THE LABOR WEDGE AS A MATCHING FRICTION

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Abstract. Variations in the labor wedge account for a large fraction of business cycle fluctuations. We provide an interpretation of the labor wedge from a search-theoretic point of view and assess the importance of job creation and job destruction shocks for business cycles. We find that the labor wedge is mainly driven by matching shocks. However, in a recession, separation shocks account for the initial increase in unemployment, while matching shocks are responsible for the slow recovery. Finally, allowing for changes in the reservation value of workers can solve Shimer's puzzle.

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I. Introduction

For the last 25 years, macro and labor economists have pointed to large, cyclical variations in the relationship between the marginal rate of substitution between leisure and consumption and the marginal product of labor as an important feature of business cycles. In their business cycle accounting framework Chari, Kehoe and McGrattan (2007, [5]) (CKM) label this relationship a "labor wedge" and show that it accounts for 60% of output fluctuations.

Many economists conjecture that changes in the labor wedge represent time-varying imperfections in the labor market, such as labor taxes, monopoly power by unions or firms, sticky wages or sticky prices, and changes in institutions and regulations. Search frictions haven't been examined as a potential explanation of the labor wedge, though shocks to job destruction and job creation have been shown to play a crucial role in the behavior of the labor market.

We develop a theory of the labor wedge and quantitatively analyze it using a model that features time-varying search and matching frictions in the spirit of Mortensen and Pissarides (1994, [18]), Shimer (2005, [19]) and Hall (2005, [12]).[1] Then, using

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[1]A growing body of literature, inspired by the Mortensen-Pissarides (1994, [18]) labor search model, has been trying to incorporate labor frictions into the standard RBC model. Early attempts, including the models of Merz (1995, [16]) and Andolfatto (1996, [2]), showed that these models are successful at explaining the behavior of hours and employment. However, Shimer (2005, [19]) pointed out that those models have a hard time explaining the volatility of unemployment and vacancies and argued that some variations in the bargaining power are necessary to explain the magnitude of fluctuations. Hall (2005, [12]) argued that a natural explanation for the bargaining power of workers to increase during recessions is some form of wage-stickiness.

a methodology similar to that of CKM we identify the sources of fluctuations and assess the importance of job creation and job destruction shocks to business cycles.

Our modeling approach follows the path of Farmer and Hollenhorst (2006, [7]) by assuming that workers can choose whether to participate in the labor force. The standard assumption that labor is traded in a spot market is replaced by a labor search friction which puts an additional constraint on how much labor can be used in production and has direct implications for the behavior of vacancies and unemployment. We introduce exogenous shocks to the separation rate and the matching efficiency that determine how many jobs are destroyed and created every period, and incorporate an exogenous process, governing fluctuations in the bargaining power, general enough to mimic the behavior of most known mechanisms generating time-varying bargaining power.

This extension allows us to decompose the labor wedge into three different shocks: the *separation shock*, the *matching shock* and the *bargaining shock*. The separation shock determines the proportion of employed workers that get separated from their jobs every period and stay such until the next. The matching shock determines the number of new matches being formed every period given the number of unemployed people searching for a job and the number of vacancies posted by firms. The bargaining shock determines the proportions in which the lifetime surplus of a newly formed match is split between the worker and the firm. We show that the labor wedge of CKM can be directly reconstructed from these three shocks.

Our methodology follows that of CKM in its main points. We use data on real GDP, consumption, investment, hours, unemployment and vacancies to recover the six shocks: TFP, investment, government consumption, separation, matching and bargaining. To measure the contributions of each shock to each of the six variables we fit the shocks back into the model one at a time and all but one at a time. The contributions are measured on average - over the whole period from 1964 to 2007, and in bad times - during the last five recession episodes.

We confirm CKM's statement that efficiency (TFP) and labor wedges explain virtually all of the variations in output; and find that the labor wedge is mainly driven by shocks to matching efficiency. However, both job creation and job destruction shocks play an important role in output and unemployment fluctuations. This last result has direct implications for the debate between Fujita and Ramey (2007, [9]) and Shimer (2005, [20]) on whether job destruction or job creation is more important for fluctuations in unemployment and output.

We find that job destruction and job creation shocks play a role at different points in time. At the beginning of a recession the decline in TFP coincides with an increase in the separation rate. So layoffs caused by low productivity in the real sector start the recession. As unemployment increases, the reservation value (threat-point) of the workers goes down while an increase in their bargaining power keeps the wage fairly constant. This increase in the bargaining power is consistent with most forms of wage stickiness proposed in the literature. The corresponding decrease in the bargaining power of the firms causes a decline in vacancies. Later the efficiency of the matching process falls (we attribute it to some form of disorganization), keeping unemployment at a high level.

Our results also provide an potential solution to Shimer's puzzle. Shimer (2005, [19]) pointed out that standard Mortensen-Pissarides-type models are incapable of generating the observed high volatility of unemployment and vacancies and low volatility of wages. Hall (2005, [13]) suggested that changes in the bargaining power - generated by fluctuations in the marginal product of labor and sticky wages - are necessary to solve the puzzle. In our model the bargaining power of the workers increases in recessions because their reservation value falls and the wage does not. This mechanism allows us to match the observed volatility of unemployment, vacancies and wages. Hence, considering changes in the reservation value of workers can solve Shimer's puzzle.

Thus our paper has three main results. First, in a search environment the labor wedge is mainly driven by matching shocks. Second, both job creation and job destruction shocks are important for business-cycle fluctuations: while separation shocks start recessions, matching shocks are responsible for its depth and slow recovery. Third, allowing for changes in the reservation value of workers can solve Shimer's puzzle.

The paper is organized as follows. Section 2 lays out the theoretical framework and introduces the six shocks, Section 3 describes the methodology we use to estimate the model and recover the shocks, Section 4 explains the results and Section 5 concludes.

II. Theoretical Framework

In this section we describe a model economy that modifies the standard one sector real business cycle model by adding a search technology for moving labor between productive activities and leisure as in Farmer and Hollenhorst (2006, [7]).

In the following subsections we first describe a decentralized version of the model economy, we then show the solution to the same model when solved as a social planner's problem. The decentralized version of the model has a missing equilibrium condition that is typically replaced with a Nash bargaining condition to fix the real wage. We use the fact that there is a missing condition to our advantage, and by comparing the social planner's solution with the decentralized version of the model, we construct a time varying bargaining shock, which will implicitly determine the wage level.

II.1. **A Decentralized Model.** The economy is populated by a continuum of families and each family operates a backyard technology. The members of the family cannot work in their own backyard. There are two market activities: head-hunting which is competitive and a productive activity where the wage is set according to a specific wage-setting rule.

There are six exogenous shocks. A total factor productivity (TFP) shock A_t, an investment specific technology shock T_t, a government expenditure shock G_t, a shock to the separation rate of employment δ_{Lt}, a shock to matching efficiency B_t and a shock to the bargaining power of workers ϕ_t. This last shock ϕ_t represents the fraction of the lifetime surplus of the match that goes to the worker, hence, as we will show later on, ϕ_t determines the wage w_t of the productive sector.

At the beginning of period t values of A_t, T_t, δ_{Lt}, G_t, B_t, ϕ_t, K_t, L^s_{t-1}, L^d_{t-1} are given, and each family - given the job finding and job filling rate - decides how many individuals search for a job U_t and how many vacancies to open V_t^d. For each vacancy they open they have to hire one head-hunter. At the same time they decide how many individuals offer their labor to work in head-hunting activities V_t^s in their neighbors' backyard. The wage q_t, clears this market.

