When Bonds Matter: Home Bias in Goods and Assets*

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February 12, 2008.
Preliminary and Incomplete. Do not distribute.

Abstract

Recent models on international equity portfolios exhibit two potential caveats: 1) Portfolios are indeterminate in the presence of bonds denominated in different currencies; 2) Equity portfolios are highly sensitive to preference parameters. We show that the addition of an additional, even small, source of risk alleviates this indeterminacy and makes equity portfolios much less sensitive to the preferences when bond returns can insure fluctuations in total consumption expenditures. We compute these 'robust portfolios' for the various set-ups explored in the literature. We document two cases where bond trading cannot hedge total consumption expenditures: in presence of nominal shocks, or preference/quality shocks. We discuss the empirical importance of these two cases.

Keywords : International risk sharing, International portfolios, Home equity bias
JEL codes: F30, F41, G11

*Pierre-Olivier Gourinchas thanks the NSF for financial support (grants SES-0519217 and SES-0519242).

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1 Introduction

[Under Construction....]

The current international financial landscape exhibits two critical features. First, the last twenty years have witnessed an unprecedented increase in cross-border financial transactions. Second, despite this massive wave of financial globalization, international portfolios remain heavily tilted toward domestic assets (see table (1) in appendix). The importance of these two features has not gone unnoticed, and has generated renewed interest for the theory of optimal international portfolio allocation.[1]

An important strand of literature, launched into orbit by the influential contribution of Obstfeld and Rogoff (2000), sets out to explore the link between the allocation of consumption expenditures and optimal portfolios in frictionless general equilibrium models à la Lucas (1982).[2] One popular approach, initially developed by Baxter et al. (1998) and extended by Coeurdacier (2005), consists in characterizing the constant equity portfolio that –locally– reproduces the complete market allocation. Typically, these models achieve perfect risk sharing solely through trades in claims to domestic and foreign equities.

As emphasized by Van Wincoop and Warnock (2006) in a recent contribution, the equity position of these optimal portfolios often reflects the hedging properties of relative equity returns against real exchange rate fluctuations.[3] For instance, with Constant Relative Risk Aversion (CRRA) preferences, the optimal equity position is related to the covariance between the excess return on domestic equity (relative to foreign equity), and the rate of change of the real exchange rate. When the CRRA coefficient exceeds unity, home equity bias arises when excess domestic equity returns are positively correlated with the real exchange rate (measured as the foreign price of the domestic basket of goods, so that an increase in the real exchange rate represents an appreciation). In that case, perfect risk sharing requires that domestic consumption expenditures increase as the real exchange rate appreciates. If domestic equity returns are high precisely at that time, domestic equity provides the perfect hedge against real exchange rate risk and the optimal equity portfolio exhibits home portfolio bias. Seen in this light, most of the theoretical literature mentioned above represents a search for the 'right' correlation between relative equity returns and real exchange rate fluctuations.

This line of research faces two serious challenges. First, as shown convincingly by Van Wincoop and Warnock (2006), the empirical correlation between excess equity returns and the real exchange rate is very low, too low to explain observed home equity bias. Further,

[1]Some of that literature dates back to the early 1970s or 1980s cite: Adler and Dumas (1983), optimal portfolio balance etc...+ surveys

[2]A chronological but non-exhaustive list of contributions –some of which precedes Obstfeld and Rogoff (2000)– includes Dellas and Stockman (1989), Baxter and Jermann (1997), Baxter, Jermann and King (1998), Coeurdacier (2005), Obstfeld (2007), Kollmann (2006), Heathcote and Perri (2007), Coeurdacier, Kollmann and Martin (2007) and Collard, Dellas, Diba and Stockman (2007).

[3]A result also emphasized in the earlier, partial equilibrium literature. See Adler and Dumas (1983).

most of the fluctuations in the real exchange rate represent movements in the nominal exchange rate, so once forward currency markets are introduced, the conditional correlation between equity returns and real exchange rates disappears. This result casts a serious doubt on the ability of this class of models to quantitatively explain the home equity bias. Second, as shown initially by Coeurdacier (2005), Kollmann (2005) and Obstfeld (2007), the optimal equity portfolios are extremely sensitive to the values of preference parameters. Whether the coefficient of relative risk aversion is smaller, bigger than or equal to unity, whether domestic and foreign goods are substitute or complements, equity portfolios can exhibit home, foreign, or no bias. In the terminology of this paper, these equity portfolios are not *stable*.

This paper addresses both issues simultaneously. We argue that many of the results in this literature are not robust to the introduction of bonds denominated in different currencies. Of course, bonds are redundant in the original set-up since risk-sharing is efficient with equities only. This creates an obvious and uninteresting indeterminacy. However, this indeterminacy is lifted once we allow for an additional -possibly trivial- source of risk that perturbates equity returns, bond returns, and income. In that case, perfect risk sharing requires simultaneous holdings of equities and bonds. This additional risk factor can take many forms that cover most cases of interest: redistributive shocks, fiscal shocks, investment shocks, preference shocks, nominal shocks.....

Of course the precise form of this additional source of risk matters for optimal portfolio holdings. Regardless, our first result is to show that optimal portfolios fall into two categories, depending on the correlation between relative bond returns, relative equity returns and relative consumption expenditures. In the case where bond returns are perfectly correlated with fluctuations in total consumption expenditures, we show that optimal equity holdings do not depend on the correlation between equity returns and the real exchange rate. This corresponds to the case where the additional source of risk represents redistributive shocks, fiscal shocks, investment shocks, or nominal shocks with extreme price rigidities. The intuition for our result is simple: in that case real bonds constitute a perfect hedge for real exchange rate movements, or equivalently for total nominal consumption expenditures. When these instruments are available, optimal hedging of real exchange rate risks does not involve equity positions.

Moreover, the optimal equity portfolio takes an extremely simple expression. In particular, and in contrast to most of the existing literature, we demonstrate that the optimal equity portfolio does not depend upon the preferences of the representative household. Equivalently, optimal equity positions coincide with the equity positions of a *log-investor* who doesn't care about hedging the real exchange rate risk. The intuition again is simple to grasp. Since the task of hedging real exchange rate risk is optimally delegated to bond holdings, the investor behaves as a log-investor that does not care about real exchange rate exposure. Instead, optimal equity portfolios are designed to hedge the remaining source of risks in the economy. We show that this depends on the equilibrium correlation between equity returns and non-financial income, *conditional* on bond returns. We call these equity portfolios 'stable' portfolios.

In turn, bond positions reflect the optimal hedge for fluctuations in total consumption expenditures, as well as a hedge for the implicit real exchange rate exposure arising from the optimal equity position. Hence bond portfolios vary with the preferences of the investor.

This simple results has important empirical implications. First, in those cases where real bonds can hedge total consumption expenditures, we should not expect equity positions to be driven by real exchange rate risk. In the presence of real bonds, home portfolio bias can only arise from hedging demands other than the real exchange rate, thus both validating the van Wincoop and Warnock (2006) result and also establishing its limits. Moreover, in these cases, home equity bias arises from the remaining hedging motives. In particular, we show that home portfolio bias arises if the correlation between relative non-financial income and relative equity return, *conditional on excess bond returns*, is negative (a generalization of Baxter, Jermann and King (1997), and a generalization of Heathcote and Perri (2007)). This condition has not been tested in the literature.

We also analyze the cases where excess bond returns do not provide a perfect hedge for total nominal consumption fluctuations. This arises in two cases: (a) preference shocks similar to Coeurdacier, Kollman and Martin (2007), and (b) nominal shocks as in Lucas (1982) (flex prices) or Obstfeld's (2006) version of Matsumoto and Engel (2007) (sticky prices). In both cases, bonds will not be used in equilibrium, while equities will be used to hedge real exchange rate risk.

- in the presence of taste/quality/variety shocks, our results break down for a simple reason: the welfare-based and actual real exchange rate move in opposite directions. Following a positive taste shock that increases the demand for domestic goods, the unit price of domestic goods increases (a real appreciation) while the demand adjusted price declines (a real depreciation of the welfare based real exchange rate). Since relative bonds pay the actual real exchange rate, they do not provide a perfect hedge against fluctuations in the welfare based real exchange rate.

- In the context of nominal shocks, nominal bonds (as opposed to real bonds) allow perfect hedging of a nominal shocks, with no effect on the real allocation of resources.

While potentially restoring the results from the earlier literature in theory, we argue that these two additional sources of shocks are unlikely to do so in practice. First, we observe that nominal bonds are strongly correlated with real bonds in industrial economies, limiting the extent to which nominal bonds are unable to hedge fluctuations in total nominal expenditures. Second, to the extent that the welfare based real exchange rate differs from the actual one, we argue that this is unlikely to resolve the home equity bias puzzle. For this would require a positive correlation between relative equity returns and the unobserved welfare based real exchange rate. We argue instead that this is not the case and that demand type shocks cannot play a substantive role in explaining observed portfolios, under the maintained assumption that risk-sharing is efficient. To establish this result, we proceed in two steps:

- first we show, under the assumption that risk-sharing is efficient, how one can recover welfare based real exchange rates purely from data on nominal consumption expenditures per capita.

