Prices as Optimal Competitive Sales Mechanisms*

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Abstract

We analyze the efficiency properties of price posting in a market where sellers compete for the buyers' business. They key feature of the approach is to investigate price posting as an equilibrium outcome even if sellers can compete with other mechanisms. When buyers are homogeneous, we show equivalence between price posting and a large class of competing mechanisms, including second price auctions. When buyers are heterogeneous, posted prices are able to perfectly screen *ex-ante* because different buyers endogenously choose to trade at different prices. As a result, search is non-random and buyers sort themselves by choosing different prices. Whether price posting is efficient depends on the matching technology. Prices are efficient and arise in equilibrium when meetings are bilateral, which is the case in competitive search models. When meetings are multilateral, as in directed search models, price posting is not an equilibrium. In that case, the efficient mechanism screens *ex post* amongst multiple buyers and necessarily involves random search. The conclusion for market design is that the prevalence of price posting over auctions depends crucially on the properties of the meeting technology.

1 Introduction

Markets with frictions due to random matching are known to be inefficient. Typically, trading partners meet each other according to some exogenous stochastic process, and determine the trading outcome after a buyer and seller meet. Given that traders meet in small groups, pricing is not competitive and the outcome of the process is often inefficient for the market as a whole. For example, the Hosios (1990) condition for optimality is generally not fulfilled when prices are determined by bargaining procedures.

Moen (1997) observes that the inefficiency stems from the random matching assumption. When sellers can compete for the buyers' business by publicly posting their price, they face a trade-off between charging a higher price and expecting fewer buyers who demand the good. As a result, the equilibrium price reflects the marginal valuation of each trader and efficiency is restored. This result has led to the popular adoption of competitive/directed search models in macro, labor and monetary economics.[1] In such models, whether search turns out to be random or not is a matter of the equilibrium visit strategies by buyers.

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[1]For recent contributions in macro/labor see e.g. Delacroix and Shi (2004), Menzio and Shi (2007) and Shimer (2005). For models using monetary theory see e.g. Berentsen, Menzio and Wright (2007) and Rocheteau and Wright (2005).

This work focusses on contracts with posted prices, and efficiency is assessed in the light of such simple pricing contracts. Yet sellers can in principle compete by posting more complicated trading arrangements such as auctions. The key feature of our analysis is to analyze posted prices as an optimal mechanism even when other mechanisms are available. Our paper thus spans the literatures of *directed/competitive search* and *competing mechanisms design*. We find that the optimality of posted price mechanisms is tightly linked to the specification of the matching technology, and in particular to the desirability of randomness in the search process.

Price posting has many desirable properties, compared to more elaborate trading protocols such as auctions. Trade is easy, fast, and potentially cost effective. It is hard for sellers to manipulate the outcome through shill bidders and other means. It has a clear analog in the Walrasian notion of markets that operate through prices, and it is a good approximation for existing selling protocols in many markets. And, not least, price posting is pervasive in many economic transactions. Even the internet auction house eBay derives 40% of its revenue from price posting. There could be many reasons why prices are pervasive, including low transaction costs.[2] Our objective is to find out under which conditions price posting is an efficient trading mechanism in the presence of search frictions, and without assuming differential transaction costs on other mechanisms.

We have three main results. Our first result concerns a revenue equivalence theorem in a setting *homogeneous* buyers. For any mechanism, seller revenues are identical conditional on leaving the same expected surplus to buyers (and conditional on inducing trade whenever viable). The novelty of this result is in the explicit treatment of the stochastic nature of the meeting process that brings buyers and sellers together. Given the revenue equivalence, the sellers do not care whether they compete in posted prices, second price auctions or other mechanisms, and therefore competition in posted prices does constitute an equilibrium. There is a continuum of other equilibria in which sellers use different mechanisms, but in all equilibria buyers search randomly under standard assumptions on the meeting process. This is due to the fact that random meetings lead to most trades in the market.

Next, we consider buyers with *heterogeneous* private values for the good. We find that price posting generates different equilibrium prices and leads to separation of buyers. Low type buyers choose to consume at low prices and corresponding low odds of trade, while high type buyers consume at high prices and enjoy a high probability of trade. Search is non-random and the buyers reveal their type ex-ante by choosing the price at which they want to trade. Due to the separation, each seller knows exactly the type of buyer he faces, and has ex-post no incentive to use type-revealing mechanisms.

Whether this separation outcome is efficient depends on the nature of the matching technology. We consider two classes of matching technologies which cover the models used in this literature: Competitive Search and Directed Search. Under *Competitive Search*, all meetings are bilateral: a seller never meets more than one buyer at a time.[3] Under Directed Search, the matching technology permits sellers to meet multiple buyers at a time. [4]

[2]See for example Wang (1993)

[3]This classification includes work by Moen (1997); Acemoglu and Shimer (1999); Mortensen and Wright (2003); Moen and Rosen (2006); Guerrieri (2007). Competitive Search models are generally formulated in continuous time where there is no recall. We condense the analysis into a one-shot environment, but the real importance for our result is the nature of the meeting technology that permits a seller to meet with at most one buyer at a time. There exist other distinctions between directed and competitive search based on the micro-foundations of the approach, which we will not consider in this work.

[4]This classification includes such work as Peters (1991, 1997a, 1997b, 1999, 2000); Montgomery (1991); Peters and Severinov

Our second result establishes efficiency of fixed price mechanisms under Competitive Search, extending the usual notion of efficiency for given fixed price mechanisms to encompass competition in larger classes of mechanisms. The key observation is that random search is not efficient because it leaves a seller unsure about the type of buyer he faces. He could screen between buyer types ex-post, but incentive compatibility induces a cost in terms of wasteful destruction. Additionally, non-random search outperforms random search because of an effect that we call "crowding-out": When low types enter the same market as high types, then the probability of meeting a seller goes down for the high types. Both effects together make it beneficial to keep buyers apart in separate markets. Ex-post screening is then no longer necessary because in the optimum sellers go to different markets, and therefore prices do a good job in allocating sales efficiently.

Our third main result shows that the efficiency of fixed price mechanisms does *not* carry over to Directed Search meeting technologies. This arises even though each seller knows exactly the type of buyers that he faces in the fixed price regime. The reason is due to the efficiency properties of random search: Random search leads to strictly more meetings than non-random search under standard assumptions on the meeting technology. Given that the seller often meets several buyers, he can use incentive schemes to elicit the buyers type that do not rely on output destruction by letting the buyers compete for the good in an auction. If there is no "crowding out" [5] then there is no efficiency losses by inducing random search. The number of trades is maximized and the good goes to the buyer with the highest valuation whenever one is present. Random search is strictly more efficient than non-search: Equilibria with ex-ante screening as in the case of posted prices induce non-random matching which is less efficient than equilibria with ex-post screening and random search. Even if all other sellers offer posted prices, a single deviant can exploit the efficiency gain of random search by having both types visit him.

Our results have immediate consequences for the question when we expect competition in terms of prices to arise and to be optimal. It builds on the following insight: Even though *matches* (i.e. realized trades) are by assumption bilateral in the economy, *meetings* do not have to be. If meetings are only one-on-one, then prices do well. Bilateral meetings always have the property that more low types in the market reduce the probability for high types to even meet a seller. In environments in which sellers meet with multiple buyers, more complicated mechanisms that screen ex post are more efficient and arise in equilibrium. The results are driven by the relative efficiency of random vs. non-random meetings.

We note at this point that an additional concerns arise when sellers are heterogeneous as well. Even using auctions, the higher cost sellers would offer worse terms of trade and attract less sellers, i.e. search would be non-random. Nevertheless it is not difficult to show that the optimal strategy would again involve buyers to visit randomly all sellers with reservation prices below some cutoff level, where the cutoff level is dependent on the buyer type (as shown by Peters (1997b) and Peters and Severinov (1997) under urn-ball matching). Therefore, again the comparison of random versus non-random search applies on a restricted part of the pool of traders.

The literature so far has focused mainly on competing in posted prices, or has considered ex-post screening mechanisms without considering whether posted prices could actually achieve the same outcome by ex-ante screening. In the next section we review that literature. The third section outlines and analyzes the model for

(1997); Burdett, Shi and Wright (2001); Shi (2001, 2002); Shimer (2005).

[5] The presence of a low type does not affect the meeting probability of a high type (though it may affect the probability that the high type trades with the seller)

homogeneous agents for a general meeting technology that allows for bilateral as well as multilateral specifications. The fourth section presents the model for heterogeneous buyers. We focus on two buyer types for tractability, yet the analysis extends to any finite number of buyers. Section five analyzes the case with multilateral matches (Directed Search) and Section six considers bilateral matches (Competitive Search). The final section concludes by discussing matching technologies that have some but not full "crowding out".

2 Relation to the Literature

Our setup is in the spirit of the competing mechanism design literature of McAfee (1993) and Peters (1997b, 1999). Sellers compete for buyers by posting some mechanism, and buyers and sellers meet according to the urn-ball matching technology (McAfee (1993), Peters (1997b, 1999)). The main insight from this literature is that if the seller is unsure about the type of buyer that he is facing in the match, he should screen them apart ex-post (after the match is formed). Ex-post screening through second price auctions is a weakly dominant strategy under this matching technology and the literature has focussed on equilibria in these mechanisms.

