Pacifying Monogamy: The Mystery Revisited*

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Abstract: In some unequal societies rich men take only one wife. This has been labelled the Mystery of Monogamy. Here I propose a new explanation for this phenomenon. In a society where a lot of resources and women are allocated to one ruler, the resulting shortage of women among the non-endowed men can induce these to rebel against the ruler. Monogamy, or "constrained" polygyny, can pacify men, making the incumbent ruler safer. It may thus serve a ruler's reproductive interests to be subject to an institution which limits the number of wives he (or anyone who successfully ousts him) can take. Moreover, our model suggests how such marriage norms can arise endogenously in the course of economic development, as an optimal response to rising population levels.

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1 Introduction

In a world with big gaps between men in incomes, wealth, status, consumption, etc., one would expect to see an unequal distribution of women too. That is, one would expect to see *polygynous* mating and marriage.[1]

We arguable do not observe this in the Western World today, despite big income gaps. Even the richest and most powerful men do not take more than one wife at a time. Many rich men do remarry (so-called serial monogamy), or have extramarital affairs, but they do not monopolize several hundred young and fertile women at a time in huge harems, as the rulers of ancient civilizations did.

Gould, Moav, and Simhon (2003) label this the *Mystery of Monogamy*. Their explanation centers on the quality-quantity trade-off in children. In a world where the returns to quality (e.g., education) of children is high, men may choose a relatively modest quantity of offspring, and prefer one well-educated wife to carry them.

This explanation works well when comparing incomes of most men. However, the richest few percent still take only one wife, despite being able to attract (and have children with) many women, and still invest large amounts in each child.

Here I propose a complementary explanation, using a model with endogenous violence. I show that it may be in the reproductive interest of an incumbent ruler to marry fewer women than would like to marry him.

Both men and women maximize their expected number of surviving offspring. Women do so by choosing a man with abundant resources to invest in their offspring; men by attracting as many women as possible, which they in turn do by acquiring resources in competition with other men.

A sizeable fraction of all resources are concentrated in the hands of one man, the "King." Other men can either be content with an equal share of

[1]Polygyny means that one man can take several wives. The term *polygamy* formally includes *polyandry* (a woman taking several husbands), which is exceedingly rare among human societies.

the remaining resources (and the women they can attract with these), or try to challenge the King in a rebellion. With his wealth the King is in principle able to attract many women, but if he exercises this ability other men end up starved of reproductive opportunities, and thus more inclined to rebel. This may reduce the King's chances of staying alive and in power, and thus potentially his expected number of offspring. A King can thus improve his reproductive success by marrying fewer women than his wealth can attract. Monogamy – or "constrained polygyny," as we shall call it – functions as a *pacifying* institution.

We examine how the degree of constraint on polygyny depends on the details of the timing of the game. If the incumbent King has the power to impose a law, or institution, which limits not only the number of wives he can take, but also that of a successful rebel who ousts him, the King will choose to be more constrained than if he cannot.

The model contains a sort of "scale effect." Larger population implies a larger number women, making the (single and despotic) King able to attract more wives. This in turn makes the returns to rebellion higher, and polygyny must therefore become more constrained to pacify the subjects. The model thus generates an endogenous transition to from less to more constrained polygyny in response to expanding population, consistent with the historical record.

The rest of this paper is organized as follows. Next, Section 2 discusses some existing literature. Section 3 sets up the model. Section 4 examines the dynamic implications, showing how (technology-driven) population growth can generate transitions to more constrained polygyny. Section 5 discusses some extensions, and Section 6 gives a concluding discussion.

2 Existing literature

Despite a big literature on marriage institutions, few papers have addressed the Mystery of Monogamy, i.e., what causes unequal societies to be monogamous. The exception is Gould et al. (2003), discussed above.[2]

My own earlier work (Lagerlöf 2005) explains how an increase in equality and can lead to the emergence of monogamy, which obviously does not address the Mystery itself.[3]

Many other papers on marriage forms are related, but do not address the Mystery itself. For example, Tertilt (2005) explains African underdevelopment by widespread polygynous marriage forms, but not why polygyny is so common in Africa. In my model, polygyny is more common (or tends to be "less constrained") in societies with small populations, thus potentially explaining polygyny in Africa, which has long been a relatively sparsely populated continent.

Edlund and Lagerlöf (2006) make a distinction between so-called love and arranged marriage forms, but do not explain changes in marriage forms along the polygyny-monogamy dimension.

Other related papers, which do not address the Mystery as such, include Bergstrom (1994a,b) and Guner (1999, Section 5.2).

There is also a political-economy literature on the rise of democracy, and the extension of the franchise (e.g., Acemoglu and Robinson 2000a,b,c). Some of the mechanisms modelled there may relate to the way in which a ruler in our model can benefit from a more equal distribution of women and the associated lower risk of a rebellion.

Outside the field of economics a large amount of work discusses why some societies are more monogamous than others. See, in particular, the work of Laura Betzig (1986, 1992, 1993, 1995, 2002); Wright (1994) gives a nice overview. The overall conclusion in that literature is that polygyny is

[2] Becker's (1991, p. 95) *Treatise on the Family* presents some informal reasoning along the lines of Gould et al. (2003).

[3] I maintain that *part* of the explanation behind the rise of monogamy in the Western World may have been an increasingly equal distribution of land, power, and incomes between men. However, one cannot deny that a Mystery remains. As argued already, observed income gaps today between different men who marry only one wife (say, Bill Gates and the average economics professor) are very big indeed.

a phenomenon of inequality among men. In early human civilizations it was the despotic ruler, and a small elite of men around him, who could take more than one wife; most marriages were monogamous. As a corollary, polygynous societies have tended to be strongly despotic and hierarchical.

Rulers with many wives (and/or sex partners) have also had far more offspring than the average man. This is well documented not only in work by anthropologists (like Betzig cited above); geneticists have found that about 8% of the male population in Central Asia today are descendents of Genghis Kahn (Zerjal et al. 2003). This should imply a high return in terms of reproductive success from becoming the ruler, suggesting both high rewards to male violence, and that reproductive equality can serve as a pacifying force.

The idea that the world today is one of "constrained" polygyny, rather than monogamy in a strict or conventional sense, is consistent with the observation that richer and more powerful men still tend to have more sex (Perusse 1993).

3 The Model

Consider a two-sex overlapping-generations model, where agents live as adults and (passive) children. In any given period, there is a continuum of P (adult) men, and equally many women. (An unbalanced sex ratio does not change the results if that ratio is exogenous.)

The size of the (adult male or female) population in the next period is denoted P', and similarly for other variables. Up until Section 4 we can interpret the model as being static, since all analysis refers to one period.

Agents maximize their reproductive success (fertility). We assume that agents (both men and women) are risk neutral with respect to their number of offspring, in the sense that they maximize *expected* fertility. Such preferences are consistent with what we should expect natural selection to have endowed us with (Robson 1996).

Women are essentially "baby plants," whose input is food and output is children. A woman who receives an amount of food c from her mate gives birth to

$$n(c) = c^{\rho}, \tag{1}$$

(surviving) children, where $\rho \in (0, 1)$.

Note that the food input is supplied by the men; women have no incomes of their own. Moreover, there is no paternal uncertainty, so a man knows he is the father of his offspring and thus invests all his income in the women he mates with. (Adults do not consume.)

Due to the diminishing marginal product in child production of each woman in (1), every man equalizes his income (the food he procures) among the women he mates with. In other words, if a man has income y, and w wives, each of his wives rears $n(y/w) = (y/w)^{\rho}$ children; the man fathers $wn(y/w) = y^{\rho}w^{1-\rho}$ children in total.

There is one unit one land, a fraction $1 - \omega$ of which belongs to a single male agent, referred to as the King. Since he is one single agent, and the other men are a continuum, the King carries zero measure.

The amount of food generated by the unit land endowment is A, which may be thought of as the level of technology. Thus, the King's income equals $(1 - \omega)A$ (if he is not ousted, as explained below).

Those men who are not the King, referred to as subjects, can either be peaceful of rebel against the King. The land not held by the King, which produces ωA units of food, is shared equally among those subjects who are peaceful. Let x be the fraction of the P subjects who choose to be peaceful. A peaceful subject's income thus equals $\omega A/xP$.

As explained, men invest all their incomes in child production. Let c^S denote the investment a woman receives if she chooses to mate with a peaceful subject, and c^K what she receives if mating with the King. Let z denote the fraction of all P women who choose to mate with a peaceful subject. Then c^S equals income per peaceful subject, divided by the number of women per peaceful subject, i.e.,

$$c^S = \frac{\frac{\omega A}{xP}}{\frac{zP}{xP}} = \frac{\omega A}{zP}. \tag{2}$$

This is just total non-King income divided by the number of women marrying subjects.

Using the same logic, we can derive an expression for the food a woman receives if mating with the King, c^K. This simply equals total income of the King, $(1-\omega)A$, divided by the total number of women mating with him:

$$c^K = \frac{(1-\omega)A}{(1-z)P}. \tag{3}$$

3.1 Women

Consider now how women behave on the marriage market. Like men, they maximize their expected number of offspring, which depends on their mates' incomes, and the total number of wives that their respective mates take. If women cannot be forced to join the King's harem, then the number of children per harem woman must not fall below what she would have if marrying a subject. The implication, summarized by the proposition below, is that the distribution of women simply mirrors that of resources.

