Introducing Financial Frictions and Unemployment into a Small Open Economy Model*

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Abstract

How important are financial and labor market frictions for the business cycle dynamics of a small open economy? What are the quantitative effects of increased financial risk on output and inflation? How important are variations in the intensive vs. the extensive margin of labor supply? What are the spillover effects of financial market disturbances to unemployment and vice versa? In order to address these question we extend the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b) in two important dimensions. First, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003, 2007). Second, we include the search and matching framework of Mortensen and Pissarides (1994), Gertler, Sala and Trigari (2006) and Christiano, Ilut, Motto, and Rostagno (2007) into a small open economy model. We estimate the full model using Bayesian techniques.

Keywords: small open economy, DSGE, financial frictions, unemployment.

JEL codes: E0, E3, F0, F4, G0, G1, J6.

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1. Introduction

How important are financial and labor market frictions for the business cycle dynamics of a small open economy? In particular, what are the quantitative effects of increased financial risk on output and inflation? Furthermore, how important are variations in the intensive and/or extensive margin of labor supply? Moreover, what are the spillover effects of financial market disturbances to unemployment and vice versa in a small open economy? In order to address these question we extend the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b) in two important dimensions.

First, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003, 2007). The financial frictions we introduce reflect fundamentally that borrowers and lenders are different people, and that they have different information. Thus, we introduce 'entrepreneurs'. These are agents who have a special skill in the operation and management of capital. Although these agents have their own financial resources, their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is a financial friction because the management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks which are observed only by them. The agents that they borrow from, 'banks', can only observe the idiosyncratic shocks by paying a monitoring cost. This type of asymmetric information implies that it is impractical to have an arrangement in which banks and entrepreneurs simply divide up the proceeds of entrepreneurial activity, because entrepreneurs have an incentive to understate their earnings. Entrepreneurs who suffer an especially bad idiosyncratic income shock and who therefore cannot afford to pay the required interest, are 'bankrupt'. Banks pay the cost of monitoring these entrepreneurs and take all of their net worth in partial compensation for the interest that they are owed.

In the model, the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Irving Fisher (1933). For example, when a shock occurs which drives the price level down, households receive a wealth transfer. Because this transfer is taken from entrepreneurs, their net worth is reduced. With the tightening in their balance sheets, their ability to invest is reduced, and this produces an economic slowdown.

Second, we include the search and matching framework of Mortensen and Pissarides (1994), Hall (2005a,b,c), Shimer (2005a,b), Gertler, Sala and Trigari (2006) and Christiano, Ilut, Motto, and Rostagno (2007) into the small open economy model. We integrate this into our specific framework - which includes capital and monetary factors - following the labor market version of Gertler, Sala and Trigari (2006) (henceforth GST). A key feature of the GST model is that there are wage-setting frictions, but they do not have a direct impact on on-going worker employer relations. However, wage-setting frictions have an impact on the effort of an employer in recruiting new employees In this sense, the setup is not vulnerable to the Barro (1977) critique of sticky wages. The model is also attractive because of the richness of its labor market implications: the model differentiates between hours worked and the quantity of people employed, it has unemployment and vacancies.

The labor market in our model is a slightly modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions. In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with our baseline model.

In the baseline model, the homogeneous labor services supplied to the competitive labor market by labor retailers (contractors) who combine the labor services supplied to them by households who monopolistically supply specialized labor services. The modified model dispenses with the specialized labor services abstraction. Labor services are instead supplied to the homogeneous labor market by 'employment agencies'.

Each employment agency retains a large number of workers. At the beginning of the period a fraction of workers is randomly selected to separate from the firm and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. After separation and new arrivals occur, the nominal wage rate is set.

The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every N periods. Each employment agency is permanently allocated to one of N different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort, $1/N$ of the agencies bargain in each period. The wage in agencies that do not bargain in the current period is updated from the previous period according to the same indexing rule used in our baseline model. The intensity of labor effort is determined by equating the worker's marginal cost to the agency's marginal benefit.

Furthermore, the employment agency in the i^{th} cohort determines how many employees it will have in period $t+1$ by choosing vacancies subject to quadratic vacancy posting costs.

In addition to the main two new features described above, we integrate the following other new features into the model compared to Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b): imported goods are directly used for export production, a unit-root investment specific technological progress, working capital loans for all monopolists, possible price and wage dispersion in steady state, capital tax timing and allowances as well as the specification and estimation of a VAR that represents the foreign economy. These new features turn out to be useful when taking the model to the data.

We estimate the full model, which contains the financial frictions as well as the and search and matching frictions, with Bayesian techniques.

The paper is organized as follows. In section 2 we describe the baseline small open economy model. Section 3 introduces financial frictions while section 4 incorporates search and matching frictions to the model. Section 5 develops the full model with both financial and search and matching frictions. Section 6 contains the estimation of the full model. Finally, section 7 summarizes the paper.

2. The Baseline Small Open Economy Model

This manuscript describes an extension of the model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b)(henceforth ALLV), and presents a way to introduce financial frictions and search and matching in the labor market. Our baseline model makes some changes on the ALLV model:

- Exports are produced by using homogeneous imported goods in addition to homogeneous domestically produced goods.
- The price of investment goods is treated as a random variable with a unit root. Thus, growth in the model is driven by two independent unit root processes, one for neutral technology shocks and the other for technology shocks in the production of investment goods.
- Capital maintenance costs are deducted from capital income taxes, and physical depreciation is deducted at historic cost.
- The capital income tax rate is realized at the time the investment decision is made, not at the time when the payoff on investment is realized.
- Wages are indexed to the steady state growth rate of the economy, rather than to the current realization of technology shocks.
- All producers of specialized goods are assumed to require working capital loans.
- We allow for price and wage dispersion in steady state and estimate the degree of this dispersion in the steady state.

- The VAR that represents the foreign economy is estimated jointly with all other parameters in the Bayesian estimation.

2.1. Scaling of Variables

We adopt the following scaling of variables. The nominal exchange rate is denoted by S_t and its growth rate is s_t :

$$s_t = \frac{S_t}{S_{t-1}}.$$

The neutral shock to technology is z_t and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

The variable, Ψ_t, is an embodied shock to technology and it is convenient to define the following combination of embodied and neutral technology:

$$\begin{aligned} z_t^+ &= \Psi_t^{\frac{\alpha}{1-\alpha}} z_t, \\ \mu_{z^+,t} &= \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}. \end{aligned} \tag{2.1}$$

Capital, $\bar{K}_t$, and investment, I_t, are scaled by $z_t^+ \Psi_t$. Foreign and domestic inputs into the production of I_t (we denote these by I_t^d and I_t^m, respectively) are scaled by z_t^+. Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods and C_t are final consumption goods) are scaled by z_t^+. Government consumption, the real wage and real foreign assets are scaled by z_t^+. Exports (X_t^m are imported intermediate goods for use in producing exports and X_t are final export goods) are scaled z_t^+. Also, υ_t is the shadow value in utility terms to the household of domestic currency and $\upsilon_t P_t$ is the shadow value of one consumption good (i.e., the marginal utility of consumption). The latter must be multiplied by z_t^+ to induce stationarity. Thus,

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t},\ \bar{k}_{t+1} = \frac{\bar{K}_{t+1}}{z_t^+ \Psi_t},\ i_t^d = \frac{I_t^d}{z_t^+},\ i_t = \frac{I_t}{z_t^+ \Psi_t},\ i_t^m = \frac{I_t^m}{z_t^+} \\ c_t^m &= \frac{C_t^m}{z_t^+}, c_t^d = \frac{C_t^d}{z_t^+},\ c_t = \frac{C_t}{z_t^+},\ g_t = \frac{G_t}{z_t^+},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t},\ a_t \equiv \frac{S_t A_{t+1}^*}{P_t z_t^+}, \\ x_t^m &= \frac{X_t^m}{z_t^+},\ x_t = \frac{X_t}{z_t^+},\ \psi_{z^+,t} = \upsilon_t P_t z_t^+,\ \tilde{y}_t = \frac{Y_t}{z_t^+},\ \tilde{p}_t = \frac{\tilde{P}_t}{P_t},\ w_t = \frac{\tilde{W}_t}{W_t}. \end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period $t+1$ as $p_{k',t}$ and we define the scaled real rental rate of capital as $\bar{r}_t^k$:

$$p_{k',t} = \Psi_t P_{k',t},\ \bar{r}_t^k = \Psi_t r_t^k.$$

where $P_{k',t}$ is in units of the domestic homogeneous good. We define the following inflation rates:

$$\begin{aligned}\pi_t &= \frac{P_t}{P_{t-1}},\ \pi_t^c = \frac{P_t^c}{P_{t-1}^c},\ \pi_t^* = \frac{P_t^*}{P_{t-1}^*},\\ \pi_t^i &= \frac{P_t^i}{P_{t-1}^i},\ \pi_t^x = \frac{P_t^x}{P_{t-1}^x},\ \pi_t^{m,j} = \frac{P_t^{m,j}}{P_{t-1}^{m,j}},\end{aligned}$$

for $j = c, x, i$. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of the domestic final consumption goods (i.e., the 'CPI'), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t. When the price is denominated in foreign currency units, we divide by P_t^*, the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods, P_t^i. This grows at a rate slower than P_t, and we therefore scale it by P_t/Ψ_t. Thus,

$$\begin{aligned}p_t^{m,x} &= \frac{P_t^{m,x}}{P_t},\ p_t^{m,c} = \frac{P_t^{m,c}}{P_t},\ p_t^{m,i} = \frac{P_t^{m,i}}{P_t},\\ p_t^x &= \frac{P_t^x}{P_t^*},\ p_t^c = \frac{P_t^c}{P_t},\ p_t^i = \frac{\Psi_t P_t^i}{P_t}.\end{aligned} \tag{2.2}$$

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case $j = x$, in the production of the final consumption good in the case of $j = c$, and in the production of final investment goods in the case of $j = i$. When there is just a single superscript the underlying good is a final good, with $j = x, c, i$ corresponding to exports, consumption and investment, respectively.

We denote the real exchange rate by q_t :

$$q_t = \frac{S_t P_t^*}{P_t^c}. \tag{2.3}$$

2.2. Production of the Domestic Homogeneous Good

A homogeneous domestic good, Y_t, is produced using

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{1}{\lambda_{d,t}}} di\right]^{\lambda_{d,t}},\ 1 \leq \lambda_{d,t} < \infty. \tag{2.4}$$

The domestic good is produced by a competitive, representative firm which takes the price of output, P_t, and the price of inputs, $P_{i,t}$, as given.

The i^{th} intermediate good producer has the following production function:

$$Y_{i,t} = (z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^{+}\phi,$$

where $K_{i,t}$ denotes the labor services rented by the i^{th} intermediate good producer. Firms must borrow a fraction of the wage bill, so that one unit of labor costs is denoted by

$$W_t R_t^f,$$

with

$$R_t^f = \nu_t^f R_t + 1 - \nu_t^f, \tag{2.5}$$

where W_t is the aggregate wage rate, R_t is the interest rate on working capital loans, and ν_t^f corresponds to the fraction that must be financed in advance.

The firm's marginal cost, divided by the price of the homogeneous good is denoted by mc_t :

$$\begin{aligned} mc_t &= \frac{\tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k P_t\right)^{\alpha} \left(W_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t}}{z_t^{1-\alpha} P_t} \\ &= \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(r_t^k\right)^{\alpha} \left(\bar{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\epsilon_t}, \end{aligned} \tag{2.6}$$

where r_t^k is the nominal rental rate of capital scaled by P_t. Also, τ_t^d is a tax-like shock, which affects marginal cost, but does not appear in a production function. In the linearization of a version of the model in which there are no price and wage distortions in the steady state, τ_t^d is isomorphic to a disturbance in λ_d, i.e., a markup shock.

Productive efficiency dictates that another expression for marginal cost must also be satisfied:

$$\begin{aligned} mc_t &= \tau_t^d \frac{1}{P_t} \frac{W_t R_t^f}{MP_{l,t}} \\ &= \tau_t^d \frac{1}{P_t} \frac{W_t R_t^f}{\epsilon_t (1-\alpha) z_t^{1-\alpha} \left(k_{i,t} z_{t-1}^{+} \Psi_{t-1} / H_{i,t}\right)^{\alpha}} \\ &= \tau_t^d \frac{\left(\mu_{\Psi,t}\right)^{\alpha} \bar{w}_t R_t^f}{\epsilon_t (1-\alpha) \left(\frac{k_{i,t}}{\mu_{z^+,t}} / H_{i,t}\right)^{\alpha}} \end{aligned} \tag{2.7}$$

The i^{th} firm is a monopolist in the production of the i^{th} good and so it sets its price. Price setting is subject Calvo frictions. With probability ξ_d the intermediate good firm cannot reoptimize its price, in which case,

$$P_{i,t} = \tilde{\pi}_{d,t} P_{i,t-1},\ \tilde{\pi}_{d,t} \equiv (\pi_{t-1})^{\kappa_d} \left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d} (\breve{\pi})^{\varkappa_d},$$

where κ_d, $\varkappa_d$, $\kappa_d + \varkappa_d \in (0,1)$ are parameters, π_{t-1} is the lagged inflation rate and $\bar{\pi}_t^c$ is the central bank's target inflation rate. Also, $\breve{\pi}$ is a scalar which allows us to capture, among other things, the case in which non-optimizing firms either do not change price at all (i.e., $\breve{\pi} = \varkappa_d = 1$) or that they index only to the steady state inflation rate (i.e., $\breve{\pi} = \bar{\pi}$, $\varkappa_d = 1$).

With probability $1 - \xi_d$ the firm can change its price. The problem of the i^{th} domestic intermediate good producer which has the opportunity to change price is to maximize discounted profits:

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} \{P_{i,t+j} Y_{i,t+j} - mc_{t+j} P_{t+j} Y_{i,t+j}\},$$

subject to the requirement that production equal demand. In the above expression, υ_t is the multiplier on the household budget constraint. It measures the marginal value to the household of one unit of profits, in terms of currency. In the profit function, we replace the firm's output with the demand function:

$$\left(\frac{P_t}{P_{i,t}}\right)^{\frac{\lambda_d}{\lambda_d - 1}} Y_t = Y_{i,t},$$

to obtain, after rearranging,

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{1 - \frac{\lambda_d}{\lambda_d - 1}} - mc_{t+j} \left(\frac{P_{i,t+j}}{P_{t+j}}\right)^{\frac{-\lambda_d}{\lambda_d - 1}} \},$$

or,

$$E_t \sum_{j=0}^{\infty} \beta^j \upsilon_{t+j} P_{t+j} Y_{t+j} \{ (X_{t,j} \tilde{p}_t)^{1 - \frac{\lambda_d}{\lambda_d - 1}} - mc_{t+j} (X_{t,j} \tilde{p}_t)^{\frac{-\lambda_d}{\lambda_d - 1}} \},$$

where

$$\frac{P_{i,t+j}}{P_{t+j}} = X_{t,j} \tilde{p}_t, \;\; X_{t,j} \equiv \begin{cases} \frac{\tilde{\pi}_{d,t+j} \cdots \tilde{\pi}_{d,t+1}}{\pi_{t+j} \cdots \pi_{t+1}}, \; j > 0 \\ 1, \; j = 0. \end{cases} .$$

The i^{th} firm maximizes profits by choice of $\tilde{p}_t$. The fact that this variable does not have an index, i, reflects that all firms that have the opportunity to reoptimize in period t solve the same problem, and hence have the same solution. Differentiating its profit function, multiplying the result by $\tilde{p}_t^{\frac{\lambda_d}{\lambda_d - 1} + 1}$, rearranging, and scaling we obtain:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} [\tilde{p}_t X_{t,j} - \lambda_d mc_{t+j}] = 0,$$

where A_{t+j} is exogenous from the point of view of the firm:

$$A_{t+j} = \psi_{z^+, t+j} \tilde{y}_{t+j} X_{t,j}.$$

After rearranging the optimizing intermediate good firm's first order condition for prices, we obtain,

$$\tilde{p}_t^d = \frac{E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} \lambda_d mc_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} X_{t,j}} = \frac{K_t^d}{F_t^d},$$

say, where

$$\begin{aligned} K_t^d &\equiv E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} \lambda_d mc_{t+j} \\ F_t^d &= E_t \sum_{j=0}^{\infty} (\beta \xi_d)^j A_{t+j} X_{t,j}. \end{aligned}$$

These objects have the following convenient recursive representations:

$$\begin{aligned} E_t \left[\psi_{z^+,t} \tilde{y}_t + \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_d}} \beta \xi_d F_{t+1}^d - F_t^d \right] &= 0 \\ E_t \left[\lambda_d \psi_{z^+,t} \tilde{y}_t mc_t + \beta \xi_d \left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_d}{1-\lambda_d}} K_{t+1}^d - K_t^d \right] &= 0. \end{aligned}$$

Turning to the aggregate price index:

$$\begin{aligned} P_t &= \left[\int_0^1 P_{it}^{\frac{1}{1-\lambda_d}} di \right]^{(1-\lambda_d)} \\ &= \left[\left(1 - \xi_p\right) \tilde{P}_t^{\frac{1}{1-\lambda_d}} + \xi_p \left(\tilde{\pi}_{d,t} P_{t-1} \right)^{\frac{1}{1-\lambda_d}} \right]^{(1-\lambda_d)} \end{aligned} \tag{2.8}$$

After dividing by P_t and rearranging:

$$\frac{1 - \xi_d \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_d} = \left(\tilde{p}_t^d \right)^{\frac{1}{1-\lambda_d}} . \tag{2.9}$$

In sum, the equilibrium conditions associated with price setting are:[1]

$$E_t\left[\psi_{z^+,t}y_t+\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{1}{1-\lambda_d}}\beta\xi_d F_{t+1}^d-F_t^d\right] = 0 \tag{2.10}$$

$$E_t\left[\lambda_d\psi_{z^+,t}y_t mc_t+\beta\xi_d\left(\frac{\tilde{\pi}_{d,t+1}}{\pi_{t+1}}\right)^{\frac{\lambda_d}{1-\lambda_d}}K_{t+1}^d-K_t^d\right] = 0, \tag{2.11}$$

$$\mathring{p}_t=\left[(1-\xi_d)\left(\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right)^{\lambda_d}+\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\mathring{p}_{t-1}\right)^{\frac{\lambda_d}{1-\lambda_d}}\right]^{\frac{1-\lambda_d}{\lambda_d}} \tag{2.12}$$

$$\left[\frac{1-\xi_d\left(\frac{\tilde{\pi}_{d,t}}{\pi_t}\right)^{\frac{1}{1-\lambda_d}}}{1-\xi_d}\right]^{(1-\lambda_d)} = \frac{K_t^d}{F_t^d} \tag{2.13}$$

$$\tilde{\pi}_{d,t}\equiv(\pi_{t-1})^{\kappa_d}\left(\bar{\pi}_t^c\right)^{1-\kappa_d-\varkappa_d}(\breve{\pi})^{\varkappa_d} \tag{2.14}$$

The domestic intermediate output good is allocated among alternative uses as follows:

$$Y_t=G_t+C_t^d+I_t^d+\int_0^1 X_{i,t}^d. \tag{2.15}$$

Here, C_t^d denotes intermediate goods used (together with foreign consumption goods) to produce final household consumption goods. Also, I_t^d is the number of intermediate domestic goods used in combination with imported foreign investment goods to produce a homogeneous investment good. Some of this good is used to add to the physical stock of capital, $\bar{K}_t$. The rest of the investment good is used in maintenance expenditures, which arise from the utilization of capital, $a(u_t)\bar{K}_t$. Here, u_t denotes the utilization rate of capital, with capital services being defined by:

$$K_t=u_t\bar{K}_t.$$

We adopt the following functional form for a :

$$a(u)=0.5\sigma_b\sigma_a u^2+\sigma_b(1-\sigma_a)u+\sigma_b((\sigma_a/2)-1), \tag{2.16}$$

[1] When we linearize about steady state and set $\varkappa_d=0$, we obtain,

$$\begin{aligned}\hat{\pi}_t-\hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_d\beta}E_t\left(\hat{\pi}_{t+1}-\hat{\bar{\pi}}_{t+1}^c\right)+\frac{\kappa_d}{1+\kappa_d\beta}\left(\hat{\pi}_{t-1}-\hat{\bar{\pi}}_t^c\right)\\ &\quad -\frac{\kappa_d\beta(1-\rho_\pi)}{1+\kappa_d\beta}\hat{\bar{\pi}}_t^c\\ &\quad +\frac{1}{1+\kappa_d\beta}\frac{(1-\beta\xi_d)(1-\xi_d)}{\xi_d}\widehat{mc}_t,\end{aligned}$$

where a hat indicates log-deviation from steady state.

where σ_a and σ_b are the parameters of this function. Finally, the integral in (2.15) denotes domestic resources allocated to exports. The determination of consumption, investment and export demand is discussed below.

2.3. Production of Final Consumption and Investment Goods

Final consumption goods are purchased by households. These goods are produced by a representative competitive firm using the following linear homogeneous technology:

$$C_t = \left[(1-\omega_c)^{\frac{1}{\eta_c}} \left(C_t^d \right)^{\frac{(\eta_c-1)}{\eta_c}} + \omega_c^{\frac{1}{\eta_c}} \left(C_t^m \right)^{\frac{(\eta_c-1)}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}. \tag{2.17}$$

The representative firm takes the price of final consumption goods output, P_t^c, as given. Final consumption goods output is produced using two inputs. The first, C_t^d, is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t. The second input, C_t^m, is the homogeneous composite of specialized consumption import goods discussed in the next subsection. The price of C_t^m is $P_t^{m,c}$. The representative firm takes the input prices, P_t and $P_t^{m,c}$ as given. Profit maximization leads to the following demand for the intermediate inputs in scaled form:

$$\begin{aligned} c_t^d &= (1-\omega_c) \left(p_t^c \right)^{\eta_c} c_t \\ c_t^m &= \omega_c \left(\frac{p_t^c}{p_t^{m,c}} \right)^{\eta c} c_t. \end{aligned} \tag{2.18}$$

In the usual way, the price of C_t is related to the price of the inputs by:

$$P_t^c = \left[(1-\omega_c) \left(P_t \right)^{1-\eta_c} + \omega_c \left(P_t^{m,c} \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}.$$

After dividing by P_t, this becomes

$$p_t^c = \left[(1-\omega_c) + \omega_c \left(p_t^{m,c} \right)^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}. \tag{2.19}$$

The rate of inflation of the consumption good is:

$$\pi_t^c = \frac{P_t^c}{P_{t-1}^c} = \pi_t \left[\frac{(1-\omega_c) + \omega_c \left(p_t^{m,c} \right)^{1-\eta_c}}{(1-\omega_c) + \omega_c \left(p_{t-1}^{m,c} \right)^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}}. \tag{2.20}$$

We define investment to be the sum of investment goods, I_t, used in the accumulation of physical capital, plus investment goods used in capital maintenance:

$$\tilde{I}_t = I_t + a(u_t) \bar{K}_t.$$

Capital maintenance are expenses that arise from the utilization of capital. We discuss maintenance in subsection 2.5 below. Investment goods are produced by a representative competitive firm using the following technology:

$$\tilde{I}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I_t^m\right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} .$$

The representative firm takes the price of the final investment good, P_t^i, as given. Investment goods are produced using two inputs. The first, I_t^d, is a one-for-one transformation of the homogeneous domestic good and therefore has price, P_t. The second input, I_t^m, is the homogeneous composite of specialized investment import goods discussed in the next subsection. The price of I_t^m is $P_t^{m,i}$. The representative firm takes the input prices, P_t and $P_t^{m,i}$ as given. To accommodate the observation that the price of investment goods relative to the price of consumption goods is declining over time, we assume that Ψ_t is a unit root process with positive drift. The details of the law of motion of this process is discussed below.

Profit maximization implies:

$$\begin{aligned} P_t^i \left(\frac{\tilde{I}_t}{\Psi_t I_t^d} \right)^{\frac{1}{\eta_i}} (1-\omega_i)^{\frac{1}{\eta_i}} &= \frac{P_t}{\Psi_t} \\ P_t^i \left(\frac{\tilde{I}_t}{I_t^m} \right)^{\frac{1}{\eta_i}} \omega_i^{\frac{1}{\eta_i}} \left(\Psi_t\right)^{\frac{\eta_i - 1}{\eta_i}} &= P_t^{m,i}, \end{aligned} \qquad (2.21)$$

or,

$$\begin{aligned} \left(p_t^i\right)^{\eta_i - 1} \left(\frac{\tilde{I}_t}{\Psi_t} \right)^{\frac{\eta_i - 1}{\eta_i}} (1-\omega_i)^{\frac{\eta_i - 1}{\eta_i}} &= \left(I_t^d\right)^{\frac{\eta_i - 1}{\eta_i}} \\ \left(\frac{p_t^i}{\Psi_t p_t^{m,i}} \right)^{(\eta_i - 1)} \tilde{I}_t^{\frac{\eta_i - 1}{\eta_i}} \omega_i^{\frac{\eta_i - 1}{\eta_i}} \left(\Psi_t\right)^{\frac{\eta_i - 1}{\eta_i}(\eta_i - 1)} &= \left(I_t^m\right)^{\frac{\eta_i - 1}{\eta_i}} \end{aligned} .$$

Substituting these expressions into the production function for the final investment good, we obtain:

$$p_t^i = \left[(1-\omega_i) + \omega_i \left(p_t^{m,i}\right)^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}} . \qquad (2.22)$$

Then, the inflation in the price of investment goods is:

$$\pi_t^i = \frac{\pi_t}{\mu_{\Psi,t}} \left[\frac{(1-\omega_i) + \omega_i \left(p_t^{m,i}\right)^{1-\eta_i}}{(1-\omega_i) + \omega_i \left(p_{t-1}^{m,i}\right)^{1-\eta_i}} \right]^{\frac{1}{1-\eta_i}} . \qquad (2.23)$$

From here on we drop the notation $\tilde{I}_t$ and only refer to I_t, so that the production function for investment is:

$$I_t + a\left(u_t\right) \bar{K}_t = \Psi_t \left[(1-\omega_i)^{\frac{1}{\eta_i}} \left(I_t^d\right)^{\frac{\eta_i - 1}{\eta_i}} + \omega_i^{\frac{1}{\eta_i}} \left(I_t^m\right)^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} .$$

Finally, the demand for imported investment, 2.21, in scaled form as follows:

$$i_t^m = \omega_i \left(\frac{p_t^i}{p_t^{m,i}} \right)^{\eta_i} \left(i_t + a\left(u_t\right) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) \tag{2.24}$$

The demand for domestic investment inputs in scaled form is as follows:

$$i_t^d = \left(p_t^i\right)^{\eta_i} \left(i_t + a\left(u_t\right) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) \left(1 - \omega_i\right) \tag{2.25}$$

2.4. Exports and Imports

This section reviews the structure of imports and exports. Both activities involve Calvo price setting frictions, and so require the presence of market power. In each case, we follow the Dixit-Stiglitz strategy of introducing a range of specialized goods. This allows there to be market power without the counterfactual implication that there is a small number of firms in the export and import sector. Thus, exports involve a continuum of exporters, each of which is a monopolist which produces a specialized export good. Each monopolist produces the export good using a homogeneous domestically produced good and a homogeneous good derived from imports. The specialized export goods are sold to foreign, competitive retailers which create a homogeneous good that is sold to foreign citizens.

