of the mid nineties, where projects with no discernible source of revenue were heavily financed and went bust a short term later. It is difficult to argue that they were good projects that due to an exogenous shock became bad. If anything, the late nineties saw a strong technology improvement in the computer and internet industries. Our explanation is that some of those projects were actually bad, and were financed since with the easy financial conditions of those times, screening had become particularly expensive.

The rest of the paper is organized as follows. Section II overviews related literature. In Section III, we introduce the general model and derive static equilibrium contracts for given prices. In Section IV, we study a fully dynamic economy with externalities in the production sector (which simplifies the analysis since it makes the price of capital constant) and show the existence of equilibrium paths along which the model economy exhibits cyclical behavior. A classical model, with no externalities, is analyzed in Section V, where we find cyclical behavior and also the possibility of indeterminacy of equilibrium on some range of state variables (due to the existence of more than one self-fulfilling belief). Finally, in Section VI, we endogenize the amount of funds available to finance entrepreneurs' projects, and find the possibility of a richer cyclical behavior. For example, for some parameters, this environment generates the stylized fact of sudden drops and slow recoveries. We conclude in Section VII.

⁵In Asea and Blomberg (2004), for example, early repayments are one of the factors consider to define a lending standard.

II. Related Literature

A number of theoretical models illustrate potentially important interactions between credit market frictions and economic fluctuations. For the purpose of our discussion, we focus on two strands of related work. One strand argues that credit market imperfections amplify exogenous shocks and make them more persistent. Another strand of literature argues that credit market imperfections are responsible for a reversion in the productivity.

A classical example of an amplification mechanism is Bernanke and Gertler (1989), where the borrowers' balance sheets amplify exogenous external shocks in a model of costly state verification. Business upturns improve borrowers' net worth, which lowers agency costs of financing investment, increases investment and hence amplifies the upturn, while the opposite happens in the presence of a downturn. Another example is Kiyotaki and Moore (1997), which assumes that loan payments cannot be enforced and hence only collateralized debt arises in equilibrium. A temporary shock that reduces a credit constrained firm's net worth reduces this firm's ability to obtain new loans and therefore its investment, thus propagating the effect of a temporary shock. In Rampini (2004), entrepreneurial activity which consists in the undertaking of risky, but in expected terms productive, projects increases at peaks. This is due to agents higher willingness to take risks when the economic situation is booming, and the smaller need for bearing such a risk in those times, due to slacker incentive constraints.

Suarez and Sussman (1997) generate a reversion mechanism that works through the effect of equilibrium prices on liquidity constraints. The model is a dynamic extension of the Stiglitz-Weiss (1982) model of lending under moral hazard in an overlapping generations model with three generations. During booms, old entrepreneurs sell high quantities and, as a consequence, prices are low and young entrepreneurs must finance a higher fraction of output externally. Because external financing generates excessive risk-taking booms are followed by high project failure rates. Though it delivers an endogenous reversion mechanism, the main channel through which this mechanism works - higher reliance on external financing at peaks - seems to be at odds with the data.

Reichlin and Siconolfi (2003) generalize the Rothschild and Stiglitz adverse selection problem to include moral hazard. Both safe and risky projects can be implemented. Entrepreneurs differ only in their return to implementing the risky project, with lower skilled entrepreneurs facing a higher fixed cost of implementing it. The safe project requires a zero fixed cost and hence yields higher expected returns. They embed this mechanism in an overlapping generations model where the opportunity cost of lending evolves endogenously. They show that endogenous cycles may arise: when loanable funds are high, equilibrium contracts are such that a large fraction of entrepreneurs engages into risky production, high setup costs decrease output and wages sending the economy into recession. This model relies heavily on risky projects being worse in expected terms than safe ones. In fact the opposite assumption (risky projects are better in expected terms) is made in Rampini (2004), and the informational friction delivers an amplification mechanism rather than a reversion mechanism.

Similar in spirit to our work is Dell'Ariccia and Marquez (2006), which consider the effect of an exogenous negative shock on screening costs. This (which they associate to a financial liberalization) leads to a

lending boom, followed by a deterioration of lending standards and finally a banking crisis as observed in many of the emerging economies. Their idea that large screening costs lead to the selection of pooling contracts is similar to ours, the main difference being that in our model the screening costs are endogenous and procyclical in equilibrium, therefore generating economic cycles.

III. The Model

A. Environment

Consider a model economy where time is discrete and indexed by $t = 0, 1, 2, \dots$. It is populated with overlapping generations of entrepreneurs who live for two periods and there exist two types of goods: consumption and capital. When young, entrepreneurs are endowed with 1 unit of time and an ability to implement projects that produces capital goods from consumption goods. We assume entrepreneurs do not suffer disutility from labor and enjoy utility from consuming both when young and old, according homothetic preferences represented by $u(c^y, c^o)$.

Consumption good is produced by an infinitely lived aggregate firm that employs labor and capital according to technology $Y_t = A_t K_t^\beta L_t^{1-\beta}$. Capital goods produced in period $t-1$ can be used as an input in production in period t , upon completion of which it fully depreciates. We assume this firm has access to perfect credit markets and borrows at a risk free rate R_f . Hence, the aggregate firm can buy capital used in time t production (at price ρ_t) from young entrepreneurs in period $t-1$. The period 0 firm is endowed with K_0 units of capital and debt in the amount of $R_f \rho_0 K_0$. The aggregate firm behaves competitively, so the wage w_t and the cost per unit of capital faced by the firm, $R_f \rho_t$, equal their marginal products $R_f \rho_t = A_t \beta k_t^{\beta-1}$ and $w_t = A_t (1-\beta) k_t^\beta$, where $k_t = K_t/L_t$.⁶

Capital goods are produced by entrepreneurs. Each generation consists of a measure μ of type G and a measure $1-\mu$ of type B . Types differ in their productivity and are private information. Each entrepreneur can implement a project within a single period, but in two stages. A fixed cost in the amount of M units of consumption goods is required at each stage of production. A project implemented by an entrepreneur of type i yields a fixed amount f_i of capital goods at the end of the first stage (we assume that investing more than M in the first stage yields a zero return) and $g_i + \frac{g_i}{M} s_i$ units of capital good at the end of the second stage, where s_i represents investment of funds beyond the fixed cost amount into the second stage of the project (by assumption 1, both types of entrepreneurs reinvest all the remaining income after the early repayment in stage 2 of their projects). We assume that type G is more productive at each stage.

Assumption 1 $f_G > f_B$ and $g_G > g_B$.

There is a competitive banking sector that loans investment funds to the young entrepreneurs. Each period, banks are endowed with $2M\mu$ loanable units of the consumption good, exactly the fixed cost amount of implementing projects of all type G entrepreneurs. This particular amount is assumed for

⁶Taking prices as given, the firm solves $\max_{\{K_t, L_t\}} \sum_{t=0}^{\infty} A_t K_t^\beta L_t^{1-\beta} - R_f \rho_t K_t - w_t L_t + \rho_{t+1} K_{t+1} - \rho_{t+1} K_{t+1}$, where in period t the bank receives $\rho_{t+1} K_{t+1}$ from the bank and pays it to capital producers; it repays $R_f \rho_{t+1} K_{t+1}$ to the bank in the next period.

analytical simplicity and is not crucial for the results. In fact, we endogenize the supply of funds in Section VI. A risk-free savings technology is available to the bank at rate R_f .

We consider contracts which are signed in the beginning of the entrepreneurs' young period. If an entrepreneur enters into a contract, he receives an amount M in the beginning of the first stage and another amount M in the beginning of the second stage, conditional on meeting a partial payment δ_t towards the loan balance. At the end of the period, the remaining loan repayment is $2MR_t - \delta_t$. Hence, a contract is characterized by (δ_t, R_t) . Whenever financing is obtained, the entrepreneur must implement the project, which is justified by the availability of a monitoring technology. We allow for default upon completion of the second stage, in which case a fraction α of wealth is kept by the entrepreneur, while a fraction $1 - \alpha$ is seized by the bank. We think of α as representing the state of available bankruptcy institutions. We assume that default can take place only at the end of the second stage. This captures the idea that it is more difficult to hide income during initial stages of production and is made only for analytical simplicity.⁷

Alternatively, we could assume that banks offer a credit line to entrepreneurs and contracts specify the credit limit and the interest rate on the borrowed funds. In the appendix, we prove that identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with credit limit $2M - \delta_t$ and interest rate $\frac{2MR_t - \delta_t}{2M - \delta_t}$.

It is instructive to examine the cash flows for an entrepreneur who enters into a contract and does not default, illustrated in the timeline below. A time t young entrepreneur obtains M units of consumption good from the bank in the beginning of the first stage of production and invests it into the project. At the end of the first stage he receives $\rho_{t+1}f_i$ in payment for his capital⁸ and w_t as labor income from supplying 1 unit of time to the aggregate firm. He pays δ_t towards the loan balance upon completion of the first stage of production. He receives M units of consumption good from the bank and invests into the second stage, along with his own income net of the loan payment, $w_t + \rho_{t+1}f_i - \delta_t$, which transforms into capital at the rate of $\frac{g_i}{M}$. At the end of the period, the entrepreneur sells his capital $g_i + (w_t + \rho_{t+1}f_i - \delta_t)\frac{g_i}{M}$ at price ρ_{t+1} and pays the remaining loan balance $2R_tM - \delta_t$ to the bank. The net worth at the end of the period, $\rho_{t+1} [g_i + (w_t + \rho_{t+1}f_i - \delta_t)\frac{g_i}{M}] - (2MR_t - \delta_t)$, is then allocated between consumption when young in t and consumption when old in $t + 1$ according to u .

Because entrepreneurs are productive for a single period only, and their capital requirement is the same for both types, two stages of production are necessary to allow the possibility of screening. In our case, the screening tool is the requirement of a partial payment towards the loan balance before production is completed. Note that screening would not be possible if capital output were purchased by the aggregate firm only at the end of the period. For that reason, we assume that the aggregate firm delivers the payment for capital goods produced in the first stage before the second stage of production starts.

⁷This essentially means that in case of default at the end of the first stage, the bank seizes the entire wealth of the defaulting entrepreneur.

⁸The price this entrepreneur observes for his capital is ρ_{t+1} since this capital is used in period $t + 1$.

Table 1: Cash Flows for the Young Entrepreneur that Seeks Financing and Repays in Full

STAGE 1	STAGE 2
$+M$	$+M$
$-M$	$-M - (w_t + \rho_{t+1}f_i - \delta_t)$
$+ \rho_{t+1}f_i + w_t$	$+ \rho_{t+1} [g_i + (w_t + \rho_{t+1}f_i - \delta_t) \frac{g_i}{M}]$
$-\delta_t$	$-(2MR_t - \delta_t)$

Thus, we capture the idea that the lenders always have the ability to screen out bad entrepreneurs, because type G can always afford a higher payment than type B . However, with liquidity (as determined by w_t or ρ_{t+1}) evolving endogenously through time, the cost of effective screening, as perceived by type G entrepreneurs, will also evolve endogenously through time. Competition in the banking sector and the endogenously evolving cost of screening will determine whether or not the screening tool is used in equilibrium at a particular point in time.

B. Entrepreneurs' Behavior

For a given contract (δ_t, R_t) and prices w_t and ρ_{t+1} , a time t young entrepreneur of type i chooses among the options summarized below.

(O1) Do not enter into the contract. End of period net worth is w_t .

(O2) Enter into the contract, meet the payment δ_t , default. End of period net worth is $\alpha \rho_{t+1} [g_i + (w_t + \rho_{t+1}f_i - \delta_t) \frac{g_i}{M}]$.

(O3) Enter into the contract, meet the payment δ_t , pay in full. End of period net worth is $\rho_{t+1} [g_i + (w_t + \rho_{t+1}f_i - \delta_t) \frac{g_i}{M}] - (2MR_t - \delta_t)$.

Because of our assumption of homothetic preferences, entrepreneurs will save a constant fraction ($\equiv \xi$) of their end of period net worth. Hence, to determine the best option, it suffices to compare end of period net worth associated with each. Note that since default is not an option before the second stage, entrepreneur i will always choose O1 if $w_t + \rho_{t+1}f_i < \delta_t$, i.e., if he cannot afford the early payment.

We are interested in equilibrium paths along which (1) type G entrepreneurs seek financing whenever $\delta_t < w_t + \rho_{t+1}f_G$ and in case of obtaining it, repay in full. (2) type B entrepreneurs seek financing whenever $\delta_t < w_t + \rho_{t+1}f_B$ and in case of obtaining it, default on the loan. The tradeoffs that the entrepreneurs face between different options depend on the endogenously determined prices ρ and w . In the proposition that follows, for each of the environments considered in this paper (Sections IV, V and VI), we state under which conditions on the set of parameters and initial conditions, the entrepreneurs' behavior along any equilibrium path is as described by (1) and (2). We emphasize that Assumptions 2, 3 and 4 are simply assumptions on the parameter space and initial conditions, although their specification relies on definitions introduced in later sections. For each environment, we will show, by providing examples, that parameters sets satisfying these assumptions are nonempty.

Assumption 2 For the environment studied in Section IV, define $k_{min} = k_p(\bar{k})$ and $k_{max} = k_s$ employing (11), (10), and Corollary 1. Assume $k_0 \in [k_{min}, k_{max}]$. Restrict the parameter space such that Condition 1 (in the appendix) holds given prices in (7).

Assumption 3 For the environment studied in Section V, define $k_{min} = k_p(\underline{k}_p)$, where $\underline{k}_p \equiv k_{li}$, and $k_{max} = k_s$ employing (22) and (21). Assume $k_0 \in [k_{min}, k_{max}]$. Restrict the parameter space such that Condition 1 (in the appendix)⁹ holds for the price definitions from (13).

