Public Information and Monetary Policy

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Abstract

We study the nature of monetary policy in a model where uncertainty can lead to a discrepancy between economic agents' beliefs and true fundamentals. Monetary policy transmits information about fundamentals. The public nature of this information can help agents to coordinate their decisions. This comes at a cost, however, since monetary policy may lead the private sector to coordinate on the wrong fundamentals and it may result in inflation. We discuss conditions under which monetary policy will be unambiguously welfare-improving. We formalize the notion that monetary policy is equivalent to information revelation by the central bank, and offer an information-based (as opposed to the standard liquidity-based) argument for why higher nominal rate hikes occur less frequently than lower ones.

1 Introduction

We investigate the role of monetary policy in a model where uncertainty can lead to a discrepancy between economic agents' beliefs and true fundamentals, and where money is essential in facilitating trade. We take the view that when the central bank adjusts monetary policy instruments, such as interest rate, it sends an important *message* to the private sector. This allows the central bank to coordinate expectations in the economy. The central bank, thus, plays a role that is somewhat related to the role of sunspots in earlier models.[1]

We build an intertemporal model in which money is essential as a medium of exchange. Agents have heterogeneous expectations about economic fundamentals and they suffer a cost if their expectations are wrong, resulting in individual actions that are different from the optimal actions under the true fundamentals. Following Morris and Shin (MS, 2002), we allow for consumption decisions to be subject to a "beauty contest:" additional consumption is more beneficial if other agents consume more, and less beneficial otherwise. We assume that trade takes place in centralized markets that are subject to frictions that make the use of money necessary for transactions. We then use the resulting model to study the nature of monetary policy in various environments in which the central bank has some private information that it can communicate to the private sector through monetary policy.

While insightful, existing game-theoretic models that are based on MS leave out much institutional detail and are, thus, of little use in guiding policy. Notably, these models abstract from the frictions that give a role to money as a medium of exchange. In addition, these models frequently abstract from prices. Both of these are important limitations. In particular, the relevance of the miscoordination scenario in these models has been criticized on the basis that prices often act as a coordination device and should reveal all the relevant information to economic participants.[2] On the other hand, policy models used by practitioners usually underplay the role for coordinating expectations. In this paper, we make an attempt to bridge the gap between these two literatures.

Our model has three novel features. First, we imbed the MS static structure in a model with an infinite horizon. Second, our model is subject to frictions that

[1] For the analysis of sunspot equilibria and of how sunspots can be used as a coordination device, see Cass and Shell (1983). An important difference is that monetary policy in our model is endogenous.

[2] See, for example, the discussion in Atkeson (2000) and Angeletos and Werning (2006).

make money essential. Thus, it is meaningful to study questions related to monetary policy. Third, our environment involves an explicit role for prices, since trade takes place in Walrasian markets.[3] Markets are incomplete due to spatial separation. This friction, which together with anonymity provides a role for money as a medium of exchange, also prevents prices from being fully revealing. This allows us to study the interplay between monetary and informational frictions from primitives in the presence of market clearing prices.

A central assumption in our paper is that monetary policy is the only way for the central bank to credibly convey its information to the private sector. In other words, we assume that a simple "announcement" through which the central bank could potentially reveal its information will be treated as "cheap-talk" and, thus, will be ignored.[4] The central bank's information is not necessarily more precise relative to the private sector's. Nevertheless, the public nature of this information can help agents to coordinate their decisions. This comes at a cost, however, since monetary policy may actually lead the private sector to coordinate on the wrong fundamentals. Moreover, our analysis introduces an additional cost from revealing information, since monetary policy creates a distortion and may result in higher inflation in the economy. Thus, monetary policy should be applied only if the central bank is fairly certain that the private sector's beliefs are away from true fundamentals.

Finally, we provide a formal criterion for the equivalence (in a topological sense) between monetary policy and information revelation by the central bank. This criterion requires that the set of feasible monetary policies is homeomorphic to the set describing the central bank's information. Thus, whenever our result applies, at the cost of imposing a distortion associated with some inflation, the central bank can use monetary policy in order to perfectly communicate its information to the private

[3] See Amador and Weill (2007) for a model where prices imperfectly aggregate information when economic fundamentals are uncertain.

[4] Although we do not explicitly model the reasons for this here, our assumption is analogous to ignoring "cheap-talk" equilibria in a communication game between the central bank and the private agents.

sector. We demonstrate this equivalence under different assumptions regarding the information of the central bank. As an application of this principle, our model offers an information-based (as opposed to the standard liquidity-based) argument for why higher nominal rate hikes occur less frequently than lower ones.

The paper proceeds as follows. The next two sections describe the environment and deal with the full information benchmark. Section 4 studies the private information case. Section 5 discusses welfare as a function of the quality of information in the economy. Section 6 studies the general case in which signal precision itself is a random variable.

2 The Environment

Our basic environment combines the monetary model of Lagos-Wright (2005), who in turn build on the model of Kiyotaki-Wright (1989), with the information-based model of Morris-Shin (2002). Other choices of a monetary model are also possible. Our choice was for the most tractable model in which money is essential.

Agents, Preferences, and technology. Time $t = 0, 1, ...,$ is infinite and discrete. The economy consists of a $[0, 1]$ continuum of infinite-lived agents. Agents are uncertain about their type. There are two possible types, denoted by n and e, and each agent becomes a type n in each period with probability $\frac{1}{2}$. Agents discount the future at rate $\beta \in (0, 1)$. There is a central bank (CB) which has the ability to print money.

Trading stages. Agents populate a large number of locations or "islands." There is a continuum of agents in each island. Agents produce and consume two non-storable goods, q and z, that are traded sequentially in two stages within each period. There is no discounting between stages. The nature of trading is somewhat different in the two stages. In the first, agents are divided across islands, with each island containing an equal measure of buyers and sellers. In this stage, good q is traded within each island in isolation. Agents are anonymous so they trade via the use of money. In the second stage, good z is traded in a general market that involves all islands together. The second stage is modelled as a frictionless Walrasian market.

Starting with the second stage, utility (disutility) of consumption (production) of the good z is assumed to be linear and is denoted by $(-)z$. Given the linearity assumption, agents will use the second stage to even their money holdings. This dramatically improves tractability. Preferences and technology in the first stage are slightly more involved. Let $r \in [0, 1)$ be a constant. Let q denote the amount of good q consumed (always by an e agent) and g denote the amount of the good produced (always by an n agent) in a given island during the first stage. We use $\mathbf{q}$ to denote the profile of such consumptions across all islands. Finally, define $L_i = \int_0^1 (q_j - q_i)^2 dj$, and $\overline{L} = \int_0^1 L_j dj$.

The disutility of production on island i is defined as:

$$U^n(g_i) = -g_i^+. \tag{1}$$

The utility of consumption on island i is defined as:

$$U^e(\mathbf{q}) = q_i^+ - (1-r)(q_i - \theta)^2 - r(L_i - \overline{L}), \tag{2}$$

where $\theta \in \mathbb{R}$ denotes the underlying fundamental and where x^+ equals x, if $x \geq 0$, and equals $-x$, if $x \leq 0$.[5] We assume that θ is a random variable which is *iid* across time. Thus, each period consumers face a preference shock whose effect is captured by θ. The first term of the utility function is standard in that the utility increases in the amount consumed. The second term, weighted by $1 - r \leq 1$, reflects the costs of being away from the fundamental value. If $r > 0$, an additional term affects the utility. This term captures complementarities in the consumption decisions.[6] The term L_i reflects the fact that additional consumption is more beneficial only if other agents consume more, and less beneficial otherwise. It is a reduced way of capturing

[5] We introduce this notation in order to ensure that production and consumption are positive, as it may be that in equilibrium, $q_i < 0$, for instance if $\theta < 0$.