In the case of imports, specialized domestic importers purchase a homogeneous foreign good, which they turn into a specialized input and sell to domestic retailers. There are three types of domestic retailers. One uses the specialized import goods to create the homogeneous good used as an input into the production of specialized exports. Another uses the specialized import goods to create an input used in the production of investment goods. The third type uses specialized imports to produce a homogeneous input used in the production of consumption goods. See Figure **??** for a graphical illustration.

We emphasize two features of this setup. First, before being passed on to final domestic users, imported goods must first be combined with domestic inputs. This is consistent with the view emphasized by Burstein, Eichenbaum and Rebelo (2005, 2007), that there are substantial distribution costs associated with imports. Second, there are pricing frictions in all sectors of the model. The pricing frictions in the homogeneous domestic good sector are standard, and perhaps do not require additional elaboration. We do need to elaborate on the pricing frictions in the part of the model related to imports and exports.

In all cases we assume that prices are set in the currency of the buyer ("pricing to market"). Pricing frictions in the case of imports help the model account for the evidence that exchange rate shocks take time to pass into domestic prices. Pricing frictions in the case of exports help the model to produce a hump-shape in the response of output to a monetary shock. To see this, it is useful to recall how a hump-shape is produced in a closed economy version of the model. In that version, the hump shape occurs because there are costs to quickly expanding consumption and investment demand. Consumption is not expanded rapidly because of the assumption of habit persistence in preferences and investment is not expanded because of the assumption that there are adjustment costs associated with changing the flow of investment. When the closed economy is opened up, another potential source of demand in the wake of a monetary policy shock is introduced, namely, exports. To explain this further it is convenient to adopt a highly simplified version of our model.

Suppose that P^x is the price of exports in foreign currency units, and the demand for exports, X, is given by

$$X = (P^x)^{-\varepsilon} A,$$

where A summarizes the impact on demand of foreign prices and output which we take as predetermined. With this type of demand curve, a monopolist who sells X will set P^x as a constant markup, μ, over marginal cost, MC. Given the denomination of P^x, we require that MC is denominated in foreign currency units. Thus,

$$P^x = \mu \times MC = \mu \times \frac{f\,(\text{domestic factor prices})}{S}.$$

The expression after the second equality emphasizes that the marginal costs of the exporter involve domestic factors of production. The costs of domestic factors of production do not rise much in the face of an expansionary monetary policy shock because of the slow response of demand and frictions in wage setting.[2] The term in the denominator is the nominal exchange rate, S. A positive monetary policy shock can be expected to result in an immediate depreciation in the currency, *i.e.*, jump in S. Thus, in the presence of sticky domestic factor costs, a depreciation in the exchange rate is expected to put downward pressure on P^x. Given the demand curve for exports, this is expected to create a surge in X. To the extent that this surge is strong enough, this can overwhelm the other factors in the model designed to create a hump-shaped response of output to an expansionary monetary policy shock. In sum, we introduce the frictions in the setting of P^x in order to help assure a hump-shape response of output to a monetary policy shock.

There are two additional observations worth making concerning the role of price frictions in the export sector. First, it is interesting to note that the price frictions in the import of goods used as inputs into the production of exports work against us. These price frictions increase the need for price frictions in the export sector to damp the response of X to a positive domestic monetary shock. The reason is that in the absence of price frictions on imports, 'domestic factor prices' in the above expression would jump in the face of an expansionary monetary policy shock, as pass through from the exchange rate to the domestic

[2] The slow expansion of demand also plays a role here.

currency price of imports of goods destined for export increases. From the perspective of achieving a hump-shaped response of output to an expansionary monetary policy shock, we suspect that it would be better to treat the import of goods destined for the export sector asymmetrically by supposing there are no price frictions in those goods.

The second observation on the role of price frictions in the export sector is related to the first. We make assumptions in the model that have the effect of also producing a hump-shape response of the exchange rate to an expansionary monetary policy shock. The model follows ALLV in capturing, in a reduced form way, the notion that holders of domestic assets require less compensation for risk in the wake of an expansionary monetary policy shock. As a result, the model does not display the classic Dornbusch 'overshooting' pattern in the exchange rate in response to a monetary policy shock. Instead, the nominal exchange rate rises slowly in response to an expansionary monetary policy shock. The slow response in the exchange rate reduces the burden on price frictions in P^x to slow the response of X in to an expansionary monetary policy shock.

2.4.1. Exports

We assume there is a total demand by foreigners for domestic exports, which takes on the following form:

$$X_t = \left(\frac{P_t^x}{P_t^*}\right)^{-\eta_f} Y_t^*.$$

In scaled form, this is

$$x_t = (p_t^x)^{-\eta_f} y_t^* \tag{2.26}$$

Here, Y_t^* is foreign GDP and P_t^* is the foreign currency price of foreign homogeneous goods. Also, P_t^x is an index of export prices, whose determination is discussed below. The goods, X_t, are produced by a representative, competitive foreign retailer firm using specialized inputs as follows:

$$X_t = \left[\int_0^1 X_{i,t}^{\frac{1}{\lambda_x}} di\right]^{\lambda_x}. \tag{2.27}$$

Here, $X_{i,t}$, $i \in (0,1)$, are exports of specialized goods. The retailer that produces X_t takes its output price, P_t^x, and its input prices, $P_{i,t}^x$, as given. Optimization leads to the following demand for specialized exports:

$$X_{i,t} = \left(\frac{P_{i,t}^x}{P_t^x}\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} X_t. \tag{2.28}$$

Combining (2.27) and (2.28), we obtain:

$$P_t^x = \left[\int_0^1 \left(P_{i,t}^x\right)^{\frac{1}{1-\lambda_{x,t}}} di\right]^{1-\lambda_{x,t}}.$$

The i^{th} specialized export is produced by a monopolist using the following technology:

$$X_{i,t} = \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x-1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x - 1}},$$

where $X_{i,t}^m$ and $X_{i,t}^d$ are the i^{th} exporter's use of the imported and domestically produced goods, respectively. We derive the marginal cost associated with the CES production function from the multiplier associated with the Lagrangian representation of the cost minimization problem:

$$C = \min \tau_t^x \left[P_t^{m,x} R_t^x X_{i,t}^m + P_t R_t^x X_{i,t}^d \right] + \lambda \left\{ X_{i,t} - \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x-1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x - 1}} \right\},$$

where $P_t^{m,x}$ is the price of the homogeneous import good and P_t is the price of the homogeneous domestic output good. The first order conditions are:

$$\begin{aligned} \tau_t^x R_t^x P_t^{m,x} &= \lambda X_{i,t}^{\frac{1}{\eta_x}} \omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{-1}{\eta_x}} \\ \tau_t^x R_t^x P_t &= \lambda X_{i,t}^{\frac{1}{\eta_x}} (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{-1}{\eta_x}} \\ X_{i,t} &= \left[\omega_x^{\frac{1}{\eta_x}} \left(X_{i,t}^m\right)^{\frac{\eta_x - 1}{\eta_x}} + (1-\omega_x)^{\frac{1}{\eta_x}} \left(X_{i,t}^d\right)^{\frac{\eta_x-1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x - 1}} \end{aligned}$$

Use the first two conditions to solve for the inputs as a function of the exogenous variables and the multiplier:

$$\left(X_{i,t}^m\right)^{\frac{\eta_x-1}{\eta_x}} = \frac{\lambda^{\eta_x - 1} X_{i,t}^{\frac{\eta_x-1}{\eta_x}} \omega_x^{\frac{\eta_x-1}{\eta_x}}}{\left(\tau_t^x R_t^x P_t^{m,x}\right)^{\eta_x - 1}} \tag{2.29}$$

$$\left(X_{i,t}^d\right)^{\frac{\eta_x-1}{\eta_x}} = \frac{\lambda^{\eta_x - 1} X_{i,t}^{\frac{\eta_x-1}{\eta_x}} (1-\omega_x)^{\frac{\eta_x-1}{\eta_x}}}{\left(\tau_t^x R_t^x P_t\right)^{\eta_x - 1}}. \tag{2.30}$$

Substitute these into the production function, to get:

$$X_{i,t} = \lambda^{\eta_x} X_{i,t} \left(\tau_t^x\right)^{-\eta_x} \left(\frac{\omega_x}{\left(R_t^x P_t^{m,x}\right)^{\eta_x - 1}} + \frac{(1-\omega_x)}{\left(R_t^x P_t\right)^{\eta_x - 1}} \right)^{\frac{\eta_x}{\eta_x - 1}}.$$

Nominal marginal cost is λ, so that real (in terms of the homogeneous final export good) marginal cost, mc_t^x, is

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{S_t P_t^x} \left[\omega_x \left(P_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x) \left(P_t\right)^{1-\eta_x} \right]^{\frac{1}{1-\eta_x}},$$

where

$$R_t^x = \nu_t^x R_t + 1 - \nu_t^x. \tag{2.31}$$

We rewrite the expression for marginal cost to get it in terms of stationary variables

$$mc_t^x = \frac{\lambda}{S_t P_t^x} = \frac{\tau_t^x R_t^x}{q_t p_t^c p_t^x}\left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{1}{1-\eta_x}}, \tag{2.32}$$

where we have used

$$\frac{S_t P_t^x}{P_t} = \frac{S_t P_t^*}{P_t^c}\frac{P_t^c}{P_t}\frac{P_t^x}{P_t^*} = q_t p_t^c p_t^x. \tag{2.33}$$

The i^{th}, $i \in (0,1)$, domestic exporting firm takes (2.28) as its demand curve. This producer sets prices subject to a Calvo sticky-price mechanism. In a given period, $1-\xi_x$ producers can reoptimize their price and ξ_x cannot. The firms that cannot optimize price, do so as follows:

$$P_{i,t}^x = \tilde{\pi}_t^x P_{i,t-1}^x,\ \tilde{\pi}_t^x = \left(\pi_{t-1}^x\right)^{\kappa_x} \left(\pi^x\right)^{1-\kappa_x-\varkappa_x} \left(\breve{\pi}\right)^{\varkappa_x}, \tag{2.34}$$

where $\kappa_x, \varkappa_x, \kappa_x + \varkappa_x \in (0,1)$.

The equilibrium conditions associated with price setting by exporters are analogous to the ones derived for domestic intermediate good producers:[3]

$$E_t\left[\psi_{z^+,t} q_t p_t^c p_t^x x_t + \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{1}{1-\lambda_x}} \beta\xi_x F_{x,t+1} - F_{x,t}\right] = 0 \tag{2.35}$$

$$E_t\left[\lambda_x \psi_{z^+,t} q_t p_t^c p_t^x x_t mc_t^x + \beta\xi_x \left(\frac{\tilde{\pi}_{t+1}^x}{\pi_{t+1}^x}\right)^{\frac{\lambda_x}{1-\lambda_x}} K_{x,t+1} - K_{x,t}\right] = 0, \tag{2.36}$$

$$\mathring{p}_t^x = \left[(1-\xi_x)\left(\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right)^{\lambda_x} + \xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\mathring{p}_{t-1}^x\right)^{\frac{\lambda_x}{1-\lambda_x}}\right]^{\frac{1-\lambda_x}{\lambda_x}} \tag{2.37}$$

$$\left[\frac{1-\xi_x\left(\frac{\tilde{\pi}_t^x}{\pi_t^x}\right)^{\frac{1}{1-\lambda_x}}}{1-\xi_x}\right]^{(1-\lambda_x)} = \frac{K_{x,t}}{F_{x,t}} \tag{2.38}$$

The quantity of the domestic homogeneous good used by specialized exporters is:

$$\int_0^1 X_{i,t}^d di,$$

[3]When we linearize around steady state and $\varkappa_{m,j} = 0$, equations (2.35)-(2.38) reduce to:

$$\begin{aligned}\hat{\pi}_t^x &= \frac{\beta}{1+\kappa_x\beta}E_t\hat{\pi}_{t+1}^x + \frac{\kappa_x}{1+\kappa_x\beta}\hat{\pi}_{t-1}^x \\ &\quad + \frac{1}{1+\kappa_x\beta}\frac{(1-\beta\xi_x)(1-\xi_x)}{\xi_x}\widehat{mc}_t^x,\end{aligned}$$

where a hat over a variable indicates log deviation from steady state.

and this needs to be expressed in terms of aggregates. Rewriting (2.30), one of the first order conditions of the foreign retailer who purchases the specialized export goods:

$$X_{i,t}^{d}=\left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x} X_{i,t}\left(1-\omega_x\right).$$

Integrating this expression:

$$\begin{aligned}\int_0^1 X_{i,t}^d di &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}\left(1-\omega_x\right)\int_0^1 X_{i,t}di \qquad (2.39)\\ &= \left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}\left(1-\omega_x\right)X_t\frac{\int_0^1\left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}di}{\left(P_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}}.\end{aligned}$$

Define $\mathring{P}_t^x$, a linear homogeneous function of $P_{i,t}^x$:

$$\mathring{P}_t^x=\left[\int_0^1\left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}di\right]^{\frac{\lambda_{x,t}-1}{-\lambda_{x,t}}}.$$

Then,

$$\left(\mathring{P}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}=\int_0^1\left(P_{i,t}^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}di,$$

and

$$\int_0^1 X_{i,t}^d di=\left(\frac{\lambda}{\tau_t^x R_t^x P_t}\right)^{\eta_x}\left(1-\omega_x\right)X_t\left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}, \qquad (2.40)$$

where

$$\mathring{p}_t^x\equiv\frac{\mathring{P}_t^x}{P_t^x},$$

and the law of motion of $\mathring{p}_t^x$ is given in (2.37).

We now simplify (2.40). Rewriting the second equality in (2.32), we obtain:

$$\frac{\lambda}{P_t\tau_t^x R_t^x}=\frac{S_t P_t^x}{P_t q_t p_t^c p_t^x}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x}+\left(1-\omega_x\right)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t\tau_t^x R_t^x}=\frac{S_t P_t^x}{P_t\frac{S_tP_t^*}{P_t^c}\frac{P_t^c}{P_t}\frac{P_t^x}{P_t^*}}\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x}+\left(1-\omega_x\right)\right]^{\frac{1}{1-\eta_x}},$$

or,

$$\frac{\lambda}{P_t\tau_t^x R_t^x}=\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x}+\left(1-\omega_x\right)\right]^{\frac{1}{1-\eta_x}}.$$

Substituting into (2.40), we obtain:

$$X_t^d=\int_0^1 X_{i,t}^d di=\left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x}+\left(1-\omega_x\right)\right]^{\frac{\eta_x}{1-\eta_x}}\left(1-\omega_x\right)\left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}}\left(p_t^x\right)^{-\eta_f}Y_t^* \qquad (2.41)$$

We also require an expression for imported inputs for exports in terms of aggregates. Using (2.29) and (2.32),

$$X_{i,t}^{m}=\omega_{x}\left(\frac{\left[\omega_{x}\left(p_{t}^{m,x}\right)^{1-\eta_{x}}+\left(1-\omega_{x}\right)\right]^{\frac{1}{1-\eta_{x}}}}{p_{t}^{m,x}}\right)^{\eta_{x}}X_{i,t}$$

The object on the left side of the equality is the quantity of the homogeneous import good used by the i^{th} specialized exporter. (This is to be distinguished from the output of the i^{th} specialized importer, which has the same notation.) The unweighted integral of $X_{i,t}^{m}$ is X_{t}^{m} because X_{t}^{m} is a homogeneous good. This is the same unweighted integral considered in (2.39). Using the result derived in (2.39), we obtain:

$$X_{t}^{m}=\omega_{x}\left(\frac{\left[\omega_{x}\left(p_{t}^{m,x}\right)^{1-\eta_{x}}+\left(1-\omega_{x}\right)\right]^{\frac{1}{1-\eta_{x}}}}{p_{t}^{m,x}}\right)^{\eta_{x}}\left(\mathring{p}_{t}^{x}\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t-1}}}X_{t},$$

where the law of motion of $\mathring{p}_{t}^{x}$ is given in (2.37). Note how the impact of price dispersion operates in the previous expression. For a given total of the homogeneous input good, X_{t}^{m}, one obtains less of the homogeneous output good, X_{t}, to the extent that there is price dispersion.

After scaling the preceding expression and substituting out for X_{t} using the demand for exports (see (2.26)), we obtain

$$x_{t}^{m}=\omega_{x}\left(\frac{\left[\omega_{x}\left(p_{t}^{m,x}\right)^{1-\eta_{x}}+\left(1-\omega_{x}\right)\right]^{\frac{1}{1-\eta_{x}}}}{p_{t}^{m,x}}\right)^{\eta_{x}}\left(\mathring{p}_{t}^{x}\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t-1}}}\left(p_{t}^{x}\right)^{-\eta_{f}}y_{t}^{*} \tag{2.42}$$

2.4.2. Imports

We now turn to a discussion of imports. Foreign firms sell a homogeneous good to domestic importers. The importers convert the homogeneous good into a specialized input (they 'brand name it') and supply that input monopolistically to domestic retailers. Importers are subject to Calvo price setting frictions. There are three types of importing firms: (i) one produces goods used to produce an intermediate good for the production of consumption, (ii) one produces goods used to produce an intermediate good for the production of investment goods, and (iii) one produces an intermediate good used for the production of an input into the production of export goods.

Consider (i) first. The production function of the domestic retailer of imported consumption goods is:

$$C_{t}^{m}=\left[\int_{0}^{1}\left(C_{i,t}^{m}\right)^{\frac{1}{\lambda^{m,C}}}di\right]^{\lambda^{m,C}},$$

where $C_{i,t}^m$ is the output of the i^{th} specialized producer and C_t^m is an intermediate good used in the production of consumption goods. Let $P_t^{m,c}$ denote the price index of C_t^m and let $P_{i,t}^{m,c}$ denote the price of the i^{th} intermediate input. The domestic retailer is competitive and takes $P_t^{m,c}$ and $P_{i,t}^{m,c}$ as given. In the usual way, the demand curve for specialized inputs is given by the domestic retailer's first order necessary condition for profit maximization:

$$C_{i,t}^m = C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda^{m,C}}{\lambda^{m,C}-1}}.$$

We now turn to the producer of $C_{i,t}^m$, who takes the previous equation as a demand curve. This producer buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $C_{i,t}^m$. The intermediate good producer's marginal cost is

$$\tau_t^{m,c} S_t P_t^* R_t^{\nu,*}, \tag{2.43}$$

where

$$R_t^{\nu,*} = \nu_t^* R_t^* + 1 - \nu_t^*, \tag{2.44}$$

and R_t^* is the foreign nominal, intratemporal rate of interest. In addition, $\tau_t^{m,c}$ is a tax term. The notion here is that the intermediate good firm must pay the inputs with foreign currency and because they have no resources themselves at the beginning of the period, they must borrow those resources if they are to buy the foreign inputs needed to produce $C_{i,t}^m$. There is no risk to this firm, because all shocks are realized at the beginning of the period, and so there is no uncertainty within the duration of the working capital loan about the realization of prices and exchanges rates.[4]

It is of interest to have a measure of the total imports of the intermediate good producers:

$$S_t P_t^* R_t^{\nu,*} \int_0^1 C_{i,t}^m di.$$

In order to relate this to C_t^m, we substitute the demand curve into the previous expression:

$$\begin{aligned}
& S_t P_t^* R_t^{\nu,*} \int_0^1 C_t^m \left(\frac{P_t^{m,c}}{P_{i,t}^{m,c}} \right)^{\frac{\lambda^{m,C}}{\lambda^{m,C}-1}} di \\
= \; & S_t P_t^* R_t^{\nu,*} C_t^m \left(P_t^{m,c} \right)^{\frac{\lambda^{m,C}}{\lambda^{m,C}-1}} \int_0^1 \left(P_{i,t}^{m,c} \right)^{\frac{-\lambda^{m,C}}{\lambda^{m,C}-1}} \\
= \; & S_t P_t^* R_t^{\nu,*} C_t^m \left(\frac{\mathring{P}_t^{m,c}}{P_t^{m,c}} \right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}},
\end{aligned}$$

[4]We are somewhat uncomfortable with this feature of the model. The fact that interest is due and matters indicates that some time evolves over the duration of the loan. Our assumption that no uncertainty is realized over a period of significant duration of time seems implausible. We suspect that a more realistic representation would involve some risk. Our timing assumptions in effect abstract away from this risk, and we conjecture that this does not affect the first order properties of the model.

where

$$\mathring{P}_t^{m,c} = \left[\int_0^1 \left(P_{i,t}^{m,c} \right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} \right]^{\frac{1-\lambda^{m,C}}{\lambda^{m,C}}}$$

We conclude that total imports account for by the consumption sector is:

$$S_t P_t^* R_t^{\nu,*} C_t^m \left(\mathring{p}_t^{m,c} \right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}}, \tag{2.45}$$

where

$$\mathring{p}_t^{m,c} = \frac{\mathring{P}_t^{m,c}}{P_t^{m,c}},$$

and $\mathring{p}_t^{m,c}$ is discussed below.

Now consider (ii). The production function for the domestic retailer of imported investment goods, I_t^m, is:

$$I_t^m = \left[\int_0^1 \left(I_{i,t}^m \right)^{\frac{1}{\lambda_t^{m,I}}} di \right]^{\lambda_t^{m,I}}.$$

The retailer of imported investment goods is competitive and takes output prices, $P_t^{m,i}$, and input prices, $P_{i,t}^{m,i}$, as given.

The producer of the i^{th} intermediate input into the above production function buys the homogeneous foreign good and converts it one-for-one into the domestic differentiated good, $I_{i,t}^m$. The marginal cost of $I_{i,t}^m$ is also (2.43). Note that this implies the importing firm's cost is P_t^* (before borrowing costs and exchange rate conversion), which is the same cost for the specialized inputs used to produce C_t^m. This may seem inconsistent with the property of the domestic economy that domestically produced consumption and investment goods have different relative prices. We assume that (2.43) applies to both types of producer in order to simplify notation. Below, we suppose that the efficiency of imported investment goods grows over time, in a way that makes our assumptions about the relative costs of consumption and investment, whether imported or domestically produced.

The total value of imports associated with the production of investment goods is analogous to what we obtained for the consumption good sector:

$$S_t P_t^* R_t^{\nu,*} I_t^m \left(\mathring{p}_t^{m,i} \right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}}, \ \mathring{p}_t^{m,i} = \frac{P_{i,t}^{m,i}}{P_t^{m,i}}, \tag{2.46}$$

where $\mathring{p}_t^{m,i}$ is discussed below.

Now consider (iii). The production function of the domestic retailer of imported goods used in the production of an input, X_t^m, for the production of export goods is:

$$X_t^m = \left[\int_0^1 \left(X_{i,t}^m \right)^{\frac{1}{\lambda_t^{m,X}}} di \right]^{\lambda_t^{m,X}}.$$

The imported good retailer is competitive, and takes output prices, $P_t^{m,x}$, and input prices, $P_{i,t}^{m,x}$, as given. The producer of the specialized input, $X_{i,t}^m$, has marginal cost, (2.43). The total value of imports associated with the production of X_t^m is:

$$S_t P_t^* R_t^{\nu,*} X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}},\ \mathring{p}_t^{m,x} = \frac{P_{i,t}^{m,x}}{P_t^{m,x}} \tag{2.47}$$

Each of the above three types of intermediate good firm is subject to Calvo price-setting frictions. With probability $1-\xi_{m,j}$, the j^{th} type of firm can reoptimize its price and with probability $\xi_{m,j}$ it sets price according to the following relation:

$$P_{i,t}^{m,j} = \tilde{\pi}_t^{m,j} P_{i,t-1}^{m,j},\ \tilde{\pi}_t^{m,j} \equiv \left(\pi_{t-1}^{m,j}\right)^{\kappa_{m,j}} \left(\bar{\pi}_t^c\right)^{1-\kappa_{m,j}-\varkappa_{m,j}} \breve{\pi}^{\varkappa_{m,j}}. \tag{2.48}$$

for $j = c, i, x$.

The equilibrium conditions associated with price setting by importers are analogous to the ones derived for domestic intermediate good producers:

$$E_t\left[\psi_{z^+,t} p_t^{m,j} \Xi_t^j + \left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}} \beta\xi_{m,j} F_{m,j,t+1} - F_{m,j,t}\right] = 0 \tag{2.49}$$

$$E_t\left[\lambda_{m,j}\psi_{z^+,t} p_t^{m,j} mc_t^{m,j} \Xi_t^j + \beta\xi_{m,j}\left(\frac{\tilde{\pi}_{t+1}^{m,j}}{\pi_{t+1}^{m,j}}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}} K_{m,j,t+1} - K_{m,j,t}\right] = 0, \tag{2.50}$$

$$\mathring{p}_t^{m,j} = \left[\left(1-\xi_{m,j}\right)\left(\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right)^{\lambda_{m,j}} + \xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\mathring{p}_{t-1}^{m,j}\right)^{\frac{\lambda_{m,j}}{1-\lambda_{m,j}}}\right]^{\frac{1-\lambda_{m,j}}{\lambda_{m,j}}} \tag{2.51}$$

$$\left[\frac{1-\xi_{m,j}\left(\frac{\tilde{\pi}_t^{m,j}}{\pi_t^{m,j}}\right)^{\frac{1}{1-\lambda_{m,j}}}}{1-\xi_{m,j}}\right]^{(1-\lambda_{m,j})} = \frac{K_{m,j,t}}{F_{m,j,t}}, \tag{2.52}$$

for $j = c, i, x$.[5] Here,

$$\Xi_t^j = \begin{cases} c_t^m & j = c \\ x_t^m & j = x \\ i_t^m & j = i \end{cases}.$$

Real after tax marginal cost is

$$\begin{aligned} mc_t^{m,j} &= \tau_t^{m,j} \frac{S_t P_t^*}{P_t^{m,j}} R_t^{\nu,*} = \tau_t^{m,j} \frac{S_t P_t^* P_t^c P_t}{P_t^c P_t^{m,j} P_t} R_t^{\nu,*} \\ &= \tau_t^{m,j} \frac{q_t p_t^c}{p_t^{m,j}} R_t^{\nu,*} \end{aligned} \tag{2.53}$$

for $j = c, i, x$.

2.5. Households

Household preferences are given by:

$$E_0^j \sum_{t=0}^{\infty} \beta^t \left[\zeta_t^c \ln \left(C_t - bC_{t-1} \right) - \zeta_t^h A_L \frac{\left(h_{j,t} \right)^{1+\sigma_L}}{1+\sigma_L} \right]. \tag{2.54}$$

The household owns the economy's stock of physical capital. It determines the rate at which the capital stock is accumulated and the rate at which it is utilized. The household owns the stock of net foreign assets and determines its rate of accumulation.