Assumption 4 For the environment studied in Section VI, define $k_{min} = \lim_{k \rightarrow \bar{k}^+} k_{t+1}(k, \frac{2M\bar{k}}{k_s^G})$ and $k_{max} = \mu k_s^G$ using equation (23). Then, if the initial state (k_0, S_0) satisfies

- $k_0 \in [k_{min}, k_{max}]$
- $\bar{k} \geq k_0 \geq \frac{S_0}{2M} k_s^G$
- $k_0 \leq \begin{cases} \frac{(2M/\xi - W_G^s)S_t}{(1-\beta)(2M - S_t)} & \text{and } S_t \leq 2M\mu \\ \frac{S_t/\xi - \mu W_G^s}{(1-\beta)(1-\mu)} & \text{and } S_t > 2M\mu \end{cases}$

Restrict the parameter space such that Condition 1 (in the appendix) holds given prices in (7).

Proposition 1 Suppose that Assumption 1 holds. The following holds for each of the environments of Sections IV, V and VI, under Assumptions 2, 3 and 4, respectively. Along every equilibrium path,

- Both types, if financed, reinvest all the proceedings from the first stage (after the early payment δ) for production in the second stage.
- If type B enters into the contract, he chooses to default.
- If type G enters into the contract, he chooses to repay in full.
- Both types choose entering into a contract (δ_t, R_t) if they can afford to repay, i.e. if $\delta_t < w_t + \rho_{t+1}f_i$.
- $k_t \in [k_{min}, k_{max}]$ for all $t = 1, 2, \dots$

Proof. See the appendix. ■

C. Equilibrium contracts

Next we characterize the equilibrium contract (δ, R) under given wages for the current period w and price of capital for the next period ρ . Our assumption of competition in the lending sector implies that there must be no profit-making opportunities. In the following proposition we use that fact to show that a situation in which both types obtain financing under different contracts cannot arise in equilibrium. In other words, either financing of both types is done under the same contract (δ, R) or type G is financed while type B accepts the null contract.

⁹Note that for the case of section 4, $\rho(k') \equiv \beta$, therefore Condition 1 simplifies significantly.

Proposition 2 *If, for some given w and ρ , positive measures of both types obtain financing, it must be the case that a single (pooling) contract is offered.*

Proof. See the appendix. ■

Again, our assumption of a competitive banking sector implies that banks earn zero profits in equilibrium. We derive the zero-profit condition that a contract (δ, R) must satisfy in this environment as a function of w and ρ . A type B entrepreneur can afford the early payment δ if and only if it is smaller than

$$(1) \quad \tilde{\delta}_{w,\rho} \equiv w + \rho f_B.$$

Note that $\tilde{\delta}_{w,\rho}$ increases in both w and ρ . Indeed, as labor or capital income of type B increases, a higher payment is needed for separation.

We must specify the zero profit condition for two situations, one in which G obtains financing and B does not, and one in which both types are financed. In what follows we invoke Proposition 1.

Separating contracts. For $\delta > \tilde{\delta}_{w,\rho}$, only type G can afford the payment, entering the contract and repaying in full. Type B stays out of the contract. Hence, the zero profit condition reduces to $R = R_f$, where we assumed that the opportunity cost of loaning out funds is the risk free return R_f .

Pooling contracts. For $\delta \leq \tilde{\delta}_{w,\rho}$, both types enter, type G repaying in full while type B defaulting. Since types are not observable and only $2M\mu$ can be loaned out, a measure μ^2 of entrepreneurs of type G and a measure $(1 - \mu)\mu$ of type B are financed. The zero profit condition in that case is given by

$$\mu^2 2MR + (1 - \mu)\mu \left[\delta + (1 - \alpha)\rho \left(g_B + (w + \rho f_B - \delta) \frac{g_B}{M} \right) \right] = R_f 2M\mu,$$

where the right hand side represents the opportunity cost of loaning out the available funds. The left hand side represents the funds obtained as loan repayment. It consists of full repayment from measure μ^2 of type G entrepreneurs and δ collected as early payment together with the fraction $1 - \alpha$ of output seized from measure $(1 - \mu)\mu$ of type B entrepreneurs.

Therefore, the zero profit condition can be written as:

$$(2) \quad R_{w,\rho}(\delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu} \left[\delta + (1 - \alpha)\rho \left(g_B + (w + \rho f_B - \delta) \frac{g_B}{M} \right) \right], & \text{if } \delta < \tilde{\delta}_{w,\rho} \\ R_f, & \text{otherwise,} \end{cases}$$

Note that $R_{w,\rho}(\delta)$ decreases in both ρ and w in the interval $[0, \tilde{\delta}]$. Intuitively, as income increases, the amount recovered from type B increases, and hence a lower interest rate is needed to ensure zero profit for the lender.

The next assumption assures that, for pooling contracts, raising δ (which effectively lowers reinvestment from type B) actually raises the total collection from type B . This implies that the segment of $R_{w,\rho}(\delta)$ corresponding to pooling contracts (i.e. $\delta < \tilde{\delta}_{\rho,w}$) is downward sloping. As δ increases, total collection

from defaulting agents increases and a lower interest rate R is needed to ensure zero profit. This assumption guarantees that every pooling contract that leaves banks with a zero profit is robust to profit making opportunities from an entrant that lowers δ and leaves R unchanged to attract type B entrepreneurs.

Assumption 5 $1 > (1 - \alpha) \rho(k_{max}) \frac{g_B}{M}$.

The next proposition considers a contract which maximizes utility of type G entrepreneur subject to the lenders' participation constraint. It states that such contract is robust to profit making opportunities. Because we assume competition in the lending sector and require that in equilibrium there must exist no profit-making opportunities for a bank who offers a different contract, we find our equilibrium contracts by solving (3).

Proposition 3 *Consider a contract (δ^*, R^*) that solves*

$$(3) \quad \max_{\delta, R} \quad \rho \left(g_G + (w + \rho f_G - \delta) \frac{g_G}{M} \right) - (2MR - \delta) \\ \text{s.t.}$$

$$(4) \quad R = R_{w, \rho}(\delta),$$

where $R_{w, \rho}(\delta)$ is given by equation (2). This contract yields a zero profit for the lender and is robust to profit-making opportunities.

Proof. See the appendix. ■

In the next lemma, we show that pooling contracts exhibit an interest rate R higher than the risk-free rate. Intuitively, this is true because type B enters into the contract to default, so it takes a higher than the risk-free interest rate to ensure zero profits for the lender.

Lemma 1 *If $\delta < \tilde{\delta}_{w, \rho}$, then $R_{w, \rho}(\delta) > R_f$.*

Proof. See Appendix. ■

The bank's maximization problem (3), summarized by the feasible set (zero-profit condition) and type G indifference curves, is depicted in Figure 3. To find the solution to this problem, i.e., to derive the equilibrium contract, we note that indifference curves describing his trade-off between R and δ , given current prices, have a slope

$$(5) \quad ICS \equiv -\frac{\rho \frac{g_G}{M} - 1}{2M} < 0.$$

ICS is negative by Condition 1,a, and indifference curves become steeper as ρ rises. This last fact is intuitive since, as capital sells for a higher price, the cost of foregone investment associated with δ rises, and type G requires a greater reduction in R to compensate for a given rise in δ . The solution to (3) depends on the relative size of ICS and the slope of the line connecting the following two points on the

lender's zero-profit curve: the point associated with $\delta = 0$ (a pooling contract) and a point associated with $\tilde{\delta}_{\rho,w}$ (a separating contract). We call this slope IPS , the first two letters referring to the iso-profit,

$$(6) \quad IPS \equiv -\frac{R_{w,\rho}(0) - R_f}{\tilde{\delta}_{w,\rho}} < 0.$$

The IPS is negative due to Lemma 1. From Figure 3, it is clear that a pooling contract $(0, R_{w,\rho}(0))$ is offered if $ICS < CS$, and a separating contract $(\tilde{\delta}_{\rho,w}, R_f)$ is offered otherwise. The following proposition summarizes this result.

Proposition 4 *If $\frac{1-\rho\frac{gG}{M}}{2M} \geq -\frac{R_{w,\rho}(0)-R_f}{\tilde{\delta}_{w,\rho}}$, then a separating contract $(\tilde{\delta}_{w,\rho}, R_f)$ is offered. Otherwise, a pooling contract, $(0, R_{w,\rho}(0))$ is offered, with $R_{w,\rho}(0)$ defined in (2).*

Proof. See the appendix. ■

IV. Simplified Mechanism

In the previous section, we derived equilibrium contracts for given prices (w, ρ) . We now turn to analyze the *dynamics* of capital and aggregate productivity, which is the main focus of this paper. The optimal contract changes over time, because the prices relevant in period t , w_t and ρ_{t+1} , depend on the capital level of the economy in periods t and $t+1$, respectively. This contract, in turn, affects aggregate productivity and capital accumulation, thus generating time-varying, sometimes cyclic, economic behavior.

In this section, we consider a simple case, where we assume the presence of a productivity externality due to the size of the economy. This makes the price of capital constant, while the wage w_t varies linearly with the capital level k_t . The reversion mechanism, which is the focus of this paper, is easy to observe in this context. In this case, liquidity changes arise only due to changes in labor income (and not changes in revenues from capital sales). Higher labor income raises the level of the early payment needed for effective screening. In addition, with higher labor income, default by type B entrepreneurs is less costly for the banks, implying that a lower interest rate charged in case of a pooling equilibrium. These two forces combined make the pooling equilibrium appear when capital is high, pushing it down for the next period. Because of the same intuition, a separating equilibrium appears for a low enough level of capital. In the general case, without a production externality, the same mechanism is present except it also works through the effect on revenues from capital sales. Its slightly more complicated dynamics are presented in Section V.

A. Prices and Equilibrium Contracts

We now consider a positive externality on the production of consumption goods, by assuming $A_t = K_t^\gamma$, $\gamma + \beta = 1$. We then have

$$(7) \quad \rho_t = \beta A_t K_t^{\beta-1} L_t^{1-\beta} = \beta \text{ and } w_t = (1-\beta) A_t \frac{K_t^\beta}{L_t^\beta} = (1-\beta) \frac{K_t^{\beta+\gamma}}{L_t^\beta} = (1-\beta) k_t.$$

The key to simplifying the analysis is that the rental price of capital is constant, making k_t the only state

variable. We restate the analysis of the previous section, which was done for given prices w and ρ , in terms of the state variable k .

Net worth associated with each strategy can be written in terms of the state variable. Option (O1), that is, not engaging in production, yields $(1 - \beta)k$. Option (O2), entering into the contract to default, yields $\alpha\beta[g_i + ((1 - \beta)k + \beta f_i - \delta) \frac{q_i}{M}]$. Finally, option (O3), entering into the contract to repay, yields $\beta[g_i + ((1 - \beta)k + \beta f_i - \delta) \frac{q_i}{M}] - (2MR - \delta)$. Again, if an entrepreneur of type i cannot afford δ , i.e., if $(1 - \beta)k + \beta f_i < \delta$, he will choose O1.

The lender's zero profit function can be written as

$$(8) \quad R_k(\delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu} [\delta + (1-\alpha)\beta(g_B + ((1-\beta)k + \beta f_B - \delta) \frac{q_B}{M})], & \text{if } \delta < \tilde{\delta}_k \\ R_f, & \text{otherwise,} \end{cases}$$

with

$$(9) \quad \tilde{\delta}_k = (1 - \beta)k + \beta f_B.$$

The *ICS* and *IPS* that were given by (5) and (6) now become

$$CS = -\frac{R_k(0) - R_f}{\tilde{\delta}_k} \text{ and } IPS \equiv -\frac{\beta \frac{q_G}{M} - 1}{2M}.$$

Note that *ICS* is independent of k , while *IPS* is not. As k increases, *IPS* becomes less negative for two reasons. First, higher labor income raises $\tilde{\delta}_k$, making it more difficult to screen out type B entrepreneurs. Second, $R_k(0)$ decreases with higher income, because a higher collection from defaulting entrepreneurs allows for a lower interest rate to guarantee a zero profit for the banks.

Proposition 4, which specifies the contracts offered in equilibrium, can be now expressed in terms of k . If $\frac{1-\beta \frac{q_G}{M}}{2M} \geq -\frac{R_k(0)-R_f}{\tilde{\delta}_k}$, then a separating contract $(\tilde{\delta}_k, R_f)$ is offered, where $\tilde{\delta}_k$ and $R_k(0)$ are given by (9) and (8). Otherwise, a pooling contract $(0, R_k(0))$ is offered.

Because *IPS* increases in k , while *ICS* is independent of k , there is a threshold $\bar{k}$, given by the unique solution to $\frac{\beta \frac{q_G}{M} - 1}{2M} = \frac{R_k(0)-R_f}{\tilde{\delta}_k}$, which divides the regions of pooling and separating equilibria.

Corollary 1 Define $\bar{k} = \left(\frac{(1-\mu)(2MR_f - g_B(1-\alpha)\beta)}{\mu(\beta \frac{q_G}{M} - 1) + \beta \frac{q_B}{M}(1-\mu)(1-\alpha)} - \beta f_B \right) / (1 - \beta)$. Then, if $k \geq \bar{k}$, a pooling equilibrium occurs, while if $k < \bar{k}$, a separating equilibrium occurs.

Proof. See the appendix. ■

As we can see, there are two effects that make the separating contract (with a high repayment rate δ but a low interest rate R_f) more attractive for type G entrepreneurs when k is high. First, with high k , unproductive entrepreneurs earn higher labor income, so a higher $\tilde{\delta}$ is needed for separation. Recalling that δ is costly for type G as it lowers the rate of reinvestment into the second stage of production, it follows that the separating contract is costlier for larger k . Second, a pooling contract does not hurt type

G as much when k is high, because the default of type B entrepreneurs is less costly to the bank (it collects more after default due to higher labor income) and hence a lower loan interest, $R_k(0)$, is needed to ensure bank participation.