[6] Imposing the complementarity on consumers rather than on producers turns out to be technically simpler but nothing important should depend on this assumption.

the financial sector involvement in the economy.[7] The kind of complementarity we consider only affects individual utility. The aggregate utility depends only on the aggregate amount consumed and on the overall discrepancy between the consumption and true fundamentals. The term $\overline{L}$ normalizes individual utility and renders the complementarity irrelevant from the aggregate perspective.[8,9]

3 θ is Common Knowledge

As a benchmark, and in order to become familiar with our main notation, we first consider the case where there is no private information; i.e., it is common knowledge that the true value of θ is observed by everyone. In this case, the first-best involves all agents coordinating to produce and consume the same level of output. Given the friction associated with the use of money in our model, the resulting allocation can be decentralized as an equilibrium outcome only if the CB follows a monetary policy that is consistent with the Friedman rule.

3.1 The Planner's Problem

The planner maximizes the normalized welfare function

$$W(\mathbf{q}, \theta) = \frac{1}{1-r}[\int_0^1 [U^n(g_i) + U^e(\mathbf{q})]di$$

[7] For example, it can be viewed as a reduced-form representation of the behavior of fund managers as described by Rajan (2005). In this case, as managers' performance is evaluated vis-a-vis their peers, they would tend to engage in correlated investments. By doing so, they would never underperform their peers. Thus, their investment decisions are not necessarily based on fundamentals but rather on what other investors in the economy do.

[8] It also insures that the equilibrium is unique. For an example of an environment where the complementarity is also present at the social level, see Angeletos and Pavan (2004).

[9] We assume that autarky (shutting down the economy) is not feasible. Alternatively, we could assume that $U(\mathbf{q}) = AI_{\{q>0 \text{ or } q<0\}} + U^e(\mathbf{q})$, where $U^e(\mathbf{q})$ is defined as above, I is an indicator function, and $A > g(0)$. This modification would not change any of the calculations that follow. In addition, it would ensure that utility from consuming amount q is always higher than the cost from producing this amount.

$$= \frac{1}{1-r}[\int_0^1 -g_i^+ di - (1-r)\int_0^1 (q_i-\theta)^2 di - r\int_0^1 (L_i - \overline{L})di + \int_0^1 q_i^+ di$$

$$= -\int_0^1 (q_i - \theta)^2 di. \quad (3)$$

Proposition 1 *The efficient allocation has agents in each island producing(consuming) $q^+ = g^+$, so that $q = g = \theta$ in the first stage of each period. The amount of the general good, x, produced in the second stage is indeterminate.*

3.2 The Equilibrium Problem

As agents are not readily identifiable during the first stage, some type of record-keeping is needed for transactions to take place. In this section we decentralize the solution to the planner's problem by means of monetary trade. Throughout we assume that money is provided exclusively by the CB. Let M be the per capita supply of money. A monetary transfer, T, takes place at the *end* of the second stage of each period.[10] The net stock of money grows as $M_{+1} = \gamma M$. We will concentrate our analysis on stationary monetary equilibria where $\phi M = \phi_{-1} M_{-1}$, where ϕ is the real price of money in terms of the numeraire good (x). Hence, γ also equals ϕ_{-1}/ϕ. We use $W(m)$ to denote the discounted lifetime utility of an agent when he enters the second stage holding m units of money, and $V(\widetilde{m})$ to denote the expected discounted lifetime utility from entering the first stage with money holdings $\widetilde{m}$. The function $W(m)$ is then defined as

$$\begin{aligned} W(m) &= \max_{z, m_{+1}} \{-z + \beta E V(\widetilde{m}_{+1} + T, \theta)\} \\ & \quad s.t. \quad \phi \widetilde{m}_{+1} + z = \phi m, \end{aligned} \quad (4)$$

[10] This is for simplicity only. If the monetary transfers were made at the start of the second stage, agents would update their prior on θ_{+1} *before* choosing their money holdings. This would induce an additional effect on the price of money. Although potentially interesting, this effect would further complicate the analysis.

where z denotes the net production of the general good. Given θ, the discounted lifetime utility of agents when they enter the first stage with $\widetilde{m}$ units of money is

$$V(\widetilde{m}) = \frac{1}{2}\left[U^n(g) + W(m^n)\right] + \frac{1}{2}\left[U^e(\mathbf{q}) + W(m^e)\right]. \tag{5}$$

where g, m^n, q and m^e are chosen optimally (we postpone the description of the agents' problem until later). It is easy to demonstrate the following

Proposition 2 *When the true value of θ is publicly observable, the Friedman rule decentralizes the efficient allocation of the previous section.*

4 Private and Public Signals about θ

We now consider the more realistic scenario in which the realization of θ is not observable. Instead, when the true state is θ, the CB receives a signal $y = \theta + \eta$, while agents in each island receive an island-specific signal $x_i = \theta + \varepsilon_i$. Following MS, we assume that the noise terms η and ε_i are normally distributed with zero mean and variances σ_η^2 and σ_ε^2, respectively. Moreover, $E(\varepsilon_i \varepsilon_j) = 0$ holds for $i \neq j$.

In the case where the central bank can *credibly* communicate its signal (say, by making a public announcement) it is easy to demonstrate that equilibrium exhibits a "classical dichotomy" and that the optimal monetary policy is to follow the Friedman rule in each period. We will not consider this case in what follows.[11] Instead, throughout the paper we impose the following.

Assumption 1 *The central bank can credibly communicate its signal only through monetary policy.*

Recall that both the private signals and the signal through monetary policy (the monetary transfer) arrive at the *end* of each second stage. Next, we contrast the

[11] Even if a public announcement by the CB is possible, some private agents might not be entirely sure about whether every other agent has heard the announcement, or if every other agent is sure that every other agent is sure... and so on. Our analysis remains valid in the presence of such higher order uncertainty.

resulting welfare in the following two special cases. In the first case, the CB does not communicate its signal. Instead, it follows the Friedman rule in each period. In this case, buyers and sellers in each island trade based solely on their island-specific signal. In the second case, we consider the other extreme whereby the CB communicates its signal, thus violating the Friedman rule, in at least some periods. In the latter case, private agents make decisions based on both, their island-specific signals as well as the public signal contained in monetary policy.

Note that the CB faces a trade-off. Communicating its information *might* lead to better coordination of actions by the private sector. However, this *always* comes at the cost of a distortion associated with inflation. We assume that monetary policy takes the following form

$$M_{+1} = f(M, y). \tag{6}$$

A special case, which we study for simplicity, is the case where $M_{+1} = \beta M$, when the CB follows the Friedman rule, while

$$M_{+1} = [\beta + H(y)]\, M, \tag{7}$$

when the CB reveals its signal. We require that $H : \mathbb{R} \rightarrow \mathbb{R}_+$ is a homeomorphism[12] so that, having observed the money stock, private agents can infer the bank's signal, y.

We remark that the restriction to a homeomorphism is not simply a mathematical issue. The existence of a function with this property allows us to think of the set describing the potential information held by the CB and the set of potential monetary policy decisions as "topologically equivalent," or homeomorphic. To put it differently, whenever such a function exists, monetary policy is, in a mathematically well defined sense, the *translation* of the information held by the central bank. If, on the contrary, such a function cannot be constructed for a given economic environment, this would be

[12]A function H is a homeomorphism if it has the following properties: (1) H is a bijection; (2) H is continuous; (3) the inverse function H^{-1} is continuous. It is well known that $\mathbb{R}$and $\mathbb{R}_+$ are homeomorphic; i.e., there exists a homeomorphism between these two sets.

an indication that monetary policy and the information held by the central bank are "distinct." Later in the paper we will use this equivalence to demonstrate that the CB can use monetary policy in order to communicate *both* its signal about fundamentals and its confidence in that signal to the private sector.