Yet the mere fact that second price auctions do well does not mean that they are necessary. If sellers compete in posted prices – i.e. they cannot screen among buyers ex-post but give the good at random to one of them – then buyers will sort themselves *ex-ante* by attempting to trade with sellers who offer different prices, as we show in this work. Therefore, the seller has no uncertainty about the type of buyer that he is facing, and ex-post screening is not necessary to figure out the buyers' types. Therefore it is not a priori clear that one has to rely on auction mechanisms, as the screening can be done ex ante by attracting different buyers to different sellers. Of real importance is how efficient ex-ante screening is relative to ex-post screening, and it turns out that under a much wider class than just urn-ball matching, random search actually leads to more matches and is therefore desirable. This in turn requires ex-post screening.

But there are also common search models that have a matching technology that does not fall into that class. Competitive Search models like Moen (1997) concentrate on fixed prices in an environment with homogeneous buyers, and Mortensen and Wright (2003) extend the setup to heterogeneous buyers under the same price posting restriction. Our paper extends the scope of their results by showing that under their meeting technology other mechanisms are not optimal.

The Directed Search literature generates endogenously a matching technology that converges to urn-ball matching in large markets (see Peters (1991, 1997a, 2000) and Burdett, Shi and Wright (2001)). While initial work has focussed exclusively on the homogeneous agent case, some of the Directed Search recent literature has focussed on buyer heterogeneity that influences the *sellers'* payoff. One can think of the buyers as workers that have a different productivity in their interaction with the seller (employer). In contrast to the present paper the buyer's type is assumed to be observable. This is important as buyers don't internalize their type and therefore cannot be screened apart if their type would be unobservable (and payoff is not output-contingent). Shi (2001) analyzes a model in which each buyer only posts one price and the type he wants to hire, while Shi (2002, 2006) and Shimer (2005) allow for a menu of type-contingent prices and priority rules in case multiple buyers of different types arrive. The setup differs substantially from ours in the sense that those papers assume a specific meeting technology, and that the types are observable and contractible. Also, they do not relate the efficiency of various contract types to the efficiency of the underlying meeting process, but they do point out that some overlap of buyers at a given seller is optimal when type-contingent menus are available.

Lang et al. (2005) and Peters (2007) investigate the same environment as described in the previous paragraph, but assume that sellers can only post one price and cannot commit to the type they hire. Therefore sellers always hire more productive buyers in case several buyer types arrive. This arises no matter how small the differences between buyers are. Therefore payoffs for different types can be very different despite small underlying differences, as Lang et al. (2005) point out. In contrast, in our setting type differences are private and payoffs cannot differ when type differences are small, as low types can always pretend to be high types.

3 The homogeneous agent model

3.1 The model

Consider an economy with a measure b of homogeneous buyers and measure s of homogeneous sellers. Sellers are endowed with one unit of a good for sale. Buyers have a valuation v for the good. If a buyer and a seller trade at a transfer t from the buyer to the seller, then the buyers payoff is $v-t$ and the seller payoff is t. Payoffs from no trade are normalized to zero. We are interested in trading procedures where search is directed.

Mechanisms. Sellers compete by posting a mechanism m from some borel-measurable set M with the following restrictions. We require that M includes the set of all fixed price mechanisms, i.e. sellers post a price and sell at this price to one of the buyers that show up. If several show up he picks one at random to whom to sell. We require the set M to include only anonymous mechanisms that do not condition on the other mechanisms that are present. In particular, a mechanism specifies for each number n of sellers some extensive form game[6] Γ_m^n that induces some expected equilibrium payoff π_n^m for the seller and some expected equilibrium payoff u_n^m for each of the buyers. Because of anonymity, the expected payoffs do not distinguish buyers by names. Also, conditional on the presence of n buyers the game's payoffs do not depend on the other mechanisms that are offered.[7] While some buyers might do better than others in those cases where they can trade while the others cannot, the lucky buyer has to be selected at random because names of buyers are not known to the seller and therefore the expected payoffs are identical to buyers. Moreover, the payoffs are not conditional on the mechanisms that other buyers post. Since only a surplus v is realized, we require that the payoffs at each end node of Γ_m^n sum to weakly less than v. This immediately implies $\pi_n^m + nu_n^m \leq 1$. The mechanisms do not have to obey any participation constraints, as sellers can always choose to stay away from any mechanism if their expected payoffs (averaged over the expectation of n, the number of other buyers that are expected to turn up) is too low.[8] Obviously if

[6] And an equilibrium selection device S_m, in case there are multiple equilibria.

[7] Since buyers observe all other mechanisms, the seller could elicit information about other sellers from the buyers. This is ruled out by assumption in most of the literature such as McAffee (1993) or Peters (1997b, 1999). In large economies or in a pure strategy equilibrium in a finite economy each seller knows the distribution of other mechanisms with certainty, and there is no benefit from conditioning on other mechanisms when no seller does so. Therefore, equilibria survive even if sellers can condition on the mechanisms offered by other sellers. Yet additional equilibria can arise. For a deeper discussion and modeling of mechanisms that condition on other mechanisms, see Epstein and Peters (1999) and Peters (2001).

[8] If they arrive they are assumed to sign an agreement to participate before n is revealed.

there are no buyers, then $u_0 = 0$ and $\pi_0 = 0$ because neither exchange nor transfers are possible.

Examples: Fixed price mechanisms where sellers post price p have payoffs, conditional on matching of $\pi_n = p$ and $u_n = [1-p]/n$. Other mechanisms that fall in this class include first price auctions, second price auctions, all-pay auctions with reserve price, as well as more esoteric mechanisms. For example: If no more than 5 buyers show up, one of them gets the good for free. If more than 5 buyers show up, each has to pay a price p and one gets the good. This yields payoffs $\pi_n = 0$ and $u_n = 1/n$ for $n \in \{1, .., 5\}$ and $\pi_n = np$ and $u_n = 1/n - p$ for $n \in \{6, 7, ...\}$.

Matching. If the ratio of buyers to sellers is λ, there is a probability $P_n(\lambda)$ that n buyers will show up at a given seller. P_n is assumed to be twice differentiable. From a buyers perspective, there is a probability $Q_n(\lambda)$ that he arrives at a seller who has n buyers, with $Q_0(\lambda)$ being the probability of not finding any seller. Since buyers and sellers meet, consistency requires that the number of sellers who have n buyers equals the number of buyers in groups with n buyers, i.e. for $n > 0$ we have

$$nP_n(\lambda) = \lambda Q_n(\lambda). \tag{1}$$

To see the logic of this equation, it is useful to consider an equal number of buyers and sellers, and to observe that the probability of a match with two buyers is then twice as likely for a buyer than for a seller. For this reason the left hand side probability has to be multiplied by n. Clearly $Q_0(\lambda) = 1 - \sum_{n=1}^{\infty} Q_n(\lambda)$. We require that for any $\lambda > 0$ there are some matches, i.e. $0 < P_1(\lambda)$, but due to frictions matching is not perfect, i.e. $0 < P_0(\lambda)$. Moreover, we require that a higher buyer-seller ratio increases the chances that a seller gets matched. i.e. we assume that the probability that no buyer shows up decreases in λ and that at any point in the distribution, the cumulative probability is lower for higher λ, This first order stochastic dominance relation on the distribution can be expressed as:

$$\begin{aligned} P_0'(\lambda) &< 0 \quad \texttt{and} \\ \sum_{n=0}^{N} P_n'(\lambda) &\leq 0 \quad \texttt{for all } N. \end{aligned} \tag{2}$$

Additionally assume that

$$P_0 \texttt{ is convex}.$$

The convexity property has to hold for some $\lambda \in [0, \infty)$ since $P_0(\lambda) \in [0, 1]$ is bounded, and we make the standard assumption that this property extends to the entire domain. For the buyers, we also assume that the distribution satisfies first order stochastic dominance, i.e. $\sum_{n=N}^{\infty} Q_n'(\lambda) \leq 0$ for all N with strict inequality for some N. This immediately implies that $\sum_{n=1}^{\infty} \frac{Q_n'(\lambda)}{n} < 0$, which in turns implies the following relationship that will be useful later on: $1 + \lambda P_0'(\lambda^*) - P_0(\lambda^*) > 0$.[9]

Examples: The standard example in the directed search literature is that buyers apply randomly among the sellers that they like, which leads to a Poisson distribution $P_n(\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$. A specification from the monetary literature is one where agents are randomly matched into pairs, and if the pair includes a buyer and a seller, a match is formed, i.e. $P_1(\lambda) = \frac{b}{b+s} = \frac{\lambda}{1+\lambda}$, $P_0(\lambda) = 1 - P_1(\lambda)$, and $P_n(\lambda) = 0$ for $n > 1$.

[9]Since $(1 - P_0(\lambda)) = \sum_{n=1}^{\infty} \lambda \frac{Q_n(\lambda)}{n}$ we have that $-\frac{1-P_0(\lambda)+\lambda P_0'(\lambda)}{\lambda^2} = \sum_{n=1}^{\infty} \frac{Q_n'(\lambda)}{n}$, and the result follows from $\sum_{n=1}^{\infty} \frac{Q_n'(\lambda)}{n} < 0$.

Directed Matching. A seller chooses a mechanism $m \in M$ to maximize profits. Since he can publicly post his mechanism he is more likely to attract a buyer when he posts a more favorable mechanism. Assume all sellers obtain utility U^* at any other seller. Then the seller who posts a mechanisms m and wants to achieve an expected queue length λ solves the following problem:

$$\max_{\lambda \in \mathbb{R}^+,\ m \in M} \sum_{n=1}^{\infty} P_n(\lambda)\pi_n^m \tag{3}$$
$$\texttt{s.t.} \quad \sum_{n=1}^{\infty} Q_n(\lambda)u_n^m \geq U^*,$$

where the constraint requires the seller to leave at least the same expected utility to the buyers as they get elsewhere. For a given mechanism m we call queue length λ feasible if it satisfies the constraint.