Figure 4 illustrates the equilibrium contract determination for two arbitrary states of the economy, k_L and k_H , where $k_L < \bar{k} < k_H$. Because $\frac{R_{k_H}(0) - R_f}{\delta_{k_H}} < \frac{\beta \frac{g_G}{M} - 1}{2M} < \frac{R_{k_L}(0) - R_f}{\delta_{k_L}}$, a pooling contract is offered for k_H and a separating contract is offered for k_L .

B. Dynamics

We have up to this point discussed the type of contracts offered for a given state of the economy. We now move on to analyze the evolution of the capital level through time. We derive the transition function, which has two segments, each corresponding to the type of equilibrium contracts.

For $k \leq \bar{k}$, we established that separating contracts with an early payment $\tilde{\delta}_k = (1 - \beta)k + \beta f_B$ arise, and thus only type G (of measure μ) engages in production of capital. Each one produces f_G in the first stage and sells it at price β , while also earning $(1 - \beta)k$ as labor income. Type G then meets the early payment $\tilde{\delta}_k$ and reinvests the remaining funds $(1 - \beta)k + \beta f_G - \tilde{\delta}_k$ into the project, which yields $g_G + [(1 - \beta)k + \beta f_G - \tilde{\delta}_k] \frac{g_G}{M} = g_G + \beta(f_G - f_B) \frac{g_G}{M}$ units of capital. Hence, the next period capital stock is given by

$$(10) \quad k_s(k) \equiv k_s = \mu \left(f_G + g_G + \beta(f_G - f_B) \frac{g_G}{M} \right).$$

Note that $\frac{dk_s(k)}{dk} = 0$ since any additional unit of capital, which translates into $1 - \beta$ additional units of labor income, is paid to the bank before the second stage takes place, and therefore it is not reinvested. This allows us to define a constant k_s .

For $k > \bar{k}$, we established that pooling contracts arise with a no repayment. Therefore both types participate in the production of capital. Measure μ^2 of type G and measure $(1 - \mu)\mu$ of type B obtain financing, and the next period capital stock is then given by

$$(11) \quad k_p(k) = \mu^2 \left(f_G + g_G + ((1 - \beta)k + \beta f_G) \frac{g_G}{M} \right) + (1 - \mu)\mu \left(f_G + g_B + ((1 - \beta)k + \beta f_B) \frac{g_B}{M} \right).$$

Note that $\frac{dk_p(k)}{dk} > 0$, because a pooling contract involves no early payment and therefore every additional unit of labor income is reinvested (due to Condition 1,a), augmenting current capital production and hence the future capital level.

The transition function for capital is thus given by

$$(12) \quad k'(k) = \begin{cases} k_s & \text{if } k \leq \bar{k} \\ k_p(k) & \text{otherwise} \end{cases}.$$

How does $k_s(\bar{k})$ relate to $k_p(\bar{k})$? On the one hand, under separation, all of the productive entrepreneurs (entire measure μ) engage in production. Type B entrepreneurs do not apply for financing and do not

crowd out type G from getting financed. On the other hand, to make separation possible, the income obtained in the first stage cannot be reinvested, so each type B entrepreneur produces less. In this paper, we focus on the set of parameters for which the composition effect dominates the per agent production effect at $\bar{k}$. We therefore make the following assumption.

Assumption 6 $k_p(\bar{k}) < k_s$

This implies that as type B gets pooled into the mix, capital output goes down due to the crowding out of type G , despite the fact that more resources are invested in production (as no resources are used for early payment). Note that since k_s is constant, setting $k_p(\bar{k}) < k_s(\bar{k})$ is sufficient to guarantee that for $k \leq \bar{k}$, we have $k_p(k) < k_s(k)$, i.e. the composition effect dominates the per agent production effect for all levels $k \leq \bar{k}$.

Finally, we make an assumption to ensure there is no perpetual growth in this economy. We make sure that for pooling contracts, an extra unit of capital, which translates into an extra $1 - \beta$ units of input into the second stage of production, results in less than one unit of additional capital.

Assumption 7 $k'_p(k) = (1 - \beta)\mu(\mu \frac{q_G}{M} + (1 - \mu) \frac{q_B}{M}) < 1$.

With the previous assumptions, equilibrium dynamics can be summarized as follows:

Proposition 5 *Case 1. If $\bar{k} < k_p(\bar{k}) < k_s$ then the capital stock converges to $k_{ss} = k_p(\bar{k})$. See Figure 5.*

Case 2. If $k_p(\bar{k}) < \bar{k} < k_s$ then the capital stock exhibits cycles, not necessarily trivial. See Figure 6.

Case 3. $k_p(\bar{k}) < k_s < \bar{k}$ then the capital stock converges to $k_{ss} = k_s$. See Figure 7.

Proof. See the appendix. ■

Case 2 is the focus of this paper. When capital is high, pooling contracts are chosen, so both types entrepreneurs seek financing, type G entrepreneurs choose to repay and type B entrepreneurs default at the end of the period. Because type B entrepreneurs participate in production thus crowding out some type G entrepreneurs, next period's capital stock is lower. When capital is low enough, separating contracts are chosen, only type G entrepreneurs seek financing and, even with the lack of reinvestment generated by the repayment requirement, a high amount of capital is produced for the next period.

Finally, the length of the cycle can be easily calculated from the primitives.

Corollary 2 *Consider an economy satisfying $k_p(\bar{k}) < \bar{k} < k_s$. If n is the smallest number such that $k_p^{(n-1)}(k_s) > \bar{k}$ but $k_p^n(k_s) < \bar{k}$, then an economy starting at $k_0 = k_s$ exhibit cycles of length $n + 1$. In each cycle the capital level for the first $n - 1$ periods and goes up to k_s in the last.*

Proof. See the appendix. ■

We give a numerical example, which shows the restrictions derived on the set of parameters and initial conditions are non-empty and that non-trivial cyclical behavior is possible.

Example 1. Parameters are $\alpha = 0.32, \beta = 0.91, \mu = 0.64, f_G = 2.07, g_G = 2, f_B = 0, g_B = 1.6, R_f = 1, M = 1$. With these parameters, $\bar{k} = 4.58, k_s = 5.02, k_p(k_s) = 4.6$. Because these parameters imply $k_p(k_s) > \bar{k}$, we obtain non-trivial cycles, capital cycle given by 5.02, 4.6, 4.55, 5.02, 4.6, 4.55... See Figure 8.

V. No externalities

We now consider the case with no externalities, that is $A_t \equiv A$. The rental price of capital goods is no longer a constant; it depends on the belief of the aggregate firm about the capital stock in the next period, which we denote by k' . Prices are given by

$$(13) \quad \rho(k') = A\beta k'^{\beta-1} \text{ and } w(k) = A(1 - \beta)k^\beta.$$

Although this extension introduces complications in the analysis of the model's dynamics, the reversion of the aggregate productivity arises according to the same intuition. High levels of capital, i.e. high liquidity, generates high costs of effective screening, inducing pooling equilibria and a lower level of capital in the next period. For levels of capital (and liquidity) low enough, screening costs are low and separating equilibria are obtained, generating higher levels of capital in the following period due to the absence of type B entrepreneurs among those engaged in capital production. Hence, capital accumulation and aggregate productivity evolve endogenously, and can possibly exhibit non-trivial cyclical dynamics.

However, a new interesting phenomenon - an indeterminacy region - appears in the case of no externalities. For certain levels of capital k , two possible forecasts about future capital k' are self-fulfilling. If the aggregate firm (the buyer of currently produced capital) believes that separating contracts will arise yielding high levels of capital production, it will pay a low price $\rho(k')$ to current period capital producers. This low price, in turn, induces a separating contract, as a low early payment is needed for separation and a high interest rate is charged in a pooling contract, thus making the firm's belief consistent with equilibrium outcome. In the same way, for k in the indeterminacy region, the belief of a pooling contract and hence lower levels of future capital induces current capital prices to be high, induces pooling equilibria making the belief consistent with equilibrium outcomes.

Later we explain the intuition for why only a single belief can be consistent for regions of capital outside the indeterminacy region.

A. Prices, Equilibrium Contracts and Consistent Beliefs

As we discussed, the aggregate firm's belief regarding capital production in the current period influences the behavior of entrepreneurs and equilibrium outcomes through its effect on the rental price of capital. We refer to (k, k') as the state of the economy. Given such state, equilibrium outcomes can be inferred. We require beliefs k' to be consistent with the equilibrium outcome.

We can write the minimum level of the early payment that ensures separation and the interest rate $R_{w,\rho}(0)$ given in (1) and (2) as functions of the economy state (k, k') ,

$$(14) \quad \tilde{\delta}_{k,k'} = w(k) + \rho(k')f_B,$$

$$(15) \quad R_{k,k'}(\delta) = \begin{cases} \frac{R_f}{\mu} - \frac{(1-\mu)}{2M\mu} [\delta + (1-\alpha)\rho(k')(g_B + (w(k) + \rho(k')f_B - \delta)\frac{g_B}{M})] & \text{if } \delta \geq \tilde{\delta}_{k,k'} \\ R_f & \text{otherwise} \end{cases}$$

As before, the existence of a pooling or separating equilibrium, and its implications for the next period capital level, can be determined as functions of the state variables. Denoting the next period's capital by $\kappa(k, k')$, we obtain

$$\kappa(k, k') = \begin{cases} \kappa_p(k, k') & \text{if } \frac{1-\rho(k')\frac{g_G}{M}}{2M} < -\frac{R_{k,k'}(0)-R_f}{\tilde{\delta}_{k,k'}} \\ \kappa_s(k') & \text{otherwise} \end{cases},$$

where

$$(16) \quad \kappa_p(k, k') = \mu^2[f_G + g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] + (1-\mu)\mu[f_B + g_B + (w(k) + \rho(k')f_B)\frac{g_B}{M}],$$

$$(17) \quad \kappa_s(k') = \mu[f_G + g_G + \rho(k')(f_G - f_B)\frac{g_G}{M}].$$

The intuition regarding the type of equilibrium that emerges is similar to that of the previous section. The effect of k on whether pooling or separating contracts arise is unchanged. The slope of type G 's indifference curves is independent of k ; a lower k makes pooling contracts less attractive due to higher interest rates associated with pooling and a lower early payment needed for separation. There appears a new force, however, induced by the belief k' . As this belief goes up, the price of capital $\rho(k')$ decreases, generating two important effects. First, an increase in the early payment δ now hurts type G less, as the benefits from extra reinvestment (due to the sale of extra units of capital in the second stage) are now smaller, and therefore type G 's indifference curves flatten in the $R - \delta$ plane. Second, the critical slope IPS becomes more negative (steeper), because the minimum early payment that ensures separation declines and the interest rate charged in case of pooling rises. As we can see, an increase in the belief k' has the same qualitative effect, i.e., makes pooling (separation) more (less) attractive, as a decline in k does.

Formally, it is easy to see that, as k' rises, the slope of type G 's indifference curves, $\frac{1-\rho(k')\frac{g_G}{M}}{2M}$, becomes less negative while the critical slope $-\frac{R_{k,k'}(0)-R_f}{\tilde{\delta}_{k,k'}}$ becomes more negative. (See equations (14) and (15).) This implies that there exists (for a fixed k) a cutoff level of beliefs, denoted by $\tilde{k}'_k$ and defined as the solution of

$$(18) \quad \frac{1 - \rho(k')\frac{g_G}{M}}{2M} = -\frac{R_{k,k'}(0) - R_f}{\tilde{\delta}_{k,k'}}.$$

For beliefs k' below this cutoff level, pooling equilibria arise, leading to the production of $\kappa_p(k, k')$ units of capital. For beliefs k' above this level, separating equilibria arise leading to the production of $\kappa_s(k, k')$

units of capital. Hence, the transition function for capital simplifies to

$$(19) \quad \kappa(k, k') = \begin{cases} \kappa_p(k, k') & \text{if } k' < \tilde{k}'_k, \\ \kappa_s(k') & \text{if } k' \geq \tilde{k}'_k. \end{cases}$$

Figure 9 illustrates $\kappa(k, k')$ as a function of beliefs k' (for a fixed k). It consists of two segments: for low beliefs k' a pooling equilibrium arises, and for high beliefs k' , separating contracts arise. Both segments are downward sloping, as higher k' reduces reinvestment and hence total capital production. Moreover, as k increases, the segment corresponding to the pooling equilibrium moves up (due to higher reinvestments), and the segment corresponding to the separating equilibrium stays constant (as the extra income is used for repayment). Finally, we explain that as k increases, a higher belief k' is needed to induce the selection of a separating contract. As k goes up, labor income $w(k)$ goes up and therefore the critical slope IPS flattens (because the early payment needed for separation increases and the interest rate charged in a pooling equilibrium declines). Hence, it takes a higher belief k' (which implies a lower price $\rho(k')$) to make this curve steeper and type G 's indifference curves flatter, to restore indifference between the two contracts for type G entrepreneurs. All these facts are summarized in the following lemma.

Lemma 2 *The following are true*

$$\frac{\partial \kappa_p(k, k')}{\partial k'} \leq 0, \frac{\partial \kappa_s(k')}{\partial k'} \leq 0, \frac{\partial \kappa_p(k, k')}{\partial k} \geq 0, \frac{d\tilde{k}'_k}{dk} \geq 0.$$

Proof. See the appendix. ■

We impose restrictions on beliefs k' . In particular, we require that in accordance with the rational expectations hypothesis, equilibrium beliefs are consistent with equilibrium outcomes, i.e.,

$$(20) \quad k' = \kappa(k, k').$$

The right hand side gives the future level of capital as a function of current capital k and beliefs about future capital k' . These beliefs, i.e., the left hand side of the equation, must equal the actual level of future capital. We define the consistent beliefs correspondence $k'(k)$ as the set of solutions to equation (20), i.e., the set of consistent beliefs about future capital when the current level is k . This correspondence is critical for determining equilibrium dynamics because, together with k , beliefs determine future capital. We now turn our attention to ensuring that this correspondence is non-empty and analyzing how many different values it can assume.

