Money and Credit with Limited Commitment

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Abstract

We study the interplay among imperfect memory, limited commitment, and theft, in an environment that can support monetary exchange and credit. Imperfect memory makes money useful, but it also permits theft to go undetected, and therefore provides lucrative opportunities for thieves. Limited commitment constrains credit arrangements, and the constraints tend to tighten with imperfect memory, as this mitigates punishment for bad behavior in the credit market. In spite of the fact that theft is a deadweight loss, theft in anonymous transactions can discipline credit market behavior, and can therefore be a good thing. We show that the Friedman rule is in general not feasible, or not optimal if it is feasible, and that there are conditions under which theft exists at the optimum.

INTRODUCTION

The goal of this paper is to study the roles of money and credit in the context of a particular set of frictions, which are limited commitment, imperfect record-keeping, and theft. We get some new results, and develop some ideas which we think are important for understanding monetary policy.

We start with the model of Rocheteau and Wright (2005) and Lagos and Wright (2005), which has quasilinear utility and alternating decentralized and centralized trading among economic agents. This lends tractability to our analysis, but we think that the basic ideas are quite general. Our approach is to add frictions to the model, one-by-one, and study the resulting implications.

First, we start with only the limited commitment friction and perfect memory, and solve for efficient allocations. As is standard in limited commitment environments, if the discount factor is sufficiently high, then any efficient allocation under limited commitment is also an efficient allocation with full commitment. In this case there are allocations that are efficient under full commitment that are not feasible under limited commitment. With a sufficiently small discount factor, no full-commitment efficient allocation is efficient with limited commitment and incentive constraints bind.

Our first step is to study monetary equilibrium in this environment with a form of imperfect memory that is standard in the monetary theory literature. That is, decentralized meetings of economic agents are not monitored, but centralized meetings are. For this case we get results similar to Andolfatto (2007), though Andolfatto does not study credit arrangements in his model. Allowing for the fact that there is limited commitment for tax liabilities, a Friedman rule is optimal when a limited-commitment efficient allocation is also efficient under full commitment. However, when full commitment efficiency is not feasible under limited commitment, then it is optimal for the money supply to fall over time, but the money growth rate is higher than at the Friedman rule. The Friedman rule is not feasible in this case, as in Andolfatto (2007), because agents would default on the tax liabilities that are required to support the Friedman rule.

We then study equilibria in this environment with perfect memory, when monetary exchange is possible. If there exists an efficient allocation with full commitment that is feasible with limited commitment, then an equilibrium with valued money does not exist. However, if no full-commitment efficient allocation is feasible with limited commitment, then monetary equilibria exist, but only at a critical money growth rate. At this money growth rate, a continuum of monetary equilibria exist, and they all yield the same equilibrium allocation as the efficient credit equilibrium where no money is held.

With limited commitment and the standard (in the monetary literature) form of imperfect memory, there is no role for credit, while with perfect memory the role for money is extremely limited. We would like to have features in our model that provide a realistic interplay between money and credit, and where both credit and monetary exchange are robust. Thus, in the remainder of the paper, we study an environment with an intermediate level of imperfect memory in exchange. We also add the feature that agents who default on their debts and are subsequently banned from credit markets can still conduct transactions forever in an anonymous fashion using money, much as in Aiyagari and Williamson (2000) or Antinolfi, Azariadis, and Bullard (2007). Now, no equilibrium with valued money exists with a negative money growth rate, and in any equilibrium with valued money and a strictly positive money growth rate, money and credit coexist.

Our last setup involves allowing theft of money balances in anonymous transactions, at a cost. Without theft, the framework of limited commitment and imperfect memory perhaps allows people who fail to pay their debts to get away with too much. With theft (and the potential for theft), default is less attractive, since default implies that a buyer has to deal with thieves in anonymous transactions with higher probability. Theft is beneficial, in that it helps to discipline debtors, but it also involves a deadweight loss. A Friedman rule is in general not optimal, but it is possible that there is some theft in equilibrium at the optimum.

THE ENVIRONMENT

Time is discrete and each period is divided into two subperiods: day and night. There are two types of agents in the economy, buyers and sellers, and there is a continuum of each type with unit measure. There is a unique perishable consumption good which is produced and consumed within each subperiod. During the day, a seller can produce one unit of the consumption good with one unit of labor. At night, a buyer is able to produce one unit of the consumption good with one unit of labor.

A buyer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) - n_t] \tag{1}$$

where q_t is consumption during the day, and n_t is labor supply at night, with $\beta \in (0,1)$ the discount factor between night and day. Assume $u(\cdot)$ is strictly concave, strictly increasing, and twice continuously differentiable with $u(0) = 0$, $u'(0) = \infty$, and define q^* to be the solution to $u'(q^*) = 1$. A seller has preferences given by

$$\sum_{t=0}^{\infty} \delta^t (-l_t + x_t) \tag{2}$$

where l_t is labor supply during the day and x_t is consumption at night, where $\delta \in (0,1)$ is the seller's discount factor.

Agents are bilaterally matched during the day and at night trade is centralized.

PLANNER'S PROBLEM

Full Commitment

First, consider what a social planner could achieve in this economy in the absence of a money. Ultimately, money will consist of perfectly divisible and durable objects that are portable at zero cost, and can be produced only by the government. In this section assume that each agent can commit at $t = 0$. To be more explicit, suppose that there is complete memory and that the planner can impose infinite utility penalties on an agent if they deviate from promises made at $t = 0$. If the planner treats all sellers the same and all buyers the same, then an allocation $\{(q_t, x_t)\}_{t=0}^{\infty}$ satisfies the participation constraints

$$\sum_{t=0}^{\infty} \beta^t [u(q_t) - x_t] \geq 0, \tag{3}$$

and

$$\sum_{t=0}^{\infty} \delta^t (-l_t + x_t) \geq 0, \tag{4}$$

which state that a buyer and a seller, respectively, each prefer to participate in the plan at $t = 0$. Then, if we confine attention to stationary allocations with $q_t = q$ and $x_t = x$ for all t, we must have

$$u(q) - x \geq 0 \tag{5}$$

and

$$-q + x \geq 0 \tag{6}$$

The set of feasible stationary allocations is given by (5) and (6), and this set is non-empty given our assumptions. Further, the set of efficient allocations is also non-empty, satisfying (5), (6) and $q = q^*$.

Limited Commitment

Now, continue to assume complete memory, but now suppose that at any agent can at any time opt out of the plan. The worst punishment that the planner can impose is zero consumption forever for an agent who deviates. Let v_t denote the expected utility of a buyer at the beginning of t, with w_t similarly denoting the expected utility of a seller. Then an allocation must satisfy the participation constraints (3) and (4) as before, as well as the incentive constraints

$$-x_t + \beta v_{t+1} \geq 0, \tag{7}$$

and

$$-q_t + x_t + \delta w_{t+1} \geq 0 \tag{8}$$

for $t = 0, 1, 2, ..., \infty$. Constraints (7) and (8) state that the buyer and seller, respectively, prefer to produce at each date rather than defecting from the plan.

Again, if we confine attention to stationary allocations, then the feasible set is defined by

$$x \leq \beta u(q)$$

and (6). Now, let q^{**} denote the solution to $\beta u(q^{**}) = q^{**}$. If $q^{**} \geq q^*$ then the efficient allocations are (q^*, x) with $x \in [q^*, \beta u(q^*)]$, as depicted in Figure 1. In the figure, the set of efficient allocations with limited commitment is AB, while the set of efficient allocations with full commitment is AD. Clearly, in this case limited commitment reduces the efficient set, but the efficient allocations with limited commitment are also efficient allocations under full commitment. The interesting case (as we will see) is when $q^{**} < q^*$, in which case the set of efficient allocations is $\{(q, x) : x = \beta u(q), q \in [\hat{q}, q^{**}], \beta u'(\hat{q}) = 1\}$. This case is depicted in Figure 2, where the set of efficient allocations is AB. That is, in the interesting case the incentive constraint for the buyer always binds for the efficient stationary allocation, and an efficient allocation with limited commitment is not efficient under full commitment.