An equilibrium $(\mu_s, \mu_b, \lambda, U^*)$ consists of probability measure μ_s and μ_b over M where $s\mu_s$ describes the distribution of mechanisms that are offered and $b\mu_b$ the measure of buyers that visit the mechanisms, $\lambda(m)$ is the queue length at the mechanisms in the support of μ_b, and $U^* \geq 0$ is the equilibrium utility, satisfying:

1. Seller Optimality: Any m in the support of μ_s and associated feasible $\lambda(m)$ solve (3).
2. Buyer Optimality: $\sum_{n=1}^{\infty} Q_n(\lambda)u_n^m = U^*$ for any m in the support of μ_b.
3. Consistency: The ratio $\lambda(m)$ indeed reflects the equilibrium behavior, i.e. there exists probability measure ν describing the buyers choice over mechanisms such that
$$s\int_x \lambda(m)d\mu_s = b\int_x d\mu_b, \texttt{ for all measurable subsets } x \subseteq M.$$

3.2 Analysis for homogeneous agents

We will show that price posting is an equilibrium in this setting, and so is every other class of mechanisms that allows the surplus to be shifted between buyers and sellers (e.g. through a reserve price in an auction).

Before we proceed, assume that the space of mechanisms includes mechanisms such that any u_n^m and π_n^m with $nu_n^m + \pi_n^m \leq 1$ is feasible.[10] For a given U^* we call a mechanism m with queue length $\lambda(m)$ a full-trade mechanism if $nu_n^m + \pi_n^m = 1$ whenever $P_n(\lambda(m)) > 0$. That is, the mechanism induces trade with certainty.

Lemma 1 *Given U^*, any mechanism $m \in M$ with queue length $\lambda(m)$ that is not a full-trade mechanism is revenue-dominated by a full-trade mechanism $m' \in M$ with queue $\lambda(m')$.*

Proof. Mechanism m has $nu_n^m + \pi_n^m < 1$ for at least some n with $P_n(\lambda(m)) > 0$. Let m' give identical utilities to buyers $u_n^{m'} = u_n^m$ but different profit $\pi'_n = 1 - nu_n^m + \pi_n^m$ to the seller. Clearly, buyers obtain identical payoffs and thus $\lambda(m') = \lambda(m)$ is feasible, but the seller achieves strictly higher payoffs when n buyers arrive. ■

Therefore, for equilibrium play we only have to restrict attention to full-trade mechanisms. For the following theorem, let $R(m, \lambda) = \sum_{n=1}^{\infty} P_n(\lambda)\pi_n^m$ be the expected seller payoff for a mechanism m with feasible queue λ and $U(m, \lambda) = \sum_{n=1}^{\infty} Q_n(\lambda)u_n^m$ the expected buyer utilities.

[10] Any sequence u_n^m and π_n^m can be implemented by a mechansism that charges price p_n to all buyers, and trade occurs with probability α_n (in which case one of the buyers is selected at random): $p_n = \pi_n^m/n$ and $\alpha_n = n[u_n^m + p - 1]$.

Theorem 2 *(Revenue Equivalence) For given U^*, consider some full-trade mechanism $m \in M$ with with feasible λ. Any other full-trade mechanism m' with $U(m', \lambda) = U(m, \lambda)$ achieves $R(m', \lambda) = R(m, \lambda)$ and λ is feasible for m'.*

Proof. The queue length λ is feasible for mechanism m means that $U(m, \lambda) \geq U^*$. Since $U(m', \lambda) = U(m, \lambda)$, λ is also feasible for mechanism m'. Since m is a full-trade mechanism, we have $1 - nu_n^m = \pi_n^m$ and therefore

$$
\begin{aligned}
R(m, \lambda) &= \sum_{n=1}^{\infty} P_n(\lambda) \pi_n^m \\
&= \sum_{n=1}^{\infty} P_n(\lambda)[1 - nu_n^m] \\
&= 1 - P_0(\lambda) - \sum_{n=1}^{\infty} P_n(\lambda) nu_n^m \\
&= 1 - P_0(\lambda) - \lambda \sum_{n=1}^{\infty} Q_n(\lambda) u_n^m \\
&= 1 - P_0(\lambda) - \lambda U(m, \lambda),
\end{aligned}
$$

where the forth line follows from (1). Similarly, since m' is also a full-trade mechanism we have $R(m', \lambda) = 1 - P_0(\lambda) - \lambda U(m', \lambda)$. Since $U(m', \lambda) = U(m, \lambda)$ we have $R(m', \lambda) = R(m, \lambda)$. ■

We call a class of mechanisms payoff-complete if for any $\lambda > 0$ and any U there exists a mechanisms such that $U(m, \lambda) = U$. Clearly the class of price posting mechanisms is complete, since $U(p, \lambda) = \sum_{n=1}^{\infty} Q_n(\lambda)[1-p]/n$ and $Q_1(\lambda) > 0$ [as $P_1(\lambda) > 0$]. Similarly, the class of second price auctions with reserve price is complete: since $Q_1(\lambda)$ is positive, the reserve price – which can be negative – can be adjusted to yield the right payoff to buyers. For any γ, the set of mechanisms that offer with probability γ a fixed price and with probability $(1-\gamma)$ give the good away for free is complete. Theorem 2 implies that for any U^*, if there is an optimal mechanism, there is an optimal mechanisms within any payoff-complete class of mechanisms.

Proposition 3 *There exists an equilibrium in any class of payoff-complete full-trade mechanisms. An equilibrium mechanism from a class of pay-off complete full-trade mechanisms remains optimal in equilibria where additional mechanisms are available.*

Proof. We will prove existence in the space of second-price auctions with reserve price. The second statement follows immediately from revenue equivalence: the same expected payoffs yield an equilibrium in any other payoff-complete class of full-trade mechanisms such as price posting. For a given reserve r the seller gets r and the buyer $1 - r$ if one buyer arrives, if more buyers arrive the seller gets 1 and the buyers get 0. Payoffs with reserve r are therefore given by $R(r, \lambda) = P_1(\lambda) r + (1 - P_1(\lambda) - P_2(\lambda))$ and $U(r\ \lambda) = Q_1(\lambda)(1 - r)$. For a given U^* it is by first order stochastic dominance condition (2) clear that the optimal λ is biding in the constraint (**??**). Sellers therefore maximize

$$
\begin{aligned}
&\max_r P_1(\lambda) r + (1 - P_0(\lambda) - P_1(\lambda)) \\
&\texttt{s.t.} \quad \frac{P_1(\lambda)}{\lambda}(1 - r) = U^*.
\end{aligned}
$$

Substituting out the constraint leaves

$$\max_{\lambda} 1 - \lambda U^* - P_0(\lambda).$$

Since P_0 is strictly convex, each seller has a unique optimum, and therefore an equilibrium has all sellers posting the same r. It is characterized by the unique first order condition

$$-P_0'(\lambda) = U^*.$$

Since all sellers in equilibrium post the same r, the will face a queue length of $\lambda^* = b/s$. Therefore equilibrium utility $U^* = -P_0'(\lambda^*)$, yielding equilibrium profits of $1 + \lambda P_0'(\lambda^*) - P_0(\lambda^*)$, which means that the equilibrium mechanism is

$$r^* = 1 + \frac{\lambda^* P_0'(\lambda^*)}{P_1(\lambda^*)}. \tag{4}$$

Assume there exists an equilibrium (potentially in other payoff-complete mechanisms) in which U^{**} obtains for buyers and some sellers have queue length λ' while others have queue length λ''. By Proposition 2 there must be an equilibrium in second-price auctions where buyers obtain U^{**} and some sellers achieve queue length λ' while others have queue length λ''. Yet in second price auctions the equilibrium is unique and all sellers have the same queue length. ■

The reserve price r^* in (4) can in general be different from zero. Therefore, Peters' (1997) conjecture that the reserve price in the homogeneous agent case may lie in $(0, 1)$ obtains for many matching specifications. For the specific urn-ball matching technology of the directed search literature this is not true, though.

Corollary 4 *Under urn-ball matching ($P_n(\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$) the equilibrium second price auction has $r^* = 0$.*

The combination of Theorem 2 and Proposition 3 readily yield the following multiplicity result.

Corollary 5 *There exists a continuum of equilibria in which sellers announce different payoff-equivalent mechanisms.*

Finally, we relate our findings to the efficiency of random matching environments.

Corollary 6 *Since in any equilibrium sellers have the same queue length, the equilibrium can be implemented by random search of buyers. Random search is constrained efficient in the sense of creating the highest number of trades.*

To see the efficiency result, observe that the number of trades is

$$s \int [1 - P_0(\lambda(m))] \, d\mu_s$$

and due to the concavity of $[1 - P_0(\lambda)]$ the optimal allocation has the same queue for all buyers, i.e. it arises under random search.

To see that all sellers always have the same queue length, suppose this is not the case and there exist at least two types of firms that each face different queue lengths for a given mechanism each announces. From revenue equivalence, these firms could post second price auctions, possibly with different reserve price. But because there is a unique solution to the first order condition of a seller posting an auction (see proof of Proposition 3), and given concavity of the seller's profit function, there must exist a reserve price and corresponding queue length that yields a higher profit. By revenue equivalence, the mechanism the firms initially announced does not maximize profits, therefore contradicting that firms face different queue lengths.