The next assumption (comparable to Assumption 6 in Section IV), guarantees existence of consistent beliefs and, therefore, an equilibrium path, as shown in the next lemma.

Assumption 8 *Consider k_{hi} , defined as the highest k for which a belief about a separating equilibrium is consistent, i.e., as a solution to $\kappa_s(\tilde{k}'_k) = \tilde{k}'_k$ (or 0 if a solution does not exist). We require that $\tilde{k}'_{k_{hi}} \geq \kappa_p(k_{hi}, \tilde{k}'_{k_{hi}})$.*

Figure 10 is a sketch of $\kappa(k_{hi}, k')$; it illustrates Assumption 8 graphically. It is easy to see graphically that there is at least one and at most two consistent beliefs for each k .

Lemma 3 *If Assumption 8 is satisfied, then, for every k , there is at least one and at most two solutions to (20).*

Proof. See the appendix. ■

Figures 10-12 illustrate three situations that may arise: a unique consistent belief about separating contracts, two consistent beliefs, one about a separating contract and one about a pooling one, and a unique consistent belief about a pooling one. Indeterminacy of equilibrium arises in the second case. From Lemma 2, we deduce the shape of the consistent belief correspondence $k'(k)$. As k increases, we always move from regions with a separating belief, to regions with indeterminacy, to regions with a pooling belief (as seen from Figures 11-13).

In fact, analogous to our definition of k_{hi} , we can define k_{li} as the lowest k for which a belief about pooling equilibrium is consistent (see Figure 14). Formally, k_{li} is defined as

$$(21) \quad k_{li} = \begin{cases} 0 & \text{if } \kappa_p(0, \tilde{k}'_0) < 0 \\ \sup\{k | \kappa_p(k, \tilde{k}'_k) \geq \tilde{k}'_k\} & \text{otherwise} \end{cases}.$$

Lemma 4 *It must be the case that $k_{li} \leq k_{hi}$.*

Proof. See the appendix. ■

We then have a transition correspondence which, for a given k , pins down the consistent belief regarding the next period's capital, which in turn represents the *actual* next period's capital level:

$$(22) \quad k'(k) = \begin{cases} k_s & \text{if } k \leq k_{li} \\ \{k_s, k_p(k)\} & \text{if } k \in \{k_{li}, k_{hi}\} \\ k_p(k) & \text{if } k \geq k_{hi} \end{cases},$$

where k_s satisfies $\kappa_s(k_s) = k_s$ and $k_p(k)$ satisfies $\kappa_p(k, k_p(k)) = k_p(k)$.¹⁰

B. Dynamics

The transition correspondence, derived above, allows us to characterize the dynamic behavior of the capital stock, aggregate productivity and total output in the economy. The next assumption is equivalent to assumption 7 in the previous section, and guarantees that there is no perpetual growth.

Assumption 9 $\frac{\partial \kappa_p(k, k')}{\partial k} < 1$.

Figure 15 illustrates one possible transition correspondence. From Lemma 2, we can show that $k_p(k)$ is increasing in k .¹¹ Also, due to Assumption 8, $k_p(k_{hi}) < k_s$.

¹⁰ Because $\kappa_s(k')$ is independent of k , we have that k_s is a constant, and not a function of k .

¹¹ Indeed, implicitly differentiating $\kappa_p(k, k_p(k)) = k_p(k)$ gives

$$\begin{aligned} k'_p(k) &= \frac{d\kappa_p(k, k')}{dk} + \frac{d\kappa_p(k, k')}{dk'} k'_p(k) \\ k'_p(k) &= \frac{d\kappa_p(k, k')}{dk} / \left(1 - \frac{d\kappa_p(k, k')}{dk'}\right) \geq 0. \end{aligned}$$

We now show two stylized cases, for which the economy may exhibit cycles. In the first case, cycles always exist, but their length and/or the amplitude changes according to the (consistent) beliefs held by the aggregate firm. In the second case, it is possible to have cycles, but it is also possible that the economy converges to a steady state, depending on the beliefs.

Case 1: (See Figure 15) $k_p(k_{li}) \leq k_{li} \leq k_s$ and $k_p(k_{hi}) \leq k_{hi} \leq k_s$. Regardless of the selection from the correspondence $k'(\cdot)$, perpetual cycles emerge. However, if the selection is “optimistic”, in the sense that $k'(k) = k_s$ for $k \in [k_{li}, k_{hi}]$, the economy exhibits potentially shorter cycles or smaller amplitude, i.e., with the drop in capital stock and total output not as pronounced.

Case 2: (See Figure 16) $k_p(k_{li}) \leq k_{li} \leq k_s$ and $k_{hi} \geq k_s$. Here, the existence of cycles depends on the selection from the correspondence $k'(\cdot)$. If the aggregate firm is optimistic, there are no cycles, the economy converging to the steady state k_s . However, if the aggregate firm is pessimistic, the economy exhibits perpetual cycles.

We give a numerical example, which shows the restrictions derived on the set of parameters and initial conditions in this section are non-empty, and that non-trivial cyclical behavior is possible.

Example 2. Parameters are $A = 2.66, \alpha = 0.82, \beta = 0.89, \mu = 0.33, f_G = 2, g_G = 2, f_B = 0, g_B = 1.6, R_f = 1, M = 1$. With these parameters, $k_s = 4.06, k_{li} = 2.316, k_{hi} = 2.63, k_p(k_s) = 2.327$. Because these parameters imply $k_p(k_s) > k_{li}$, we obtain non-trivial cycles ($k = 4.06, 2.301, 2.114, 4.06 \dots$) as long as beliefs in the indeterminacy region correspond to pooling equilibrium in the future. The dynamics for this case is illustrated in Figure 16. If the beliefs for k in the indeterminacy region are that separating equilibrium arises, then the cycle for this example is trivial.

VI. Endogenous Savings

Recall the setup used up to now. Entrepreneurs allocated their end of period net worth between consumption in period t and consumption in period $t + 1$ according to their preferences. We then assumed that entrepreneurs could save at the risk-free rate in international markets. The amount of loanable funds available for financing the young entrepreneurs of each generation was always fixed at the level of $2M\mu$. In this section, we endogenize the supply of loanable funds and require that loans to the young entrepreneurs must be financed with the old entrepreneurs' savings. To keep the analysis tractable, we maintain that the risk-free rate is fixed as determined by the international markets; and whenever present, excess supply of funds earns R_f .

Making the supply of loanable funds endogenous not only confirms the possibility of cyclical economic behavior, but it also gives rise to the possibility of sudden drops and slow recoveries, which have been widely documented in the literature. The intuition for slow recoveries is as follows. For low enough levels of capital, again, separating equilibria emerge. However, if the supply of funds is also low, only a small fraction of type G entrepreneurs is financed. Aggregate productivity and capital production increase, and so do the savings. In the next period, a higher fraction of type G is financed and the slow recovery

continues until capital reaches a high enough level to warrant a pooling equilibrium. As pooling emerges, however, high levels of funds, which increase individual production, may not be enough to counteract the decline in the capital level generated by the mix of entrepreneurs engaged in production. Hence, in contrast to the previous setup, endogenous supply of funds can give rise to non-trivial cycles through slow expansions and sudden recessions. Even though in this section we focus on the case of slow recoveries and sudden drops, which we find particularly interesting, in general endogenous evolution of loanable funds allows for smoother aggregate fluctuations.

In what follows, we extend the analysis of the the production externality case presented in Section IV to endogenize the supply of loanable funds. First, we derive the dynamical system describing the evolution of the two state variables (capital and supply of funds). We then analyze their behavior using the phase diagram, subdivided into the regions of pooling and separating equilibria. We finally illustrate the possibility of cycles with sudden drops and slow recoveries.

A. Transition Functions

The supply of funds (S_t) used to finance the young generation is given by the savings of the old generation. If funds are not sufficient to finance all applicants, crowding out occurs, with the unfinanced entrepreneurs staying out of capital production and obtaining only $(1 - \beta)k$ as labor income. If funds are in excess of applicants' demand, every applicant gets financed; the excess funds are saved in a foreign bank that pays an interest rate R_f .

Referring to Section IV, we note that whether pooling or separating contracts emerge depends on k and not on S . In turn, S affects the measure of entrepreneurs financed and influences capital in the next period. In case of a pooling contract, the available funds are used to finance both type G and type B entrepreneurs, the total demand for funds is $2M$. In case of a separating contract, only type G is financed, the total demand for funds equal to $2M\mu$. Capital stock in period $t + 1$, given today's state variables k_t, S_t , is then

$$(23) \quad k_{t+1}(k_t, S_t) = \begin{cases} \frac{S_t}{2M} [\mu k_p^G(k_t) + (1 - \mu)k_p^B(k_t)] & \text{if } k_t \geq \bar{k} \text{ and } S_t \leq 2M \\ \mu k_p^G(k_t) + (1 - \mu)k_p^B(k_t) & \text{if } k_t \geq \bar{k} \text{ and } S_t > 2M \\ \frac{S_t}{2M} k_s^G & \text{if } k_t \leq \bar{k} \text{ and } S_t \leq 2M\mu \\ \mu k_s^G & \text{if } k_t \leq \bar{k} \text{ and } S_t > 2M\mu \end{cases},$$

where

$$\begin{aligned} k_s^G &= f_G + g_G + \beta(f_G - f_B) \frac{g_G}{M}, \\ k_p^G(k_t) &= f_G + g_G + ((1 - \beta)k_t + \beta f_G) \frac{g_G}{M}, \\ k_p^B(k_t) &= f_B + g_B + ((1 - \beta)k_t + \beta f_B) \frac{g_B}{M} \end{aligned}$$

denote individual capital production of type G entrepreneur under separation, type G entrepreneur under pooling, and type B entrepreneur under pooling, respectively.

Given the entrepreneurs' preferences, their savings are always a constant fraction ($\equiv \xi$) of their end of period net worth. In periods with a pooling contract, both types engage in production, but because the types differ in productivity and undertake different decisions regarding loan repayment, their per-capita savings differ. Recall that the aggregate savings today represent the supply of funds tomorrow. Hence, we obtain the supply of funds tomorrow, as a function of the current state,

$$(24) \quad S_{t+1}(k_t, S_t) = \begin{cases} \xi \left[\frac{S_t}{2M} (\mu W_p^G(k_t) + (1-\mu)W_p^B(k_t)) + (1 - \frac{S_t}{2M}) (1-\beta)k_t \right] & \text{if } k_t \geq \bar{k} \text{ and } S_t \leq 2M \\ \xi [\mu W_p^G(k_t) + (1-\mu)W_p^B(k_t)] & \text{if } k_t \geq \bar{k} \text{ and } S_t > 2M \\ \xi \left[\frac{S_t}{2M} W_s^G + (1 - \frac{S_t}{2M}) (1-\beta)k_t \right] & \text{if } k_t \leq \bar{k} \text{ and } S_t \leq 2M\mu \\ \xi [\mu W_s^G + (1-\mu)(1-\beta)k_t] & \text{if } k_t \leq \bar{k} \text{ and } S_t > 2M\mu \end{cases},$$

where

$$\begin{aligned} W_s^G &= \beta[g_G + \beta(f_G - f_B)\frac{g_G}{M}] - 2MR_f, \\ W_p^G(k_t) &= \beta \left[g_G + ((1-\beta)k_t + \beta f_G)\frac{g_G}{M} \right] - \frac{2MR_f}{\mu} + \frac{1-\mu}{\mu}(1-\alpha)\beta \left[g_B + ((1-\beta)k_t + \beta f_B)\frac{g_B}{M} \right], \\ W_p^B(k_t) &= \alpha\beta \left[g_B + ((1-\beta)k_t + \beta f_B)\frac{g_B}{M} \right] \end{aligned}$$

represent the net worth of type G entrepreneur under separation, type G entrepreneur under pooling, and type B entrepreneur under pooling, respectively.

B. The Phase Diagram

As usual for the analysis of such a dynamical system, we now divide the state space into regions where it is possible to sign the changes $k_{t+1} - k_t$ and $S_{t+1} - S_t$.

We put S on the X-axis and k on the Y-axis. Because only k matters for whether separating or pooling contracts emerge, the state space diagram is divided into two regions by $k = \bar{k}$. For all points (k, S) such that $k \leq \bar{k}$, separating contracts emerge in equilibrium. For other points, pooling contracts emerge.

First, consider the region of separation ($k_t \leq \bar{k}$). From (23) and (24), we derive that $k_{t+1} = k_t$ iff

$$(25) \quad k_t = \begin{cases} \frac{S_t}{2M} k_s^G & \text{and } S_t \leq 2M\mu \\ \mu k_s^G & \text{and } S_t > 2M\mu \end{cases}$$

and $S_{t+1} = S_t$ iff

$$(26) \quad k_t = \begin{cases} \frac{(2M/\xi - W_s^G)S_t}{(1-\beta)(2M - S_t)} & \text{and } S_t \leq 2M\mu \\ \frac{S_t/\xi - \mu W_s^G}{(1-\beta)(1-\mu)} & \text{and } S_t > 2M\mu \end{cases}.$$

Note that (25) is an upward sloping linear curve for $S_t \leq 2M\mu$, at which point it connects to the zero-sloped line. Referring back to (23), we see that $k_{t+1} - k_t < 0$ for points to the left of this curve, and above the curve on the region $S_t > 2M\mu$, while $k_{t+1} - k_t > 0$ to the right of this curve.

Now consider the hyperbola (26) on the range $S_t \leq 2M\mu$. If $2M/\xi > C_G^s$, then the curve is in the first quadrant and is convex; otherwise, it lies in the fourth quadrant. In either case, at point $S_t = 2M\mu$, the curve connects to an upward sloping line. From (24), we obtain that $S_{t+1} - S_t > 0$ for points above the resulting curve and $S_{t+1} - S_t < 0$ for points below the curve.