EQUILIBRIUM ALLOCATIONS WITH PERFECT MEMORY

To establish a benchmark, we first assume that there is perfect memory. As we would expect from the work of Kochertakota (1998), this will severely limit the role of money in this economy. So that we can focus on the limited commitment friction, we will assume that the buyer makes a take-it-or-leave-it offer in bilateral daytime meetings between a buyer and seller. Therefore, there will be no bargaining inefficiencies.

Credit Equilibrium

Here, the daytime take-it-or-leave-it offers of buyers consist of credit contracts with a loan made during the day and repayment at night. Again, confine attention to stationary equilibria, and let l denote the loan quantity offered by the buyer to the seller in the day. Then, we have

$$v = \beta \max_{l} \left[u(l) - l + v \right] \tag{9}$$

subject to the incentive constraint

$$l \leq v.$$

and

$$l \geq 0.$$

As in the previous section, default triggers autarky for a debtor. Letting $\psi(v)$ denote the right-hand side of (9), we get

$$\psi_c(v) = \beta u(v), \text{ for } 0 \leq v \leq q^*,$$

and

$$\psi_c(v) = \beta \left[u(q^*) - q^* + v \right], \text{ for } v \geq q^*.$$

An equilibrium is then a solution to $v = \psi(v)$. If $q^{**} < q^*$ then there are two equilibria. In the first, $v = 0$, and in the second $v > 0$, which are both solutions to $v = \beta u(v)$. Note that v is also the consumption of the buyer during the day, and of the seller during the night, with $v < q^*$. In this case, the incentive constraint for the buyer binds in either equilibrium. If $q^{**} \geq q^*$ then the $v = 0$ equilibrium still exists, and the equilibrium with $v > 0$ has

$$v = \frac{\beta}{1-\beta} \left[u(q^*) - q^* \right],$$

in which case consumption is q^* for any agent consuming at any date and the incentive constraint does not bind.

In Figure 3, panel (a) shows the case where the incentive constraint binds in equilibrium, and panel (b) the case where the incentive constraint does not bind. The nonmonetary equilibrium, by virtue of the bargaining solution we use, just picks out the efficient stationary allocation that gives all of the surplus to the buyer.

Monetary Equilibrium

Assume that money is uniformly distributed across buyers at the beginning of the first day. Subsequently the government makes lump-sum transfers in the night market, so that the money stock grows at rate μ. Confine attention to stationary monetary equilibria, and consider only cases where $\mu \geq \beta$, as otherwise a monetary equilibrium does not exist. Let m denote the real money balances acquired by a buyer in the night, and γ the real value of a lump-sum transfer received from the government during the night. Suppose that the buyer receives the lump-sum transfer before acquiring money balances during the night, and let v denote the continuation utility for the buyer after receiving the lump sum transfer. The government sets the gross money growth rate at

μ. As in Andolfatto (2007), we treat the government symmetrically with the private sector, in the there is limited commitment with respect to tax liabilities as well as private liabilities. Continue to assume complete memory, and assume that if a buyer defaults on a loan contract or on tax liabilities that he or she never gets to trade again (no one will even take his money in exchange). Then, analogous to what we wrote above,

$$v = \max_{l,m} \left\{ -m + \beta \left[u\left(\frac{1}{\mu} m + l \right) - l + \gamma + v \right] \right\} \tag{10}$$

subject to

$$l \leq \gamma + v.$$

$$l \geq 0$$

Here, note that we need to be careful about the lump-sum transfer the buyer receives. Should the buyer default on his or her debt, he or she will also not receive the transfer, or will default on the current and all future tax liabilities if $\gamma < 0$. In equilibrium, we have

$$\gamma = m\left(1 - \frac{1}{\mu}\right)$$

For $\mu > \beta$, the right-hand side of equation (10) is given by

$$\psi_m(v) = -v_1 + \beta u(v_1) + v, \text{ for } \max\left[0, m^*\left(1 - \frac{1}{\mu}\right)\right] \leq v \leq v_1,$$

$$\psi_m(v) = \beta u(v), \text{ for } v_1 \leq v \leq q^*,$$

$$\psi_m(v) = \beta\left[u(q^*) - q^* + v\right], \text{ for } v \geq q^*.$$

Here, m^* solves

$$u'\left(\frac{m^*}{\mu}\right) = \frac{\mu}{\beta},$$

and v_1 satisfies

$$u'\left[v_1\right] = \frac{\mu}{\beta},$$

For $\mu = \beta$, we have

$$\psi_m(v) = \beta[u(q^*) - q^*] - m(1-\beta) + \beta v, \text{ for } v \geq m^* \left(1 - \frac{1}{\beta}\right),$$

where

$$m \in [q^* - \min(q^*, v), \beta q^*]$$

Proposition 1 *If $q^{**} \geq q^*$, then a monetary equilibrium does not exist if $\mu \neq \beta$.*

Proposition 2 *If $q^{**} \geq q^*$ and $\mu = \beta$, then a continuum of monetary equilibria exists with $v \in [\frac{\beta[u(q^*)-q^*]}{1-\beta} - \beta q^*, \frac{\beta[u(q^*)-q^*]}{1-\beta})$. All of these equilibria yield expected utility for the buyer of $\frac{u(q^*)-q^*}{1-\beta}$.*

Proposition 3 *If $q^{**} < q^*$, then a monetary equilibrium does not exist if $\mu \neq \beta u'(q^{**})$.*

Proposition 4 *If $q^{**} < q^*$ and $\mu = \beta u'(q^{**})$, then a continuum of monetary equilibria exists with $v \in [0, q^{**})$. All of these equilibria yield expected utility for the buyer of $\frac{u(q^{**})-q^{**}}{1-\beta}$.*

In Figure 4, we show the case where $q^{**} \geq q^*$ and $\mu > \beta$. Here, note that $\psi_m(v)$ is linear for the values of v where the buyer will choose to hold money. In this case there are two equilibria, at points A and B in the figure, which are identical to the credit equilibria in the previous section. Here, the $v > 0$ equilibrium has the feature that q^* is always traded in the day market, and this is supported as an equilibrium with credit, and without valued money. In the case where $q^{**} \geq q^*$ a Friedman rule is feasible, and for $\mu = \beta$, buyers are indifferent in equilibrium to purchasing goods in the day market with money or credit, and the incentive constraint does not bind in equilibrium. At the Friedman rule there exists a continuum of equilibria, which all yield the same allocation, i.e. consumption of the buyer in the day is q^*. These equilibria are indexed by the quantity of real balances held.

Now, when $q^{**} < q^*$, first suppose that $\mu > \beta u'(q^{**})$,as in Figure 5. Then, no monetary equilibrium exists and we just recover the two credit equilibria as in the previous section. Here, money is not held because the rate of return on money is sufficiently low that no agent will hold money to relax their incentive constraint, in spite of the fact that the incentive constraint binds. In Figure 6, we have $q^{**} < q^*$ and $\mu < \beta u'(q^{**})$. Here, in spite of the fact that the return on money is high, and that holding money could potentially relax the buyer's incentive constraint, the taxes required to support a low money growth rate imply that the buyer would want to default on his or her private liabilities and tax liabilities, and therefore an equilibrium does not exist with valued money. Again, we just recover the two credit equilibria at A and B. The case where money can be valued in equilibrium is when $\mu = \beta u'(q^{**})$, as in Figure 7. Here, the two credit equilibria, at A and B still exist, but now there is a continuum of equilibria with valued money, between A and B in the figure. Note that all of these monetary equilibria yield the identical allocation that is achieved in the efficient credit equilibrium with $v = q^{**}$. As v increases across monetary equilibria, the real quantity of money declines and the quantity of lending increases.