Let $\mathcal{I}^i$ denote the information available to each agent in a generic island, i, (note that there is no asymmetric information *within* an island). Hence, $\mathcal{I}^i = (y, x^i)$. Thus, the agents' choices of q^b, q^s, m^b, m^s in each island i must be measurable with respect to $\mathcal{I}^i$. Under this restriction, $W(m)$ and $V(m)$ are defined as before. Let $T(y)$ denote the monetary transfer prescribed by H. We then have that

$$\begin{aligned} W(m) &= \max_{z, m_{+1}} \{-z + \beta E V(\widetilde{m}_{+1} + T(y), \theta)\} \\ & \quad s.t. \quad \phi \widetilde{m}_{+1} + z = \phi m. \end{aligned} \tag{8}$$

As before, the discounted lifetime utility of agents when they enter the first stage with $\widetilde{m}$ units of cash is

$$V(\widetilde{m}) = \frac{1}{2}\left[U^n(g) + W(m^n)\right] + \frac{1}{2}\left[U^e(\mathbf{q}) + W(m^e)\right]. \tag{9}$$

where g, m^n, q and m^e are chosen optimally as follows. The problem of a type n agent is

$$\max_g [U^n(g) + EW(\widetilde{m} + pg^+)]. \tag{10}$$

The FOC for the producer of type n with respect to g (when his constraint is not-binding) gives[13]

$$1 - pE\left[\phi_{+1}\right] = 0. \tag{11}$$

Thus, the price on island i in this case is given by

$$p_i = \frac{1}{E_i\left[\phi_{+1}\right]}. \tag{12}$$

[13] Here, we adopt the usual convention that, for any variable v, $\partial(v^+)/\partial v = 1$ if $v \geq 0$, and $\partial(v^+)/\partial v = -1$ if $v < 0$. Also, note that "more" production when $g < 0$, actually means that g becomes "more" negative, so that g^+ increases.

The type e consumer solves

$$\max_{q}[U^{e}(\mathbf{q})+EW(\widetilde{m}-pq^{+})]$$
$$s.t.\ \widetilde{m}-pq^{+}\geq 0. \tag{13}$$

The FOC for the consumer of type e with respect to q gives

$$U^{e\prime}(\mathbf{q})-E[\phi_{+1}]p-\lambda^{e}p=0. \tag{14}$$

Market clearing conditions in the first stage require

$$q=g. \tag{15}$$

Combining the FOC for q and g and using the equilibrium conditions, the equilibrium consumption on island i is then given by:

$$q_{i}=(1-r)\,E_{i}\,(\theta)+rE_{i}\,(\overline{q})+p\lambda_{i}^{e}. \tag{16}$$

Under the Friedman rule the budget constraint is slack, so that $\lambda_{i}^{e}=0$, and we have

$$q_{i}=(1-r)\,E_{i}\,(\theta)+rE_{i}\,(\overline{q})\,. \tag{17}$$

One question is under what conditions the CB will find it optimal to reveal information about the state θ. As mentioned before, such revelation is costly, since a distortion is introduced in any period where the Friedman rule is violated. Thus, the CB must optimally choose whether the benefits from the additional information exceed the costs. The answer to this question depends in an interesting way on the relative informativeness of the public versus the private signals. Next, assume that

$$M_{+1}=[\beta+H(y)]\,M. \tag{18}$$

The value function at the beginning of the first stage can be written as

$$V(\widetilde{m})=\frac{1}{2}[U^{n}(g)+EW(\widetilde{m}+pg^{+})]+\frac{1}{2}\left[U^{e}(\mathbf{q})+EW(\widetilde{m}-pq^{+})\right]. \tag{19}$$

The first order conditions with respect to $\widetilde{m}$ give

$$\begin{aligned} V'(\widetilde{m}) &= \frac{1}{2}\left[\frac{\partial g}{\partial \tilde{m}} + EW'(\widetilde{m}+pg^{+})\left(1+p\frac{\partial g^{+}}{\partial \tilde{m}}\right)\right] \\ &+\frac{1}{2}\left[U^{e\prime}(\mathbf{q})\frac{\partial q}{\partial \tilde{m}} + EW'(\widetilde{m}-pq^{+})\left(1-p\frac{\partial q^{+}}{\partial \tilde{m}}\right)\right]. \end{aligned} \tag{20}$$

Since the budget constraint of each buyer is binding, we have $\frac{\partial q^{+}}{\partial \tilde{m}} = \frac{1}{p}$, while $\frac{\partial g^{+}}{\partial \tilde{m}} = 0$. Hence, the FOC simplifies to

$$V'(\widetilde{m}) = \frac{1}{2}\left[EW'(\widetilde{m}+pg^{+})\right] + \frac{1}{2}\left[U^{e\prime}(\mathbf{q})\frac{1}{p}\right]. \tag{21}$$

Using the envelope condition, $W_m(m) = \phi$, we have that $EW'(\widetilde{m}-pq^{+}) = E\left[\phi_{+1}\right]$. The first order condition gives $\beta EV_m(\widetilde{m}_{+1},\theta) = \phi$. Thus, we have

$$\frac{\phi}{\beta} = \frac{1}{2}E\left[\phi_{+1}\right] \; + \frac{1}{2}\left[U^{e\prime}(\mathbf{q})\frac{1}{p}\right]. \tag{22}$$

Substituting the expression for the prices in the first stage market on island i, we obtain

$$p = \frac{1}{E_i[\phi_{+1}]}. \tag{23}$$

Thus, the previous expression becomes

$$\begin{aligned} \frac{\phi}{\beta} &= \frac{1}{2}E\left[\phi_{+1}\right] \; + \frac{1}{2}E_i[\phi_{+1}]U^{e\prime}(\mathbf{q}), \text{ or,} \quad &(24)\\ \frac{2\phi}{\beta E\left[\phi_{+1}\right]} &= 1 + U^{e\prime}(\mathbf{q}). &(25) \end{aligned}$$

This implies that $U^{e\prime}(\mathbf{q}) \geq 1$ in the optimum with strict inequality for $\frac{\phi}{\beta E_i\left[\phi_{+1}\right]} > 1$. The equilibrium quantity q_i must be unique since there exists a $q_i > 0$ such that $U^{e\prime}(\mathbf{q}) = 1$ and $U^{e\prime}(\mathbf{q})$ is strictly decreasing for all q_i.

