Optimal Monetary Policy with Credit Augmented Liquidity Cycles

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Abstract

The optimal response of monetary policy to financial instability is a long standing question whose policy relevance is now emphasized by the increase in available liquidity and in firms' financial exposure. Bernanke, Gertler and Gilchrist (1998) build a model in which credit frictions occur on the demand for capital investment and induce *demand driven fluctuations* which exacerbate shock transmission. In this context the policy maker does not face any trade-off as output stabilization is achieved through inflation targeting. I build a sticky price DSGE model in which the demand for working capital is affected both by a cost channel and an external finance premium. In this context the policy instrument affects the cost of collateralizable loans which in turn affects firms' marginal cost and inflation dynamics (*supply side driven fluctuations)*. The optimal monetary policy design is based upon both constrained and global Ramsey policies. Results show that: a) the optimal inflation level lies between zero and the one prescribed by the Friedman rule, b) the optimal dynamic path features deviations from price stability, c) the optimal rule features asset price targeting.

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1 Introduction

The optimal response of monetary policy to financial conditions, mostly in presence of lending distortions, is a long standing question whose policy relevance became cogent with the impressive increase in credit availability and firms' financial exposure. Those issues have received considerable attention since the time of the Great Depression when Friedman-Schwartz and Fisher argued that the Federal Reserve should have eased liquidity to avoid the financial turmoil. Nowadays most central banks have a mandate of strict price stability or inflation targeting, but those monetary policy strategies raise the doubt that central bankers might be helpless in the face of financial crises which occurred especially in recent years.

It is well-known in the literature that credit frictions have a significant impact on the transmission of shocks. A prominent role has been played by the *financial accelerator* mechanism of Bernanke, Gertler and Gilchrist 1998 - BGG 1998 hereafter - which assumes the presence of credit frictions, in the form of an external finance premium, on the demand for capital investment. Those frictions generate *demand driven fluctuations which* do not induce policy trade-offs as the policy maker can achieve output stabilization by pursuing inflation targeting[1]. As it is in the standard new keynesian model, in the BGG (1998) model positive productivity shocks induce an increase in output (beyond the one under flexible prices) and a decrease in inflation. The presence of an external finance premium which depends on collateralizable wealth tends to exacerbate output and inflation fluctuations without changing the qualitative responses. In this context it is still true, as in the Clarida, Gali' and Gertler 1998, that a policy maker acting under optimality can close both the output and the inflation gap by targeting either of the two. In face of a positive productivity shock this is achieved for instance by raising the nominal interest rate. Suppose instead that credit frictions apply to the funding of firms' working capital. In presence of a cost channel a' la Barth and Ramey 2001 or Christiano, Eichembaum and Evans 2001 the policy instrument affects firms' marginal costs and inflation. This implies that in face of a negative productivity shock, an increase in the nominal interest rate implemented to close the output gap will ultimately raise the inflation gap. In this context the monetary authority faces a meaningful output-inflation trade-off. As emphasized in also Ravenna and Walsh 2006, in this context productivity shocks act as cost-push shocks as counteracting movements in output invariably implies fluctuations in the inflation gap.

Against this background I build a sticky price DSGE model where credit frictions (in the form of costly state verification debt contracts) occur on the demand for working capital. More

[1] See also Bernanke and Gertler 1998 and Faia and Monacelli 2006.

specifically my model economy features: 1) a liquidity channel on working capital which allows for pass-through of movements in the nominal interest rate onto marginal costs, 2) sticky prices which provide a link between marginal cost and inflation via a conventional Phillips curve, 3) an external finance premium on working capital working capital which exacerbate response to shocks. By augmenting the cost channel with information asymmetry and moral hazard I induce acceleration in *supply side driven fluctuations.* Overall this mechanism can produce quantitatively meaningful policy trade-offs.

In this economy I study the optimal design of monetary policy by solving both *constrained* and *global Ramsey policies.* The first case (constrained Ramsey policy) allows us to solve the optimal targeting problem as the allocation is constrained by a well-specified set of operational policy rules. The second case (global solution to the Ramsey policy) allows us to characterize the optimal path of all variables.

Three results stand out. First, the long-run level of inflation lies between zero and the one associated with the Friedman rule. Second, an optimal policy rule features asset price targeting alongside with inflation targeting. Third, the optimal dynamic path features deviations from price stability and more muted response of nominal interest rates. Overall the activist role of monetary policy is revived.

Importantly in our context the optimal policy design follows the classical Ramsey approach (Ramsey 1927, Atkinson and Stiglitz 1980, Lucas and Stokey 1983, Chari, Christiano and Kehoe 1992) in which a social planner maximizes household's welfare subject to a resource constraint and to the constraints describing the equilibrium in the private sector economy. Such approach allows to study the design of optimal policy in presence of wedges that affect both the long run steady state of the economy and the dynamic path of variables[2]. In the context of the present model the cost channel induces an efficiency wedge both in the labour market equilibrium condition and in the optimal allocation of capital, while the external finance premium tends to amplify fluctuations driven by the abovementioned efficiency distortions.

The rest of the paper is divided as follows. Section 2 presents the model economy. Section 3 presents the dynamic properties of the model under standard Taylor rules. Section 4 shows the implementation of the optimal monetary policy design and the results concerning the optimal targeting rule. Section 5 solves for the long run policy. Section 6 presents the dynamic properties of the optimal policy and section 7 concludes.

[2]See Adao, Correia and Teles 2003, Schmitt-Grohe and Uribe 2003, Khan, King and Wolman 2003, Faia 2007a and Faia and Monacelli 2007a for further applications of this approach to new keynesian models.

2 The Model Economy

The laboratory economy is populated by representative agents who consume, supply labour and capital to firms and invest in money and deposits. Monopolistically competitive firms produce different varieties merging capital and labour through a Cobb-Douglas technology. Firms' face both adjustment costs a' la Rotemberg 1982 on their pricing decisions and costs for working capital as factors of production must be paid before the proceeds from the sale of output are received. In addition the external funds raised to pay for the factors of production are provided by a risk neutral intermediary that is unable to observe the ex-post realization of firms' revenues. Due to both an asymmetric information and a moral hazard problem, raising external funds requires the payment of an external finance premium as a result of a costly state verification debt contract a' la Gale and Hellwig 1983.