- we then show that -for reasonable values of the risk aversion coefficient- these welfare based real exchange rates are in fact very correlated with observed real exchange rate.

This implies that the optimal loadings of real bonds and equities are similar to the ones one would observe in a model without these shocks.

We then show that the results are robust to the introduction of non-traded goods. In fact, when non-traded goods are introduced, we show that real bonds still load on the real exchange rate. Domestic equities then hedge the additional sources of risks: fluctuations in the distribution of income, and movements in the terms of trade. As before, the overall home bias (across traded and non-traded equities) is independent of preferences. However, we now also show that the optimal holdings of traded relative to non-traded domestic equity depend upon their hedging properties of movements in the terms of trade.

Finally, the paper turns to a discussion of the optimal bond holdings. We show that the model generally predicts that domestic investors will go long in bonds denominated in their own currency. This conflicts with the empirical evidence on bond holdings as well as the evidence on the direction of transfer effects (that a country gains when the currency depreciates), showing that many industrial countries hold short bond positions in their currency and long positions in foreign currency. Nevertheless, we document that short private bond positions can arise once we introduce a non-zero government bond supply. Crucially, for this result to obtain, we show that we need government revenues to be assessed against both domestic and foreign residents, creating a departure from Ricardian equivalence.

2 A Benchmark Model with Traded Goods Only

2.1 Goods and preferences

We consider first a two-period ($t = 0, 1$) endowment economy. The benchmark model is essentially identical to Coeurdacier (2005) and Coeurdacier, Kollmann, Martin (2007). There are two symmetric countries, Home (H) and Foreign (F), each with a representative household. Each country produces one tradable good. Agents consume both goods with a preference towards the local good. In period $t = 0$, no output is produced and no consumption takes place, but agents trade financial claims (stocks and bonds). In period $t = 1$, country i receives an exogenous endowment y_i of good i. Countries are symmetric and we normalize $E_0(y_i) = 1$ for both countries, where E_0 is the conditional expectation operator, given date $t = 0$ information. A share δ of the endowment in country i is distributed to stockholders as dividend, while a share $(1 - \delta)$ is not capitalized and is distributed to households of country i. At the simplest level, one can think of the share $1 - \delta$ as representing 'labor income', but more general interpretations are also possible. At a generic level, $1 - \delta$ represents the share of output that cannot be capitalized into financial claims. This could be due to domestic financial frictions, capital income taxation or poor property right enforcement. In our symmetric setting, δ is common to both countries (see Caballero et al (2008) for a model where δ differs across countries). Once stochastic endowments are realized at period 1, households consume using the revenues from their portfolio chosen in period 0 and their endowment received in period 1.

The country i household has the standard CRRA preferences, with a coefficient of relative risk aversion σ:

$$U_i = E_0\left[\frac{C_i^{1-\sigma}}{1-\sigma}\right], \tag{1}$$

where C_i is an aggregate consumption index in period 1. C_i, for $i = H, F$ is given by:

$$C_H = \left[a^{1/\phi}\left(c_{HH}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(c_{HF}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{2}$$

$$C_F = \left[a^{1/\phi}\left(c_{FF}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(c_{FH}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{3}$$

where c_{ij} is country i's consumption of the good from country j at date 1. ϕ is the elasticity of substitution between the two goods and $a > 1/2$ represents preference for the home good (mirror-symmetric preferences).

The ideal consumer price index that correspond to these preferences is for $i = H, F$:

$$P_H = \left[a\left(p_H\right)^{1-\phi} + (1-a)\left(p_F\right)^{1-\phi}\right]^{1/(1-\phi)} \tag{4}$$

$$P_F = \left[(1-a)\left(p_H\right)^{1-\phi} + a\left(p_F\right)^{1-\phi}\right]^{1/(1-\phi)}, \tag{5}$$

where p_H (resp. p_F) denotes the price of the Home (resp. Foreign) good in terms of the numeraire.

Resource constraints are given by:

$$c_{HH} + c_{FH} = y_H \tag{6}$$

$$c_{FF} + c_{HF} = y_F. \tag{7}$$

We denote Home terms of trade, i.e. the relative price of the Home tradable good in terms of the Foreign tradable good, by q:

$$q \equiv \frac{p_H}{p_F} \tag{8}$$

2.2 Financial markets

There is trade in stocks and bonds, in period 0. In each country there is one stock a la Lucas (1978). Each stock represents a claim to a share δ of the future endowment of the tree. The supply of each type of share is normalized at unity. We assume also that there is a CPI-indexed bond in each country denominated in the composite good of country i. Buying one unit of the Home (Foreign) bond in period 0 gives one unit of the Home composite (Foreign) good at $t = 1$. Both bonds are in zero net supply.

Each household fully owns the local stock of tradable and the local stock of non-tradable, at birth, and has zero initial foreign assets. The country i household thus faces the following budget constraint, at $t = 0$:

$$p_S S_{ii} + p_S S_{ij} + b_{ii} + b_{ij} = p_S, \quad \text{with } j \neq i \tag{9}$$

where S_{ij} is the number of shares of stock j held by country i at the end of period 0, while b_{ij} represents claims (held by i) to future unconditional payments of the good j. p_S is the share price of stock that is identical for both stocks due to symmetry[4].

Market clearing in asset markets for stocks and bonds requires:

$$S_{HH} + S_{FH} = S_{FF} + S_{HF} = 1 \tag{10}$$

$$b_{HH} + b_{FH} = b_{HF} + b_{FF} = 0 \tag{11}$$

Symmetry of preferences and shock distributions implies that equilibrium portfolios are symmetric: $S_{HH} = S_{FF}$, $b_{HH} = b_{FF}$ and $b_{FH} = b_{HF}$. In what follows, we denote a country's holdings of local stock by S, and its holdings of bonds denominated in its local composite good by b. The vector $(S; b)$ thus describes international portfolios. $S > \frac{1}{2}$ means that there is equity home bias on stocks, while $b < 0$ means that the country issues bonds denominated in its local composite good, and that the country is lending in units of the foreign composite good.

2.3 Characterization of world equilibrium with efficient risk-sharing

We characterize first the equilibrium with (effectively) complete markets. As shown below, markets are complete when the number of shocks is at least equal to the number of assets. In a world with just endowment shocks, markets will be complete but portfolios will be indeterminate (i.e. the number of assets is larger than the dimension of the shocks). We will show that this is a knife-edge case and adding a tiny additional source of risk pins down both the international bond and equity portfolios.

2.3.1 Efficient consumption and relative prices

After the realization of uncertainty in period 1, the representative consumer in country i maximizes $\frac{C_i^{1-\sigma}}{1-\sigma}$ subject to a budget constraint (for $j \neq i$):

$$P_i C_i = p_i c_{ii} + p_j c_{ij} \leq I_i \qquad (\lambda_i)$$

where I_i represent the (given) total income of the representative agent in country i and λ_i is the Lagrange-Multiplier associated with the budget constraint.

[4] Bond prices are also identical due to symmetry.

The intratemporal equilibrium conditions are as follows:

$$c_{ii} = a\left(\frac{p_i}{P_i}\right)^{-\phi} C_i \tag{12}$$

$$c_{ij} = (1-a)\left(\frac{p_j}{P_i}\right)^{-\phi} C_i \tag{13}$$

Using equations (12) and (13) for both countries and market-clearing conditions for tradable goods (6) and (7) gives:

$$q^{-\phi}\Omega_a\left[(\frac{P_F}{P_H})^{\phi}\frac{C_F}{C_H}\right] = \frac{y_H}{y_F} \tag{14}$$

where $\Omega_u(x)$ is a continuous function of two variables (u, x) such that: $\Omega_u(x) = \frac{1+x(\frac{1-u}{u})}{x+(\frac{1-u}{u})}$.

When markets are complete, the ratio of Home to Foreign marginal utilities of aggregate consumption is linked to the consumption-based real exchange rate by the following, familiar condition:

$$\left(\frac{C_H}{C_F}\right)^{-\sigma} = \frac{P_H}{P_F}. \tag{15}$$

Hence, any shock that raises Home aggregate consumption relative to Foreign aggregate consumption must be associated with a Home real exchange rate depreciation. Thus, under complete markets, we can rewrite (14) as follows:

$$\frac{y_H}{y_F} = q^{-\phi}\Omega_a\left[(\frac{P_F}{P_H})^{\phi-1/\sigma}\right] \tag{16}$$

2.3.2 Budget constraints

Recall that each household holds shares S and $1-S$ of local and foreign stocks, while b denotes her holding of bonds denominated in her local good; also, stock $j's$ dividend is $p_j y_j$. The period 1 budget constraints of countries H and F are, thus:

$$P_H C_H = S\delta p_H y_H + (1-S)\delta p_F y_F + P_H b - P_F b + (1-\delta) p_H y_H \tag{17}$$

$$P_F C_F = S\delta p_F y_F + (1-S)\delta p_H y_H + P_F b - P_H b + (1-\delta) p_F y_F \tag{18}$$

These constraints imply:

$$P_H C_H - P_F C_F = [\delta\,(2S-1) + (1-\delta)](p_H y_H - p_F y_F) + 2b(P_H - P_F) \tag{19}$$

which says that the difference between countries' consumption spending equals the difference between their incomes.