Next, we consider the region of pooling ($k_t \geq \bar{k}$). From (23) and (24), we obtain that $k_{t+1} = k_t$ iff

$$(27) \quad k_t = \begin{cases} \frac{A'S_t}{2M-B'S_t} & \text{if } S_t \leq 2M \\ \frac{A'}{1-B'} & \text{if } S_t > 2M \end{cases},$$

with $A' = \mu(f_G + g_G + \beta f_G \frac{g_G}{M}) + (1-\mu)(f_B + g_B + \beta f_B \frac{g_B}{M})$, and $B' = \mu(1-\beta)\frac{g_G}{M} + (1-\mu)(1-\beta)\frac{g_B}{M}$, and $S_{t+1} = S_t$ iff

$$(28) \quad k_t = \begin{cases} \frac{(2M/\xi-A)S_t}{BS_t+(2M-S_t)(1-\beta)} & \text{if } S_t \leq 2M \\ \frac{S_t/\xi-A}{B} & \text{if } S_t > 2M \end{cases},$$

with $A = \beta \left[\mu \left(g_G + \frac{\beta f_G g_G}{M} \right) + (1-\mu)(g_B + \frac{\beta f_B g_B}{M}) \right] - 2MR_f$ and $B = \beta(1-\beta)(\mu \frac{g_G}{M} + (1-\mu)\frac{g_B}{M})$.

Note that hyperbola (27) is a convex curve in the region $S_t \leq 2M/B'$, i.e., to the left of its asymptote. If $B' < 1$, then the curve is positive on the entire region $S_t \leq 2M$, connecting to a zero-sloped line at $S_t = 2M$. In this case, $k_{t+1} - k_t < 0$ to the left and above of the curve and $k_{t+1} - k_t > 0$ to the right and below the curve. To eliminate the possibility of perpetual growth, which is not the focus of this paper, we limit our attention to parameters such that $B' < 1$, which is equivalent to the previous Assumption 7.¹²

Now consider (28). If $2M/\xi > A$, then it is a concave and positive curve, connecting to a positively sloped line at $S_t = 2M$. For points above of the curve, $S_{t+1} - S_t > 0$, and for points below, the opposite true. If $2M/\xi < A$, the curve is negative, connecting to a positively sloping line at $S_t = 2M$. In this case, we have $S_{t+1} - S_t > 0$ for all points in the range $S_t \leq 2M$ and above the line on the range $S_t > 2M$; the opposite is true for points below the line.

It is possible to derive the necessary conditions for endogenous cycles. In particular, we need that equations (25) – (26) when solved for S_t in terms of k_t and then evaluated at $k_t = \bar{k}$, yield savings levels in the following order: have (28) < (25) < (26) < (27).¹³ Another set of necessary conditions is the same as above, except the last two levels switch order.

¹²In fact, if $B' > 1$, then the curve is negative in the range $S_t \in (2M/B', 2M)$, connecting to a zero-sloped line at $S_t = 2M$. In that case, $k_{t+1} - k_t < 0$ to the left of the convex curve in range $S_t \in (0, 2M/B')$ and $k_{t+1} - k_t > 0$ for all the points to its right in the first quadrant, thus allowing for perpetual growth in that region.

¹³Note that the condition that (25) is less than (27) at $k = \bar{k}$ is written as

$$\frac{2M\bar{k}}{(f_G + g_G + \beta(f_G - f_B)\frac{g_G}{M})} < \frac{2M\bar{k}}{[\mu(f_G + g_G + ((1-\beta)\bar{k} + \beta f_G)\frac{g_G}{M}) + (1-\mu)(f_B + g_B + ((1-\beta)\bar{k} + \beta f_B)\frac{g_B}{M})]},$$

this is the same condition as was necessary to assure the jump in the transition function for the simple model.

We give a numerical example, which shows the restrictions derived on the set of parameters and initial conditions in this section are non-empty.

Example 3. Parameters are $\xi = 0.75$, $\alpha = 0.44$, $\beta = 0.975$, $\mu = 0.26$, $f_G = 2.41$, $g_G = 1.83$, $f_B = 0.77$, $g_B = 1.825$, $R_f = 1$, $M = 1$. $\bar{k} = 1.6$. Cycles are given by an 11 period cycle with slow recoveries and sudden drops ($s_t = 0.35, 0.362, 0.374, 0.385, 0.397, 0.41, 0.422, 0.435, 0.448, 0.46, \dots 0.35\dots$ and $k_t = 1.197, 1.26, 1.29, 1.34, 1.38, 1.42, 1.46, 1.51, 1.55, 1.6003, \dots 1.197$. Because these parameters generate such a slow recovery, the highest level of capital is very close to $\bar{k}$. It slowly rises and as soon as it is above $\bar{k}$, it drops. (See Figure 17a,b)

VII. Conclusion

There is a widespread disagreement with regard to the importance of information frictions in credit market in driving economic fluctuations. We propose a novel reversion mechanism that is consistent with several empirical findings outlined above. We build a dynamic screening model with fully rational entrepreneurs of different types and competitive lenders. When the economy is productive (productivity evolves endogenously), signals about entrepreneurs' quality become endogenously less informative. Consequently, screening out the bad projects becomes more costly. Contracts that try to do that simply do not survive in equilibrium. Instead, contracts financing a mix of good and bad projects emerge. The composition effect due to both good and bad projects being implemented sets off a recession. The opposite happens at troughs.

Our model can give rise to endogenous (and non-trivial) fluctuations via the lending standards channel, with the strength of the entrepreneurs' signal of the quality of their project and thus the cost of screening evolving endogenously over the cycle. Aggregate productivity over this cycle changes due to the composition of types of projects being financed and implemented. This mechanism can also work in a model with exogenous shocks to productivity.

Future research can benefit from embedding our reversion mechanism within the general equilibrium model with infinitely lived agents and assessing its importance quantitatively. Overall, there is still very little quantitative work present in this discussion, with an exception of Carlstrom and Fuerst (1997).

VIII. Appendix

Credit Line as the Alternative Form of Contracts

Identical equilibrium outcomes are achieved under the assumption of contracts in the form of a credit line with the credit limit $2M - \delta_t$ and interest rate $\frac{2MR_t - \delta_t}{2M - \delta_t}$.

In the first stage the entrepreneur borrows M and starts production (he does not borrow more since it would not be productive to do so). In the second stage he taps the credit line for the remaining $M - \delta$ and complements it with his own savings, so his disposable income is $M - \delta + \rho f_i + w$, exactly as in the paper. Finally, for this credit line he repays, in the last stage $2MR - \delta$, so the effective interest rate is $\frac{2MR - \delta}{2M - \delta}$.

Note that this analysis holds true if the bank can monitor the reinvestment in the second stage, not only of the amount tapped in the credit line, but also of at least $\rho f_B + w$ extra.

Condition 1

- a. $\frac{\rho(k_{max})g_B}{M} > 1$
- b. $w(k_{max}) + \rho(k_{min})f_B + (1 - \alpha)\rho(k_{min})g_B \leq 2MR_f$
- c. If $\mu((1 - \alpha)\frac{\rho(k_{max})g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho(k_{max})g_B}{M} - 1) \leq 0$, then

$$(29) \quad \begin{aligned} 2MR_f \leq & (1 - \alpha)\rho(k_p(k_{max}))[\mu[g_G + (w(k_{min}) + \rho(k_p(k_{max}))f_G)\frac{g_G}{M}] \\ & + (1 - \mu)[g_B + (w(k_{min}) + \rho(k_p(k_{max}))f_B)\frac{g_B}{M}]]. \end{aligned}$$

If $\mu((1 - \alpha)\frac{\rho(k_{min})g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho(k_{min})g_B}{M} - 1) \geq 0$, then

$$(30) \quad \begin{aligned} 2MR_f \leq & \mu \left[(1 - \alpha)\rho(k_p(k_{max}))[g_G + \beta(f_G - f_B)\frac{g_G}{M}] + \rho(k_p(k_{max}))f_B + w(k_{min}) \right] \\ & + (1 - \mu) [\rho(k_p(k_{max}))f_B + w(k_{min}) + (1 - \alpha)\beta g_B] \end{aligned}$$

If neither of the conditions is satisfied¹⁴, then both (29) and (30) must hold.

- d. $(1 - \alpha)\rho(k_{max})[g_G + \rho(k_{max})(f_G - f_B)\frac{g_G}{M}] \geq 2MR_f - w(k_{min}) - \rho(k_{max})f_B$
- e. $\alpha\rho(k_p(k_{max}))g_B \geq w(k_{max})$
- f. $\rho(k_{max})[g_G + \rho(k_{max})(f_G - f_B)\frac{g_G}{M} + f_B] \geq 2MR_f$
- g. $1 > (1 - \alpha)\frac{\rho(k_{min})g_B}{M}$

Proof of Proposition 1

► One extra unit of consumption good reinvested into the second stage of production yields g_B/M units of capital, generating $\rho(k')g_B/M$ units of the consumption good. Therefore, for reinvestment to be profitable, we must have $\rho(k')g_B/M > 1$. Because $\rho(k')$ is a decreasing function (or constant in Sections IV, VI), this is implied for all k' by

$$\frac{\rho(k_{max})g_B}{M} > 1,$$

which is assumed (Condition 1,a).

► We now show that type B defaults whenever he enters the contract. We must check this only for $\delta \leq \rho(k')f_B + w(k)$,¹⁵ and for every interest rate $R \geq R_f$.¹⁶ The constraint is written as

$$\alpha\rho(k')[g_B + (w(k) + \rho(k')f_B - \delta)\frac{g_B}{M}] \geq \rho(k')[g_B + (w(k) + \rho(k')f_B - \delta)\frac{g_B}{M}] - (2MR_{k,k'}(\delta) - \delta).$$

¹⁴One of them is always satisfied in section 4, where $\rho(k') \equiv \beta$

¹⁵Otherwise, B would not enter into the contract.

¹⁶In fact, if for some interest rate (and any delta) this does not hold, the pooling contract could be offered at the risk-free interest rate, and would be preferred by type G .

Noting that the most restrictive case is when $R = R_f$ and $\delta = \tilde{\delta} = \rho(k')f_B + w(k)$ (this last fact is due to Condition 1,g), the constraint is implied by

$$\begin{aligned} \alpha\rho(k')[g_B + (w(k) + \rho(k')f_B - \tilde{\delta})\frac{g_B}{M}] &\geq \rho(k')[g_B + (w(k) + \rho(k')f_B - \tilde{\delta})\frac{g_B}{M}] - (2MR_f - \tilde{\delta}), \text{ i.e.,} \\ \alpha\rho(k')g_B &\geq \rho(k')g_B - 2MR_f + w(k) + \rho(k')f_B, \text{ i.e.,} \\ w(k) + \rho(k')f_B + (1 - \alpha)\rho(k')g_B &\leq 2MR_f. \end{aligned}$$

Since $\rho(k')$ is decreasing, the assumption $w(k_{max}) + \rho(k_{min})f_B + (1 - \alpha)\rho(k_{min})g_B \leq 2MR_f$, given in Condition 1,b, suffices to ensure the default of type B .

► We now prove that type G always chooses to repay in full, for both pooling ($\delta \in [0, w(k) + \rho(k')f_B]$) and separating ($\delta = w(k) + \rho(k')f_B$) contracts.

Case 1 (pooling)

For type G not to default, we need

$$\begin{aligned} \alpha\rho(k')[g_G + (w(k) + \rho(k')f_G - \delta)\frac{g_G}{M}] &\leq \rho(k')[g_G + (w(k) + \rho(k')f_G - \delta)\frac{g_G}{M}] - (2MR_{k,k'}(\delta) - \delta), \text{ i.e.,} \\ (1 - \alpha)\rho(k')[g_G + (w(k) + \rho(k')f_G - \delta)\frac{g_G}{M}] &\geq 2MR_{k,k'}(\delta) - \delta, \text{ i.e.,} \\ (1 - \alpha)\rho(k')[g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] &\geq (1 - \alpha)\rho(k')\delta\frac{g_G}{M} + 2MR_{k,k'}(\delta) - \delta. \end{aligned}$$

It is easy to see that the derivative of the right hand side with respect to δ is given by

$$\mu((1 - \alpha)\frac{\rho(k')g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho(k')g_B}{M} - 1),$$

thus, there are two possible cases.

Case 1.1: $\mu((1 - \alpha)\frac{\rho(k_{max})g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho(k_{max})g_B}{M} - 1) \leq 0$. Then the most restrictive case is when $\delta = 0$ and we have, using the definition of $R_{k,k'}(0)$,

$$2MR_f \leq (1 - \alpha)\rho(k') \left[\mu[g_G + (w(k) + \rho(k')f_G)\frac{g_G}{M}] + (1 - \mu)[g_B + (w(k) + \rho(k')f_B)\frac{g_B}{M}] \right],$$

which is satisfied if (29) holds.

Case 1.2: $\mu((1 - \alpha)\frac{\rho(k_{min})g_G}{M} - 1) + (1 - \mu)((1 - \alpha)\frac{\rho(k_{min})g_B}{M} - 1) \geq 0$. Then the most restrictive case is when $\delta \rightarrow \tilde{\delta}$, with the corresponding $R_{k,k'}(\delta)$ and we have,

$$2MR_f \leq \mu \left[(1 - \alpha)\rho(k')[g_G + \beta(f_G - f_B)\frac{g_G}{M}] + \rho f_B + w(k) \right] + (1 - \mu) [\rho(k')f_B + w(k) + (1 - \alpha)\beta g_B]$$

which is satisfied if (30) holds. Hence, Condition 1,c implies that G repays in full in case of pooling contracts.

Case 2 (separation) In case of a separating equilibrium ($\delta = w(k) + \rho(k')f_B$, $R = R_f$), for type G not to

default we need

$$\begin{aligned} \alpha \rho(k') [g_G + \rho(k')(f_G - f_B) \frac{g_G}{M}] &\leq \rho(k') [g_G + \rho(k')(f_G - f_B) \frac{g_G}{M}] - (2MR_f - w(k) - \rho(k')f_B), \text{ i.e.,} \\ (1 - \alpha) \rho(k') [g_G + \rho(k')(f_G - f_B) \frac{g_G}{M}] &\geq 2MR_f - w(k) - \rho(k')f_B, \end{aligned}$$

which is satisfied due to Condition 1,d.