This result then implies that whenever the market utility assumption holds as the limit of finite economies (see for example Peters 2000), then the mechanism used does not matter. A mechanism is an equilibrium as long as it delivers random matching and all agents obtain the market utility. Of course, this need not be the case in a finite economy where different mechanisms induce different outcomes, and multiple equilibria are possible. [11]

4 The heterogeneous agent model

Consider a directed search model in which measure $\underline{b}$ of low type buyers, measure $\overline{b}$ of high type buyers and measure s of sellers who are homogeneous. The low buyer type has a valuation $\underline{v} > 0$ for the good, the high buyer type 2 has valuation $\overline{v} > \underline{v}$, and sellers have no value or cost of production for the good. If a buyer and a seller trade at a transfer t from the buyer to the seller, then the buyers payoff is $v - t$ and the seller payoff is t. Payoffs from no trade are normalized to zero.

Mechanisms. A mechanism now specifies for a given number $\underline{n}$ of low type buyers and $\overline{n}$ of high type buyers the expected payoff $\pi^m_{\underline{n},\overline{n}}$ for the seller and $\underline{u}^m_{\underline{n},\overline{n}}$ ($\overline{u}^m_{\underline{n},\overline{n}}$) for low (high) valuation buyers. The payoffs have to be implemented in a way such that buyers are willing to truthfully reveal their type. To specify this constraint, it is useful to decompose the payoffs into the probability to trade $\underline{x}^m_{\underline{n},\overline{n}}$ ($\overline{x}^m_{\underline{n},\overline{n}}$) for low (high) valuation buyers when $\underline{n}$ low valuation and $\overline{n}$ high valuation buyers are present. Clearly probabilities cannot add to more than unity, i.e. $\underline{n}\ \underline{x}^m_{\underline{n},\overline{n}} + \overline{n}\ \overline{x}^m_{\underline{n},\overline{n}} \leq 1$, and the seller cannot give out more surplus than is created

$$\pi^m_{\underline{n},\overline{n}} \ + \ \underline{n}\ \underline{u}^m_{\underline{n},\overline{n}} \ + \ \overline{n}\ \overline{u}^m_{\underline{n},\overline{n}} \ \leq \ \underline{n}\ \underline{x}^m_{\underline{n},\overline{n}}\ \underline{v} + \overline{n}\ \overline{x}^m_{\underline{n},\overline{n}}\ \overline{v}. \tag{5}$$

Incentive compatibility constraints will apply to the expected payoffs. First, we therefore specify the matching technology.

Matching. Denote the ratio of low type buyers to all sellers by $\underline{\lambda}$, and the ratio of high type buyers to all sellers by $\overline{\lambda}$, then the overall ratio of buyers to sellers in $\lambda = \underline{\lambda} + \overline{\lambda}$. The probabilities $P_n(\lambda)$ and $Q_n(\lambda)$ are defined as in the previous section, and we impose the same conditions. Clearly, from the seller's perspective, the probability of being in a match with $\underline{n}$ low and $\overline{n}$ high types is $\tilde{P}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda}) = P_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})B_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})$ where $B_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})$ is the binomial probability of drawing $\underline{n}$ low types out of all $\underline{n}+\overline{n}$ buyers, when the probability of drawing one low type is $\underline{\lambda}/(\underline{\lambda}+\overline{\lambda})$. For a buyer who meets a seller, the probability of being in a match with $\underline{n}+\overline{n}$ *other* buyers of which $\underline{n}$ are low types and $\overline{n}$ high types is $\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda}) = Q_{\underline{n}+\overline{n}+1}(\underline{\lambda},\overline{\lambda})B_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})$. Let $\tilde{Q}_{\overline{n}}(\underline{\lambda},\overline{\lambda}) = \sum_{\underline{n}=0}^{\infty} \tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})$ be the marginal of that distribution.

[11] For a finite agent model with a continuum of equilibria, including asymmetric ones where the queue length of one firm is larger than the other, see Coles and Eeckhout (2003).

A low type buyer is then willing to reveal his type if the payoff he receives is higher than the payoff when pretending to be a high type:[12]

$$\sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\underline{u}^m_{\underline{n}+1,\overline{n}} \geq \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\left[\overline{u}^m_{\underline{n},\overline{n}+1} + \overline{x}^m_{\underline{n},\overline{n}+1}(\underline{v}-\overline{v})\right] \tag{6}$$

Similarly, the high type is willing to reveal his type if

$$\sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\left[\underline{u}^m_{\underline{n}+1,\overline{n}} + \underline{x}^m_{\underline{n}+1,\overline{n}}(\overline{v}-\underline{v})\right] \leq \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\overline{u}^m_{\underline{n},\overline{n}+1} \tag{7}$$

Directed Matching. A seller chooses his mechanisms $m \in M$ to maximize his profits. Since he can publicly post his mechanism he is more likely to attract a buyer when he posts a more favorable mechanism. Assume all low (high) type buyers obtain utility $\underline{U}^*$ ($\overline{U}^*$) in expectation at other sellers. Then the seller solves the following problem for a given $\underline{U}^*$ and $\overline{U}^*$

$$\max_{(\underline{\lambda},\overline{\lambda})\in\mathbb{R}^2_+,\ \ m\in M} \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty} P_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\pi^m_{\underline{n},\overline{n}} \tag{8}$$

such that

$$\sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\underline{u}^m_{\underline{n}+1,\overline{n}} \quad \geq \quad \underline{U}^* \qquad \texttt{if } \underline{\lambda} > 0, \tag{9}$$

$$\sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty}\tilde{Q}_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\overline{u}^m_{\underline{n},\overline{n}+1} \quad \geq \quad \overline{U}^*. \qquad \texttt{if } \overline{\lambda} > 0, \tag{10}$$

and incentive compatibility constraints for truthful type revelation (6) and (7) and the resource constraint (5) have to hold.

An equilibrium $(\mu_s, \mu_{\underline{b}}, \mu_{\overline{b}}, \underline{\lambda}, \overline{\lambda}, \underline{U}^*, \overline{U}^*)$ consists of probability measures μ_s and μ_b over M such that $s\mu_s$ describes the distribution of mechanisms that are offered, $b\mu_b$ the measure of each buyer type that visits the mechanisms, a queue length $\lambda(m)$ at the mechanisms in the support of μ_b, and equilibrium utilities $\underline{U}^* \geq 0$ and $\overline{U}^* \geq 0$ such that

1. Seller Optimality: Any m in the support of μ_b and associated $\underline{\lambda}(m)$ and $\overline{\lambda}(m)$ solve (8).

2. Buyer Optimality: if $\underline{U}^* > 0$ constraint (9) holds with equality for any $m \in \text{supp}\mu_{\underline{b}}$, if $\overline{U}^* \geq 0$ constraint (10) holds with equality for for any $m' \in \text{supp}\mu_{\overline{b}}$.

3. Consistency: The ratio $\lambda(m)$ indeed reflects the equilibrium behavior, i.e. if the equilibrium utility for each buyer type is strictly positive then

$$s\int_x \lambda(m)d\mu_s = b\int_x d\mu_b \texttt{ for all measurable subsets } x \subseteq M,$$

   for $\lambda \in \{\underline{\lambda}, \overline{\lambda}\}$ and $\mu_b \in \{\mu_{\underline{b}}, \mu_{\overline{b}}\}$.

[12]Interim incentive compatibility sufficies when the buyers can be asked about their type prior to knowing how many other buyers have arrived at the seller.

5 Price Posting Mechanisms

Consider first the set of equilibria in a price posting environment. Under price posting, the good is allocated indiscriminatory and with equal probability to any one of the buyers that arrives. The surplus then is

$$\pi^m_{\underline{n},\overline{n}} + \underline{n}\ \underline{u}^m_{\underline{n},\overline{n}} + \overline{n}\ \overline{u}^m_{\underline{n},\overline{n}} = \frac{\underline{n}}{\underline{n}+\overline{n}}\ \underline{v} + \frac{\overline{n}}{\underline{n}+\overline{n}}\ \overline{v} \leq \underline{n}\ \underline{v} + \overline{n}\ \overline{v}.$$

Full surplus is only realized if $\underline{n}$ or $\overline{n}$ is equal to zero. A price p induces payoffs $\pi_{\underline{n},\overline{n}}(p) = p$ if either $\underline{n}$ or $\overline{n}$ or both are strictly positive, and $\underline{u}_{\underline{n},\overline{n}}(p) = \frac{\underline{v}-p}{\underline{n}+\overline{n}}$ and $o^m_{\underline{n},\overline{n}} = \frac{\overline{v}-p}{\underline{n}+\overline{n}}$. Taking $\underline{U}^*$ and $\overline{U}^*$ as given, we can consider (8) for an individual firm. For price posting mechanisms the maximization becomes particularly simple: the seller only cares about the probability of trade times the price $[1 - P_0(\underline{\lambda} + \overline{\lambda})]p$, while the buyers only care about their probability to trade times their gain $\sum_{n=1}^{\infty} Q_n(\underline{\lambda} + \overline{\lambda})\frac{v-p}{n}$. Using $nP_n(\lambda) = \lambda Q_n(\lambda)$ from (1) we get $\sum_{n=1}^{\infty} Q_n(\underline{\lambda} + \overline{\lambda})/n = [1 - P_0(\underline{\lambda} + \overline{\lambda})]/(\underline{\lambda} + \overline{\lambda})$ and therefore

$$\max_{(\underline{\lambda},\overline{\lambda})\in\mathbb{R}^2_+, p\in\mathbb{R}_+} [1 - P_0(\underline{\lambda} + \overline{\lambda})]p \tag{11}$$

such that

$$\frac{1 - P_0(\underline{\lambda} + \overline{\lambda})}{\underline{\lambda} + \overline{\lambda}} [\underline{v} - p] \quad \geq \quad \underline{U}^* \qquad \texttt{if } \underline{\lambda} > 0, \tag{12}$$

$$\frac{1 - P_0(\underline{\lambda} + \overline{\lambda})}{\underline{\lambda} + \overline{\lambda}} [\overline{v} - p] \quad \geq \quad \overline{U}^*. \qquad \texttt{if } \overline{\lambda} > 0. \tag{13}$$