2.3.3 Log-linearization of the model

Henceforth, we write $y \equiv \frac{y_H}{y_F}$ to denote relative outputs in both countries. We log-linearize the model around the symmetric steady-state where y equal unity, and use Jonesian hats ($\widehat{x} \equiv log(x/\bar{x})$) to denote the log deviation of a variable x from its steady state value $\overline{x}$.

The log-linearization of the Home country's real exchange rate $RER \equiv \frac{P_H}{P_F}$ gives:

$$\widehat{RER} = \widehat{\frac{P_H}{P_F}} = (2a-1)\widehat{q}. \tag{20}$$

Log-linearizing (16) and using (20) implies:

$$\widehat{y} = -\lambda\widehat{q} \tag{21}$$

where $\lambda \equiv \phi\left(1-(2a-1)^2\right) + \frac{(2a-1)^2}{\sigma}$. Note that $\lambda > 0$ as $1/2 < a < 1$. A relative increase in the supply of the home good ($\hat{y} > 0$) is always associated with a worsening of the terms of trade ($\hat{q} < 0$) with an elasticity $-1/\lambda$. Note that without home bias in preferences ($a = \frac{1}{2}$), λ is simply the elasticity of substitution between Home and Foreign goods (ϕ).

Note also that from (21), we get that relative equity returns $\widehat{R}$ (relative dividends) are equal to:

$$\widehat{R} = \widehat{q} + \widehat{y} = (1-\lambda)\widehat{q} \tag{22}$$

When $\lambda > 1$, an increase in relative output is associated with an improvement in relative equity returns. Conversely, when $\lambda < 1$, an increase in Home relative output is associated with a relative decrease in Home dividends. This happens when either $\phi < 1$ or the preference for the home good is sufficiently strong.[5]

We next log-linearize equation (19); using (15) we obtain:

$$\widehat{P_H C_H} - \widehat{P_F C_F} = \left(1-\frac{1}{\sigma}\right)(2a-1)\,\widehat{q} = \left[\delta\,(2S-1) + (1-\delta)\right](\widehat{q}+\widehat{y}) + 2b\,(2a-1)\,\widehat{q} \tag{23}$$

The first equality shows the Pareto optimal reaction of relative consumption spending to a change of the welfare based real exchange rate. This reaction depends on the coefficient of relative risk aversion. In a Pareto efficient equilibrium, a shock that appreciates the (welfare based) real exchange rate of country H, induces an increase in country H relative consumption spending when $\sigma > 1$ (as assumed in the analysis here). The risk-sharing condition (15) shows that when the (welfare based) real exchange rate appreciates by 1%, then relative aggregate country H consumption $\left(\frac{C_H}{C_F}\right)$ decreases by $1/\sigma$ %. Hence, efficient relative consumption spending by H $(\frac{P_H C_H}{P_F C_F})$ increases by $(1-\frac{1}{\sigma})$%. The expression to the right

[5] Specifically, when $\phi > 1$ and $\sigma > 1$ (the empirically plausible case), we need:

$$a > \frac{1}{2}\left[1 + \left(\frac{1-\phi}{\frac{1}{\sigma}-\phi}\right)^{1/2}\right]$$

of the second equal sign in (23) shows the change in country H relative income (compared to the income of F) necessary to obtain the Pareto optimal allocation. Given $\sigma > 1$, the efficient portfolio has to be such that a real appreciation (welfare based) is associated with an increase in relative spending and income.

2.4 The Instability of Optimal Equity Portfolios

Financial markets are effectively complete (up to a first order approximation) when there exists a portfolio (S, b) such that ((21) and (23) both hold for arbitrary realizations of the relative shocks $\widehat{y}$. Clearly, here portfolios are undetermined since the dimension of 'relative' shocks exceeds the dimension of 'relative assets'. Much of the literature focuses on the case where bonds are not available and efficient risk sharing is implemented with equities only (Coeurdacier (2005), Obstfeld (2006), Kollmann (2006b)).

Substituting $b = 0$ into (23), we obtain the optimal equity portfolio position:

$$S = \frac{1}{2}\left[\frac{2\delta - 1}{\delta} - \frac{\left(1 - \frac{1}{\sigma}\right)(2a - 1)}{\delta(\lambda - 1)}\right] \tag{24}$$

When $\delta = 1$, this expression coincides with the optimal equity position of Coeurdacier (2005) and Obstfeld (2006). In the more general case where $\delta < 1$, the optimal equity portfolio has two components. The first term inside the brackets represents the optimal position of a log-investor ($\sigma = 1$). As in Baxter and Jermann (1997), the domestic investor is already endowed with an implicit equity position equal to $(1 - \delta)/\delta$ through non-financial income. Offsetting this implicit equity holding and diversifying optimally implies a position $S = (2\delta - 1)/2\delta < 1/2$ for $\delta < 1$. As is well known, this component of the optimal portfolio implies foreign equity bias.

The second component of the optimal equity portfolio is a hedge against real exchange rate fluctuations. It only applies when $\sigma \neq 1$, i.e. when total consumption expenditures fluctuate with the real exchange rate. Looking more closely at the structure of this hedging component calls for a number of observations. First, this hedging demand is a complex and non-linear function of the structure of preferences summarized by the parameters σ, ϕ and a. As Obstfeld (2006) and Coeurdacier (2005) note, for reasonable parameter values, this hedging demand can contribute to home equity bias only when $\lambda < 1$, i.e. when the terms of trade impact of relative supply shocks is large.[6] The non-linearity implies that small changes in preferences can have a large impact on this hedging demand. This is most apparent if we consider the optimal portfolio in the neighborhood of $\lambda = 1$. As figures XXX make clear, small and reasonable changes in a, σ or ϕ have a large and disproportionate impact on optimal portfolio holdings, from large foreign bias ($S < 0$) to unrealistically high domestic bias ($S > 1$). To the extent that we don't know precisely what value these parameters take, one is left with the unescapable conclusion that this model does not provide enough guidance

[6] When $\lambda = 1$, this component is indeterminate since the relative return on equities is independent of the real exchange rate (and constant).

to pin down equity portfolios, or a-fortiori, explain the home portfolio bias.[7]

Second, the model implies that equities pay-offs are perfectly correlated with terms of trade and real exchange rates in all states of nature (see equation 22). This feature is quite unrealistic, as argued by Warnock and van Wincoop (2006). Indeed, in the case of the US, they show that relative equity are poorly correlated with the real exchange rate (some other citations here).

Third, given the constant sharing rule δ, the model also predicts perfect correlation between equity returns and non-financial income. While this correlation might be positive, it is hard to believe that it is perfect and many paper found it pretty low (see Fama and Schwert (1977) for earlier work and Bottazzi, Pesenti and van Wincoop (1996), Julliard (2002 and 2004), Lustig and van Nieuwerburgh (2005)[8].

Overall, we will show that the sensitivity of the equity portfolios is a knife-edge results. Once additional sources of uncertainty are introduced, and bond trading is allowed to load on the welfare based real exchange rate, equity portfolio holdings will be pinned down precisely, independently from the structure of preferences.

3 Equity and Bond Equilibrium Portfolios

This paper's main objective is to show that optimal equity portfolios are in fact stable and well defined once we introduce bonds. Of course, introducing bonds in the model of the previous section yields an indeterminacy since markets are already complete. We approach this issue by adding one additional (and orthogonal) source of uncertainty in the model. With one additional shock, and one additional asset (the bonds), the markets remain complete and we can use an extension of the previous method to characterize optimal portfolio holdings. This calls for two remarks. First, relative endowment or supply shocks are unlikely to represent the only source of uncertainty in the economy. In that sense, adding another source of uncertainty is quite general. Second, we introduce these additional shocks in a very generic fashion and show that the portfolio implications are broadly similar and fall into two categories depending on whether relative bond returns are sufficient to insure fluctuations in relative total consumption expenditures or not.

In particular, we show that optimal portfolios have the same structure when the additional source of uncertainty arises from shocks to investment, shocks to government expenditures, or redistributive shocks that affect the breakdown between financial and non-financial income. A different set of results obtains when the additional source of uncertainty represents either relative demand shocks, or nominal shocks.

[7]As emphasized by Obstfeld (2006), and as figure XXX make clear, things are even worse since the benchmark model cannot deliver home equity holdings between $S = 1 - 1/2\delta < 0.5$ and $S = 1$ thus excluding the relevant empirical range.

[8]see Baxter and Jermann (1997) for an opposite view.

3.1 Case I: Relative bond returns insure fluctuations in total consumption expenditures

Assume that a shock ε_i affects country i in period $t = 1$. Again, denote $\varepsilon = \frac{\varepsilon_H}{\varepsilon_F}$ the relative shock and assume $E_0(\varepsilon) = 1$. The only assumption we make is that the stochastic properties of ε_i are symmetric across countries and that $\widehat{\varepsilon} = \ln \varepsilon$ is not perfectly correlated with $\widehat{y}$. To characterize optimal portfolio, we only need to specify how this additional shock impacts equity returns $\hat{R}$ and non-financial income $\hat{w}$. That is, we assume the following:

$$
\begin{aligned}
\widehat{R} &= (1-\bar{\lambda})\widehat{q} + \widehat{\varepsilon} &\quad (25)\\
\widehat{w} &= (1-\bar{\lambda})\widehat{q} + \theta\widehat{\varepsilon} &\quad (26)
\end{aligned}
$$

where $\bar{\lambda}$ is a positive number ($\bar{\lambda}$ is model dependant but will be closely related to the previous λ and reflect preference parameters; see the examples below) and the shock $\hat{\varepsilon}$ is normalized by its impact on equity returns $\hat{R}$. The parameter θ, that can be positive or negative, represents the impact of $\hat{\varepsilon}$ on non-financial income. Different models will have different implications on what θ and $\bar{\lambda}$ should be, and will be explored in mode details in the next section.