Proposition 1 *$n(c^K) \geq (<) n(c^S)$ if, and only if, $z \geq (<) \omega$.*

The proof is in the Appendix.

From the King's perspective, $n(c^K) \geq n(c^S)$ constitutes a participation constraint, stating that women must not want to marry a subject instead. This constraint need not bind: the King may choose to marry fewer women than he can attract (i.e., setting $z > \omega$), in which case harem membership must be rationed among women. If so, we may assume that women are allocated harem membership through a lottery.

3.2 Subjects

As described, subjects maximize their expected number of children. If peaceful, the average subject has z/x wives (a number typically less than one, but we abstract from integer restrictions; or we can think of this as the probability that he marries one woman). From (1) and (2) we see that each of the z/x wives rears $(\omega A/zP)^\rho$ children. The subject's expected number of children, if he chooses to be peaceful, thus becomes $(z/x) \times (\omega A/zP)^\rho$.

A subject who chooses to rebel faces a stochastic outcome: he either gets killed, or he becomes the new King. Let the function $G(R)$ denote the probability that the King is ousted and killed. This probability is a non-decreasing function of $R = (1-x)P$, i.e., the number of rebels (recall that xP subjects are peaceful). The probability of becoming the new King if the incumbent King is killed equals $1/R$, so the unconditional probability of a rebelling subject becoming the new King is $G(R)/R$.

If a rebel wins and becomes the new King he may in principle choose a number of wives independently of that which the incumbent had chosen. Let $1-\widetilde{z}$ denote the fraction of the women a successful rebel would marry. Each of his $(1-\widetilde{z})P$ wives then has $([1-\omega]A/[1-\widetilde{z}]P)^\rho$ children [see (1) and (3)].

Unsuccessful rebels have no children. Thus, a subject's excepted number of children if he chooses to rebel simply equals the probability of becoming the new King, times the number of children he would have if that happened: $[G(R)/R] \times (1-\widetilde{z})P \times ([1-\omega]A/[1-\widetilde{z}]P)^\rho$.

Analyzing the subjects' decisions boils down to comparing two expected payoffs: that of rebelling, and that of not rebelling. Using the expressions for the payoffs derived above the equilibrium condition can thus be written:

$$\frac{z^{1-\rho}\omega^\rho}{x}\left(\frac{A}{P}\right)^\rho \geq \frac{G(R)}{R}(1-\widetilde{z})^{1-\rho}(1-\omega)^\rho A^\rho P^{1-\rho}. \tag{4}$$

If the inequality in (4) is strict when $x = 1$, all subjects choose to be peaceful in equilibrium. Alternatively, we can have an equilibrium where all subjects are indifferent between rebelling and being peaceful, which happens if (4) holds with equality for some $x > 0$.

The analysis of the subjects' behavior is complicated by the fact that the equilibrium number of rebels, $R = (1 - x)P$, determines the probability that the King is ousted, $G(R)$, thus potentially influencing the equilibrium fraction of the subjects choosing to rebel. We are going to use a specification for $G(R)$ which ensures that the equilibrium is unique:

$$G(R) = \min\{1, \varepsilon R\}, \tag{5}$$

where $\varepsilon > 0$ is an exogenous parameter.

3.3 The King

The King chooses how many women to marry with the aim of maximizing his expected number of offspring, taking into account how this decision affects the equilibrium behavior of the subjects. To set up the King's maximization problem we must therefore solve for the subjects' behavior in equilibrium.

There are two ways to do this. One approach is to let the incumbent King, when choosing his number of wives, also force the successful rebel to choose the same number of wives. We shall refer to this as the optimal approach (Section 3.3.1). The other approach is to let the successful rebel choose his number of wives independently of what the King chooses, which we refer to as the equilibrium approach (Section 3.3.2).

3.3.1 Optimal constraints on polygyny

Consider first the optimal approach. This means that the King effectively writes a kind of rule, or constitution, which determines the fraction women both he and a potentially successful rebel can marry. That is, when the King chooses z he knows that $\widetilde{z}$ equals whatever he sets z to. This is not necessarily a realistic description of the choices faced by the rulers of any ancient society (although e.g. religion may have served such a role). However, the approach is interesting because the results can be compared to the equilibrium approach later (Section 3.3.2), and it is also analytically much simpler.

Let S be a variable denoting whether a subject chooses to rebel ($S = \mathcal{R}$), or be peaceful ($S = \mathcal{P}$). Using the expressions in (4), and setting $\widetilde{z} = z$, we can then write the subject's expected number of offspring as:

$$m(S; z, x, A, P) = \begin{cases} \frac{z^{1-\rho}\omega^{\rho}}{x}\left(\frac{A}{P}\right)^{\rho} & \text{if } S = \mathcal{P}, \\ \left[\frac{G([1-x]P)}{(1-x)P}\right](1-z)^{1-\rho}(1-\omega)^{\rho}A^{\rho}P^{1-\rho} & \text{if } S = \mathcal{R}. \end{cases} \tag{6}$$

Next define the following:

$$\overline{z} = \frac{(\varepsilon P)^{\frac{1}{1-\rho}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}}{1 + (\varepsilon P)^{\frac{1}{1-\rho}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}}, \tag{7}$$

and

$$\widehat{z} = \frac{(\varepsilon P - 1)^{\frac{1}{1-\rho}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}}{1 + (\varepsilon P - 1)^{\frac{1}{1-\rho}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}}, \tag{8}$$

where we note that $\widehat{z}$ is defined only for $\varepsilon P \geq 1$, and that $\overline{z} \geq \widehat{z}$ (if $\widehat{z}$ is defined).

We can now state the following.

Proposition 2 *At any given level of z, there exists a unique equilibrium where the fraction of the subjects being peaceful, x^*, is given as follows:*
(a) If $\varepsilon P > 1$, then

$$x^* = \begin{cases} 1 & \textit{if } z \in [\overline{z}, 1], \\ \left(\frac{1}{\varepsilon P}\right)\left(\frac{z}{1-z}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho} & \textit{if } z \in [\widehat{z}, \overline{z}], \\ \frac{\left(\frac{z}{1-z}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho}}{1+\left(\frac{z}{1-z}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho}} & \textit{if } z \in [0, \widehat{z}]. \end{cases} \tag{9}$$

(b) If $\varepsilon P \leq 1$, then

$$x^* = \begin{cases} 1 & \textit{if } z \in [\overline{z}, 1], \\ \left(\frac{1}{\varepsilon P}\right)\left(\frac{z}{1-z}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho} & \textit{if } z \in [0, \overline{z}]. \end{cases} \tag{10}$$

The proof is in the Appendix.

Intuitively, a high z implies a large fraction women allocated to the peaceful subjects (and a low fraction women being taken by the King). This makes it relatively less tempting to rebel and try to seize power. If $z \geq \overline{z}$, all subjects are peaceful in equilibrium. If z falls below $\widehat{z}$, so many men rebel that the King is ousted with probability one; note that this can only occur if there are sufficiently many men to rebel, $\varepsilon P > 1$.

Given the equilibrium fraction of the subjects rebelling, we can derive the probability that the King is ousted.

Proposition 3 *At any given level of z, there exists a unique equilibrium where the probability that the King is ousted, $G(R^*) = \min\{1, \varepsilon[1 - x^*]P\}$, is given as follows:*
(a) If $\varepsilon P > 1$, then

$$G(R^*) = \begin{cases} 0 & \textit{if } z \in [\overline{z}, 1], \\ \varepsilon P - \left(\frac{z}{1-z}\right)^{1-\rho} \left(\frac{\omega}{1-\omega}\right)^{\rho} & \textit{if } z \in [\widehat{z}, \overline{z}], \\ 1 & \textit{if } z \in [0, \widehat{z}]. \end{cases} \tag{11}$$

(b) If $\varepsilon P \leq 1$, then

$$G(R^*) = \begin{cases} 0 & \textit{if } z \in [\overline{z}, 1], \\ \varepsilon P - \left(\frac{z}{1-z}\right)^{1-\rho} \left(\frac{\omega}{1-\omega}\right)^{\rho} & \textit{if } z \in [0, \overline{z}]. \end{cases} \tag{12}$$

The proof is in the Appendix.

Proposition 3 essentially states that the King is safer if he is constrained to a lower share of the women (i.e., if z is high) because this pacifies the male subjects. This is the trade-off that generates an optimal degree of polygyny for the King. From a reproductive perspective a low z directly benefits the King, by giving him more women if he stays in power. On the other hand, a low z also lowers the probability he will stay in power, and have any children at all.

To explore this tradeoff, next we shall derive the King's expected number of offspring as a function of z, denoted $k(z)$. First define:

$$\phi(z) = (1 - \varepsilon P)(1 - \omega)^{\rho}(1 - z)^{1-\rho} + \omega^{\rho} z^{1-\rho}. \tag{13}$$

We can now state the following.