IMPERFECT MEMORY (I)

As we have seen, with perfect memory there is essentially no social role for money, and it will only be held under special circumstances. As is well-known, particularly given the work of Kocherlakota, we need some imperfections in record-keeping in order for money to be useful and to help it survive as a valued object. We will start by assuming that, during the day, there is no memory in some bilateral meetings, and perfect memory in other meetings. In any day, a given buyer or seller has probability ρ of being in a meeting where there is no memory, in that each agent in such a meeting has no knowledge of his or her trading partner's history, and nothing about the meeting will be recorded. With probability $1-\rho$ a given buyer or seller will be in a meeting with perfect memory during the day. Here, $0 < \rho \leq 1$. We will also assume that there is perfect monitoring of activity during the night. Here, the case where $\rho = 1$ is the standard one in monetary models with random matching such as Lagos-Wright (2005).

Credit Equilibrium

Just as in the previous section, first consider stationary equilibria where money is not valued, so that all exchanges in the day market involve credit. Here, v is determined by

$$v = \beta \left\{ (1-\rho) \max_{l} \left[u(l) - l \right] + v \right\}$$

subject to the incentive constraint

$$l \leq v,$$

and

$$l \geq 0,$$

or

$$v = \phi_c(v),$$

with

$$\phi_c(v) = \beta \left[(1-\rho) u(v) + \rho v \right], \text{ for } 0 \leq v \leq q^*$$

and

$$\phi_c(v) = \beta \left\{ (1-\rho)[u(q^*) - q^*] + v \right\}, \text{ for } v \geq q^*.$$

Let q^{***} denote the solution to

$$\frac{\beta(1-\rho)}{1-\rho\beta} u(q^{***}) = q^{***}$$

Proposition 5 *If $q^{***} < q^*$ then there is are two credit equilibria, one where $v = 0$, and one where the incentive constraint binds, $l < q^*$ and $v = q^{***}$.*

Proposition 6 *If $q^{***} \geq q^*$ then there are two credit equilibria, one where $v = 0$ and one where the incentive constraint does not bind, $l = q^*$, and*

$$v = \frac{\beta(1-\rho)[u(q^*) - q^*]}{1-\beta}.$$

Monetary Equilibrium

Here, we have

$$v = \max_{m,l} \left(-m + \beta \left\{ \rho u \left(\frac{m}{\mu} \right) + (1-\rho) \left[u \left(\frac{m}{\mu} + l \right) - l \right] + \gamma + v \right\} \right) \tag{11}$$

subject to

$$l \leq \gamma + v,$$

$$l \geq 0.$$

In equilibrium, tax liabilities are

$$\gamma = m \left(1 - \frac{1}{\mu} \right) \tag{12}$$

Here, let $\phi_m(v)$ denote the right-hand side of (11), where (12) is satisfied, so that a stationary monetary equilibrium involves looking for the fixed point of $\phi_m(v)$.

Binding Incentive Constraint.—

In this case, we have

$$\phi_m(v) = -m(v) + \beta \left\{ \rho u \left(\frac{m(v)}{\mu} \right) + (1-\rho) u \left[m(v) + v \right] + \rho m(v) \left(1 - \frac{1}{\mu} \right) + \rho v \right\}$$

where $m(v)$ is the quantity of real balances given continuation value v, and $m(v)$ solves the first-order condition

$$-1 + \frac{\beta}{\mu} \left\{ \rho u' \left(\frac{m(v)}{\mu} \right) + (1-\rho) u' \left[m(v) + v \right] \right\} = 0$$

If $\mu \leq 1$, then the incentive constraint binds for

$$0 \leq v \leq \hat{v},$$

where $\hat{v}$ solves

$$\rho u'\left(\frac{q^* - \hat{v}}{\mu}\right) = \frac{\mu}{\beta} - 1 + \rho,$$

while if $\mu \geq 1$, then the incentive constraint binds for

$$m(0)\left(1 - \frac{1}{\mu}\right) \leq v \leq \hat{v}.$$

Non-binding Incentive Constraint.—

If the incentive constraint does not bind, then

$$\phi_m(v) = -m^* + \beta\left\{\rho\left[u\left(\frac{m^*}{\mu}\right) - \frac{m^*}{\mu}\right] + (1-\rho)\left[u\left(q^*\right) - q^*\right] + m^* + v\right\},$$

for $v \geq \hat{v}$, where m^* solves the first-order condition

$$-1 + \frac{\beta}{\mu}\left[\rho u'\left(\frac{m^*}{\mu}\right) + 1 - \rho\right] = 0.$$

Equilibrium.—

It is straightforward to show that $\phi_m(v)$ is a concave and continuously differentiable function for $v \geq 0$ if $\mu \geq 1$, and $v \geq m(0)\left(1 - \frac{1}{\mu}\right)$ if $\mu \leq 1$. Further, when $\mu \geq 1$, then $\phi_m(0) > 0$. When $q^{**} \geq q^*$ and $\beta \leq \mu \leq 1$, then

$$\phi_m\left(m(0)\left(1 - \frac{1}{\mu}\right)\right) \geq m(0)\left(1 - \frac{1}{\mu}\right).$$

But when $q^{**} < q^*$, then

$$\phi_m\left(m(0)\left(1 - \frac{1}{\mu}\right)\right) \geq m(0)\left(1 - \frac{1}{\mu}\right) \text{ for } \beta u'(q^{**}) \leq \mu \leq 1$$

and

$$\phi_m\left(m(0)\left(1 - \frac{1}{\mu}\right)\right) < m(0)\left(1 - \frac{1}{\mu}\right) \text{ for } \beta \leq \mu < \beta u'(q^{**}).$$

Finally, we can show that, when $v \geq 0$ if $\mu \geq 1$, and $v \geq m(0)\left(1 - \frac{1}{\mu}\right)$ if $\mu \leq 1$, we have

$$\phi_m'(v) \leq \frac{(1-\rho)\mu + \rho\beta}{(1-\rho)\mu + \rho} < 1.$$

This gives us the following propositions.

Proposition 7 *If $q^{**} \geq q^*$ then a unique stationary monetary equilibrium exists for $\mu \geq \beta$.*

Proposition 8 *If $q^{**} < q^*$ then a unique stationary monetary equilibrium exists for $\mu \geq \beta u'(q^{**})$.*

If $q^{***} \geq q^*$, which guarantees that $q^{**} > q^*$, as $q^{***} < q^{**}$then the incentive constraint does not bind for all $\mu \geq \beta$. In this case, the buyer consumes q^* in all monitored trades where credit is used during the day, and consumes $\frac{m}{\mu}$ in nonmonitored trades, where $\frac{m}{\mu} \leq q^*$ and $\frac{m}{\mu}$ is decreasing in μ. Therefore, the welfare of the buyer is decreasing in μ, while the seller receives zero utility in each period for all μ.

If $q^{**} \geq q^* > q^{***}$, then the incentive constraint binds for $\mu > \hat{\mu}$, where

$$u\left(\frac{m(\hat{\mu})}{\hat{\mu}}\right) - \frac{m(\hat{\mu})}{\hat{\mu}} = \frac{-\beta(1-\rho)u(q^*) + q^*(1-\rho\beta)}{\rho\beta}$$

with $m(\hat{\mu})$ the solution to

$$-1 + \frac{\beta}{\hat{\mu}}\left[\rho u'\left(\frac{m(\hat{\mu})}{\hat{\mu}}\right) + 1 - \rho\right] = 0.$$

The incentive constraint does not bind for $\beta \leq \mu \leq \hat{\mu}$.

Finally, if $q^{**} < q^*$, then the incentive constraint will always bind in a stationary monetary equilibrium.