2.1 Households

In this economy there is continuum of households, each indexed by $i \in (0, 1)$. They consume the final good, c_t,invest in safe bank deposits, d_t, real money balances, m_t, and capital, k_t, supply labor, n_t, and own shares of a monopolistic competitive sector that produces differentiated varieties of goods. The representative worker chooses the set of processes $\{c_t,\, n_t\}_{t=0}^{\infty}$ and $\{d_t, k_t\}_{t=0}^{\infty}$, taking as given the set of processes $\{p_t, w_t,\, (1 + r_t^n), q_t\}_{t=0}^{\infty}$and the initial condition d_0, k_0 to maximize[3]:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \right\} \tag{1}$$

subject to the sequence of budget constraints:

$$p_t c_t + p_t d_{t+1} + m_{t+1} \leq (1 + r_t^n) p_t d_t + m_t + w_t n_t + \Theta_t + t_t + q_t k_t - p_t i_t \tag{2}$$

w_t is the nominal wage while p_t is the CPI price index, r_t^d is the nominal interest rate paid on deposits, Θ_t are the real profits that households receive from running production in the monopolistic sector, t_t are lump sum taxes/transfers from the fiscal authority and i_t is investment in capital that evolves according to the following law of motion:

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{3}$$

[3] In this context I follow the convention of considering real money balances as numeraire.

and to the cash in advance constraint:

$$p_t(c_t + i_t) = m_t + w_t n_t - d_t \tag{4}$$

which states that consumers can finance consumption using resources left over from the previous period. Let's define λ_t as the lagrange multiplier on constraint (2) and η_t as the lagrange multiplier on constraint (4). The first order conditions of the above problem read as follows:

$$u_{c,t} = \lambda_t + \eta_t \tag{5}$$

$$u_{c,t} = \beta(1 + r_t^d)E_t\left\{u_{c,t+1}\frac{p_t}{p_{t+1}}\right\} \tag{6}$$

$$u_{c,t}\frac{w_t}{p_t} = -u_{n,t} \tag{7}$$

$$u_{c,t} = \beta E_t\left\{u_{c,t+1}[(1-\delta)p_{t+1} + q_{t+1}]\right\} \tag{8}$$

Equation (5) defines the marginal utility on consumption. Equation (6) gives the optimal intertemporal allocation of consumption. Equation (7) gives the optimal allocation of labour supply and equation (8) gives the optimal allocation of capital. To achieve optimality this set of first order conditions must be satisfied together with a No- Ponzi condition on total asset allocation and with cash in advance constraint, (4).

Let's define the real interest rate as:

$$(1 + r_t) = (1 + r_t^d)E_t\left\{\frac{p_t}{p_{t+1}}\right\} \tag{9}$$

2.2 Final Good Sector

The aggregate final good y is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods, indexed by i, via the aggregate production function:

$$y_t \equiv \left(\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{10}$$

where ε is the demand elasticity for variety i. Maximization of profits yields typical demand functions:

$$y_t(i) = \left(\frac{p_t(i)}{p_t}\right)^{-\varepsilon} y_t \tag{11}$$

for all i, where $p_t \equiv \left(\int_0^1 p_t(i)^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}}$ is the price index consistent with the final good producers earning zero profits.

2.3 Firms

Each firm borrows external funds to finance input demand up to an amount:

$$l_t \leq \frac{w_t}{p_t} n_t + q_t k_t \tag{12}$$

External funds are available from a risk neutral intermediary at a nominal gross rate $(1 + r_t^l)$. Each firm assembles labor and capital to operate a constant return to scale production function for the variety i of the intermediate good:

$$y_t(i) = a_t \omega_t n_t(i)^{1-\alpha} k_t(i)^{\alpha} \tag{13}$$

where a_t is a productivity shifter common to all entrepreneurs and ω_t is an idiosyncratic productivity shifter that follows a uniform distribution, $f(\omega_t)$over the interval $[a, b]$[4]. For notational convenience I am assuming that the idiosyncratic shock does not have a firm index: it will be shown later that the index can be dropped due to the linear nature of the monitoring technology characterizing the optimal contract. The idiosyncratic shock is observable in advance only to the firm while the intermediary must pay a monitoring cost to observe realized output. As all firms in equilibrium will charge the same price and produce the same output we can now skip the index i. Cost minimization is obtained through the following maximization problem:

$$Min \frac{w_t}{p_t} n_t + q_t k_t + (1 + r_t^l)(\frac{w_t}{p_t} n_t + q_t k_t) \tag{14}$$

subject to equation (13) and (12). Let's define mc_t, the lagrange multiplier on (13), as the real marginal cost. First order conditions for this problem give:

- n_t :

$$\frac{w_t}{p_t}(1 + (1 + r_t^l)) = mc_t a_t \omega_t (1 - \alpha) n_t^{-\alpha} k_t^{\alpha} \tag{15}$$

- k_t :

$$q_t(1 + (1 + r_t^l)) = mc_t a_t \omega_t \alpha n_t^{1-\alpha} k_t^{\alpha-1} \tag{16}$$

The above conditions can be written as follows:

- n_t :

$$\frac{w_t}{p_t}(1 + (1 + r_t^l)) = mc_t (1 - \alpha) \frac{y_t}{n_t} \tag{17}$$

[4]This assumption is needed to guarantee uniqueness of the solution to the debt contract.

- k_t :

$$q_t(1+(1+r^l_t)) = mc_t\alpha\frac{y_t}{k_t} \tag{18}$$

By substituting (17) into (16) we obtain the following expression for the marginal cost:

$$\frac{w_t}{p_t}\frac{(1+(1+r^l_t))}{a_t\omega_t(1-\alpha)}(\frac{n_t}{k_t})^\alpha = mc_t \tag{19}$$

The above expression manifest the mechanism through which movements in the nominal interest rate by affecting the cost of loans have an impact on firms' marginal costs.

Later on it will prove useful for notational convenience to express the link between factor inputs and firms' revenues on the one side and loan demand on the other. Merging equation (19) with equation (16) and rearranging we obtain the optimal ratio of factor inputs:

$$q_tk_t = \frac{\alpha}{1-\alpha}\frac{w_t}{p_t}n_t \tag{20}$$

Recalling the liquidity constraint, (12), and rearranging gives the following optimal input demands:

$$k_t = \alpha\frac{l_t}{q_t} \tag{21}$$

$$n_t = (1-\alpha)\frac{l_t}{\frac{w_t}{p_t}} \tag{22}$$

Substituting those input demands into the production functions we obtain optimal firms revenues as function of loan demand:

$$\mathcal{G}(l_t) = p_ty_t = p_ta_t\omega_t\left(\frac{(1-\alpha)}{\frac{w_t}{p_t}}\right)^{(1-\alpha)}\left(\frac{\alpha}{q_t}\right)^\alpha l_t \tag{23}$$

Besides from the cost minimization task each firm i has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to $\frac{\theta_p}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}-1\right)^2$, where the parameter θ_p measures the degree of nominal price rigidity. The problem of each domestic monopolistic firm is the one of choosing the sequence $\{p_t(i)\}_{t=0}^{\infty}$, given optimal input demands, in order to maximize expected discounted real profits:

$$E_0\left\{\sum_{t=0}^{\infty}\beta^tu_{c,t}\Theta_t(i)\right\} \tag{24}$$

where:

$$\Theta_t(i) \equiv y_t(i) - \left(\frac{w_t}{p_t}n_t(i) + q_tk_t(i)(1+(1+r^l_t))\right) - \frac{\theta_p}{2}\left(\frac{p_t(i)}{p_{t-1}(i)}-1\right)^2 \tag{25}$$

subject to demand constraint, (11) and the production constraint (13). Let's denote by mc_t, the lagrange multiplier on (13), as the real marginal cost, by $\widetilde{p}_t \equiv \frac{P_t(i)}{P_t}$ the relative price of variety i and $\pi_t = \frac{p_t}{p_{t-1}}$ as the inflation rate. The first order condition of the above problem reads as follows:

$$\begin{aligned} 0 &= y_t \widetilde{p}_t^{\,-\varepsilon} \left((1-\varepsilon) + \varepsilon mc_t\right) - \theta_p \left(\pi_t \frac{\widetilde{p}_t}{\widetilde{p}_{t-1}} - 1\right) \frac{\pi_t}{\widetilde{p}_{t-1}} \\ &\quad + \beta \theta_p \left(\pi_{t+1} \frac{\widetilde{p}_{t+1}}{\widetilde{p}_t} - 1\right) \pi_{t+1} \frac{\widetilde{p}_{t+1}}{\widetilde{p}_t^{\,2}} \end{aligned} \tag{26}$$

where I have suppressed the superscript i since all firms employ an identical capital/labor ratio in equilibrium. After imposing symmetry the above condition can be written as follows:

$$0 = y_t \left((1-\varepsilon) + \varepsilon mc_t\right) - \theta_p \left(\pi_t - 1\right) \pi_t + \beta \theta_p \left(\pi_{t+1} - 1\right) \pi_{t+1} \tag{27}$$

Equation (27) gives a standard Phillips curve relating current inflation to expected one and the real marginal cost.