Proposition 4 *The King's expected number of offspring in equilibrium, $k(z)$, is given as follows.*
(a) If $\varepsilon P > 1$, then

$$k(z) = \begin{cases} (1 - z)^{1-\rho}(1 - \omega)^{\rho} A^{\rho} P^{1-\rho} & \text{if } z \in [\overline{z}, 1], \\ \\ \phi(z) A^{\rho} P^{1-\rho} & \text{if } z \in [\widehat{z}, \overline{z}], \\ \\ 0 & \text{if } z \in [0, \widehat{z}]. \end{cases} \tag{14}$$

(b) If $\varepsilon P \leq 1$, then

$$k(z) = \begin{cases} (1 - z)^{1-\rho}(1 - \omega)^{\rho} A^{\rho} P^{1-\rho} & \text{if } z \in [\overline{z}, 1], \\ \\ \phi(z) A^{\rho} P^{1-\rho} & \text{if } z \in [0, \overline{z}]. \end{cases} \tag{15}$$

Figure 1 illustrates the results in Propositions 2, 3, and 4, by graphing x^*, $G(R^*)$, and $k(z)$ against z. As seen, $k(z)$ reaches a maximum at $\overline{z}$, where $x^* = 1$ and $G(R^*) = 0$.

However, the King chooses z subject to a constraint. Recall from Proposition 1 that the women's participation constraint states that $z \geq \omega$. Since z is a fraction it must also hold that $z \leq 1$. The King thus seeks to maximize $k(z)$ subject to $z \in [\omega, 1]$. Let the optimal choice of z be denoted z^*, defined as

$$z^* = \arg\max_{z \in [\omega, 1]} k(z). \tag{16}$$

We then make the following assumption to simplify the analysis.

Assumption 1 *$\omega > 1/2$.*

We can now state the following.

Proposition 5 *Under Assumption 1, the King's optimal z is given by*

$$z^* = \begin{cases} \omega & \text{if } \varepsilon P \le \frac{\omega}{1-\omega}, \\ \overline{z} = \frac{(\varepsilon P)^{\frac{1}{1-\rho}} \left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}}{1+(\varepsilon P)^{\frac{1}{1-\rho}} \left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}} & \text{if } \varepsilon P \ge \frac{\omega}{1-\omega}. \end{cases} \tag{17}$$

The proof is in the Appendix. Assumption 1 ensures that $z^* = \omega$ when $\varepsilon P \le \omega/(1-\omega)$.

Proposition 5 states that for low levels of population, $\varepsilon P \le \omega/(1-\omega)$, *unconstrained polygyny* is optimal, meaning that the King marries as many wives as he can attract. A low P means that there are relatively few women around. Thus, becoming the new King, and conquering some fraction of all women, does not generate a terribly large number of offspring. Therefore, it is not very tempting for a subject to rebel, even under an institution where the King is allowed to marry the maximum number of women that he can attract, $1-\omega$.

For higher levels of population, $\varepsilon P \ge \omega/(1-\omega)$, the King is better off with restricting the number of wives he (and any potentially successful rebel) can take. The optimal norm from the incumbent King's perspective is to allocate just enough women to the subjects so that all of them stay peaceful and the probability that the King is ousted equals zero; cf. Proposition 2 and 3. This is what we call *constrained polygyny.*

The insight in Proposition 5 is illustrated in Figure 2. The position of $k(z)$ shifts in response to an increase in P, thus pushing the maximum from $z^* = \omega$ to $z^* = \overline{z}$.

Note also that $\overline{z}$ is increasing in P. Intuitively, as population grows, the total number of women that the King can marry grows. This inflates to reproductive payoff for subjects to seizing power. As a result, the peace-inducing marriage norm becomes one of more constrained polygyny.

3.3.2 Equilibrium constraints on polygyny

In the setting considered thus far we imposed the assumption that the z chosen by the incumbent King determined the fraction women that a rebel who successfully ousted the King could marry. That is, a successful rebel could not alter the marriage norm that the predecessor had decided on.

Now we relax this assumption. As before, z denotes the fraction women that the King chooses to marry, and $\widetilde{z}$ denotes the fraction women an agent who successfully ousts him would marry. The analytical approach is in principle straightforward: the King chooses z, taking as given $\widetilde{z}$; this gives a best-response function, determining the King's optimal z for any given $\widetilde{z}$. A fixed point to that best-response function (denoted z^{EQ} below) is the equilibrium marriage norm.

As before, we let S be a variable denoting whether the subject chooses to rebel ($S = \mathcal{R}$), or be peaceful ($S = \mathcal{P}$). We can then write the subject's expected number of offspring as a function of $\widetilde{z}$ and z:

$$m(S; z, \widetilde{z}, x, A, P) = \begin{cases} \frac{z^{1-\rho}\omega^{\rho}}{x}\left(\frac{A}{P}\right)^{\rho} & \text{if } S = \mathcal{P}, \\ \left[\frac{G([1-x]P)}{(1-x)P}\right](1-\widetilde{z})^{1-\rho}(1-\omega)^{\rho}A^{\rho}P^{1-\rho} & \text{if } S = \mathcal{R}. \end{cases} \tag{18}$$

We next define the following functions of $\widetilde{z}$:

$$\begin{aligned} \beta(\widetilde{z}) &= (1-\widetilde{z})(\varepsilon P)^{\frac{1}{1-\rho}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}, \\ \alpha(\widetilde{z}) &= (1-\widetilde{z})(\varepsilon P - 1)^{\frac{1}{1-\rho}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}. \end{aligned} \tag{19}$$

Note that $\overline{z}$ in (7) and $\widehat{z}$ in (8) are fixed points to $\beta(\cdot)$ and $\alpha(\cdot)$, respectively.

As before, the equilibrium number of rebels, $R^{*} = (1-x^{*})P$, determines the probability that the King is ousted, $G(R^{*})$. A subject's decision about whether to rebel, or not, is given by weighing the excepted number of children if he is peaceful, against that of rebelling. The equilibrium fraction of the subjects who choose to be peaceful is derived in the following proposition.

Proposition 6 *At any given levels of z and $\widetilde{z}$ there exists a unique equilibrium where the fraction of the subjects being peaceful, x^*, is given as follows:*
(a) If $\varepsilon P > 1$, then

$$x^* = \begin{cases} 1 & \text{if } z \in [\beta(\widetilde{z}), 1], \\ \left(\frac{1}{\varepsilon P}\right)\left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho} & \text{if } z \in [\alpha(\widetilde{z}), \beta(\widetilde{z})], \\ \frac{\left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho}}{1+\left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho}} & \text{if } z \in [0, \alpha(\widetilde{z})]. \end{cases} \tag{20}$$

(b) If $\varepsilon P \leq 1$, then

$$x^* = \begin{cases} 1 & \text{if } z \in [\beta(\widetilde{z}), 1], \\ \left(\frac{1}{\varepsilon P}\right)\left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho} & \text{if } z \in [0, \beta(\widetilde{z})]. \end{cases} \tag{21}$$

The proof is in the Appendix and follows along the same lines as that of Proposition 2. The only alteration in (20) and (21), compared to (2) and (3), is that z is replaced by $\widetilde{z}$ in a couple of places.

Given the equilibrium fraction of the subjects rebelling, we can derive the probability that the King is ousted.

Proposition 7 *At any given levels of z and $\widetilde{z}$, there exists a unique equilibrium where the probability that the King is ousted, $G(R^*) = \min\{1, \varepsilon[1 - x^*]P\}$, is given as follows:*
(a) If $\varepsilon P > 1$, then

$$G(R^*) = \begin{cases} 0 & \text{if } z \in [\beta(\widetilde{z}), 1], \\ \varepsilon P - \left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho}\left(\frac{\omega}{1-\omega}\right)^{\rho} & \text{if } z \in [\alpha(\widetilde{z}), \beta(\widetilde{z})], \\ 1 & \text{if } z \in [0, \alpha(\widetilde{z})]. \end{cases} \tag{22}$$

(b) If $\varepsilon P \leq 1$, then

$$G(R^*) = \begin{cases} 0 & \text{if } z \in [\beta(\widetilde{z}), 1], \\ \varepsilon P - \left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho} \left(\frac{\omega}{1-\omega}\right)^{\rho} & \text{if } z \in [0, \beta(\widetilde{z})]. \end{cases} \tag{23}$$

The proof is in the Appendix.

Proposition 7 states that the King is safer if he constrains himself to a lower share of the women (i.e., if he chooses a high z). Note that this is true while holding fixed the potentially successful rebel's action, $\widetilde{z}$.

We can now derive the King's expected number of offspring as a function of z and $\widetilde{z}$, denoted $k(z, \widetilde{z})$. First define:

$$\lambda(z, \widetilde{z}) = \left[1 - \varepsilon P + \left(\frac{\omega}{1-\omega}\right)^{\rho} \left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho}\right] (1-z)^{1-\rho}(1-\omega)^{\rho}. \tag{24}$$

Note that (24) boils down to (13) when $z = \widetilde{z}$, i.e., $\lambda(z, z) = \phi(z)$. We can now state the following.