3.1.1 Equilibrium Portfolios

Under the assumption -verified below- that markets are complete, the budget constraint (23) can be rewritten as follows:

$$
(1-\frac{1}{\sigma})(2a-1)\widehat{q} = \delta\,(2S-1)\,((1-\bar{\lambda})\widehat{q} + \widehat{\varepsilon}) + (1-\delta)((1-\bar{\lambda})\widehat{q} + \theta\widehat{\varepsilon}) + 2b(2a-1)\widehat{q} \quad (27)
$$

The financial market is still effectively complete (up to a first order approximation) since one can always find a portfolio (S, b) such that (27) holds for arbitrary realizations of the shocks $\widehat{y}$ and $\widehat{\varepsilon}$. Clearly, here portfolios are uniquely determined since the dimension of 'relative shocks' equals the dimension of 'relative assets'. The unique portfolio (S^*, b^*) that satisfies (27) for all realization of shocks is given by:

$$
\begin{aligned}
S^* &= \frac{1}{2}\left[1 - \frac{\theta\,(1-\delta)}{\delta}\right] &\quad (28)\\
b^* &= \frac{1}{2}\left(1-\frac{1}{\sigma}\right) + \frac{1}{2}\,(2a-1)^{-1}\left(\bar{\lambda}-1\right)(1-\delta)\,(1-\theta) &\quad (29)
\end{aligned}
$$

This is the unique portfolio compatible with efficient risk-sharing. Let's concentrate on each term in turn. The optimal equity portfolio (28) presents a number of interesting characteristics. First, and contrary to most of the literature, it does not depend on the 'tradability' of goods in consumption, as measured by a.[9] Second, it is independent of

[9] For models where the equity portfolio share depends on the preference for the home good or trade costs in goods, see Coeurdacier (2005), Kollmann (2006b), Obstfeld (2000, 2006). See section (???) for an extension of this result with non-tradable goods.

preference parameters such as the elasticity of substitution across goods ϕ or the degree of risk aversion σ. Hence the complex and non-linear dependence of optimal equity portfolios as a function of preferences disappears once we introduce trade in bonds and a genuine additional source of uncertainty that impacts both equity and non-financial income. This independence of equity positions from preference parameters implies that the optimal equity holdings would be the same for a log-investor ($\sigma = 1$). Hence our result has the simple implication that the optimal portfolio is the portfolio of the log-investor. This has two implications. First, since we know that log-investors do not care about fluctuations in the real exchange rate, what determines this optimal equity portfolio is *not* the correlation between equity returns and the real exchange rate as in Warnock and van Wincoop (2006). Second, the portfolio of the log-investor is in general easier to characterize, hence potentially simplifying the calculation of optimal portfolios.

Instead, the optimal equity holdings insulate total income (both financial and non-financial) from the $\widehat{\varepsilon}$ shocks only. Conditional on the terms of trade (or equivalently on the real exchange rate or relative bond returns), the domestic investor is endowed with an implicit equity exposure through the impact of the $\hat{\varepsilon}$ shock on nonfinancial income, equal to $\theta(1-\delta)/\delta$. Offsetting this implicit conditional equity position and diversifying optimally implies a position $S^* = 0.5\left(1 - \theta(1-\delta)/\delta\right)$. When $\theta = 0$, so that the implicit conditional exposure is zero, the optimal equity portfolio is perfectly diversified: $S^* = 0.5$. More generally, for equity portfolio holdings to exhibit home bias requires a negative θ, i.e. a negative covariance between non-financial and financial income (conditional on the real exchange rate, or equivalently conditional on bond returns). A couple of remarks are necessary at this stage. First, a negative θ arises naturally when the additional shock reallocates income between its financial and nonfinancial components. We will show below that this occurs with redistributive shocks but also with investment shocks (as in Heathcote and Perri) or government expenditures.

Second, and more importantly, it is obvious that this invalidates the results of much of the previous literature that emphasized the hedging properties of equity returns for real exchange rate risk. In particular, in our model, home portfolio bias can arise independently of the correlation between equity returns and the real exchange rate. The finding that $cor\left(\hat{R}, \hat{q}\right) = 0$, as emphasized by Warnock and van Wincoop (2006), has no bearing on the optimal portfolio holdings. Instead, equity portfolio bias arises only when $cor_{\hat{q}}\left(\hat{R}, \hat{w}\right) = \theta < 0$, a condition that has not been looked at in the empirical literature (see section XXX). Moreover, in our set-up, since nominal and real bonds coincide perfectly, we would find by construction a zero correlation between relative equity returns and real exchange rates conditional on bond returns, a finding consistent with the empirical results of Warnock and van Wincoop (2006).

Since our results are so different from the previous literature, one is entitled to wonder why the optimal equity portfolio in (28) isn't loading on the real exchange rate? After all, (25) shows that relative equity returns fluctuate with the terms of trade, or equivalently with the real exchange rate? The answer is that real exchange rate risk is best taken care of through bond holdings since the latter load perfectly on the real exchange rate, and not on the $\widehat{\varepsilon}$ shocks. Hence, once the $\widehat{\varepsilon}$ shocks have been hedged by equity positions, the bond portfolio will be structured such that financial and non-financial income have the appropriate exposure to real exchange rate changes.

Looking at (29), we can decompose this optimal bond portfolio as the sum of two components. The second term on the right hand side represents the bond portfolio of the log-investor (term $(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\theta)$). This term represents a hedge for the implicit real exchange rate exposure of relative income arising from the optimal equity position and non-financial income. The log-investor wants to neutralize the exposure of his total income to real exchange movements. It does so by structuring his bond portfolio such that any capital gains on financial and non-financial incomes are offset by capital losses on the bond portfolio. To understand this result, consider a combination of shocks that leads to a 1% increase in the Home terms-of-trade.[10] Given (25) and (27), relative equity returns and non-financial incomes changes are equal to $(1-\bar{\lambda})\%$. At the optimal equity portfolio (S^*), capital gains/losses on equity positions and non-financial incomes for the Home investor (relative to the Foreign one) are equal to $(\bar{\lambda}-1)[\delta\,(2S^*-1)+(1-\delta)]\% = (\bar{\lambda}-1)(1-\delta)(1-\theta)\%$. In these states of the world, Home bond excess returns over Foreign bonds are equal to $(2a-1)\%$.[11] Then, holding $b = \frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\theta)$ Home bonds and $(-b)$ Foreign bonds generates capital gains/losses on the bond position necessary to insulate relative incomes from real exchange rate changes.

The first term on the right hand side of (29) is the optimal hedge for fluctuations in total consumption expenditures when $\sigma \neq 1$ (the term $\frac{1}{2}\left(1-\frac{1}{\sigma}\right)$). Investors more risk averse than the log-investor want to have a positive exposure of their incomes to real exchange rate changes. They do so by increasing their holding of Home bonds (and decreasing their holdings of Foreign bonds) since Home bonds have higher pay-offs when the real exchange rate appreciates.[12]

Overall, the sign of the bond position in (29) depends upon the relative strength of the two components. The hedge for fluctuations in total consumption expenditures always leads to a long position ($b^* > 0$) in the domestic bond for the plausible case $\sigma > 1$. The sign of the log-investor position depends upon the sign of $\bar{\lambda}$ and θ. When $\bar{\lambda} > 1$ and $\theta < 1$ this component is also positive. Otherwise, this component will become negative. We explore in section XXX the empirical implications of the model for bond holdings.

Finally, notice that the bond portfolio depends upon preference parameters σ, a and potentially $\bar{\lambda}$ in a complex and non-linear way. A natural question then, is whether this bond portfolio inherits the instability of the equity portfolio of the previous model. To answer this question requires that we flesh out some of the details of the model, as we do next.

[10] Of course in this model, terms-of-trade are endogenous but it is always possible to find a combination of shocks that leads to a 1% increase in the Home terms-of-trade.

[11] Since the real exchange rate appreciation following a 1% increase in the Home terms-of-trade is $(2a-1)\%$.

[12] This result is closely related to Adler and Dumas (1983) and Krugman (1980).

3.1.2 Examples

Redistributive shocks

The distribution of total income between financial and non-financial income is controlled by the parameter δ. Variations in δ redistribute income from its financial to non-financial components or vice versa. If we interpret non-financial income as labor income, shocks to δ affect the labor share of total income. Fluctuations in the labor share can occur in a model where capital and labor enter into the production function with a non-unit elasticity in presence of capital and labor augmenting productivity shocks (REFERENCES?). Alternatively, movements in the labor share occur if we move away perfect competition in the goods markets and firm profits are shared between shareholders and workers based on their (stochastic) bargaining power or reservation utility (REFERENCES).