Each family maximizes the discounted present value of their utility function, subject to a budget constraint and labor supply and demand accumulation constraints:

$$\max_{\{C_t, L_t^s, L_t^d, V_t^s, V_t^d, U_t, , K_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t^s, V_t^s, U_{t,}\right) \tag{1}$$

subject to

$$C_t + \frac{1}{T_t}\left(K_{t+1} - (1-\delta_K)K_t\right) + G_t \leq A_t F(K_t, L_t^d) + w_t(L_t^s - L_t^d) + q_t(V_t^s - V_t^d) \tag{2}$$

$$L_t^s = (1-\delta_{Lt})L_{t-1}^s + U_t \frac{\bar{M}_t}{\bar{U}_t} \tag{3}$$

$$L_t^d = (1-\delta_{Lt})L_{t-1}^d + V_t^d \frac{\bar{M}_t}{\bar{V}_t} \tag{4}$$

where $\bar{M}_t$ is the total number of matches formed in the economy in period t. Labor supply in period t depends on last period's labor supply minus the workers that got separated from their job plus the new formed matches; $\frac{\bar{M}_t}{\bar{U}_t}$ is the job finding rate and represents the increase in employment when there is one more individual searching for a job (U_t increases by one unit). Labor demand accumulates in the same way as labor supply with the difference that the term $V_t^d \frac{\bar{M}_t}{\bar{V}_t}$ is the vacancy filling rate times the number of head-hunters demanded and means that for every new individual that works as a head-hunter V_t^d, the stock of employed workers increases by $\frac{\bar{M}_t}{\bar{V}_t}$.

The market clearing conditions of this economy are given by:

$$L_t^s = L_t^d = L_t$$

$$V_t^s = V_t^d = V_t$$

and in equilibrium:

$$\bar{M}_t = M_t = B_t M(U_t, V_t)$$
$$\bar{U}_t = U_t$$
$$\bar{V}_t = V_t$$

The resource constraint is given by

$$C_t + \frac{1}{T_t} X_t + G_t = Y_t \tag{5}$$

where

$$Y_t = A_t F(K_t, L_t^d) \tag{6}$$

and

$$X_t = K_{t+1} - (1-\delta_K)K_t \tag{7}$$

Finally, the optimality conditions of the model are

$$\frac{1}{T_t} = \beta E_t \frac{U'_{C_{t+1}}}{U'_{C_t}} \left(A_{t+1} F'_{K_{t+1}}(K_t, L_t) + \frac{1}{T_t}(1-\delta_K) \right) \tag{8}$$

$$w_t + \frac{U'_{L_t}}{U'_{C_t}} = \mu_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \mu_{t+1} \left(1 - \delta_{Lt+1}\right) \right) \tag{9}$$

$$A_t F'_{L_t}(K_t, L_t) - w_t = \eta_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \eta_{t+1} \left(1 - \delta_{Lt+1}\right) \right) \tag{10}$$

$$\frac{U'_{V_t}}{U'_{C_t}} + q_t = 0 \tag{11}$$

$$\eta_t \frac{M_t}{V_t} - q_t = 0 \tag{12}$$

$$-\frac{U'_{U_t}}{U'_{C_t}} = \mu_t \frac{M_t}{U_t} \tag{13}$$

In the equations above μ_t is the Lagrange multiplier associated with the labor supply accumulation constraint and η_t is the Lagrange multiplier associated with the labor demand accumulation constraint. Together with equations (5),(6),(7), the aggregate labor accumulation equation (14) and the matching function (15) they completely describe the solution to the decentralized problem. We shall discuss the first order conditions of the model later.

$$L_t = (1 - \delta_{Lt}) L_{t-1} + M_t \tag{14}$$

$$M_t = B_t M(U_t, V_t) \tag{15}$$

In equation (15) B_t represents the efficiency of the matching technology, determining the number of matches formed for each combination of the numbers of workers and head-hunters seeking for a match. Since T_t, A_t, δ_{Lt}, G_t and B_t are exogenous, we have a system of eleven equations and twelve variables, $\{K_{t+1}, L_t, C_t, M_t, Y_t, V_t, M_t, \mu_t, \eta_t, X_t, w_t\}$. The model is missing an equilibrium condition because equations (9) and (10) determine two different ways of moving labor between leisure and employment in productive activities and there is only one price w_t. Therefore the model has multiple equilibria and every wage corresponds to one of them. We shall introduce a bargaining shock, as mentioned earlier on, to determine which equilibrium is chosen.

II.2. **The Social Planner's Problem.** To compare competitive allocations with the efficient ones we solve the social planning problem. The social planner maximizes the discounted present value of the utility function:

$$\max_{\{C_t, L_t, V_t, U_{t,}, K_{t+1}\}} E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, L_t, V_t, U_{t,}\right) \tag{16}$$

subject to

$$C_t + \frac{1}{T_t}\left(K_{t+1} - (1-\delta_K)K_t\right) + G_t \leq A_t F(K_t, L_t) \tag{17}$$

$$L_t = (1-\delta_L)L_{t-1} + M_t \tag{18}$$

The optimality conditions of the planner are given by:

$$\frac{1}{T_t} = \beta E_t \frac{U'_{C_{t+1}}}{U'_{C_t}} \left(A_{t+1} F'_{K_{t+1}}(K_t, L_t) + \frac{1}{T_t}(1-\delta_K) \right) \tag{19}$$

$$A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}} = \Gamma_t - \beta E_t \left(\frac{U'_{C_{t+1}}}{U'_{C_t}} \Gamma_{t+1}(1-\delta_{Lt}) \right) \tag{20}$$

$$-\frac{U'_{V_t}}{U'_{C_t}} = \Gamma_t \frac{\partial M_t}{\partial V_t} \tag{21}$$

$$-\frac{U'_{U_t}}{U'_{C_t}} = \Gamma_t \frac{\partial M_t}{\partial U_t} \tag{22}$$

Together with equations (5),(6),(7), (14) and (15) they describe the allocations a social planner would choose. Γ_t is the Lagrange multiplier associated with the labor accumulation constraint. Given that T_t, A_t, δ_{Lt}, G_t and B_t are exogenous, we have a system of nine equations and nine unknowns $\{K_{t+1}, L_t, C_t, M_t, Y_t, V_t, U_t, \Gamma_t, X_t\}$.

II.3. **Constructing a Time Varying Bargaining Shock.** By comparing the social planner's optimality conditions with those of the decentralized problem, we can find the necessary assumptions to make the decentralized problem efficient.

By putting equations (11) and (12) together we get that $-\frac{U'_{V_t}}{U'_{C_t}} = \eta_t \frac{M_t}{V_t}$, and if we compare this expression with equation (21), and equation (13) with equation (22), we need

$$\eta_t \frac{M_t}{V_t} = \Gamma_t \frac{\partial M_t}{\partial V_t} \tag{23}$$

$$\mu_t \frac{M_t}{U_t} = \Gamma_t \frac{\partial M_t}{\partial U_t} \tag{24}$$

so that the optimality conditions on vacancies and unemployment are the same in the decentralized and planner's problem. Furthermore, if we assume that the matching function has constant returns to scale

$$\frac{\partial M(U_t, V_t)}{\partial U_t} U_t + \frac{\partial M(U_t, V_t)}{\partial V_t} V_t = M(U_t, V_t)$$

then $\Gamma_t = \mu_t + \eta_t$ and the decentralized outcome is Pareto-optimal. Hence, the Hosios condition for efficiency is given by:

$$\eta_t = \Gamma_t \frac{\partial M_t}{\partial V_t} \frac{V_t}{M_t} \tag{25}$$

$$\mu_t = \Gamma_t \frac{\partial M_t}{\partial U_t} \frac{U_t}{M_t} \tag{26}$$

As an illustration, assume $\frac{\partial M_t}{\partial U_t} \frac{U_t}{M_t} = \theta$ and $\frac{\partial M_t}{\partial V_t} \frac{Vt}{M_t} = (1-\theta)$, which together with conditions (25) and (26) give

$$\eta_t = (1-\theta)\Gamma_t \tag{27}$$

and

$$\mu_t = \theta\Gamma_t \tag{28}$$

Notice that if we replace equations (27) and (28) in equations (9) and (10) and sum them up, we get equation (20), hence the optimality conditions for labor in the decentralized version become equal to the optimality condition for labor in the planners problem.