Proposition 8 *The King's expected number of offspring in equilibrium, $k(z, \widetilde{z})$, is given as follows.*
(a) If $\varepsilon P > 1$, then

$$k(z, \widetilde{z}) = \begin{cases} (1-z)^{1-\rho}(1-\omega)^{\rho} A^{\rho} P^{1-\rho} & \text{if } z \in [\beta(\widetilde{z}), 1], \\ \lambda(z, \widetilde{z}) A^{\rho} P^{1-\rho} & \text{if } z \in [\alpha(\widetilde{z}), \beta(\widetilde{z})], \\ 0 & \text{if } z \in [0, \alpha(\widetilde{z})]. \end{cases} \tag{25}$$

(b) If $\varepsilon P \leq 1$, then

$$k(z, \widetilde{z}) = \begin{cases} (1-z)^{1-\rho}(1-\omega)^{\rho} A^{\rho} P^{1-\rho} & \text{if } z \in [\beta(\widetilde{z}), 1], \\ \lambda(z, \widetilde{z}) A^{\rho} P^{1-\rho} & \text{if } z \in [0, \beta(\widetilde{z})]. \end{cases} \tag{26}$$

The King chooses z to maximize $k(z, \widetilde{z})$ subject to $z \in [\omega, 1]$, taking as given $\widetilde{z}$. (Recall the women's marriage participation constraint, $z \geq \omega$, from Proposition 1.) The King's optimal choice of z can then be defined as a best-response function of $\widetilde{z}$, given by

$$\Psi(\widetilde{z}) = \arg \max_{z \in [\omega, 1]} k(z, \widetilde{z}). \tag{27}$$

We can now define the equilibrium degree of constrained polygyny, z^{EQ}, as a fixed-point to $\Psi(\widetilde{z})$, that is:

$$z^{EQ} = \Psi(z^{EQ}). \tag{28}$$

The following assumption is sufficient to ensure that z^{EQ} exists and is unique:

Assumption 2 *$\omega > \rho$.*

Under Assumptions 1 and 2 we can now state the following.

Proposition 9 *There exists a unique equilibrium level of z^{EQ}, given as follows:*
(a) If $\varepsilon P \leq \omega/(1-\omega)$, then $z^{EQ} = \omega$.
(b) If $\varepsilon P \geq \omega/(1-\omega)$, then z^{EQ} is defined from $F(z^{EQ}, P) \equiv 0$, where

$$F(z, P) = (2z - 1) \left(\frac{\omega}{1 - \omega} \right)^{\rho} - (\varepsilon P - 1)\, z^{\rho} (1 - z)^{1-\rho}. \tag{29}$$

Note that the equilibrium and optimal degrees of monogamy coincide when $\varepsilon P \leq \omega/(1-\omega)$ (i.e., $z^{EQ} = z^* = \omega$). For small population, unconstrained polygyny is both the optimal and equilibrium outcome. However, for larger population it can be seen that polygyny is less constrained in equilibrium than is optimal:

Proposition 10 *For $\varepsilon P > \omega/(1-\omega)$, the following holds:*
(a) $z^{EQ} < z^*$.
(b) z^{EQ} is a (strictly) increasing function of P.

Part (b) states that (once population exceeds a threshold) society becomes more monogamous in equilibrium as population expands. [Recall that the same holds for the optimal degree of monogamy, z^*, as seen from differentiating (17) with respect to P.] Using this relationship, in Section 4 we examine the endogenous evolution of population, technology, and the marriage system.

To simplify matters, we first show that when $\rho = 1/2$, we can derive an explicit expression for z^{EQ}:

Proposition 11 *If $\rho = 1/2$, then*

$$z^{EQ} = \begin{cases} \omega & \text{if } \varepsilon P \leq \omega/(1-\omega), \\ \frac{1}{2}\left[1 + \sqrt{\frac{U}{4+U}}\right] & \text{if } \varepsilon P \geq \omega/(1-\omega), \end{cases} \tag{30}$$

where

$$U = (\varepsilon P - 1)^2 \left(\frac{1-\omega}{\omega}\right). \tag{31}$$

Consistent with Proposition 10, this verifies that z^{EQ} is increasing in P (and U) for $\varepsilon P > \omega/(1-\omega)$, as well as z^{EQ} going to one as P goes to infinity.

4 Dynamics

To illustrate the dynamics of this economy we simulate it; see Figure 3. We assume that the marriage norm in any period equals its equilibrium level, i.e., $z = z^{EQ}$, and that technology progresses at an exogenously given rate, $g > 0$, i.e., $A' = (1+g)A$.

Half of all born children are girls, and half are boys. It can then be seen that the (female and male) population who enter as adults in the next period, P', is given by

$$P' = \left[(z^{EQ})^{1-\rho}\omega^{\rho} + \left(1 - z^{EQ}\right)^{1-\rho}(1-\omega)^{\rho}\right] \frac{A^{\rho} P^{1-\rho}}{2}, \tag{32}$$

where z^{EQ} would generally be an implicit function of the state variables A and P, defined in Proposition 9. The paths generated in Figure 3 refer to the simple case when $\rho = 1/2$, where we have an explicit expression for z^{EQ}; recall Proposition 11.

Note the Malthusian features of the population dynamics: population growth, P'/P, is increasing in A and decreasing in P; holding fixed A the economy would converge to a Malthusian steady state with constant population. This Malthusian feature comes from the diminishing marginal product of labor imposed on the production function for children in (1), i.e., $\rho \in (0, 1)$.

Together with initial conditions, $A' = (1 + g)A$, (32), (**??**), and (**??**), define the dynamic paths for A and P, and thus the evolution of z^{EQ}. To upward population path in Figure 3 is driven by rising levels of A. At early stages of development, when levels of technology and population density are low, the optimal marriage norm is that of unconstrained polygyny. The King marries $(1 - \omega)P$ women.

One day εP exceeds $\omega/(1 - \omega)$, at which point the equilibrium marriage form becomes that of constrained polygyny. Note that the optimally constrained polygyny, z^*, exceeds the equilibrium constrained polygyny, z^{EQ}.

As population levels grow further polygyny becomes increasingly constrained, i.e., z^{EQ} rises. In the limit, z^{EQ} approaches one, and the share of the women married to the King, $1 - z^{EQ}$, approaches zero. In that sense, society becomes fully monogamous.

5 Extensions

5.1 Sharing land

In the framework used thus far, the fraction of the land held by the subjects, ω, is treated as given. Now consider a setting where the King is allowed to allocate some of his land to the subjects as a means to pacify them.

Let q denote the fraction of the land that the King allocates to the subjects, himself keeping $1 - q$. The King cannot take a larger fraction of the land than $1 - \omega$. (We may think of $1 - \omega$ as a maximum tax rate.) Thus, the King chooses q on $[\omega, 1]$.

We follow the approach in Section 3.3.1 above, under which the King chooses a rule which constrains also a potentially succesful rebel. This rule now pins down both z and q. First define the following function:

$$\psi(z,q) = (1-\varepsilon P)(1-q)^{\rho}(1-z)^{1-\rho} + q^{\rho} z^{1-\rho}, \tag{33}$$

where we note from (13) that $\psi(z,\omega) = \phi(z)$. Then define

$$\begin{aligned} \gamma(q) &= \frac{(\varepsilon P)^{\frac{1}{1-\rho}}\left(\frac{1-q}{q}\right)^{\frac{\rho}{1-\rho}}}{1+(\varepsilon P)^{\frac{1}{1-\rho}}\left(\frac{1-q}{q}\right)^{\frac{\rho}{1-\rho}}}, \\ \delta(q) &= \frac{(\varepsilon P-1)^{\frac{1}{1-\rho}}\left(\frac{1-q}{q}\right)^{\frac{\rho}{1-\rho}}}{1+(\varepsilon P-1)^{\frac{1}{1-\rho}}\left(\frac{1-q}{q}\right)^{\frac{\rho}{1-\rho}}}, \end{aligned} \tag{34}$$

where we note from (7) and (8) that $\gamma(\omega) = \overline{z}$ and $\delta(\omega) = \widehat{z}$.

That is, if we set $q = \omega$ this formulation is identical to that in Section 3.3.1. Therefore, Propositions 2, 3, and 4 apply, but with q replacing ω. The only news is essentially that the King's control variables are now q and z. Analogously to (14) and (15) we can write the King's expected number of offspring, now denoted $k(z,q)$, as follows: for $\varepsilon P > 1$,

$$k(z,q) = \begin{cases} (1-z)^{1-\rho}(1-q)^{\rho} A^{\rho} P^{1-\rho} & \text{if } z \in [\gamma(q), 1], \\ \psi(z,q) A^{\rho} P^{1-\rho} & \text{if } z \in [\delta(q), \gamma(q)], \\ 0 & \text{if } z \in [0, \delta(q)], \end{cases} \tag{35}$$

and for $\varepsilon P \leq 1$,

$$k(z,q) = \begin{cases} (1-z)^{1-\rho}(1-q)^{\rho} A^{\rho} P^{1-\rho} & \text{if } z \in [\gamma(q), 1], \\ \psi(z,q) A^{\rho} P^{1-\rho} & \text{if } z \in [0, \gamma(q)]. \end{cases} \tag{36}$$

The King maximizes $k(z,q)$ over z and q in the choice set

$$\Omega = \left\{(z,q) \in [0,1]^2 : q \geq \omega, z \geq q\right\}, \tag{37}$$

where $z \geq q$ follows from the women's participation constraint (recall Proposition1 again). Define the King's optimal levels of z and q as z^* and q^*, i.e.,

$$(z^*, q^*) = \arg \max_{(z,q) \in \Omega} k(z, q). \tag{38}$$

We can now state the following.

Proposition 12 *Under Assumption 1, z^* and q^* are given by*

$$z^* = q^* = \begin{cases} \omega & \text{if } \varepsilon P \leq \frac{\omega}{1-\omega}, \\ \\ \frac{\varepsilon P}{1+\varepsilon P} & \text{if } \varepsilon P \geq \frac{\omega}{1-\omega}. \end{cases} \tag{39}$$

The proof is in the Appendix.