In terms of the previous set-up, we can interpret ε_i as shocks to the share that his distributed as dividend, with $E_0(\varepsilon_i) = \delta$. One can verify that financial and non-financial incomes satisfy:

$$\widehat{R} = (1-\lambda)\widehat{q} + \widehat{\varepsilon} \tag{30}$$

$$\widehat{w} = (1-\lambda)\widehat{q} - \frac{\delta}{1-\delta}\widehat{\varepsilon} \tag{31}$$

Then, the optimal portfolio can be easily derived from (28) and (**??**) with $\theta = -\delta/(1-\delta)$:

$$S^* = 1 \tag{32}$$

$$b^* = \frac{1}{2}(1-\frac{1}{\sigma}) + \frac{1}{2}(2a-1)^{-1}(\lambda-1) \tag{33}$$

The implications for portfolios are similar to Coeurdacier, Kollmann, Martin (2007). Since purely redistributive shocks only affect the distribution of total output, but not its size, the optimal hedge is for the representative domestic household to hold all the domestic equity. This perfectly offsets the impact of $\hat{\varepsilon}$ shocks on total income. The equity portfolio exhibits full equity home bias. Once the redistributive shocks have been hedged by holding local equity, the bond portfolio takes care of the exposure of incomes to real exchange rate movements. Notice that this result does not depend upon the size of the redistributive shock: even an infinitesimal amount of redistributive variation leads to full equity home bias.[13]

The bond position is negative when $\lambda < 1 - (1-\frac{1}{\sigma})(2a-1)$ and positive otherwise. A negative bond position (borrowing in domestic bonds and investing in foreign bonds) is possible only for sufficiently low values for λ. This condition echoes the condition for home equity bias in the equity only model of section XX. However, unlike (**??**) inspection of (33) reveals that the optimal bond positions are nicely behaved as a function of the underlying preference parameters (for $\sigma > 1$ and $a > 0.5$). Figure XXX reports the variation

[13]This argument is only valid up to the first order approximation. Redistributive shocks smaller than that first order would drop out from the derivation and portfolios would once again become essentially indeterminate.

in b^* in the range XXX. Hence, unlike equity positions in the equity only model (see figure XXX), the portfolios positions vary smoothly with preferences parameters. This implies that uncertainty about the true preference parameters translates into uncertainty of the same order regarding optimal portfolio positions.

Government expenditures shocks

Government spending shocks constitute another potential source of uncertainty. They break the link between private consumption and output and can also affect revenues from both financial and non-financial incomes depending on the way fiscal expenditures are financed. This will also severe the link between the real exchange rate and relative equity returns *net of taxes*.

Assume that in each country i, a government must finance period 1 government expenditures $E_{G,i}$ equal to $P_{g,i}G_i$, where G_i is the aggregate consumption index of the government and $P_{g,i}$ is the price index for government consumption, potentially different from the price index for private consumption. G_i is stochastic and symmetrically distributed, with $E_0(G_i) = \bar{G}$.

We denote $E_G = \frac{P_{g,H}G_H}{P_{g,F}G_F}$ the ratio of Home to Foreign government expenditures and $\widehat{E_G}$ the log deviation from its steady-state symmetric value of one.

Preferences of the government are similar to that of the consumers:[14]

$$G_H = \left[a_g^{1/\phi}(g_{HH})^{(\phi-1)/\phi} + (1-a_g)^{1/\phi}(g_{HF})^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{34}$$

$$G_F = \left[a_g^{1/\phi}(g_{FF})^{(\phi-1)/\phi} + (1-a_g)^{1/\phi}(g_{FH})^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \tag{35}$$

where g_{ij} is country i government's consumption of the good from country j in period 1 and $a_g > 1/2$ represents the preference for the home good of the government (mirror-symmetric preferences) that may differ from the bias in household preferences ($a_g \neq a$).

Government expenditures in country $i = \{H, F\}$ are financed through taxes on financial income (for a share δ_g), $T_{R,i} = \delta_g E_{G,i}$, and through taxes on non-financial incomes (for a share $(1-\delta_g)$), $T_i^w = (1-\delta_g)E_{G,i}$, so as to ensure budget balance in period 1.

Market-clearing conditions for both goods are now:

$$c_{HH} + c_{FH} + g_{HH} + g_{FH} = y_H \tag{36}$$

$$c_{FF} + c_{HF} + g_{FF} + g_{HF} = y_F. \tag{37}$$

Following similar steps as before, relative demand of Home over Foreign goods by governments ($y_G = (g_{HH} + g_{FH}) / (g_{HF} + g_{FF})$) satisfies (in log-linearized terms):

$$\widehat{y_G} = -\lambda_g \widehat{q} + (2a_g - 1)\widehat{E_G} \tag{38}$$

[14] One can also allow for a different elasticity of substitution between Home and Foreign goods for government consumption. This extension is straightforward and does not add much substance.

where $0 \leq \lambda_g = \phi(1-(2a_g-1)^2)+(2a_g-1)^2 \leq \phi$ represents the impact of fluctuations in the terms of trade on relative government consumption, after controlling for relative expenditures $\widehat{E_G}$.

Relative demand of Home over Foreign goods by consumers $(y_C = (c_{HH}+c_{FH})\,/\,(c_{HF}+c_{FF}))$ still satisfies equation (21) since the private allocation across goods has not changed:

$$\widehat{y_C} = -\lambda\widehat{q} \tag{39}$$

Equation (38) and (39) together with market clearing conditions of both goods (36) and (37) implies the following equilibrium on the goods market:

$$\widehat{y} = (1-s_G)\widehat{y_C} + s_G\widehat{y_G} = -\bar{\lambda}\widehat{q} + s_G(2a_g-1)\widehat{E_G} \tag{40}$$

where s_G is the steady-state ratio of government spending over GDP and $\bar{\lambda} = (1-s_G)\lambda + s_G\lambda_g$. Note that, intuitively, efficient terms-of-trade $\widehat{q}$ are decreasing with the relative supply of goods $\widehat{y}$ (with an elasticity $1/\bar{\lambda}$) and increasing with relative government expenditure shocks (due to the presence of government home bias in preferences a_g), which act as relative demand shocks in this set-up.

This gives the following relative equity returns and non-financial incomes net-of-taxes:

$$\begin{aligned} \widehat{R} &= (1-\bar{\lambda})\widehat{q} + s_G(2a_g - 1 - \frac{\delta_g}{\delta})\widehat{E_G} && (41)\\ \widehat{w} &= (1-\bar{\lambda})\widehat{q} + s_G(2a_g - 1 - \frac{1-\delta_g}{1-\delta})\widehat{E_G} && (42)\end{aligned}$$

Direct inspection of the returns reveals that in general markets are complete and that the system (41)-(42) is similar to (25)-(26), where:[15]

$$\begin{aligned} \widehat{\varepsilon} &= s_G(2a_g - 1 - \frac{\delta_g}{\delta})\widehat{E_G} && (43)\\ \theta &= \frac{2a_g - 1 - \frac{1-\delta_g}{1-\delta}}{2a_g - 1 - \frac{\delta_g}{\delta}} = -\frac{\delta}{1-\delta}\left(1 - \frac{2\,(1-a_g)}{2\,(1-a_g)\,\delta - (\delta-\delta_g)}\right) && (44)\end{aligned}$$

The impact of the fiscal shock on relative equity returns depends on the fluctuations in relative government expenditures $\widehat{E_G}$, as well as the government preferences for the home good a_G, the steady state share of government expenditures in output s_G, and the relative fiscal incidence of the shocks δ_G/δ. In turn, the incidence of fiscal shocks on non-financial income depends only on a_g and δ_g/δ. Importantly, θ does not depend upon the preferences of the representative household, only on the preferences of the government in terms of consumption (a_g) and taxation (δ_g). Notice that in the case where government consume only

[15] The exception is the very peculiar case where $2a_g = 1 + \delta_g/\delta$. In that case, government expenditures shocks do not modify equity returns conditionally on bond returns and then cannot be hedged perfectly. This rules out the case where government expenditures fall entirely on the domestic good $(a_G = 1)$ and the fiscal incidence is equally distributed on financial and non-financial income $(\delta_G = \delta)$.

domestic goods ($a_G = 1$), $\theta = -\delta / (1-\delta)$, as in the case of purely redistributive shocks.[16] In the case where $\delta_g = \delta$, $\theta = 1$.

In this set-up, (27) needs to be slightly modified since private consumption in steady-state does not equal total consumption. (27) can be rewritten as follows:

$$(1-s_G)(1-\frac{1}{\sigma})(2a-1)\widehat{q} = \delta\,(2S-1)\,((1-\bar{\lambda})\widehat{q}+\widehat{\varepsilon})+(1-\delta)((1-\bar{\lambda})\widehat{q}+\theta\widehat{\varepsilon})+2b(2a-1)\widehat{q} \quad (45)$$

With the value of θ defined in (44), equilibrium portfolios are given by:

$$S^* = \frac{1}{2}\left(1-\frac{\theta\,(1-\delta)}{\delta}\right) \quad (46)$$

$$b^* = \frac{1}{2}(1-s_G)(1-\frac{1}{\sigma})+\frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\theta) \quad (47)$$

Once again, portfolios are uniquely determined and the equity portfolio is independent from consumer preferences (ϕ, σ and a). Note that here, we have restricted ourselves to cases where the *marginal* and *average* shares of taxes on financial and non-financial income in total fiscal revenies are the same (and equal to δ_g and $1-\delta_g$ respectively). However, what matters for equity portfolios is how marginal changes in government expenditures are financed, not how they are financed on average. So δ_g must be understood as the contribution of taxes on financial income to finance a marginal increase in government expenditures.