► We now prove that whenever they can afford the repayment, both types seek financing.

We first check that this is true for type B , which compares (whenever he can afford it) entering to default with the option of not undertaking his project. We need that, for any $\delta \in [0, w(k) + \rho(k')f_B]$,

$$\alpha \rho(k') [g_B + (w(k) + \rho(k')f_B - \delta) \frac{g_B}{M}] \geq w(k).$$

Since this condition is the most restrictive when $\delta = w(k) + \rho(k')f_B$, it suffices to have

$$\alpha \rho(k') g_B \geq w(k),$$

which is implied by Condition 1,e.

We now check that G also seeks financing whenever he can afford it. Type G compares entering and repaying in full with the option of not undertaking his project. There are two cases to check.

Case 1. Pooling. First note that the utility of type G under a pooling contract is that of repaying, and this is larger than the utility of defaulting, as shown earlier in the current proof. Since his utility of default is larger than the utility of type B entrepreneur under default (due to Assumption 1), and that utility in turn is larger than $w(k)$, we conclude without extra assumptions that agent G always participates when there is a pooling equilibrium.

Case 2. Separation. We need that

$$\begin{aligned} \rho(k') [g_G + \rho(k')(f_G - f_B) \frac{g_G}{M}] - (2MR_f - w(k) - \rho(k')f_B) &\geq w(k), \text{ i.e.,} \\ \rho(k') [g_G + \rho(k')(f_G - f_B) \frac{g_G}{M} + f_B] &\geq 2MR_f, \end{aligned}$$

which is implied by Condition 1,f.

We now prove that if $k_t \in [k_{min}, k_{max}]$ then $k_{t+1} \in [k_{min}, k_{max}]$. To do so we use the results shown earlier in the current proof, i.e., results concerning equilibrium behavior of entrepreneurs. For each environment studied (Sections IV,V,VI), there is a minimum level of capital at which pooling equilibria occurs, which we denote by $\underline{k}_p$, defined in Assumptions 2, 3 and 4. Then, given that $k_t \in [k_{min}, k_{max}]$, the minimum value that k_{t+1} can take is $k_{min} = k_p(\underline{k}_p)$, since $k_p(k)$ is increasing and smaller than k_s . For the same reason, the maximum value attained by k_{t+1} is $k_{max} = k_s$. Recall that we assume that $k_0 \in [k_{min}, k_{max}]$. By induction, we have $k_t \in [k_{min}, k_{max}]$ for all $t = 1, 2, 3 \dots$

Proof of Proposition 2

Suppose instead type-dependent contracts $(\delta_t^G, R_t^G) \neq (\delta_t^B, R_t^B)$ were offered and positive measures of each type self-select those contracts. Then it must be the case that $0 \leq \delta_t^B \leq \delta_t^G$, otherwise, type B

would select (δ_t^G, R_t^G) as the interest rate is irrelevant for their decision (by Proposition 1) while the early payment is costly (by Condition 1a).

Case 1. If $\delta_t^G = 0$, then $R_t^G = R_f$ for contract (δ_t^G, R_t^G) to be robust to cream skimming. For B to pick (δ_t^B, R_t^B) it is necessary that $\delta_t^B = 0$. For G to pick (δ_t^G, R_t^G) , it is necessary that $R_t^B > R_t^G$. By Proposition 1 Type B prefers to default. He also chooses contract (δ_t^B, R_t^B) . So, his payoff from default (which is the same under both contracts here), must be greater than paying in full under both contracts. Since paying in full yields greater utility under (δ_t^G, R_t^G) , it must be the case that

$$\begin{aligned} \alpha \rho_{t+1} \left[g_B + (w_t + \rho_{t+1} f_B - 0) \frac{g_B}{M} \right] &> \rho_{t+1} \left[g_B + (w_t + \rho_{t+1} f_B - 0) \frac{g_B}{M} \right] - (2MR_t^G - 0), \text{ i.e.,} \\ 2MR_t^f &> (1 - \alpha) \rho_{t+1} \left[g_B + (w_t + \rho_{t+1} f_B) \frac{g_B}{M} \right], \end{aligned}$$

which means the bank makes negative profits from lending to type B. Because the bank makes zero profits lending to type G, overall profit is negative. Such contracts cannot be sustained in equilibrium.

Case 2. If $\delta_t^G > 0$, then $R_t^G < R_f$ for contract (δ_t^G, R_t^G) to be robust to cream skimming. For B to pick (δ_t^B, R_t^B) , it is necessary that $\delta_t^B \leq \delta_t^G$. For G to pick (δ_t^G, R_t^G) , it is necessary that $R_t^B \geq R_t^G$. Then the zero lenders' profit Assumption is given by

$$\mu^2 2MR_t^G + (1 - \mu) \mu \left[\delta_t^B + (1 - \alpha) \rho_{t+1} \left(g_B + (w_t + \rho_{t+1} f_B - \delta_t^B) \frac{g_B}{M} \right) \right] = 2\mu MR_f,$$

Replacing R_t^G with R_f yields a strict inequality

$$\delta_t^B + (1 - \alpha) \rho_{t+1} \left(g_B + (w_t + \rho_{t+1} f_B - \delta_t^B) \frac{g_B}{M} \right) > 2MR_f,$$

which means that there is a profit opportunity for the lender, as it can offer type B a contract with a lower δ^B and still make positive profit. Such contracts cannot be sustained.

Proof of Proposition 3

A contract that solves the maximization problem yields zero profits by feasibility, so we just must check that there is no other contract that would be preferred by a group of entrepreneurs and give positive profits to the bank who offers it. In fact, contracts (δ, R) with $\delta > \tilde{\delta}$ do not offer a profit making opportunity, since for $R > R_f$ they would not be preferred by type G , which is the only one than can afford them, and for $R < R_f$ they would incur in losses.

For $\delta < \tilde{\delta}$, since by assumption the bank collects more from type B when it increases δ , a contract (δ, R) with $R < R(\delta)$ leads to losses (even if both types migrate to that contract). Now, if $R > R(\delta)$, this contract will not be selected by type G , since it is dominated by $(\delta, R(\delta))$, and by attracting only type G it incurs in a loss.

Proof of Lemma 1

If $\delta_t \leq w_t + \rho_{t+1}f_B$, then by Proposition 1, Type B chooses O2 over O3, that is, O2 yields higher consumption,

$$\begin{aligned} \alpha \rho_{t+1} [g_B + (w_t + \rho_{t+1}f_B - \delta_t) \frac{g_B}{M}] &> \rho_{t+1} \left(g_B + (w_t + \rho_{t+1}f_B - \delta_t) \frac{g_B}{M} \right) - (2MR_t - \delta_t), \text{ i.e.,} \\ 2MR_t - \delta_t &> (1 - \alpha) \rho_{t+1} \left(g_B + (w_t + \rho_{t+1}f_B - \delta_t) \frac{g_B}{M} \right). \end{aligned}$$

Now rearranging the expression for R_t given by (4) and then using the above inequality gives the result,

$$\begin{aligned} \mu 2MR_t + (1 - \mu) \left[\delta_t + (1 - \alpha) \rho_{t+1} \left(g_B + (w_t + \rho_{t+1}f_B - \delta_t) \frac{g_B}{M} \right) \right] &= 2MR_f, \\ \mu 2MR_t + (1 - \mu) [\delta_t + (2MR_t - \delta_t)] &> 2MR_f, \\ R_t &> R_f. \end{aligned}$$

Proof of Proposition 4

Since prices ρ and w are fixed, we drop the subindices. We consider the case where $(1 - (\rho g_G)/M)/2M \leq (R_f - R(0))/\tilde{\delta}$, the other is analogous.

From the payment to agent G , who always repays, we can write $U(\delta, R) = Z + (1 - \frac{\rho g_G}{M})\delta - 2MR$, with Z a constant independent of R and δ . Then

$$\begin{aligned} U(0, R(0)) &= Z - 2MR(0) \\ &\geq Z - \tilde{\delta} \left(1 - \frac{\rho g_G}{M} \right) - 2MR_f \\ &= U(\tilde{\delta}, R_f) \end{aligned}$$

so the pooling contract beats the separating one. Moreover, in this case, and because of lemma 1, $(1 - (\rho g_G)/M)/2M \leq -((1 - \mu)/(2M\mu))(1 + (1 - \alpha)\rho g_B/M)$, therefore

$$\begin{aligned} U(0, R(0)) &= Z - 2MR(0) \\ &\geq Z - 2MR(\delta) - \frac{1 - \mu}{\mu} (1 - (1 - \alpha)\rho \frac{g_B}{M}) \\ &\geq Z - 2MR(\delta) - (1 - \frac{\rho g_G}{M})\delta \\ &= U(\delta, R(\delta)) \end{aligned}$$

so the pooling equilibrium also beats any other contract that satisfies the zero-profit constraint.

Proof of Proposition 5

Cases 1 and 3 are simple since there is a unique stationary point, either $k_{ss} = k_s$, or k_{ss} characterized by $k_P(k_{ss}) = k_{ss}$. The fact that there cannot be more than 1 is direct from the fact that k_s is constant and

$\frac{k_p(k)}{dk} < 1$ (by assumption 7).

For case 2, consider $k_0 < \bar{k}$ (the other case is analogous). Then $k_1 = k_s > \bar{k}$. It only rests to prove that there exists N , such that $k_p^N(k_1) < \bar{k}$. For this, note that because of assumption 7 and the linearity of k_p , $k - k_p(k) \geq (1 - \beta)\bar{k} - Z > 0$, with Z a constant. Therefore $k_p^{(n)}(k) \leq k - [(1 - \beta)\bar{k} - Z]n$, and becomes smaller than $\bar{k}$ for a finite N .

Then, $k_{N+1} = k_p^N(k_s) < \bar{k}$, and therefore $k_{N+2} = k_s$, continuing the cycle.

Proof of Corollary 2

We know that $k_0 = k_s > \bar{k}$ and therefore $k_1 = k_p(k_s)$. For all $m \leq n$ we have then $k_m = k_p^{(m)}(k_s)$ (since by hypothesis $k_p^{(m-1)}(k_s) > \bar{k}$). Since, also by hypothesis, $k_m > k_s$, we have that $k_{m+1} = k_s$ and the result follows.

Proof of Lemma 2

The first four results are obvious from straightforward differentiation. We obtain $\frac{dk'_k}{dk} \geq 0$ by differentiating equation (18) implicitly. We obtain

$$-\frac{\rho'(k') \frac{g_G}{M} \frac{dk'_k}{dk}}{2M} = - \left[\frac{\left(\frac{dR_{k,k'}(0)}{dk'} \frac{dk'_k}{dk} + \frac{dR_{k,k'}(0)}{dk} \right) \delta_{k,k'} - R_{k,k'}(0) \left(w'(k) + \rho'(k') f_B \frac{dk'_k}{dk} \right)}{\delta_{k,k'}^2} \right],$$

$$\frac{w'(k) R_{k,k'}(0)}{\delta_{k,k'}^2} - \frac{\frac{dR_{k,k'}(0)}{dk}}{\delta_{k,k'}} = \frac{dk'_k}{dk} \left[-\frac{\rho'(k') \frac{g_G}{M}}{2M} + \frac{\frac{dR_{k,k'}(0)}{dk'}}{\delta_{k,k'}} - \frac{\rho'(k') f_B R_{k,k'}(0)}{\delta_{k,k'}^2} \right].$$

Because $\rho'(k') < 0$, $\frac{dR_{k,k'}(0)}{dk'} > 0$ and $\frac{dR_{k,k'}(0)}{dk} < 0$, the result follows directly.

Proof of Lemma 3

The fact that there are at most two solutions is direct in the light of lemma 2, since both segments of $\kappa(k, k')$ are decreasing in k' . Existence comes from the fact that both segments of $\kappa(k, k')$ decrease in k' together with Assumption 8. In fact, if $k \leq k_{hi}$, by definition of k_{hi} , the belief about a separating equilibrium, given by the solution to $\kappa_s(k, k') = k'$, is consistent. For $k > k_{hi}$, we know that $\kappa_s(k, \tilde{k}'_k) < \tilde{k}'_k$, therefore $\kappa_p(k, \tilde{k}'_k) < \tilde{k}'_k$ (by Assumption 8). Then, since $\kappa_p(k, 0) > 0$, the existence of a consistent belief about a pooling equilibrium follows from the continuity of κ_p .

Proof of Lemma 4

The result is trivial if $k_{li} = 0$. Consider $k_{li} > 0$. Because of Assumption 8, for $k = k_{hi}$ we have $\kappa_p(k_{hi}, \tilde{k}'_{k_{hi}}) < \tilde{k}'_{k_{hi}}$. Because $\kappa_p(0, \tilde{k}'_0) \geq 0$, the continuity of $\kappa_p(\cdot, \cdot)$ guarantees a solution to $\kappa_p(k, \tilde{k}'_k) = \tilde{k}'_k$ for a $k \in [0, k_{hi}]$.

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FIGURE 1.

Charge-off and delinquency rates lag after the cycle.