Substituting for $U^{e\prime}(\mathbf{q})$ yields

$$\begin{aligned} \frac{2\phi}{\beta E\left[\phi_{+1}\right]} &= 1 + 2\left(1-r\right)E_i\left(\theta\right) + 2rE_i\left(\overline{q}\right) + 1 - 2q_i, \text{ or,} \quad &(26)\\ \frac{\phi}{\beta E\left[\phi_{+1}\right]} - 1 &= -q_i + \left(1-r\right)E_i\left(\theta\right) + rE_i\left(\overline{q}\right). &(27) \end{aligned}$$

Therefore,

$$\begin{aligned} q_i &= (1-r) E_i(\theta) + r E_i(\overline{q}) + \left[1 - \frac{\phi}{\beta E_i[\phi_{+1}]}\right] \\ &= (1-r) E_i(\theta) + r E_i(\overline{q}) - \frac{\gamma(y) - \beta}{\beta}. \end{aligned} \tag{28}$$

At the Friedman rule, $1_y = 0$, so that $\phi_{+1}/\phi = 1/\beta$, and $\lambda_i^n = \lambda_i^e = 0$. Note that away from the Friedman rule, $\frac{\phi}{\beta E_i[\phi_{+1}]} > 1$, hence, $q \neq \theta$, even if the true value of θ is publicly revealed. In equilibrium, ϕ_{+1} can be inferred by every agent, so that $\frac{\phi}{E_i[\phi_{+1}]} = \frac{\phi}{\phi_{+1}} = \gamma$. In general, the expression for the equilibrium quantity in the first stage market on island i is given by

$$q_i = (1-r) E_i(\theta | x_i, \gamma(y)) + r E_i(\overline{q} | x_i, \gamma(y)) - \frac{\gamma(y) - \beta}{\beta}. \tag{29}$$

Note that monetary policy, represented by $\gamma(y)$, has real effects whenever the CB deviates from the Friedman rule. In addition, the equilibrium quantity on island i depends not only on the fundamental state θ, but also on the expectation about the average quantity produced by the other islands. Since all other islands are known to be facing the same decision, the quantity produced on island i is also affected by higher-order expectations; i.e., by the expectation about the average expectation of other islands, the expectation about the average expectation of that average expectation..., and so on.[14]

[14] The effects of higher-order expectations were emphasized in a different context by Phelps (1983). His insight concerned Lucas's (1972) "island economy" model. This literature studies the question of how uncertainty about the money supply affects prices on individual islands and under what conditions can monetary shocks have real effects. Monetary policy is shown to have real effects due to spatial separation (Lucas, 1972). Those effects can be persistent when the effect of higher-order expectations is taken into account and when agents have limited capacity to pay attention to all the information in their environment (see Woodford (2001)). In our model, the change in monetary policy is known and it occurs in response to the real (preference) shock. Monetary policy has real effects, as a change in monetary policy stance reveals information on economic fundamentals, and money is essential as a medium of exchange.

4.1 Perfectly Informative CB signal ($\eta = 0$ and $y = \theta$)

Here we consider the benchmark case in which the CB receives a *perfectly informative* signal. In that case, the CB can communicate the exact value for θ to all agents through monetary policy by simply adopting the rule $\gamma(\theta) = \beta + H(\theta)$, where $H(\bullet) : \mathbb{R} \to \mathbb{R}_+$ is a homeomorphism. Thus, $E(\theta | x_i, \gamma(\theta)) = \theta$, for all i. Then all agents produce the same q as given by

$$\begin{aligned} q &= (1-r)\theta + rE(\overline{q}|x_i, \gamma(\theta)) - \frac{\gamma(\theta) - \beta}{\beta} \\ &= (1-r)\theta + rq - \frac{H(\theta)}{\beta}. \end{aligned} \tag{30}$$

Assuming, for example, that $H(y = \theta) = h(\theta)\beta(1-r)$, the above gives

$$q = \theta - h(\theta). \tag{31}$$

At the other extreme, when the CB does not reveal its information, the equilibrium quantity on island i in the first stage market is given by

$$\begin{aligned} q_i &= (1-r)E(\theta|x_i) + rE(\overline{q}|x_i) \\ &= (1-r)E(\theta|x_i) + rq_i \\ &= E(\theta|x_i) \\ &= x_i. \end{aligned} \tag{32}$$

Thus, if the CB does not signal its information, period-t expected welfare in state θ is given by

$$\begin{aligned} E\left[W_t\left(q_t^i, \theta\right) \mid \theta\right] &= -\int_0^1 (q_t^i - \theta)^2 di \\ &= -\int_0^1 \varepsilon_i^2 di \\ &= -\sigma_\varepsilon^2. \end{aligned} \tag{33}$$

In contrast, when the CB signals through monetary policy (still assuming $\gamma(\theta) = \beta + H(\theta)$), period-$t$ expected welfare is given by

$$\begin{aligned} & -\int_0^1 h^2(\theta)\, di \\ = & -h^2(\theta). \end{aligned} \tag{34}$$

Hence, we have the following result.

Proposition 3 *The central bank deviates from the Friedman rule in period t if and only if $h^2(\theta) < \sigma_\varepsilon^2$.*

All proofs are relegated to the Appendix. Note that the degree to which deviation from the Friedman rule distorts production away from the efficient output level depends on the fundamental θ. Since $h(\theta)$ is strictly monotone in θ, the inequality above is more likely to hold the larger is the magnitude of the realization of θ and the noisier is the information held by the private sector. As the left hand side is always positive, if the precision of private information is nearly perfect, it is almost never optimal for the CB to signal its information.

5 Welfare and Signal Precision

We begin by deriving an expression for the quantity produced on island i as a function of precisions of the private and public signals and of monetary policy. It is useful to define the precision of the two types of information by

$$\alpha = \frac{1}{\sigma_\eta^2}, \delta = \frac{1}{\sigma_\varepsilon^2}. \tag{35}$$

Based on both the signal x_i and $H(y)$, the expected value of θ on island i is given by

$$E_i(\theta) = \frac{\alpha y + \delta x_i}{\alpha + \delta}. \tag{36}$$

Recall that, as a function of the information available on island i, the quantity produced is given by

$$q_i = (1-r) E_i(\theta | x_i, \gamma(y)) + r E_i(\overline{q} | x_i, \gamma(y)) + \left[1 - \frac{\gamma(y)}{\beta}\right]. \tag{37}$$

In order to solve for the quantity produced in each island as a function of α, δ, and y, we follow the same method as MS (see Appendix for details). We then obtain

$$q_i(\mathcal{I}_i) = \frac{\delta(1-r)}{\alpha + \delta(1-r)} x_i + \left(1 - \frac{\delta(1-r)}{\alpha + \delta(1-r)}\right) y - \frac{1}{(1-r)} \frac{\gamma(y) - \beta}{\beta}. \tag{38}$$

When $\gamma(y) = \beta + H(y)$, with $H(y) = h(y)\beta(1-r)$, we have

$$q_i(\mathcal{I}_i) = \frac{\delta(1-r)}{\alpha+\delta(1-r)}x_i + \left(1 - \frac{\delta(1-r)}{\alpha+\delta(1-r)}\right)y - h(y). \tag{39}$$

Note that, for the case where $\alpha \to \infty$ (perfectly informative CB signal), we have

$$q_i = \theta - h(y). \tag{40}$$

Simplifying the expression for $q_i(\mathcal{I}_i)$, we obtain

$$q_i(\mathcal{I}_i) = \frac{\alpha y + \delta(1-r)x_i}{\alpha+\delta(1-r)} - h(y). \tag{41}$$

In terms of θ, ε, and η, the equilibrium quantity on island i can be written as

$$q_i = \theta + \frac{\alpha E(\eta|x_i, y) + \delta(1-r)\varepsilon_i}{\alpha+\delta(1-r)} - h(y). \tag{42}$$

Proposition 4 *The linear equilibrium above is the unique equilibrium. Furthermore,* $\partial q_i/\partial\gamma < 0$*.*

In order to investigate the effects of increased accuracy of the signals, recall that period-t welfare in this economy, (given a public signal y) is given by

$$\begin{aligned} E\left[W_t(\mathbf{q}_t, \theta_t, y_t) \mid \theta_t, y_t\right] &= -\int_0^1 (q_t^i - \theta_t)^2 di \\ &= -\left[\int_0^1 h^2(y)\,di + \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2} - \frac{2\alpha\int_0^1 E(\eta|x_i,y)\,h(y)\,di}{\alpha+\delta(1-r)}\right] \\ &= -\left[\int_0^1 h^2(y)\,di + \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2} - \frac{2\alpha h(y)\int_0^1 E(\eta|x_i,y)\,di}{\alpha+\delta(1-r)}\right] \\ &= -h^2(y) - \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2}. \end{aligned} \tag{43}$$

In addition, recall from the previous section that, if the CB does not signal its information, period-t expected welfare in state θ is given by

$$E\left[W_t\left(q_t^i, \theta\right) \mid \theta\right] = -\sigma_\varepsilon^2. \tag{44}$$

Thus, it is optimal that the CB reveals its information if and only if

$$h^2(y) + \frac{\alpha+\delta(1-r)^2}{[\alpha+\delta(1-r)]^2} < \frac{1}{\delta}, \tag{45}$$

or after some manipulation if and only if

$$h^2(y) < \alpha \frac{\alpha + \delta(1 - 2r)}{\delta[\alpha + \delta(1 - r)]^2} \equiv \kappa. \tag{46}$$

Hence, (since $h(y) > 0$ for all y), we have the following.