Furthermore, if we divide equation (9) by equation (10) and use equations (27) and (28) we get

$$\frac{w_t + \frac{U'_{L_t}}{U'_{C_t}}}{A_t F'_{L_t}(K_t, L_t) - w_t} = \frac{\mu_t - \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}\mu_{t+1}\left(1-\delta_{Lt+1}\right)\right)}{\eta_t - \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}\eta_{t+1}\left(1-\delta_{Lt+1}\right)\right)} = \frac{\theta}{1-\theta}$$

Given that w_t is the wage earned by the worker, and $-\frac{U'_{L_t}}{U'_{C_t}}$ is his reservation utility, the term $w_t + \frac{U'_{L_t}}{U'_{C_t}}$ represents the value of the match earned by the worker. Since the bargaining power of the worker is constant and equal to θ, the optimal wage rate satisfies

$$\left(w_t + \frac{U'_{L_t}}{U'_{C_t}}\right) = \theta\left(A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}}\right)$$

where $A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}}$ is the difference between the marginal product of labor and the marginal disutility of labor. This term represents the instantaneous marginal value of the match, and a fraction θ goes to the worker.

To introduce the time varying bargaining shock we build on this result, re-parameterize and substitute θ by ϕ_t. ϕ_t is time varying and will follow an exogenous autorregressive process that will be defined later. Notice that replacing θ by ϕ_t implies that allocations are suboptimal whenever $\phi_t \neq \theta$.

Equations (27) and (28) are replaced by

$$\eta_t = (1-\phi_t)\Gamma_t \tag{29}$$

$$\mu_t = \phi_t\Gamma_t \tag{30}$$

once again if we substitute equations (29) and (30) in (9) and (10) we get equation (20) so it is still true that the optimality conditions for labor of the decentralized version imply the optimality condition for labor of the planner's problem.

Dividing equation (9) by equation (10) and using equations (29) and (30) we get

$$\frac{w_t + \frac{U'_{L_t}}{U'_{C_t}}}{A_t F'_{L_t}(K_t, L_t) - w_t} = \frac{\phi_t\Gamma_t - \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}\phi_{t+1}\Gamma_{t+1}\left(1-\delta_{Lt+1}\right)\right)}{(1-\phi_t)\Gamma_t - \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}(1-\phi_{t+1})\Gamma_{t+1}\left(1-\delta_{Lt+1}\right)\right)}$$

hence, the optimal wage rate satisfies

$$\left(w_t + \frac{U'_{L_t}}{U'_{C_t}}\right) = \frac{\phi_t - \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}\phi_{t+1}J_{t+1}\right)}{1-\beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}J_{t+1}\right)}\left(A_tF'_{L_t}(K_t,L_t) + \frac{U'_{L_t}}{U'_{C_t}}\right)$$

where $J_{t+1} = \frac{\Gamma_{t+1}}{\Gamma_t}(1-\delta_{Lt+1})$ and can be interpreted as a stochastic discount factor for labor.

Equations (20), (9) and (10) can be iterated forward to solve for the corresponding multipliers:

$$\Gamma_t = A_tF'_{L_t}(K_t,L_t) + \frac{U'_{L_t}}{U'_{C_t}} + E_t\sum_{s=t+1}^{\infty}\beta^{s-t}\left(A_sF'_{L_s}(K_s,L_s) + \frac{U'_{L_s}}{U'_{C_s}}\right)\prod_{k=t+1}^{s}(1-\delta_{Lk})$$

$$\eta_t = A_tF'_{L_t}(K_t,L_t) - w_t + E_t\sum_{s=t+1}^{\infty}\beta^{s-t}\left(A_sF'_{L_s}(K_s,L_s) - w_t\right)\prod_{k=t+1}^{s}(1-\delta_{Lk})$$

$$\mu_t = \frac{U'_{L_t}}{U'_{C_t}} + w_t + E_t\sum_{s=t+1}^{\infty}\beta^{s-t}\left(\frac{U'_{L_s}}{U'_{C_s}} + w_t\right)\prod_{k=t+1}^{s}(1-\delta_{Lk})$$

The Lagrange multiplier on the resource constraint for labor Γ_t is the expected sum of instantaneous marginal values of the match discounted and adjusted for the probability of the match being dissolved in any given period. Similarly, the Lagrange multiplier in the labor demand (supply) accumulation equation is the expected sum of instantaneous marginal values of the match for the representative firm (worker), discounted and adjusted for the probability of the match being dissolved in any given period.

The variable ϕ_t represents the fraction of the gross value of the match going to the worker, and $(1-\phi_t)$ the fraction of the gross value of the match going to the firm. Hence, we call ϕ_t the bargaining power of the worker as well as the bargaining shock.

II.4. **Equilibrium Conditions for the Model Economy and Functional Forms.** The competitive equilibrium of the model economy is a solution to the following equations

$$\frac{1}{T_t} = \beta E_t\frac{U'_{C_{t+1}}}{U'_{C_t}}\left(A_{t+1}F'_{K_{t+1}}(K_t,L_t) + \frac{1}{T_{t+1}}(1-\delta_K)\right) \tag{31}$$

$$\Gamma_t = A_tF'_{L_t}(K_t,L_t) + \frac{U'_{L_t}}{U'_{C_t}} + \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}\Gamma_{t+1}(1-\delta_{Lt+1})\right) \tag{32}$$

$$(1-\phi_t)\Gamma_t\frac{M_t}{V_t} = -\frac{U'_{V_t}}{U'_{C_t}} \tag{33}$$

$$\phi_t\Gamma_t\frac{M_t}{U_t} = -\frac{U'_{U_t}}{U'_{C_t}} \tag{34}$$

$$\left(w_t + \frac{U'_{L_t}}{U'_{C_t}}\right) = \frac{\phi_t - \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}\phi_{t+1}J_{t+1}\right)}{1 - \beta E_t\left(\frac{U'_{C_{t+1}}}{U'_{C_t}}J_{t+1}\right)}\left(A_t F'_{L_t}(K_t, L_t) + \frac{U'_{L_t}}{U'_{C_t}}\right) \tag{35}$$

$$J_{t+1} = \beta\frac{\Gamma_{t+1}}{\Gamma_t}\left(1 - \delta_{Lt+1}\right) \tag{36}$$

$$C_t + \frac{1}{T_t}\left(K_{t+1} - (1-\delta_K)K_t\right) + G_t = A_t F(K_t, L_t) \tag{37}$$

$$Y_t = A_t F(K_t, L_t) \tag{38}$$

$$X_t = K_{t+1} - (1-\delta_K)K_t \tag{39}$$

$$L_t = (1-\delta_{Lt})L_{t-1} + M_t \tag{40}$$

$$M_t = B_t M(U_t, V_t) \tag{41}$$

where $\{K_{t+1}, C_t, L_t, V_t, U_t, \Gamma_t, w_t, J_t, Y_t, X_t, M_t\}$ are endogenous variables and $\{A_t, T_t, \delta_{Lt}, G_t, B_t, \phi_t\}$ are the exogenous shocks of the model. The exogenous variables behave according to stochastic processes to be defined later.