Note that in this case we can omit the truth-telling constraint as the seller will always choose to trade with the buyer type where he can achieve the highest queue length $\lambda = \underline{\lambda} + \overline{\lambda}$ and the other type then is either indifferent or strictly prefers not to appear. We first show:

Lemma 7 *For given $\underline{U}^*$ and $\overline{U}^*$ there exists a price $\hat{p}$ such that the seller will strictly prefer to attract low types at prices strictly below $\hat{p}$ and strictly prefers to trade with high types at prices strictly above $\hat{p}$. Moreover, it is never optimal to trade at price $\hat{p}$, and a seller restricted to the set of prices $[0, \hat{p}]$ (or restricted to $[\hat{p}, \infty)$) has a unique optimal price within that set.*

Proof. For a given price p, the optimal choice of queue length is to maximize $\lambda = \underline{\lambda} + \overline{\lambda}$ such that at least one constraints is still met at equality. If the constraint holds with equality we have $\frac{(1-P_0(\lambda))}{\lambda}[v - p] = C$ for some C which implicitly defines λ, and implicit differentiation gives

$$\frac{\partial \lambda}{\partial p} = -\frac{(1 - P_0(\lambda))\lambda}{(1 - P_0(\lambda) + \lambda P_0'(\lambda))\,(v - p)},$$

which is negative and strictly increasing in v. This single crossing property immediately implies that there exists some price $\hat{p}$ such that at price $p < \hat{p}$ the optimal λ is binding at the low type's constraint (12) while at $p > \hat{p}$ the optimal λ is binding at the high type's constraint (13). It also implies that no seller would like to cater to both low and high types by offering $\hat{p}$: if it is profitable to increase one's price up to $\hat{p}$ even though the queue length is falling a lot when trading with low types, it will clearly be profitable to increase it strictly above $\hat{p}$ because there the queue length is falling less drastic due to a price change. We will make this argument precise below.

We know that a seller who sells at price below $\hat{p}$ will have a queue length determined by (12) holding with equality. Substituting the constraint, the optimal utility of such a seller is given by the optimization problem

$$\max_{\underline{\lambda}\in[\hat{\lambda},\infty)} [1-P_0(\underline{\lambda})]\underline{v}-\underline{\lambda}\,\underline{U}^*, \tag{14}$$

where $\hat{\lambda}$ is the queue length at $\hat{p}$ if (12) holds with equality. This problem is strictly concave. Therefore every buyer will choose the same queue length $\underline{\lambda}^*(\underline{U}^*,\overline{U}^*)$ dependent on $\underline{U}^*$ (and associated price) when trading with low types. The dependence on $\overline{U}^*$ only arises because it might affect $\hat{\lambda}$. Similarly, the optimization problem for a seller that considers trading with high buyer types is

$$\max_{\overline{\lambda}\in[0,\hat{\lambda}]} [1-P_0(\overline{\lambda})]\overline{v}-\overline{\lambda}\,\overline{U}^* \tag{15}$$

which is also strictly concave. Therefore every buyer will choose the same queue length $\overline{\lambda}^*(\underline{U}^*,\overline{U}^*)$ dependent on $\overline{U}^*$ and $\underline{U}^*$ (and associated price) when trading with low types.

It is easy to see that it is not optimal to trade at $\hat{\lambda}$. For an individual firm, trading at $\hat{\lambda}$ is only optimal if profits at higher queue length according to (14) are not profitable, i.e. the right derivative has to be $-P_0'(\hat{\lambda})\underline{v}-\underline{U}^*\leq 0$. By a similar logic the left derivative of (15) has to be positive, i.e. $-P_0'(\hat{\lambda})\overline{v}-\overline{U}^*\geq 0$, which together imply $-P_0'(\hat{\lambda})\,[\overline{v}-\underline{v}]\geq \overline{U}^*-\underline{U}^*$. Since $\hat{\lambda}$ is the point where both (12) and (13) hold, we have $\overline{U}^*-U^*=[\overline{v}-\underline{v}](1-P_0(\hat{\lambda}))/\hat{\lambda}$. Therefore the prior inequality becomes after some rearranging $1-P_0(\hat{\lambda})+\hat{\lambda}P_0'(\hat{\lambda})\leq 0$, which delivers a contradiction. Therefore no seller caters to both buyer types. ■

This leads to the following result:

Proposition 8 *Assume the set of seller mechanisms includes price posting only. A unique equilibrium exists with one "market" for each buyer type that trades: If the low types trade in equilibrium, then exactly two prices are offered in equilibrium and all low type buyers trade at the low price and all high type buyers trade at the high price. If only high type sellers trade in equilibrium, only one price is offered at which they trade.*

Proof. Since $\hat{\lambda}$ is never an optimal choice for an individual seller, and neither zero nor infinity can be optimal choices, in equilibrium the optimal choice of a firm has to be characterized by the first order condition. For (14) this is

$$-P_0'(\underline{\lambda})\underline{v}=\underline{U}^*.$$

Since all sellers will chose the same queue length, we have $\underline{\lambda}=\underline{\lambda}^*(\gamma)=\frac{b}{\gamma s}$, where γ is the fraction of sellers that trade with low types. Profits are then

$$\underline{\pi}(\gamma)=[1-P_0(\underline{\lambda}^*(\gamma))+\underline{\lambda}^*(\gamma)P_0'(\underline{\lambda}^*(\gamma))]\,\underline{v}. \tag{16}$$

Similar logic for high types implies $\overline{U}^*=-P_0'(\overline{\lambda}^*(\gamma))\overline{v}$ and $\overline{\lambda}^*(\gamma)=\frac{\overline{b}}{(1-\gamma)s}$ and

$$\overline{\pi}(\gamma)=\left[1-P_0(\overline{\lambda}^*(\gamma))+\underline{\lambda}^*(\gamma)P_0'(\overline{\lambda}^*(\gamma))\right]\overline{v}. \tag{17}$$

In equilibrium it cannot be that only the low type buyers trade, as then $\overline{U}^*=0$ and it would be optimal to trade with the high types. If $\underline{\pi}(0)\leq\overline{\pi}(0)$ then even if it is most attractive to trade with low types it is not worthwhile

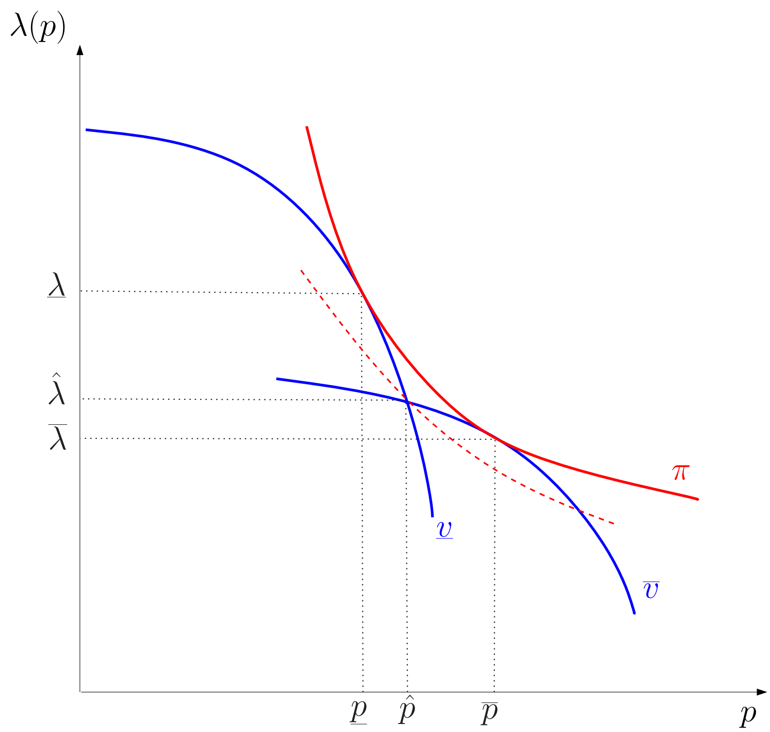

Figure 1: The Separation of Buyer Types

and therefore in equilibrium $\gamma^* = 0$. Otherwise the equilibrium has γ^* uniquely determined by $\underline{\pi}(\gamma^*) = \overline{\pi}(\gamma^*)$. This uniquely characterizes the equilibrium. ■

This separation result is illustrated in Figure 1. Indifference curves satisfy single crossing as derived in the proof of Lemma 7. For high valuation buyers $\overline{v}$, the IC is everywhere flatter than that of low valuation buyers $\underline{v}$. All buyers are identical and in equilibrium they will obtain equal profits catering to both types of buyers. Observe that if the price $\hat{p}$ is offered to both types of buyers, a seller has an incentive to deviate by offering either a strictly higher (or strictly lower) price, thereby catering exclusively to the high type (low type) buyers. Such a deviation is profitable since the isoprofit curve at $\hat{p}$ (dashed line) is not tangent, and maintaining the same utility level, a seller is strictly better off. This continuous to be the case as long as the isoprofit curve is not tangent to both indifference curves simultaneously. In equilibrium, a seller makes equal profits from both types of buyers, and the buyer types have strict preferences over which p, λ pair to choose. High type buyers prefer high prices and low queue lengths, and low type buyers prefer low prices and high queue lengths.