Proposition 5 *It is optimal for the central bank to reveal its information in period t if and only if $h(y) < \sqrt{\kappa}$.*

The above proposition suggests that it is possible for the CB to design monetary policy so that the above equality holds for all y. In that case, it is always optimal for the CB to reveal its information to the private sector. In addition, note that the higher the precision of private information, the smaller the chance that the above inequality holds. Finally we show that the optimal $h(y)$ is independent of α.

Proposition 6 *For all $\alpha \neq \alpha'$, suppose $h(y, \alpha)$ and $h(y, \alpha')$ are optimal, then $h(y, \alpha) = h(y, \alpha')$ for almost all y.*

Next, suppose the CB finds it optimal to reveal its information. How does the period-t welfare change as the precision of the CB's information changes? Similarly, does smaller noise in the private signal necessarily lead to a higher period-t welfare?

Let us first consider the impact of changes in the precision of the private signal. We have

$$\begin{aligned}
\frac{\partial E[W_t \mid \theta_t, y_t]}{\partial \delta} &= -\frac{(1-r)}{[\alpha + \delta(1-r)]^3}\left[(1-r)[\alpha + \delta(1-r)] - 2\left[\alpha + \delta(1-r)^2\right]\right] \\
&= -\frac{(1-r)}{[\alpha + \delta(1-r)]^3}\left[\alpha(1-r) + \delta(1-r)^2 - 2\alpha - 2\delta(1-r)^2\right] \\
&= -\frac{(1-r)}{[\alpha + \delta(1-r)]^3}\left[-\alpha(1+r) - \delta(1-r)^2\right] > 0.
\end{aligned} \tag{47}$$

Thus, we have the following.

Proposition 7 *Period-t welfare is always increasing in the precision of the private signal δ.*

An increase in the precision of public information in *unambiguously* welfare-improving only when there is no complementarities, i.e. $r = 0$. For $r > 0$, like in MS, an increase in the public signal's precision is beneficial only in some cases. More precisely, we have

$$\frac{\partial E\left[W_t \mid \theta_t, y_t\right]}{\partial \alpha} = -\frac{\alpha + \delta(1-r) - 2\left[\alpha + \delta(1-r)^2\right]}{\left[\alpha + \delta(1-r)\right]^3} = \frac{\alpha - \delta(2r-1)(1-r)}{\left[\alpha + \delta(1-r)\right]^3}. \quad (48)$$

If $r = 0$, we have $\frac{1}{(\alpha+\delta)^2} > 0$. For $r > 0$, an increase in the precision of the public signal is welfare improving if and only if

$$\frac{\alpha}{\delta} \geq (2r-1)(1-r). \quad (49)$$

Proposition 8 *Period-t welfare is increasing in the precision of the public signal, α, if and only if $\frac{\alpha}{\delta} \geq (2r-1)(1-r)$.*

The reader should note that this result always holds when $r = 0$. Hence, when the "beauty contest" is negligible in agents' utility, revealing the public signal is always welfare improving.

6 Signaling Confidence

Here we consider the case where the public signal precision itself is a random variable. In this case, the CB might want to use monetary policy in order to signal *both* the realized value of its signal and its confidence in the signal.

To study this issue, suppose that the precision of the public signal, α, can take on one of the two values: $\{\alpha_L, \alpha_H\}$, where $\alpha_L < \alpha_H$. The probability that $\alpha = \alpha_L$ is denoted by π. As before, the CB receives signal y about the value of θ. In addition, we assume that the CB knows the realization of α. The CB can choose to signal the value of y and α to the public through monetary policy. This can be accomplished via a rule that takes the following form:

$$\gamma(y, \alpha) = \beta + H(y, \alpha) = \beta + h(y, \alpha)\beta(1-r), \quad (50)$$

with $H(y,\alpha) = 0$, if the CB does not reveal its information in state (y,α). The analysis from Section 5 continues to apply. More precisely,

$$q_i(\mathcal{I}_i) = \frac{\alpha y + \delta(1-r)x_i}{\alpha + \delta(1-r)} - h(y,\alpha). \tag{51}$$

Given $h(y,a)$, two cases are possible, as we showed in the previous section. First, it could be optimal not to reveal any information. This is true if the following inequality is satisfied for all y and both α_L and α_H:

$$\int_0^1 h^2(y,\alpha)\,di + \frac{\alpha + \delta(1-r)^2}{[\alpha + \delta(1-r)]^2} - \frac{2\alpha}{\alpha + \delta(1-r)} \int_0^1 \eta h(y,\alpha)\,di \geq \frac{1}{\delta}. \tag{52}$$

Alternatively, it could be optimal to reveal the CB's information. This is true if

$$\int_0^1 h^2(y,\alpha)\,di + \frac{\alpha + \delta(1-r)^2}{[\alpha + \delta(1-r)]^2} - \frac{2\alpha}{\alpha + \delta(1-r)} \int_0^1 \eta h(y,\alpha)\,di < \frac{1}{\delta} \tag{53}$$

holds for some y and α. In what follows, we will only consider the case where the last inequality is satisfied for all pairs (y,α).

Here, we attempt to answer the question we studied in an earlier section. Can the CB use monetary policy in order to communicate its information about both its signal and that signal's precision? Put differently, is the (expanded) information set of the CB still equivalent to the set of monetary policies in can apply? As we discussed earlier, one contribution of our paper is to formulate the above as a mathematical question. We need to investigate whether there exist a homeomorphism between these two sets. The next proposition provides us with an affirmative answer.

Since the CB needs to signal both y and α to the public, it must be the case that $H(y,\alpha_L) \neq H(y',\alpha_H)$ for any y, y'. Since $H(y,\alpha)$ is strictly increasing given α, this inequality implies that $H(\infty,\alpha_j) < H(-\infty,\alpha_i)$ must hold for $i \neq j \in \{L,H\}$. One candidate policy for $i \neq j \in \{H,L\}$ is depicted in Figure 1 and is defined as follows

$$H(y,\alpha) = \begin{cases} h(y) \text{ if } \alpha = \alpha_i \\ h(y) + \Delta \text{ if } \alpha = \alpha_j \end{cases}, \tag{54}$$

where $\Delta \geq \sup h(y)$. Indeed, for this policy, we have the following.

Proposition 9 *The function* $H(y,\alpha)$ *defined above,* $H(y,\alpha) : R \times \{\alpha_L, \alpha_H\} \rightarrow (0,\Delta) \cup (\Delta, 2\Delta)$ *is a homeomorphism.*

The proof, which appears in the Appendix, readily generalizes to a countable set of possible precision values. The above Proposition demonstrate the (topological) equivalence between the set describing the information potentially held by the CB and the set of feasible monetary policies at the CB's disposal. This implies that, at the cost of imposing a distortion associated with some inflation, the CB can use monetary policy in order to communicate both its signal about fundamentals and its confidence in that signal to the private sector. Note that, whenever $\Delta > 0$, the monetary distortion will be larger if $\alpha = \alpha_j$ as compared to $\alpha = \alpha_i$, for the same realization of y. Therefore, the monetary policy rule that maximizes welfare has to take into account the relative frequency of the precision realizations. We state this in the next Proposition.