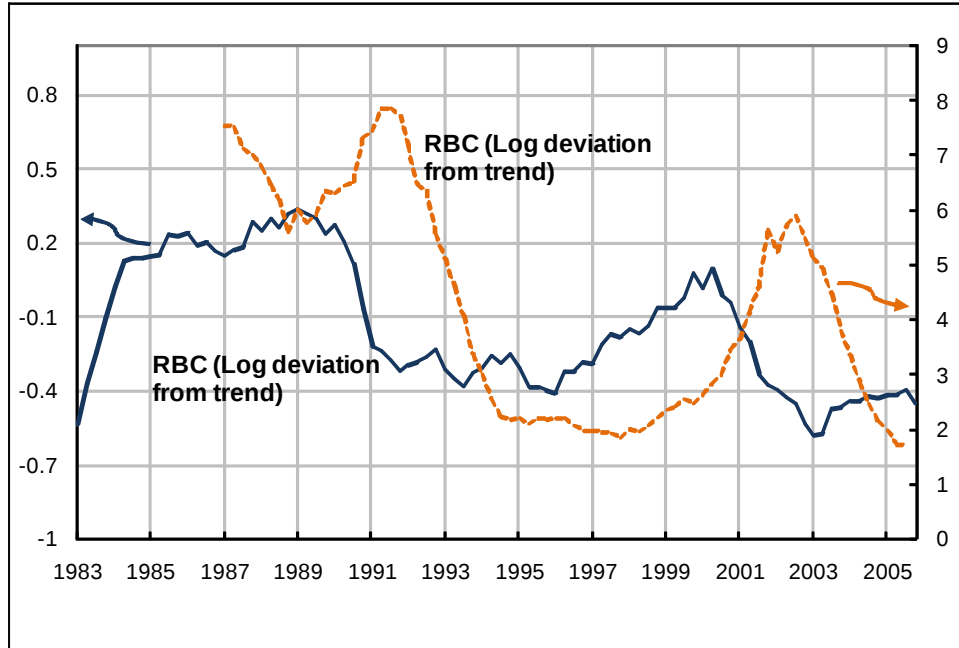


FIGURE 2.

Value of outstanding commercial and industrial loans is procyclical.

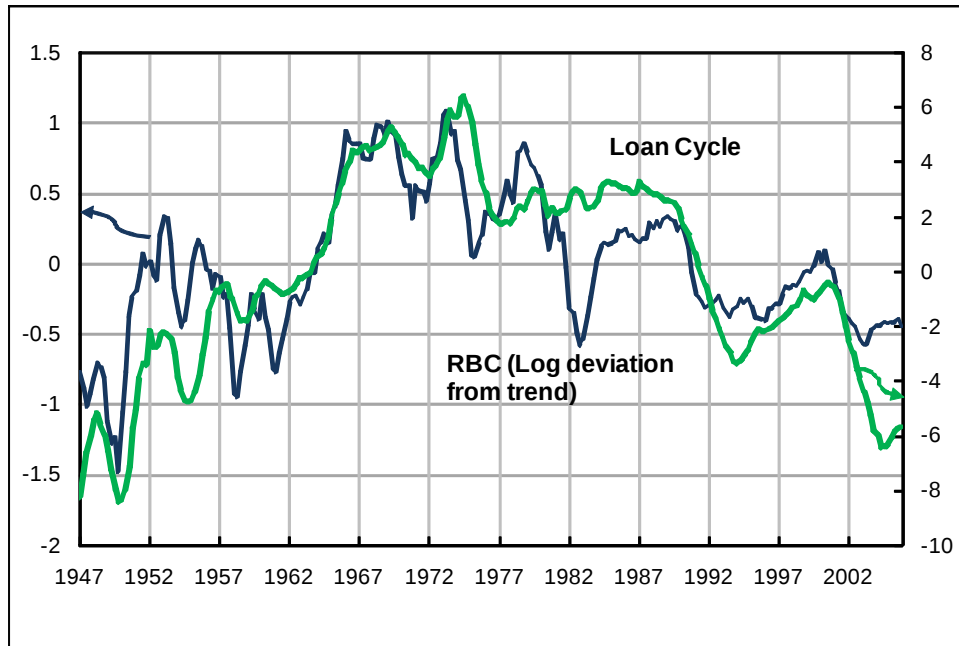


FIGURE 3.

General Model. Equilibrium Contract Determination (for given w_t and ρ_{t+1}).

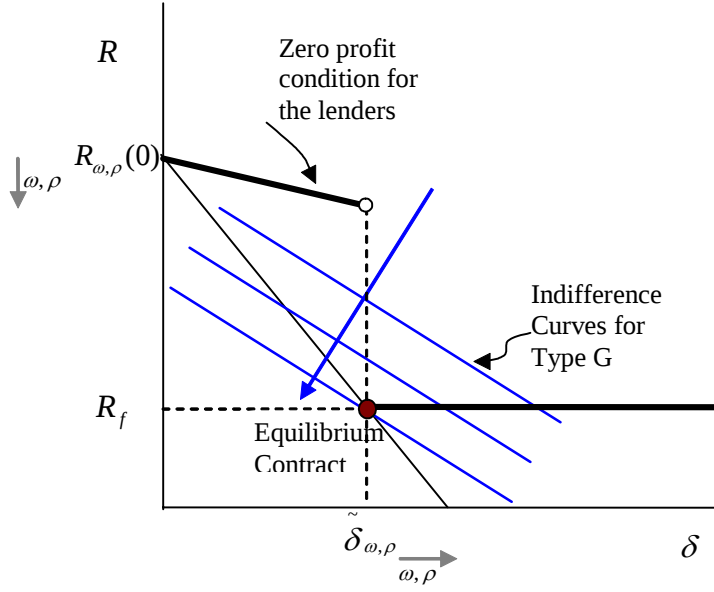


FIGURE 4.

Simplified Model. Equilibrium Contract Determination, 2 states, $k_H > k_L$.

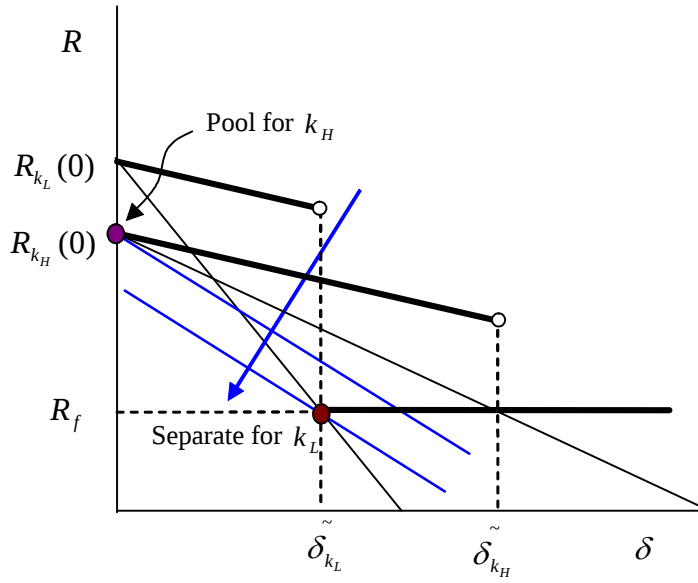


FIGURE 5.

Transition function $k'(k)$ in the simplified model,
 $\bar{k} < k_P(\bar{k}) < k_S$, only pooling contracts emerge in the long run.

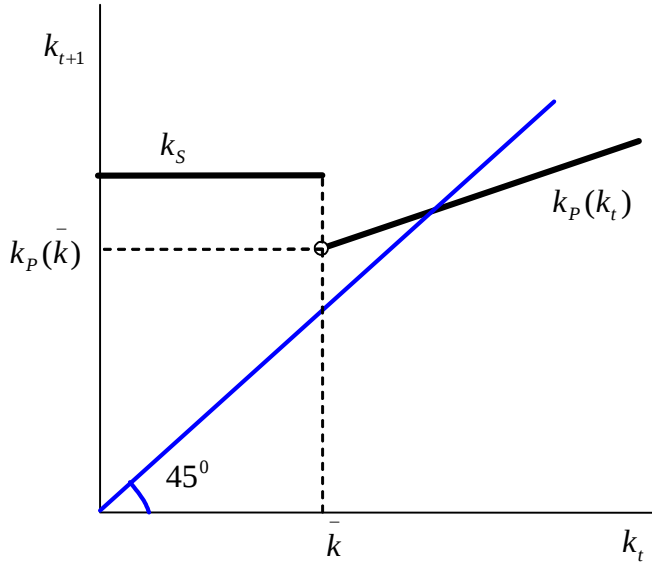


FIGURE 6.

Transition function $k'(k)$ in the simplified model,
 $k_P(\bar{k}) < \bar{k} < k_S$, endogenous cycles.

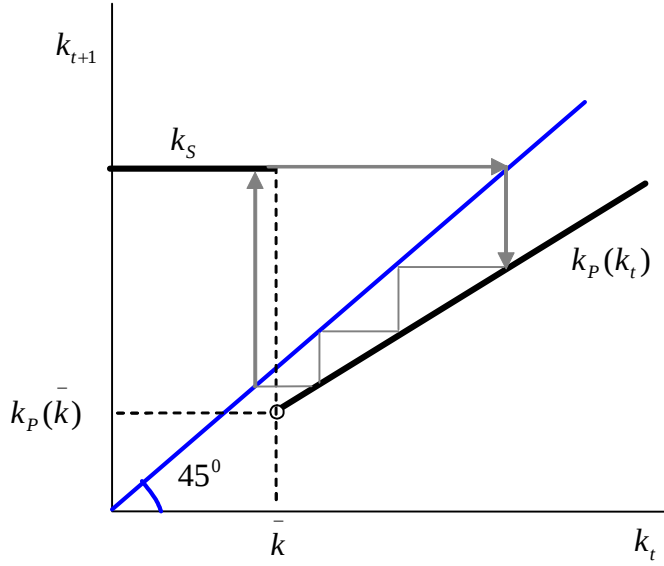


FIGURE 7.

Transition function $k'(k)$ in the simplified model,
 $k_P(\bar{k}) < k_S < \bar{k}$, only separating contracts emerge in the long run.

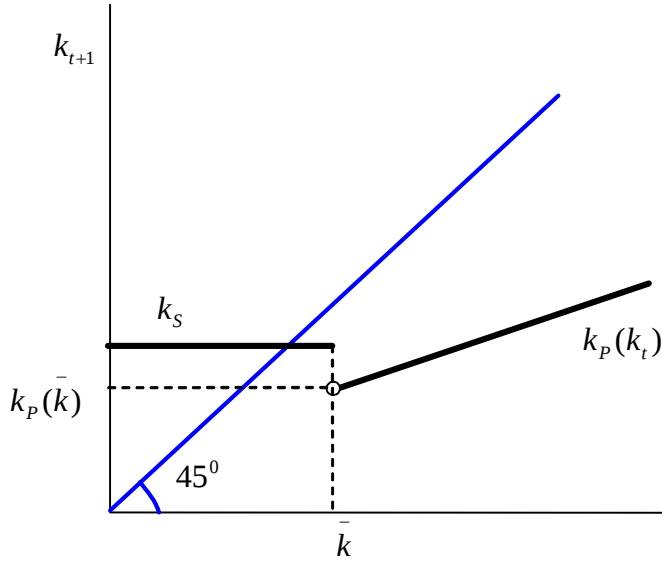


FIGURE 8.

Example 1: Cycles for the simple model with production externalities (Section IV).

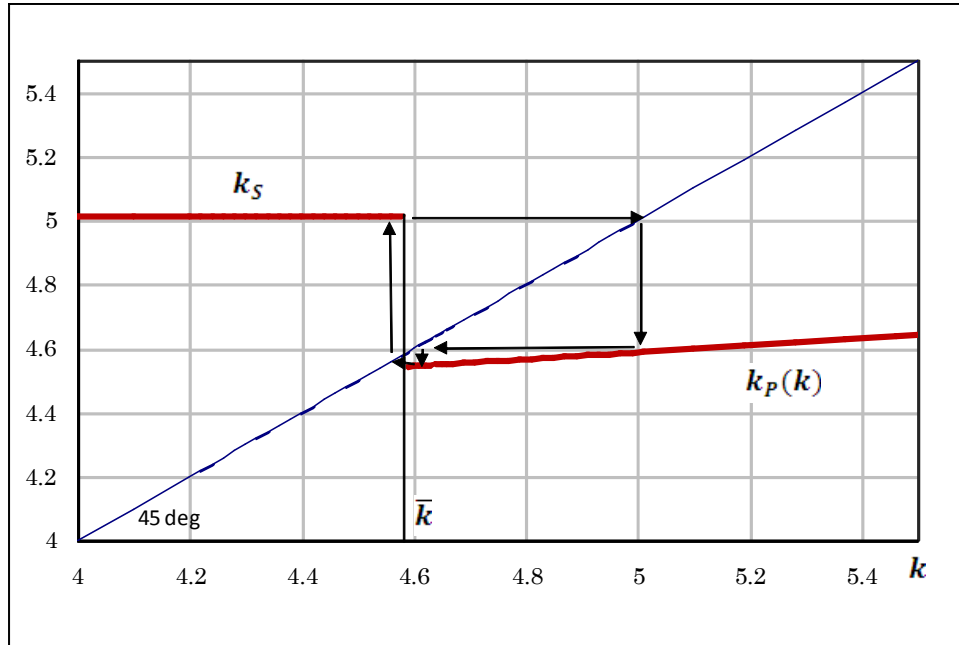


FIGURE 9.

General Model. Illustration of $\kappa(k, k')$, for a fixed k , $\tilde{k}'_k$ is defined as a solution to $\frac{1-\rho(k')\frac{gG}{M}}{2M} = -\frac{R_{k,k'}(0)-R_f}{\delta_{k,k'}}$.