To establish constrained efficiency, we consider a planner that takes the frictions as given and can only use mechanisms that give the good to one of the buyers at random. The planner assigns to each seller σ in $[0, s]$ the queue lengths $\underline{\lambda}^p(\sigma)$ and $\overline{\lambda}^p(\sigma)$ of low and high buyer types. These functions are feasible if they are measurable and indeed reflect the ratio of buyers to sellers in the sense that

$$\int_0^s \underline{\lambda}^p(\sigma) d\sigma \leq \underline{b} \texttt{ and } \int_0^s \overline{\lambda}^p(\sigma) d\sigma \leq \bar{b}. \tag{18}$$

The realized surplus is

$$S(\underline{\lambda}^p, \overline{\lambda}^p) = \int_0^s \left[1 - P_0\left(\underline{\lambda}^p(\sigma) + \overline{\lambda}^p(\sigma)\right)\right]\left[\frac{\underline{\lambda}^p(\sigma)\underline{v} + \overline{\lambda}^p(\sigma)\overline{v}}{\underline{\lambda}^p(\sigma) + \overline{\lambda}^p(\sigma)}\right] d\sigma, \tag{19}$$

where the first brackets reflects the probability that the seller has a buyer, in which case his probability of selecting a high type is $\overline{\lambda}^p(\sigma)/(\underline{\lambda}^p(\sigma) + \overline{\lambda}^p(\sigma))$ and $\overline{v}$ is realized, and with the complementary probability $\underline{v}$ is realized.

The price posting equilibrium in which a fraction γ caters to the low valuation sellers can be represented as a planners strategy $\underline{\lambda}^p(\sigma) = \underline{\lambda}^*(\gamma)$ for all $\sigma \in [0, \gamma s]$ and zero otherwise, and $\overline{\lambda}^p(\sigma) = \overline{\lambda}^*(\gamma)$ for $\sigma \in (\gamma s, s]$ and zero otherwise. We will show that this "price posting" assignment is constrained efficient in the space of mechanisms that give away the good at random.

Proposition 9 *The realized surplus under price posting is higher than under any other feasible queue length functions $\underline{\lambda}(.)$ and $\overline{\lambda}(.)$.*

Proof. Consider some queue length functions $\underline{\lambda}^p(.)$ and $\overline{\lambda}^p(.)$. We will first show that it is sufficient to concentrate on queue length function that assign strictly positive $\underline{\lambda}^p(\sigma)$ only if $\overline{\lambda}^p(.) = 0$. To see this, write the surplus as

$$\begin{aligned} S(\underline{\lambda}^p, \overline{\lambda}^p) &= \underline{v}\int_0^s \left[1 - P_0\left(\check{\lambda}(\sigma)\right)\right] \underline{f}(\sigma)d\sigma + \overline{v}\int_0^s \left[1 - P_0\left(\check{\lambda}(\sigma)\right)\right] \overline{f}(\sigma)d\sigma \\ &= \underline{v}\int_0^s \left[1 - P_0\left(\check{\lambda}(\sigma)\right)\right] d\underline{F}(\sigma) + \overline{v}\int_0^s \left[1 - P_0\left(\check{\lambda}(\sigma)\right)\right] d\overline{F}(\sigma) \end{aligned}$$

where $\check{\lambda}(\sigma) = \underline{\lambda}^p(\sigma) + \overline{\lambda}^p(\sigma)$, $\underline{f}(\sigma) = \frac{\underline{\lambda}^p(\sigma)}{\underline{\lambda}^p(\sigma) + \overline{\lambda}^p(\sigma))}$ and $\underline{F}(\sigma) = \int_0^s \underline{f}(\sigma)d\sigma$, likewise for $\overline{f}, \overline{F}$. The first term can be interpreted as assigning all low valuation buyers to a measure $\underline{F}(s)$ of firms and all high valuation buyers to a measure $\overline{F}(s)$ if firms according to the assignment function $\check{\lambda}(\sigma)$. Another way of seeing this is to rewrite the term as

$$S(\underline{\lambda}^p, \overline{\lambda}^p) = \underline{v}\int_0^{\underline{F}(s)} \left[1 - P_0\left(\check{\lambda}(\underline{F}(x))\right)\right] dx + \overline{v}\int_{\underline{F}(s)}^s \left[1 - P_0\left(\check{\lambda}(\overline{F}(x - \underline{F}(s)))\right)\right] dx$$

which is identical to a feasible assignment function that assigns queue length $\underline{\lambda}(\sigma) = \check{\lambda}(\underline{F}(\sigma))$ to all firms in $[0, \underline{F}(s)]$ and $\underline{\lambda}(\sigma) = 0$ otherwise, and $\overline{\lambda}(\sigma) = \check{\lambda}(\overline{F}(\sigma - \underline{F}(s)))$ to all firms in $[\underline{F}(s), s]$ and $\underline{\lambda}(\sigma) = 0$ otherwise.

Next, observe that $1 - P_0(\lambda)$ is strictly concave, and therefore it is optimal to assign an identical queue length $\underline{\lambda}$ to all firms that sell to low valuation buyers and an identical queue length $\overline{\lambda}$ to all firms that sell to high valuation buyers. If a fraction α of the firms sell to low valuation buyers, the feasibility constraint then requires that $\underline{\lambda}(\alpha) = \underline{b}/(\alpha s)$ and $\overline{\lambda}(\alpha) = \overline{b}/(1-\alpha)$. The maximization problem of the planner reduces to

$$\max_{\alpha \in [0,1]} \alpha(1 - P_0(\underline{\lambda}(\alpha)))\underline{v} + (1-\alpha)(1 - P_0(\overline{\lambda}(\alpha)))\overline{v}.$$

The planner clearly chooses $\alpha < 1$. If he chooses to trade with low valuation buyers, the first order condition for optimality is

$$(1 - P_0(\underline{\lambda}(\alpha)) + \underline{\lambda}(\alpha)P_0'(\underline{\lambda}(\alpha)))\underline{v} = (1 - P_0(\overline{\lambda}(\alpha)) + \overline{\lambda}(\alpha)P_0'(\overline{\lambda}(\alpha)))\overline{v}, \tag{20}$$

where we used the fact that $(\partial\underline{\lambda}(\alpha)/\partial\alpha) = -\underline{\lambda}(\alpha)/\alpha$ and $(\partial\overline{\lambda}(\alpha)/\partial\alpha) = \overline{\lambda}(\alpha)/(1-\alpha)$. Note that this is identical to the condition that profits according to (16) and (17) have to be equal. Moreover, it is easy to show that the $\alpha = 0$ if the left hand side of (20) evaluated as $\alpha = 0$ is smaller then the right hand side of (20) evaluated at $\alpha = 0$, which again coincides with the equilibrium. ■

6 Competition in Mechanisms

6.1 Directed Search

Here we consider matching technologies in which a seller can meet multiple buyers simultaneously. More specifically, we assume that buyer types do not crowd out one-another: the marginal $Q_n(\underline{\lambda}, \overline{\lambda})$ is not affected by $\underline{\lambda}$, i.e. the probability that a high type seller is in a group with n other high type sellers does not depend on the number of low type sellers. Moreover, when more low type buyers arrive this does not diminish the meeting probability for high type buyers. It might diminish their trading probability e.g. in the case when prices are used and a seller tries to attract both types of buyers. Yet in terms of simply meeting one of the sellers we assume that there are no spillovers. This means that $Q_0'(\lambda) = 0$. This is for example true if buyers choose sellers at random, as in this case the probability of not meeting any seller is zero. The probability of trading will change when there are more buyers around because it is harder to obtain the good, but the probability of meeting is constant. This is the standard assumption of the competing mechanism design literature (Peters (1997, 1999)).

Under this assumption we will prove that an equilibrium in which all sellers use second price auctions with a reserve below $\underline{v}$ is strictly more efficient than an equilibrium in which sellers use price posting mechanisms. With slight modification it can be shown that any class of mechanisms that does not allow for ex-post screening will not be constrained efficient. Our efficiency criterion is the sum of surplus given the matching frictions. The result obtains even though we have seen that sellers can perfectly screen between buyers by using prices, and therefore no seller in a price posting environment has any uncertainty about the type of seller he is facing. Note that under no crowding out, the probability of having a match with n other high type depends only on $\overline{\lambda}$ and is $Q_n(\overline{\lambda})$.

Proposition 10 *The equilibrium in a price posting environment, i.e. an environment where M only includes fixed price mechanisms, is not constrained efficient in a larger class of mechanisms: All sellers posting identical second price auctions with reserve below $\underline{v}$ is constrained efficient.*

Proof. We compare the outcome of the price posting equilibrium to the outcome when all sellers post the same second price auction with reserve price $r < \underline{v}$ and all sellers try to trade. We first consider the case when both buyer types can trade in a price posting environment. The proof has two steps. First, we prove that there are strictly more trades in the auction environment. Second, we prove that high valuation buyers have strictly more trades. Together this establishes that the auction environment is more efficient.