Proposition 10 *If* $\pi > 1/2$, *welfare is maximized if* $h(y,\alpha_L) = h(y)$ *and* $h(y,\alpha_H) = h(y) + \Delta$. *If* $\pi \leq 1/2$, *welfare is maximized if* $h(y,\alpha_H) = h(y)$ *and* $h(y,\alpha_L) = h(y) + \Delta$.

The above Proposition obtains an interesting interpretation in the context of monetary policy. Consider the case where "most of the time" the precision of the CB's signal is rather low relative to that of the private sector; i.e., π is close to 1. In that case, given a signal y, a benevolent CB will find it optimal to choose $h(y,\alpha_L) = h(y)$. On the other hand, when the CB frequently receives a high quality signal (π is low), then it optimally adopts $h(y,\alpha_L) = h(y) + \Delta$. This interpretation implies that higher nominal rate hikes occur less frequently than lower ones, which is in line with actual observations.

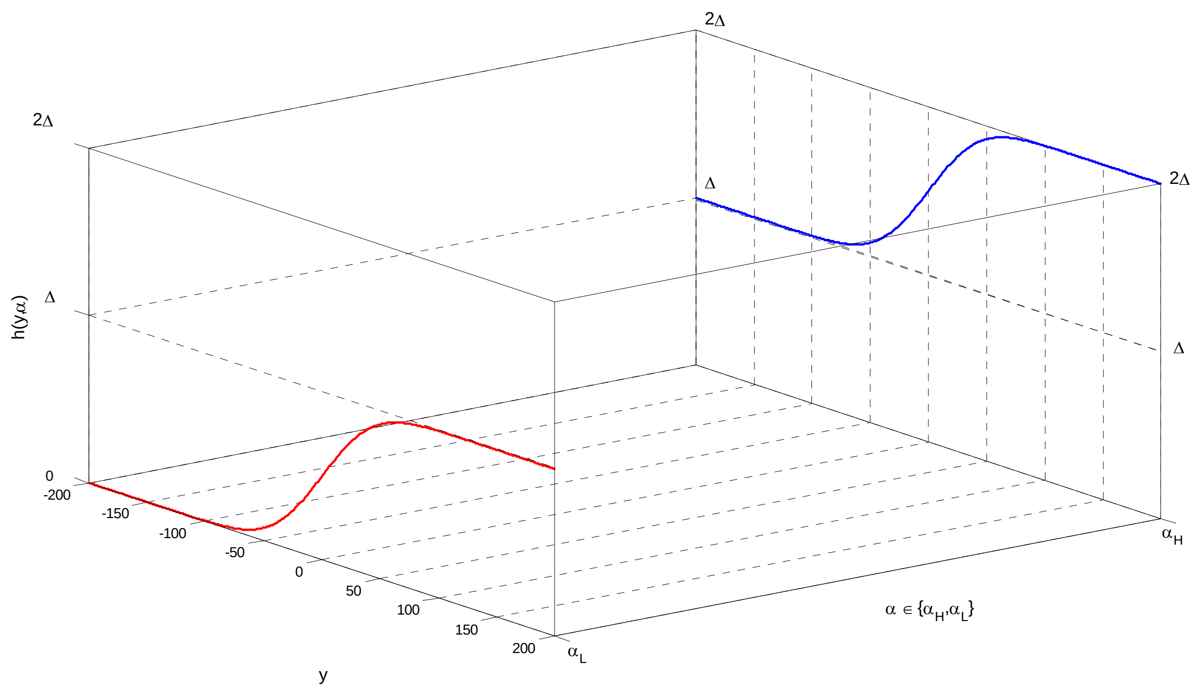

Figure 1: $h\left(y,\delta\right)$ is a homeomorphism.

7 Discussion

We studied monetary policy in a dynamic model where uncertainty can lead to a discrepancy between economic agents' beliefs and true fundamentals. Existing game-theoretic models of beauty contests under private information do not involve monetary trade and abstract from prices. These limitations make it difficult to use such models in order to answer monetary policy questions. At the same time, traditional macro-policy models do no usually involve an explicit need for coordinating expectations. In this paper, we attempted to bridge this gap. Our model involves three novel features. First, we imbedded the static structure of MS in an infinite horizon model. Second, our model is subject to frictions that make money, and monetary policy, essential. Third, our environment involves an explicit role for prices, since trade takes place in Walrasian markets. We formalized a criterion for the equivalence between the information held by the central bank and the monetary policy chosen as a function of that information. Our analysis has some novel predictions about monetary policy. For example, in line with actual observations, higher interest rate hikes in our model could occur less frequently than lower ones.

8 Appendix

8.1 Derivation of q_i:

Following Morris and Shin, we first conjecture that

$$q_i(\mathcal{I}_i) = \kappa x_i + (1-\kappa)y + (A + B\gamma(y)), \tag{55}$$

where $\gamma(y) = \beta + H(y)$. Then, the above two expressions imply

$$\begin{aligned} E_i(\overline{q}) &= \kappa\left(\frac{\alpha y + \delta x_i}{\alpha+\delta}\right) + (1-\kappa)y + (A + B\gamma(y)) \\ &= \left(\frac{\kappa\delta}{\alpha+\delta}\right)x_i + \left(1 - \frac{\kappa\delta}{\alpha+\delta}\right)y + (A + B\gamma(y)). \end{aligned} \tag{56}$$

Recall that, as a function of the information available on island i, the quantity produced is given by

$$q_i = (1-r)\,E_i(\theta|x_i, \gamma(y)) + rE_i(\overline{q}|x_i, \gamma(y)) - \frac{\gamma(y) - \beta}{\beta}. \tag{57}$$

Combining the above two, we obtain

$$\begin{aligned} q_i &= (1-r)\left(\frac{\alpha y + \delta x_i}{\alpha+\delta}\right) + r\left[\frac{\kappa\delta}{\alpha+\delta}x_i + (1 - \frac{\kappa\delta}{\alpha+\delta})y + A + B\gamma(y)\right] - \frac{\gamma(y)-\beta}{\beta} \\ &= \frac{\delta(r\kappa + 1 - r)}{\alpha+\delta}x_i + \left(1 - \frac{\delta(r\kappa+1-r)}{\alpha+\delta}\right)y + r(A + B\gamma(y)) - \frac{\gamma(y)-\beta}{\beta}. \end{aligned} \tag{58}$$

Equating coefficients we obtain

$$\kappa = \frac{\delta(1-r)}{\alpha + \delta(1-r)},\ \ A = \frac{1}{1-r},\ \ B = -\frac{1}{\beta(1-r)}. \tag{59}$$

Thus, the equilibrium action as a function of the signal precisions is given by

$$q_i(\mathcal{I}_i) = \frac{\delta(1-r)}{\alpha+\delta(1-r)}x_i + \left(1 - \frac{\delta(1-r)}{\alpha+\delta(1-r)}\right)y + \frac{1}{1-r}\left[1 - \frac{\gamma(y)}{\beta}\right]. \tag{60}$$

8.2 Proof of Proposition ??:

Proof. The proof follows closely Morris and Shin (2003). Recall that the equilibrium quantity on island i is given by:

$$q_i = (1-r)\,E_i(\theta|x_i, \gamma(y)) + rE_i(\overline{q}|x_i, \gamma(y)) + \left[1 - \frac{\gamma(y)}{\beta}\right]. \tag{61}$$