The King shares both land and women in order to pacify his subjects. With the functional forms used here the optimal shares of land and women allocated to the subjects become identical. Note also that z^* and q^* are increasing in P. The intuition is the same as when the King shares only women: the more he shares, the less inclined are the subjects to rebel, and the need to pacify subjects becomes more acute when population is larger, because that makes the returns to rebellion higher.

6 Conclusions

There is an ongoing debate in the social sciences over what caused the rise of monogamy. One proposition has been that monogamy serves to pacify men. The purpose of this paper has been to set up a formal model of such a pacifying role of monogamy. The central mechanism is that a more equal distribution of women makes subjects less inclined to rebel, and thus the ruler (the King) safer. Monogamy, or rather what we call constrained polygyny, can be optimal to a King who seeks to maximize his reproductive success.

We consider two ways set up the problem. One is to assume that the King, when he decides how many wives to take, constrains also any potentially successful rebel to take the same number of wives. That is, if the King is ousted the marriage norm cannot be altered by the new King. This what we refer to as the optimal approach.

Another (perhaps more realistic) assumption is that the King cannot influence the number of wives a rebel who successfully ousts him chooses to marry. However, taking that number as given, the King may still have an incentive to constrain his own number of wives, and the same holds for a successful rebel in the same situation. We can thus examine an equilibrium where the incumbent King, and a potentially successful rebel, choose the same number of wives. This is what we call the equilibrium approach.

We show that the King chooses a less constrained form of polygyny under the equilibrium approach compared to the optimal approach. The intuition is that an incumbent King faces a higher return to constraining himself if he thereby constrains also the potentially successful rebel. Sharing women with the subjects make them peaceful not only because it makes them more likely to get married, but also because a rebelling subject would be forced to share his women if he became King.

This may suggest why monogamy (or constrained polygyny) is typically imposed as a law or social norm, rather than chosen at the King's own discretion.

This model exhibits a "scale effect" in the reproductive value of being the King. The larger is population, and thus the number of women, the more wives the King can attract. Therefore, the returns to rebellion are higher, and polygyny must be more constrained in order to pacify the subjects.

This has some interesting dynamic implications. Letting technology grow over time (at some exogenous rate) generates population expansion in a standard Malthusian fashion, which in turn leads to increasingly constrained forms of polygyny. The model can in that sense replicate an endogenous rise of monogamy in the course of long-run development.

APPENDIX

Proof of Proposition 1: Using (1), (2), and (3) gives $z = \omega$. $\|$

Proof of Proposition 2: There are four possible types of equilibria: (I) $\varepsilon R^* \geq 1$, $x^* = 1$; (II) $\varepsilon R^* \geq 1$, $x^* < 1$; (III) $\varepsilon R^* < 1$, $x^* = 1$; (IV) $\varepsilon R^* < 1$, $x^* < 1$. Below we examine for what values of z any one of these equilibria exists. Also, if equilibria of type I and/or III exist, we examine what x^* equals in that equilibrium.

Equilibria of type I: $\varepsilon R^* \geq 1$, $x^* = 1$. If $x^* = 1$, then $\varepsilon R^* = \varepsilon(1 - x^*)P = 0 < 1$. We conclude that an equilibrium of type I cannot exist.

Equilibria of type II: $\varepsilon R^* \geq 1$, $x^* < 1$. If $\varepsilon R^* \geq 1$, (5) shows that $G(R^*) = 1$. If $x^* < 1$, agents must be indifferent between rebelling and being peaceful, i.e., $m(\mathcal{P}; z, x^*, A, P) = m(\mathcal{R}; z, x^*, A, P)$ in (6), setting $G(R^*) = 1$. We can then solve for a unique x^*, which becomes $x^* = W/(1 + W)$, where

$$W = \left(\frac{z}{1 - z}\right)^{1-\rho} \left(\frac{\omega}{1 - \omega}\right)^{\rho}, \tag{A1}$$

which verifies that $x^* = W/(1 + W)$, as stated in (9), and that $x^* < 1$ holds. For $\varepsilon R^* \geq 1$ to hold requires that $\varepsilon R^* = \varepsilon(1 - x^*)P = \varepsilon P/[1 + W] \geq 1$, or $\varepsilon P - 1 \geq W$. If $\varepsilon P \leq 1$, this never holds (for $z > 0$). If $\varepsilon P > 1$, we see that $\varepsilon P - 1 \geq W$, together with (A1), implies $z \leq \widehat{z}$, where $\widehat{z}$, is defined in (8). We thus conclude that an equilibrium of type II exists if and only if $\varepsilon P > 1$ and $z \leq \widehat{z}$.

Equilibria of type III: $\varepsilon R^* < 1$, $x^* = 1$. If $\varepsilon R^* < 1$, it holds that $G(R^*) = \varepsilon R^*$. We see right away that $x^* = 1$ ensures that $\varepsilon R^* = \varepsilon(1 - x^*)P = 0 < 1$. For $x^* = 1$ to hold in equilibrium no agent must strictly prefer to rebel when $x^* = 1$, i.e., $m(\mathcal{P}; z, 1, A, P) \geq m(\mathcal{R}; z, 1, A, P)$. Setting $G(R^*) = \varepsilon R^*$ in (6) this implies $z \geq \overline{z}$, where $\overline{z}$, is defined in (7). We thus conclude that an equilibrium of type III exists if and only if $z \geq \overline{z}$.

Equilibria of type IV: $\varepsilon R^* < 1$, $x^* < 1$. If $\varepsilon R^* < 1$, it holds that $G(R^*) = \varepsilon R^*$. If $x^* < 1$, agents are indifferent between rebelling and being peaceful, i.e., $m(\mathcal{P}; z, x^*, A, P) = m(\mathcal{R}; z, x^*, A, P)$ in (6), setting $G(R^*) = \varepsilon R^*$. We can then solve for a unique x^*, namely $x^* = W/(\varepsilon P)$, where W is given by (A1). For $x^* = W/(\varepsilon P) < 1$ to hold requires that $z < \overline{z}$, where $\overline{z}$, is defined in (7). For $\varepsilon R^* = \varepsilon(1-x^*)P < 1$ to hold requires $\varepsilon P x^* = W > \varepsilon P - 1$. This always holds if $\varepsilon P \leq 1$. If $\varepsilon P > 1$, for $W > \varepsilon P - 1$ to hold requires that $z > \widehat{z}$, where $\widehat{z}$ is defined in (8). We thus conclude that an equilibrium of type IV exists if and only if the following holds: either $\varepsilon P \leq 1$ and $z < \overline{z}$; or $\varepsilon P > 1$ and $z \in (\widehat{z}, \overline{z})$.

Finally, we note that x^* in the equilibria of type II and IV coincide when $z = \widehat{z}$, and that x^* in the equilibria of type III and IV coincide when $z = \overline{z}$. It follows that $x^* = W/(\varepsilon P)$ when $z \in [\widehat{z}, \overline{z}]$, as stated in (9). ‖

Proof of Proposition 3: The result follows from inserting the expressions for x^* in Proposition 2 into $G(R^*) = \min\{1, \varepsilon[1 - x^*]P\}$. ‖

Proof of Proposition 4: The King is ousted with probability $G(R^*)$, in which case he is killed and/or leaves no offspring. If he is not ousted, he has $(1-z)P$ wives, each of whom has $[(1-\omega)A/(1-z)P]^\rho$ children [see (1) and (3)], so if he is not ousted he has $(1-z)P \times [(1-\omega)A/(1-z)P]^\rho = (1-z)^{1-\rho}(1-\omega)^\rho A^\rho P^{1-\rho}$ children. We can thus write his expected number of children as

$$[1 - G(R^*)] \times (1-z)^{1-\rho}(1-\omega)^\rho A^\rho P^{1-\rho} + G(R^*) \times 0. \tag{A2}$$

Inserting the equilibrium expression for $G(R^*)$ in Proposition 3 gives $k(z)$ as in Proposition 4. ‖

Proof of Proposition 5: Consider first the case when $\varepsilon P > 1$, so that $k(z)$ is given by (14). Note from (14) that $k(z) = 0$ for all $z \leq \widehat{z}$, whereas $k(z) > 0$ for all $z > \widehat{z}$, so z^* must be strictly greater than $\widehat{z}$. Next note from (13) that (for $\varepsilon P > 1$) $\phi'(z) > 0$. It then follows from (14) that $k'(z) > 0$ for $z \in (\widehat{z}, \overline{z})$, and $k'(z) < 0$ for $z > \overline{z}$. Thus, $k(z)$ is maximized at $z = \overline{z}$ if $\omega > \overline{z}$, which holds if $\varepsilon P \geq \omega/(1-\omega)$; else $k(z)$ is maximized at $z = \omega$.