Proposition 9 *If $q^{**} \geq q^*$, then $\mu = \beta$ is optimal, and this implies that $l = 0$, the incentive constraint does not bind, and the buyer consumes q^* in all trades during the day.*

Proposition 10 *If $q^{**} < q^*$, then $\mu = \beta u'(q^{**})$ is optimal, and this implies that $l = 0$, the incentive constraint binds, and the buyer consumes q^{**} in all trades during the day.*

Here, we have essentially generalized the results of Andolfatto (1997) to the case where credit is permitted in some types of bilateral trades. If the discount factor is sufficiently small, then the Friedman rule is not feasible and the incentive constraint binds at the optimum. A perhaps undesirable feature of this setup is that an optimal monetary policy drives credit out of the economy. Here, the only inefficiency in monetary exchange is due to the fact that buyers in general hold too little real money balances in equilibrium, and this inefficiency can be corrected in the usual way, with the caveat that too much deflation can cause agents to default on their tax liabilities. Ultimately, at the optimum money is equivalent to memory, in that an appropriate monetary policy achieves the same allocation that could be achieved by a social planner with perfect record keeping.

IMPERFECT MEMORY (II)

In the previous section, given the limited commitment friction, optimal monetary policy will yield an equilibrium allocation where credit is not used. Credit seems to be more robust than this in practice, so we would like to study frictions that potentially imply that money and credit coexist, even when monetary policy is conducted efficiently.

In this section, we will continue to assume that some bilateral trades during the day are monitored, while others are not. Here, however, we will assume that buyers always have the option of meeting a seller in a non-monitored trade, though with probability ρ a non-monitored trading opportunity is the only option. Note that in equilibrium a buyer who has the choice of being monitored will always weakly prefer that option. However, with out-of-equilibrium defection, a buyer will choose not to be monitored in the future.

Further, we want to assume that agents can trade money for goods during the night in a non-monitored fashion. An agent with an outstanding debt at the beginning of the night can choose to make himself or herself known and repay the debt, or can make himself or herself known when receiving a transfer or paying a tax. However, it is always an option to remain anonymous in the night. Note the difference here from our first assumptions about imperfect memory, where we assumed that all transactions were monitored in the night.

Now, at the beginning of the night, if a buyer chooses to default on his or her private liabilities and tax liabilities, he or she faces a continuation value

$$\frac{1}{1-\beta}\omega(\mu)=\frac{1}{1-\beta}\max_{m}\left[-m+\beta u\left(\frac{m}{\mu}\right)\right].$$

That is, a buyer who defects can exchange money for goods indefinitely in anonymous trades, and does not receive transfers from the government or pay taxes.

Now, the continuation value v is the solution to

$$v=\max_{m,l}\left(-m+\beta\left\{\rho u\left(\frac{m}{\mu}\right)+(1-\rho)\left[u\left(\frac{m}{\mu}+l\right)-l\right]+\gamma+v\right\}\right) \tag{13}$$

subject to

$$l\leq\gamma+v-\frac{1}{1-\beta}\omega(\mu). \tag{14}$$

$$l\geq 0.$$

In equilibrium, we have

$$\gamma=m\left(1-\frac{1}{\mu}\right).$$

Note that, when money is not valued in equilibrium, we have $\omega(\mu)=0$, and the credit equilibria are identical to what we obtained in the previous section.

Binding Incentive Constraint

Now, suppose that the incentive constraint binds with $v\geq-\gamma+\frac{1}{1-\beta}\omega(\mu)$. Then, the problem on the right-hand side of (13) is

$$\max_{m}\left(-m+\beta\left\{\begin{array}{c}\rho\left[u\left(\frac{m}{\mu}\right)\right]+(1-\rho)u\left(\frac{m}{\mu}+\gamma+v-\frac{1}{1-\beta}\omega(\mu)\right)\\+\rho(\gamma+v)+\left(\frac{1-\rho}{1-\beta}\right)\omega(\mu)\end{array}\right\}\right),$$

and the first-order condition for an optimum is

$$-1 + \frac{\beta}{\mu}\left\{\rho u'\left(\frac{m}{\mu}\right) + (1-\rho)u'\left(\frac{m}{\mu} + \gamma + v - \frac{1}{1-\beta}\omega(\mu)\right)\right\} = 0. \qquad (15)$$

Now, letting $m(v)$ denote the value of m that solves this first order condition, the fixed point problem we want to solve is $v = \eta(v)$, where

$$\eta(v) = -m(v) + \beta\left\{\begin{array}{c} \rho u\left(\frac{m(v)}{\mu}\right) + (1-\rho)u\left(m(v) + v - \frac{1}{1-\beta}\omega(\mu)\right) \\ +\rho m(v)\left(1 - \frac{1}{\mu}\right) + \rho v + \left(\frac{1-\rho}{1-\beta}\right)\omega(\mu) \end{array}\right\} \qquad (16)$$

for $-m^*\left(1 - \frac{1}{\mu}\right) + \frac{1}{1-\beta}\omega(\mu) \leq v \leq \tilde{v}$, where m^* solves

$$-1 + \frac{\beta}{\mu}u'\left(\frac{m^*}{\mu}\right) = 0,$$

and $\tilde{v}$ solves

$$\rho u'\left(\frac{q^* - \tilde{v} + \frac{\omega(\mu)}{1-\beta}}{\mu}\right) = \frac{\mu}{\beta} - 1 + \rho$$

Non-Binding Incentive Constraint

When $v \geq \tilde{v}$, the incentive constraint does not bind and $l = q^* - \frac{m}{\mu}$, so the problem on the right-hand side of (13) is

$$\max_m \left(-m + \beta\left\{\rho u\left(\frac{m}{\mu}\right) + (1-\rho)\left[u\left(q^*\right) - q^* + \frac{m}{\mu}\right] + \gamma + v\right\}\right),$$

and the first-order condition for an optimum is

$$-1 + \frac{\beta}{\mu}\left[\rho u'\left(\frac{m}{\mu}\right) + 1 - \rho\right] = 0 \qquad (17)$$

Letting m^{**} denote the quantity of real balances that solves (17), which gives, in equilibrium,

$$\eta(v) = -m^{**}(1-\beta) + \beta\left\{\rho\left[u\left(\frac{m^{**}}{\mu}\right) - \frac{m^{**}}{\mu}\right] + (1-\rho)\left[u\left(q^*\right) - q^*\right] + v\right\} \qquad (18)$$

Equilibrium with Valued Money

Proposition 11 *An equilibrium with valued money exists if and only if* $\mu \geq 1$, *and it is unique. If* $\mu = 1$, *then there is no credit in equilibrium, and with* $\mu > 1$ *money and credit coexist.*

Proof. It is straighforward to show that $\eta(v)$ is concave and continuously differentiable for $v \geq -m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu)$. Now,

$$\eta\left[-m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu)\right]=\frac{1}{1-\beta}\omega(\mu),$$

and so

$$\eta\left[-m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu)\right]<-m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu), \text{ if } \mu<1,$$

$$\eta\left[-m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu)\right]=-m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu), \text{ if } \mu=1,$$

$$\eta\left[-m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu)\right]>-m^*\left(1-\frac{1}{\mu}\right)+\frac{1}{1-\beta}\omega(\mu), \text{ if } \mu>1.$$

Finally, we have

$$\eta'(v)\leq\frac{(1-\rho)\mu+\beta\rho}{(1-\rho)\mu+\rho}<1.$$

■

Figures 8 and 9 show the function $\eta(v)$ for the cases $\mu < 1$ and $\mu > 1$, respectively. For the case $\mu < 1$, there is no equilibrium with valued money, as the continuation value could never be high enough in equilibrium that buyers would not default on their private liabilities and tax liabilities. However, when $\mu > 1$, there is a unique equilibrium where valued money and credit coexist.

Now, what is the optimal money growth factor μ in this setup? We can show that there are conditions under which strictly positive money growth is optimal. That is, define $W(\mu)$ to be the utility of a buyer in equilibrium as a function of the money growth factor, and note that a seller's utility is always zero in equilbrium. Then, we have

$$W(\mu) = m + v$$

Now, since the incentive constraint binds in equilibrium for μ close to 1, use (15) and (16) to obtain

$$W'(1) = \tilde{m}\left[1 - \rho - \frac{u'(\tilde{m})}{\tilde{m}u''(\tilde{m})}\right],$$

where $\tilde{m}$ solves

$$u'(\tilde{m}) = \frac{1}{\beta}.$$

That is, a small increase in the money growth rate above zero is Pareto-improving if the inverse of the coefficient of relative risk aversion of $u(\cdot)$, evaluated at the level of consumption received by the buyer in the day when $\mu = 1$, is less than $1 - \rho$. Here, higher money growth can increase welfare because this acts to reduce the value of defaulting on private liabilities and tax liabilities, much as in Aiyagari and Williamson (2000) or Antinolfi, Azariadis, and Bullard (2007). To get this result here requires that money not be too important in exchange in the economy, that is we need ρ to be sufficiently small. Note that is necessary for a welfare improvement from positive money growth that the coefficient of relative risk aversion be greater than 1.