2.5.1. Technology for Capital Accumulation

The law of motion of the physical stock of capital is:

$$\bar{K}_{t+1} = (1-\delta) \bar{K}_t + \Upsilon_t F \left(I_t, I_{t-1} \right),$$

where

$$F \left(I_t, I_{t-1} \right) = \left(1 - \tilde{S} \left(\frac{I_t}{I_{t-1}} \right) \right) I_t,$$

and

$$\begin{aligned} \tilde{S}(x) &= \frac{1}{2} \left\{ \exp \left[\sqrt{\tilde{S}''} \left(x - \mu_{z^+} \mu_{\Psi} \right) \right] + \exp \left[-\sqrt{\tilde{S}''} \left(x - \mu_{z^+} \mu_{\Psi} \right) \right] - 2 \right\} \\ &= 0, \ x = \mu_{z^+} \mu_{\Psi}. \end{aligned} \tag{2.55}$$

[5] When we linearize around steady state and $\varkappa_{m,j} = 0$,

$$\begin{aligned} \hat{\pi}_t^{m,j} - \hat{\bar{\pi}}_t^c &= \frac{\beta}{1+\kappa_{m,j}\beta} E_t \left(\hat{\pi}_{t+1}^{m,j} - \hat{\bar{\pi}}_{t+1}^c \right) + \frac{\kappa_{m,j}}{1+\kappa_{m,j}\beta} \left(\hat{\pi}_{t-1}^{m,j} - \hat{\bar{\pi}}_t^c \right) \\ &\quad - \frac{\kappa_{m,j}\beta \left(1 - \rho_\pi \right)}{1+\kappa_{m,j}\beta} \hat{\bar{\pi}}_t^c \\ &\quad + \frac{1}{1+\kappa_{m,j}\beta} \frac{\left(1 - \beta\xi_{m,j} \right) \left(1 - \xi_{m,j} \right)}{\xi_{m,j}} \widehat{mc}_t^{m,j}, \end{aligned}$$

Also,

$$\begin{aligned}\tilde{S}'(x) &= \frac{1}{2}\sqrt{\tilde{S}''}\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]-\exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\} \\ &= 0,\ x=\mu_{z^+}\mu_\Psi.\end{aligned} \tag{2.56}$$

and

$$\begin{aligned}\tilde{S}''(x) &= \frac{1}{2}\tilde{S}''\left\{\exp\left[\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]+\exp\left[-\sqrt{\tilde{S}''}(x-\mu_{z^+}\mu_\Psi)\right]\right\} \\ &= \tilde{S}'',\ x=\mu_{z^+}\mu_\Psi.\end{aligned}$$

Also,

$$\begin{aligned}F_1(I_t,I_{t-1}) &= \left(1-\tilde{S}\left(\frac{I_t}{I_{t-1}}\right)\right)-\tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}} \\ &= 1,\ \frac{I_t}{I_{t-1}}=\mu_{z^+}\mu_\Psi,\end{aligned}$$

and,

$$\begin{aligned}F_2(I_t,I_{t-1}) &= \tilde{S}'\left(\frac{I_t}{I_{t-1}}\right)\left(\frac{I_t}{I_{t-1}}\right)^2 \\ &= 0,\ \frac{I_t}{I_{t-1}}=\mu_{z^+}\mu_\Psi.\end{aligned}$$

Scaling,

$$\begin{aligned}F(I_t,I_{t-1}) &= \left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)z_t^+\Psi_t i_t \\ F_1(I_t,I_{t-1}) &= \left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)-\tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}} \\ F_2(I_t,I_{t-1}) &= \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)^2\end{aligned}$$

In this notation, the law of motion of capital is written,

$$\bar{k}_{t+1}z_t^+\Psi_t=(1-\delta)\bar{K}_t z_{t-1}^+\Psi_{t-1}+\Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)z_t^+\Psi_t i_t,$$

or,

$$\bar{k}_{t+1}=\frac{1-\delta}{\mu_{z^+,t}\mu_{\Psi,t}}\bar{k}_t+\Upsilon_t\left(1-\tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\right)i_t. \tag{2.57}$$

2.5.2. Household Consumption and Investment Decisions

The first order condition for consumption is:

$$\frac{\zeta_t^c}{c_t - bc_{t-1}\frac{1}{\mu_{z^+,t}}} - \beta b E_t \frac{\zeta_{t+1}^c}{c_{t+1}\mu_{z^+,t+1} - bc_t} - \psi_{z^+,t} p_t^c \left(1+\tau_t^c\right) = 0. \tag{2.58}$$

To define the intertemporal Euler equation associated with the household's capital accumulation decision, we need to define the rate of return on a period t investment in a unit of physical capital, R_{t+1}^k :

$$R_{t+1}^k = \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}}a(u_{t+1})\right]P_{t+1} + (1-\delta)P_{t+1}P_{k',t+1} + \tau_t^k \delta P_t P_{k',t}}{P_t P_{k',t}}, \tag{2.59}$$

where it is convenient to recall

$$\frac{p_t^i}{\Psi_t}P_t = P_t^i,$$

the date t price of the homogeneous investment good. Here, $P_{k',t}$ denotes the price of a unit of newly installed physical capital, which operates in period $t+1$. This price is expressed in units of the homogeneous good, so that $P_t P_{k',t}$ is the domestic currency price of physical capital. The numerator in the expression for R_{t+1}^k represents the period $t+1$ payoff from a unit of additional physical capital. The timing of the capital tax rate reflects the assumption that the relevant tax rate is known at the time the investment decision is made. The expression in square brackets in (2.59) captures the idea that maintenance expenses associated with the operation of capital are deductible from taxes. The last expression in the numerator expresses the idea that physical depreciation is deductible at historical cost. It is convenient to express R_t^k in terms of scaled variables:

$$\begin{aligned}
R_{t+1}^k &= \frac{P_{t+1}\Psi_{t+1}}{P_t\Psi_{t+1}} \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - \frac{p_{t+1}^i}{\Psi_{t+1}}a(u_{t+1})\right] + (1-\delta)P_{k',t+1} + \tau_t^k \delta \frac{P_t}{P_{t+1}} P_{k',t}}{P_{k',t}} \\
&= \pi_{t+1} \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)\Psi_{t+1}P_{k',t+1} + \tau_t^k \delta \frac{P_t}{P_{t+1}} \Psi_{t+1} P_{k',t}}{\Psi_{t+1}P_{k',t}}.
\end{aligned}$$

so that

$$R_{t+1}^k = \frac{\pi_{t+1}}{\mu_{\Psi,t+1}} \frac{\left(1-\tau_t^k\right)\left[u_{t+1}\bar{r}_{t+1}^k - p_{t+1}^i a(u_{t+1})\right] + (1-\delta)p_{k',t+1} + \tau_t^k \delta \frac{\mu_{\Psi,t+1}}{\pi_{t+1}} p_{k',t}}{p_{k',t}}. \tag{2.60}$$

Capital is a good hedge against inflation, except for the way depreciation is treated. A rise in inflation effectively raises the tax rate on capital because of the practice of valuing depreciation at historical cost. The first order condition for capital implies:

$$\psi_{z^+,t} = \beta E_t \psi_{z^+,t+1} \frac{R_{t+1}^k}{\pi_{t+1}\mu_{z^+,t+1}}. \tag{2.61}$$

We differentiate the Lagrangian representation of the household's problem as displayed in ALLV, with respect to I_t :

$$-\upsilon_t P_t^i + \omega_t \Upsilon_t F_1\left(I_t, I_{t-1}\right) + \beta\omega_{t+1}\Upsilon_{t+1}F_2\left(I_{t+1}, I_t\right) = 0,$$

where υ_t denotes the multiplier on the household's nominal budget constraint and ω_t denotes the multiplier on the capital accumulation technology. In addition, the price of capital is the ratio of these multipliers:

$$P_t P_{k',t} = \frac{\omega_t}{\upsilon_t}.$$

Expressing the investment first order condition in terms of scaled variables,

$$\begin{aligned} -\frac{\psi_{z^+,t}}{z_t^+}\frac{p_t^i}{\Psi_t} + \upsilon_t P_t P_{k',t}\Upsilon_t \left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] & \\ +\beta\upsilon_{t+1}P_{t+1}P_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)^2 &= 0. \end{aligned}$$

Now multiply by $z_t^+\Psi_t$

$$\begin{aligned} -\psi_{z^+,t}p_t^i + \psi_{z^+,t}p_{k',t}\Upsilon_t\left[1 - \tilde{S}\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right) - \tilde{S}'\left(\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right)\frac{\mu_{z^+,t}\mu_{\Psi,t}i_t}{i_{t-1}}\right] & \\ +\beta\psi_{z^+,t+1}p_{k',t+1}\Upsilon_{t+1}\tilde{S}'\left(\frac{\mu_{z^+,t+1}\mu_{\Psi,t+1}i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\mu_{\Psi,t+1}\mu_{z^+,t+1} &= 0. \end{aligned} \tag{2.62}$$

Our first order condition for I_t appears to differ slightly from the first order condition in ALLV, equation (2.55), but the two actually coincide when we take into account the definition of f.

The first order condition associated with capital utilization is:

$$\Psi_t r_t^k = p_t^i a'\left(u_t\right),$$

or, in scaled terms,

$$\bar{r}_t^k = p_t^i a'\left(u_t\right). \tag{2.63}$$

The tax rate on capital income does not enter here because of the deductibility of maintenance costs.

2.5.3. Financial Assets

The household does the economy's saving. Period t saving occurs by the acquisition of net foreign assets, A_{t+1}^*, and a domestic asset. The domestic asset is used to finance the working capital requirements of firms. This asset pays a nominally non-state contingent return from t to $t+1$, R_t. The first order condition associated with this asset is:

$$-\psi_{z^+,t} + \beta E_t \frac{\psi_{z^+,t+1}}{\mu_{z^+,t+1}}\left[\frac{R_t - \tau_t^b\left(R_t - \pi_{t+1}\right)}{\pi_{t+1}}\right] = 0, \tag{2.64}$$

where τ_t^b is the tax rate on the real interest rate on bond income (for additional discussion of τ^b, see section 2.6.) A consequence of our treatment of the taxation on domestic bonds is that the steady state real after tax return on bonds is invariant to π.

In the model the tax treatment of domestic agents' earnings on foreign bonds is the same as the tax treatment of agents' earnings on foreign bonds. The scaled date t first order condition associated with A_{t+1}^*:

$$\upsilon_t S_t = \beta E_t \upsilon_{t+1}[S_{t+1} R_t^* \Phi_t - \tau^b \left(S_{t+1} R_t^* \Phi_t - \frac{S_t}{P_t} P_{t+1} \right)]. \tag{2.65}$$

Recall that S_t is the domestic currency price of a unit of foreign currency. On the left side of this expression, we have the cost of acquiring a unit of foreign assets. The currency cost is S_t and this is converted into utility terms by multiplying by the multiplier on the household's budget constraint, υ_t. The term in square brackets is the after tax payoff of the foreign asset, in domestic currency units. The first term is the period $t+1$ pre-tax interest payoff on A_{t+1}^*, which is $S_{t+1} R_t^* \Phi_t$. Here, R_t^* is the foreign nominal rate of interest, which is risk free in foreign currency units. The term, Φ_t represents a risk adjustment, so that a unit of the foreign asset acquired in t pays off $R_t^* \Phi_t$ units of foreign currency in $t+1$. The determination of Φ_t is discussed below. The remaining term in parentheses pertains to the impact of taxation on the return on foreign assets. If we ignore the term after the minus sign in parentheses, then we see that taxation is applied to the whole nominal payoff on the bond, including principle. The term after the minus sign is designed to ensure that the principal is deducted from taxes. The principal is expressed in nominal terms and is set so that the real value at $t+1$ coincides with the real value of the currency used to purchase the asset in period t. In particular, recall that S_t is the period t domestic currency cost of a unit (in terms of foreign currency) of foreign assets. So, the period t real cost of the asset is S_t/P_t. The domestic currency value in period $t+1$ of this real quantity is $P_{t+1} S_t / P_t$.

We scale the first order condition, (2.65), by multiplying both sides by $P_t z_t^+ / S_t$:

$$\psi_{z^+,t} = \beta E_t \frac{\psi_{z^+,t+1}}{\pi_{t+1} \mu_{z^+,t+1}} [s_{t+1} R_t^* \Phi_t - \tau_t^b \left(s_{t+1} R_t^* \Phi_t - \pi_{t+1} \right)], \tag{2.66}$$

where, recall,

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+, \; s_t = \frac{S_t}{S_{t-1}}.$$

The risk adjustment term has the following form:

$$\Phi_t = \Phi \left(a_t, E_t s_{t+1} s_t, \tilde{\phi}_t \right) = \exp \left(-\tilde{\phi}_a \left(a_t - \bar{a} \right) - \tilde{\phi}_s \left(E_t s_{t+1} s_t - s^2 \right) + \tilde{\phi}_t \right), \tag{2.67}$$

where, recall,

$$a_t = \frac{S_t A_{t+1}}{P_t z_t^+},$$

and $\tilde{\phi}_t$ is a mean zero shock whose law of motion is discussed below. In addition, $\tilde{\phi}_a$, $\tilde{\phi}_s$, $\bar{a}$ are positive parameters. In the steady state discussion in the appendix, we derive the equilibrium outcomes that a_t coincides with $\bar{a}$ and $\Phi_t = 1$ in non-stochastic steady state.

The dependence of Φ_t on a_t ensures, in the usual way, that there is a unique steady state value of a_t that is independent of the initial net foreign assets and capital of the economy. The dependence of Φ_t on the anticipated growth rate of the exchange rate is designed to allow the model to reproduce two types of observations. The first concerns observations related uncovered interest parity. The second concerns the hump-shaped response of output to a monetary policy shock.

We first consider interest rate parity. To understand this, consider the standard text book representation of uncovered interest parity:

$$R_t - R_t^* = E_t \log S_{t+1} - \log S_t + \phi_t,$$

where ϕ_t denotes the risk premium on domestic assets. A log linear approximation of our model implies the above expression in which ϕ_t corresponds to the log deviation of Φ_t about its steady state value of unity. Consider first the case in which $\phi_t \equiv 0$. In this case, a fall in R_t relative to R_t^* produces an anticipated appreciation of the currency. This drop in $E_t \log S_{t+1} - \log S_t$ is accomplished in part by an instantaneous depreciation in $\log S_t$. The idea behind this is that asset holders respond to the unfavorable domestic rate of return by attempting to sell domestic assets and acquire foreign exchange for the purpose of acquiring foreign assets. This selling pressure pushes $\log S_t$ up, until the anticipated appreciation precisely compensates traders in international financial assets holding domestic assets.

There are two types of evidence that the preceding scenario does not hold in the data. Vector autoregression evidence on the response of financial variables to an expansionary domestic monetary policy shock suggests that $E_t \log S_{t+1} - \log S_t$ actually rises for a period of time (see, e.g., Eichenbaum and Evans (1995)). Also, regressions of realized future exchange rate changes on current interest rate differentials fail to produce the expected value of unity. Indeed, the typical result is a statistically significant negative coefficient.

One interpretation of these results is that when the domestic interest rate is reduced, say by a monetary policy shock, then risk in the domestic economy falls and that alone makes traders happier to hold domestic financial assets in spite of their lower nominal return and the losses they expect to make in the foreign exchange market. Our functional form for ϕ_t ALLV is designed to capture this idea. According to this functional form, when a shock occurs which causes an anticipated depreciation in the level of the exchange rate, then the assessment of risk in the domestic economy, Φ_t, falls. Later, when we introduce financial frictions, we will have access to additional mechanism for achieving this outcome. A concern we have with the current model is its unfortunate implication that any shock that creates an expectation of a depreciation in the currency makes domestic financial assets seem less risky. As a general proposition, this seems implausible. When we consider financial frictions, we will have variables such as the bankruptcy rate, which falls in the wake of an expansionary monetary shock, and which may accomplish the same equilibrium outcome as the ALLV specification, though perhaps the mechanism is more plausible in this case.

When we turn to the regression interpretation of the uncovered interest parity result, it is useful to consider the regression coefficient:

$$\gamma = \frac{cov\left(\log S_{t+1} - \log S_t, R_t - R_t^*\right)}{var\left(R_t - R_t^*\right)} = \frac{cov\left(R_t - R_t^* - \phi_t, R_t - R_t^*\right)}{var\left(R_t - R_t^*\right)},$$

according to our linearized expression above. Then,

$$\gamma = 1 - \frac{cov\left(R_t - R_t^*, \phi_t\right)}{var\left(R_t - R_t^*\right)}.$$

Thus, any specification of ϕ_t which causes it to have a positive covariance with the interest rate differential will help in accounting for the regression coefficient specification of the uncovered interest rate puzzle. That is, such a covariance could result in γ being negative. This motivates an alternative to the risk specification in (2.67):

$$\Phi_t = \Phi\left(a_t, R_t^* - R_t, \tilde{\phi}_t\right) = \exp\left(-\tilde{\phi}_a\left(a_t - \bar{a}\right) - \tilde{\phi}_s\left(R_t^* - R_t - \left(R^* - R\right)\right) + \tilde{\phi}_t\right), \quad (2.68)$$

where a variable without time subscript denotes the corresponding value in nonstochastic steady state. We use this specification in our benchmark model.

We now turn to the connection between Φ_t and the hump-shape response of output to a monetary policy shock. As explained in section 2.4, a key ingredient in obtaining this type of response lies in factors that slow the response of demand to an expansionary monetary policy shock. The response of foreign purchases of domestic goods in the wake of such a shock depends on how much the exchange depreciates. The mechanism we have described slows the depreciation, and this simultaneously reduces the expansion of foreign demand.

2.5.4. Wage Setting

Finally, we consider wage setting. We suppose that the specialized labor supplied by households is combined by labor contractors into a homogeneous labor service as follows:

$$H_t = \left[\int_0^1 \left(h_{j,t}\right)^{\frac{1}{\lambda_w}} dj\right]^{\lambda_w}, \ 1 \leq \lambda_w < \infty,$$

where h_j denotes the j^{th} household supply of labor services. Households are subject to Calvo wage setting frictions as in Erceg, Henderson and Levin (2000) (EHL). With probability

$1-\xi_w$ the j^{th} household is able to reoptimize its wage and with probability ξ_w it sets its wage according to:

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{2.69}$$
$$\tilde{\pi}_{w,t+1} = \left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}, \tag{2.70}$$

where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$. The wage updating factor, $\tilde{\pi}_{w,t+1}$, is sufficiently flexible that we can adopt a variety of interesting schemes.

Consider the j^{th} household that has an opportunity to reoptimize its wage at time t. We denote this wage rate by $\tilde{W}_t$. This is not indexed by j because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t^j \sum_{i=0}^{\infty} \left(\beta \xi_w\right)^i \left[-\zeta_{t+i}^h A_L \frac{\left(h_{j,t+i}\right)^{1+\sigma_L}}{1+\sigma_L} + \upsilon_{t+i} W_{j,t+i} h_{j,t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w} \right],$$

where τ_t^y is a tax on labor income and τ_t^w is a payroll tax. Also, υ_t is the multiplier on the household's period t budget constraint. The demand for the j^{th} household's labor services, conditional on it having optimized in period t and not again since, is:

$$h_{j,t+i} = \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i}.$$

Here, it is understood that $\tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \equiv 1$ when $i = 0$. Substituting this into the objective function,

$$E_t^j \sum_{i=0}^{\infty} \left(\beta \xi_w\right)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L}$$
$$+\upsilon_{t+i} \tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1} \left(\frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}],$$

It is convenient to recall the scaling of variables:

$$\psi_{z^+,t} = \upsilon_t P_t z_t^+, \; \bar{w}_t = \frac{W_t}{z_t^+ P_t}, \; \tilde{y}_t = \frac{Y_t}{z_t^+}, \; w_t = \tilde{W}_t / W_t, \; z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t.$$

Then,

$$\begin{aligned} \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{W_{t+i}} &= \frac{\tilde{W}_t \tilde{\pi}_{w,t+i} \cdots \tilde{\pi}_{w,t+1}}{\bar{w}_{t+i} z_{t+i}^+ P_{t+i}} = \frac{\tilde{W}_t}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} \\ &= \frac{W_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i} z_t^+ P_t} X_{t,i} = \frac{\bar{w}_t \left(\tilde{W}_t / W_t\right)}{\bar{w}_{t+i}} X_{t,i} = \frac{w_t \bar{w}_t}{\bar{w}_{t+i}} X_{t,i}, \end{aligned}$$

where

$$\begin{aligned} X_{t,i} &= \frac{\tilde{\pi}_{w,t+i}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+i}\pi_{t+i-1}\cdots\pi_{t+1}\mu_{z^+,t+i}\cdots\mu_{z^+,t+1}},\ i>0 \\ &= 1,\ i=0. \end{aligned}$$

It is interesting to investigate the value of $X_{t,i}$ in steady state, as $i\to\infty$. Thus,

$$X_{t,i}=\frac{\left(\pi_t^c\cdots\pi_{t+i-1}^c\right)^{\kappa_w}\left(\bar{\pi}_{t+1}^c\cdots\bar{\pi}_{t+i}^c\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}^i\right)^{\varkappa_w}\left(\mu_{z^+}^i\right)^{\vartheta_w}}{\pi_{t+i}\pi_{t+i-1}\cdots\pi_{t+1}\mu_{z^+,t+i}\cdots\mu_{z^+,t+1}}$$

In steady state,

$$\begin{aligned} X_{t,i} &= \frac{\left(\bar{\pi}^i\right)^{\kappa_w}\left(\bar{\pi}^i\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}^i\right)^{\varkappa_w}\left(\mu_{z^+}^i\right)^{\vartheta_w}}{\bar{\pi}^i\mu_{z^+}^i} \\ &= \left(\frac{\breve{\pi}^i}{\bar{\pi}^i}\right)^{\varkappa_w}\left(\mu_{z^+}^i\right)^{\vartheta_w-1} \\ &\to 0, \end{aligned}$$

in the no-indexing case, when $\breve{\pi}=1$, $\varkappa_w=1$ and $\vartheta_w=0$.

Simplifying using the scaling notation,

$$\begin{aligned} &E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\frac{\left(\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\ &+\upsilon_{t+i}W_{t+i}\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}], \end{aligned}$$

or,

$$\begin{aligned} &E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\frac{\left(\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L} \\ &+\psi_{z^+,t+i}w_t\bar{w}_tX_{t,i}\left(\frac{w_t\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}], \end{aligned}$$

or,

$$\begin{aligned} &E_t^j\sum_{i=0}^{\infty}\left(\beta\xi_w\right)^i[-\zeta_{t+i}^h A_L\frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\right)^{1+\sigma_L}}{1+\sigma_L}w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \\ &+\psi_{z^+,t+i}w_t^{1+\frac{\lambda_w}{1-\lambda_w}}\bar{w}_tX_{t,i}\left(\frac{\bar{w}_t}{\bar{w}_{t+i}}X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}}H_{t+i}\frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}], \end{aligned}$$

Differentiating with respect to w_t,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \frac{\left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{1+\sigma_L} \lambda_w (1+\sigma_L) w_t^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)-1}$$
$$+\psi_{z^+,t+i} w_t^{\frac{\lambda_w}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}] = 0$$

Dividing and rearranging,

$$E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i [-\zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}$$
$$+ \frac{\psi_{z^+,t+i}}{\lambda_w} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}] = 0$$

Solving for the wage rate:

$$\begin{aligned} w_t^{\frac{1-\lambda_w(1+\sigma_L)}{1-\lambda_w}} &= \frac{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h A_L \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L}}{E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} \bar{w}_t X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}} \\ &= \frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}} \end{aligned}$$

where

$$\begin{aligned} K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\ F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w}. \end{aligned}$$

Thus, the wage set by reoptimizing households is:

$$w_t = \left[\frac{A_L K_{w,t}}{\bar{w}_t F_{w,t}} \right]^{\frac{1-\lambda_w}{1-\lambda_w(1+\sigma_L)}}.$$

We now express $K_{w,t}$ and $F_{w,t}$ in recursive form:

$$
\begin{aligned}
K_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \zeta_{t+i}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \right)^{1+\sigma_L} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w \zeta_{t+1}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \right)^{1+\sigma_L} \\
&\quad + (\beta\xi_w)^2 \zeta_{t+2}^h \left(\left(\frac{\bar{w}_t}{\bar{w}_{t+2}} \frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \\
&\quad + ...
\end{aligned}
$$

or,

$$
\begin{aligned}
K_{w,t} &= \zeta_t^h H_t^{1+\sigma_L} + E_t \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} \{\zeta_{t+1}^h H_{t+1}^{1+\sigma_L} \\
&\quad + \beta\xi_w \left(\left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}} \frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}} \right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \right)^{1+\sigma_L} \zeta_{t+2}^h + ...\} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\bar{w}_t}{\bar{w}_{t+1}} \frac{\left(\pi_t^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}\right)^{\varkappa_w} \left(\mu_{z^+}\right)^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \\
&= \zeta_t^h H_t^{1+\sigma_L} + \beta\xi_w E_t \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1},
\end{aligned}
$$

using,

$$
\pi_{w,t+1} = \frac{W_{t+1}}{W_t} = \frac{\bar{w}_{t+1} z_{t+1}^+ P_{t+1}}{\bar{w}_t z_t^+ P_t} = \frac{\bar{w}_{t+1}\mu_{z^+,t+1}\pi_{t+1}}{\bar{w}_t} \tag{2.71}
$$

Also,

$$
\begin{aligned}
F_{w,t} &= E_t^j \sum_{i=0}^{\infty} (\beta\xi_w)^i \frac{\psi_{z^+,t+i}}{\lambda_w} X_{t,i} \left(\frac{\bar{w}_t}{\bar{w}_{t+i}} X_{t,i}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_{t+i} \frac{1-\tau_{t+i}^y}{1+\tau_{t+i}^w} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} \\
&\quad + \beta\xi_w \frac{\psi_{z^+,t+1}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+1} \frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w} \\
&\quad + (\beta\xi_w)^2 \frac{\psi_{z^+,t+2}}{\lambda_w} \left(\frac{\bar{w}_t}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \\
&\quad \times \left(\frac{\left(\pi_t^c \pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+1}^c \bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} \left(\breve{\pi}^2\right)^{\varkappa_w} \left(\mu_{z^+}^2\right)^{\vartheta_w}}{\pi_{t+2}\pi_{t+1}\mu_{z^+,t+2}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} H_{t+2} \frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w} \\
&\quad + \ldots
\end{aligned}
$$

or,

$$
\begin{aligned}
F_{w,t} &= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_t}{\bar{w}_{t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{(\pi_t^c)^{\kappa_w} \left(\bar{\pi}_{t+1}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+1}\mu_{z^+,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \{ \frac{\psi_{z^+,t+1}}{\lambda_w} H_{t+1} \frac{1-\tau_{t+1}^y}{1+\tau_{t+1}^w} \\
&\quad + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_{t+2}}\right)^{\frac{\lambda_w}{1-\lambda_w}} \left(\frac{\left(\pi_{t+1}^c\right)^{\kappa_w} \left(\bar{\pi}_{t+2}^c\right)^{(1-\kappa_w-\varkappa_w)} (\breve{\pi})^{\varkappa_w} (\mu_{z^+})^{\vartheta_w}}{\pi_{t+2}\mu_{z^+,t+2}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} \frac{\psi_{z^+,t+2}}{\lambda_w} H_{t+2} \frac{1-\tau_{t+2}^y}{1+\tau_{t+2}^w} \\
&\quad + \ldots \} \\
&= \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
\end{aligned}
$$

so that

$$
F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} H_t \frac{1-\tau_t^y}{1+\tau_t^w} + \beta\xi_w E_t \left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right) \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1},
$$

We obtain a second restriction on w_t using the relation between the aggregate wage rate and the wage rates of individual households:

$$
W_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{1}{1-\lambda_w}} + \xi_w \left(\tilde{\pi}_{w,t} W_{t-1}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w}.
$$