Consider next the case when $\varepsilon P \leq 1$, so that $k(z)$ is given by (15). Here we use Assumption 1, which implies that $\omega/(1-\omega) > 1$, to note that if $\varepsilon P \leq 1$, then $\varepsilon P < \omega/(1-\omega)$, which in turn implies that $\omega > \overline{z}$. Thus, from (15) we see that $k'(z) < 0$ for $z \in [\omega, 1]$, so $k(z)$ is maximized at $z = \omega$. $\|$

Proof of Proposition 6: Like in the proof of Proposition 2, there are four possible types of equilibria: (I) $\varepsilon R^* \geq 1$, $x^* = 1$; (II) $\varepsilon R^* \geq 1$, $x^* < 1$; (III) $\varepsilon R^* < 1$, $x^* = 1$; (IV) $\varepsilon R^* < 1$, $x^* < 1$. It thus remains to examine for what values of z and $\widetilde{z}$ which equilibrium exists. Also, if equilibria of type I and/or III exist, we examine what x^* equals in that equilibrium.

Equilibria of type I: $\varepsilon R^* \geq 1$, $x^* = 1$. If $x^* = 1$, then $\varepsilon R^* = \varepsilon(1 - x^*)P = 0 < 1$. We conclude that an equilibrium of type I cannot exist.

Equilibria of type II: $\varepsilon R^* \geq 1$, $x^* < 1$. If $\varepsilon R^* \geq 1$, (5) shows that $G(R^*) = 1$. If $x^* < 1$, agents must be indifferent between rebelling and being peaceful, i.e., $m(\mathcal{P}; z, \widetilde{z}, x^*, A, P) = m(\mathcal{R}; z, \widetilde{z}, x^*, A, P)$ in (18), setting $G(R^*) = 1$. We can then solve for a unique x^*, which becomes $x^* = W/(1 + W)$, where

$$W = \left(\frac{z}{1-\widetilde{z}}\right)^{1-\rho} \left(\frac{\omega}{1-\omega}\right)^{\rho}, \tag{A3}$$

which verifies that $x^* = W/(1+W)$, as stated in (20), and that $x^* < 1$ holds. For $\varepsilon R^* \geq 1$ to hold requires that $\varepsilon R^* = \varepsilon(1 - x^*)P = \varepsilon P/[1 + W] \geq 1$, or $\varepsilon P - 1 \geq W$. If $\varepsilon P \leq 1$, this never holds (for $z > 0$). If $\varepsilon P > 1$, we see that $\varepsilon P - 1 \geq W$, together with (A1), implies $z \leq \beta(\widetilde{z})$, where $\beta(\widetilde{z})$, is defined in (19). We thus conclude that an equilibrium of type II exists if and only if $\varepsilon P > 1$ and $z \leq \beta(\widetilde{z})$.

Equilibria of type III: $\varepsilon R^* < 1$, $x^* = 1$. If $\varepsilon R^* < 1$, it holds that $G(R^*) = \varepsilon R^*$. We see right away that $x^* = 1$ ensures that $\varepsilon R^* = \varepsilon(1-x^*)P = 0 < 1$. For $x^* = 1$ to hold in equilibrium no agent must strictly prefer to rebel when $x^* = 1$, i.e., $m(\mathcal{P}; z, \widetilde{z}, 1, A, P) \geq m(\mathcal{R}; z, \widetilde{z}, 1, A, P)$. Setting $G(R^*) = \varepsilon R^*$ in (18) this implies $z \geq \alpha(\widetilde{z})$, where $\alpha(\widetilde{z})$, is defined in (19). We thus conclude that an equilibrium of type III exists if and only if $z \geq \alpha(\widetilde{z})$.

Equilibria of type IV: $\varepsilon R^* < 1$, $x^* < 1$. If $\varepsilon R^* < 1$, it holds that

$G(R^*) = \varepsilon R^*$. If $x^* < 1$, agents are indifferent between rebelling and being peaceful, i.e., $m(\mathcal{P}; z, \widetilde{z}, x^*, A, P) = m(\mathcal{R}; z, \widetilde{z}, x^*, A, P)$ in (18), setting $G(R^*) = \varepsilon R^*$. We can then solve for a unique x^*, namely $x^* = W/(\varepsilon P)$, where W is given by (A3). For $x^* = W/(\varepsilon P) < 1$ to hold requires that $z < \beta(\widetilde{z})$, where $\beta(\widetilde{z})$, is defined in (19). For $\varepsilon R^* = \varepsilon(1 - x^*)P < 1$ to hold requires $\varepsilon P x^* = W > \varepsilon P - 1$, which always holds if $\varepsilon P \leq 1$. If $\varepsilon P > 1$, for $W > \varepsilon P - 1$ to hold requires that $z > \alpha(\widetilde{z})$, where $\alpha(\widetilde{z})$ is defined in (19). We thus conclude that an equilibrium of type IV exists if and only if the following holds: either $\varepsilon P \leq 1$ and $z < \alpha(\widetilde{z})$; or $\varepsilon P > 1$ and $z \in (\alpha(\widetilde{z}), \beta(\widetilde{z}))$.

Finally, we note that x^* in the equilibria of type II and IV coincide when $z = \beta(\widetilde{z})$, and that x^* in the equilibria of type III and IV coincide when $z = \alpha(\widetilde{z})$. It follows that $x^* = W/(\varepsilon P)$ when $z \in [\alpha(\widetilde{z}), \beta(\widetilde{z})]$, as stated in (20). $\|$

Proof of Proposition 7: The result follows from inserting the expressions for x^* shown in Proposition *6 into* $G(R^*) = \min\{1, \varepsilon[1 - x^*]P\}$. $\|$

Proof of Proposition 8: The King is ousted with probability $G(R^*)$, in which case he is killed and/or leaves no offspring. If he is not ousted, he has $(1 - z)P$ wives, each of whom has $[(1 - \omega)A/(1 - z)P]^\rho$ children, so if he is not ousted he has $(1-z)P \times [(1-\omega)A/(1-z)P]^\rho = (1-z)^{1-\rho}(1-\omega)^\rho A^\rho P^{1-\rho}$ children. We can thus write his expected number of children as

$$[1 - G(R^*)] \times (1 - z)^{1-\rho}(1 - \omega)^\rho A^\rho P^{1-\rho} + G(R^*) \times 0. \qquad \text{(A4)}$$

Inserting the equilibrium expression for $G(R^*)$ in Proposition 7 gives $k(z, \widetilde{z})$ as in Proposition 8. $\|$

Proof of Proposition 9: Recall from (27) and (28) that z^{EQ} is a fixed-point to $\Psi(\widetilde{z})$, where $\Psi(\widetilde{z})$ gives the z that maximizes $k(z, \widetilde{z})$, and $k(z, \widetilde{z})$ is given by (25) or (26).

First note that, if $\varepsilon P > 1$, $\Psi(\widetilde{z}) > \alpha(\widetilde{z})$. This follows from $k(z, \widetilde{z}) = 0$ for $z \leq \alpha(\widetilde{z})$, and $k(z, \widetilde{z}) > 0$ for $z > \alpha(\widetilde{z})$.

Next note that $\Psi(\widetilde{z}) \leq \beta(\widetilde{z})$. This follows from $k(z, \widetilde{z})$ being decreasing in z for $z > \beta(\widetilde{z})$.

Therefore, the problem of maximizing $k(z, \widetilde{z})$ with respect to z has either a corner solution, $\Psi(\widetilde{z}) = \beta(\widetilde{z})$; or an interior solution. The interior solution would lie on the interval $(\alpha(\widetilde{z}), \beta(\widetilde{z})]$ if $\varepsilon P > 1$, or $(0, \beta(\widetilde{z})]$ if $\varepsilon P \leq 1$.

Since $k(z, \widetilde{z}) = \lambda(z, \widetilde{z}) A^\rho P^{1-\rho}$ on the relevant interval, the task is thus to find a z that it maximizes $\lambda(z, \widetilde{z})$ at $z = \widetilde{z}$, where $z = \widetilde{z} \in [\omega, 1]$. Using (24), and some algebra, we see that $\partial\lambda(z, \widetilde{z})/\partial z > (=) 0$ at $z = \widetilde{z}$ if, and only if, $F(z, P) < (=) 0$, where $F(z, P)$ is given by (29). Next we note that

$$\frac{\partial F(z,P)}{\partial z} = 2\left(\frac{\omega}{1-\omega}\right)^{\rho} - (\varepsilon P - 1)\left(\frac{1-z}{z}\right)^{1-\rho}\left[\frac{\rho(1-z)-(1-\rho)z}{(1-z)}\right]. \tag{A5}$$

Consider now first case (b), where $\varepsilon P \geq \omega/(1-\omega) > 1$ (since $\omega > 1/2$; see Assumption 1). Recalling Assumption 2 ($\omega > \rho$), we can deduce that the expression in square brackets in (A5) is negative for all $z \in [\omega, 1]$. Thus, since $\varepsilon P - 1 > 0$, we know that $F(z, P)$ is increasing in z on the interval $[\omega, 1]$. It can also be seen that $F(\omega, P) \leq 0$ [which follows from $\varepsilon P \geq \omega/(1-\omega)$], and $F(1, P) = (\omega/[1-\omega])^\rho > 0$. This implies that there exists a unique z^{EQ} such that $F(z^{EQ}, P) = 0$.