2.4 The Financial Contract between Firms and Intermediaries

The financial contract between firms and the intermediaries assumes the form of an optimal debt contract à la Gale and Hellwig 1983. When the idiosyncratic shock to production is above the cut-off value which determines the default states the entrepreneurs repay an amount $(1+r_t^l)l_t^j$[5]. On the contrary, in the default states, the bank monitors the production activity (and pays a monitoring cost μ) and repossesses the firms' revenues. Default occurs when the revenues from production $\omega_t^j \mathcal{G}(l_t^j)$ falls short of the amount that needs to be repaid $(1+r_t^l)l_t^j$[6]. Hence the *default space* is implicitly defined as the range for ω such that :

$$\omega_t^j < \varpi_t^j \equiv \frac{(1+r_t^l)l_t^j}{\mathcal{G}(l_t^j)} \tag{28}$$

where ϖ_t^j is a cutoff value for the idiosyncratic productivity shock. Let's define by $\Gamma(\varpi^j) \equiv \int_0^{\varpi_t^j} \omega_t^j f(\omega) d\omega + \varpi_t^j \int_{\varpi_t}^{\infty} f(\omega) d\omega$ and $1 - \Gamma(\varpi_t^j)$ the fractions of net capital output received by the intermediary and the firm respectively. Expected bankruptcy costs are defined as $\mu M(\varpi_t^j) \equiv \mu \int_0^{\varpi_t^j} \omega_t^j f(\omega) d\omega$ with the *net share* accruing to the intermediary being $\Gamma(\varpi_t^j) - \mu M(\varpi_{t+1}^j)$. The

[5] In every period t this amount must be independent from the idiosyncratic shock in order to satisfy incentive compatibility conditions.

[6] Notice that this contract starts and ends in a single period as it finances working capital which is used for production in the same period. This differ from BGG 1998 that have a intra-period contract as loans are required to finance capital which pays returns in future periods.

real return paid on deposits is given by the safe rate, $(1+r_t^n)$, which as such corresponds, for the intermediary, to the opportunity cost of financing capital. The intermediary must hold a fraction of deposits in the form of reserves required by the central bank for insurance purposes. Let's define ξ as the fraction of required reserves. Therefore in equilibrium:

$$l_t = d_t(1-\xi)$$

The *participation constraint* for the intermediary states that the expected return from the lending activity should not fall short of the opportunity cost of finance:

$$\mathcal{G}(l_t^j)(\Gamma(\varpi_t^j) - \mu M(\varpi_t^j)) \geq \frac{(1+r_t^n)}{(1-\xi)} l_t \tag{29}$$

The contract specifies a pair $\left\{\varpi_t^j, l_t^j\right\}$ which solves the following maximization problem:

$$Max\ (1-\Gamma(\varpi_t^j))\mathcal{G}(l_t^j) \tag{30}$$

subject to the participation constraint (29). Since unitary monitoring costs are constant this will give raise to linear relations that allow aggregation, hence we can skip the index j from now on. Using the first order conditions with respect $\{\varpi_t, l_t\}$ and aggregating yield a wedge between marginal revenues and the safe return paid on deposits:

$$\rho(\varpi_t) = \left[\frac{(1-\Gamma(\varpi_t))(\Gamma'(\varpi_t) - \mu M'(\varpi_t))}{\Gamma'(\varpi_t)} + (\Gamma(\varpi_t) - \mu M(\varpi_t))\right]^{-1}$$

which is positively related to the default threshold. Let's define $(1+r_t^p) = \mathcal{G}'(l_t)$, the marginal revenues from the production activity, as the return from working capital investment. We can now define the *external finance premium* as:

$$efp_t \equiv \frac{(1+r_t^p)}{(1+r_t^n)(1-\xi)} = \rho(\varpi_t) \tag{31}$$

By combining the above expression with (29) and with the expression for $\rho(\varpi_t)$ it is possible to write a relation between the external finance premium, efp_t, and firms leverage, $\frac{l_t}{\mathcal{G}(l_t^j)}$[7]:

$$efp_t = \rho(\frac{l_t}{\mathcal{G}(l_t)}) \tag{32}$$

A decrease in the leverage ratio reduces the optimal cut-off value, as shown by equation (28). A fall in the cut-off value, by reducing the size of the default space, implies a fall in the size of the bankruptcy costs and of the external finance premium.

[7] Notice that I abstract from the presence of internal funding. This implies that the external finance premium represents a wedge between investment in working capital or in alternative risk free activities.

2.5 The Credit Augmented Cost Channel

Before turning to the analysis of optimal policy it is useful to disentangle the effects that characterize the monetary transmission mechanism in this model. First, monetary policy has a supply-side effect arising from the pure cost channel. To see this let's for the time being shut-off the external finance premium and let's work with the relations describing the flexible price equilibrium. Particularly the latter assumption implies the marginal cost is equal to the inverse of the mark-up, $mc = \frac{\varepsilon-1}{\varepsilon}$. In this case the labour demand schedule is given by:

$$\frac{w_t}{p_t} = \frac{\frac{\varepsilon-1}{\varepsilon} a_t(1-\alpha) n_t^{-\alpha} k_t^{\alpha}}{(1+(1+r_t^n))} \tag{33}$$

This schedule can be depicted as a downward-sloping demand schedule in the real wage-employment space. Other things being equal a decrease in the nominal interest rate shifts the labour demand to the right. The same line of reasoning holds for the demand of capital:

$$q_t = \frac{\frac{\varepsilon-1}{\varepsilon} a_t \alpha n_t^{1-\alpha} k_t^{\alpha-1}}{(1+(1+r_t^n))} \tag{34}$$

The equilibrium response of employment to changes in the nominal interest rate can be recovered by looking at the equilibrium condition in the labour market which is obtained by merging (19) with (7):

$$mc_t = -\frac{u_{n,t}}{u_{c,t}} \frac{(1+(1+r_t^n))}{a_t(1-\alpha)} (\frac{n_t}{k_t})^{\alpha} \tag{35}$$

The last equation also provides an expression for the marginal cost in this economy. As marginal costs are given by the inverse of the mark-up and since the latter must be kept constant in an equilibirum with flexible prices, it follows that a decrease in the nominal interest rate must be accompanied by an increase in employment.