Equation (31) is the standard Euler equation. Equation (32) governs the behavior of the total value of the match. Today's value is equal to the instantaneous gain from a newly formed match plus the future value, discounted and adjusted for the possibility of being destroyed tomorrow.

Equations (33) and (34) equate the marginal benefits and marginal costs of head-hunting and searching for a job respectively and thus pin down vacancies and unemployment. Equations (35) and (36) determine the real wage and the discount factor for labor. Equations (37), (38) and (39) are the resource constraints for consumption, investment and output. In turn equations (40) and (41) govern the dynamics of hours and the new matches.

Most of the functional forms we use are standard in the literature. We assume that the production function is Cobb-Douglas with constant returns to scale:

$$F(K, L) = K^\alpha L^{1-\alpha}$$

We assume that the matching function is also Cobb-Douglas and consistent with constant returns to scale mentioned above:

$$M(U, V) = U^\theta V^{1-\theta}$$

We postulate a utility function consistent with a balanced growth path and where fractions of time spent head-hunting and searching for a job enter symmetrically with the time spent on the production activity:

$$U(C, L, U, V) = \log C - \chi\frac{(L + V + U)^{1+\gamma}}{1+\gamma}$$

The functional form for hours is the only non-standard assumption we make. It implies that workers get the same disutility from working in market activities as when searching for a job. This assumption may seem somewhat extreme, but we assume that although individuals spend only a few hours per week searching for a job, they also spend time in other activities that generate disutility: in expanding their network by making phone calls, getting technical training, continuing their education, helping their relatives or working in home production.

In the decomposition we perform, this utility function affects the behavior of the reservation value of the workers. The parameter γ determines how much the reservation value falls as they spend less time in the labor market. Given these functional forms and stochastic processes for the shocks, to be explained later, the shocks are uniquely identified. Appendix A explains step by step how given data on output, consumption, investment, hours, unemployment and vacancies one can recover the shocks.

III. Estimation Procedure

This section describes the details of the estimation procedure. Our estimation strategy is different from that of CKM in three dimensions. First, though some of the parameters can be calibrated, others have no analogs in the literature. We estimate them using the Kalman filter and what is becoming standard - Markov Chain Monte-Carlo (MCMC) methods.

Second, estimation results can depend significantly (see Cogley and Nason (1995) [6]) on the the way the data is pre-filtered. In order to avoid certain filtering biases we minimize the extent to which we alter the data. We embed the trends into the model. We describe in detail a procedure of detrending the model around a non-stationary stochastic trend, which we borrow from Fernandez-Villaverde and Rubio-Ramirez (2006 [8]).

Finally, because many of the parameters of the model are estimated using MCMC methods, we use the same approach to recover the shocks. We apply the Kalman filter to the linearized version of the model to compute the stochastic innovations of the shocks.

III.1. **Processes for the shocks.** In the data real output, consumption and investment are nonstationary even with respect to a log-linear trend. To make the data comparable to the model the business cycle literature commonly uses the Hodrick-Prescott (HP) filter. However, Cogley and Nason (1995 [6]) and Canova (1998 [4]) show that the use of the filter introduces significant biases into the data by amplifying business-cycle frequencies even if it does not have any. To avoid using any kind of filter we follow the approach presented in Fernandez-Villaverde and Rubio-Ramirez (2006 [8]) and assume random walks for the two processes that are commonly thought to be extremely persistent: the TFP and investment shocks.

$$\log A_t = \log a_{ss} + \log A_{t-1} + \sigma_A \varepsilon_{At} \tag{42}$$

$$\log T_t = \log \tau_{ss} + \log T_{t-1} + \sigma_T \varepsilon_{Tt} \tag{43}$$

Here a_{ss} is the mean growth rate of TFP, τ_{ss} can be interpreted as the mean growth rate of the investment-specific technology. We assume that the rest of the shocks follow first-order autoregressive processes. Here lower-case variable g_t corresponds to detrended government consumption to be defined later.

$$\log \delta_{Lt} = (1-\rho_S)\log \delta_{Lss} + \rho_S \log \delta_{Lt-1} + \sigma_S \varepsilon_{St} \tag{44}$$

$$\log B_t = (1-\rho_M)\log B_{ss} + \rho_M \log B_{t-1} + \sigma_M \varepsilon_{Mt} \tag{45}$$

$$\log \phi_t = (1-\rho_B)\log \phi_{ss} + \rho_B \log \phi_{t-1} + \sigma_B \varepsilon_{Bt} \tag{46}$$

$$\log g_t = (1-\rho_G)\log g_{ss} y_{ss} + \rho_G \log g_{t-1} + \sigma_G \varepsilon_{Gt} \tag{47}$$

where δ_{Lss} is the separation rate in steady-state, B_{ss} determines the fraction of hours spent on head-hunting in steady-state, ϕ_{ss} is the average bargaining power which determines the vacancy-unemployment ratio, g_{ss} is the steady-state fraction of GDP consumed by the government. Also $\sigma.$ and $\rho.$ denote the standard deviations and autocorrelations of the shocks. All innovations $\varepsilon_{\cdot t}$ are assumed to be standard normal. We do not put restrictions on the correlations of these innovations.

III.2. **Detrending.** Here we describe how to detrend the model with respect to a pair of nonstationary trends. From the optimality conditions of the model we can see that all variables except capital grow at a factor $(a_{ss}\tau_{ss}^{\alpha})^{\frac{1}{1-\alpha}}$. Then, if we take the first differences of the TFP and investment shocks by defining $a_t = \frac{A_t}{A_{t-1}} = a_{ss}\exp(\sigma_A \varepsilon_{At})$, $\tau_t = \frac{T_t}{T_{t-1}} = \tau_{ss}\exp(\sigma_T \varepsilon_{Tt})$., we can derive an aggregate trend $Z_t^{1-\alpha} = A_t T_t^{\alpha}$, which will be common to all the variables except capital. Hence, we can define detrended variables of the form: $x_t = \frac{X_t}{Z_{t-1}}$. Capital grows at a factor $(a_{ss}\tau_{ss})^{\frac{1}{1-\alpha}}$, so it is detrended as follows: $k_{t+1} = \frac{K_{t+1}}{Z_t T_t}$.

Similarly to the result of King et. al. (1988 [15]) given separability of the utility function, we need to assume logarithmic utility for there to exist a balanced growth path. This will not affect our results since Hansen (1986, [14]) has shown that quantitatively the degree of risk aversion has almost no effect on the behavior of quantities in real business cycle models.