Consider next case (a), where $\varepsilon P \leq \omega/(1-\omega)$. We first show that, in case (a), $F(z, P) \geq 0$ for all $z \in [\omega, 1]$. To see this, note that $F(z, P)$ is decreasing in P. Thus, if $F(z, \omega/[\varepsilon(1-\omega)]) \geq 0$, then $F(z, P) \geq 0$ for all $\varepsilon P \leq \omega/(1-\omega)$. Now define

$$H(z) \equiv F(z, \omega/[\varepsilon(1-\omega)]) = (2z-1)\left(\frac{\omega}{1-\omega}\right)^{\rho} + \left(\frac{2\omega-1}{1-\omega}\right) z^{\rho}(1-z)^{1-\rho}. \tag{A6}$$

We must show that $H(z) \geq 0$ for all $z \in [\omega, 1]$. From (A6) we see that is true, because $H(\omega) = 0$, and $H'(z) > 0$ for all $z \in [\omega, 1]$; the latter follows from Assumption 2 ($\omega > \rho$).

We have thus shown that $F(z, P) \geq 0$ for all $z \in [\omega, 1]$. It is now easy to verify that: if $\varepsilon P < \omega/(1-\omega)$, the only equillibirum on $[\omega, 1]$ is $z^{EQ} = \omega$, such that $F(z^{EQ}, P) > 0$; the special case $\varepsilon P = \omega/(1-\omega)$ implies that $F(\omega, P) = 0$, so which also gives $z^{EQ} = \omega$. $\|$

Proof of Proposition 10: Consider first part (a). For $\varepsilon P > \omega/(1-\omega)$, Proposition 9 tells us that z^{EQ} is defined from $F(z^{EQ}, P) \equiv 0$, where $F(z, P)$ is given by (29), and Proposition 5 tells us that z^* is given by $\overline{z}$ in (17). Under Assumptions 1 ($\omega > 1/2$) and 2 ($\omega > \rho$), and for $\varepsilon P > \omega/(1-\omega) > 1$, it can be seen that $F(z, P)$ is strictly increasing in z on $[\omega, 1]$, cf. (A5). To show that $z^{EQ} < z^*$ we must show that $F(z^*, P) = F(\overline{z}, P) > 0$. Using (17) and (29) we see that $F(\overline{z}, P) > 0$ is equivalent to the following series of inequalities:

$$\begin{gathered}
(2\overline{z}-1)\left(\tfrac{\omega}{1-\omega}\right)^{\rho} > (\varepsilon P - 1)\,\overline{z}^{\rho}\,(1-\overline{z})^{1-\rho}, \\
\left(\tfrac{2\overline{z}-1}{\overline{z}}\right)\left(\tfrac{\omega}{1-\omega}\right)^{\rho} > (\varepsilon P - 1)\left(\tfrac{1-\overline{z}}{\overline{z}}\right)^{1-\rho}, \\
\left(\tfrac{2\overline{z}-1}{\overline{z}}\right)\left(\tfrac{\overline{z}}{1-\overline{z}}\right)^{1-\rho}\left(\tfrac{\omega}{1-\omega}\right)^{\rho} > \varepsilon P - 1, \\
\left(\tfrac{2\overline{z}-1}{\overline{z}}\right)\varepsilon P > \varepsilon P - 1, \\
\left(1-\tfrac{1}{\overline{z}}\right)\varepsilon P > -1, \\
1-\tfrac{1}{\overline{z}} = -\tfrac{1}{(\varepsilon P)^{\frac{1}{1-\rho}}\left(\frac{1-\omega}{\omega}\right)^{\frac{\rho}{1-\rho}}} > -\tfrac{1}{\varepsilon P}, \\
\varepsilon P > \tfrac{\omega}{1-\omega}.
\end{gathered} \tag{A7}$$

Thus, $F(\overline{z}, P) > 0$ (and $z^{EQ} < z^*$) is equivalent to the stated condition, $\varepsilon P > \omega/(1-\omega)$. This proves part (a)

To prove part (b), use $F(z^{EQ}, P) \equiv 0$ to implicitly differentiate z^{EQ} with respect to P. This gives $dz^{EQ}/dP = -[\partial F(z,P)/\partial P]/[\partial F(z,P)/\partial z]$. It is straightforward to see that $\partial F(z,P)/\partial P < 0$. It can also be seen that $\partial F(z,P)/\partial z > 0$ holds if $\varepsilon P \geq \omega/(1-\omega) > 1$ [recall Assumptions 1 and 2; see (A5) above]. Thus, $dz^{EQ}/dP > 0$. $\|$

Proof of Proposition 11: The proof is done by applying Proposition 9, setting $\rho = 1/2$. For $\varepsilon P \leq \omega/(1-\omega)$, $z^{EQ} = \omega$, as stated in (30). For $\varepsilon P \geq \omega/(1-\omega)$, we use $\rho = 1/2$, $F(z^{EQ}, 0) \equiv 0$, and the definition of $F(z, P)$ in (29), to write

$$(2z^{EQ}-1)^2 = 4\left(z^{EQ}\right)^2 + 1 - 4z^{EQ} = Uz^{EQ}(1-z^{EQ}) \tag{A8}$$

where U is given in (31). Solving the quadratic equation in (A8) for z^{EQ} gives $z^{EQ} = (1/2)[1 + \sqrt{U/(4+U)}]$, as stated in (30). Note that we can disregard the solution $z^{EQ} = (1/2)[1 - \sqrt{U/(4+U)}]$, since it implies $z^{EQ} < \rho = 1/2$; for $\varepsilon P \geq \omega/(1-\omega) > 1$ (cf. Assumption 1) this would imply $F(z^{EQ}, P) < 0$, which is false by definition. $\|$

Proof of Proposition 12: Consider first the case when $\varepsilon P > 1$, and $k(z, q)$ is given by (35). Note that $k(z, q) = 0$ for all $z \leq \delta(q)$, so z^* must be strictly greater than $\delta(q^*)$. It also follows from (35) that $\partial k(z, q)/\partial z > 0$ for $z \in (\delta(q), \gamma(q))$, and $\partial k(z, q)/\partial z < 0$ for $z > \gamma(q)$. Thus, $k(z, q)$ is maximized when $z = \gamma(q)$. Using (35), the maximization problem then boils down to maximizing $\psi(z, q)$, subject to $z = \gamma(q)$ and $q \in [\omega, 1]$. This in turn amounts to maximizing $\pi(q) \equiv \psi(\gamma(q), q)$ over $q \in [\omega, 1]$. Using (33), (34), and some algebra, it can be seen that

$$\pi(q) = \left[\frac{(1-q)^{\frac{\rho}{1-\rho}}}{1 + (\varepsilon P)^{\frac{\rho}{1-\rho}} \left(\frac{1-q}{q}\right)^{\frac{\rho}{1-\rho}}} \right]^{1-\rho}. \tag{A9}$$

The task is now to maximize $\pi(q)$, subject to $q \in [\omega, 1]$. After some algebra, the first-order condition for an interior solution, $\pi'(q) = 0$, gives $q = \varepsilon P/(1 + \varepsilon P)$. Using (34) gives the optimal z as $\gamma(\varepsilon P/[1 + \varepsilon P]) = \varepsilon P/(1 + \varepsilon P)$. The problem has an interior solution if $\varepsilon P/(1 + \varepsilon P) \geq \omega$, or $\varepsilon P \geq \omega/(1-\omega)$. If instead $\varepsilon P \leq \omega/(1-\omega)$, then $\pi(q)$ is maximized at $q = \omega$. Thus, for $\varepsilon P \leq \omega/(1-\omega)$ the model boils down to the one where q was exogenous and equal to ω. From Proposition 5 we then recall that the optimal level of z equals ω.

Consider next the case when $\varepsilon P \leq 1$, and $k(z, q)$ is given by (36). Here we use Assumption 1, which implies that $\omega/(1-\omega) > 1$, to note that if $\varepsilon P \leq 1$, then $\varepsilon P < \omega/(1-\omega)$. This in turn implies that $\omega > \gamma(q)$ for all $q \geq \omega$. [To see this, note that $\omega > \gamma(\omega)$ is equivalent to $\varepsilon P < \omega/(1-\omega)$, and that $\gamma'(q) < 0$. Thus, if $\omega > \gamma(\omega)$, then $\omega > \gamma(q)$ for all $q \geq \omega$.] Now, from (36) we see that $\partial k(z, q)/\partial z < 0$ and $\partial k(z, q)/\partial q < 0$ for $(z, q) \in [q, 1] \times [\omega, 1]$.