Dividing both sides by W_t and rearranging,

$$
w_t = \left[\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w}.
$$

Substituting, out for w_t from the household's first order condition for wage optimization:

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)} \bar{w}_t F_{w,t} = K_{w,t}.$$

We now derive the relationship between aggregate homogeneous hours worked, H_t, and aggregate household hours,

$$h_t \equiv \int_0^1 h_{j,t} dj.$$

Substituting the demand for $h_{j,t}$ into the latter expression, we obtain,

$$\begin{aligned} h_t &= \int_0^1 \left(\frac{W_{j,t}}{W_t}\right)^{\frac{\lambda_w}{1-\lambda_w}} H_t dj \\ &= \frac{H_t}{(W_t)^{\frac{\lambda_w}{1-\lambda_w}}} \int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj \\ &= \mathring{w}_t^{\frac{\lambda_w}{1-\lambda_w}} H_t, \end{aligned} \tag{2.72}$$

where

$$\mathring{w}_t \equiv \frac{\mathring{W}_t}{W_t},\ \mathring{W}_t = \left[\int_0^1 (W_{j,t})^{\frac{\lambda_w}{1-\lambda_w}} dj\right]^{\frac{1-\lambda_w}{\lambda_w}}.$$

Also,

$$\mathring{W}_t = \left[(1-\xi_w)\left(\tilde{W}_t\right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\tilde{\pi}_{w,t}\mathring{W}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}},$$

so that,

$$\begin{aligned} \mathring{w}_t &= \left[(1-\xi_w)(w_t)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}} \\ &= \left[(1-\xi_w)\left(\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right)^{\lambda_w} + \xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\mathring{w}_{t-1}\right)^{\frac{\lambda_w}{1-\lambda_w}}\right]^{\frac{1-\lambda_w}{\lambda_w}}. \end{aligned} \tag{2.73}$$

In addition to (2.73), we have following equilibrium conditions associated with sticky wages[6]:

$$F_{w,t}=\frac{\psi_{z^+,t}}{\lambda_w}\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}}h_t\frac{1-\tau_t^y}{1+\tau_t^w}+\beta\xi_w E_t\left(\frac{\bar{w}_{t+1}}{\bar{w}_t}\right)\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}}F_{w,t+1} \tag{2.75}$$

$$K_{w,t}=\zeta_t^h\left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}}h_t\right)^{1+\sigma_L}+\beta\xi_w E_t\left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)}K_{w,t+1} \tag{2.76}$$

$$\frac{1}{A_L}\left[\frac{1-\xi_w\left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w}\right]^{1-\lambda_w(1+\sigma_L)}\bar{w}_t F_{w,t}=K_{w,t}. \tag{2.77}$$

2.6. Fiscal and Monetary Authorities

For purposes of estimating our models, we must make assumptions about how policy was conducted in the historical sample. In the case of Sweden there was a break in policy in 1992. In the decade before 1992, the value of the Krona in relation to a basket of currencies was held fixed.[7] After 1992, there are three ways to represent monetary policy. One is to imagine that the Riksbank conducted policy with commitment with the object of maximizing the

[6] Log linearizing these equations about the nonstochastic steady state and under the assumption of $\varkappa_w=0$, we obtain

$$E_t\left[\begin{array}{c}\eta_0\widehat{\bar{w}}_{t-1}+\eta_1\widehat{\bar{w}}_t+\eta_2\widehat{\bar{w}}_{t+1}+\eta_3\left(\hat{\pi}_t-\widehat{\bar{\pi}}_t^c\right)+\eta_4\left(\hat{\pi}_{t+1}-\rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right)\\+\eta_5\left(\hat{\pi}_{t-1}^c-\widehat{\bar{\pi}}_t^c\right)+\eta_6\left(\hat{\pi}_t^c-\rho_{\widehat{\bar{\pi}}^c}\widehat{\bar{\pi}}_t^c\right)\\+\eta_7\hat{\psi}_{z^+,t}+\eta_8\hat{H}_t+\eta_9\hat{\tau}_t^y+\eta_{10}\hat{\tau}_t^w+\eta_{11}\hat{\zeta}_t^h\\+\eta_{12}\hat{\mu}_{z^+,t}+\eta_{13}\hat{\mu}_{z^+,t+1}\end{array}\right]=0, \tag{2.74}$$

where

$$b_w=\frac{[\lambda_w\sigma_L-(1-\lambda_w)]}{[(1-\beta\xi_w)(1-\xi_w)]}$$

and

$$\begin{pmatrix}\eta_0\\\eta_1\\\eta_2\\\eta_3\\\eta_4\\\eta_5\\\eta_6\\\eta_7\\\eta_8\\\eta_9\\\eta_{10}\\\eta_{11}\\\eta_{12}\\\eta_{13}\end{pmatrix}=\begin{pmatrix}b_w\xi_w\\\left(\sigma_L\lambda_w-b_w\left(1+\beta\xi_w^2\right)\right)\\b_w\beta\xi_w\\-b_w\xi_w\\b_w\beta\xi_w\\b_w\xi_w\kappa_w\\-b_w\beta\xi_w\kappa_w\\(1-\lambda_w)\\-(1-\lambda_w)\sigma_L\\-(1-\lambda_w)\frac{\tau^y}{(1-\tau^y)}\\-(1-\lambda_w)\frac{\tau^w}{(1+\tau^w)}\\-(1-\lambda_w)\\-b_w\xi_w\\b_w\beta\xi_w\end{pmatrix}.$$

[7] There was some adjustment to the exchange because the basket of currencies was adjusted.

following criterion:

$$E_t \sum_{j=0}^{\infty} \beta^j \{ \left(100 \left[\pi_t^c \pi_{t-1}^c \pi_{t-2}^c \pi_{t-3}^c - (\bar{\pi}_t^c)^4\right]\right)^2 + \lambda_y \left(100 \log\left(\frac{y_t}{y}\right)\right)^2 + \lambda_{\Delta R} \left(400 \left[R_t - R_{t-1}\right]\right)^2$$
$$+\lambda_s \left(S_t - \bar{S}\right)^2 \}$$

This approach takes the parameters in the criterion, λ_y, $\lambda_{\Delta R}$ and λ_s as unknown parameters to be estimated. A second approach is to suppose that policy was Ramsey-optimal, that is that it was chosen with commitment to maximize the discounted social welfare criterion, (**??**). A virtue of this approach is that there are no policy parameters to be estimated. A third approach is to suppose that policy was conducted according to a Taylor rule of the following form (see ALLV):

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_R)\left[\log\left(\frac{\bar{\pi}_t^c}{\bar{\pi}^c}\right) + r_\pi \log\left(\frac{\pi_{t-1}^c}{\bar{\pi}_t^c}\right)\right. \\ &\left. + r_y \log\left(\frac{y_{t-1}}{y}\right) + r_q \log\left(\frac{q_{t-1}}{q}\right)\right] + r_{\Delta\pi} \Delta \log\left(\frac{\pi_t^c}{\pi^c}\right) + r_{\Delta y} \Delta \log\left(\frac{y_t}{y}\right) + \varepsilon_{R,t}. \end{aligned} \tag{2.78}$$

Here too, the parameters would be taken as unknowns to be estimated. In addition, $\bar{\pi}_t^c$ is an exogenous process that characterizes the central bank's consumer price index inflation target and its steady state value corresponds to the steady state of actual inflation.

For estimation to be done using a sample beginning with the 1980s, we have to take into account the break in policy in 1992. In order to avoid transition effects, we plan to drop data beginning with 1992Q4 and ending with a quarter after the new policy regime is in place. In formulating the likelihood function of the whole dataset, we plan to treat the pre- and post-break samples as independent datasets. Currently, we estimate the model on data from 1995q1-2008q1. The data discussed below refers to this period.

We model government consumption expenditures as

$$G_t = g_t z_t^+,$$

where g_t is an exogenous stochastic process, orthogonal to the other shocks in the model. We suppose that

$$\log g_t = \left(1 - \rho_g\right) \log g + \rho_g \log g_{t-1} + \varepsilon_t^g.$$

where $g = \eta_g Y$. We set $\eta_g = 0.3$, the sample average of government consumption as a fraction of GDP.

The tax rates in our model are:

$$\tau_t^k,\ \tau_t^b,\ \tau_t^y,\ \tau_t^c,\ \tau_t^w.$$

We briefly discuss the treatment of these tax rates. In our model, capital is accumulated and capital income accrues directly to the household. However, an observationally equivalent representation of the model has these activities occurring in the firm. This latter interpretation is the convenient one, when thinking about the data and, in particular, the measurement of τ^k. We set the tax rate on capital income, τ^k, to 0.25. We arrived at this number as follows. The statutory rate on household capital income is 30 percent and the statutory rate on corporate income is 28 percent. Combining these two numbers we conclude that the statutory rate on corporate and household income is 50 percent. Indirect evidence from Devereux, Griffith and Klemm (2002) suggests to us that the effective tax on capital income may be one half this amount, and this is why we set $\tau^k = 0.25$ in the model. We reach this conclusion because of Devereux, Griffith and Klemm observation that the effective corporate income tax is roughly 1/2 of the statutory rate and we adopt the rough approximation that the same applies to the household tax rate. Our assumption that τ^k is constant is also motivated by Devereux, Griffith and Klemm. Their measure of the corporate component of the effective capital income tax rate exhibits very little variation over our sample which is 1995-2005.

Now we turn to the tax rate on bonds, τ^b. For now, we set $\tau^b = 0$ to match the pre-tax real rate on bonds of roughly 2%. We plan to investigate empirical measures of the effective tax rate on bonds in the future.

For evidence on τ^w we use the data collected by ALLV. Based on these data, we set the payroll tax rate, τ^w, to 0.35. Data on the value-added tax on consumption, τ^c, and the personal income tax rate that applies to labor, τ^y, are available from Statistics Sweden and indicate $\tau^c = 0.25$ and $\tau^y = 0.3$. These are the average values of the corresponding tax rates over the period 1995-2004. We hold these tax rates constant because they exhibit very little variability over this period.

2.7. Foreign variables

Below, we describe the stochastic process driving the foreign variables. Our representation takes into account our assumption that foreign output, Y_t^*, is affected by disturbances to z_t^+, just as domestic variables are. In particular, our model of Y_t^* is:

$$\begin{aligned}\log Y_t^* &= \log z_t^+ + \log y_t^* \\ &= \log y_t^* + \frac{\alpha}{1-\alpha} \log \psi_t + \log z_t,\end{aligned}$$

where $\log(y_t^*)$ is assumed to be a stationary process. We assume:

$$\begin{pmatrix} \log\left(\frac{y_t^*}{y^*}\right) \\ \pi_t^* - \pi^* \\ R_t^* - R^* \\ \log\left(\frac{\mu_{z,t}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right) \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & \frac{a_{24}\alpha}{1-\alpha} \\ a_{31} & a_{32} & a_{33} & a_{34} & \frac{a_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \rho_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \rho_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \log\left(\frac{y_{t-1}^*}{y^*}\right) \\ \pi_{t-1}^* - \pi^* \\ R_{t-1}^* - R^* \\ \log\left(\frac{\mu_{z,t-1}}{\mu_z}\right) \\ \log\left(\frac{\mu_{\psi,t-1}}{\mu_\psi}\right) \end{pmatrix}$$

$$+ \begin{bmatrix} \sigma_{y^*} & 0 & 0 & 0 & 0 \\ c_{21} & \sigma_{\pi^*} & 0 & c_{24} & \frac{c_{24}\alpha}{1-\alpha} \\ c_{31} & c_{32} & \sigma_{R^*} & c_{34} & \frac{c_{34}\alpha}{1-\alpha} \\ 0 & 0 & 0 & \sigma_{\mu_z} & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mu_\psi} \end{bmatrix} \begin{pmatrix} \varepsilon_{y^*,t} \\ \varepsilon_{\pi^*,t} \\ \varepsilon_{R^*,t} \\ \varepsilon_{\mu_z,t} \\ \varepsilon_{\mu_\psi,t} \end{pmatrix},$$

where the ε_t's are mean zero, unit variance, iid processes uncorrelated with each other. In matrix form,

$$X_t^* = AX_{t-1}^* + C\varepsilon_t,$$

in obvious notation. Note that the matrix C has 10 elements, so that the order condition for identification is satisfied, since CC' has 10 independent parameters.

We now briefly discuss the intuition underlying the zero restrictions in A and C. First, we assume that the shock, $\varepsilon_{y^*,t}$, affects the first three variables in X_t^*, while $\varepsilon_{\pi^*,t}$ only affects the second two and $\varepsilon_{R^*,t}$ only affects the third. The assumption about $\varepsilon_{R^*,t}$ corresponds to one strategy for identifying monetary policy shock, in which it is assumed that inflation and output are predetermined relative to the monetary policy shock. Under this interpretation of $\varepsilon_{R^*,t}$, our treatment of the foreign monetary policy shock and the domestic one are inconsistent. In our model as currently formulated domestic prices are not predetermined in the period of a monetary policy shock. Second, note from the zeros in the last two columns of the first row in A and C, that the technology shocks do not affect y_t^*. This reflects our assumption that the impact of technology shocks on Y_t^* is completely taken into account by z_t^+, while all other shocks to Y_t^* are orthogonal to z_t^+ and they affect Y_t^* via y_t^*. Third, the A and C matrices capture the notion that innovations to technology affect foreign inflation and the interest rate via their impact on z_t^+. Fourth, our assumptions on A and C imply that $\log\left(\frac{\mu_{\psi,t}}{\mu_\psi}\right)$ and $\log\left(\frac{\mu_{z,t}}{\mu_z}\right)$ are univariate first order autoregressive processes driven by $\varepsilon_{\mu_\psi,t}$ and $\varepsilon_{\mu_z,t}$, respectively. This is a standard assumption made on technology shocks in DSGE models.

2.8. Resource Constraints

We begin by deriving a relationship between total output of the domestic homogeneous good, Y_t, and aggregate factors of production. Next, we consider the resource constraint.

2.8.1. Domestic Homogeneous Output: Production

Consider the unweighted average of the intermediate goods:

$$\begin{aligned}
Y_t^{sum} &= \int_0^1 Y_{i,t} di \\
&= \int_0^1 \left[(z_t H_{i,t})^{1-\alpha} \epsilon_t K_{i,t}^{\alpha} - z_t^+ \phi \right] di \\
&= \int_0^1 \left[z_t^{1-\alpha} \epsilon_t \left(\frac{K_{i,t}}{H_{it}} \right)^{\alpha} H_{it} - z_t^+ \phi \right] di \\
&= z_t^{1-\alpha} \epsilon_t \left(\frac{K_t}{H_t} \right)^{\alpha} \int_0^1 H_{it} di - z_t^+ \phi
\end{aligned}$$

where K_t is the economy-wide average stock of capital services and H_t is the economy-wide average of homogeneous labor. The last expression exploits the fact that all intermediate good firms confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. This follows from cost minimization, and holds for all firms, regardless whether or not they have an opportunity to reoptimize. Then,

$$Y_t^{sum} = z_t^{1-\alpha} \epsilon_t K_t^{\alpha} H_t^{1-\alpha} - z_t^+ \phi.$$

Recall that the demand for $Y_{j,t}$ is

$$\left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} = \frac{Y_{i,t}}{Y_t},$$

so that

$$\mathring{Y}_t \equiv \int_0^1 Y_{i,t} di = \int_0^1 Y_t \left(\frac{P_t}{P_{i,t}} \right)^{\frac{\lambda_d}{\lambda_d - 1}} di = Y_t P_t^{\frac{\lambda_d}{\lambda_d - 1}} \left(\mathring{P}_t \right)^{\frac{\lambda_d}{1 - \lambda_d}},$$

say, where

$$\mathring{P}_t = \left[\int_0^1 P_{i,t}^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}}. \tag{2.79}$$

Dividing by P_t,

$$\mathring{p}_t = \left[\int_0^1 \left(\frac{P_{it}}{P_t} \right)^{\frac{\lambda_d}{1-\lambda_d}} di \right]^{\frac{1-\lambda_d}{\lambda_d}},$$

or,

$$\mathring{p}_t = \left[(1 - \xi_p) \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \right)^{\frac{1}{1-\lambda_d}}}{1 - \xi_p} \right)^{\lambda_d} + \xi_p \left(\frac{\tilde{\pi}_{d,t}}{\pi_t} \mathring{p}_{t-1} \right)^{\frac{\lambda_d}{1-\lambda_d}} \right]^{\frac{1-\lambda_d}{\lambda_d}}, \tag{2.80}$$

after using (**??**).

The preceding discussion implies:

$$Y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \mathring{Y}_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[z_t^{1-\alpha} \epsilon_t K_t^\alpha H_t^{1-\alpha} - z_t^+ \phi \right],$$

or, after scaling by z_t^+,

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha H_t^{1-\alpha} - \phi \right],$$

where

$$k_t = \bar{k}_t u_t. \tag{2.81}$$

We replace aggregate homogeneous labor, H_t, with aggregate household labor, h_t, as follows:

$$y_t = (\mathring{p}_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\epsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{z^+,t}} k_t \right)^\alpha \left(\mathring{w}_t^{-\frac{\lambda_w}{1-\lambda_w}} h_t \right)^{1-\alpha} - \phi \right]. \tag{2.82}$$

2.8.2. Trade Balance

We begin by developing the link between net exports and the current account. Expenses on imports and net new purchases of foreign assets, A_{t+1}, must equal income from exports and from previously purchased net foreign assets:

$$S_t A_{t+1} + \textit{expenses} \text{ on imports}_t = \textit{receipts} \text{ from exports}_t + R_{t-1}^* \Phi_{t-1} S_t A_t^*,$$

where Φ_t is as defined in (2.68) in our benchmark model. Expenses on imports correspond to the purchases of specialized importers for the consumption, investment and export sectors:

$$\textit{expenses} \text{ on imports}_t = S_t P_t^* R_t^{\nu,*} \left(C_t^m \left(\mathring{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + I_t^m \left(\mathring{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + X_t^m \left(\mathring{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right),$$

using (2.45), (2.46) and (2.47). Note the presence of the price distortion terms here. To understand these terms, recall that, for example, C_t^m is produced as a linear homogeneous function of specialized imported goods. Because the specialized importers only buy foreign goods, it is their total expenditures that interests us here. When the imports are distributed evenly across differentiated goods, then the total quantity of those imports is C_t^m, and the value of imports associated with domestic production of consumption goods is $S_t P_t^* R_t^{\nu,*} C_t^m$. When there are price distortions among imported intermediate goods, then the sum of the homogeneous import goods is higher for any given value of C_t^m. This is captured by the price distortion terms in the above expression.

We conclude that the current account can be written as follows:

$$
\begin{aligned}
& S_t A^*_{t+1} + S_t P^*_t R^{\nu,*}_t \left(C^m_t \left(\mathring{p}^{m,c}_t\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + I^m_t \left(\mathring{p}^{m,i}_t\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + X^m_t \left(\mathring{p}^{m,x}_t\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\
= \; & S_t P^x_t X_t + R^*_{t-1} \Phi_{t-1} S_t A^*_t,
\end{aligned}
$$

where Φ_t is defined in section 2.5.3. Expressing the current account in scaled form,

$$
\begin{aligned}
a_t P_t z^+_t + S_t P^*_t z^+_t R^{\nu,*}_t \left(c^m_t \left(\mathring{p}^{m,c}_t\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + i^m_t \left(\mathring{p}^{m,i}_t\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x^m_t \left(\mathring{p}^{m,x}_t\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\
= z^+_t S_t P^x_t x_t + R^*_{t-1} \Phi_{t-1} S_t \frac{P_{t-1} z^+_{t-1} a_{t-1}}{S_{t-1}},
\end{aligned}
$$

where, recall, $a_t = S_t A^*_{t+1}/(P_t z^+_t)$. Dividing by $P_t z^+_t$, we obtain

$$
\begin{aligned}
a_t + q_t p^c_t R^{\nu,*}_t \left(c^m_t \left(\mathring{p}^{m,c}_t\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} + i^m_t \left(\mathring{p}^{m,i}_t\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} + x^m_t \left(\mathring{p}^{m,x}_t\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \right) \\
= q_t p^c_t p^x_t x_t + R^*_{t-1} \Phi_{t-1} s_t \frac{a_{t-1}}{\pi_t \mu_{z^+,t}},
\end{aligned}
\tag{2.83}
$$

using (2.33).

We have already defined real, scaled GDP in terms of aggregate factors of production. It is convenient to also have an expression that exhibits the uses of domestic homogeneous output. Using (2.41),

$$
z^+_t y_t = G_t + C^d_t + I^d_t + \left[\omega_x \left(p^{m,x}_t\right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}^x_t\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p^x_t\right)^{-\eta_f} Y^*_t,
$$

or, after scaling by z^+_t and using (2.18):

$$
\begin{aligned}
y_t \;=\; & g_t + (1-\omega_c) \left(p^c_t\right)^{\eta_c} c_t + \left(p^i_t\right)^{\eta_i} \left(i_t + a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1-\omega_i) \\
& + \left[\omega_x \left(p^{m,x}_t\right)^{1-\eta_x} + (1-\omega_x) \right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(\mathring{p}^x_t\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t}-1}} \left(p^x_t\right)^{-\eta_f} y^*_t.
\end{aligned}
\tag{2.84}
$$

We now consider the restrictions across inflation rates implied by our relative price formulas. In terms of the expressions in (2.2) there are the restrictions implied by $p^{m,j}_t / p^{m,j}_{t-1}$, $j = x, c, i$, and p^x_t. The restrictions implied by the other two relative prices in (2.2), p^i_t and p^c_t, have already been exploited in (2.23) and (2.57), respectively. Finally, we also exploit the restriction across inflation rates implied by q_t/q_{t-1} and (2.3). Thus,

$$
\frac{p^{m,x}_t}{p^{m,x}_{t-1}} = \frac{\pi^{m,x}_t}{\pi_t} \tag{2.85}
$$

$$\frac{p_t^{m,c}}{p_{t-1}^{m,c}} = \frac{\pi_t^{m,c}}{\pi_t} \tag{2.86}$$

$$\frac{p_t^{m,i}}{p_{t-1}^{m,i}} = \frac{\pi_t^{m,i}}{\pi_t} \tag{2.87}$$

$$\frac{p_t^{x}}{p_{t-1}^{x}} = \frac{\pi_t^{x}}{\pi_t^*} \tag{2.88}$$

$$\frac{q_t}{q_{t-1}} = \frac{s_t\pi_t^*}{\pi_t^c}. \tag{2.89}$$

This completes the description of the baseline model.

2.9. Endogenous Variables of the Baseline Model

In the above sections we derived following 70 equations,

$$2.5, 2.6, 2.7, 2.10, 2.11, 2.12, 2.13, 2.14, 2.16, 2.18, 2.19, 2.20, 2.22, 2.23, 2.24, 2.26, 2.31, 2.32,$$
$$2.34, 2.35, 2.36, 2.37, 2.38, 2.42, 2.44, 2.49, 2.50, 2.51, 2.52, 2.48, 2.53, 2.55, 2.56, 2.57, 2.58,$$
$$2.60, 2.61, 2.62, 2.63, 2.64, 2.66, 2.75, 2.76, 2.77, 2.73, 2.70, 2.71, 2.72, 2.78, 2.81, 2.82, 2.83,$$
$$2.84, 2.85, 2.86, 2.87, 2.88, 2.89$$

which can be used to solve for the following 70 unknowns:

$$\bar{r}_t^k, \bar{w}_t, R_t^{\nu,*}, R_t^f, R_t^x, R_t, mc_t, mc_t^x, mc_t^{m,c}, mc_t^{m,i}, mc_t^{m,x}, \pi_t, \pi_t^x, \pi_t^c, \pi_t^i, \pi_t^{m,c}, \pi_t^{m,i}, \pi_t^{m,x},$$
$$p_t^c, p_t^x, p_t^i, p_t^{m,x}, p_t^{m,c}, p_t^{m,i}, p_{k',t}, k_{t+1}, \bar{k}_{t+1}, u_t, h_t, H_t, q_t, i_t, c_t, x_t, a_t, s_t, \psi_{z^+,t}, y_t$$
$$K_t^d, F_t^d, \tilde{\pi}_{d,t}, \mathring{p}_t, K_{x,t}, F_{x,t}, \tilde{\pi}_t^x, \mathring{p}_t^x, \{K_{m,j,t}, F_{m,j,t}, \tilde{\pi}_t^{m,j}, \mathring{p}_t^{m,j}; j = c, i, x\}, K_{w,t}, F_{w,t}, \tilde{\pi}_t^w, R_t^k$$
$$\tilde{S}_t, \tilde{S}_t', a(u_t), \mathring{w}_t, c_t^m, i_t^m, x_t^m, \pi_w.$$

3. Introducing Financial Frictions into the Model

A number of the activities in the model of the previous section require financing. Producers of specialized inputs must borrow working capital within the period. The management of capital involves financing because the construction of capital requires a substantial initial outlay of resources, while the return from capital comes in over time as a flow. In the model of the previous section financing requirements affect the allocations, but not very much. This is because none of the messy realities of actual financial markets are present. There is no asymmetric information between borrower and lender, there is no risk to lenders. In the case of capital accumulation, the borrower and lender are actually the same household, who puts up the finances and later reaps the rewards. When real-world financial frictions are introduced into a model, then intermediation becomes distorted by the presence of balance sheet constraints and other factors.

Although the literature shows how to introduce financial frictions much more extensively, here we proceed by assuming that only the accumulation and management of capital involves frictions. We will continue to assume that working capital loans are frictionless. At the end of this introduction, we briefly discuss the idea of introducing financial frictions into working capital loans. Our strategy of introducing frictions in the accumulation and management of capital follows the variant of the Bernanke, Gertler and Gilchrist (1999) (henceforth BGG) model implemented in Christiano, Motto and Rostagno (2003) The discussion here borrows heavily from the derivation in Christiano, Motto and Rostagno (2007) (henceforth CMR).

The financial frictions we introduce reflect fundamentally that borrowers and lenders are different people, and that they have different information. Thus, we introduce 'entrepreneurs'. These are agents who have a special skill in the operation and management of capital. Although these agents have their own financial resources, their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support, by borrowing additional funds. There is a financial friction because the management of capital is risky. Individual entrepreneurs are subject to idiosyncratic shocks which are observed only by them. The agents that they borrow from, 'banks', can only observe the idiosyncratic shocks by paying a monitoring cost. This type of asymmetric information implies that it is impractical to have an arrangement in which banks and entrepreneurs simply divide up the proceeds of entrepreneurial activity, because entrepreneurs have an incentive to understate their earnings. An alternative arrangement that is more efficient is one in which banks extend entrepreneurs a 'standard debt contract', which specifies a loan amount and a given interest payment. Entrepreneurs who suffer an especially bad idiosyncratic income shock and who therefore cannot afford to pay the required interest, are 'bankrupt'. Banks pay the cost of monitoring these entrepreneurs and take all of their net worth in partial compensation for the interest that they are owed. For a graphical illustration of the financing problem in the capital market, see Figure **??**.

The amount that banks are willing to lend to an entrepreneur under the standard debt contract is a function of the entrepreneur's net worth. This is how balance sheet constraints enter the model. When a shock occurs that reduces the value of the entrepreneur's assets, this cuts into their ability to borrow. As a result, they acquire less capital and this translates into a reduction in investment and ultimately into a slowdown in the economy.