While optimal equity portfolio are independent from household preferences, they depend on government preferences (a_G and δ_g) through θ. Some specific calibrations of the parameters help to understand the equity portfolio.

- When $\delta_g = \delta$, *i.e* when increases in government expenditures fall on financial incomes proportionally to their share in gross GDP, $\theta = 1$ and the equity portfolio is the one of Baxter and Jermann (1997); in particular, investors exhibit foreign bias in equities.

$$S^* = \frac{1}{2}\frac{2\delta-1}{\delta} \quad (48)$$

  The reason is simple. Conditionally on relative bond returns, shocks to Home government expenditures exactly decrease Home equity returns and Home labor incomes by the same amount, making financial and non financial incomes perfectly correlated. Being over-exposed on government expenditures shocks due to their non financial incomes, investors will reduce their holdings of local stocks and increase their holdings of foreign stocks to share optimally government expenditures risks.

- When $\delta_g = 1$, *i.e* changes in government expenditures are entirely financed by taxes on financial incomes, $\theta = \frac{2a_g-1}{2a_g-1-\frac{1}{\delta}}$ and the equity portfolio is:

$$S^* = \frac{1}{2}\left[1+\frac{(2a_g-1)(1-\delta)}{1-\delta(2a_g-1)}\right] > \frac{1}{2} \quad (49)$$

[16]Although in that case $\bar{\lambda} = (1-s_G)\,\lambda + s_G$.

The equity portfolio always exhibits Home bias and is very similar to the one described in Heathcote and Perri (2007). Here, government expenditures play the same role as investment in their set-up. Indeed, conditional on bond returns, an increase in Home government expenditures decreases dividends net of taxes at Home and raises wages by raising the relative demand for Home goods. To neutralize the effect of government expenditures on private consumption expenditures, the Home investor biases its portfolio towards local stocks. When Home government expenditures are high, so are wages but this increase in wages is offset by capital losses on the equity portfolio, leaving consumption expenditures unchanged.

- When $a_g = 1$, *i.e* government expenditures are fully biased towards local goods, the equity portfolio is fully biased towards local stocks and $S = 1$.[17] The reason is simple, from (40), a 1% increase in Home government expenditures raises Home dividends and Home non-financial income before taxes by s_G%. With a portfolio fully biased towards local equity, the Home investor will have an increase of taxes of s_G%. Then, such a portfolio insulates completely his revenues from changes in government expenditures (and taxes) and allow efficient risk-sharing of government expenditures shocks.

Investment shocks

Like government spending, investment breaks the link between private consumption and output; moreover, when dividends are reinvested, the link between contemporary dividends and wages is also broken. This is another potential candidate to solve the indeterminacy. However, our two-period model is not very convenient to model investment and a dynamic model with endogenous investment would be more appropriate. We will use a short cut for investment in our set-up by assuming some exogenous sunk costs that have to be financed out of dividends in period 1 in order to produce. This exogenous sunk-cost I_i in country i is stochastic (symmetric distribution) with equal mean in both countries ($E_0(I_i) = I$) and represents shocks to investment expenditures. The sunk costs in both countries are paid using a constant fraction of local and foreign goods: we assume that in country i, a share a_I of the sunk cost I_i is spent on goods i (and a share $(1 - a_I)$ on foreign goods). We assume $a_I \geq \frac{1}{2}$ to model some bias in investment expenditures towards local inputs (as in Heathcote and Perri (2007)).

We denote $I = \frac{I_H}{I_F}$ the ratio of Home over Foreign sunk costs (relative investment expenditures) and by $\widehat{I}$ the deviation from its steady state-value of one.

From a portfolio perspective, these stochastic sunk costs are isomorphic in a dynamic set-up to stochastic changes of investment that might be driven by investment specific technological change (Greenwood, Hercowitz and Krusell (1997)), changes in the depreciation rate or some stochastic 'news' about future productivity (Cochrane (1994), Beaudry and Portier (2003)). One can show that the structure of steady-state (time-invariant) portfolios

[17] Again, this is true except for a unique calibration where equity and bonds have the same pay-offs, which occurs here when $\delta_g = \delta$.

would be the same in a dynamic model where investment is affected by some shocks that are not fully correlated with contemporaneous productivity shocks (as long as financial assets available are similar to the ones of this model.

The ratio of investment spending on Home goods relative to Foreign goods ($q\frac{y_{I,H}}{y_{I,F}}$) depends only on the relative sunk cost across countries:

$$q\frac{y_{I,H}}{y_{I,F}} = \frac{a_I I_H + (1-a_I)I_F}{a_I I_F + (1-a_I)I_H} = \Omega_{a_I}\left[\frac{I_F}{I_H}\right] \tag{50}$$

Or in log-linearized terms:

$$\widehat{y_I} = -\widehat{q} + (2a_I - 1)\widehat{I} \tag{51}$$

As before, relative demand of Home over Foreign goods by consumers (y_C) satisfies equation (21):

$$\widehat{y_C} = -\lambda\widehat{q} \tag{52}$$

Like for government expenditures, equation (51) and (39) together with market clearing conditions of both goods implies the following equilibrium on the goods market:

$$\widehat{y} = (1-s_I)\widehat{y_C} + s_I\widehat{y_I} = -\bar{\lambda}\widehat{q} + s_I(2a_I - 1)\widehat{I} \tag{53}$$

where s_I is the steady-state ratio of investment expenditures over GDP and $\bar{\lambda} = (1-s_I)\lambda + s_I$. Note that like for government expenditures, efficient terms-of-trade $\widehat{q}$ are decreasing with the relative supply of goods $\widehat{y}$ and increasing with shocks to relative sunk costs (due to $a_I \geq \frac{1}{2}$). Higher sunk costs at home crowds out Home goods for private consumption and raises their relative price.

This gives the following relative equity returns (net of the financing of sunk costs) and non-financial incomes:

$$\widehat{R} = (1-\bar{\lambda})\widehat{q} + s_I(2a_I - 1 - \frac{1}{\delta})\widehat{I} \tag{54}$$

$$\widehat{w} = (1-\bar{\lambda})\widehat{q} + s_I(2a_I - 1)\widehat{I} \tag{55}$$

Hence the model with investment shocks is isomorphic to the model with government expenditures in the case where government expenditures are financed entirely from financial incomes ($\delta_g = 1$). Then, obviously, we are back to a system similar to equations (25) and (26) where:

$$\widehat{\varepsilon} = s_I(2a_I - 1 - \frac{1}{\delta})\widehat{I} \tag{56}$$

$$\theta = \frac{2a_I - 1}{2a_I - 1 - \frac{1}{\delta}} \tag{57}$$

and the equilibrium equity and bond portfolios are:

$$S^* = \frac{1}{2}[1 + \frac{(2a_I - 1)(1-\delta)}{1-\delta(2a_I - 1)}] \tag{58}$$

$$b^* = (1-s_I)(1-\frac{1}{\sigma}) + (2a-1)^{-1}(\bar{\lambda}-1)[\delta\,(2S^*-1) + (1-\delta)] \tag{59}$$

As noted above, the optimal portfolio have exactly the same structure as the portfolio with government shocks financed out of dividends. This is very intuitive since sunk costs act here as a tax on dividends. Moreover, as previously, in presence of bias towards local inputs ($a_I \geq \frac{1}{2}$), an increase in investment expenditures at Home raises Home wages while decreasing Home dividends. Consequently, an equity portfolio biased towards Home equity insulates consumption expenditures from changes in investment expenditures.

Note that the equity portfolio in (58) is exactly the one described by Heathcote and Perri (2007). The similarity deserves a bit of attention. Heathcote and Perri (2007) have a dynamic model with investment and only one source of uncertainty, productivity shocks. Then, obviously, in their model, real bonds are redundant and financial markets are complete with equities only.

Why is their portfolio similar to ours then? The reason is that they assumed very specific preferences. More specifically, they assume a unit elasticity of substitution between Home and Foreign goods ($\phi = 1$), as well as logarithmic preferences over consumption ($\sigma = 1$).

One can easily check that these preference assumptions imply $\bar{\lambda} = 1$ and $b^* = 0$ in our model. So with their calibration or preferences, bonds holdings are zero in our model. There is no need to hedge the real exchange rate risk when $\sigma = 1$, and there is no exposure of relative equity returns to the real exchange rate when $\bar{\lambda} = 1$.