Substituting detrended variables into the model, we come to the following representation:

$$E_t j_{t+1}\left(\alpha \frac{y_{t+1}}{k_{t+1}} - \frac{1-\delta_K}{\tau_{t+1}}\right) = 1 \tag{48}$$

$$y_t = a_t k_t^{\alpha} L_t^{1-\alpha} \tag{49}$$

$$c_t + z_t k_{t+1} - (1-\delta_K)\frac{k_t}{\tau_t} + g_t = y_t \tag{50}$$

$$\Gamma_t = \left((1-\alpha)\frac{y_t}{L_t} - \kappa_t\right) + E_t j_{t+1}\Gamma_{t+1}(1-\delta_{Lt}) \tag{51}$$

$$\left(B_t U_t^{\theta} V_t^{1-\theta}\right)\Gamma_t = (V_t + U_t)\kappa_t \tag{52}$$

$$\phi_t V_t = (1-\phi_t)\, U_t \tag{53}$$

$$L_t = (1-\delta_{Lt})\, L_{t-1} + B_t U_t^{\theta} V_t^{1-\theta} \tag{54}$$

$$z_t^{1-\alpha} = a_t \tau_t^{\alpha} \tag{55}$$

$$m_t = (1-\delta_{Lt})\, \frac{\gamma_t}{\gamma_{t-1}} \tag{56}$$

$$j_t = \beta \left(\frac{c_{t-1}}{c_t} \right) \frac{1}{z_{t-1}} \tag{57}$$

$$\kappa_t = \chi c_t \left(L_t + U_t + V_t \right)^{\gamma} \tag{58}$$

$$q_t = (1-\phi_t)\, \Gamma_t B_t \left(\frac{U_t}{V_t} \right)^{\theta} \tag{59}$$

$$(1 - E_t m_{t+1} j_{t+1})\, (w_t - \kappa_t) = (\phi_t - E_t m_{t+1} j_{t+1} \phi_{t+1}) \left((1-\alpha)\, \frac{y_t}{L_t} - \kappa_t \right) \tag{60}$$

$$a_t = a_{ss} \exp\left(\sigma_{At} \varepsilon_{At} \right) \tag{61}$$

$$\tau_t = \tau_{ss} \exp\left(\sigma_{Tt} \varepsilon_{Tt} \right) \tag{62}$$

$$\log \delta_{Lt} = (1-\rho_S) \log \delta_{Lss} + \rho_S \log \delta_{Lt-1} + \sigma_S \varepsilon_{St} \tag{63}$$

$$\log B_t = (1-\rho_M) \log b_{ss} + \rho_M \log B_{t-1} + \sigma_M \varepsilon_{Mt} \tag{64}$$

$$\log \phi_t = (1-\rho_B) \log \phi_{ss} + \rho_B \log \phi_{t-1} + \sigma_B \varepsilon_{Bt} \tag{65}$$

$$\log g_t = (1-\rho_G) \log g_{ss} y_{ss} + \rho_G \log g_{t-1} + \sigma_G \varepsilon_{Gt} \tag{66}$$

This model possesses a unique steady-state. We describe the algorithm of computing the steady-state in Appendix B. We linearize the model around the steady-state, compute the state-space representation and estimate.

III.3. **Measurement Equations.** We use six variables in our estimation procedure: 1) real per capita GDP, 2) real per capita nondurable consumption expenditures, 3) real per capita gross private domestic investment (including durable consumption), 4) an index of aggregate weekly per capita hours worked in private industries, 5) the unemployment rate and 6) the Conference Board help-wanted advertising index as a proxy for vacancies.

All data are seasonally adjusted. Monthly data is averaged to make it quarterly. We divide by population to obtain per capita values. This corresponds to modeling the economy using a representative household/firm. We remove an extremely low frequency trend from hours, unemployment and vacancies, using an hp-filter with a smoothing parameter 100000 (we follow Shimer (2005, [19])). This removes long-run secular trends, which are a result of demographic and other factors unrelated to business cycles. We normalize the resulting detrended indexes of hours and vacancies to one on average. We take logs of GDP, consumption and investment, and then take the first difference. All data we use is for the period 1964:I-2007:III.

To be able to estimate the model we need to add six measurement equations corresponding to the six variables that we observe. Since the data for real output, consumption and investment are modeled as nonstationary, we take the first differences of the data to make it comparable to the model. In addition, the definition of output in our model includes time spent head-hunting. In the real economy firms are paying head-hunters a wage and it is measured as part of GDP. To account for this, we derive the shadow price of time spent head-hunting, multiply it by the amount of time spent in this activity and include the product in our definition of GDP.

$$d\log GDP_t = \log \frac{y_t + q_t V_t}{y_{t-1} + q_{t-1} V_{t-1}} z_{t-1} \tag{67}$$

$$d\log Cons_t = \log \frac{c_t}{c_{t-1}} z_{t-1} \tag{68}$$

$$d\log Inv_t = \log \frac{k_{t+1} z_t \tau_t - (1-\delta_K)\, k_t}{k_t z_{t-1} \tau_{t-1} - (1-\delta_K)\, k_{t-1}} z_{t-1} \tau_{t-1} \tag{69}$$

Hours in our model correspond to the total time spend on the productive activity and head-hunting. This index corresponds closely to employment, since most of the cyclical variation in hours is on the extensive margin (see Gertler, Sala, Trigari (2006, [10]) and Hall (2005. [13])).

$$Hours_t = \frac{L_t + V_t}{L_{ss} + V_{ss}} \tag{70}$$

Due to the above correspondence between hours and employment, the fraction of time spent by the representative agent searching for a job of the total time spent in the labor market also corresponds to the number of people searching for a job as a fraction of people participating in the labor market - the unemployment rate.

$$Unemp_t = \frac{U_t}{L_t + V_t + U_t}. \tag{71}$$

Changes in the help-wanted advertising index proxy changes in the number of vacancies posted by firms in the real economy. To make the model over-identified[2] we add a measurement error to the variable which is measured worst - the index of vacancies.

$$HWant_t = \frac{V_t}{V_{ss}} \exp\left(\sigma_\varepsilon \varepsilon_{\varepsilon t}\right) \tag{72}$$

III.4. **Calibration and Estimation.** Our model has 9 structural parameters and 13 parameters that characterize the shocks. The scale parameter L_{ss} does not affect the log-linearized representation of the model. There are three parameters standard to the business-cycle literature that we calibrate. We set the share of capital in the Cobb-Douglas production function α to 0.34, the discount factor β to 0.99, the depreciation rate δ_K to 2.5 % per quarter. We set the steady-state value of the government shock to 22% of GDP, the average value in the data. We also set the elasticity of matches to unemployment θ to 0.7, the value used by Shimer (2005, [19]) and falls within the range of values plausible from a microeconomic perspective reported by Blanchard and Diamond (1989, [3]). We calibrate this parameter because it is not well-identified. We find that this value of θ helps us match the volatility of wages, which are not used in our estimation procedure.

From the average growth rates of investment, consumption and output we infer the means of innovations to TFP and investment shocks. We calibrate them to be 0.16 percent and 0.12 percent per quarter respectively. Table 1 summarizes the calibrated parameters.

Table 1: Calibrated parameters

α	β	δ_K	g_{ss}	θ	a_{ss}	τ_{ss}
0.34	0.99	0.025	0.22	0.7	1.0016	1.0012

We estimate the model using Bayesian methods (see An and Schorfheide (2006, [1])). Linearized equations of the model (48-66) combined with the linearized measurement equations (67-72) form a state-space representation of the model. We apply the Kalman filter to compute the likelihood of the data given the model and to obtain smoothed estimates of the innovations to the wedges. We combine the likelihood function $L\left(Y^{Data}|p\right)$, where p is the parameter vector, with the priors $\pi_0\left(p\right)$ to obtain the posterior distribution of the parameters $\pi\left(p|Y^{Data}\right) = L\left(Y^{Data}|p\right)\pi_0\left(p\right)$. Draws from the posterior distribution are generated using the Markov-Chain Monte-Carlo (MCMC) algorithm. We use the Random-Walk Metropolis-Hastings implementation.

Table 2 reports the prior and posterior distributions of each structural parameter. We estimate the elasticity of the utility function with respect to labor to be 3.54. The high elasticity leads to large variations in the value of non-market activity to be discussed later.

[2]We need more shocks than observed variables to be able to construct a non-degenerate likelihood and be able to estimate the parameters of the model.