The ultimate source of funds for lending to entrepreneurs is the household. The standard debt contracts extended by banks to entrepreneurs are financed by issuing liabilities to households. Although individual entrepreneurs are risky, banks themselves are not. We suppose that banks lend to a sufficiently diverse group of entrepreneurs that the uncertainty that exists in individual entrepreneurial loans washes out across all loans. Extensions of the model that introduce risk into banking have been developed, but it is not clear that the added complexity is justified.

In the model, the interest rate that households receive is nominally non state-contingent. This gives rise to potentially interesting wealth effects of the sort emphasized by Irving Fisher (1933). For example, when a shock occurs which drives the price level down, households receive a wealth transfer. Because this transfer is taken from entrepreneurs, their net worth is reduced. With the tightening in their balance sheets, their ability to invest is reduced, and this produces an economic slowdown.

At the level of abstraction of the model, the capital stock includes both housing and business capital. As a result, the entrepreneurs can also be interpreted as households in their capacity of homeowners. An expanded version of the model would pull apart the household and business sectors to study each individually. Another straightforward expansion of the model would apply the model of financial frictions to working capital loans.

With this model, it is typically the practice to compare the net worth of entrepreneurs with a stock market quantity such as the Dow Jones Industrial Average. Whether this is really appropriate is uncertain. A case can be made that the 'bank loans' of entrepreneurs in the model correspond well with actual bank loans *plus* actual equity. It is well known that dividend payments on equity are very smooth. Firms work hard to accomplish this. For example, during the US Great Depression some firms were willing to sell their own physical capital in order to avoid cutting dividends. That this is so is perhaps not surprising. The asymmetric information problems with actual equity are surely as severe as they are for the banks in our model. Under these circumstances one might expect equity holders to demand a payment that is not contingent on the realization of uncertainty within the firm (payments could be contingent upon publicly observed variables). Under this vision, the net worth in the model would correspond not to a measure of the aggregate stock market, but to the ownership stake of the managers and others who exert most direct control over the firm. The 'bank loans' in this model would, under this view of things, correspond to the actual loans of firms (i.e., bank loans and other loans such as commercial paper) plus the outstanding equity. While this is perhaps too extreme, these observations highlight that there is substantial uncertainty over exactly what variable should be compared with net worth in the model. It is important to emphasize, however, that whatever the right interpretation is of net worth, the model potentially captures balance sheet problems very nicely.

Finally, we make some remarks on the introduction of financial frictions into working capital loans. It is possible to accomplish this with relatively little modification to the model, by following the strategy described in Fisher (1998). However, with this strategy the effects of financial frictions are quite modest, because the firms in the model which use working capital do not have assets. As a result, the balance sheet channel does not operate. We conjecture that for financial frictions in working capital to be interesting, the borrowing firms would need to have assets. One way this could be accomplished would be to assume that they use and own capital that is specific to their firm. In this way, fluctuations in the value of that capital induced by changes in asset prices would change their ability to borrow, and hence to produce. This strategy is algebra-intensive because of the fact that these firms also set their prices subject to Calvo frictions.

3.1. Modifying the Baseline Model

The financial frictions bring a net increase of two equations over the equations in the model of the previous section. In addition, they introduce two new endogenous variables, one related to the interest rate paid by entrepreneurs as well as their net worth. The financial frictions also allow us to introduce two new random variables. We now provide a formal discussion of the model.

As we shall see, entrepreneurs all have different histories, as they experience different idiosyncratic shocks. Thus, in general, solving for the aggregate variables would require also solving for the distribution of entrepreneurs according to their characteristics and for the law of motion for that distribution. However, as emphasized in BGG, the right functional form assumptions have been made in the model, which guarantee the result that the aggregate variables associated with entrepreneurs are not a function of distributions. The loan contract specifies that all entrepreneurs, regardless of their net worth, receive the same interest rate. Also, the loan amount received by a entrepreneur is proportional to his level of net worth. These are enough to guarantee the aggregation result.

3.1.1. The Individual Entrepreneur

At the end of period t each entrepreneur has a level of net worth, N_{t+1}. The entrepreneur's net worth, N_{t+1}, constitutes his state at this time, and nothing else about his history is relevant. We imagine that there are many entrepreneurs for each level of net worth and that for each level of net worth, there is a competitive bank with free entry that offers a loan contract. The contract is defined by a loan amount and by an interest rate, both of which are derived as the solution to a particular optimization problem.

Consider a type of entrepreneur with a particular level of net worth, N_{t+1}. The entrepreneur combines this net worth with a bank loan, B_{t+1}, to purchase new, installed physical capital, $\bar{K}_{t+1}$, from capital producers. The loan the entrepreneur requires for this is:

$$B_{t+1} = P_t P_{k',t} \bar{K}_{t+1} - N_{t+1}. \tag{3.1}$$

The entrepreneur is required to pay a gross interest rate, Z_{t+1}, on the bank loan at the end of period $t+1$, if it is feasible to do so. After purchasing capital the entrepreneur experiences an idiosyncratic productivity shock which converts the purchased capital, $\bar{K}_{t+1}$, into $\bar{K}_{t+1}\omega$. Here, ω is a unit mean, lognormally and independently distributed random variable across entrepreneurs. The variance of $\log \omega$ is σ_t^2. The t subscript indicates that σ_t is itself the realization of a random variable. This allows us to consider the effects of an increase in the riskiness of individual entrepreneurs. We denote the cumulative distribution function of ω by F_t.

After observing the period $t+1$ shocks, the entrepreneur sets the utilization rate, u_{t+1}, of capital and rents capital out in competitive markets at nominal rental rate, $P_{t+1} r_{t+1}^k$. In choosing the capital utilization rate, the entrepreneur takes into account that operating one unit of physical capital at rate u_{t+1} requires $a(u_{t+1})$ of domestically produced investment goods for maintenance expenditures, where a is defined in (**??**). The entrepreneur then sells the undepreciated part of physical capital to capital producers. Per unit of physical capital purchased, the entrepreneur who draws idiosyncratic shock ω earns a return (after taxes), of $R_{t+1}^k \omega$, where R_{t+1}^k is defined in (2.59). Because the mean of ω across entrepreneurs is unity, the average return across all entrepreneurs is R_{t+1}^k.

After entrepreneurs sell their capital, they settle their bank loans. At this point, the resources available to an entrepreneur who has purchased $\bar{K}_{t+1}$ units of physical capital in period t and who experiences an idiosyncratic productivity shock ω are $P_t P_{k',t} R_{t+1}^k \omega \bar{K}_{t+1}$. There is a cutoff value of ω, $\bar{\omega}_{t+1}$, such that the entrepreneur has just enough resources to pay interest:

$$\bar{\omega}_{t+1} R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}. \tag{3.2}$$

Entrepreneurs with $\omega < \bar{\omega}_{t+1}$ are bankrupt and turn over all their resources,

$$R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

to the bank, which is less than $Z_{t+1} B_{t+1}$. In this case, the bank monitors the entrepreneur, at cost

$$\mu R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1},$$

where $\mu \geq 0$ is a parameter.

We note briefly that the definition of R_{t+1}^k lacks some realism because it does not take into account the deductibility of interest payments. With the more realistic treatment of interest, the after tax rate of return on capital would be changed from (2.59) to include deductability of interest payments for firms:

$$
\begin{aligned}
R_{t+1}^{k} &= \frac{\left(1-\tau_{t}^{k}\right)\left[u_{t+1} r_{t+1}^{k}-\frac{p_{t+1}^{i}}{\Psi_{t+1}} a\left(u_{t+1}\right)-\left(Z_{t+1}-1\right) \frac{B_{t+1}}{P_{t} P_{k^{\prime}, t} \bar{K}_{t+1}}\right] P_{t+1}}{P_{t} P_{k^{\prime}, t}} \\
&+\frac{(1-\delta) P_{t+1} P_{k^{\prime}, t+1}+\tau_{t}^{k} \delta P_{t} P_{k^{\prime}, t}}{P_{t} P_{k^{\prime}, t}} \\
&= \frac{\left(1-\tau_{t}^{k}\right)\left[u_{t+1} r_{t+1}^{k}-\frac{1}{\Psi_{t+1}} a\left(u_{t+1}\right)-\bar{\omega}_{t+1} R_{t+1}^{k}+\frac{B_{t+1}}{P_{t} P_{k^{\prime}, t} \bar{K}_{t+1}}\right] P_{t+1}}{P_{t} P_{k^{\prime}, t}} \\
&+\frac{(1-\delta) P_{t+1} P_{k^{\prime}, t+1}+\tau_{t}^{k} \delta P_{t} P_{k^{\prime}, t}}{P_{t} P_{k^{\prime}, t}},
\end{aligned}
$$

by (3.2). With this representation, R_t^k is a function of features of the loan contract. This will change the choice of optimal contract, discussed below. We plan to explore the implications of this in future work.

Banks obtain the funds loaned in period t to entrepreneurs by issuing deposits to households at gross nominal rate of interest, R_t. The subscript on R_t indicates that the payoff to households in $t+1$ is not contingent on the period $t+1$ uncertainty. This feature of the relationship between households and banks is simply assumed. There is no risk in household bank deposits, and the household Euler equation associated with deposits is exactly the same as (2.64).

We suppose that there is competition and free entry among banks, and that banks participate in no financial arrangements other than the liabilities issued to households and the loans issued to entrepreneurs.[8] It follows that the bank's cash flow in each state of period $t+1$ is zero, for each loan amount.[9] For loans in the amount, B_{t+1}, the bank receives gross interest, $Z_{t+1}B_{t+1}$, from the $1-F_t\left(\bar{\omega}_{t+1}\right)$ entrepreneurs who are not bankrupt. The bank takes all the resources possessed by bankrupt entrepreneurs, net of monitoring costs. Thus, the state-by-state zero profit condition is:

$$
\left[1-F_{t}\left(\bar{\omega}_{t+1}\right)\right] Z_{t+1} B_{t+1}+(1-\mu) \int_{0}^{\bar{\omega}_{t+1}} \omega d F_{t}(\omega) R_{t+1}^{k} P_{t} P_{k^{\prime}, t} \bar{K}_{t+1}=R_{t} B_{t+1},
$$

or, after making use of (3.2) and rearranging,

$$
\left[\Gamma_{t}\left(\bar{\omega}_{t+1}\right)-\mu G_{t}\left(\bar{\omega}_{t+1}\right)\right] \frac{R_{t+1}^{k}}{R_{t}} \varrho_{t}=\varrho_{t}-1 \tag{3.3}
$$

[8] If banks also had access to state contingent securities, then free entry and competition would imply that banks earn zero profits in an ex ante expected sense from the point of view of period t.

[9] Absence of state contingent securities markets guarantee that cash flow is non-negative. Free entry guarantees that ex ante profits are zero. Given that each state of nature receives positive probability, the two assumptions imply the state by state zero profit condition quoted in the text.

where

$$G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF_t(\omega).$$
$$\Gamma_t(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1}\left[1 - F_t(\bar{\omega}_{t+1})\right] + G_t(\bar{\omega}_{t+1})$$
$$\varrho_t = \frac{P_t P_{k',t} \bar{K}_{t+1}}{N_{t+1}}.$$

The expression, $\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})$ is the share of revenues earned by entrepreneurs that borrow B_{t+1}, which goes to banks. Note that $\Gamma_t'(\bar{\omega}_{t+1}) = 1 - F_t(\bar{\omega}_{t+1}) > 0$ and $G_t'(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} F_t'(\bar{\omega}_{t+1}) > 0$. It is thus not surprising that the share of entrepreneurial revenues accruing to banks is non-monotone with respect to $\bar{\omega}_{t+1}$. BGG argue that the expression on the left of (3.3) has an inverted 'U' shape, achieving a maximum value at $\bar{\omega}_{t+1} = \omega^*$, say. The expression is increasing for $\bar{\omega}_{t+1} < \omega^*$ and decreasing for $\bar{\omega}_{t+1} > \omega^*$. Thus, for any given value of ϱ_t and R_{t+1}^k / R_t, generically there are either no values of $\bar{\omega}_{t+1}$ or two that satisfy (3.3). The value of $\bar{\omega}_{t+1}$ realized in equilibrium must be the one on the left side of the inverted 'U' shape. This is because, according to (3.2), the lower value of $\bar{\omega}_{t+1}$ corresponds to a lower interest rate for entrepreneurs which yields them higher welfare. As discussed below, the equilibrium contract is one that maximizes entrepreneurial welfare subject to the zero profit condition on banks. This reasoning leads to the conclusion that $\bar{\omega}_{t+1}$ falls with a period $t+1$ shock that drives R_{t+1}^k up. The fraction of entrepreneurs that experience bankruptcy is $F_t\left(\bar{\omega}_{t+1}\right)$, so it follows that a shock which drives up R_{t+1}^k has a negative contemporaneous impact on the bankruptcy rate. According to (2.59), shocks that drive R_{t+1}^k up include anything which raises the value of physical capital and/or the rental rate of capital.

As just noted, we suppose that the equilibrium debt contract maximizes entrepreneurial welfare, subject to the zero profit condition on banks and the specified required return on household bank liabilities. The date t debt contract specifies a level of debt, B_{t+1} and a state $t+1-$contingent rate of interest, Z_{t+1}. We suppose that entrepreneurial welfare corresponds to the entrepreneur's expected wealth at the end of the contract. It is convenient to express welfare as a ratio to the amount the entrepreneur could receive by depositing his net worth in a bank:

$$\frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[R_{t+1}^k \omega P_t P_{k',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1}\right] dF_t(\omega)}{R_t N_{t+1}}$$
$$= \frac{E_t \int_{\bar{\omega}_{t+1}}^{\infty} \left[\omega - \bar{\omega}_{t+1}\right] dF_t(\omega) R_{t+1}^k P_t P_{k',t} \bar{K}_{t+1}}{R_t N_{t+1}}$$
$$= E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1})\right] \frac{R_{t+1}^k}{R_t} \right\} \varrho_t,$$

after making use of (3.1), (3.2) and

$$1=\int_0^\infty \omega dF_t(\omega)=\int_{\bar{\omega}_{t+1}}^\infty \omega dF_t(\omega)+G_t(\bar{\omega}_{t+1}).$$

We can equivalently characterize the contract by a state-$t+1$ contingent set of values for $\bar{\omega}_{t+1}$ and a value of ϱ_t. The equilibrium contract is the one involving $\bar{\omega}_{t+1}$ and ϱ_t which maximizes entrepreneurial welfare (relative to R_tN_{t+1}), subject to the bank zero profits condition. The Lagrangian representation of this problem is:

$$\max_{\varrho_t,\{\bar{\omega}_{t+1}\}} E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}\varrho_t+\lambda_{t+1}\left([\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1\right)\right\},$$

where λ_{t+1} is the Lagrange multiplier which is defined for each period $t+1$ state of nature. The first order conditions for this problem are:

$$\begin{aligned}
E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}+\lambda_{t+1}\left([\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}-1\right)\right\} &= 0\\
-\Gamma_t'(\bar{\omega}_{t+1})\frac{R^k_{t+1}}{R_t}+\lambda_{t+1}\left[\Gamma_t'(\bar{\omega}_{t+1})-\mu G_t'(\bar{\omega}_{t+1})\right]\frac{R^k_{t+1}}{R_t} &= 0\\
[\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}\varrho_t-\varrho_t+1 &= 0,
\end{aligned}$$

where the absence of λ_{t+1} from the complementary slackness condition reflects that we assume $\lambda_{t+1}>0$ in each period $t+1$ state of nature. Substituting out for λ_{t+1} from the second equation into the first, the first order conditions reduce to:

$$[\Gamma_{t-1}(\bar{\omega}_t)-\mu G_{t-1}(\bar{\omega}_t)]\frac{R^k_t}{R_{t-1}}\varrho_{t-1}-\varrho_{t-1}+1=0 \tag{3.4}$$

$$E_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}+\frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1})-\mu G_t'(\bar{\omega}_{t+1})}\left([\Gamma_t(\bar{\omega}_{t+1})-\mu G_t(\bar{\omega}_{t+1})]\frac{R^k_{t+1}}{R_t}-1\right)\right\} = 0,$$

for $t=0,1,2,...\infty$.

Since N_{t+1} does not appear in the last two equations, we conclude that ϱ_t and $\bar{\omega}_{t+1}$ are the same for all entrepreneurs, regardless of their net worth. The results for ϱ_t implies that

$$\frac{B_{t+1}}{N_{t+1}}=\varrho_t-1,$$

that an entrepreneur's loan amount is proportional to his net worth. Rewriting (3.1) and (3.2) we see that the rate of interest paid by the entrepreneur is

$$Z_{t+1}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{N_{t+1}}{P_tP_{k',t}\bar{K}_{t+1}}}=\frac{\bar{\omega}_{t+1}R^k_{t+1}}{1-\frac{1}{\varrho_t}}, \tag{3.5}$$

which is the same for all entrepreneurs, regardless of their net worth.

3.1.2. Aggregation Across Entrepreneurs and the Risk Premium

Let $f(N_{t+1})$ denote the density of entrepreneurs with net worth, N_{t+1}. Then, aggregate average net worth, $\bar{N}_{t+1}$, is

$$\bar{N}_{t+1} = \int_{N_{t+1}} N_{t+1} f(N_{t+1})\, dN_{t+1}.$$

We now derive the law of motion of $\bar{N}_{t+1}$. Consider the set of entrepreneurs who in period $t-1$ had net worth N. Their net worth after they have settled with the bank in period t is denoted V_t^N, where

$$V_t^N = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N - \Gamma_{t-1}(\bar{\omega}_t) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t^N, \tag{3.6}$$

where $\bar{K}_t^N$ is the amount of physical capital that entrepreneurs with net worth N_t acquired in period $t-1$. Clearing in the market for capital requires:

$$\bar{K}_t = \int_{N_t} \bar{K}_t^N f(N_t)\, dN_t.$$

Multiplying (3.6) by $f(N_t)$ and integrating over all entrepreneurs,

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \Gamma_{t-1}(\bar{\omega}_t) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t.$$

Writing this out more fully:

$$\begin{aligned} V_t &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ [1 - F_{t-1}(\bar{\omega}_t)]\, \bar{\omega}_t + \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &= R_t^k P_{t-1} P_{k',t-1} \bar{K}_t \\ &- \left\{ [1 - F_{t-1}(\bar{\omega}_t)]\, \bar{\omega}_t + (1-\mu) \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) + \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \right\} R_t^k P_{t-1} P_{k',t-1} \bar{K}_t. \end{aligned}$$

Note that the first two terms in braces correspond to the net revenues of the bank, which must equal $R_{t-1}(P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t)$. Substituting:

$$V_t = R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left\{ R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right\} (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t).$$

After V_t is determined, each entrepreneur faces an identical and independent probability $1-\gamma_t$ of being selected to exit the economy. With the complementary probability, γ_t, each entrepreneur remains. Because the selection is random, the net worth of the entrepreneurs who survive is simply $\gamma_t \bar{V}_t$. A fraction, $1-\gamma_t$, of new entrepreneurs arrive. Entrepreneurs who survive or who are new arrivals receive a transfer, W_t^e. This ensures that all entrepreneurs, whether new arrivals or survivors that experienced bankruptcy, have sufficient funds to obtain at least some amount of loans. The average net worth across all entrepreneurs after the W_t^e transfers have been made and exits and entry have occurred, is $\bar{N}_{t+1} = \gamma_t \bar{V}_t + W_t^e$, or,

$$\begin{aligned}\bar{N}_{t+1} &= \gamma_t \{ R_t^k P_{t-1} P_{k',t-1} \bar{K}_t - \left[R_{t-1} + \frac{\mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) R_t^k P_{t-1} P_{k',t-1} \bar{K}_t}{P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t} \right] (P_{t-1} P_{k',t-1} \bar{K}_t - \bar{N}_t) \} \\ &\quad + W_t^e. \qquad (3.7)\end{aligned}$$

3.2. Solving the Financial Frictions Model

In this subsection we indicate how the equilibrium conditions of the baseline model must be modified to accommodate financial frictions. We then consider the problem of solving for the model's steady state.

3.2.1. Equilibrium Conditions

Consider the households. Households no longer accumulate physical capital, and the first order condition, (2.61), must be dropped. No other changes need to be made to the household first order conditions. Equation (2.64) can be interpreted as applying to the household's decision to make bank deposits. The household equations, (2.57) and (2.62), pertaining to investment can be thought of as reflecting that the household builds and sells physical capital, or it can be interpreted as the first order condition of many identical, competitive firms that build capital (note that each has a state variable in the form of lagged investment). We must add the three equations pertaining to the entrepreneur's loan contract: the law of motion of net worth, the bank's zero profit condition and the optimality condition. Finally, we must adjust the resource constraints to reflect the resources used in bank monitoring and in consumption by entrepreneurs.

We adopt the following scaling of variables:

$$n_{t+1} = \frac{\bar{N}_{t+1}}{P_t z_t^+}, \; w_t^e = \frac{W_t^e}{P_t z_t^+}.$$

Dividing both sides of (3.7) by $P_t z_t^+$, we obtain the scaled law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{z^+,t}} \left[R_t^k p_{k',t-1} \bar{k}_t - R_{t-1} \left(p_{k',t-1} \bar{k}_t - n_t \right) - \mu G_{t-1} \left(\bar{\omega}_t \right) R_t^k p_{k',t-1} \bar{k}_t \right] + w_t^e, \qquad (3.8)$$

for $t = 0, 1, 2, ...$. Equation (3.8) has a simple intuitive interpretation. The first object in square brackets is the average gross return across all entrepreneurs in period t. The two negative terms correspond to what the entrepreneurs pay to the bank, including the interest paid by non-bankrupt entrepreneurs and the resources turned over to the bank by the bankrupt entrepreneurs. Since the bank makes zero profits, the payments to the bank by entrepreneurs must equal bank costs. The term involving R_{t-1} represents the cost of funds loaned to entrepreneurs by the bank, and the term involving μ represents the bank's total expenditures on monitoring costs.

The zero profit condition on banks, in terms of the scaled variables, is:

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{R_t}{R_{t+1}^k}\left(1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}\right), \tag{3.9}$$

for $t = -1, 0, 1, 2, \ldots$. The optimality condition for bank loans is (3.4).

The household's first order condition associated with the accumulation of capital, (2.61), must be dropped. The output equation, (2.82), does not have to be modified. Instead, the resource constraint for domestic homogenous goods (2.84) needs to be adjusted for the monitoring costs:

$$\begin{aligned} y_t = {} & g_t + (1-\omega_c)\left(p_t^c\right)^{\eta_c} c_t + \left(p_t^i\right)^{\eta_i}\left(i_t + a\left(u_t\right)\frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}}\right)(1-\omega_i) \\ & + \left[\omega_x\left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}}(1-\omega_x)\left(\mathring{p}_t^x\right)^{\frac{-\lambda_{x,t}}{\lambda_{x,t-1}}}\left(p_t^x\right)^{-\eta_f} y_t^* + d_t, \end{aligned} \tag{3.10}$$

where

$$d_t = \frac{\mu G_{t-1}(\bar{\omega}_t) R_t^k p_{k',t-1}\bar{k}_t}{\pi_t \mu_{z^+,t}}.$$

Account has to be taken of the consumption by exiting entrepreneurs. The net worth of these entrepreneurs is $(1-\gamma_t)V_t$ and we assume a fraction, $1-\Theta$, is taxed and transferred in lump-sum form to households, while the complementary fraction, Θ, is consumed by the exiting entrepreneurs. This consumption can be taken into account by subtracting

$$\Theta\frac{1-\gamma_t}{\gamma_t}(n_{t+1} - w_t^e)z_t^+ P_t$$

from the right side of (2.17). In practice we do not make this adjustment because we assume Θ is sufficiently small that the adjustment is negligible.

We now turn to the risk premium on entrepreneurs. The cost to the entrepreneur of internal funds (i.e., his own net worth) is the interest rate, R_t, which he loses by applying it to capital rather than just depositing it in the bank. The average payment by all entrepreneurs to the bank is the entire object in square brackets in equation (3.7). So, the term involving μ represents the excess of external funds over the internal cost of funds. As a result, this is one measure of the risk premium in the model. Another is the excess of the interest rate paid by entrepreneurs who are not bankrupt, over R_t :

$$Z_{t+1} - R_t = \frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}} - R_t,$$

according to (3.5).

The financial frictions brings a net increase of 2 equations (we add (3.4), (3.8) and (3.9), and delete (2.61)) and two variables, n_{t+1} and $\bar{\omega}_{t+1}$. This increases the size of our system to 72 equations in 72 variables. The financial frictions also introduce additional shocks, σ_t and γ_t.

4. Introducing Unemployment into the Model

This section replaces the model of the labor market in our baseline model with the search and matching framework of Mortensen and Pissarides (1994) and, more recently, Hall (2005a,b,c) and Shimer (2005a,b). We integrate the framework into our specific framework - which includes capital and monetary factors - following the version of the Gertler, Sala and Trigari (2006) (henceforth GST) strategy implemented in Christiano, Ilut, Motto, and Rostagno (2007). A key feature of the GST model is that there are wage-setting frictions, but they do not have a direct impact on on-going worker employer relations. However, wage-setting frictions have an impact on the effort of an employer in recruiting new employees In this sense, the setup is not vulnerable to the Barro (1977) critique of sticky wages. The model is also attractive because of the richness of its labor market implications: the model differentiates between hours worked and the quantity of people employment, it has unemployment and vacancies.

The labor market in our alternative labor market model is a slightly modified version of the GST model. GST assume wage-setting frictions of the Calvo type, while we instead work with Taylor-type frictions. In addition, we adopt a slightly different representation of the production sector in order to maximize comparability with our baseline model. In what follows, we first provide an overview and after that we present the detailed decision problems of agents in the labor market.

4.1. Sketch of the Model

As in the discussion of section 2.2, we adopt the Dixit-Stiglitz specification of homogeneous goods production. A representative, competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists, who hire labor and capital services in competitive factor markets. The intermediate good firms are assumed to be subject to the same Calvo price setting frictions in the baseline model.

In the baseline model, the homogeneous labor services supplied to the competitive labor market by labor retailers (contractors) who combine the labor services supplied to them by households who monopolisticly supply specialized labor services (see section 2.2). The modified model dispenses with the specialized labor services abstraction. Labor services are instead supplied to the homogeneous labor market by 'employment agencies'. See Figure **??** for a graphical illustration. The change leaves the equilibrium conditions associated with the production of the homogeneous good unaffected.[10]

Each employment agency retains a large number of workers. At the beginning of the period a fraction, $1-\rho$, of workers is randomly selected to separate from the firm and go into unemployment.[11] Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. After separation and new arrivals occur, the nominal wage rate is set.

The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every N periods. The employees of an agency is represented by a union at negotiations. This assumption has no consequences except that it makes clear which wage (i.e. the collectively negotiated wage) will apply to workers arriving at the agency during the duration of the wage contract. Each employment agency is permanently allocated to one of N different cohorts. Cohorts are differentiated according to the period in which they renegotiate their wage. Since there is an equal number of agencies in each cohort, $1/N$ of the agencies bargain in each period. The wage in agencies that do not bargain in the current period is updated from the previous period according to the same rule used in our baseline model.