Let's now consider the implications of the sticky price assumption. For the time being we are still assuming that there are no informational frictions on the lending activity. The presence of sticky prices links marginal costs to inflation dynamic through endogenous mark-up movements. To see this we can substitute the above expression for the marginal cost, (35), into the Phillips curve relation to obtain:

$$0 = a_t n_t^{1-\alpha} k_t^{\alpha} \left((1-\varepsilon) + \varepsilon \left[-\frac{u_{n,t}}{u_{c,t}} \frac{(1+(1+r_t^n))}{a_t(1-\alpha)} (\frac{n_t}{k_t})^{\alpha} \right] \right) - \theta_p (\pi_t - 1) \pi_t + \beta \theta_p (\pi_{t+1} - 1) \pi_{t+1} \tag{36}$$

This expression manifests the link between movements in the nominal interest rate and inflation. An increase in the nominal interest, implemented for instance to close the output and the inflation gaps, works as a cost-push shock as by raising marginal cost also raises inflation.

Let's now reintroduce the credit channel. The above reduced form expression for the Phillips curve will now read as follows:

$$0 = a_t \omega_t n_t^{1-\alpha} k_t^{\alpha} \left((1-\varepsilon) + \varepsilon \left[-\frac{u_{n,t}}{u_{c,t}} \frac{(1+(1+r_t^l))}{a_t \omega_t (1-\alpha)} (\frac{n_t}{k_t})^{\alpha} \right] \right) - \theta_p \left(\pi_t - 1 \right) \pi_t + \beta \theta_p \left(\pi_{t+1} - 1 \right) \pi_{t+1} \tag{37}$$

As it stand clear the role of the credit frictions is that of reinforcing the effects of the cost channel. Overall the transmission mechanism in this model is characterized by *supply side driven fluctuations* which are exacerbated by the credit channel.

2.6 Definition of finance premia and interest rate differentials

As emphasized in Goodfriend and McCallum 2006 when setting its policy the monetary authority should take into account the effects of changes in the policy instrument on the full array of asset returns that are involved in the monetary transmission mechanism. So for instance in our model changes in the nominal interest rate will have an impact not only on the return of real money balances but also on the cost of working capital and on the costs of servicing collateralizable loans. In fact ultimately the size and the speed of the monetary policy transmission mechanism depends on the various channels that characterize the asset markets. Any of these channels can actually be described by referring at the finance premia that emerge by relating the different asset returns.

In the context of the present model we can define at least three different spreads between asset returns.

The first is a *liquidity term premium* which is given by the differential between the interest rate on deposits, $(1+r_t^d)$, and the cost of real money balances. This differential is simply given by the fraction of held reserve deposits:

$$\frac{(1+r_t^d)}{(1+r_t^n)} = (1-\xi) \tag{38}$$

As the fraction, ξ, is assumed constant in our case such spread should not have a qualitative impact on the transmission of monetary policy.

The second spread is a *finance premium* linking the interest rate on deposits and the one on loans, which is obtained by merging equations (29) and (28):

$$\frac{(1+r_t^l)}{(1+r_t^d)} = \frac{\varpi_t^j}{\Gamma(\varpi_t^j) - \mu M(\varpi_t^j)} \tag{39}$$

This spread depends on the ratio between the optimal default threshold and the fraction of surplus accruing to the intermediary as, for given interest rate on deposits, the cost of servicing a collateralizable loan increases when the default probability raises or when the expected net surplus for the intermediary falls. The ratio in (39) manifests the countercyclical nature of this spread which decreases in response to an aggregate productivity that by increasing revenues tends to decrease the default threshold (see equation (28)).

The third spread is given by a pure *external finance premium* obtained as differential between the rate on collateralizable assets and the interest rate on working capital investment:

$$\frac{(1+r_t^p)}{(1+r_t^l)} = \rho(\varpi_t)\frac{\varpi_t^j}{\Gamma(\varpi_t^j) - \mu M(\varpi_t^j)} \tag{40}$$

Both the components, $\rho(\varpi_t)$and $\frac{\varpi_t^j}{\Gamma(\varpi_t^j) - \mu M(\varpi_t^j)}$, characterizing this spread behave countercyclically, as both of them are positively related to the default threshold.

Countercyclical spreads in general tend to exacerbate business cycle fluctuations in response to shocks. This is so since productivity shocks have both a direct and an indirect effect on factor demands. To understand this mechanism, let's rely on the following intuitive argument. Let's assume that a technology improvement occurs in a flexible price equilibrium. In this context an increase in technology has two effects. First, it raises input demand on impact as it shifts to the right the input demand schedules. Second, by reducing both the *finance and external finance premia* it allows to protract and amplify the positive boost on input demands.

2.6.1 Market Clearing Conditions and Competitive Equilibrium

Equilibrium in the final good market requires that the production of the final good be allocated to private consumption by households, investment and to resource costs that originate from the adjustment of prices as well as from the lender's monitoring of the investment activity. Under standard aggregation assumptions the resource constraint reads as follows:

$$y_t = c_t + i_t + \frac{\theta_p}{2}(\pi_t - 1)^2 + \mu M(\varpi_t)\mathcal{G}(l_t^j) \tag{41}$$

Definition 1. *A distorted competitive equilibrium for this economy is a sequence of allocation and prices* $\{c_t, n_t, m_t, k_{t+1}, i_t, l_t, \varpi_t, mc_t, \pi_t, q_t, w_t, r_t^n, r_t^p\}_{t=0}^{\infty}$ *which, for given initial* d_0,k_0, m_0, *satisfies equations*

(6), (7),(8), (4), (12), (13),(17),(18),(27),(28),(29),(31),

(41).

3 Dynamics Responses of the Competitive Equilibrium Allocation

Before turning to the optimal monetary policy design it is instructive to consider how the economy behaves when confronted with simple operational Taylor rules of the following type:

$$\ln\left(\frac{1+r_t^n}{1+r^n}\right) = \phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) \tag{42}$$

where $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$. The analysis of the dynamic responses under simple rules allows us to disentangle the effects induced by the credit augmented liquidity cycle.

3.1 Computation and Calibration

To solve for the dynamic responses of the competitive economy I compute *second order approximations* of the equilibrium conditions. The use of those methods is particularly useful in economies with credit frictions as they allow us to account for the effects of volatility on mean levels of variables therefore providing more accurate solution (see Schmitt-Grohe and Uribe 2001 for details). Parameters' calibration is as follows.

Preferences. I set the discount factor $\beta = 0.99$, so that the annual interest rate is equal to 4 percent. Utility is chosen separable in consumption and labor:

$$U(c_t, n_t) = \log(c_t) - \gamma \log(1 - n_t)$$

The parameter γ is set equal to 3 as this guarantees that in steady state one third of time is spent working. Sensitivity analysis is done to assess the robustness of the results.

Technology. I set the share of capital in the production functions equal to $\alpha = 0.35$ as in Christiano 1988, the quarterly depreciation rate $\delta = 0.021$ as in Christiano 1991, the steady state mark-up value to $\frac{\varepsilon}{\varepsilon-1} = 1.2$ which corresponds to a value for the elasticity of demand, $\varepsilon = 11$ as in Khan, King and Wolman 2003. Following Sbordone 2004 the adjustment cost parameter, θ_p, is set 17.5.