Table 2. Prior and posterior distributions of structural parameters								
Parameter	Prior			Posterior				
	Distribution	Mean	S.D.	Mode	S.D.	Mean	5%	95%
γ	log Normal	0.00	2.000	3.54	0.49	3.62	2.97	4.43
δ_{Lss}	Gamma	0.02	0.010	0.038	0.004	0.037	0.030	0.043
ω_{ss}	Gamma	0.35	0.150	0.62	0.08	0.61	0.49	0.73
ϕ_{ss}	Beta	0.50	0.200	0.56	0.04	0.57	0.49	0.64

We estimate the steady-state separation rate to be 3.7%. This is much lower than Shimer's (2005, [19]) quarterly estimate of the separation probability for employed workers. This difference comes from the fact that our separation rate corresponds to the average fraction of productive time destroyed and remaining such until next quarter. Assuming (following Shimer) that the average job finding rate is 40% per month and the separation rate to be 3% per month, the effective number of people becoming and staying unemployed until next quarter should be around 2-3%, which is consistent with our estimate. Our model implies a 61% average job-finding rate which is also comparable to Shimer's estimates.

We estimate the steady-state bargaining power ϕ_{ss} to be 0.56, which is close to the value of 0.5 common in the literature (see Mortensen and Nagypal (2007, [17]) and Hall (2005, [13])). The estimates of the two parameters ω_{ss} and ϕ_{ss} jointly imply, that the average reservation utility is approximately 80% of the marginal product. This is close to Hagedorn and Manovsky's (2007, [11]) calibration of the value of non-market activity (0.95) and much higher than the calibration of Hall (0.4). Our estimate of the parameter ω_{ss} also pins down the ratio of time spent head-hunting to time spent in the production activity which turns out to be 4%. Taking into account the proximity of the shadow prices of different allocations of time, this mimics closely Hagedorn and Manovsky's estimate of the cost of vacancies being 3-4.5% of the quarterly wage. However, unlike their model, most of the variation in the bargaining set comes from variations in the value of non-market activity, not the marginal product.

Table 3. Prior and posterior distributions of wedge parameters								
Parameter	Prior			Posterior				
	Distribution	Mean	S.D.	Mode	S.D.	Mean	5%	95%
ρ_S	Beta	0.50	0.20	0.72	0.05	0.72	0.65	0.80
ρ_M	Beta	0.80	0.10	0.87	0.01	0.86	0.84	0.88
ρ_B	Beta	0.80	0.10	0.98	0.01	0.98	0.96	0.99
ρ_G	Beta	0.80	0.10	0.90	0.01	0.90	0.87	0.92
σ_A	IGamma	0.02	0.010	0.0066	0.0004	0.0067	0.0062	0.0073
σ_T	IGamma	0.02	0.010	0.0073	0.0004	0.0074	0.0067	0.0081
σ_S	IGamma	0.25	0.100	0.176	0.019	0.180	0.154	0.212
σ_M	IGamma	0.10	0.050	0.085	0.009	0.087	0.070	0.099
σ_B	IGamma	0.10	0.050	0.046	0.005	0.046	0.038	0.055
σ_G	IGamma	0.08	0.040	0.025	0.002	0.025	0.022	0.028
σ_ε	IGamma	0.02	0.010	0.012	0.003	0.015	0.009	0.021

Table 3 reports the prior and posterior distributions of the persistence and variance parameters of the shocks. The separation rate is the least persistent with a quarterly autoregressive parameter equal to 0.72. The matching and government shocks are more persistent, but still significantly less persistent than a random walk. The persistence of government consumption is 0.90 - exactly like in the data. The bargaining shock is close to a random walk. See Figures in Appendix D to compare the prior and posterior distributions of the parameters. Our model explains 100% of the variation in the six variables and thus provides a decomposition we need for the business cycle accounting exercise. The measurement error explains only negligible 1 percent of variation in vacancies and does not affect other variables.

IV. Results

We start with the analysis and interpretation of the behavior of the labor wedge produced by the model and the underlying shocks. Following most of the literature we define the labor wedge as the ratio of the marginal rate of substitution between leisure and consumption (MRS) and marginal product of labor (MP). Figure 1 depicts the behavior of these two determinants of the labor wedge. The picture forces one to conclude that most of the volatility of the labor wedge comes from variations in the marginal rate of substitution, rather than the marginal product. Though we estimate the elasticity of the utility function to be high, this result is true for most values of the elasticity used in the macro literature.

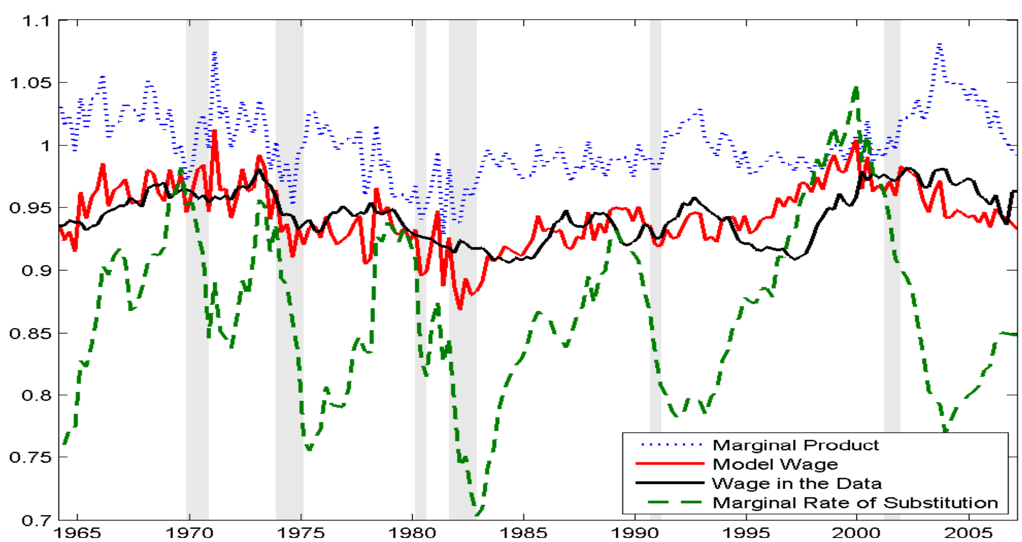

Figure 1. Variations in the bargaining set

In our model the labor wedge has a new interpretation. The MRS represents the reservation value (outside option) of workers when bargaining over the wage, which implies that the difference between the MP and the MRS represents the instantaneous welfare gain of a new match. It is clear from Figure 1 that the bargaining set narrows in good times and widens significantly in recessions. Thus, in our model increases in the labor wedge manifest decreases in the value of new matches and vice versa.

The same Figure depicts the behavior of wages predicted by the model and compares it to data (adjusted for the stochastic trend). The model predicts wages which are as volatile as in the data and the correlation between the two is high (0.56). The wage level splits the instantaneous value of the match between the worker and the firm in the proportion of their bargaining weights.

Figure 1 demonstrates that while the reservation value of workers falls in recessions, wages stay fairly constant, thus, indicating that the bargaining power of workers increases in recessions. This result supports wage stickiness as a mechanism behind the large changes in the bargaining power of the workers. However, unlike previous models, where increases in the bargaining power in recessions were a result of declines in the marginal product, in our model they are a consequence of declines in the reservation value (MRS).