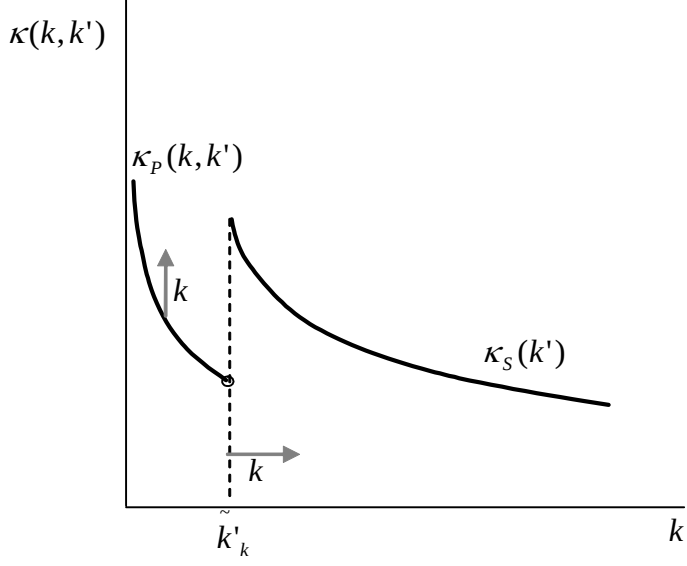


FIGURE 10.

Illustration of Assumption 8, i.e., $\tilde{k}'_{k_{hi}} \geq \kappa_p(k_{hi}, \tilde{k}'_{k_{hi}})$, where k_{hi} is the max capital for which a separating contract can emerge, defined as a solution to $\kappa_s(\tilde{k}'_k) = \tilde{k}'_k$.

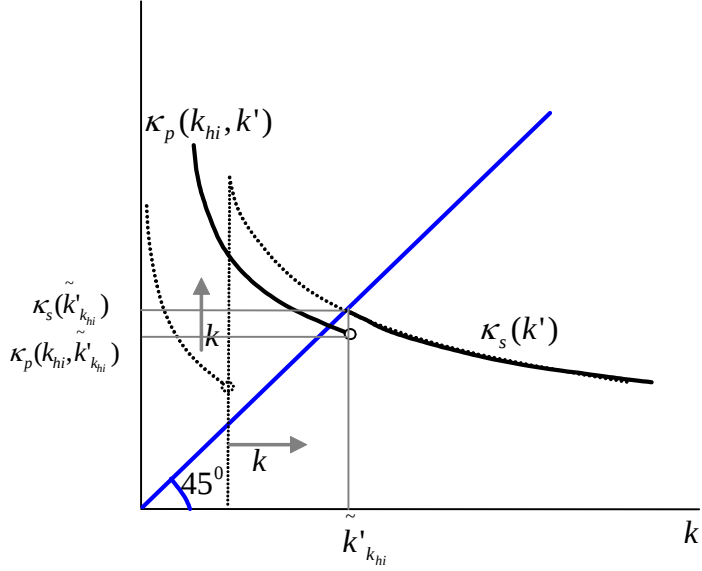


FIGURE 11.

General Model. Determination of $k'(k_L)$, for some low level k_L .

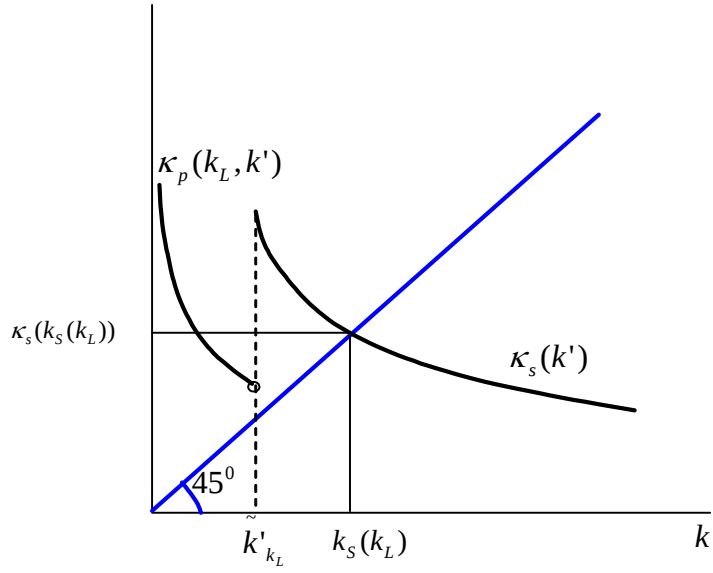


FIGURE 12.

General Model. Determination of $k'(k_M)$, for some level k_M in the region of indeterminacy.

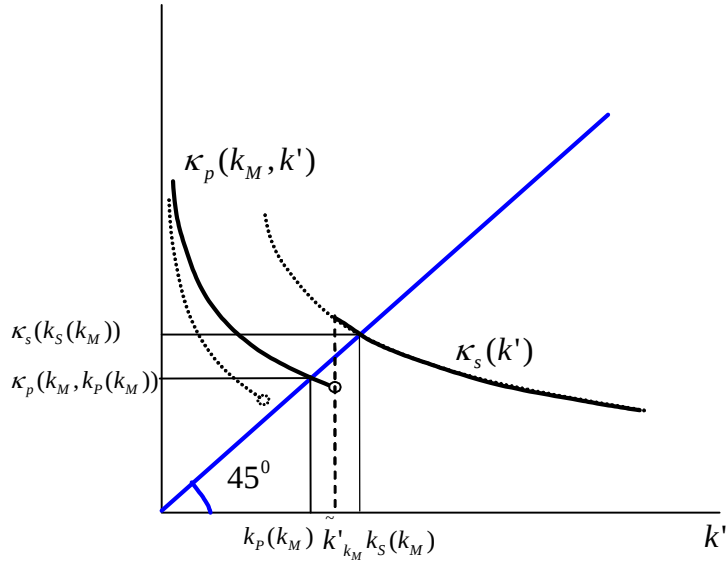


FIGURE 13.

General Model. Determination of $k'(k_H)$, for some high level k_H .

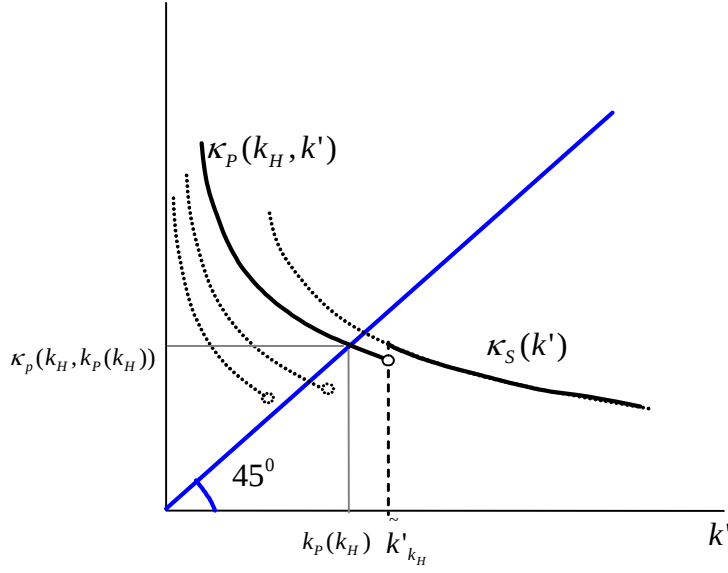


FIGURE 14.

k_{li} is the min capital for which a pooling contract can emerge, defined as a solution to $\kappa_p(k, \tilde{k}'_k) = \tilde{k}'_k$.

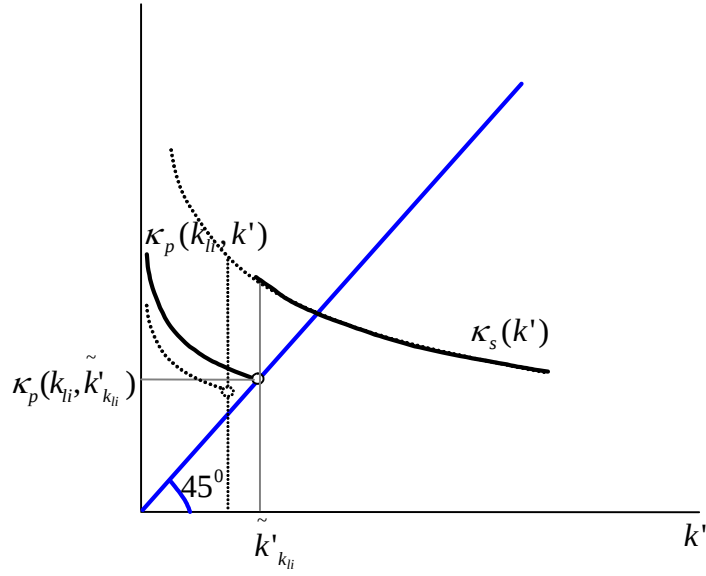


FIGURE 15.
General Model. Transition Correspondence.

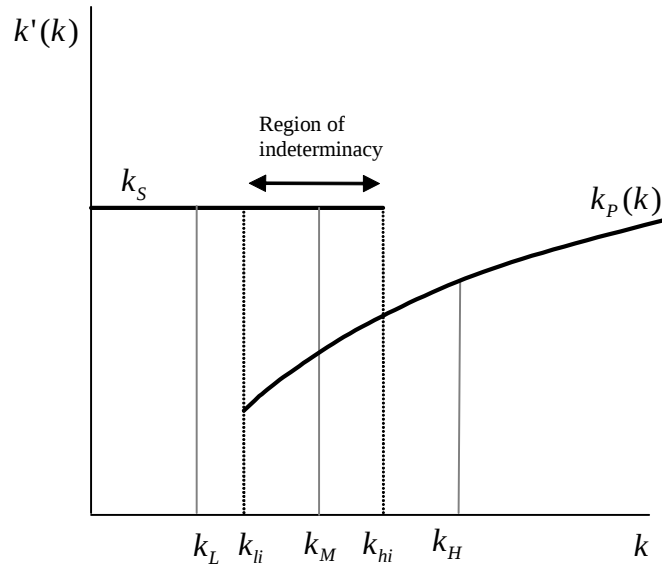


FIGURE 16.
Example 2. Cycles for the general model with no externalities (Section V).

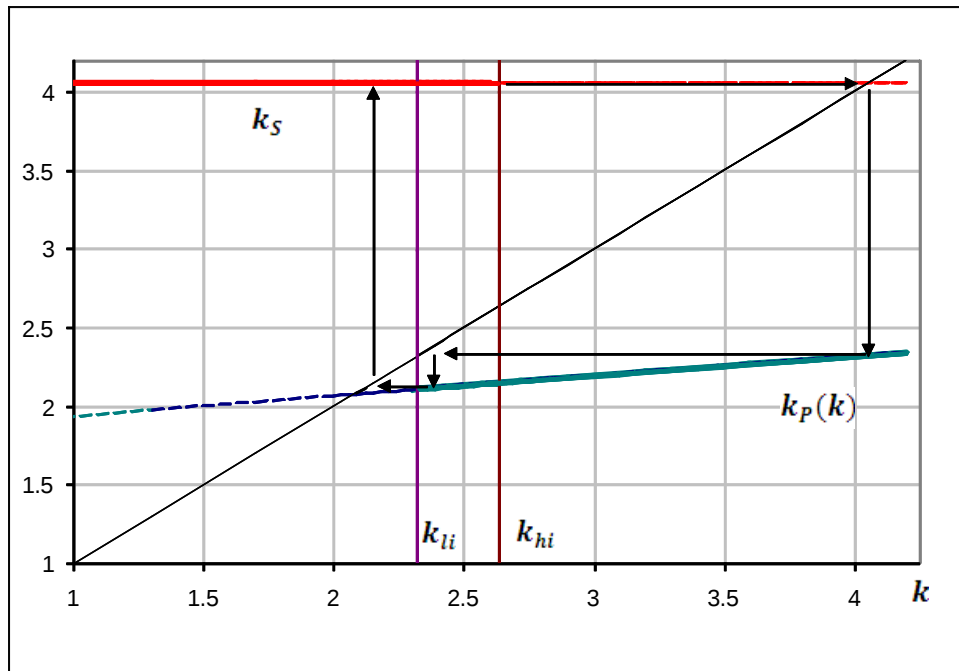
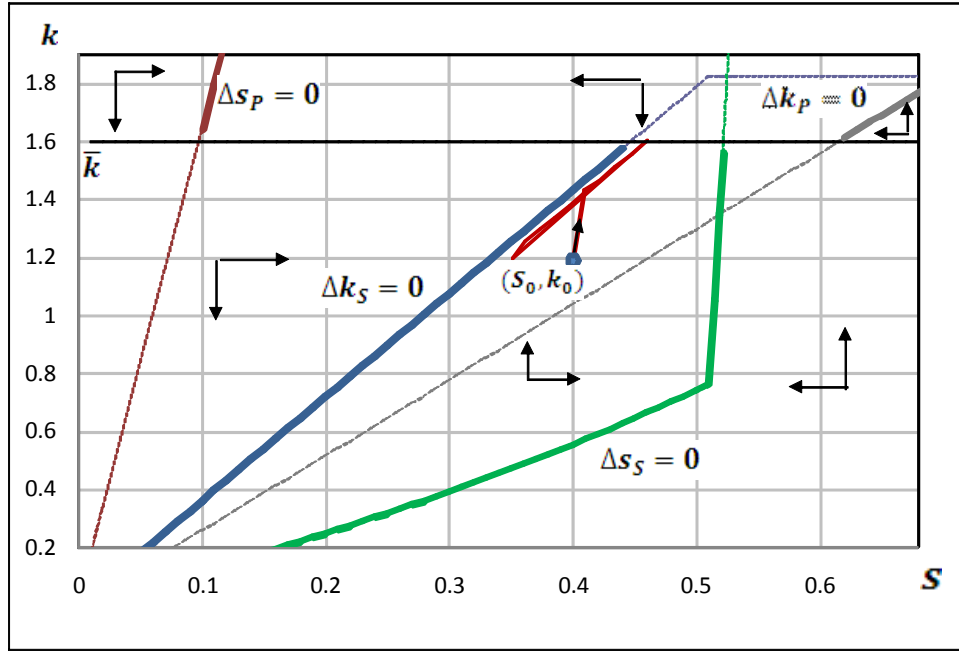


FIGURE 17

Example 3. Cycles for the model with the endogenous supply of funds (Section VI).

17A. The phase diagram.



17B. Example 3 cont. Capital and the supply of funds dynamics.