For the first part of the argument, observe that in a price posting environment some fraction d of sellers has a low queue length $\underline{\lambda} = \underline{b}/ds$ while a fraction $1-d$ has a high queue length $\overline{\lambda} = \overline{b}/(1-d)s$. The total number of matches in a price posting economy is

$$M^p(d) = ds\left[1 - P_0\left(\frac{\underline{b}}{ds}\right)\right] + (1-d)s\left[1 - P_0\left(\frac{\overline{b}}{(1-d)\,s}\right)\right]$$

The first order condition for the optimal number of matches is

$$\left[1 - P_0\left(\frac{\underline{b}}{ds}\right)\right] + \frac{\underline{b}}{ds}P_0'\left(\frac{\underline{b}}{ds}\right) - \left[1 - P_0\left(\frac{\overline{b}}{(1-d)\,s}\right)\right] - \frac{\overline{b}}{(1-d)\,s}P_0'\left(\frac{\overline{b}}{(1-d)\,s}\right) = 0,$$

which is, under P_0 convex, only satisfied if $\underline{b}/ds = \overline{b}/(1-d)s$, i.e. when agents' visiting probabilities are the same at all firms. This is not achieved in a price posting environment but is achieved under random matching. Since $M^{p\prime\prime}(d) < 0$ at all d, the first order condition indeed characterizes the optimal number of matches.

For the second part of the argument, we show that the high valuation buyers have strictly more matches under random search with auctions than in a price posting environment. Under auctions, the queue length for high buyer types is $\overline{\lambda}^a = \frac{\overline{b}}{s}$. The matching probability is

$$\sum_{\overline{n}=1}^{\infty} Q_{\overline{n}}(\overline{\lambda}^a)\frac{1}{\overline{n}} = \sum_{\overline{n}=1}^{\infty} \frac{P_{\overline{n}}(\overline{\lambda}^a)}{\overline{\lambda}^a} = \frac{1-P_0(\overline{\lambda}^a)}{\overline{\lambda}^a}.$$

Similarly, under price posting only a fraction $(1-d)$ of firms attract high types which induces a queue of $\overline{\lambda}^p = \overline{b}/[(1-d)s]$. A high buyers matching probability is then by a similar logic $(1-P_0(\overline{\lambda}^p))/\overline{\lambda}^p$. Since P_0 is convex we have the matching probability increasing in λ, and since $\overline{\lambda}^a > \overline{\lambda}^p$ the high types can trade more often in the auction environment and therefore more often the high valuation surplus is realized.

Finally, consider the case when only the high valuation buyers can trade in a price posting equilibrium. They do not trade more often than in the auction environment, as we have shown that the auction environment maximizes the number of trades for high types and in fact achieves an identical amounts of trade compared to price posting. Moreover, the auction environment allows some low types to trade when no high type is present, which strictly improves efficiency. Constrained efficiency of the auction environment follows trivially from the fact that auctions maximize the overall number of matches as well as the number of matches for high valuation buyers. ■

Observe that any mechanism that does not screen buyers ex post will be dominated by an equilibrium in second price auctions. Price posting by all sellers (or a mechanism without ex post screening) is not an equilibrium when a larger set of ex-post screening mechanisms is allowed. The reason is that a seller can offer buyers the same payoff as under price posting, while enjoying exactly the efficiency gains that an auction environment implements.

Proposition 11 *If all sellers post prices, it is strictly profitable for a deviating seller to post a second price auction coupled with a show-up fee.*

Proof. Consider the equilibrium when sellers cannot screen buyers ex post, which delivers some utility $\underline{U}^*$ and $\overline{U}^*$ for the low and high buyer type, as well as some profit π^* for the sellers. Now consider a seller who posts a second price auction with show-up fee $-f$ and reserve r. The reserve r is due only when a single buyer is present, and we assume that buyers commit to paying this reserve when they are the only buyers, even if $r > \underline{v}$. With such a mechanism, the seller obtains a payoff $\pi^{f,r}_{\underline{n},\overline{n}} = r - f$ when there is one low or high buyer, obtains $\pi^{f,r}_{\underline{n},\overline{n}} = \underline{v} - (\underline{n}+\overline{n})f$ when $\overline{n} = 1, \underline{n} \geq 1$, or when $\underline{n} \geq 2$ and $\overline{n} = 0$; and obtains $\pi^{f,r}_{\underline{n},\overline{n}} = \overline{v} + (\underline{n}+\overline{n})f$ when $\overline{n} \geq 2$. In all other cases his payoff is zero. The seller solves the following program

$$\max_{(\underline{\lambda},\overline{\lambda})\in\mathbb{R}^2_+,(f,r)\in\mathbb{R}^2} \sum_{\underline{n}=0}^{\infty}\sum_{\overline{n}=0}^{\infty} P_{\underline{n},\overline{n}}(\underline{\lambda},\overline{\lambda})\pi^{f,r}_{\underline{n},\overline{n}} \tag{21}$$

such that

$$(1-Q_0(\underline{\lambda}+\overline{\lambda}))f + \tilde{Q}_{0,0}(\underline{\lambda},\overline{\lambda})\,[\underline{v}-r] \quad \geq \quad \underline{U}^* \qquad \texttt{if } \underline{\lambda} > 0, \tag{22}$$

$$(1-Q_0(\underline{\lambda}+\overline{\lambda}))f + \tilde{Q}_{0,0}(\underline{\lambda},\overline{\lambda})\,[\overline{v}-r] + \left[\tilde{Q}_0(\underline{\lambda},\overline{\lambda}) - \tilde{Q}_{0,0}(\underline{\lambda},\overline{\lambda})\right][\overline{v}-\underline{v}] \quad \geq \quad \overline{U}^*. \qquad \texttt{if } \overline{\lambda} > 0. \tag{23}$$

where the first constrained arises because the low type buyers obtain the good only when they are alone. The second constraint arises because high types only make positive profits when there are no other high types in the auctions, in which case they either pay the bid of the low type buyer if one is present, or they pay the reserve price.

Clearly the seller can implement the average queue length $\underline{\lambda} = \underline{b}/s$ and $\overline{\lambda} = \overline{b}/s$ with some combination of his instruments f and r. Now assume all sellers would use such an auction. Then the average queue length would be implemented at all sellers, which is feasible. We know from Proposition 10 that the surplus in this environment is higher than under price posting. But we have constructed the environment in such a way that the buyers obtains exactly the same utility as in the price posting environment, which means that the sellers must get higher profits. This in turn means that our deviating seller enjoys strictly higher profits than those sellers who do not ex post screen buyers. ■

We know from McAfee (1993) and Peters (1997) that an equilibrium in second price auctions exists under urn-ball matching, and their approach can be extended to our setting by generalizing the matching function.

6.2 Competitive Search

Here we assume that only bilateral matches are feasible. That is, $P_n(\lambda) = 0$ for all $n > 1$. This immediately implies that only $Q_0(\lambda)$ and $Q_1(\lambda)$ can be strictly positive. In this case we have

$$1 - Q_0(\lambda) = Q_1(\lambda) = \frac{P_1(\lambda)}{\lambda} = \frac{1 - P_0(\lambda)}{\lambda}.$$

Since $P_0'(\lambda) < 0$ we cannot have $Q_0'(\lambda) = 0$, and therefore the meeting probability of e.g. high types changes when more low types enter the market. This is unavoidable in bilateral matching models.

We want to show that price posting is optimal, i.e., no other mechanisms performs better, and it is an equilibrium. Other mechanisms could still elicit buyers' types via wasteful destruction, yet the key insight in directed search is that ex-ante separation of types achieves type revelation without wasteful destruction.

Proposition 12 *Under bilateral matching, price posting is always a best response by a seller.*

Proof. A seller now solves the program

$$\begin{aligned} & \max_{(\underline{\lambda},\overline{\lambda})\in\mathbb{R}^2_+,\ m\in M} P_1(\underline{\lambda}+\overline{\lambda}) \left[\frac{\underline{\lambda}\pi^m_{1,0} + \overline{\lambda}\pi^m_{0,1}}{\underline{\lambda}+\overline{\lambda}}\right] && (24) \\ = & \max_{(\underline{\lambda},\overline{\lambda})\in\mathbb{R}^2_+,\ m\in M} Q_1(\underline{\lambda}+\overline{\lambda}) \left[\underline{\lambda}\pi^m_{1,0} + \overline{\lambda}\pi^m_{0,1}\right] && (25) \end{aligned}$$

such that

$$\begin{aligned} Q_1(\underline{\lambda}+\overline{\lambda})\underline{u}^m_{1,0} &\geq \underline{U}^* \quad \texttt{if } \underline{\lambda} > 0, && (26) \\ Q_1(\underline{\lambda}+\overline{\lambda})\overline{u}^m_{0,1} &\geq \overline{U}^*. \quad \texttt{if } \overline{\lambda} > 0, && (27) \end{aligned}$$

and incentive compatibility constraints for truthful type revelation have to hold (for some x and y):

$$Q_1(\underline{\lambda}+\overline{\lambda})\underline{u}^m_{1,0} \geq Q_1(\underline{\lambda}+\overline{\lambda}) \left[\overline{u}^m_{0,1} - y^m_{0,1}(\overline{v} - \underline{v})\right]$$

$$Q_1(\underline{\lambda}+\overline{\lambda})\left[\underline{u}^m_{1,0}+x^m_{1,0}(\overline{v}-\underline{v})\right]\leq Q_1(\underline{\lambda}+\overline{\lambda})o^m_{0,1}$$

as well as the resource constraints $\pi^m_{1,0}+\underline{u}^m_{1,0}\leq x^m_{0,1}\underline{v}$ and $\pi^m_{0,1}+\overline{u}^m_{0,1}\leq y^m_{0,1}\overline{v}$. The incentive compatibility conditions can be rewritten as

$$y^m_{0,1}\geq\frac{\overline{u}^m_{0,1}-\underline{u}^m_{1,0}}{\overline{v}-\underline{v}}\geq x^m_{0,1}$$