Therefore $k(z, q)$ is maximized at $z = q = \omega$. $\|$

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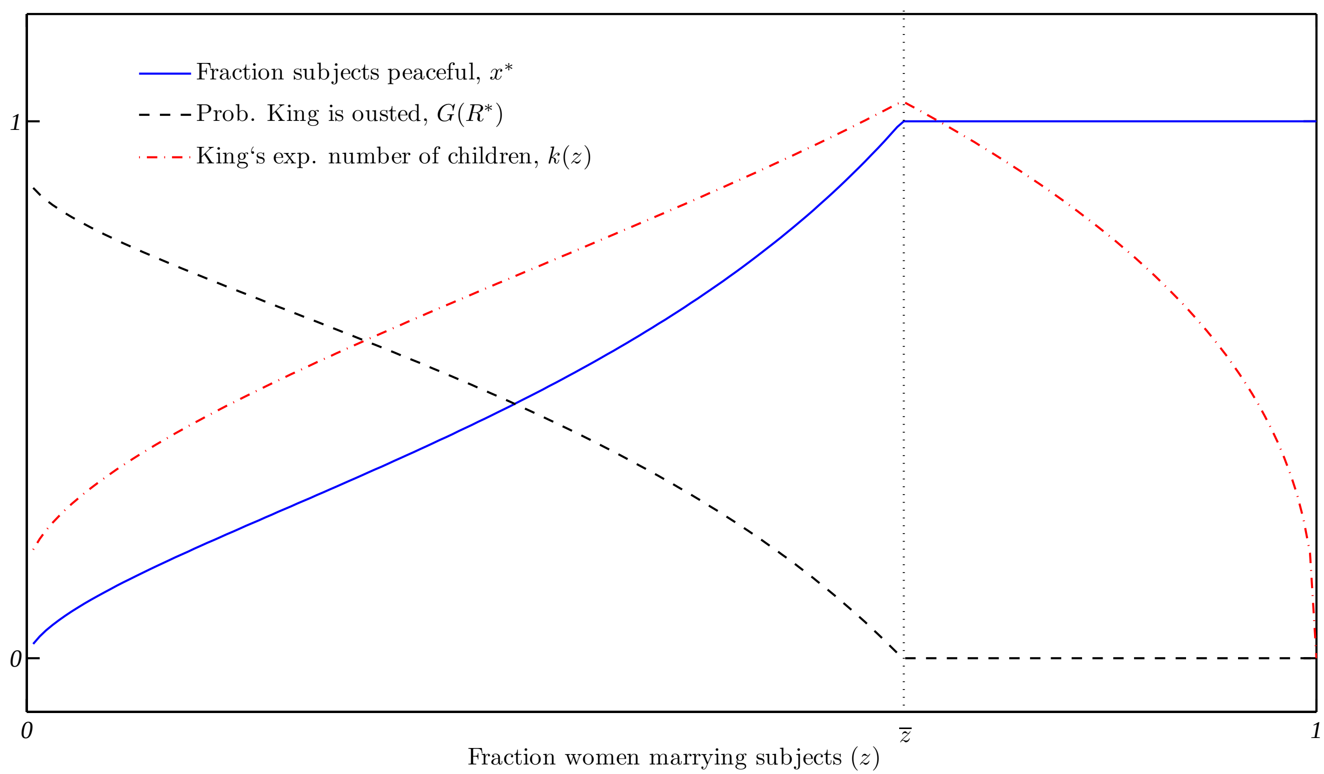

Figure 1: Illustration of the level of z that maximizes $k(z)$.

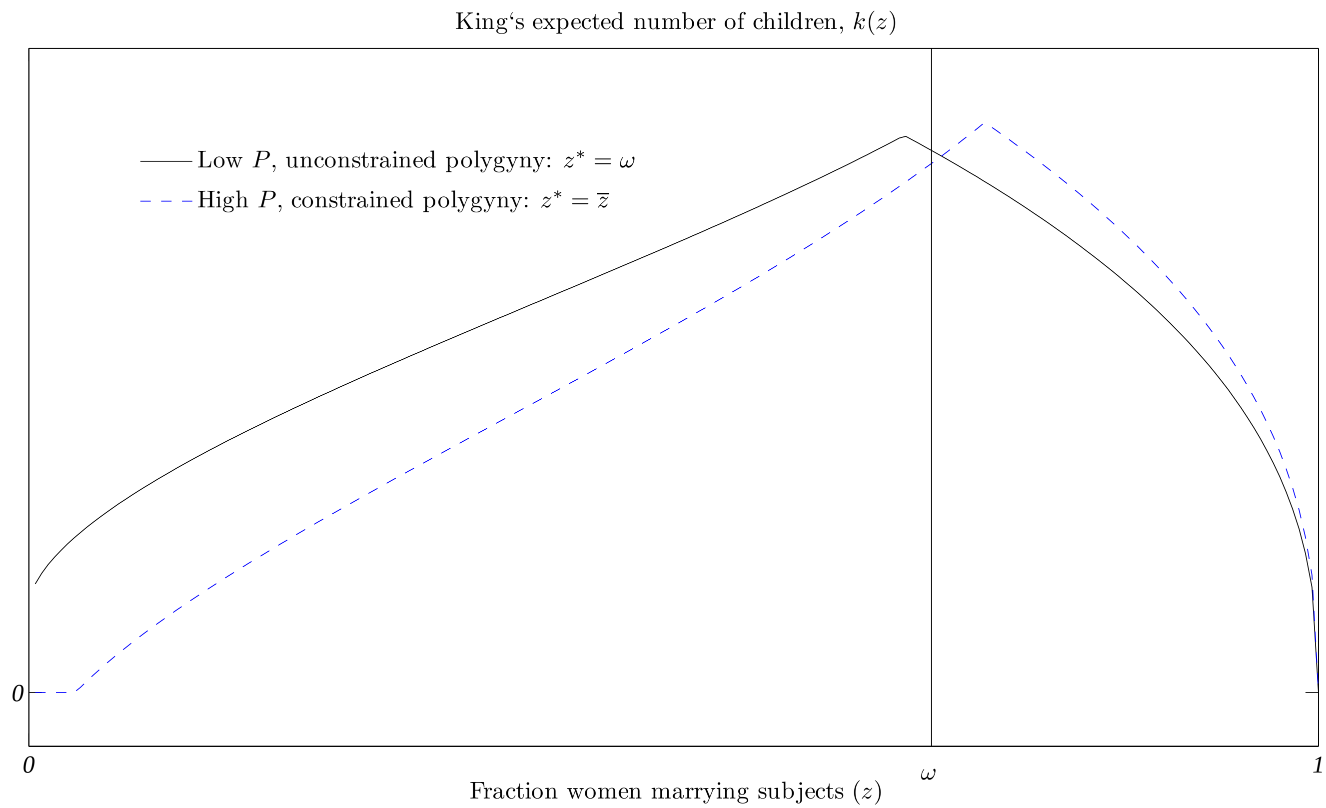

Figure 2: Illustration of constrained and unconstrained polygyny. The King chooses a z on the interval $[\omega, 1]$.

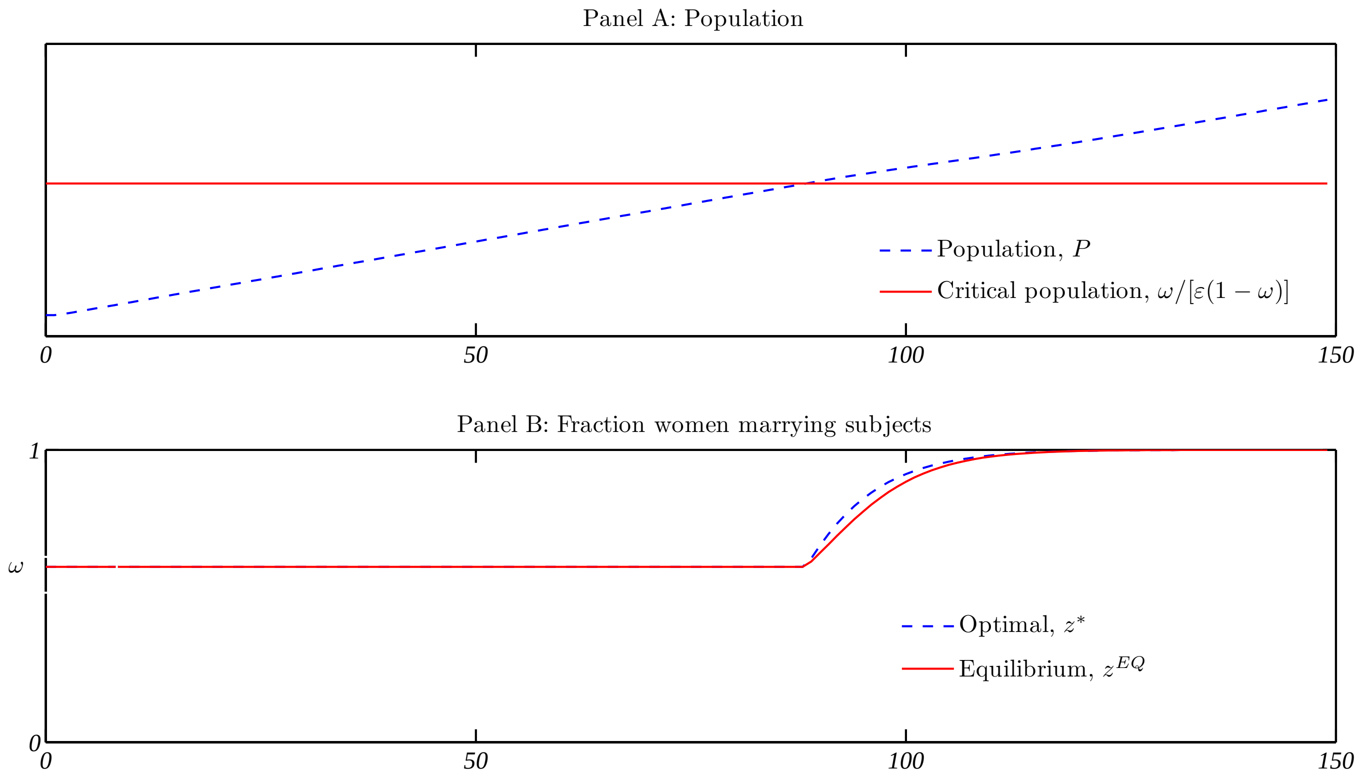

Figure 3: Simulation of the paths of P, z^*, and z^{EQ} over 150 generations.