THEFT

One aspect of monetary exchange is that, due to anonymity, theft is easier in some respects than it is with exchange using credit. It seems useful to think about a framework where limited commitment makes credit arrangements difficult, and theft makes monetary exchange difficult. However, the fact that theft makes monetary exchange difficult may lessen the limited commitment friction in the credit market, as this will make default less enticing.

Start from the imperfect memory setup above, and assume that theft can take place only in a non-monitored bilateral meeting. Suppose that a seller first learns whether he or she will be in a monitored or non-monitored meeting during the day, and then before the meeting takes place he or she must commit to pay a fixed cost τ to acquire a technology (a "gun") that allows the theft of all of the buyer's money balances. Assume that $0 < \tau < q^*$.

Let α denote the fraction of unmonitored trades where there is theft, and note that we want to determine α endogenously. In equilibrium,

$$\begin{aligned} \text{If } \alpha = 1, \text{ then } \tau &\leq \frac{m}{\mu}, \\ \text{If } \alpha \in (0,1), \text{ then } \tau &= \frac{m}{\mu}, \\ \text{If } \alpha = 0, \text{ then } \tau &\geq \frac{m}{\mu}. \end{aligned} \tag{19}$$

Now, start with the value of default for an agent who chooses to repudiate his or her debt. This is given by $\frac{\omega(\mu,\alpha)}{1-\beta}$, where

$$\omega(\mu, \alpha) = \max_m \left[-m + \beta(1-\alpha)u\left(\frac{m}{\mu}\right)\right] \tag{20}$$

Here, note that defecting in general becomes a worse deal than in the previous section, as the defector chooses to trade in only unmonitored meetings, and therefore must face thieves more frequently. Note that α in the above expression is endogenous and is the α in the equilibrium arrangement, not the α we would obtain if everyone defected.

Next, our equilibrium problem is now

$$v = \max_{m,l} \left(-m + \beta\left\{\rho(1-\alpha)u\left(\frac{m}{\mu}\right) + (1-\rho)\left[u\left(\frac{m}{\mu} + l\right) - l\right] + \gamma + v\right\}\right) \tag{21}$$

subject to

$$l \leq \gamma + v - \frac{1}{1-\beta}\omega(\mu, \alpha). \tag{22}$$

$$l \geq 0$$

There will be another condition determining α, which will state that m is chosen optimally given α (see (19) above). It's convenient to do this separately for each case.

Monetary Equilibria With No Credit

With $l = 0$, we must have $v = -\gamma + \frac{1}{1-\beta}\omega(\mu,\alpha)$ in equilibrium. The right-hand side of (21) is

$$\max_{m,l}\left\{-m+\beta\left[(1-\rho\alpha)u\left(\frac{m}{\mu}\right)+v+\gamma\right]\right\},$$

and a first-order condition for an optimum is

$$-1+\frac{\beta}{\mu}(1-\rho\alpha)u'\left(\frac{m}{\mu}\right)=0.$$

Then, in equilibrium we have

$$\text{If } \mu \leq (1-\rho)\beta u'(\tau),\ \text{ then } \alpha = 1.$$

$$\text{If } (1-\rho)\beta u'(\tau) \leq \mu \leq \beta u'(\tau),\ \text{ then } \alpha = \frac{\beta u'(\tau)-\mu}{\rho\beta u'(\tau)} \equiv \alpha_1.$$

$$\text{If } \mu \geq \beta u'(\tau), \text{ then } \alpha = 0.$$

From the defection problem (20), we get

$$w(\mu,1)=0$$

$$\omega(\mu,\alpha_1)=\max_m\left\{-m+\left[\frac{\mu-(1-\rho)\beta u'(\tau)}{\rho u'(\tau)}\right]u\left(\frac{m}{\mu}\right)\right\},\ \text{ when } 0<\alpha<1.$$

$$\omega(\mu,0)=\max_m\left[-m+\beta u\left(\frac{m}{\mu}\right)\right]$$

Binding Incentive Constraint

First, consider the case where the incentive constraint binds. Then $l = \gamma + v - \frac{1}{1-\beta}\omega(\mu,\alpha)$. If $\alpha = 1$, then we have $\omega(\mu,1) = 0$. Letting $\eta(v)$ denote the right-hand side of (21), we have

$$\eta(v)=-m+\beta\left[(1-\rho)u\left(m+v\right)+\rho m\left(1-\frac{1}{\mu}\right)+\rho v\right],$$

where m satisfies the first-order condition

$$-1+\frac{\beta}{\mu}(1-\rho)u'(m+v)=0.$$

and for $\alpha=1$ to be optimal we require $m \geq \mu\tau$ or

$$-1+\frac{\beta}{\mu}(1-\rho)u'(\mu\tau+v)\geq 0$$

Note that we must have

$$v \geq -m^*\left(1-\frac{1}{\mu}\right)$$

where m^* is the solution to

$$-1+\frac{\beta}{\mu}(1-\rho)u'\left(\frac{m^*}{\mu}\right)=0.$$

Next, if $0 \leq \alpha \leq 1$, then we have

$$\eta(v)=-\mu\tau+\beta\left\{\begin{array}{c}\rho(1-\alpha)u\left(\tau\right)+(1-\rho)\left[u\left(\mu\tau+v-\frac{1}{1-\beta}\omega(\mu,\alpha)\right)+\frac{1}{1-\beta}\omega(\mu,\alpha)\right]\\ +\rho m\left(1-\frac{1}{\mu}\right)+\rho v\end{array}\right\},$$

where the following must hold, to determine α :

$$-1+\frac{\beta}{\mu}\left[\rho(1-\alpha)u'\left(\tau\right)+(1-\rho)u'\left(\mu\tau+v-\frac{1}{1-\beta}\omega(\mu,\alpha)\right)\right]=0. \tag{23}$$

Finally, if $\alpha=0$, then $\eta(v)$ is identical to what it was in the previous section when we did not allow for theft. However, to have no theft in equilibrium here requires that

$$\mu \geq \beta\left\{\rho u'\left(\tau\right)+(1-\rho)u'\left[\mu\tau+v-\frac{1}{1-\beta}\omega(\mu,0)\right]\right\}.$$

Money and Credit With a Non-Binding Incentive Constraint

Now, consider the case where the incentive constraint does not bind. Then, $l=q^*-\frac{m}{\mu}$. First, if $\alpha=1$, then this requires that $\mu=\beta(1-\rho)$, and we have

$$\eta(v)=m\left[\beta-\frac{1}{(1-\rho)}\right]+\beta\left\{(1-\rho)[u(q^*)-q^*]+v\right\}$$

and we require

$$m \geq \mu\tau$$

for $\alpha = 1$ to be optimal.