Optimality clearly has $y^m_{0,1}=1$. Thus, any difference $\Delta u=\overline{u}^m_{0,1}-\underline{u}^m_{1,0}<\overline{v}-\underline{v}$ involves no trade with low types with probability $\frac{\Delta u}{\overline{v}-\underline{v}}$. Therefore, $\frac{\underline{\lambda}\left[\pi^m_{1,0}+\underline{u}^m_{1,0}\right]+\overline{\lambda}\left[\pi^m_{0,1}+\overline{u}^m_{0,1}\right]}{\underline{\lambda}+\overline{\lambda}}\leq\frac{\underline{\lambda}\Delta u\underline{v}+\overline{\lambda}\ \overline{v}}{\underline{\lambda}+\overline{\lambda}}$, it is easy to see that it is optimal to have this hold with equality. This reduces the problem to

$$\max_{(\underline{\lambda},\overline{\lambda})\in\mathbb{R}^2_+,\ m\in M} Q_1(\underline{\lambda}+\overline{\lambda})\left[\underline{\lambda}\frac{\overline{u}^m_{0,1}-\underline{u}^m_{1,0}}{\overline{v}-\underline{v}}\left(\underline{v}-\underline{u}^m_{1,0}\right)+\lambda_2\left(\overline{v}-\overline{u}^m_{0,1}\right)\right]\tag{28}$$

such that

$$\begin{aligned}Q_1(\underline{\lambda}+\overline{\lambda})\underline{u}^m_{1,0} &\geq \underline{U}^* \quad \texttt{if}\ \underline{\lambda}>0, &(29)\\ Q_1(\underline{\lambda}+\overline{\lambda})\overline{u}^m_{0,1} &\geq \overline{U}^* \quad \texttt{if}\ \overline{\lambda}>0, &(30)\\ 0 &\leq \overline{u}^m_{0,1}-\underline{u}^m_{1,0}\leq\overline{v}-\underline{v}. &(31)\end{aligned}$$

Assume the optimal program has some contract m and $\underline{\lambda}>0$ and $\overline{\lambda}>0$. If $\frac{\overline{u}^m_{0,1}-\underline{u}^m_{1,0}}{\overline{v}-\underline{v}}\left(\underline{v}-\underline{u}^m_{1,0}\right)\leq\lambda\overline{v}-\overline{u}^m_{0,1}$ then there exists an optimal contract m' and $\overline{\lambda}'>0$ and $\underline{\lambda}'=0$. The reason is that at least the same payoff can be obtained by choosing $\overline{\lambda}'=\underline{\lambda}+\overline{\lambda}$ and $\overline{u}^{m'}_{0,1}=\overline{u}^m_{0,1}$ and $\overline{u}^m_{0,1}-\underline{u}^{m'}_{1,0}=\overline{v}-\underline{v}$. Yet this can be achieved with a price posting contract where the posted price p is such that $\overline{u}^m_{0,1}=\overline{v}-p$.

If $\frac{\overline{u}^m_{0,1}-\underline{u}^m_{1,0}}{\overline{v}-\underline{v}}\left(\underline{v}-\underline{u}^m_{1,0}\right)<\lambda\overline{v}-\overline{u}^m_{0,1}$ then there exists an optimal contract m' and $\overline{\lambda}'=0$ and $\underline{\lambda}'>0$. The reason is again that at least the same payoff can be obtained by choosing $\underline{\lambda}'=\underline{\lambda}+\overline{\lambda}$ and $\underline{u}^{m'}_{0,1}=\underline{u}^m_{0,1}$ and $\overline{u}^{m'}_{0,1}-\underline{u}^{m'}_{1,0}=\overline{v}-\underline{v}$. This again can be achieved with a price posting contract with a price such that $\underline{u}^m_{0,1}=\underline{v}-p$. ■

Given that an equilibrium exists when sellers can only use price posting strategies, we have

Corollary 13 *Under bilateral matching, an equilibrium exists if the set of mechanisms M includes all price posting mechanisms. One of the equilibria is identical to the price posting equilibrium of Section 5.*

Our final result concerns the efficiency of the equilibrium. Again, constrained efficiency involves finding functions $\underline{\lambda}^p(\sigma)$ and $\overline{\lambda}^p(\sigma)$ such that (18) is fulfilled. We could additionally specify functions that destroy some of the surplus in order to induce truthful type revelation, but it is clear that a social planner would always want the seller to trade once a buyer shows up. The realized surplus that is to be maximized is then

$$S(\underline{\lambda}^p,\overline{\lambda}^p)=\int_0^s P_{1,0}(\underline{\lambda}^p(\sigma)+\overline{\lambda}^p(\sigma))v_1+P_{0,1}(\underline{\lambda}^p(\sigma)+\overline{\lambda}^p(\sigma))d\sigma.\tag{32}$$

Proposition 14 *The price posting equilibrium is constrained efficient.*

Proof. Since $P_{1,0}(\underline{\lambda},\overline{\lambda})=P_1(\underline{\lambda}+\overline{\lambda})\frac{\underline{\lambda}}{\underline{\lambda}+\overline{\lambda}}$ and $P_{0,1}(\underline{\lambda},\overline{\lambda})=P_1(\underline{\lambda}+\overline{\lambda})\frac{\overline{\lambda}}{\underline{\lambda}+\overline{\lambda}}$ and under bilateral matching $P_1(\underline{\lambda}+\overline{\lambda})=1-P_0(\underline{\lambda}+\overline{\lambda})$ the surplus (32) is identical to the surplus specified in (19) in Section 5, for which we have shown that the queue length that arise under price posting are optimal. ■

7 Conclusion

Posted prices have attractive efficiency properties in an economy with frictions. Buyers use prices as a signal to direct their search, and Moen (1997) shows that this is efficient when buyers are homogeneous. We show that this result continuous to hold even if sellers have access to a whole class of other mechanisms such as auctions.

Whether price posting is efficient with heterogeneous buyers depends on the matching technology. If matching is bilateral and sellers meet at most one buyer at the time, price posting is an efficient mechanism. Posted prices lead buyers to separate themselves ex ante (before they meet the sellers) by choosing sellers with different prices. This ex ante screening is efficient, because ex post screening (after sellers and buyers meet) involves inefficient destruction of resources that can be avoided by ex ante screening. There are other mechanisms that implement the same outcome, but none of them allows for random matching, which is strictly less efficient. Under these circumstances, Moen's result continues to hold even with heterogeneous buyers.

When sellers can meet multiple buyers and they have the capacity to screen those buyers ex post, price posting is no longer an equilibrium when sellers have access to other mechanisms. Sellers are better off posting identical second price auctions. This results in buyers randomizing over sellers, and sellers ex post screen the buyers with the highest valuation. The efficient mechanism necessarily involves random matching because it maximizes the total number of meetings generated, and there is no role for ex ante screening since it is entirely absorbed ex post.

We derive the result with multiple meetings under the assumption that the matching function has no crowding out, i.e., the meeting probability of one buyer type is not affected by the queue length of another buyer type. This is a common assumption whenever there is matching with multiple meetings. Bilateral matching implicitly requires crowding out. Having both low and high buyer types in the same market makes it harder for high types to even get in contact with a seller. Therefore, it is efficient separate buyers into different markets, i.e. to have them visit different sets of sellers. Once separation is desirable, then price posting does well because in each market only one type of buyer trades. Without crowding out things reverse. The presence of low types does not reduce the chance that a high type can trade. Since random meetings lead to more matches, it is optimal to have buyers search randomly and sort out ex-post which one of them has a high type. This maximizes the number of matches as well as the number of matches for high types.

We have considered two polar cases: multilateral matching without crowding out and bilateral matching with crowding out. The investigation of intermediate cases, which are absent in the present literature, is left for future work. To give an example of such intermediate cases, consider buyers who can send one letter to one of the buyers indicating that they want to trade (i.e., think of workers that have time to fill out one job application). But assume that sellers only have time to open and read up to N envelopes.[13] The case $N = 1$ amounts to bilateral meetings: The seller can only see one buyer and cannot screen between any applicants any longer. With $N \to \infty$ we are in the standard urn-ball meeting environment with multilateral matching without crowding out, and sellers have full control whom to give the object to by choosing the appropriate auction format. We conjecture that for N low price posting remains optimal, because the chances of opening the envelops of only low types without even

[13]The sellers probability of seeing n buyers is given by $P_n(\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}$ for $n < N$, and $P_n(\lambda) = \sum_{n=N}^{\infty} \frac{\lambda^n e^{-\lambda}}{n!}$ for $n = N$ and is zero for larger n.

seeing the high types remains too high to render mixing of types at one seller optimal. For N large but finite we conjecture that it will become dominant to distribute buyers randomly and give the good only to low types in the residual case that the high types are not present.

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