Once a wage rate is determined - whether by Nash bargaining or not - we assume that each matched worker-firm pair finds it optimal to proceed with the relationship in that period. In our calculations, we verify that this assumption is correct, by confirming that the wage rate in each worker-agency relationship lies inside the bargaining set associated with that relationship.

Next, the intensity of labor effort is determined according to a particular efficiency criterion. To explain this, we discuss the implications of increased intensity for the worker and for the employment agency. The utility function of the household in the present labor market model is a modified version of (2.54):

$$E_t \sum_{l=0}^{\infty} \beta^{l-t} \{ \zeta_{t+l}^c \log(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l}^h A_L \frac{\varsigma_{t+l}^{1+\sigma_L}}{1+\sigma_L} L_{t+l} \}, \tag{4.1}$$

where L_t is the fraction of members of the household that are working and ς_t is the intensity with with each worker works. As in GST, we follow the family household construct of Merz (1995) in supposing that each household has a large number of workers. Although

[10] An alternative (perhaps more natural) formulation would be for the intermediate good firms to do their own employment search. We instead separate the task of finding workers from production of intermediate goods in order to avoid adding a state variable to the intermediate good firm, which would complicate the solution of their price-setting problem.

[11] We thus specify that the job separation rate is constant. This is consistent with the findings reported in Hall (2005b,c) and Shimer (2005a,b), who report that the job separation rate is relatively acyclical.

the individual worker's labor market experience - whether employed or unemployed - is determined in part by idiosyncratic shocks, the household has sufficiently many workers that the total fraction of workers employed, L_t, as well as the fractions allocated among the different cohorts, l_t^i, $i = 0, ..., N-1$, is the same for each household. We suppose that all the household's workers are supplied inelasticly to the labor market (i.e., labor force participation is constant). Each worker passes randomly from employment with a particular agency to unemployment and back to employment according to the endogenous probabilities described below.

The household's currency receipts arising from the labor market are:

$$\left(1 - L_t\right) P_t^c b^u z_t^+ + \sum_{i=0}^{N-1} W_t^i l_t^i \varsigma_t \frac{1-\tau_t^y}{1+\tau_t^w} \tag{4.2}$$

where W_t^i is the nominal wage rate earned by workers in cohort $i = 0, ..., N-1$. The index, i, indicates the number of periods in the past when bargaining occurred most recently. As in our baseline model, there is a labor income tax τ_t^y and a payroll tax τ_t^w that affect the after-tax wage. Note that we implicitly assume that labor intensity is the same in each employment agency, regardless of cohort. This is explained below. The presence of the term involving b^u indicates the assumption that unemployed workers receive a payment of $b^u z_t^+$ final consumption goods. The unemployment benefits are financed by lump sum taxes.

Let the price of labor services, W_t, denote the marginal gain to the employment agency that occurs when an individual worker raises labor intensity by one unit. Because the employment agency is competitive in the supply of labor services, W_t is taken as given and is the same for all agencies, regardless of which cohort it is in. Labor intensity equates the worker's marginal cost to the agency's marginal benefit:

$$W_t = \zeta_t^h A_L \varsigma_t^{\sigma_L} \frac{1}{\upsilon_t \frac{1-\tau_t^y}{1+\tau_t^w}}. \tag{4.3}$$

To understand the expression on the right of the equality, note that the marginal cost, in utility terms, to an individual worker who increases labor intensity by one unit is $\zeta_t^h A_L \varsigma_t^{\sigma_L}$. This is converted to after-tax currency units by dividing by the multiplier, υ_t, on the household's nominal budget constraint as well as by the tax wedge due to labor income taxes and payroll taxes.

Labor intensity is the same for all cohorts because none of the variables in (4.3) is indexed by cohort. When the wage rate is determined by Nash bargaining, it is taken into account that labor intensity is determined according to (4.3).

Finally, the employment agency in the i^{th} cohort determines how many employees it will have in period $t+1$ by choosing vacancies, v_t^i. The vacancy posting costs associated with v_t^i

are:

$$\frac{\kappa z_t^+}{2}\left(\frac{Q_t^\iota v_t^i}{l_t^i}\right)^2 l_t^i,$$

units of the domestic homogeneous good. Here, l_t^i denotes the number of employees in the i^{th} cohort and $\kappa z_t^+/2$ is a cost parameter which is assumed to grow at the same rate as the overall economic growth rate. Also, Q_t is the probability that a posted vacancy is filled. The functional form of our cost function nests GT and GST when $\iota = 1$. With this parameterization the cost function is in terms of the number of people hired, not the number of vacancies per se. We interpret this as reflecting that the GT and GST specifications emphasize internal costs (such as training and other) of adjusting the work force, and not search costs. In models used in the search literature (see, e.g., Shimer (2005a)), vacancy posting costs are independent of Q_t, i.e., $\iota = 0$. We also plan to investigate this latter case. We suspect that the model implies less amplification in response to expansionary shock in the case, $\iota = 0$. In a boom, Q_t can be expected to fall, so that with $\iota = 1$, costs of posting vacancies decrease in the GT specification.

4.2. Model Details

An employment agency in the i^{th} cohort which does not renegotiate its wage in period t sets the period t wage, $W_{i,t}$, as in (2.69):

$$W_{i,t} = \tilde{\pi}_{w,t} W_{i-1,t-1},\ \tilde{\pi}_{w,t} \equiv \left(\pi_{t-1}^c\right)^{\kappa_w}\left(\bar{\pi}_t^c\right)^{(1-\kappa_w-\varkappa_w)}\left(\breve{\pi}\right)^{\varkappa_w}\left(\mu_{z^+}\right)^{\vartheta_w}, \tag{4.4}$$

for $i = 1, ..., N-1$ (note that an agency that was in the i^{th} cohort in period t was in cohort $i-1$ in period $t-1$) where $\kappa_w, \varkappa_w, \vartheta_w, \kappa_w + \varkappa_w \in (0,1)$. After wages are set, employment agencies in cohort i supply labor services, $l_t^i \varsigma_t$, into competitive labor markets. In addition, they post vacancies to attract new workers in the next period.

4.2.1. The Employment-Agency Problem

To understand how agencies bargain and how they make their employment decisions, it is useful to consider $F\left(l_t^0, \omega_t\right)$, the value function of the representative employment agency in the cohort that negotiates its wage in the current period. The arguments of F are the agency's workforce after beginning-of-period separations and new arrivals, l_t^0, and an arbitrary value for the nominal wage rate, ω_t. We are thus interested in the firm's problem after the wage rate has been set, when vacancy decisions remain to be made. To simplify notation, we leave out arguments of F that correspond to economy-wide variables. We find it convenient to adopt a change of variables. We suppose that the firm chooses a particular monotone transform of vacancy postings, which we denote by $\tilde{v}_t^i$:

$$\tilde{v}_t^i \equiv \frac{Q_t^\iota v_t^i}{l_t^i}.$$

The agency's hiring rate is related to $\tilde{v}_t^i$ by:

$$\chi_t^i = Q_t^{1-\iota} \tilde{v}_t^i. \tag{4.5}$$

In this notation, the agency's objective is to solve:

$$\begin{aligned} F\left(l_t^0, \omega_t\right) &= \sum_{j=0}^{N-1} \beta^j E_t \frac{\upsilon_{t+j}}{\upsilon_t} \max_{\tilde{v}_{t+j}^j} \left[\left(W_{t+j} - \Gamma_{t,j}\omega_t\right) \varsigma_{t+j} - P_{t+j} \frac{\kappa z_{t+j}^+}{2} \left(\tilde{v}_t^j\right)^2 \right] l_{t+j}^j \\ & +\beta^N E_t \frac{\upsilon_{t+N}}{\upsilon_t} F\left(l_{t+N}^0, \tilde{W}_{t+N}\right), \end{aligned} \tag{4.6}$$

where ς_t is assumed to satisfy (4.3). Here,

$$\Gamma_{t,j} = \begin{cases} \tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1}, & j > 0 \\ 1 & j = 0 \end{cases}. \tag{4.7}$$

Also, $\tilde{W}_{t+N}$ denotes the Nash bargaining wage rate that will be negotiated when the agency next has an opportunity to do so. At time t, the agency takes $\tilde{W}_{t+N}$ as given. The law of motion of an agency's work force is:

$$l_{t+1}^{i+1} = \left(\chi_t^i + \rho\right) l_t^i, \tag{4.8}$$

for $i = 0, 1, ..., N-1$, with the understanding here and throughout that $i = N$ is to be interpreted as $i = 0$. Expression (4.8) is deterministic, reflecting the assumption that the agency employs a large number of workers.

The firm chooses vacancies to solve the problem in (4.6). It is easy to verify:

$$F\left(l_t^0, \omega_t\right) = J\left(\omega_t\right) l_t^0, \tag{4.9}$$

where $J\left(\omega_t\right)$ is not a function of l_t^0. The function, $J\left(\omega_t\right)$, is the surplus that a firm bargaining in the current period enjoys from a match with an individual worker, when the current wage is ω_t. For convenience, we omit the expectation operator E_t below. Let

$$\begin{aligned} J\left(\omega_t\right) &= \max_{\left\{v_{t+j}^j\right\}_{j=0}^{N-1}} \{\left(W_t - \omega_t\right) \varsigma_t - P_t z_t^+ \frac{\kappa}{2} \left(\tilde{v}_t^0\right)^2 \\ & +\beta \frac{\upsilon_{t+1}}{\upsilon_t} \left[\left(W_{t+1} - \Gamma_{t,1}\omega_t\right) \varsigma_{t+1} - P_{t+1} z_{t+1}^+ \frac{\kappa}{2} \left(\tilde{v}_{t+1}^1\right)^2 \right] \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \\ & +\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t} \left[\left(W_{t+2} - \Gamma_{t,2}\omega_t\right) \varsigma_{t+2} - P_{t+2} z_{t+2}^+ \frac{\kappa}{2} \left(\tilde{v}_{t+2}^2\right)^2 \right] \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho\right) \\ & +... + \\ & +\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right) \left(\tilde{v}_t^0 Q_t^{1-\iota} + \rho\right) \left(\tilde{v}_{t+1}^1 Q_t^{1-\iota} + \rho\right) \cdots \left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho\right)\}. \end{aligned}$$

Differentiate with respect to $\tilde{v}_t^0$ and multiply the result by $\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}$, to obtain:

$$\begin{aligned}
0 &= -P_t z_t^+ \kappa \tilde{v}_t^0 \left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&\quad +\beta \frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}-\Gamma_{t,1}\omega_t\right)\varsigma_{t+1}-P_{t+1}z_{t+1}^+\frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right) \\
&\quad +\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}-\Gamma_{t,2}\omega_t\right)\varsigma_{t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right) \\
&\quad +\ldots+ \\
&\quad +\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho\right)\} \\
&= J\left(\omega_t\right)-\left(W_t-\omega_t\right)\varsigma_t+P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2-P_t z_t^+\kappa \tilde{v}_t^0\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota}
\end{aligned}$$

Since the latter expression must be zero, we conclude:

$$\begin{aligned}
J\left(\omega_t\right) &= \left(W_t-\omega_t\right)\varsigma_t-P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2+P_t z_t^+\kappa \tilde{v}_t^0\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)/Q_t^{1-\iota} \\
&= \left(W_t-\omega_t\right)\varsigma_t+P_t z_t^+\frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2+P_t z_t^+\kappa \tilde{v}_t^0\frac{\rho}{Q_t^{1-\iota}}
\end{aligned}$$

Next, we obtain simple expressions for the vacancy decisions from their first order necessary conditions for optimality. Multiplying the first order condition for $\tilde{v}_{t+1}^1$ by

$$\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}},$$

we obtain:

$$\begin{aligned}
0 &= -\beta \frac{\upsilon_{t+1}}{\upsilon_t}P_{t+1}z_{t+1}^+\kappa \tilde{v}_{t+1}^1\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+1}^{1-\iota}} \\
&\quad +\beta^2 \frac{\upsilon_{t+2}}{\upsilon_t}\left[\left(W_{t+2}-\Gamma_{t,2}\omega_t\right)\varsigma_{t+2}-P_{t+2}z_{t+2}^+\frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\right]\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota}+\rho\right) \\
&\quad +\ldots+ \\
&\quad +\beta^N \frac{\upsilon_{t+N}}{\upsilon_t} J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0 Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1 Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota}+\rho\right)
\end{aligned}$$

Substitute out the period $t+2$ and higher terms in this expression using the first order condition for $\tilde{v}_t^0$. After rearranging, we obtain,

$$\frac{P_t z_t^+\kappa \tilde{v}_t^0}{Q_t^{1-\iota}}=\beta \frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}-\Gamma_{t,1}\omega_t\right)\varsigma_{t+1}+P_{t+1}z_{t+1}^+\kappa\left(\frac{\left(\tilde{v}_{t+1}^1\right)^2}{2}+\frac{\tilde{v}_{t+1}^1\rho}{Q_{t+1}^{1-\iota}}\right)\right].$$

Following the pattern set with $\tilde{v}_{t+1}^1$, multiply the first order condition for $\tilde{v}_{t+2}^2$ by

$$\left(\tilde{v}_{t+2}^2 Q_{t+2}^{1-\iota}+\rho\right)\frac{1}{Q_{t+2}^{1-\iota}}.$$

Substitute the period $t+3$ and higher terms in the first order condition for $\tilde{v}_{t+2}^2$ using the first order condition for $\tilde{v}_{t+1}^1$ to obtain, after rearranging,

$$P_{t+1}z_{t+1}^{+}\kappa\tilde{v}_{t+1}^{1}\frac{1}{Q_{t+1}^{1-\iota}}=\beta\frac{\upsilon_{t+2}}{\upsilon_{t+1}}\left[\left(W_{t+2}-\Gamma_{t,2}\omega_t\right)\varsigma_{t+2}+P_{t+2}z_{t+2}^{+}\kappa\left(\frac{\left(\tilde{v}_{t+2}^{2}\right)^2}{2}+\frac{\tilde{v}_{t+2}^{2}\rho}{Q_{t+2}^{1-\iota}}\right)\right]$$

Continuing in this way, we obtain,

$$P_{t+j}z_{t+j}^{+}\kappa\tilde{v}_{t+j}^{j}\frac{1}{Q_{t+j}^{1-\iota}}=\beta\frac{\upsilon_{t+j+1}}{\upsilon_{t+j}}\left[\left(W_{t+j+1}-\Gamma_{t,j+1}\omega_t\right)\varsigma_{t+j+1}+P_{t+j+1}z_{t+j+1}^{+}\kappa\left(\frac{\left(\tilde{v}_{t+j+1}^{j+1}\right)^2}{2}+\frac{\tilde{v}_{t+j+1}^{j+1}\rho}{Q_{t+j+1}^{1-\iota}}\right)\right],$$

for $j=0,1,...,N-2$. Now consider the first order necessary condition for the optimality of $\tilde{v}_{t+N-1}^{N-1}$. After multiplying this first order condition by

$$\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+N-1}^{1-\iota}},$$

we obtain,

$$\begin{aligned}0 \quad = \quad & -\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}P_{t+N-1}z_{t+N-1}^{+}\kappa\tilde{v}_{t+N-1}^{N-1}\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_{t+1}^{1-\iota}+\rho\right)\cdots\\ & \cdots\left(\tilde{v}_{t+N-2}^{N-2}Q_{t+N-2}^{1-\iota}+\rho\right)\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\frac{1}{Q_{t+N-1}^{1-\iota}}\\ & +\beta^N\frac{\upsilon_{t+N}}{\upsilon_t}J\left(\tilde{W}_{t+N}\right)\left(\tilde{v}_t^0Q_t^{1-\iota}+\rho\right)\left(\tilde{v}_{t+1}^1Q_t^{1-\iota}+\rho\right)\cdots\left(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho\right)\}\end{aligned}$$

or,

$$P_{t+N-1}z_{t+N-1}^{+}\kappa\tilde{v}_{t+N-1}^{N-1}\frac{1}{Q_{t+N-1}^{1-\iota}}=\beta\frac{\upsilon_{t+N}}{\upsilon_{t+N-1}}J\left(\tilde{W}_{t+N}\right).$$

Making use of our expression for J, we obtain:

$$P_{t+N-1}z_{t+N-1}^{+}\kappa\tilde{v}_{t+N-1}^{N-1}\frac{1}{Q_{t+N-1}^{1-\iota}}=\beta\frac{\upsilon_{t+N}}{\upsilon_{t+N-1}}\left[\left(W_{t+N}-\tilde{W}_{t+N}\right)\varsigma_{t+N}+P_{t+N}z_{t+N}^{+}\kappa\left(\frac{\left(\tilde{v}_{t+N}^{0}\right)^2}{2}+\frac{\tilde{v}_{t+N}^{0}\rho}{Q_{t+N}^{1-\iota}}\right)\right].$$

The above first order conditions apply over time to a group of agencies that bargain at date t. We now express the first order conditions for a fixed date and different cohorts:

$$\begin{aligned}P_tz_t^{+}\kappa\tilde{v}_t^{j}\frac{1}{Q_t^{1-\iota}} \quad = \quad & \beta\frac{\upsilon_{t+1}}{\upsilon_t}\left[\left(W_{t+1}-\Gamma_{t-j,j+1}\tilde{W}_{t-j}\right)\varsigma_{t+1}+P_{t+1}z_{t+1}^{+}\kappa\left(\frac{\left(\tilde{v}_{t+1}^{j+1}\right)^2}{2}+\frac{\tilde{v}_{t+1}^{j+1}\rho}{Q_{t+1}^{1-\iota}}\right)\right],\\ & \text{for } j=0,...,N-2\end{aligned}$$

Divide both sides by $P_tz_t^{+}$ and express the result in terms of scaled variables:

$$\begin{aligned}\kappa\tilde{v}_t^{j}\frac{1}{Q_t^{1-\iota}} \quad = \quad & \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\left[\left(\bar{w}_{t+1}-G_{t-j,j+1}w_{t-j}\bar{w}_{t-j}\right)\varsigma_{t+1}+\kappa\left(\frac{\left(\tilde{v}_{t+1}^{j+1}\right)^2}{2}+\frac{\tilde{v}_{t+1}^{j+1}\rho}{Q_{t+1}^{1-\iota}}\right)\right]\\ & \text{for } j=0,...,N-2\end{aligned} \tag{4.10}$$

where

$$\begin{aligned} G_{t-i,i+1} &= \frac{\tilde{\pi}_{w,t+1}\cdots\tilde{\pi}_{w,t-i+1}}{\pi_{t+1}\cdots\pi_{t-i+1}}\left(\frac{1}{\mu_{z^+,t-i+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+1}}\right),\ i\geq 0, \\ w_t &= \frac{\tilde{W}_t}{W_t},\ \bar{w}_t = \frac{W_t}{z_t^+ P_t}. \end{aligned}$$

Also,

$$G_{t,j} = \begin{cases} \frac{\tilde{\pi}_{w,t+j}\cdots\tilde{\pi}_{w,t+1}}{\pi_{t+j}\cdots\pi_{t+1}}\left(\frac{1}{\mu_{z^+,t+1}}\right)\cdots\left(\frac{1}{\mu_{z^+,t+j}}\right) & j>0 \\ 1 & j=0 \end{cases}. \tag{4.11}$$

The scaled vacancy first order condition of agencies that are in the last period of their contract is:

$$\kappa\tilde{v}_t^{N-1}\frac{1}{Q_t^{1-\iota}} = \beta\frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}\left[\left(\bar{w}_{t+1} - w_{t+1}\bar{w}_{t+1}\right)\varsigma_{t+1} + \kappa\left(\frac{\left(\tilde{v}_{t+1}^0\right)^2}{2} + \frac{\tilde{v}_{t+1}^0\rho}{Q_{t+1}^{1-\iota}}\right)\right]. \tag{4.12}$$

We require the derivative of J with respect to ω_t. By the envelope condition, we can ignore the impact of a change in ω_t on the vacancy decisions and only be concerned with the direct impact of ω_t on J :

$$\begin{aligned} J_{w,t} &= -\varsigma_t \\ &\quad -\beta\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{t+1}\left(\chi_t^0+\rho\right) \\ &\quad -\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{t+2}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right) \\ &\quad -\ldots-\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{t+N-1}\left(\chi_t^0+\rho\right)\left(\chi_{t+1}^1+\rho\right)\cdots\left(\chi_{t+1}^{N-2}+\rho\right). \end{aligned}$$

Let,

$$\Omega_{t,j} = \begin{cases} \prod_{l=0}^{j-1}\left(\chi_{t+l}^l+\rho\right) & j>0 \\ 1 & j=0 \end{cases}. \tag{4.13}$$

so that

$$\begin{aligned} J_{w,t} &= -\varsigma_t\Omega_{t,0} \\ &\quad -\beta\frac{\upsilon_{t+1}}{\upsilon_t}\Gamma_{t,1}\varsigma_{t+1}\Omega_{t,1} \\ &\quad -\beta^2\frac{\upsilon_{t+2}}{\upsilon_t}\Gamma_{t,2}\varsigma_{t+2}\Omega_{t,2} \\ &\quad -\ldots-\beta^{N-1}\frac{\upsilon_{t+N-1}}{\upsilon_t}\Gamma_{t,N-1}\varsigma_{t+N-1}\Omega_{t,N-1} \\ &= -\sum_{j=0}^{N-1}\beta^j\frac{\upsilon_{t+j}}{\upsilon_t}\Gamma_{t,j}\Omega_{t,j}\varsigma_{t+j}. \end{aligned}$$

Then, in terms of scaled variables,

$$J_{w,t} = -\sum_{j=0}^{N-1} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} G_{t,j} \Omega_{t,j} \varsigma_{t+j}. \tag{4.14}$$

The following is an expression for J_t evaluated at $\omega_t = \tilde{W}_t$, in terms of scaled variables. Dividing by $P_t z_t^+$:

$$\begin{aligned}
J_{z^+,t} &= \frac{J\left(\tilde{W}_t\right)}{P_t z_t^+} = \frac{\left(W_t - \tilde{W}_t\right)}{P_t z_t^+}\varsigma_t - \frac{\kappa}{2}\left(\tilde{v}_t^0\right)^2 \\
&+\beta \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} \left[\frac{W_{t+1} - \Gamma_{t,1}\tilde{W}_t}{P_{t+1} z_{t+1}^+}\varsigma_{t+1} - \frac{\kappa}{2}\left(\tilde{v}_{t+1}^1\right)^2\right]\left(\chi_t^0 + \rho\right) \\
&+\beta^2 \frac{\psi_{z^+,t+2}}{\psi_{z^+,t}} \left[\frac{W_{t+2} - \Gamma_{t,2}\tilde{W}_t}{P_{t+2} z_{t+2}^+}\varsigma_{t+2} - \frac{\kappa}{2}\left(\tilde{v}_{t+2}^2\right)^2\right]\left(\chi_t^0 + \rho\right)\left(\chi_{t+1}^1 + \rho\right) \\
&+ ... + \\
&+\beta^N \frac{\psi_{z^+,t+N}}{\psi_{z^+,t}} J_{z^+,t+N}\left(\chi_t^0 + \rho\right)\left(\chi_{t+1}^1 + \rho\right)\cdots\left(\chi_{t+N-1}^{N-1} + \rho\right)
\end{aligned}$$

or,

$$\begin{aligned}
J_{z^+,t} &= \sum_{j=0}^{N-1} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \left[\left(\bar{w}_{t+j} - G_{t,j} w_t \bar{w}_t\right)\varsigma_{t+j} - \frac{\kappa}{2}\left(\tilde{v}_{t+j}^j\right)^2\right]\Omega_{t,j} \\
&+\beta^N \frac{\psi_{z^+,t+N}}{\psi_{z^+,t}} J_{z^+,t+N}\Omega_{t,N}.
\end{aligned} \tag{4.15}$$

4.2.2. The Worker Problem

We now turn to the worker. The period t value of being a worker in an agency in cohort i is V_t^i :

$$V_t^i = \Gamma_{t-i,i}\tilde{W}_{t-i}\varsigma_t \frac{1-\tau_t^y}{1+\tau_t^w} - \zeta_t^h A_L \frac{\varsigma_t^{1+\sigma_L}}{(1+\sigma_L)\,\upsilon_t} + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t}[\rho V_{t+1}^{i+1} + (1-\rho)\,U_{t+1}], \tag{4.16}$$

for $i = 0, 1, ..., N-1$. Here, ρ is the probability of remaining with the firm in the next period and U_t is the value of being unemployed in period t. The values, V_t^i and U_t, pertain to the beginning of period t, after job separation and job finding has occurred. Scaling V_t^i by $P_t z_t^+$, we obtain:

$$V_{z^+,t}^i = G_{t-i,i} w_{t-i} \bar{w}_{t-i}\varsigma_t \frac{1-\tau_t^y}{1+\tau_t^w} - \zeta_t^h A_L \frac{\varsigma_t^{1+\sigma_L}}{(1+\sigma_L)\,\psi_{z^+,t}} + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}}[\rho V_{z^+,t+1}^{i+1} + (1-\rho)\,U_{z^+,t+1}], \tag{4.17}$$

for $i = 0, 1, ..., N-1$, where

$$\frac{V_t^i}{P_t z_t^+} = V_{z^+,t}^i, \; U_{z^+,t+1} = \frac{U_{z^+,t+1}}{P_{t+1} z_{t+1}^+}.$$

In our analysis of the Nash bargaining problem, we must have the derivative of V_t^0 with respect to the wage rate. Let this derivative, evaluated at an arbitrary wage rate, ω_t, be denoted $V_w^0(\omega_t)$:

$$\begin{aligned} V_w^0(\omega_t) &= \sum_{j=0}^{N-1} (\beta\rho)^j E_t \varsigma_{t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} \Gamma_{t,j} \frac{\upsilon_{t+j}}{\upsilon_t} \\ &= \sum_{j=0}^{N-1} (\beta\rho)^j E_t \varsigma_{t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} \Gamma_{t,j} \frac{\psi_{z^+,t+j} P_t z_t^+}{\psi_{z^+,t} P_{t+j} z_{t+j}^+} \\ &= \sum_{j=0}^{N-1} (\beta\rho)^j E_t \varsigma_{t+j} \frac{1-\tau_{t+j}^y}{1+\tau_{t+j}^w} G_{t,j} \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}}. \end{aligned} \quad (4.18)$$

Note ω_t has no impact on the intensity of labor effort, ς_t. This is determined by (4.3), independent of the wage rate paid to workers.

The value of being an unemployed worker is U_t :[12]

$$U_t = P_t z_t^+ b^u (1-\tau_t^y) + \beta E_t \frac{\upsilon_{t+1}}{\upsilon_t} [f_t V_{t+1}^x + (1-f_t) U_{t+1}], \quad (4.19)$$

where f_t is the probability that an unemployed worker will land a job in period $t+1$. Also, V_t^x is the period $t+1$ value function of a worker who finds a job, before it is known which agency the job is found with:

$$V_{z^+,t}^x = \sum_{i=0}^{N-1} \frac{\chi_{t-1}^i l_{t-1}^i}{m_{t-1}} V_{z^+,t}^{i+1}, \quad (4.20)$$

after scaling. Here, the total number of new matches is given by:

$$m_t = \sum_{j=0}^{N-1} \chi_t^j l_t^j. \quad (4.21)$$

In (4.20),

$$\frac{\chi_{t-1}^i l_{t-1}^i}{m_{t-1}}$$

is the probability of finding a job in an agency which was of type i in the previous period, conditional on being a worker who finds a job in t.