Next, suppose that $0 < \alpha < 1$. Then

$$\eta(v) = -\mu\tau + [\mu - \beta(1-\rho)]\frac{u(\tau)}{u'(\tau)} + \beta(1-\rho)\left[u(q^*) - q^* + \tau\right] + \beta v$$

and

$$\alpha = \frac{-\mu + \beta\left[\rho u'(\tau) + 1 - \rho\right]}{\beta\rho u'(\tau)} \equiv \alpha_2.$$

For this case we require

$$\beta(1-\rho) \leq \mu \leq \beta[\rho u'(\tau) + 1 - \rho],$$

and

$$v \geq \frac{1}{1-\beta}\omega(\mu, \alpha_2) + q^* - \tau$$

Finally, suppose that $\alpha = 0$. Then, $\eta(v)$ is determined as in the previous section, but for $\alpha = 0$ to be optimal requires

$$\mu \geq \beta[\rho u'(\tau) + 1 - \rho],$$

As well, we must have

$$v \geq \frac{1}{1-\beta}\omega(\mu, 0) + q^* - \frac{\mu}{\beta}$$

Now, with theft, note there are three effects of inflation on the incentive constraint. First, the value of defecting, $\frac{1}{1-\beta}\omega(\mu, \alpha)$, is decreasing in μ, so more inflation tends to relax the incentive constraint. Second, more inflation reduces real money balances in equilibrium, which tends to reduce theft (i.e. α falls), which increases the value of defecting, tightening the incentive constraint. Third, since real money balances decrease in equilibrium with an increase in μ, the buyer has lower real money balances in a monitored trade, which tightens the incentive constraint.

Proposition 12 *α is decreasing in v, and strictly decreasing in v for values of v such that the incentive constraint binds.*

Proposition 13 *If $\mu \geq \beta u'(\tau)$ then $\alpha = 0$.*

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Figure 1 : Efficiency When $q^{**} > q^{*}$

(a)

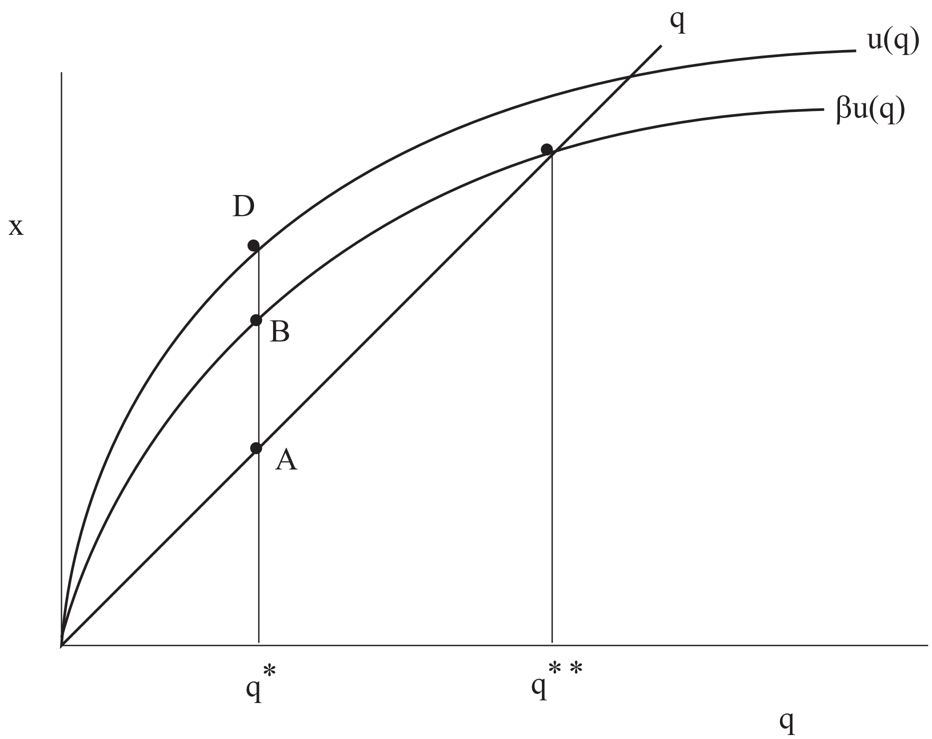


Figure 2: Efficiency When $q^{**} < q^{*}$

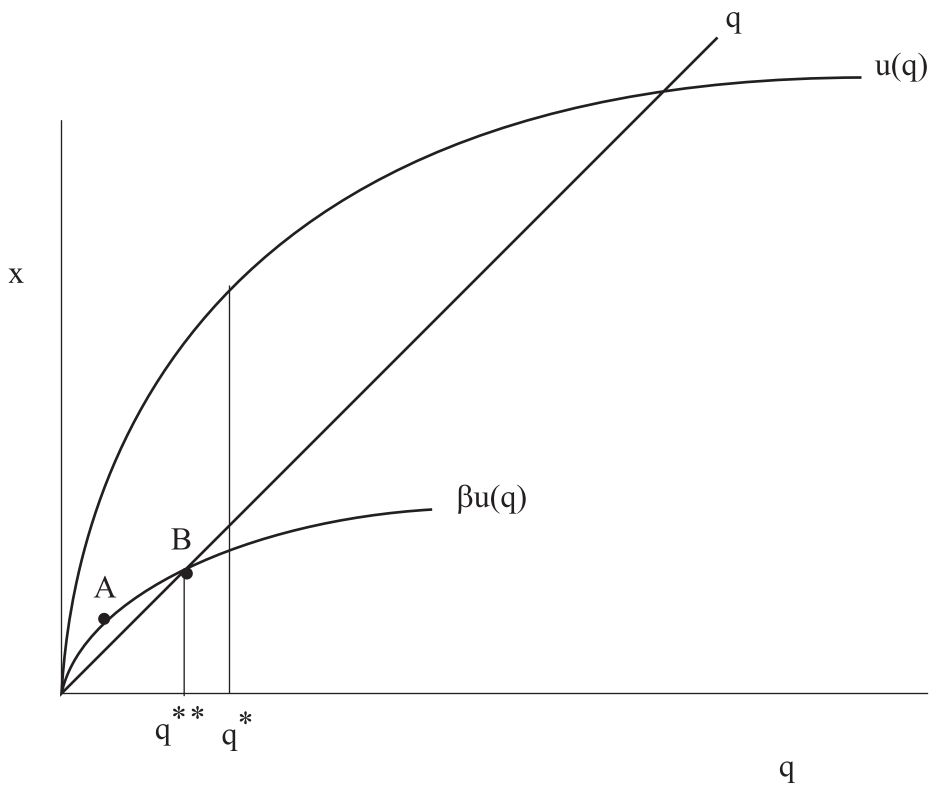
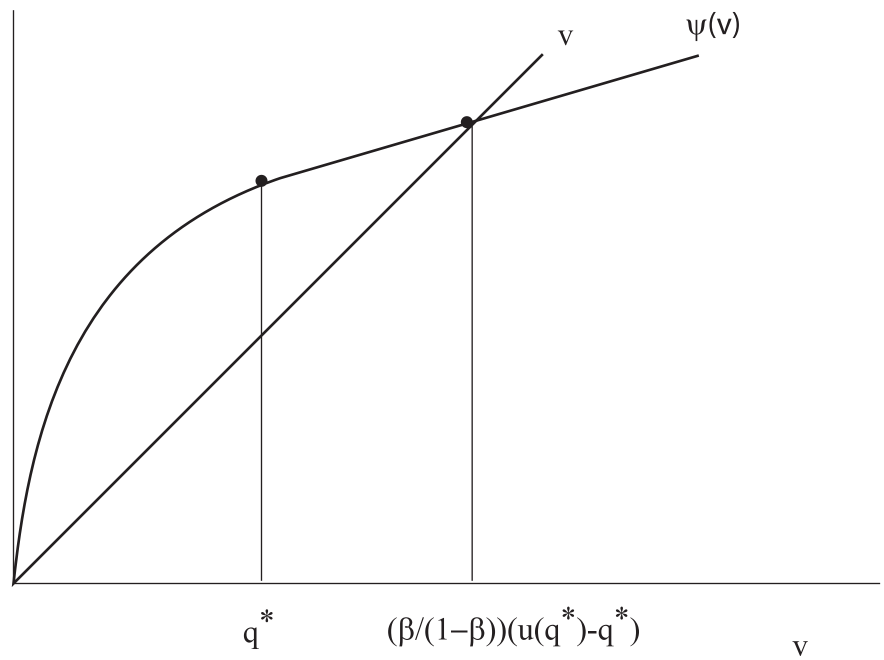