In effect, their calibration makes their model completely isomorphic to a model where the only source of risk is associated with investment shocks. Of course, as soon as $\sigma, \phi \neq 1$, total consumption expenditures will fluctuate with the real exchange rate risk and relative equity returns will load on the real exchange rate too. With one source of uncertainty, equity portfolios will hedge simultaneously the real exchange rate and investment risks. However, these optimal equity portfolios will be quite sensitive to the preference parameters as illustrated in section XXX and potentially far away from their benchmark calibration in (58).

What our results show is that the initial Heathcote and Perri (2007) equity portfolios are in fact much more robust than previously thought once (a) investment shocks are not perfectly correlated with output shocks and (b) we introduce real bonds.

The three preceding examples constitute a broad class of shocks. Yet they share the same feature that optimal equity positions will be 'robust' to changes in household preferences. This is an important result and it is important to understand why this is so. There are two main reasons:

1. Changing the risk aversion induces a change in the exposure of total consumption expenditures to the real exchange rate (the left hand side of (27). This is optimally taken care of by bonds holdings (term $\frac{1}{2}\left(1 - s_i\right)\left(1 - \frac{1}{\sigma}\right)$ where $i \in \{G, I\}$).

2. Changing the elasticity of substitution across goods changes the response of the terms of trade to output shocks ($\bar{\lambda}$). This will be also taken care of by optimal bond holdings in our model (term $\frac{1}{2}(2a-1)^{-1}(\bar{\lambda}-1)(1-\delta)(1-\theta)$).

As a consequence equity portfolio are independent on the preferences.

3.2 Case II: Relative bond returns do not insure fluctuations in total consumption expenditures

Our previous results hinge on the key-assumption that bond returns differential across countries load *perfectly* on the real exchange rate. In practice, this might not be true for at least two reasons: first, real bonds might not exist in practice.[18] Most bonds available to investors are nominal and nominal bonds returns differential across countries might not load perfectly on the real exchange rate in presence of nominal shocks. While nominal bonds may load pretty well on the real exchange rate in practice (see section XXX for some evidence), one might still want to know what are the predictions of our benchmark model in presence of nominal shocks.

Second, even in the absence of nominal shocks, the bond return differential might not load perfectly on the *welfare-based* real exchange rate, the one that matters from the investor's point of view. This happens for instance in presence of shocks to the quality of goods (or equivalently changes in the number of varieties available to consumers) as in Corsetti, Martin, Pesenti (2007) or Coeurdacier, Kollmann, Martin (2007).

We explore these two cases sequentially. But we first illustrate these cases in a generic reduced-form model where bond returns are allowed to deviate from the welfare based real-exchange rate.

3.2.1 Equilibrium Portfolios

We keep the benchmark model under complete markets, ignoring any additional source of risk on relative equity returns. This gives the following set of equations for the efficient terms-of-trade, relative equity returns and relative non-financial incomes (see section 2.3):

$$\begin{aligned} \widehat{y} &= -\lambda\widehat{q} && (60) \\ \widehat{R} &= (1-\lambda)\widehat{q} && (61) \\ \widehat{w} &= (1-\lambda)\widehat{q} && (62) \end{aligned}$$

The (welfare-based) real exchange rate is still defined by the following equation:

$$\widehat{RER} = (2a-1)\widehat{q} \tag{63}$$

We now assume that bonds returns are not fully indexed on the (welfare-based) price index of the local composite good. In other words, bond returns differential across countries R_b do not exactly load on the real exchange rate. We add a potentially small source of risk ε on bond returns differential; ε is stochastic with a symmetric distribution across country and we assume $E_0(\varepsilon) = 1$. We denote $\widehat{\varepsilon}$ its deviation from its steady-state value. Then, bond returns differentials across countries R_b can be expressed as follows (in log deviation):

$$\widehat{R_b} = (2a-1)\widehat{q} + \widehat{\varepsilon} \tag{64}$$

[18] Note that hey do exist in most developed markets: US, Euro zone, UK, Sweden are some well known examples were inflation-indexed bonds have been created. See ....

As justified in the examples below, $\widehat{\varepsilon}$ can be due to the presence of nominal risk (bonds are nominal and not real) or by changes in the quality of goods and/or preference shocks (see Coeurdacier, Kollmann, Martin (2007)).

Under the maintained hypothesis that markets are complete, the relative budget constraint (23) becomes:

$$(1-\frac{1}{\sigma})(2a-1)\widehat{q}=\delta\,(2S-1)\,(1-\lambda)\widehat{q}+(1-\delta)(1-\lambda)\widehat{q}+2b((2a-1)\widehat{q}+\widehat{\varepsilon}) \tag{65}$$

Financial markets are still effectively complete given that the representative investor still has two 'relative assets' to hedge two 'relative shocks'. Note that in this set-up, changes in relative incomes due to capital gains and losses on bond return differentials are not purely driven by changes in the real exchange rate. Since in turn relative equity returns load perfectly on the real exchange rate but bonds do not (due to $\widehat{\varepsilon}$), portfolios will be unique since the two 'relative assets' do not have the same pay-offs in all states of nature.

Moreover, equities will be used to hedge changes in relative consumption expenditures and real exchange risk, contrary to bonds. The optimal equity position will be the same as in (**??**) while bond positions will be zero:

$$S^{*} = \frac{1}{2}\left[\frac{2\delta-1}{\delta}-\frac{\left(1-\frac{1}{\sigma}\right)(2a-1)}{\delta\,(\lambda-1)}\right] \tag{66}$$

$$b^{*} = 0 \tag{67}$$

The equity portfolio shares the same difficulties as in previous literature: it is highly dependant on preference parameters and involves for most parameter values shorting Home or Foreign equities. Bond holdings are zero to insulate relative consumption expenditures from changes in $\widehat{\varepsilon}$.

3.2.2 Examples

The case of nominal shocks

Following Obstfeld (2006) and Engel and Matsumoto (2006), we add money in our benchmark model by assuming that money enters the utility function. To simplify matters, we assume that consumption and real money balances are separable in the utility function. The expected utility at date 0 of a representative agent in country i is now:

$$U_i = E_0\left[\frac{C_i^{1-\sigma}}{1-\sigma}+\chi log(\frac{M_i}{P_i})\right] \tag{68}$$

where M_i denotes money holdings in country i by agent i, χ is a positive parameter. We assume a log- utility for real money balance to simplify some of the calculus but our main results are essentially unaffected by this assumption.[19]

[19] Supposing CRRA utility in money generates an additional hedging demand in the portfolio that goes towards zero once we converge towards a cashless economy.

Nominal shocks will be simply shocks to the money supply M_i in country i. We introduce $M = \frac{M_H}{M_F}$ the relative money supply and $\widehat{M}$ its deviation from its steady state value of one.

Bonds in country i are nominal bonds that pay one unit of country i's currency. s is the nominal exchange rate, defined as the number of Foreign currency units per unit of Home currency. A rise in s represents a nominal appreciation of the Home currency. We express all variables in Home currency terms. We assume that $E_0(s) = 1$ and denotes $\widehat{s}$ the deviations of the nominal exchange rate from its steady-sate value.

The log-linearization of the Home country's real exchange rate $RER \equiv \frac{sP_H}{P_F}$ gives:

$$\widehat{RER} = \widehat{\frac{sP_H}{P_F}} = (2a-1)\widehat{q} \tag{69}$$

where $\widehat{q} = \widehat{\frac{s\ p_H}{p_F}}$ denotes the Home terms-of-trade and p_i is now the price of good i in units of currency i. Note that an increase in the Home terms-of-trade is an appreciation of the Home real exchange rate.

With only relative nominal shocks ($\widehat{M}$) and relative output shocks ($\widehat{y}$) and two 'relative assets' (stocks and nominal bonds), markets will still be (locally) complete and the usual Backus and Smith condition will obtain (in log-linearized terms):

$$-\sigma(\widehat{C_H} - \widehat{C_F}) = \widehat{\frac{sP_H}{P_F}} \tag{70}$$

First-order conditions for the demand for money are as follows (in log-linearized terms):

$$\sigma(\widehat{C_H} - \widehat{C_F}) = \widehat{M} - (\widehat{P_H} - \widehat{P_F}) \tag{71}$$

Using (70) and (71), we get the rate of depreciation of the nominal exchange rate is equal to relative money supply shocks:

$$\hat{s} = -\widehat{M} \tag{72}$$

In our benchmark flex-price model, the efficient consumption allocation is unchanged and so are the efficient terms-of-trade and consequently (21) still holds:

$$\widehat{y} = -\lambda\widehat{q} \tag{73}$$

And relative equity returns $\widehat{R}$, relative non-financial incomes $\widehat{w}$ and bond returns differentials $\widehat{R_b}$ can be rewritten as:

$$\widehat{R} = (1-\lambda)\widehat{q} \tag{74}$$

$$\widehat{w} = (1-\lambda)\widehat{q} \tag{75}$$

$$\widehat{R_b} = \widehat{s} = (2a-1)\widehat{q} + \widehat{\frac{P_H}{P_F}} \tag{76}$$

Introducing $\widehat{\varepsilon} = \widehat{\frac{P_H}{P_F}}$, we are back to our reduced form model and equity and bond portfolios will be the one described in (66)-(67). In particular, as might have been expected, the bond return differential fails to load on the real exchange because of the inflation differentials across countries $\widehat{\frac{P_H}{P_F}}$. This additional source of risk on bond returns shifts bond position towards zero (to hedge inflation risk). It then remains for equities to hedge real exchange rate exposure efficiently.