[12] Note that in the model, as in the Swedish data, unemployment benefits are subject to (labor) income tax.

Scaling (4.19),

$$U_{z^+,t} = b^u \left(1-\tau_t^y\right) + \beta E_t \frac{\psi_{z^+,t+1}}{\psi_{z^+,t}} [f_t V_{z^+,t+1}^x + (1-f_t) U_{z^+,t+1}] \tag{4.22}$$

Total job matches must also satisfy the following matching function:

$$m_t = \sigma_{m,t} \left(1-L_t\right)^{\sigma} v_t^{1-\sigma}, \tag{4.23}$$

where

$$L_t = \sum_{j=0}^{N-1} l_t^j. \tag{4.24}$$

and $\sigma_{m,t}$ is a time-varying exogenous productivity parameter.[13]

Total hours worked is:

$$H_t = \varsigma_t \sum_{j=0}^{N-1} l_t^j. \tag{4.25}$$

The job finding rate is:

$$f_t = \frac{m_t}{1-L_t}. \tag{4.26}$$

The probability of filling a vacancy is:

$$Q_t = \frac{m_t}{v_t}. \tag{4.27}$$

The $i=0$ cohort of agencies in period t solve the following Nash bargaining problem:

$$\max_{\omega_t} \left(V^0\left(\omega_t\right) - U_t\right)^{\eta_t} J\left(\omega_t\right)^{(1-\eta_t)}, \tag{4.28}$$

where $V^0\left(\omega_t\right) - U_t$ is the match surplus enjoyed by a worker and η_t is the bargaining power of workers which we allow to follow an exogenous time-varying process. We denote the wage that solves this problem by $\tilde{W}_t$. Note that $\tilde{W}_t$ takes into account that intensity will be chosen according to (4.3) as well as (4.4). The first order condition associated with this problem is:

$$\eta_t V_{w,t} J_{z^+,t} + (1-\eta_t) \left[V_{z^+,t}^0 - U_{z^+,t}\right] J_{w,t} = 0, \tag{4.29}$$

after division by $z_t^+ P_t$.

We assume that the posting of vacancies uses the homogeneous domestic good. We leave the production technology equation, (2.82), unchanged, and we alter the resource constraint:

$$\begin{aligned} y_t &= g_t + c_t^d + i_t^d \\ &\quad + \left(R_t^x\right)^{\eta_x} \left[\omega_x \left(p_t^{m,x}\right)^{1-\eta_x} + (1-\omega_x)\right]^{\frac{\eta_x}{1-\eta_x}} (1-\omega_x) \left(p_t^x\right)^{-\eta_f} y_t^* + \frac{\kappa}{2} \sum_{j=0}^{N-1} \left(\tilde{v}_t^j\right)^2 l_t^j. \end{aligned} \tag{4.30}$$

[13] Time variation of σ_m is necessary if we want to include vacancies among the observed variables, otherwise we have stochastic singularity as unemployment tomorrow, in that case, is fully determined by vacancies and unemployment today.

Total vacancies v_t are related to vacancies posted by the individual cohorts as follows:

$$v_t = \frac{1}{Q_t^\iota} \sum_{j=0}^{N-1} \tilde{v}_t^j l_t^j.$$

Note however, that this equation does not add a constraint to the model equilibrium. In fact, it can be derived from the equilibrium equations (4.27), (4.21) and (4.5).

4.3. Solving the Unemployment Model

The new endogenous variables compared to the baseline model are (apart from (4.30)):

$$G_{t,j}, \Omega_{t,j}, \tilde{v}_t^j, l_t^j, \chi_t^j, V_{z^+,t}^j, V_{w,t}, J_{z^+,t}, V_{z^+,t}^x, U_{z^+,t}, J_{w,t}, L_t, m_t, \varsigma_t, v_t, f_t, w_t, Q_t$$

These are 12 variables not indexed by j, plus 6 variables indexed by j. Equations (4.5), (4.10), (4.12), (4.17), (4.8), (4.11), (4.13) are the ones associated with the variables indexed by j. Not counting the resource constraint, (4.30), there are 13 additional equations, (4.14), (4.15), (4.24), (4.23), (4.18), (4.20), (4.21), (4.22), (4.25), (4.26), (4.3), (4.29), (4.27). Note that the above equations also contain the variables, $\psi_{z^+,t}, \pi_t, \bar{w}_t, H_t$ but they appear in other equations in the baseline model. Given that we have 13 new equations, there is an extra one relative to the new variables. However, one of the equations in the baseline model (i.e., wage Phillips curve in the case of the linearized dynamics, and the labor intratemporal Euler equation in the case of the steady state) is lost. So, it appears that we have an equal number of equations and unknowns. The unemployment part also brings two new shocks $\sigma_{m,t}$ and η_t into the model.

5. The Full Model including Financial Frictions and Unemployment

Finally, in this section, we integrate financial frictions and unemployment together in our baseline model.

5.1. Equilibrium equations

The equations which describe the dynamic behavior of the model are those of the baseline model discussed in section 2 plus those discussed in the financial frictions model specified in section 3.2.1 plus those discussed in the unemployment model presented in section 4.3. Finally, the resource constraint needs to be adjusted to include monitoring as well as vacancy posting costs.

6. Estimation

We estimate the full model which includes both financial and labor market frictions using Bayesian techniques.

6.1. Calibration

We calibrate and later estimate our model to Swedish data. The time unit is a quarter. Parameters that are related to "great ratios" and other fractions that are observable in the data are calibrated. These include the discount factor β and the tax rate on bonds τ_b that is calibrated to match a real interest of rate of slightly above 2 percent annually. We also calibrate capital share α and δ to match the ratio of investment over output, i/y.

Sample averages are used when available, e.g. for the various import shares ω_i, ω_c, ω_x (obtained from input-output tables), the remaining tax rates, the government consumption share of GDP, η_g, and several other parameters. We also calibrate the steady value of the inflation target, and the inflation target persistence.

We let the markup of export good producers λ_x be low so as to avoid double marking up of these goods. We require full working capital financing in all appropriate sectors. We set ϑ_w so that there is no indexation of wages to the steady state real growth. The indexation parameters $\varkappa^j$, $j = d, x, mc, mi, mx, w$ are set so that there is no indexation to the inflation target, and $\breve{\pi}$ is set to unity. This implies that we allow for partial indexation, which results in steady state price and wage dispersion.

For the financial block of the model we set $F(\bar{\omega})$ to match the bankruptcy rate in the data.

For the labor block, $1 - L$ is set to the sample average unemployment rate, the length of a wage contract N to annual negotiation frequency, ρ is set so that it takes an unemployed person, on average, 4 quarters to find a job (i.e. $f = 1/4$) , in line with the evidence presented in Holmlund (2006). We set the replacement rate for unemployed workers, $bshare$, based on the average statutory replacement rate, after tax, for this time period.[14] The matching function parameter σ is set so that number of unemployed and vacancies have equal factor shares in the production of matches. σ_m is calibrated to match the probability $Q = 0.9$ of filling a vacancy in a quarter. Finally, A_L is set so that employed individuals spend roughly a fraction 0.3 of their time working. The calibrated values are displayed in Table 1.

[14] This is only a rough calibration as the replacement rate in the data does not include the utility of leisure time of unemployed people that is present in the model.

Parameter	Value	Description
α	0.36	Capital share in production
β	0.999	Discount factor
δ	0.02	Depreciation rate of capital
ω_i	0.43	Import share in investment goods
ω_c	0.25	Import share in consumption goods
ω_x	0.35	Import share in export goods
η_g	0.3	Government consumption share on GDP
τ_k	0.25	Capital tax rate
τ_w	0.35	Payroll tax rate
τ_c	0.25	Consumption tax rate
τ_y	0.30	Labor income tax rate
$\bar{\pi}$	1.005	Steady state gross inflation target
$\rho_{\bar{\pi}}$	0.975	Persistence of inflation target shock
λ_x	1.045	Export price markup
$\nu_t^*, \nu_t^x, \nu_t^f$	1	Working capital shares
ϑ_w	0	Wage indexation to real growth trend
$\varkappa^j$	$1-\kappa^j$	No indexation to inflation target for $j = d, x, mc, mi, mx, w$
$\breve{\pi}$	1	Third indexing base
$F(\bar{\omega})$	0.0063	Steady state bankruptcy rate
L	1-0.075	Steady state fraction of employment
N	4	Number of agency cohorts/Length of wage contracts
ρ	0.97975	Survival rate of a match
$bshare$	0.71	Replacement rate for unemployed.
σ	0.5	Unemployment share in matching technology
σ_m	0.475	Level parameter in matching function
A_L	4.5	Scaling parameter disutility of working

Table 1. Calibrated parameters.

6.2. Choice of priors

The priors are displayed in a table in the Appendix. The general approach has been to choose diffuse priors, with four exceptions.

First, for the exogenous technology processes where we use tight priors (a standard deviation of 0.075) on the persistence parameters and a mode at 0.85. Second, for the Calvo price stickiness parameters we use a mode of 0.75 (corresponding to annual price setting) and tight priors. Third, we use tight priors for the persistence and shock size of government consumption process. The priors for this process is based on pre-estimating a univariate process for government consumption. Finally, the priors for γ and W^e are tight because of their highly-nonlinear effect on the model.

We center σ_a at 2, following the result in Altig, Christiano, Eichenbaum and Linde (2004). For the labor supply curvature σ_L we use a mode of unity. For the prior mode of the habit formation we follow a wide literature by setting it at 0.65.

For the Taylor rule we use the same priors as ALLV. We also let all estimated gross markups have a prior mode at 1.2. Regarding the parameters for indexation to past inflation we are agnostic and use a diffuse beta prior centered at 0.5.

For the financial block we choose a prior mode of μ corresponding to a 2% annual default premium.

The persistence of the entrepreneurial parameters γ_t and σ_t have the same priors as the technological processes.

For the labor block ,we let the mode of the fraction of GDP spent on vacancy costs be 0.25%.[15] Regarding ι, which determines the degree of search vs. hiring costs, we are agnostic and use a diffuse beta prior around 0.5. See the table in the Appendix for a complete list of our priors.

6.3. Data

We estimate the model on Swedish data. Our sample period is 1995Q1-2008Q1. All real quantities are in per capita terms. We use the same 15 macro variables as in ALLV. We use 3 additional data series. First, we add a time series on stock prices (the 'OMX Stockholm PI' index, formerly 'SAX All Shares') scaled by the domestic price level as a measure of real net worth. Second, we match a proxy for the spread between the risk-free rate and the loan rate entrepreneurs face. In particular, we compute the spread between the interest rate on all outstanding loans to non-financial corporations and the 6 month interest rate on government bonds.[16] Third, we include a time series for the unemployment rate.

We match the levels of the following 6 (nominal) time series:

$$R_t^{data}, \pi_t^{data}, \pi_t^{c,data}, \pi_t^{i,data}, \pi_t^{*,data}, R_t^{*,data}.$$

For the remaining 12 time series we take logs and first differences. In addition we demean each first-differenced time series:

$$\Delta \ln(W_t/P_t)^{data}, \Delta \ln C_t^{data}, \Delta \ln I_t^{data}, \Delta \ln q_t^{data}, \Delta \ln H_t^{data}, \Delta \ln Y_t^{data}, \Delta \ln X_t^{data}$$
$$\Delta \ln M_t^{data}, \Delta \ln Y_t^{*,data}, \Delta \ln N_t^{data}, \Delta \ln Spread_t^{data}, \Delta \ln Unemprate_t^{data}.$$

We demean the first-differenced time series because in our sample from 1995Q1-2008Q1 variables such as output, consumption, real wages, investment, exports, imports, stock prices

[15] Formally the steady state recruitment share is defined as

$$recruitshare = \frac{\frac{\kappa}{2} N \tilde{v}^2 l}{y}$$

[16] Ideally one would like to match interest data on newly issued loans for the same maturity as in the model. Unfortunately, such data is not available for Sweden.

grow on average at substantially different rates. The model, however, allows for two different growth rates only. In order to match these different trends in the data the estimation is likely to result in a series of negative or positive shocks for some stationary exogenous process. We want to avoid this and therefore demean the data. After the estimation we compare the growth rates of the data with those implied by the model.

See Figure A in the Appendix for plots of the above data used for the estimation.

6.4. Shocks

In total, there are 21 exogenous stochastic variables in the model. 10 variables evolve according to AR(1) processes:

$$\epsilon, \Upsilon, \bar{\pi}^c, \zeta^c, \zeta^h, \tilde{\phi}, \sigma, \gamma, g, \eta.$$

Further, we have 6 variables that are iid:

$$\tau^d, \tau^x, \tau^{mi}, \tau^{mc}, \tau^{mx}, \varepsilon_R.$$

Finally, the last 5 variables are assumed to form a VAR(1):

$$y^*, \pi^*, R^*, \mu_z, \mu_\Psi.$$

6.5. Measurement equations

Below we report the measurement equations we use to link the model to the data. First differences are written in percentages so model variables are multiplied by 100 accordingly. Furthermore our data series for inflation and interest rates are annualized, so we make the same transformation for the model variables i.e. multiplying by 400:

$$\begin{aligned}
\pi_t^{data} &= 400 \log \pi_t \\
R_t^{data} &= 400 \log(R_t) \\
\pi_t^{c,data} &= 400 \log \pi_t^c \\
\pi_t^{i,data} &= 400 \log \pi_t^i \\
\pi_t^{*,data} &= 400 \log \pi_t^* \\
R_t^{*,data} &= 400 \log(R_t^*)
\end{aligned}$$

We use demeaned first-differenced data for the remaining variables. Accordingly, we need to demean the variables in the model that grow with their respective growth factors in order to obtain:

$$
\begin{aligned}
\Delta \ln Y_t^{data} &= 100\Delta \ln y_t \\
\Delta \ln Y_t^{*,data} &= 100\Delta \ln y_t^* \\
\Delta \ln C_t^{data} &= 100\Delta \ln c_t \\
\Delta \ln X_t^{data} &= 100\Delta \ln x_t \\
\Delta \ln q_t^{data} &= 100\Delta \ln q_t \\
\Delta \ln H_t^{data} &= 100\Delta \ln H_t \\
\Delta \ln M_t^{data} &= 100\Delta \ln \text{ Imports}_t \\
&= 100\Delta \ln \begin{pmatrix} c_t^m \left(\overset{\circ}{p}_t^{m,c}\right)^{\frac{\lambda^{m,C}}{1-\lambda^{m,C}}} \\ +i_t^m \left(\overset{\circ}{p}_t^{m,i}\right)^{\frac{\lambda^{m,i}}{1-\lambda^{m,i}}} \\ +x_t^m \left(\overset{\circ}{p}_t^{m,x}\right)^{\frac{\lambda^{m,x}}{1-\lambda^{m,x}}} \end{pmatrix}
\end{aligned}
$$

In addition, demeaned investment in the data is measured as the sum of demeaned model investment plus investment goods used for capital maintenance:

$$
\Delta \ln I_t^{data} = 100\Delta \ln \left(i_t + a\left(u_t\right) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right)
$$

The demeaned real wage in the model is measured by the demeaned Nash bargaining wage in the model:

$$
\Delta \ln(W_t/P_t)^{data} = 100\Delta \ln \frac{\tilde{W}_t}{z_t^+ P_t} = 100\Delta \ln \bar{w}_t w_t
$$

Finally, we measure demeaned net worth, the interest spread and unemployment as follows:

$$
\begin{aligned}
\Delta \ln N_t^{data} &= 100\Delta \ln n_t \\
\Delta \ln Spread_t^{data} &= 100\Delta \ln(Z_{t+1} - R_t) \\
&= 100\Delta \ln \left(\frac{\bar{\omega}_{t+1} R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}\bar{k}_{t+1}}} - R_t \right) \\
\Delta \ln Unemp_t^{data} &= 100\Delta \ln(1 - L_t).
\end{aligned}
$$

6.6. Estimation results

The appendix provides preliminary estimation results. In particular, we provide parameter estimates at the posterior mean. In addition, we present the asymptotic variance decomposition as well as a comparison of data vs. model means and standard deviations for the observed time series. Finally, we plot impulse responses at the posterior mean for some selected shocks.

7. Analysis

To be written.

8. Conclusion

This paper describes several important extensions of the small open economy model presented in Adolfson, Laséen, Lindé and Villani (2005, 2007a, 2007b). First, we introduce the feature that exports are produced by using imported goods in addition to domestically produced goods. Second, we incorporate financial frictions in the accumulation and management of capital similar to Bernanke, Gertler and Gilchrist (1999) and Christiano, Motto and Rostagno (2003, 2007). Third, we include the search and matching framework of Mortensen and Pissarides (1994), Gertler, Sala and Trigari (2006) and Christiano, Ilut, Motto, and Rostagno (2007).

We estimate the full model, which contains the financial frictions as well as the and search and matching frictions, with Bayesian techniques.

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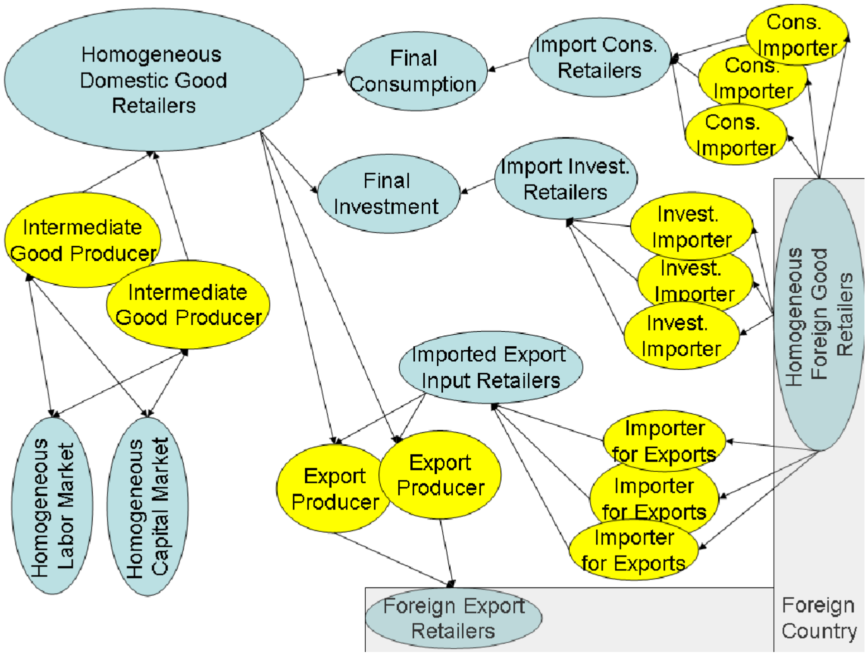
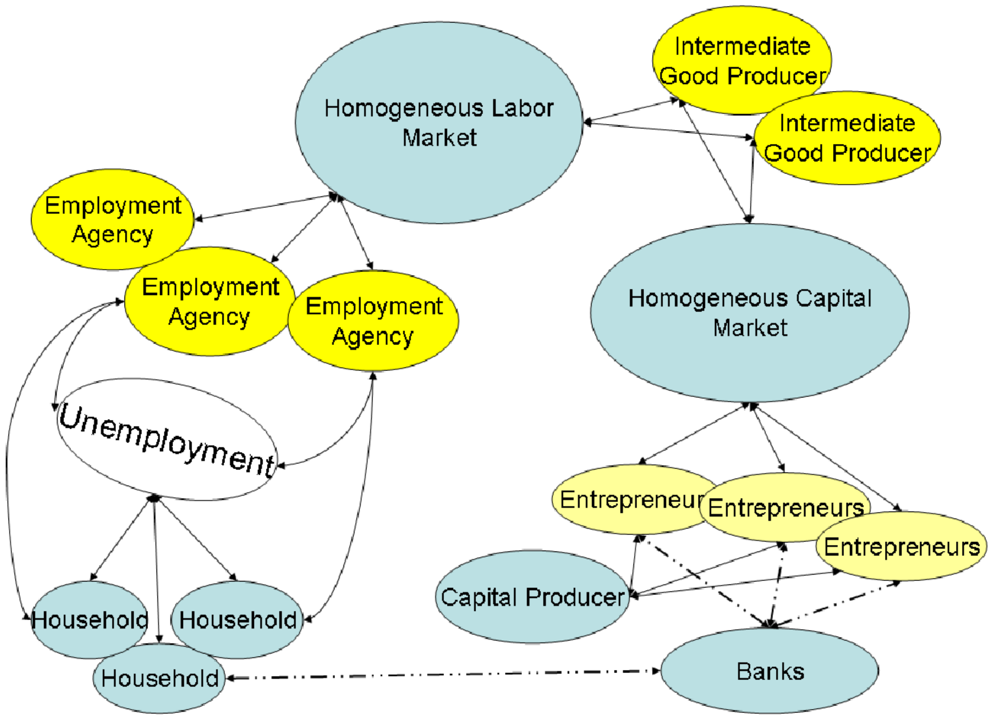
A. Tables and Figures



Graphical illustration of the goods production part of the model.



Graphical illustration of the labor and capital markets of the model.

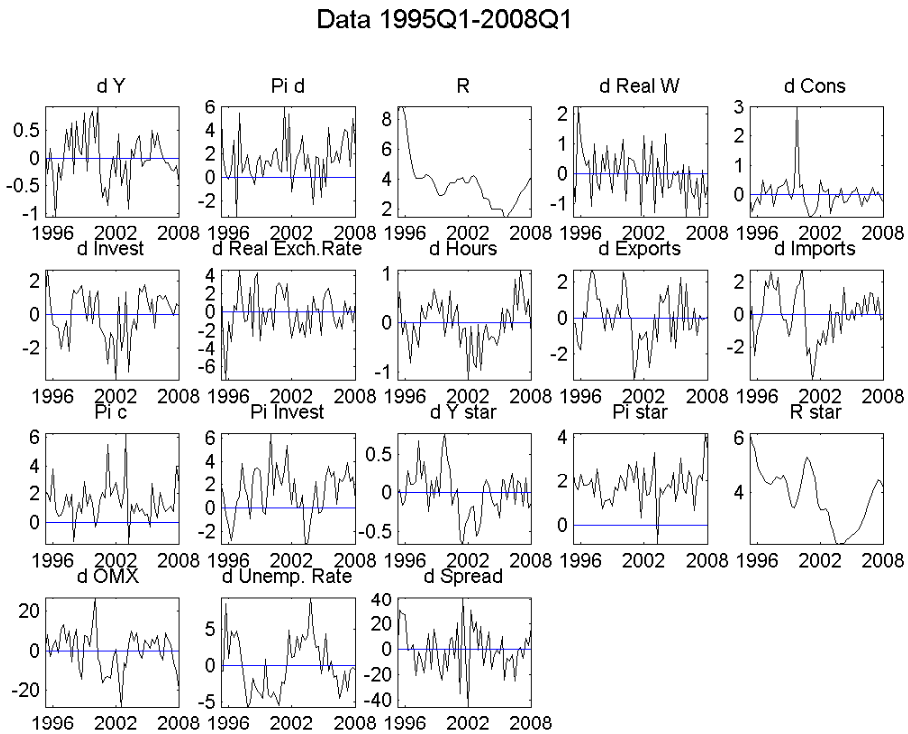
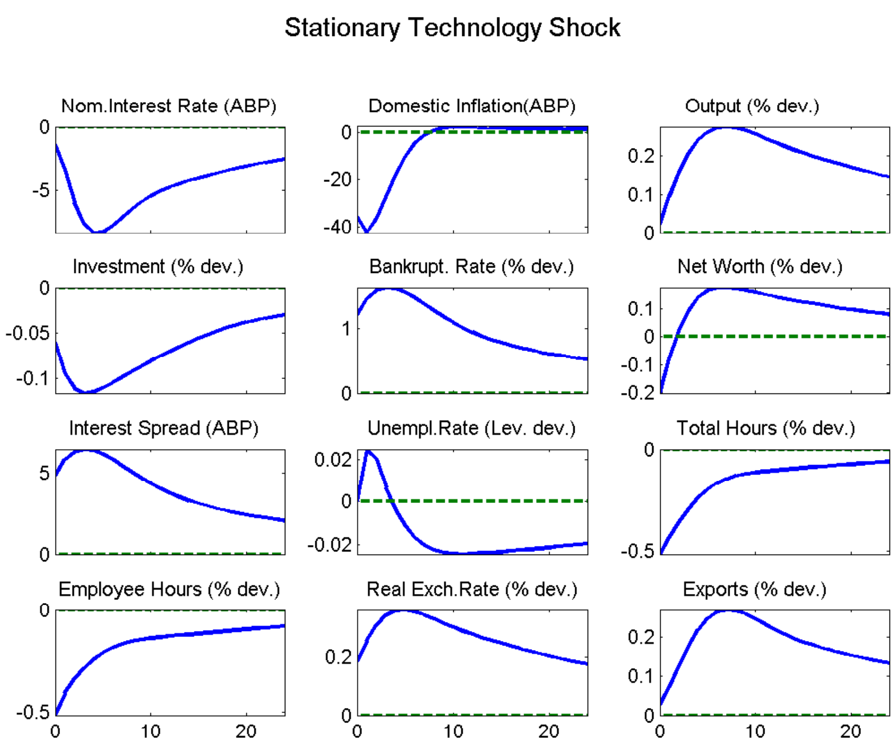
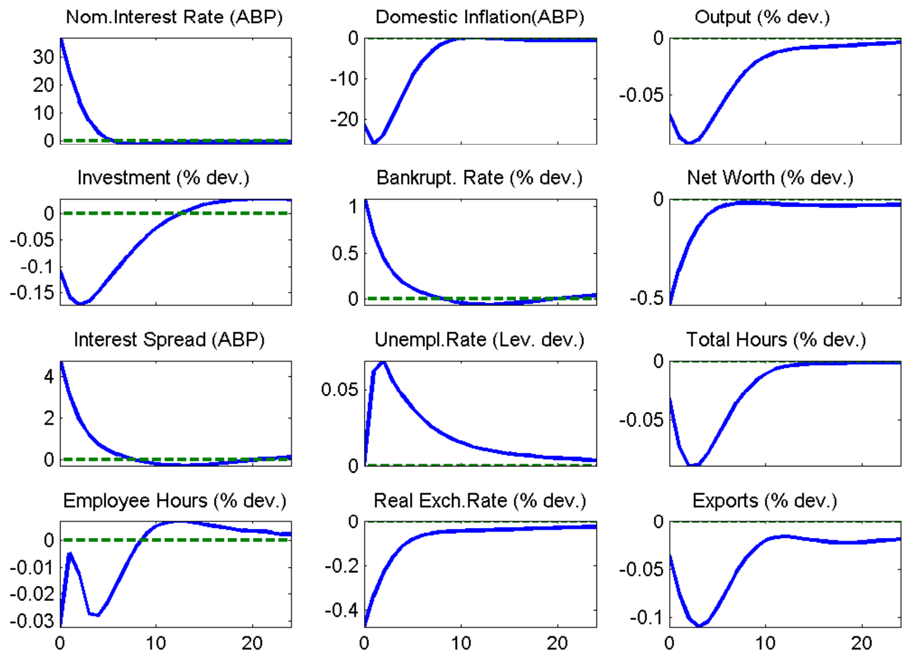


Figure C. Data series used in estimation.