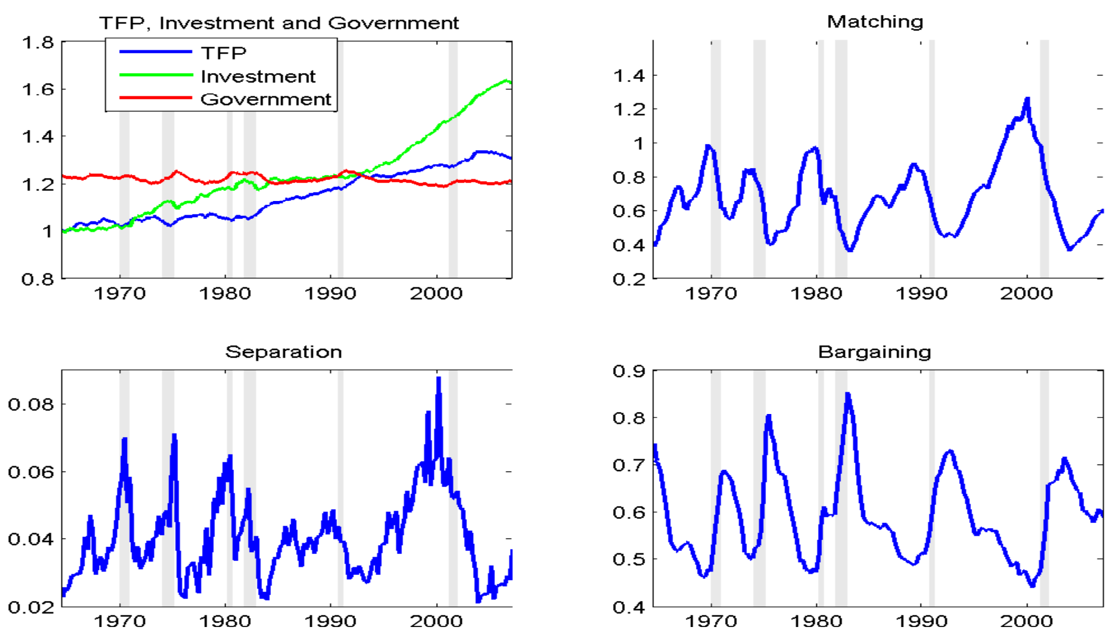

FIGURE 2. The six shocks

Now let us look at the behavior of the underlying shocks. Figure 2 describes the behavior of the six shocks over the whole forty-five year period. The shaded vertical areas correspond to the official recession periods according to NBER. Note that TFP and investment shocks are random walks with drifts, while the rest of the processes are stationary. We find that total factor productivity slows down at the beginning of each recession. The investment specific technology tends to rather increase in recessions and has a negligible effect on output and the labor market variables. This supports the main finding of CKM, that the investment wedge plays only a tertiary role in U.S. business cycles. The government shock as well as the investment shock only affect consumption and investment. Because we are primarily interested in the behavior of output, hours, unemployment and vacancies, for the rest of the exposition we shall abstract from the behavior of investment and government shocks.

From Figure 2 one can also see, that the separation shock tends to be high at the beginning of each recession. The spikes in the separation rate typically coincide with spikes observed in Shimer's data. The wave of separations usually starts earlier than the recession itself and dies out quickly - within a year after the start of a recession. As we conjectured from the behavior of the bargaining set, the bargaining power of workers increases significantly during recessions manifesting wage stickiness as a major source of inefficiency in the labor market. Notice also the large decreases in the matching efficiency during recessions.

Now let us turn to the question of why we get these large swings in the marginal rate of substitution and in the matching efficiency and the bargaining power. We argue that any model, where workers and firms decide on the margin whether to search for a job and whether to open a vacancy, would predict changes in the incentives and shocks of a similar magnitude.

First, notice that when workers choose whether to search for a job (equation (13)), they equate the cost of searching for a job, equal to the MRS in our model, with the potential benefits of forming a match times the probability of finding a party to match. The benefits are equal to the present discounted value of the wages minus the cost of working, which is also equal to MRS:

$$MRS_t = \phi_t \Gamma_t \frac{M_t}{U_t} = PV\left(W_t - MRS_t\right) \frac{M_t}{U_t} \tag{73}$$

Given that in the data the job finding probability $\frac{M_t}{U_t}$ declines significantly in recessions (documented by Shimer (2005, [19])) and the wage is fairly constant, equation (73) implies that the MRS has to fall by a fair amount. This is exactly what we get as a result of the estimate of the elasticity of the utility function of 3.54 - a significantly higher value than values of around 0.5 typically assumed in the RBC literature. This result leads us to obtain a much more volatile series for the labor wedge, which, nonetheless, behaves very similarly to previous estimates. For a comparison of our labor wedge with the labor wedge of CKM see Appendix E.

Secondly, notice that when firms choose whether to open a new vacancy, they also equate the competitive salary they pay to a head-hunter with the potential benefits of forming a match times the probability of finding a worker to fill the vacancy. The benefits are equal to the present discounted value of the marginal product minus the wage, they pay to the worker:

$$MRS_t = \left(1 - \phi_t\right) \Gamma_t \frac{M_t}{V_t} = PV\left(MP_t - W_t\right) B_t \left(\frac{U_t}{V_t}\right)^{\theta} \tag{74}$$

Given that we have already established the significant decreases in the MRS in recessions, and taking into account that in the data unemployment increases, while the number of vacancies falls and both the wage and the marginal product are not very volatile, equation (74) implies that the matching efficiency has to fall significantly in recessions.

Combining equations (73) and (74) one can find that the bargaining power of the workers is directly pinned down by the market tightness:

$$\frac{1-\phi_t}{\phi_t} = \frac{V_t}{U_t} \tag{75}$$

Thus, when unemployment increases and there are fewer vacancies, the bargaining power of workers has to increase by a comparable amount. Variations in the separation rate that we estimate are a residual of the labor accumulation equation in the productive sector.

To summarize, by allowing for changes in the marginal rate of substitution between consumption and leisure and consequently for changes in the reservation value of workers, our model both matches the volatile behavior of unemployment and vacancies and predicts absence of significant fluctuations in wages, just like in the data. Thus, our model provides an alternative mechanism which can explain Shimer's puzzle.

	Correlation of X with Y at lag k				
Shocks (X, Y)	-2	-1	0	1	2
TFP, Separation	-0.12	-0.13	-0.14	-0.11	-0.06
TFP, Matching	-0.01	0.02	0.06	0.10	0.15
TFP, Bargaining	0.23	0.18	0.11	0.05	0.00
Separation, Bargaining	-0.58	-0.50	-0.38	-0.26	-0.16
Separation, Matching	0.86	0.84	0.73	0.61	0.50
Bargaining, Matching	-0.62	-0.70	-0.76	-0.77	-0.73

Table 4. Cross Correlations of Shocks and Their Lags

Let us now take a closer look at the timing and contributions of the shocks to the variables of interest. Table 4 displays cross correlations of the shocks calculated using log deviations from steady-states. From the table it is clear that declines in TFP slightly precede increases in the separation rate. An increase in the separation rate is typically followed by an increase in the bargaining power of workers which precedes or coincides with a decrease in the matching shock. This implies that shocks to the separation rate are important at the early stages of recessions and matching and bargaining shocks come into play later.

Shutting down innovations to each one of the shocks reveals a striking picture. Figure 3 shows that absence of separation and bargaining shocks leaves the labor wedge essentially unchanged, while absence of shocks to matching efficiency produces an essentially constant labor wedge. Thus, most of the fluctuations in the labor wedge are explained by matching shocks.