The case of changes in quality/preference shocks

We follow Coeurdacier, Kollmann, Martin (2007) by adding preference shocks to the utility provided by Home goods and Foreign goods to the consumers of both countries. In that case, the aggregate consumption index C_i, for $i = H, F$, is now given by:

$$C_H = \left[a^{1/\phi}\left(\Psi_H c_{HH}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(\Psi_F c_{HF}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \quad (77)$$

$$C_F = \left[a^{1/\phi}\left(\Psi_F c_{FF}\right)^{(\phi-1)/\phi} + (1-a)^{1/\phi}\left(\Psi_H c_{FH}\right)^{(\phi-1)/\phi}\right]^{\phi/(\phi-1)} \quad (78)$$

where Ψ_i, $i = H, F$ with $E_0(\Psi_i) = 1$ is an exogenous worldwide shocks to the (relative) preference for the country i good. Note that the shock Ψ_i can also have a more supply oriented interpretation, as a shock to the quality of good i. We denote $\Psi \equiv \frac{\Psi_H}{\Psi_F}$ the relative preference shocks and $\widehat{\Psi}$ its deviation from its steady-state value of one.

As shown by Coeurdacier, Kollmann, Martin (2007), the welfare-based real exchange rate in this case is equal to (up to the first-order):

$$\widehat{RER} = (2a-1)\left(\widehat{\frac{p_H}{p_F}} - \widehat{\frac{\psi_H}{\psi_F}}\right) \quad (79)$$

We adjust the terms-of-trade for quality/preference shocks by scaling q as follows: $q = \frac{p_H/\psi_H}{p_F/\psi_F}$. Then, we still have the following equality:

$$\widehat{RER} = (2a-1)\,\widehat{q} \quad (80)$$

As shown by Coeurdacier, Kollmann, Martin (2007), under the assumption of complete financial markets, efficient terms-of-trade (adjusted for quality/preference shocks) satisfy:

$$\widehat{\psi y} = -\lambda \widehat{q} \quad (81)$$

where $\widehat{\psi y}$ are relative endowment adjusted for quality/preference shocks. Relative equity returns and relative non-financial incomes still load perfectly on the real exchange rate adjusted for the quality/preference shocks (see also Coeurdacier, Kollmann, Martin (2007)):

$$\widehat{R} = (1-\lambda)\widehat{q} \quad (82)$$

$$\widehat{w} = (1-\lambda)\widehat{q} \quad (83)$$

However, if we assume that CPI-indexed bonds returns are not adjusted for quality (a case where the observed real exchange rate deviates from the welfare-based one), then bond returns differentials can be rewritten as follows:

$$\widehat{R_b} = (2a-1)\widehat{\frac{p_H}{p_F}} = (2a-1)\left(\widehat{q} + \widehat{\psi}\right) \quad (84)$$

Then, introducing $\widehat{\varepsilon} = (2a-1)\,\widehat{\psi}$, we are back to our model in his reduced form and equity and bond portfolios will be again the same one. In particular, as it might have been expected, bond returns differential fails to load on the welfare-based real exchange because of relative change in quality/preference shocks $\widehat{\psi}$. This additional source of risk on bond returns shift bond position towards zero and risk-sharing is done by equities only.

4 The Role of Nontradable

[To be written]

Where we show that the results from the previous sections extend readily to the case where there are non-tradable.

5 The General Case: Bond and Equity Holdings under Incomplete Markets

[To be written]

Where we use the Tille and van Wincoop approach to characterize the optimal equity and bond positions when markets are incomplete (i.e. there are many ε type shocks). Our conjecture is that what matters is the relative variance of the additional shocks, conditional on the welfare-based real exchange rate. We would expect equities to be used more heavily to hedge real exchange rate fluctuations when equity returns load 'more' on total consumption expenditures and the opposite in the case where bond returns do the job.

6 Empirical Analysis: What do we know about optimal bond and equity returns?

[Under Heavy Construction!]

The results of the previous sections indicate that the pivotal factor is the ability of bond returns to hedge fluctuations in total consumption expenditures. When this is the case, i.e. in presence of redistributive, government or fiscal shocks, equity portfolios are independent of preference parameters and are driven by the correlation between equity returns and non financial income, conditional on bond returns. When this is not the case, i.e. in the presence of nominal, quality or preference shocks, bond holdings are essentially useless and equity holdings will be driven by the correlation between equity returns and real exchange rate changes.

In this section, we explore these two cases empirically. We do so in the following way. We consider the hypothesis that nominal shocks prevent the use of bonds as a hedge against total nominal consumption expenditures. To test this hypothesis, we consider the loading of relative bond returns $\hat{R}_b$ and relative equity $\hat{R}$ on relative consumption expenditures $\widehat{sP_HC_H} - \widehat{P_FC_F}$. We show that the correlation is much stronger for relative bond holdings than for bonds. Hence one would reasonably expect to use bond holdings to diversify that component of the overall exposure. Notice that this approach allows us to test both the nominal and the preference shocks hypothesis. In other words, if the home equity bias results from nominal or preference shocks, then relative equity returns should have higher loadings than relative bonds on total consumption expenditures. Noticing, under the maintained assumption of complete markets, relative consumption expenditures are directly related to the fluctuations in the -welfare based- real exchange rate, this allows to but evaluate directly the strength of that hedging motive.

7 Conclusion

[To be written]

References

Adler, Michael and Bernard Dumas, "International Portfolio Choice and Corporation Finance: A Survey," *Journal of Finance*, 1983, *38* (3), 925–84.

Baxter, Marianne and Urban J. Jermann, "The International Diversification Puzzle Is Worse Than You Think," *American Economic Review*, 1997, *87* (2), 170–80.

_ , _ , and Robert G. King, "Nontraded goods, nontraded factors, and international non-diversification," *Journal of International Economics*, April 1998, *44* (2), 211–229.

Coeurdacier, Nicolas, "Do trade costs in goods markets lead to home bias in equities?," 2005. mimeo, ESSEC.

_ , Robert Kollmann, and Philippe Martin, "International Portfolios with Supply, Demand and Redistributive Shocks," Working Paper 13424, National Bureau of Economic Research, September 2007.

Collard, Fabrice, Harris Dellas, Behzad Diba, and Alan Stockman, "Home Bias in Goods and Assets," Working Paper, IDEI, June 2007.

Dellas, Harris and Alan Stockman, "International Portfolio Nondiversication and Exchange Rate Variability," *Journal of International Economics*, 1989, *26*, 271–289.

Heathcote, Jonathan and Fabrizio Perri, "The International Diversification Puzzle Is Not As Bad As You Think," Working Paper 13483, National Bureau of Economic Research, October 2007.

Kollmann, Robert, "International Portfolio Equilibrium and the Current Account," 2005. mimeo Paris University.

_ , "International Portfolio Equilibrium and the Current Account," Discussion Paper 5512, C.E.P.R., February 2006.

Lucas, Robert, "Interest Rates and Currency Prices in a Two-Country World," *Journal of Monetary Economics*, 1982, *10*, 335–359.

Obstfeld, Maurice, "International Risk Sharing and the Costs of Trade," Ohlin Lectures, Stockholm School of Economics May 2007.

_ **and Kenneth Rogoff**, "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," in Ben Bernanke and Kenneth Rogoff, eds., *N.B.E.R. Macroeconomics Annual*, MIT Press Cambridge MA 2000, pp. 73–103.

Van Wincoop, Eric and Francis E. Warnock, "Is Home Bias in Assets Related to Home Bias in Goods?," Working Paper 12728, National Bureau of Economic Research, December 2006.

8 Appendix

Source Country	Domestic Market in % of World Market Capitalization (1)	Share of Portfolio in Domestic Equity in % (2)	Degree of Home Bias = HB_i (3)
Australia	1.9	83.6	0.832
Austria	0.3	58.5	0.583
Belgium	0.7	49.8	0.494
Canada	3.5	76.6	0.757
Denmark	0.4	62.7	0.625
Finland	0.5	63.3	0.631
France	4.2	68.8	0.674
Germany	2.9	57.5	0.562
Greece	0.3	93.4	0.933
Italy	1.9	57.1	0.562
Japan	13.2	91.9	0.906
Netherlands	1.4	32.1	0.311
New-Zealand	0.1	59.8	0.597
Norway	0.5	52	0.517
Portugal	0.2	77.8	0.777
Spain	2.3	86.3	0.859
Sweden	1	59.4	0.589
Switzerland	2.2	59.9	0.589
United Kingdom	7.3	65	0.622
United States	40.5	82.2	0.700
Average	3.67	70.58	0.70

Table 1: Home Bias in Equities in 2005 (from Sercu and Vanpee (2007); source CPIS).
HB_i=1-$\frac{\text{Share of Foreign Equities in Country i Equity Holdings}}{\text{Share of Foreign Equities in the World Market Portfolio}}$.
By definition HB_i is equal to zero if the share of domestic equities in country i's portfolio is equal to the share of domestic equities in the world market portfolio and HB_i is equal to 1 if there is full equity home bias.