Figure 3 : Equilibrium with Credit

a) $q^{**} > q^{*}$



b) $q^{**} < q^{*}$

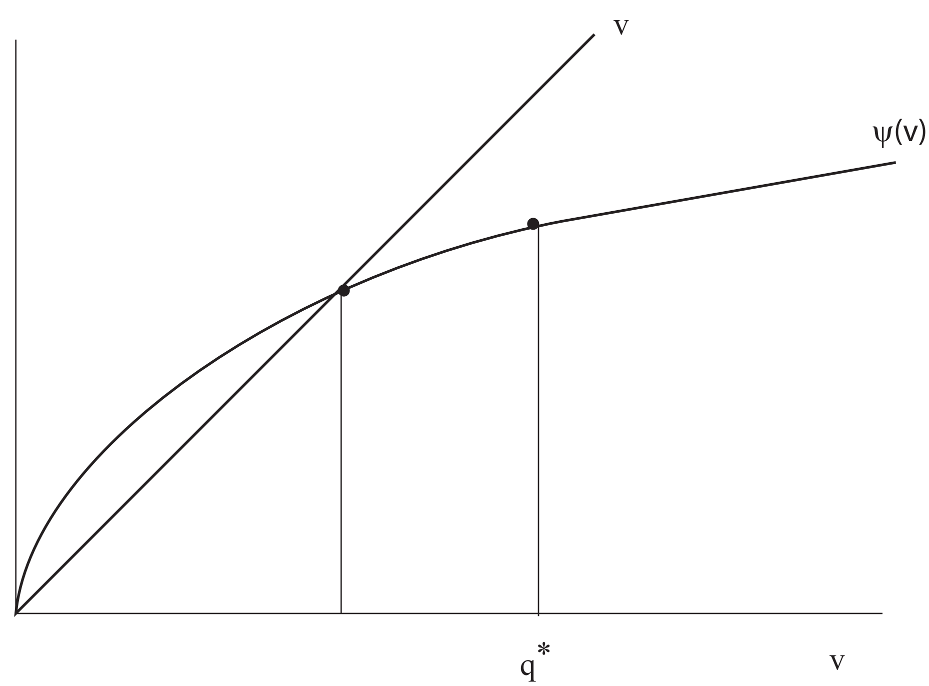


Figure 4: No Monetary Equilibrium with $q^* < q^{**}$

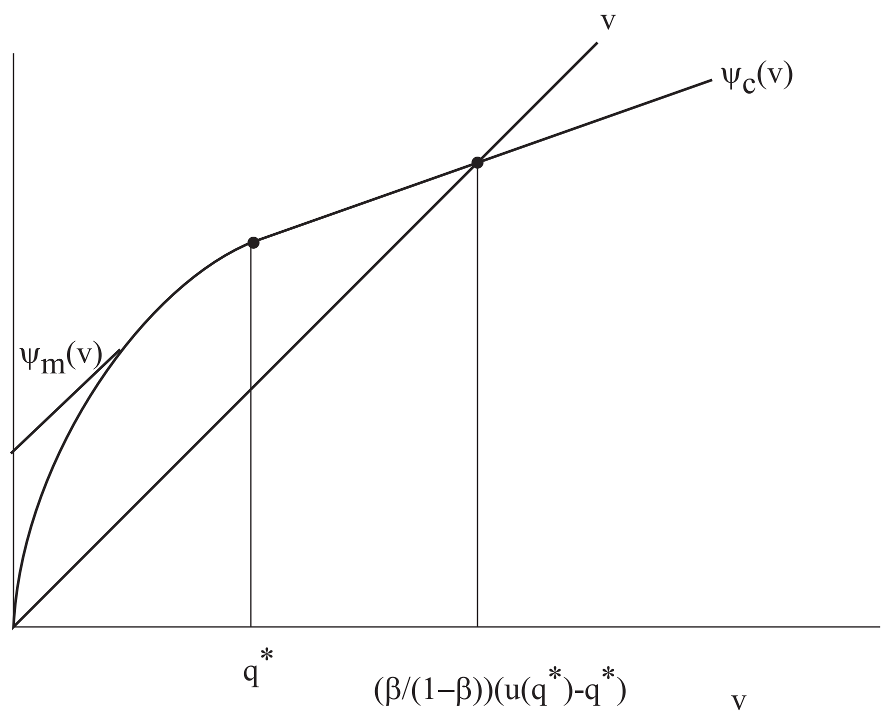


Figure 5 : No Monetary Equilibrium with $q^{*} > q^{**}$ and $\mu > \beta u'(q^{**})$

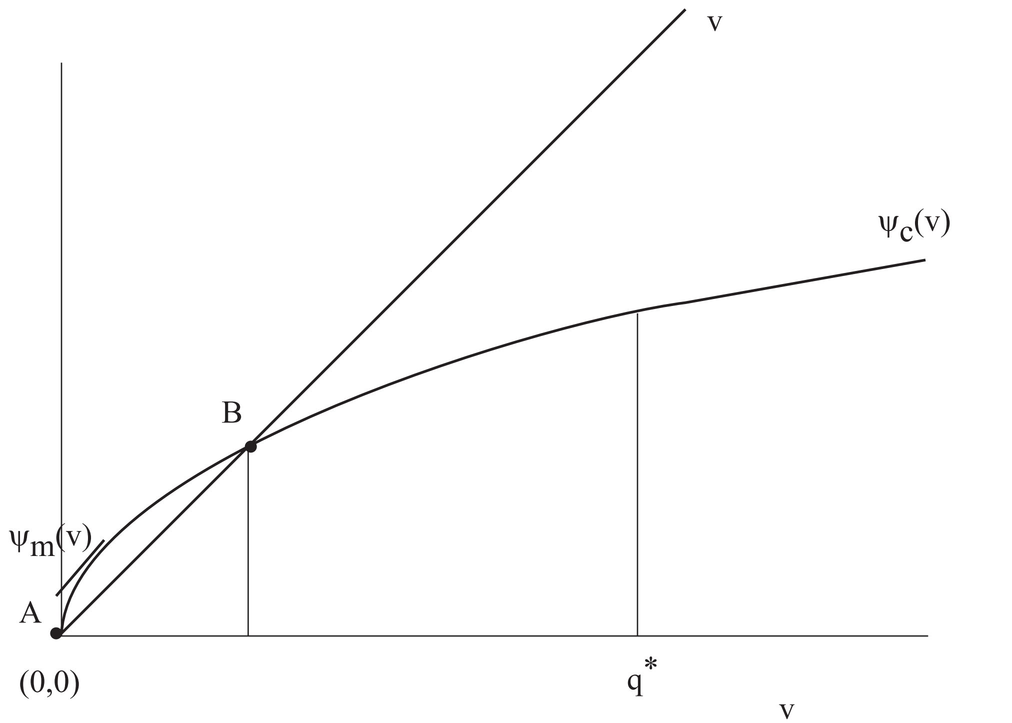


Figure 6 : No Monetary Equilibrium with $q^* > q^{**}$ and $\mu < \beta u'(q^{**})$