Writing out $\overline{q}$ and substituting yields:

$$\begin{aligned} q_i &= (1-r)\,E_i\left(\theta|x_i,\gamma\left(y\right)\right) + (1-r)\,rE_i\left[\overline{E}\left(\theta|x_i,\gamma\left(y\right)\right)\right] + r\left[1-\frac{\gamma\left(y\right)}{\beta}\right] + \\ &\quad (1-r)\,r^2E_i\left[\overline{E}^2\left(\theta|x_i,\gamma\left(y\right)\right)\right] + r^2\left[1-\frac{\gamma\left(y\right)}{\beta}\right] + \ldots \\ &= (1-r)\sum_{k=0}^{\infty} r^kE_i\left[\overline{E}^k\left(\theta|x_i,\gamma\left(y\right)\right)\right] + \sum_{k=0}^{\infty} r^k\left[1-\frac{\gamma\left(y\right)}{\beta}\right] \\ &= (1-r)\sum_{k=0}^{\infty} r^kE_i\left[\overline{E}^k\left(\theta|x_i,\gamma\left(y\right)\right)\right] + \frac{1}{1-r}\left[1-\frac{\gamma\left(y\right)}{\beta}\right]. \end{aligned} \tag{62}$$

Now, we solve explicitly for $E_i\left[\overline{E}^k\left(\theta|x_i,\gamma\left(y\right)\right)\right]$. Recall that $E_i(\theta|x_i,\gamma\left(y\right)) = \frac{\alpha y+\delta x_i}{\alpha+\delta}$. Then,

$$\overline{E}\left(\theta|x_i,\gamma\left(y\right)\right) = \int_0^1 E_i(\theta|x_i,\gamma\left(y\right)) = \frac{\alpha y+\delta\theta}{\alpha+\delta}, \tag{63}$$

$$E_i\left[\overline{E}\left(\theta|x_i,\gamma\left(y\right)\right)\right] = E_i\left(\frac{\alpha y+\delta\theta}{\alpha+\delta}\right) = \frac{\alpha y+\delta\frac{\alpha y+\delta x_i}{\alpha+\delta}}{\alpha+\delta} = \frac{\left[(\alpha+\delta)^2-\delta^2\right]y+\delta^2x_i}{(\alpha+\delta)^2}, \tag{64}$$

and

$$\overline{E}^2\left(\theta|x_i,\gamma\left(y\right)\right) = \overline{E}\left[\overline{E}\left(\theta|x_i,\gamma\left(y\right)\right)\right] = \frac{\left[(\alpha+\delta)^2-\delta^2\right]y+\delta^2\theta}{(\alpha+\delta)^2}. \tag{65}$$

The following claim can now be stated. For any k,

$$\overline{E}^k\left(\theta|x_i,\gamma\left(y\right)\right) = \left(1-\frac{\delta^k}{(\alpha+\delta)^k}\right)y + \frac{\delta^k}{(\alpha+\delta)^k}\theta \tag{66}$$

and

$$E_i\left[\overline{E}^k\left(\theta|x_i,\gamma\left(y\right)\right)\right] = \left(1-\frac{\delta^{k+1}}{(\alpha+\delta)^{k+1}}\right)y + \frac{\delta^{k+1}}{(\alpha+\delta)^{k+1}}x_i. \tag{67}$$

The proof is by induction on k and is omitted. For details, see Morris and Shin (2003), p. 1527. Substituting into the equation for q_i gives:

$$\begin{aligned} q_i &= (1-r)\sum_{k=0}^{\infty} r^k\left[\left(1-\frac{\delta^{k+1}}{(\alpha+\delta)^{k+1}}\right)y + \frac{\delta^{k+1}}{(\alpha+\delta)^{k+1}}x_i\right] + \frac{1}{1-r}\left[1-\frac{\gamma\left(y\right)}{\beta}\right] \\ &= \left(1-\frac{(1-r)\frac{\delta}{\alpha+\delta}}{1-r\frac{\delta}{\alpha+\delta}}\right)y + \frac{(1-r)\frac{\delta}{\alpha+\delta}}{1-r\frac{\delta}{\alpha+\delta}}x_i + \frac{1}{1-r}\left[1-\frac{\gamma\left(y\right)}{\beta}\right] \\ &= \left(1-\frac{\delta(1-r)}{\alpha+\delta\left(1-r\right)}\right)y + \frac{\delta(1-r)}{\alpha+\delta\left(1-r\right)}x_i + \frac{1}{1-r}\left[1-\frac{\gamma\left(y\right)}{\beta}\right]. \end{aligned} \tag{68}$$

This is the linear equilibrium from above. Finally, it is easy to check that $\partial q_i / \partial \gamma < 0$. ■

8.3 Proof of Proposition ??:

Proof. Suppose first $h(y, \alpha)$ and $h(y, \alpha')$ are both increasing functions of y (the proof is identical when both functions are decreasing). Assume to the contrary that there is a set Y of positive measure such that $h(\tilde{y}, \alpha) \neq h(\tilde{y}, \alpha')$ for all $\tilde{y} \in Y$. We will build a new function $h'(y, \alpha')$ such that it achieves a higher welfare than $h(y, \alpha)$ or $h(y, \alpha')$. Notice that welfare is decreasing in $h(.)$. Hence, for all $\tilde{y} \in Y$, set $h'(\tilde{y}, \alpha') = \min\{h(\tilde{y}, \alpha), h(\tilde{y}, \alpha')\}$. Then it is obvious that $h'(y, \alpha') \leq \min\{h(y, \alpha), h(y, \alpha')\}$ for all y, while still being strictly increasing. Hence, $h(y, \alpha)$ or $h(y, \alpha')$ can't be optimal which contradicts the original claim. Now suppose $h(y, \alpha)$ is an increasing function of y while $h(y, \alpha')$ is a decreasing function of y. Note first that there must be y^* such that $h(y^*, \alpha) = h(y^*, \alpha')$ since otherwise, one of these two functions always lies above the other one and hence can't be optimal. Now, consider the function $h'(y) = \pi h(y, \alpha) + (1 - \pi) h(y, \alpha')$ for some $\pi \in (0, 1)$. Then $h'(y) < h(y, \alpha')$ for all $y < y^*$. Then for all $y \geq y^*$, set $h'(y) = h(y, \alpha')$. Then $h'(y) \leq h(y, \alpha')$ for all y with a strict inequality for some y. Furthermore, for π close enough to one, $h'(y)$ is strictly decreasing. Hence $h(y, \alpha')$ can't be optimal. ■

8.4 Homeomorphism

Here we prove:

Proposition 11 *Let $h : \mathbb{R} \to (0, \Delta)$ be a homeomorphism where $\Delta > 0$. Then, the function $H(y, \alpha) : \mathbb{R} \times \{L, H\} \to (0, \Delta) \cup (\Delta, 2\Delta)$ given by*

$$H(y, \alpha) = \begin{cases} h(y) & \text{if } \alpha = \alpha_L \\ h(y) + \Delta & \text{if } \alpha = \alpha_H \end{cases}, \tag{69}$$

is a homeomorphism.

Proof. We impose the usual topology on $\mathbb{R}$, and the usual subspace topologies on $(0, \Delta)$ and on $(0, \Delta) \cup (\Delta, 2\Delta)$. We impose the discrete topology on $\{\alpha_L, \alpha_H\}$. The resulting topology on $\mathbb{R} \times \{\alpha_L, \alpha_H\}$ is the product topology. We need to show that $H(y, \alpha)$ satisfies all the conditions for a homeomorphism.

Claim 1: $H(y, \alpha)$ is injective.