Financial frictions parameters: The financial frictions parameters are obtained by solving the steady state version of the competitive economy under the optimal contracting problem. Some primitive parameters are set so as to match values for industrialized countries. The idiosyncratic shock is assumed to follow a uniform distribution within the range $[1 - A, 1 + A]$: the value for A is chosen to as to generate a external finance premium of 187 basis points. The monitoring cost is varied between 0.03 and 0.4, as those values are compatible with data for bankruptcy cost for industrialized countries.

Shocks. I simulate the model under productivity shocks which follow $AR(1)$ processes. Persistence and volatility are calibrated as in the RBC literature.

3.2 Responses to Productivity Shocks

In figure 1 I show dynamic responses of selected variables to a one percent productivity shock under two different parametrization of the monitoring cost (within an admissible empirical range). In response to a negative productivity shock we observe a raise in inflation which induces a raise in the nominal interest rate as the monetary authority aims at closing the gap of actual output with potential output. However in our context and due to the cost channel on working capital, fluctuations in the policy instrument have an impact on firms' marginal cost, as a raise in the nominal interest rate induces increases in the cost for external funding. The increase in firms' marginal cost brings about a raise in inflation. A monetary policy focused only on price stability targets fails to internalize the perverse effects that increases in the nominal interest rate can have on inflation. As the liquidity channel in our context is augmented with a credit channel, such perverse effects are amplified by the tightness of the external funding availability. Indeed figure 1 shows that the raise in firms' marginal cost and inflation is larger the higher the value of the monitoring cost. This in turn implies that consumption is lower when the monitoring costs are higher.

To assess the impact of the cost channel figure 2 shows impulse responses to negative productivity shocks under two different parametrization of the degree of price stickiness, as measured by the cost of adjusting prices, ω_p. Some observations stand out. First, as expected a high degree of price stickiness induces a smoother response of inflation as the sensitivity of inflation to marginal cost falls. On the other side marginal costs are more responsive under higher degrees of price stickiness. As the cost of adjusting prices raises, the monetary authority tends to respond more aggressively to fluctuations in the inflation gap. This induces larger swings in the nominal interest rate which in presence of a cost channel are transferred to firms' marginal costs.

4 The Optimal Policy Problem

To analyze the optimal policy problem I will rely on two different aspects of the monetary policy design. In general the optimal policy problem is solved by following the Ramsey approach in which the planner maximizes agents' utility subject to the constraints of the competitive equilibrium. As I first step I analyze the optimal targeting problem by solving a Ramsey plan in which the allocation is constrained also by a well specified set of operational monetary policy rules. As a second step I study the optimal dynamic path of variables by solving a full-fledged Ramsey plan. Alternatively one could see the first step as the implementation, through well specified operational policy rules, of the optimal policy plan as designed in step two.

4.1 Constrained Ramsey Plan

As specified above the optimal policy problem in this context is solved by assuming that the monetary authority maximizes households welfare subject to the competitive equilibrium conditions and the class of monetary policy rules represented by:

$$\begin{aligned} \ln\left(\frac{1+r_t^n}{1+r^n}\right) &= (1-\phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) + \phi_q \ln\left(\frac{q_t}{q}\right)\right) \\ &\quad +\phi_r \ln\left(\frac{1+r_{t-1}^n}{1+r^n}\right) \end{aligned} \tag{43}$$

where ϕ_q represents a response to asset price. I also consider the alternative specification given by:

$$\begin{aligned} \ln\left(\frac{1+r_t^n}{1+r^n}\right) &= (1-\phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_y \ln\left(\frac{y_t}{y}\right) + \phi_{lk} \ln\left(\frac{l_t/k_t}{(l/k)}\right)\right) \\ &\quad +\phi_r \ln\left(\frac{1+r_{t-1}^n}{1+r^n}\right) \end{aligned} \tag{44}$$

where ϕ_{lk} represents the response to the leverage ratio. I therefore search for parametrization of interest rate rules that satisfy the following 3 conditions: a) they are simple since they involve only observable variables, b) they guarantee uniqueness of the rational expectation equilibrium, c) they maximize the expected life-time utility of the representative agent.

Some observations on the computation of welfare in this context are in order. First, one cannot safely rely on standard first order approximation methods to compare the relative welfare associated to each monetary policy arrangement. Indeed in an economy with a distorted steady state stochastic volatility affects both first and second moments of those variables that are critical for welfare. Hence policy arrangements can be correctly ranked only by resorting to a higher order approximation of the policy functions[8]. Additionally one needs to focus on the *conditional* expected discounted utility of the representative agent. This allows to account for the transitional effects from the deterministic to the different stochastic steady states respectively implied by each alternative policy rule[9]. Define Ω as the fraction of household's consumption that would be needed

[8] See Kim and Kim (2003) for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies.

[9] See Kim and Levin (2004) for a detailed analysis on this point.

to equate conditional welfare $\mathcal{W}_0$ under a generic interest rate policy to the level of welfare $\widetilde{\mathcal{W}_0}$ implied by the optimal rule. Hence Ω should satisfy the following equation:

$$\mathcal{W}_{0,\Omega} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1+\Omega)c_t, n_t) \right\} = \widetilde{\mathcal{W}_0}$$

Under a given specification of utility one can solve for Ω and obtain:

$$\Omega = \exp\left\{ \left(\widetilde{\mathcal{W}_0} - \mathcal{W}_0 \right) (1-\beta) \right\} - 1$$

Given this welfare metric I simulate the model economy under the two sources of aggregate uncertainty, productivity and government consumption shocks. I then conduct two experiments. First, I compute welfare under different (ad hoc) specifications of the monetary policy rule. The rules are the following:

(i) *Simple Taylor rule*, with $\phi_\pi = 1.5$, $\phi_y = 0.5/4, \phi_q = \phi_r = 0$;

(ii) *Simple Taylor rule with smoothing*, with $\phi_\pi = 1.5$ and $\phi_y = 0.5/4, \phi_q = 0, \phi_r = 0.9$;

(iii) *Strict inflation response,* $\phi_\pi = 1.5, \phi_y = \phi_q = \phi_r = 0$;

(v) *Response to inflation and asset prices*, with $\phi_\pi = 1.5$, $\phi_q = 0.6/4$, $\phi_y = \phi_r = 0$;

(vi) *Strong response to inflation and response to asset prices*, with $\phi_\pi = 3$, $\phi_q = 0.6/4$, $\phi_y = 0, \phi_r = 0$;

(vi) *Response to inflation and leverage ratio*, with $\phi_\pi = 3$, $\phi_q = 0$, $\phi_y = \phi_r = 0, \phi_{lk} = 0.5/4$.

Secondly, I search in the grid of parameters $\{\phi_\pi, \phi_y, \phi_q, \phi_r\}$ for the rule which delivers the highest level of welfare, which is defined as the optimal policy rule[10] and I compare welfare under optimal policy and simple rules.