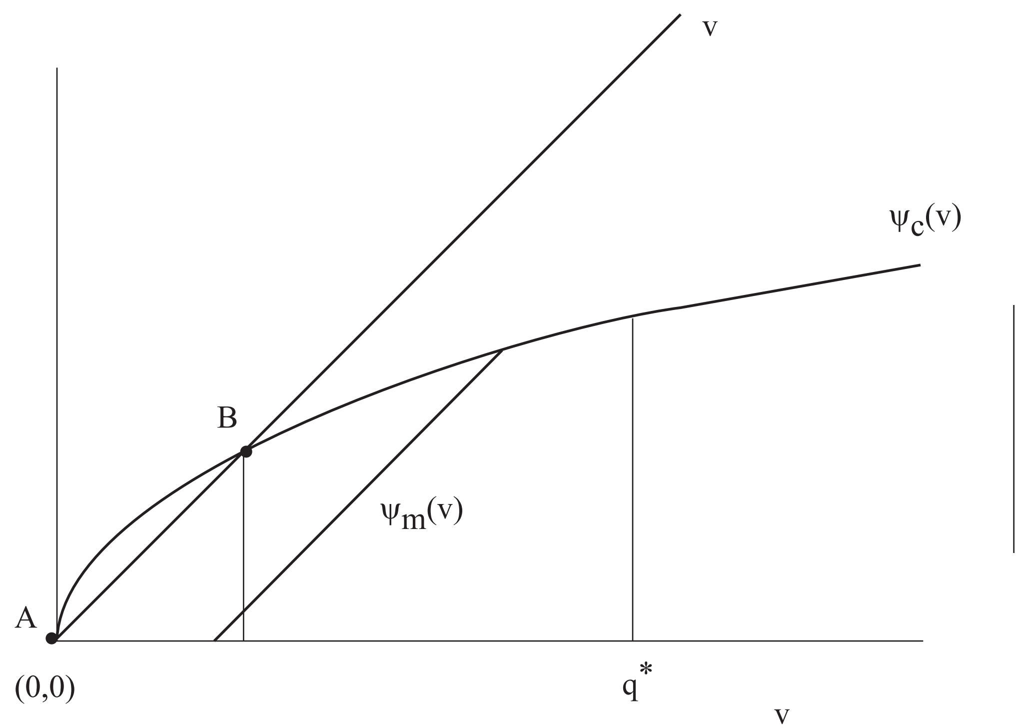


Figure 7: Monetary Equilibrium with $q^{*} > q^{**}$ and $\mu = \beta u'(q^{**})$

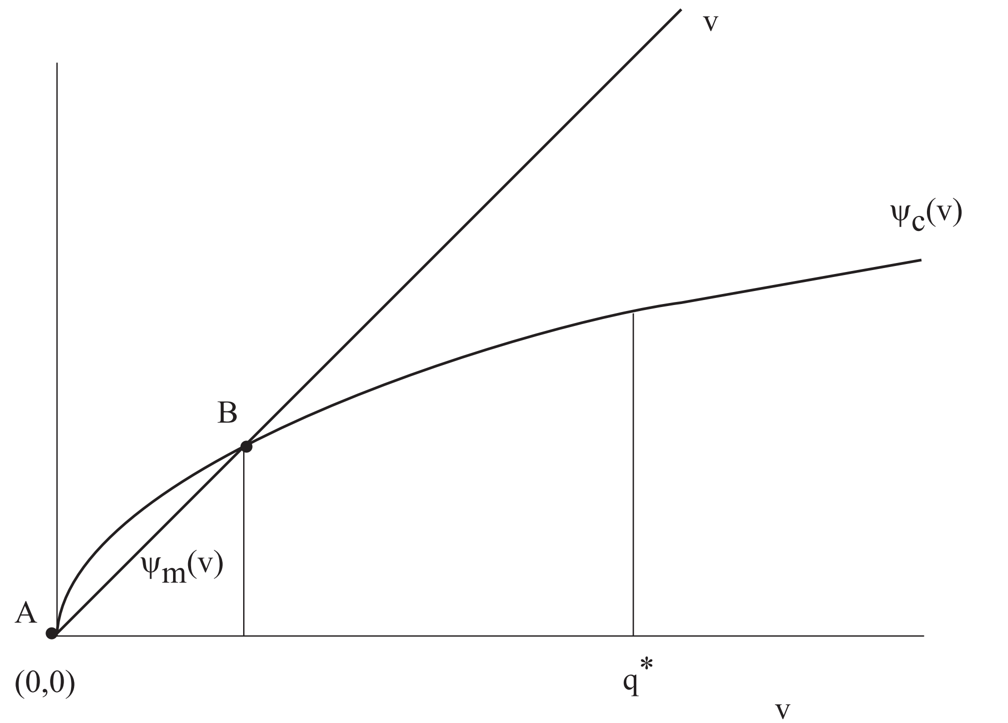


Figure 8 : $\mu < 1$

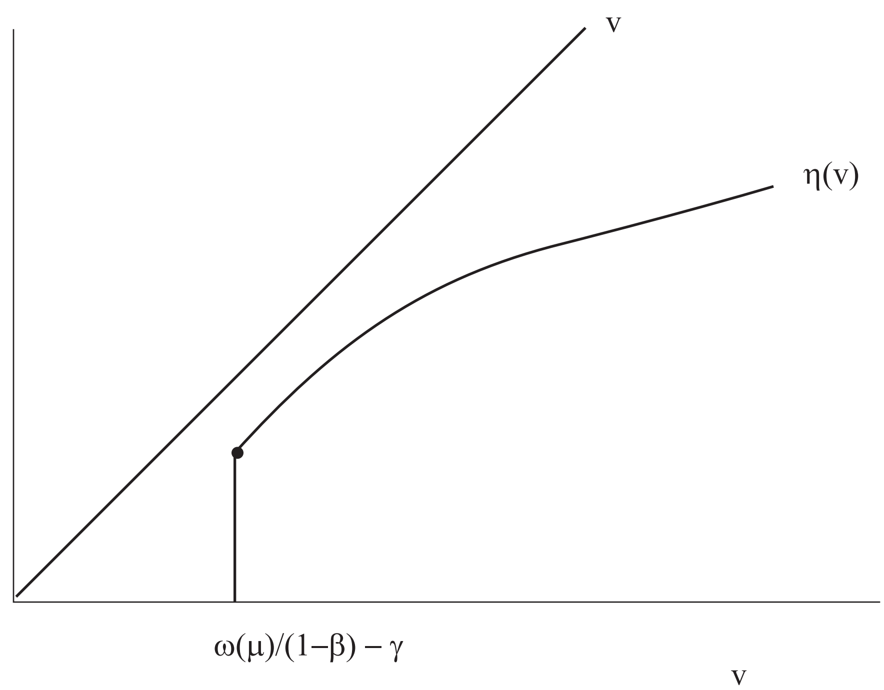


Figure 9 : $\mu > 1$

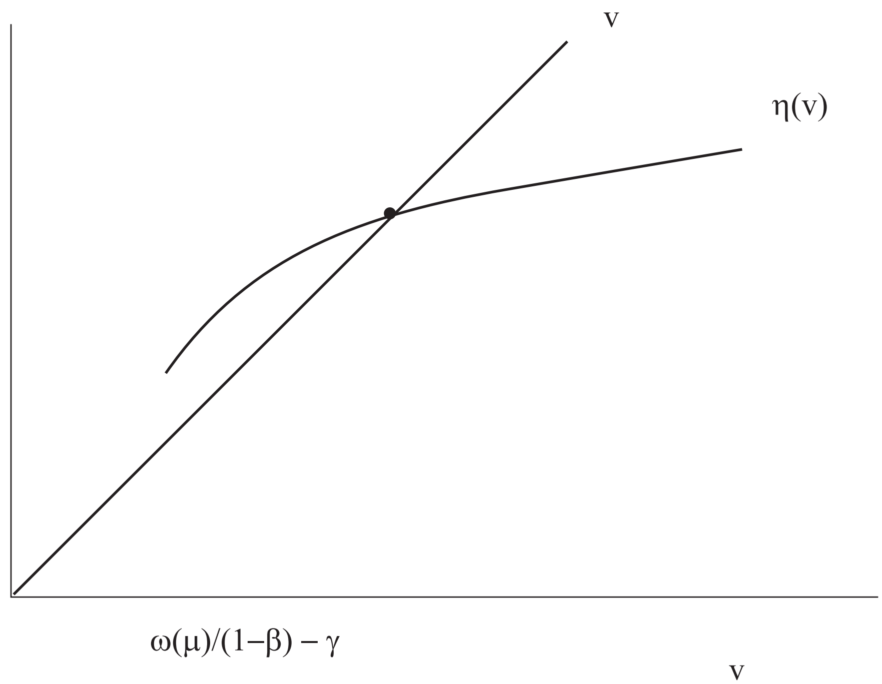


Figure 10: Theft with $\mu < \beta u'(\tau)$