We need to show that $(y_1, \alpha_1) \neq (y_2, \alpha_2) \Rightarrow H(y_1, \alpha_1) \neq H(y_2, \alpha_2)$. Let $(y_1, \alpha_1) \neq (y_2, \alpha_2)$. We need to consider two cases. First, let $\alpha_1 = \alpha_L$ and $\alpha_2 = \alpha_H$. Then,

$$\begin{aligned} H(y_1, \alpha_1) &= H(y_1, \alpha_L) = h(y_1) < \Delta \\ H(y_2, \alpha_2) &= H(y_2, \alpha_H) = h(y_2) + \Delta > \Delta. \end{aligned} \tag{70}$$

Thus, $H(y_1, \alpha_1) \neq H(y_2, \alpha_2)$. Next, consider the case where $y_1 \neq y_2$ and $\alpha_1 = \alpha_2$. If $\alpha_1 = \alpha_2 = \alpha_L$, then $H(y_1, \alpha_1) = H(y_1, \alpha_L) = h(y_1)$, and $H(y_2, \alpha_2) = H(y_2, \alpha_L) = h(y_2)$. Since h is injective, $H(y_1, \alpha_1) = h(y_1) \neq h(y_2) = H(y_2, \alpha_2)$. Thus, in both cases $H(y_1, \alpha_1) \neq H(y_2, \alpha_2)$.■

Claim 2: $H(y, \alpha)$ is surjective.

Here we have to show that $\forall z \in (0, \Delta) \cup (\Delta, 2\Delta)$, $\exists (y, \alpha)$ such that $H(y, \alpha) = z$. Let $z \in (0, \Delta) \cup (\Delta, 2\Delta)$. Then, either $z \in (0, \Delta)$ or $z \in (\Delta, 2\Delta)$. If $z \in (0, \Delta)$, since h is surjective, there exists $y \in \mathbb{R}$ such that $h(y) = z$. Thus, $H(y, \alpha_L) = h(y) = z$. Next, suppose that $z \in (\Delta, 2\Delta)$. Then, $\Delta < z < 2\Delta$, thus, $0 < z - \Delta < \Delta$. Again, since h is surjective, there exists $y \in \mathbb{R}$ such that $h(y) = z - \Delta$. Thus, $H(y, \alpha_H) \equiv h(y) + \Delta = z$. Thus, $H(y, \alpha)$ is surjective.■

Claim 3: $H(y, \alpha)$ is continuous.

It suffices to show that the inverse image of every basis (open) set is open in the respective topology. Note that any open set in the range of H is of the form $(a, b) \cap \{(0, \Delta) \cup (\Delta, 2\Delta)\}$, with $a, b \in \mathbb{R}$. First, consider any open set, B, such that

$B \subset (0, \Delta)$. Then, $h^{-1}(B)$ is open in $\mathbb{R}$, since h is continuous. We have

$$\begin{aligned}
& (y, \alpha) \in H^{-1}(B) \\
\Leftrightarrow \quad & H(y, \alpha) \in B \subset (0, \Delta) \\
\Leftrightarrow \quad & \alpha = L, H(y, \alpha) = h(y) \in B \\
\Leftrightarrow \quad & \alpha = \alpha_L, y \in h^{-1}(B) \\
\Leftrightarrow \quad & (y, \alpha) \in h^{-1}(B) \times \{\alpha_L\}. \qquad (71)
\end{aligned}$$

Thus, $H^{-1}(B) = h^{-1}(B) \times \{\alpha_L\}$, thus, whenever B is open, we have that $H^{-1}(B)$ is open in the product topology, since $h^{-1}(B)$ is open in $\mathbb{R}$, and $\{\alpha_L\}$ is open in $\{\alpha_L, \alpha_H\}$. Proceeding in the same fashion, we can show that the inverse image of B is open for any open set $B \subset (\Delta, 2\Delta)$. The only remaining case involves a set $B = B_1 \cup B_2$, where $B_1 \subset (0, \Delta)$ and $B_2 \subset (\Delta, 2\Delta)$. This easily follows from combining the above two cases.■

Finally, we have the following.

Claim 4: $H^{-1}(y, \alpha)$ is continuous.

We must show that $H(B)$ is open for every basis (open) set B. Let $B = (y_1, y_2) \times \{\alpha\}_{\alpha \in \{\alpha_L, \alpha_H\}}$ be an open set in the product topology $\mathbb{R} \times \{\alpha_L, \alpha_H\}$. First, suppose that $\alpha = \alpha_L$; i.e., $B = (y_1, y_2) \times \{\alpha_L\}$. Then,

$$H(B) = H\left((y_1, y_2), \{\alpha_L\}\right) = h\left((y_1, y_2)\right). \qquad (72)$$

Thus, $H(B)$ is open in $\mathbb{R}$, since h^{-1} is continuous and $h((y_1, y_2))$ is open. Similarly, if $\alpha = \alpha_H$; i.e., $B = (y_1, y_2) \times \{\alpha_H\}$, we have

$$H(B) = H\left((y_1, y_2) \times \{\alpha_H\}\right) = \{h(y) + \Delta : y \in (y_1, y_2)\}. \qquad (73)$$

Let $z = h(y) + \Delta$ and $y \in (y_1, y_2)$. Then, $z - \Delta = h(y) \in h((y_1, y_2))$. Since h^{-1} is continuous, $h((y_1, y_2))$ is open in $\mathbb{R}$. Thus, there exists an open interval (z_1, z_2) such that

$$z - \Delta \in (z_1, z_2) \subset h((y_1, y_2)). \qquad (74)$$

This implies that

$$z \in (z_1 + \Delta, z_2 + \Delta) \subset \{h(y) + \Delta : y \in (y_1, y_2)\}. \tag{75}$$

Thus, $H(B) = \{h(y) + \Delta : y \in (y_1, y_2)\}$ is open in $\mathbb{R}$.■

We conclude that $H(y, \alpha)$ is a homeomorphism. ■

8.5 Proof of Proposition ??:

Proof. When there are two possible precision values, the central bank maximizes

$$\begin{aligned} E\left[W_t\left(\mathbf{q}_t, \theta_t\right) \mid \theta_t\right] &= \pi\left[-\int h^2\left(\theta_t+\eta, \alpha_L\right) f\left(\eta\right) d\eta - \frac{\alpha_L+\delta\left(1-r\right)^2}{\left[\alpha_L+\delta\left(1-r\right)\right]^2}\right] + \\ &\quad + \left(1-\pi\right)\left[-\int h^2\left(\theta_t+\eta, \alpha_H\right) f\left(\eta\right) d\eta - \frac{\alpha_H+\delta\left(1-r\right)^2}{\left[\alpha_H+\delta\left(1-r\right)\right]^2}\right] \end{aligned} \tag{76}$$

It is easy to see that when $\pi > 1/2$, the prescribed policy gives a welfare function

$$\begin{aligned} &\pi\left[-\int h^2\left(\theta_t+\eta\right) f\left(\eta\right) d\eta - \frac{\alpha_L+\delta\left(1-r\right)^2}{\left[\alpha_L+\delta\left(1-r\right)\right]^2}\right] + \\ &+ \left(1-\pi\right)\left[-\Delta - \int h^2\left(\theta_t+\eta\right) f\left(\eta\right) d\eta - \frac{\alpha_H+\delta\left(1-r\right)^2}{\left[\alpha_H+\delta\left(1-r\right)\right]^2}\right] \end{aligned} \tag{77}$$

which is higher than when setting $h\left(y, \alpha_H\right) = h\left(y\right)$ and $h\left(y, \alpha_L\right) = h\left(y\right) + \Delta$. The rest of the proof goes along the exact same lines. ■

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