The choice of including asset prices or leverage ratios as independent targets is motivated by the consideration that the monetary authority might want to de-amplify fluctuations in marginal costs which ultimately depend on the cost of working capital. As both the price of working capital and the leverage ratio are proxies of the cost for working capital it seems reasonable to include them as independent targets. The fact that optimality might require assigning a positive weight

[10] The search is made over the following ranges: $[1.5, 4]$ for ϕ_π, $[0, 2]$ for ϕ_q, $[0, 2]$ for ϕ_y. Notice that the parameters ϕ_y and ϕ_q are divided by four given the standard assumption on the length of a period (quarterly) and given that inflation in Taylor type rules is expressed at annual rates. I also compare rules with interest rate smoothing ($\phi_r = 0.9$) to rules without smoothing ($\phi_r = 0$). It is judged as admissible a combination of policy parameters that delivered a unique rational expectations equilibrium.

to any of those additional targets alongside with inflation implies that strict inflation targeting or price stability policies are suboptimal.

Table 1 summarizes the findings in terms of the welfare loss Ω (relative to the optimal policy) of alternative simple rules.

Results are as follows. First, responding to asset prices along with inflation is the optimal rule. More specifically the optimal rule features the following coefficients: $\phi_\pi = 3$, $\phi_q = 0.6/4$, $\phi_y = 0, \phi_r = 0$. This is so since the policy maker faces a trade-off between stabilizing inflation, as this would allow to close the gap between the flexible and the sticky price allocation, and easing up liquidity, which by reducing the cost of working capital would stabilize fluctuations in marginal cost. The optimal management of this policy trade-off is resolved by targeting the price of working capital on top and above inflation as this guarantees a certain degree of marginal cost stabilization.

Secondly, responding to output along with inflation is welfare detrimental. This result is consistent with the one obtained by Schmitt-Grohe and Uribe 2003, Faia and Monacelli 2007, Faia 2007 by using alternative specification of the new keynesian economy. The intuition for this result in the context of the present paper lies in the fact that the policy maker aims at stabilizing only variables or gaps which proxy more closely the inefficiency. Asset prices are a better proxies for the cost of working capital than output is.

Third, interest smoothing is welfare detrimental. In this model the policy maker does not want to protract the effects of fluctuations in the nominal interest rate as they are reflected in similar fluctuations of firms' marginal costs.

Notice that the table shows that same calculations for two different values of the monitoring cost, a low value of 0.2 and a high value of 0.4. As expected under all rules higher monitoring costs induce higher welfare costs as the deadweight loss of the lending activity is higher.

4.2 Globally Optimal Ramsey Plan Under Commitment

I now turn to the specification of a general set-up for the optimal policy conduct. In this section I take full advantage of the characterization of the equilibrium conditions in terms of a minimal set of relations involving only the choice of allocations. The optimal policy plan is determined by a monetary authority that maximizes the discounted sum of utilities of all agents given the constraints of the competitive economy. The next task is to select the relations that represent the relevant constraints in the planner's optimal policy problem. This amounts to describing the competitive equilibrium in terms of a minimal set of relations involving only real allocations, in the spirit of the primal approach described in Lucas and Stokey 1983. There is a fundamental difference, though, between that classic approach and the one followed here, which stems from the impossibility, in the presence of sticky prices and other frictions, of reducing the planner's problem to a maximization only subject to a single implementability constraint. Khan, King and Wolman 2003 adopt a similar structure to analyze optimal monetary policy in a closed economy with market power, price stickiness and monetary frictions, while Schmitt-Grohe and Uribe 2002 to analyze a problem of joint determination of optimal monetary and fiscal policy.

The optimality conditions for the consumer are summarized into the Euler condition:

$$u_{c,t} = \beta(1+r_t^n)E_t\left\{\frac{u_{c,t+1}}{\pi_{t+1}}\right\} \tag{45}$$

I start by to rearranging the optimality conditions for the production sector. First by merging (19) with (7) we obtain an expression for the marginal cost associated with equilibrium in the labour market, hence by merging the latter with the Phillips curve relation in (27) and with the production function we obtain:

$$0 = a_t\omega_t n_t^{1-\alpha}k_t^{\alpha}\left((1-\varepsilon)+\varepsilon\left[-\frac{u_{n,t}}{u_{c,t}}\frac{(1+(1+r_t^l))}{a_t\omega_t(1-\alpha)}(\frac{n_t}{k_t})^{\alpha}\right]\right) - \theta_p\left(\pi_t-1\right)\pi_t + \beta\theta_p\left(\pi_{t+1}-1\right)\pi_{t+1} \tag{46}$$

where:

$$(1+r_t^l) = \frac{\varpi_t^j(1+r_t^n)(1-\xi)}{\Gamma(\varpi_t^j)-\mu M(\varpi_t^j)} \tag{47}$$

Finally we need to include the feasibility condition which together with equation (3) describing capital accumulation becomes:

$$a_t\omega_t n_t^{1-\alpha}k_t^{\alpha} = c_t + k_{t+1} - (1-\delta)k_t + \frac{\theta_p}{2}\left(\pi_t-1\right)^2 + \mu M(\varpi_t)\mathcal{G}(l_t^j) \tag{48}$$

In our context we do not need to include the government resource constraints among the equilibrium conditions as it is assumed the absence of distortionary taxation.

Definition 2. *Let* $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}\}_{t=0}^{\infty}$ *represent sequences of Lagrange multipliers on the constraints (45), (46), (48) respectively. Let* k_0*, be given. Then for given stochastic process* $\{a_t, \omega_t\}_{t=0}^{\infty}$*, plans for the control variables* $\Xi_t^n \equiv \{c_t, n_t, k_{t+1}, \pi_t, r_t^n, \varpi_t\}_{t=0}^{\infty}$ *and for the co-state variables* $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}\}_{t=0}^{\infty}$ *represent a first best constrained allocation if they solve the following maximization problem:*

$$Min_{\{\Lambda_t^n\}_{t=0}^{\infty}}\ Max_{\{\Xi_t^n\}_{t=0}^{\infty}}\ E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t, n_t)\right\} \tag{49}$$

subject to (45), (46), (48).

4.2.1 Non-recursivity and Initial Conditions

As a result of the constraint (45) and (46) exhibiting future expectations of control variables, the maximization problem as spelled out in (49) is intrinsically non-recursive. As first emphasized in Kydland and Prescott 1980, and then developed by Marcet and Marimon 1999, a formal way to rewrite the same problem in a recursive stationary form is to enlarge the planner's state space with additional (pseudo) co-state variables. Such variables, that I denote $\chi_{1,t}$ and $\chi_{2,t}$ for (45) and (46) respectively, bear the crucial meaning of tracking, along the dynamics, the value to the planner of committing to the pre-announced policy plan. Another aspect concerns the specification of the law of motion of these lagrange multipliers. For in this case both constraints feature a simple one period expectation, the same co-state variables have to obey the laws of motion:

$$\begin{aligned}\chi_{1,t+1} &= \lambda_{1,t} \\ \chi_{2,t+1} &= \lambda_{2,t}\end{aligned} \tag{50}$$

Using the new co-state variable so far described I amplify the state space of the Ramsey allocation to be $\{a_t, \chi_{1,t}, \chi_{2,t}\}_{t=0}^{\infty}$ and I define a new saddle point problem which is recursive in the new state space:

Definition 3. *Let $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}\}_{t=0}^{\infty}$ represent sequences of Lagrange multipliers on the constraints (45), (46), (48), respectively. Let k_0, be given. Then given the state space $\{A_t, \chi_{1,t}, \chi_{2,t}\}_{t=0}^{\infty}$, plans for the control variables $\Xi_t^n \equiv \{c_t, n_t, k_{t+1}, \pi_t, r_t^n, \varpi_t\}_{t=0}^{\infty}$ and for the co-state variables $\Lambda_t^n = \{\lambda_{1,t}, \lambda_{2,t}, \lambda_{3,t}\}_{t=0}^{\infty}$ represent a first best constrained allocation if they solve the following maximization problem:*

$$\begin{aligned}&Min_{\{\Lambda_t^n\}_{t=0}^{\infty}}\, Max_{\{\Xi_t^n\}_{t=0}^{\infty}}\, E_0 \left\{\sum_{t=0}^{\infty} \beta^t \mathfrak{u}(c_t, n_t, \pi_t)\right\} + \\ &+\lambda_{1,t}\left\{\frac{u_{c,t}}{(1+r_t^n)}\right\} + \\ &+\lambda_{2,t}\left\{a_t\omega_t n_t^{1-\alpha}k_t^{\alpha}\left((1-\varepsilon)+\varepsilon\left[-\frac{u_{n,t}}{u_{c,t}}\frac{(1+(1+r_t^l))}{a_t\omega_t(1-\alpha)}\left(\frac{n_t}{k_t}\right)^{\alpha}\right]\right) - \theta_p\left(\pi_t-1\right)\pi_t\right\} + \\ &+\lambda_{2,t}\left\{a_t\omega_t n_t^{1-\alpha}k_t^{\alpha} - c_t - k_{t+1} + (1-\delta)k_t - \frac{\theta_p}{2}\left(\pi_t-1\right)^2 - \mu M(\varpi_t)\mathcal{G}(l_t^j)\right\}\end{aligned} \tag{51}$$

where $\mathfrak{u}(c_t, n_t, \pi_t) = u(c_t, n_t) + \chi_{1,t}(\frac{u_{c,t}}{\pi_t}) + \chi_{2,t}(\theta_p\left(\pi_t-1\right)\pi_t)$ and $(1+r_t^l) = \frac{\varpi_t^j(1+r_t^n)(1-\xi)}{\Gamma(\varpi_t^j)-\mu M(\varpi_t^j)}$. To avoid time consistency problems and consistently with a *timeless perspective* I set the values of the three co-state variables at time zero equal to their solution in the steady state. I will return on this point in the next subsection.

4.3 Monetary Policy Trade-offs

Before turning to the solution of the optimal plan it is instructive to analyze the constraints faced by the monetary authority to highlight the relevant policy trade-offs. In the standard new keynesian framework with sticky prices but in absence of the liquidity effects associated with the cost channel and in absence of creit frictions the policy maker achieves optimality by replicating the flexible price allocation. The relevant costs in this case are represented by the resource wasted in adjusting prices, therefore implying that optimality requires closing the *gaps* between the flexible price and the sticky price allocation. To close the gaps the monetary authority must simply set $\frac{\theta_p}{2}(\pi_t - 1)^2 = 0$ which implies following price stability rules[11]. In presence of a cost channel the nominal interest rate enters the marginal cost which in turn affects inflation dynamics through the Phillips curve. The liquidity effect induced by the cost channel works as an endogenous cost push shock (see also Ravenna and Walsh 2006). This is evident by inspecting the augmented Phillips curve:

$$0 = y_t \left((1-\varepsilon) + \varepsilon \left[-\frac{u_{n,t}}{u_{c,t}} \frac{n_t{}^{\alpha}}{a_t \omega_t (1-\alpha) k_t^{\alpha}} (1 + (1 + r_t^l)) \right] \right) - \theta_p (\pi_t - 1) \pi_t \tag{52}$$

In response for instance to a positive productivity shock the policy maker is confronted with a trade-off. On the one side he would like to raise nominal interest rates to bring output back to the potential level and set inflation to zero. On the other side the increase in the nominal interest rate would raise the cost of collateralizable loans therefore raising marginal cost and inflation. Indeed if confronted only with the cost channel the policy maker might be tempted to set nominal interest rate to zero (Friedman rule) in order to reduce the liquidity cost for firms and push labour demand. Ultimately there is a tension between the price stability prescription and the Friedman rule. As the policy maker is endowed with a single instrument he will choose to trade-off among those conflicting forces.

[11] This is certainly true in presence of productivity shocks (see Clarida, Gali' and Gertler (1998)). A caveat must be made for government expenditure shocks as by allowing for fluctuations in the ratio of demand to output they endow the policy maker with a leverage represented by countercyclical mark-up (see King and Wolman (1998), Adao, Correia and Teles (2003), Khan, King and Wolman (2003) and Ravenna and Walsh (2006)).

5 Long Run Optimal Policy

Before turning to the optimal stabilization policy in response to shocks we need to characterize the log-run optimal policy, which is the one to which the policy maker would like to converge. To develop an analogy with the Ramsey-Cass-Koopmans model, this amounts to computing the *modified golden rule* steady state. To determine the long-run inflation rate associated to the optimal policy problem above, one needs to solve the steady-state version of the set of efficiency conditions. Notice in particular that the first order condition with respect to inflation reads as follows:

$$\chi_{1,t}(\frac{u_{c,t}}{\pi_t^2}) + (\lambda_{2,t} - \chi_{2,t})\theta_p(2\pi_t - 1) - \lambda_{3,t}\theta_p\,(\pi_t - 1) = 0 \tag{53}$$

For the whole set of optimality conditions of the Ramsey plan to be satisfied in the steady state a necessary condition is that equation (53) is satisfied in the steady state. In that steady-state, we have $\lambda_{2,t} = \lambda_{2,t-1} = \chi_{2,t}$. Hence condition (53) immediately implies:

$$\chi_1(\frac{u_c}{\pi^2}) + \lambda_3\theta_p\,(\pi - 1) = 0 \tag{54}$$

The above expression shows clearly that the zero (net) inflation policy is not a solution to the above condition. The Ramsey planner faces a tension between closing the gap with the flexible price allocation and setting the nominal interest rate to zero in order to reduce the transaction cost of real money balances. In particular positive levels of the nominal interest rate (which imply deflation) by increasing the cost of funds for working capital tend to increase firms' marginal cost and inflation. This second channel acquires stronger relevance when considering that firms' are subject to additional costs of external finance due to informational asymmetries. Overall the optimal level of (net) inflation stems between zero and the one satisfying the Friedman rule.

6 Dynamic Properties of the Optimal Plan (To be completed)

Here I evaluate the dynamic properties of the optimal plan based on impulse response functions, optimal volatilities and welfare costs. A quantitative assessment is indeed necessary in order to evaluate a series of things among which the importance of deviations from price stability.