Monetary Policy Shock



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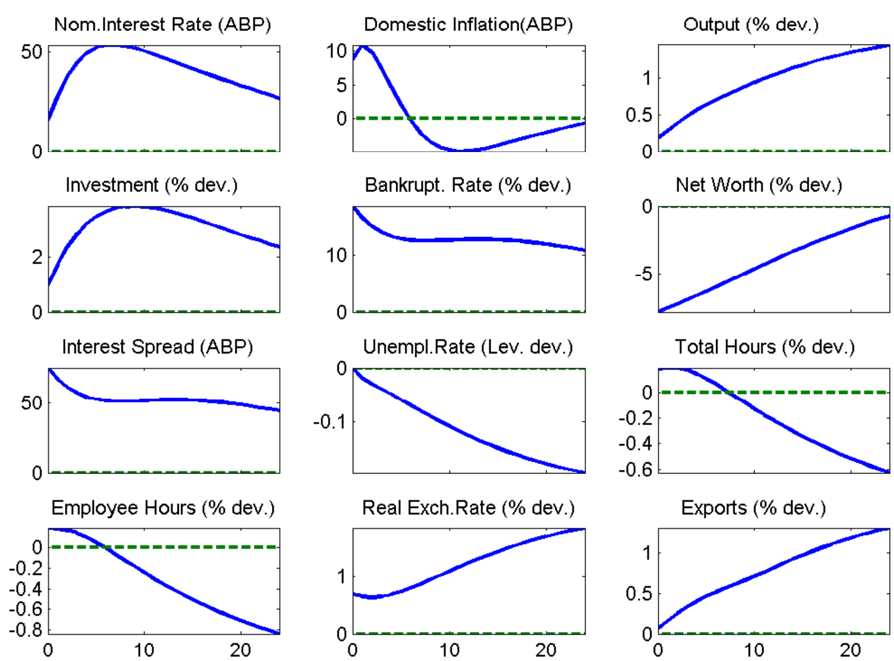
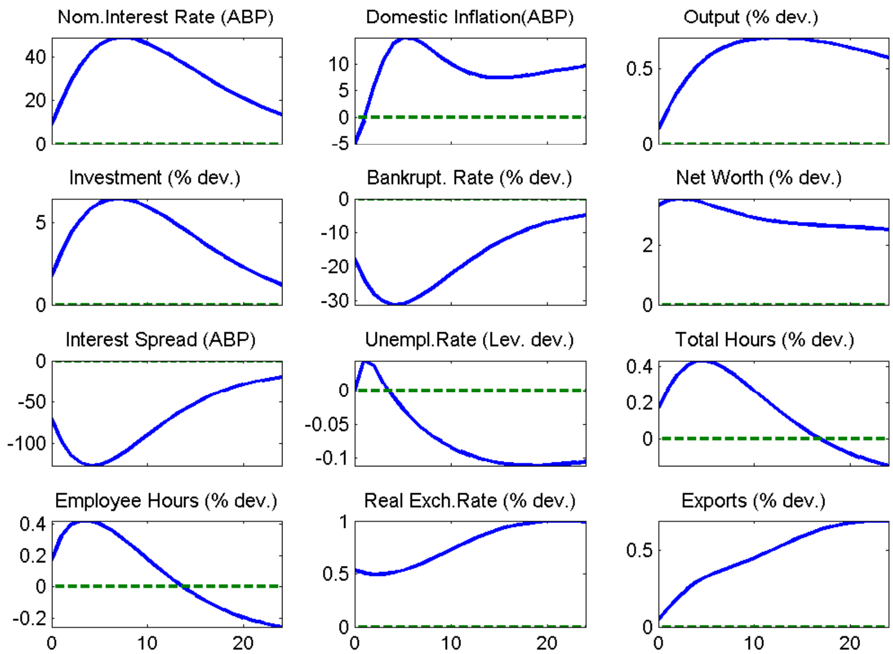
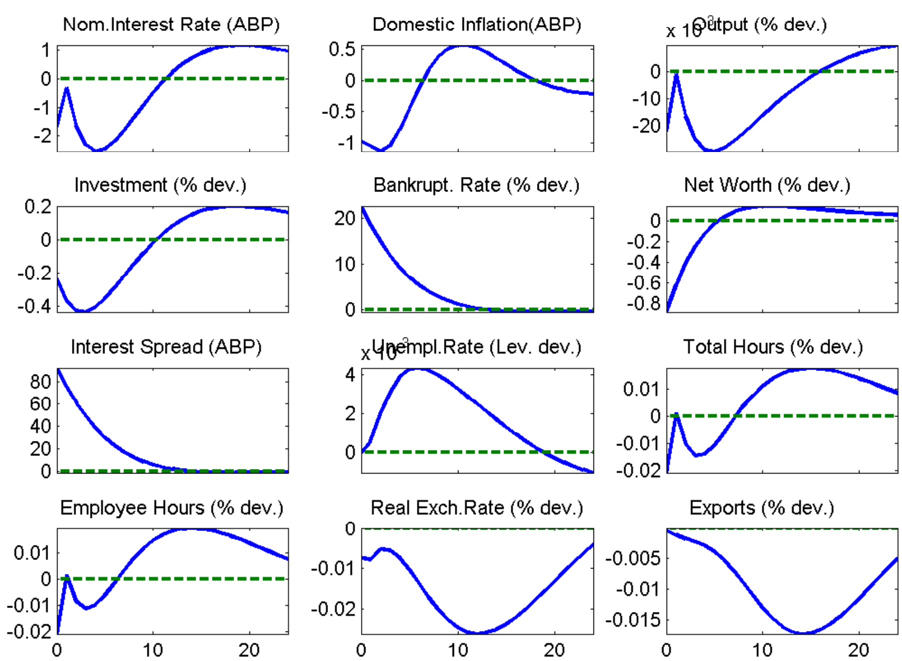


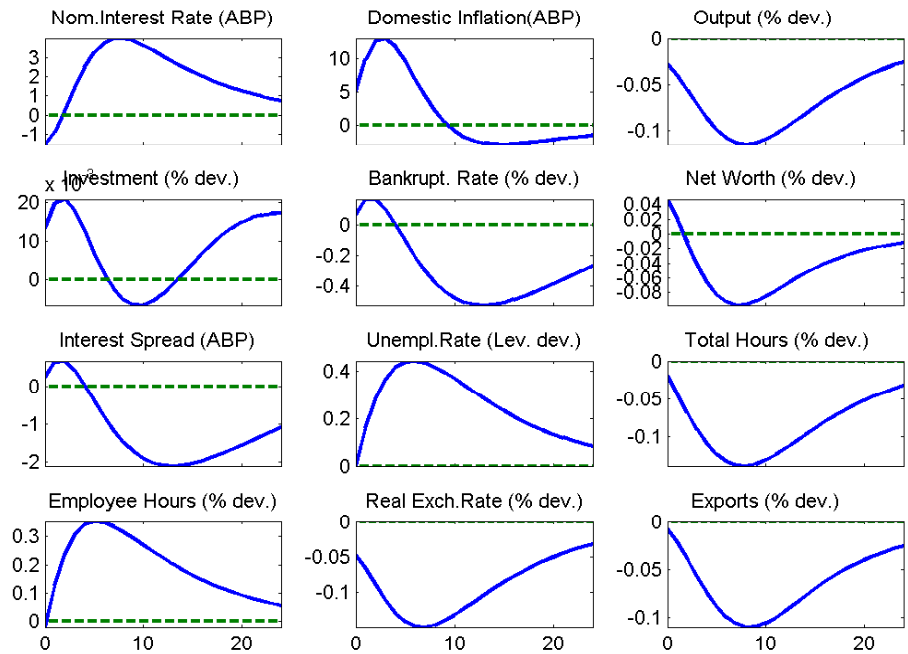
Entrepreneur Survival Probability Shock



Shock to Standard Deviation of Entrepreneurial Productivity



Bargaining Power of Workers Shock



Model versus Data moments (in percent)

	Means		Standard Deviations	
	Data	Model	Data	Model
Domestic. Inflation	1.56	2.00	1.97	3.20
CPI inflation	1.48	2.00	1.38	2.98
Invest. inflation	1.42	1.27	2.03	3.22
Nom. interest rate	3.82	4.57	1.73	3.31
GDP growth	0.61	0.54	0.44	0.83
Real wage growth	0.69	0.54	0.79	1.48
Consumption growth	0.52	0.54	0.53	0.86
Investment growth	1.09	0.72	1.42	3.66
Real exch. rate growth	0.04	0.00	2.30	2.70
Total hours growth	0.05	0.00	0.46	0.93
Exports growth	1.52	0.54	1.36	1.59
Import growth	1.37	0.54	1.48	1.46
Stock market growth	1.96	0.54	9.94	8.76
Interest spread growth	-0.32	0.00	17.39	26.55
Unemployment growth	-1.16	0.00	3.61	4.19
Foreign GDP growth	0.45	0.54	0.30	0.32
Foreign Inflation	1.84	2.00	0.81	0.82
Foreign nom. Int. rate	3.89	4.57	1.04	1.24

Variance Decomposition (asymptotic, in percent)

Shocks/Variabes	Pi	Pic	Pii	R	dy	dw	dc	di	dq	dH	dexp	dimp	dn	dspread	dunemp
Unit-root invest. tech.	0.9	0.8	3.6	0.4	0.7	0.7	0.2	0.8	0.3	0.8	0.6	0.3	0.0	0.0	0.6
Unit-root neutr. tech.	1.2	1.3	1.8	0.7	6.7	5.4	1.5	0.6	1.5	0.7	3.8	1.3	0.1	0.0	3.1
Stat. neutr. tech.	5.7	2.9	1.2	0.7	2.2	2.4	3.3	1.1	0.6	33.3	0.5	0.7	0.1	0.0	1.1
Stat. invest. tech.	0.9	6.9	13.8	50.1	18.1	7.1	23.4	19.4	8.0	8.6	3.5	5.4	82.4	4.2	2.1
Consumption pref.	1.1	1.3	1.2	3.6	7.8	0.1	45.3	0.1	0.1	7.4	0.1	5.6	0.0	0.1	0.3
Labor preference	3.5	1.9	0.9	0.5	1.2	18.5	1.5	1.1	0.3	1.5	0.2	0.7	0.1	0.1	10.4
Risk premium	2.9	4.9	9.9	2.6	1.3	0.2	1.0	0.1	68.2	1.6	3.7	2.8	0.1	0.1	0.5
Nom. Interest rate	2.4	2.6	2.7	2.0	0.8	7.3	1.0	0.6	3.6	0.4	0.2	0.7	0.5	0.1	4.6
Inflation target	0.5	0.6	0.5	0.3	0.0	0.1	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0	0.1
Gov. Consumption	0.2	0.2	0.1	0.6	9.9	0.2	0.1	0.2	0.1	7.8	0.0	0.1	0.0	0.0	0.2
Domestic markup	71.1	39.2	18.7	2.1	10.3	40.6	3.9	1.8	4.2	4.7	0.9	2.6	0.8	0.0	7.5
Export markup	1.3	1.5	1.5	3.0	9.3	0.2	0.9	0.2	0.1	7.7	81.1	4.2	0.0	0.1	1.1
Cons. import mkup	0.2	25.4	0.2	2.3	0.2	1.7	2.1	0.1	4.4	0.2	0.0	0.9	0.1	0.0	1.4
Invest. import mkup	0.0	0.0	31.2	0.0	0.0	0.0	0.0	0.5	0.0	0.2	0.0	0.6	0.0	0.0	0.0
Export import mkup	1.3	0.6	0.2	2.2	20.7	1.9	0.2	0.7	1.7	16.1	2.2	54.0	0.0	0.2	2.6
Entrepreneur risk	0.0	0.0	0.0	0.1	0.2	0.0	0.0	0.9	0.0	0.1	0.0	0.5	1.2	67.8	0.0
Entrepre. survival	5.6	8.9	10.8	28.2	10.0	7.3	15.0	71.6	4.4	8.1	1.4	19.0	14.6	27.1	4.5
Bargaining power	0.8	0.4	0.2	0.1	0.4	6.2	0.5	0.0	0.1	0.4	0.1	0.0	0.0	0.0	59.9
Foreign output	0.4	0.7	1.4	0.6	0.3	0.0	0.3	0.1	2.1	0.2	1.1	0.7	0.0	0.0	0.1
Foreign inflation	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.1	0.5	0.0	0.0	0.0	0.0
Foreign nom. Int.	0.0	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.3	0.0	0.1	0.1	0.0	0.0	0.0

	Prior distribution	Prior mean	Prior s.d.	Posterior mean	Posterior s.d.	HPD inf	HPD sup
rhomuz	beta	0.500	0.0750	0.487	0.0549	0.3979	0.5752
rhoepsilon	beta	0.850	0.0750	0.962	0.0170	0.9350	0.9898
rhoUpsilon	beta	0.850	0.0750	0.982	0.0076	0.9705	0.9952
rhozetac	beta	0.850	0.0750	0.733	0.0495	0.6549	0.8135
rhozetah	beta	0.850	0.0750	0.872	0.0453	0.7998	0.9399
rhophitilde	beta	0.850	0.0750	0.809	0.0497	0.7277	0.8859
xid	beta	0.750	0.0750	0.766	0.0304	0.7115	0.8114
xix	beta	0.750	0.0750	0.671	0.0375	0.6126	0.7285
ximc	beta	0.750	0.0750	0.798	0.0313	0.7373	0.8414
ximi	beta	0.750	0.0750	0.769	0.0478	0.6977	0.8469
kappad	beta	0.500	0.2500	0.786	0.1255	0.6079	0.9881
kappaw	beta	0.500	0.2500	0.254	0.1505	0.0047	0.4621
kappamc	beta	0.500	0.2500	0.204	0.1108	0.0154	0.3637
kappami	beta	0.500	0.2500	0.313	0.1984	0.0171	0.6183
kappax	beta	0.500	0.2500	0.872	0.0846	0.7513	0.9977
kappamx	beta	0.500	0.2500	0.696	0.1608	0.4625	0.9748
varphi	gamm	1.000	0.2000	0.955	0.2640	0.5812	1.3776
phitildea	gamm	0.010	0.0050	0.009	0.0041	0.0028	0.0154
phitildes	beta	0.500	0.2500	0.056	0.0439	0.0007	0.1142
rhoR	beta	0.850	0.1000	0.843	0.0192	0.8105	0.8726
rpi	norm	1.700	0.1000	1.638	0.1041	1.4708	1.7979
rdeltapi	norm	0.300	0.1000	0.150	0.0329	0.0955	0.2006
rq	norm	0.000	0.1000	-0.031	0.0139	-0.0526	-0.0069
ry	norm	0.125	0.1000	0.110	0.0239	0.0717	0.1464
rdeltay	norm	0.050	0.1000	0.093	0.0365	0.0341	0.1562
rhomupsi	beta	0.500	0.0750	0.527	0.0802	0.3895	0.6507
ximx	beta	0.750	0.0750	0.689	0.0363	0.6312	0.7517
lambdad	norm	1.200	0.0750	1.317	0.0605	1.2127	1.4108
lambdamx	norm	1.200	0.0750	1.140	0.0367	1.0821	1.1991
lambdamc	norm	1.200	0.0750	1.241	0.0686	1.1257	1.3512
lambdami	norm	1.200	0.0750	1.253	0.0719	1.1427	1.3796
a11	norm	0.500	0.5000	0.457	0.1892	0.1409	0.7471
a22	norm	0.500	0.5000	0.043	0.1356	-0.1764	0.2830
a33	norm	0.500	0.5000	1.018	0.1376	0.7909	1.2239
a12	norm	0.000	0.5000	-0.015	0.0818	-0.1576	0.1132
a13	norm	0.000	0.5000	0.228	0.2264	-0.1030	0.5782
a21	norm	0.000	0.5000	-0.194	0.2500	-0.6172	0.2193
a23	norm	0.000	0.5000	0.163	0.1662	-0.0791	0.4481
a24	norm	0.000	0.5000	0.193	0.0955	0.0401	0.3532
a31	norm	0.000	0.5000	-0.381	0.1420	-0.6122	-0.1558
a32	norm	0.000	0.5000	0.077	0.0608	-0.0173	0.1866
a34	norm	0.000	0.5000	0.148	0.0384	0.0781	0.2021
c21	norm	0.000	0.5000	-0.057	0.4203	-0.7117	0.6771
c31	norm	0.000	0.5000	-0.141	0.1329	-0.3655	0.0680
c32	norm	0.000	0.5000	-0.009	0.0489	-0.0862	0.0718
c24	norm	0.000	0.5000	-0.222	0.1501	-0.4714	0.0175
c34	norm	0.000	0.5000	0.015	0.0530	-0.0722	0.1002
rhog	beta	0.760	0.0250	0.757	0.0234	0.7159	0.7917
gamma	beta	0.970	0.0150	0.979	0.0016	0.9759	0.9811
rhosigma	beta	0.850	0.0750	0.883	0.0429	0.8091	0.9517
rhogamma	beta	0.850	0.0750	0.904	0.0202	0.8729	0.9397
rhoeta	beta	0.850	0.0750	0.827	0.0725	0.7090	0.9421
recruitsharePercent	beta	0.250	0.1500	0.043	0.0163	0.0180	0.0643
mu	beta	0.700	0.1000	0.769	0.0661	0.6633	0.8759
SppScaled	norm	0.800	0.2000	0.898	0.1625	0.6211	1.1339
b	beta	0.650	0.1000	0.697	0.0488	0.6239	0.7786
etai	gamm	1.500	0.7500	0.495	0.2284	0.1389	0.8405
etaf	gamm	1.500	0.7500	0.736	0.1694	0.4648	1.0112
etac	gamm	1.500	0.7500	0.349	0.1492	0.1128	0.5809
etax	gamm	1.500	0.7500	2.204	0.8079	1.1790	3.5158
iota	beta	0.500	0.2500	0.611	0.2463	0.2439	0.9913
muzPercent	norm	0.460	0.0250	0.437	0.0226	0.3971	0.4717
mupsiPercent	norm	0.260	0.0250	0.192	0.0208	0.1608	0.2270
sigmaa	gamm	2.000	0.7500	3.334	0.6959	2.2143	4.5436
sigmaL	gamm	1.000	0.1500	0.962	0.1247	0.7776	1.1881

Table A.1: Results from Metropolis-Hastings (parameters)

	Prior distribution	Prior mean	Prior s.d.	Posterior mean	Posterior s.d.	HPD inf	HPD sup
muz_eps	invg	0.200	Inf	0.233	0.0335	0.1770	0.2869
$epsilon_eps$	invg	0.700	Inf	0.405	0.0416	0.3384	0.4754
$epsR_eps$	invg	0.150	Inf	0.113	0.0119	0.0923	0.1313
$Upsilon_eps$	invg	0.500	Inf	0.909	0.0851	0.7597	1.0311
$zetac_eps$	invg	0.200	Inf	0.198	0.0321	0.1439	0.2491
$zetah_eps$	invg	1.000	Inf	1.165	0.1614	0.9020	1.4062
$phitilde_eps$	invg	0.500	Inf	0.451	0.1274	0.2488	0.6413
$ystar_eps$	invg	0.250	Inf	0.093	0.0190	0.0625	0.1236
$pistar_eps$	invg	0.250	Inf	0.189	0.0244	0.1491	0.2295
$Rstar_eps$	invg	0.050	Inf	0.028	0.0075	0.0158	0.0404
$mupsi_eps$	invg	0.200	Inf	0.180	0.0418	0.1008	0.2346
$taud_eps$	invg	0.250	Inf	0.168	0.0466	0.0971	0.2369
$taux_eps$	invg	0.250	Inf	0.330	0.0797	0.2035	0.4569
$taumc_eps$	invg	0.250	Inf	0.493	0.1449	0.2554	0.7153
$taumi_eps$	invg	0.250	Inf	0.264	0.1315	0.1046	0.4761
$taumx_eps$	invg	0.250	Inf	0.478	0.1226	0.2765	0.6639
g_eps	invg	0.660	0.0025	0.796	0.0381	0.7390	0.8612
$gamma_eps$	invg	0.500	Inf	0.455	0.0615	0.3474	0.5530
$sigma_eps$	invg	1.000	Inf	2.395	0.2362	2.0298	2.7931
eta_eps	invg	0.500	Inf	0.368	0.1116	0.2080	0.5447
$pitarget_eps$	invg	0.250	Inf	0.211	0.1524	0.0579	0.4349

Table A.2: Results from Metropolis-Hastings (standard deviation of structural shocks)

B. Technical Details for the Financial Frictions Block

Consider the steady state version of equation (3.4):

$$[1-\Gamma(\bar{\omega})]\,\frac{R^k}{R}+\frac{\Gamma'(\bar{\omega})}{\Gamma'(\bar{\omega})-\mu G'(\bar{\omega})}\left([\Gamma(\bar{\omega})-\mu G(\bar{\omega})]\,\frac{R^k}{R}-1\right)=0. \tag{B.1}$$

Note that in the limiting case, $\mu\rightarrow 0$, the only equilibrium is one in which $R^k=R$, and there is no wedge between these two variables. Interestingly, this model does not converge to the baseline model in the limiting case. In the baseline model, it is after tax R that is equated to R^k.

The log normal cdf, F, has two parameters, the mean and variance of the normal distribution of $\log\omega$. We solve for the two parameters and for $\bar{\omega}$ by imposing: (i) $E\omega=1$, (ii) $F\left(\bar{\omega}\right)$ (the bankruptcy rate) is equal to some specified calibrated value, and (iii) (B.1) holds.

We now provide some technical observations on the computations surrounding (3.4), (B.1) and conditions (i)-(iii) above. We first develop straightforward expressions for computing

$$G(\bar{\omega})=\int_0^{\bar{\omega}}\omega dF(\omega),\ \Gamma\left(\bar{\omega}\right)=\bar{\omega}\left[1-F(\bar{\omega})\right]+G(\bar{\omega}),$$

and their derivatives for a given value of $\bar{\omega}$. Consider $G\left(\bar{\omega}\right)$ first. Let $x=\log\left(\omega\right)$, so that

$$\int_0^{\bar{\omega}}\omega dF(\omega)=\int_{-\infty}^{\log\bar{\omega}}e^x f\left(x\right)dx,$$

where f is the Normal density function. Writing this explicitly:

$$\begin{aligned}\int_0^{\bar{\omega}}\omega dF(\omega) &= \int_{-\infty}^{\log\bar{\omega}}e^x f\left(x\right)dx\\ &= \frac{1}{\sigma_x\sqrt{2\pi}}\int_{-\infty}^{\log\bar{\omega}}e^x\exp^{\frac{-(x-Ex)^2}{2\sigma_x^2}}dx,\end{aligned}$$

where σ_x^2 is the variance of x. Now, $E\omega = 1$ implies $Ex = -(1/2)\sigma_x^2$, so that

$$\begin{aligned}
\int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} e^x \exp^{\frac{-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx \\
&= \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} \exp^{\frac{x2\sigma_x^2-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx \\
&= \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\log\bar{\omega}} \exp^{\frac{-\left(x-\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}} dx.
\end{aligned}$$

Now, make the change of variable,

$$\begin{aligned}
v &= \frac{x-\frac{1}{2}\sigma_x^2}{\sigma_x} = \frac{x+\frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x \\
\bar{v} &= \frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x \\
dv &= \frac{1}{\sigma_x}dx
\end{aligned}$$

so that

$$\begin{aligned}
\int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1}{\sigma_x\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x} \exp^{\frac{-v^2}{2}} \sigma_x dv \\
&= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}-\sigma_x} \exp^{\frac{-v^2}{2}} dv,
\end{aligned}$$

or,

$$G(\bar{\omega}) = prob\left[v < \frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x} - \sigma_x\right],$$

for a standard normal variable, v. The expression on the right of the equality can be evaluated using the cdf of a standard normal distribution. Similarly, $\bar{\omega}[1-F(\bar{\omega})]$ can be evaluated using the cdf of a log normal distribution, with log mean $-\sigma_x^2/2$ and log standard deviation, σ_x. Both these cdf's are available as part of standard computer packages.

By Leibniz's rule, $G'(\bar{\omega}) = \bar{\omega}F'(\bar{\omega})$, where F' is the pdf of a log normal. This pdf is available in standard computer packages. Regarding the derivative of Γ, note that

$$\begin{aligned}
\Gamma'(\bar{\omega}) &= 1 - F(\bar{\omega}) - \bar{\omega}F'(\bar{\omega}) + G'(\bar{\omega}) \\
&= 1 - F(\bar{\omega}),
\end{aligned}$$

which again is evaluated using the cdf of a log normal. We find the value of σ_x that ensures $F(\bar{\omega})$ equals some calibrated value by trying different values of σ_x and using a lognormal cdf to evaluate $F(\bar{\omega})$.

We also require

$$\frac{R^k}{R}\left[\Gamma\left(\bar{\omega}\right)-\mu G\left(\bar{\omega}\right)\right]=1-\frac{n}{p_{k'}\bar{k}},$$

where

$$\Gamma\left(\bar{\omega}\right)=\bar{\omega}\left[1-F\left(\bar{\omega}\right)\right]+G\left(\bar{\omega}\right),$$

so that

$$\Gamma\left(\bar{\omega}\right)-\mu G\left(\bar{\omega}\right)=\bar{\omega}\left[1-F\left(\bar{\omega}\right)\right]+\left(1-\mu\right)G\left(\bar{\omega}\right).$$

To obtain the derivative, $F'\left(\bar{\omega}\right)$, note

$$F\left(\bar{\omega}\right)=\int_0^{\bar{\omega}}dF(\omega)=\frac{1}{\sigma_x\sqrt{2\pi}}\int_{-\infty}^{\log\bar{\omega}}\exp^{\frac{-\left(x+\frac{1}{2}\sigma_x^2\right)^2}{2\sigma_x^2}}dx.$$

We make a change of variable similar to the one above:

$$\begin{aligned}
v &= \frac{x+\frac{1}{2}\sigma_x^2}{\sigma_x}\\
\bar{v} &= \frac{\log\left(\bar{\omega}\right)+\frac{1}{2}\sigma_x^2}{\sigma_x}\\
dv &= \frac{1}{\sigma_x}dx
\end{aligned}$$

so that

$$\begin{aligned}
F\left(\bar{\omega}\right) &= \int_0^{\bar{\omega}}dF(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}}\exp^{\frac{-v^2}{2}}dv\\
&= prob\left[v<\frac{\log\left(\bar{\omega}\right)+\frac{1}{2}\sigma_x^2}{\sigma_x}\right]
\end{aligned}$$

Differentiate with respect to $\bar{\omega}$:

$$\begin{aligned}
F'\left(\omega\right) &= \frac{1}{\bar{\omega}\sigma_x}\frac{1}{\sqrt{2\pi}}\exp^{\frac{-\left[\frac{\log(\bar{\omega})+\frac{1}{2}\sigma_x^2}{\sigma_x}\right]^2}{2}}\\
&= \frac{1}{\bar{\omega}\sigma_x}\text{Standard Normal pdf }\left(\frac{\log\left(\bar{\omega}\right)+\frac{1}{2}\sigma_x^2}{\sigma_x}\right),
\end{aligned}$$

where the first equality uses Leibniz's rule.