6.1 Dynamic Responses to Shocks of the Optimal Plan

To solve for the optimal stabilization policy I compute second order approximations[12] of the first order conditions of the Lagrangian problem described in definition 3. Technically I compute the stationary allocation that characterize the deterministic steady state of the first order conditions to the Ramsey plan. I then compute a second order approximation of the respective policy functions in the neighborhood of the same steady state. This amounts to implicitly assuming that the economy has been evolving and policy has been conducted around such a steady already for a long period of time (under timeless perspective).

7 Conclusions

This paper studies optimal monetary policy in a model that combines a cost channel with a credit channel. The presence of a cost channel implies that fluctuations in nominal interest rate have an impact on firms marginal costs and inflation dynamics therefore inducing a policy trade-off. The addition of credit frictions, generated by informational asymmetries and moral hazard, tends to amplify those types of supply side driven fluctuations.

Results show that the monetary authority should deviate from the price stability prescription both in the long run and in the short run. An optimal monetary policy rule should feature asset price targeting alongside with inflation targeting.

The current analysis focused on the role of liquidity and credit frictions for the firm problem. One of the feature of the recent crises on the sub-prime lending market is that much of the lending activity has involved households investment in durable goods. One promising avenue of research comes from exploring the role of those type of frictions for the households problem.

[12] Second order approximation methods have the particular advantage of accounting for the effects of volatility of variables on the mean levels of the same. See Schmitt-Grohe and Uribe (2004a,b) among others.

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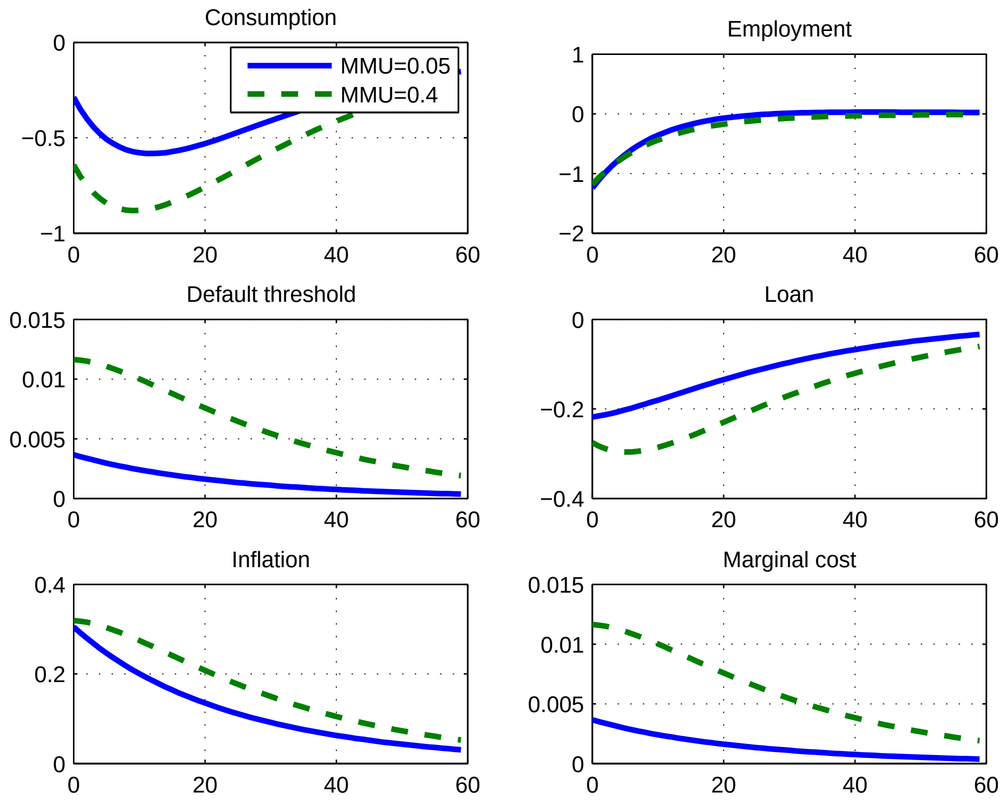

Figure 1: **Dynamic responses of selected variables to negative productivity shocks under two different values for the monitoring cost.**

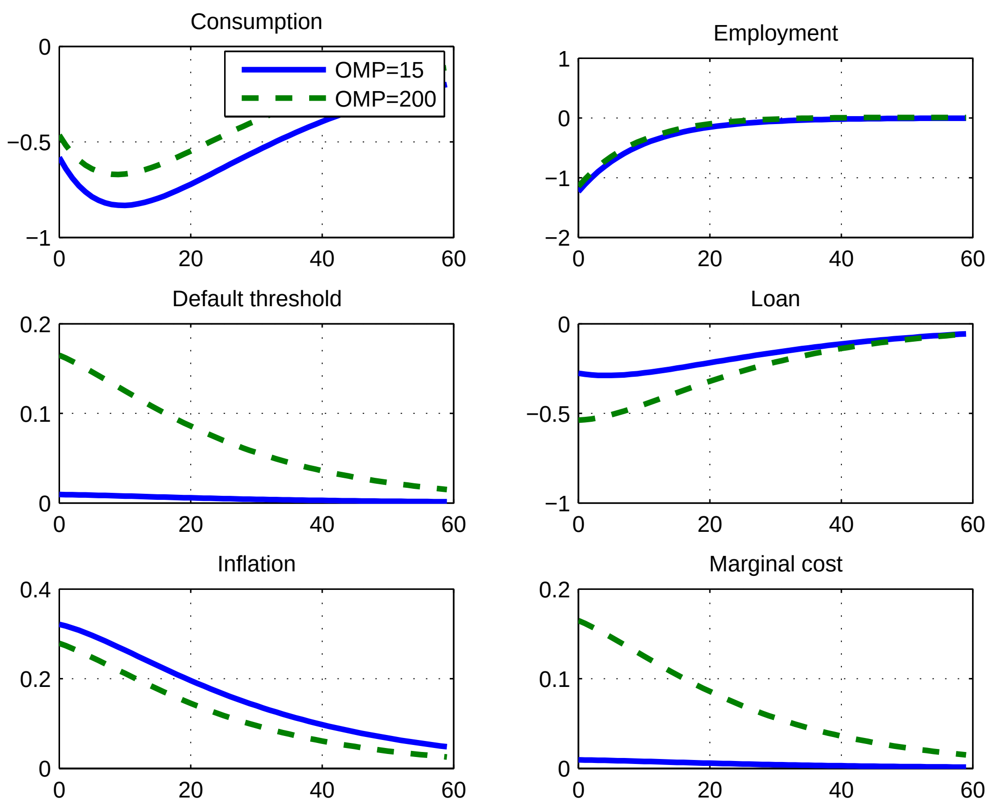

Figure 2: **Dynamic responses of selected variables to negative productivity shocks under two different values for the monitoring cost.**

Table 1: **Welfare comparison of alternative monetary policy rules.**

Monetary Policy Rule	% Loss relative to optimal rule	
	$\mu = 0.2$	$\mu = 0.4$
Taylor rule	1.28	0.77
Taylor rule with smoothing	0.14	0.0919
Strict response to inflation	0.0013	0.0016
Inflation and asset price response	0.35	0.1929
Strong Inflation and asset price response	0	0
Inflation and leverage ratio response	0	0