To analyze the effects of each shock on output and unemployment and the timing patterns, we shall focus on the 2001 recession episode which is the last recession in our sample. Although only matching shocks matter for the labor wedge, the impact on output is not so clear-cut. Figure 4 illustrates the effects of TFP shocks, investment shocks and the labor wedge. Here by the effect of the labor wedge we mean the joint effect of separation, bargaining and matching shocks. Appendix C provides a closed form expression for the labor wedge as a combination of these three shocks, thus, demonstrating that TFP, investment and government shocks do not affect the behavior of the labor wedge. The vertical axes measures percentage deviations from the path that output would have followed, if all the wedges were constant (the random walks would preserve their drifts). The solid line depicts the actual path of output in the data. The rest of the lines depict the paths of output if we shut down innovations to just one of the shocks, eliminating its effect on the economy.

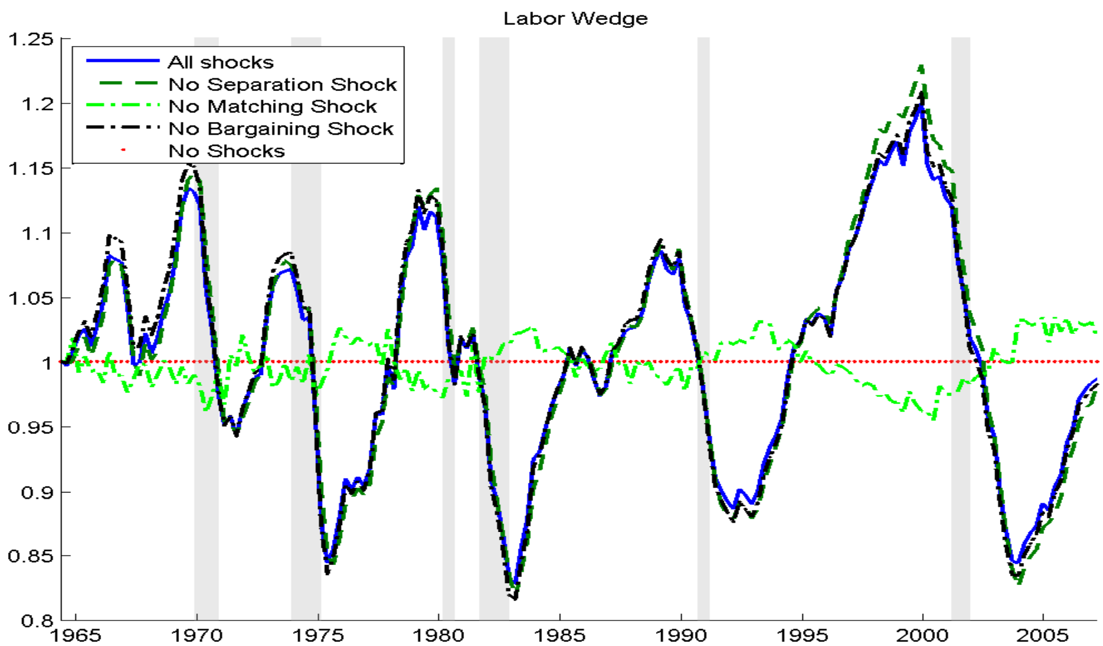

FIGURE 3. The decomposition of the labor wedge

Figure 4 shows that if there was no change in the labor wedge, the recession would have been much shorter (if at all noticeable) and twice less severe. If there was no change in total factor productivity, the recession probably wouldn't have started. Absence of investment shocks would have almost no effect on the path of output. Thus the TFP shock is at work mostly at the start of the recession of 2001. The labor wedge explains the bulk of fluctuations in output after the recession has started.

Figure 5 is the key to our paper. It decomposes the effect of the labor wedge into three pieces: the separation shock, the matching shock and the bargaining shock. Similarly to the previous graph, the solid line shows the deviation of output from the path it would have taken, if all the wedges were kept constant starting in the fall quarter of 1999. The three other lines depict the paths of output if only one of the three labor shocks returned to its steady-state level.

It is clear from Figure 4, that if there were no separation shocks at the early stage of the recession, output would have fallen by twice as little. The bargaining shocks add to the depth of the recession while the matching shock is key to the slow recovery: in absence of the adverse matching shocks the economy would have fully recovered from the recession by summer of 2003.

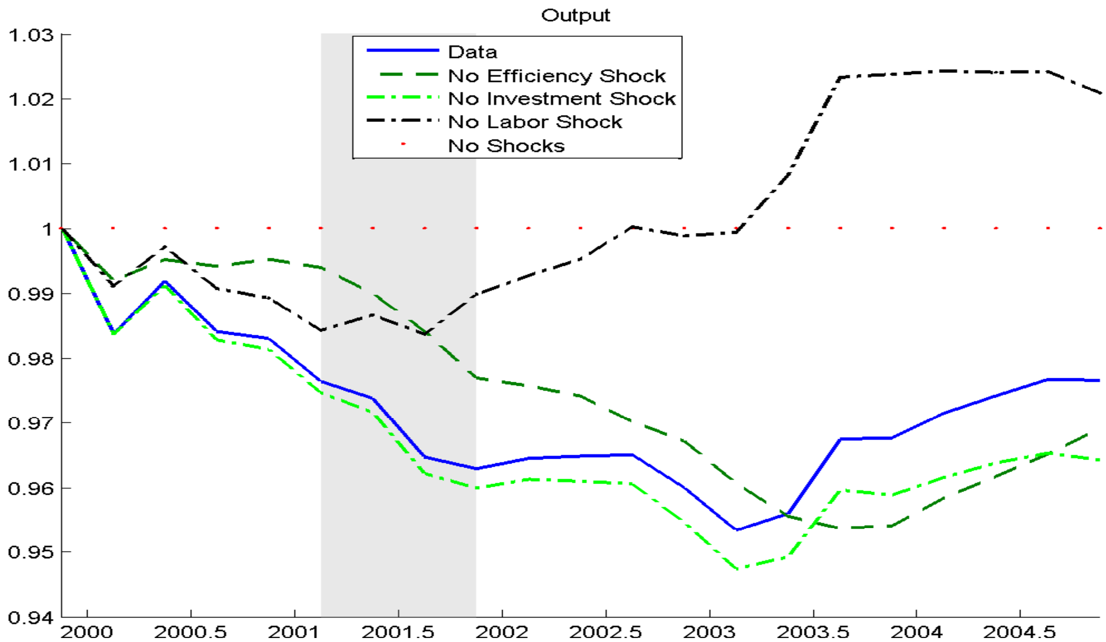

FIGURE 4. Output with all but one shock: TFP, investment, labor wedge

Figure 5 demonstrates, that although matching shocks explain a huge fraction of changes in output, they also start impacting output only after the recession has already started. As we conjectured from Table 4, shocks to TFP and the separation rate start recessions by accounting for the initial slowdown in output, while the role of bargaining and matching shocks is to deepen the recession and delay the recovery.

Figure 6 depicts a similar decomposition of unemployment. It follows from Figure 6 that separation shocks are responsible for the initial increase in unemployment. Increases in the bargaining power of workers start playing a role only once the economy is already in a recession, reinforcing this initial increase in unemployment. Declines in matching efficiency leave unemployment at a high level for a longer period of time after the recession has already ended, thus accounting for the so-called jobless recovery.

Thus, after some firms in the economy have become less productive, the role of the separation wedge is to create the initial pool of unemployed people. As the number of unemployed goes up, the reservation value of workers goes down significantly - they desperately need jobs. The sluggish response of wages drives up the bargaining power of the workers while the firm is now in a worsened position. As a result firms start posting less vacancies, and there are more and more unemployed in the market. Consistent with this explanation, the sharp increase in the bargaining power of workers accounts for the bulk of changes in unemployment and vacancies in the second phase of the recession.

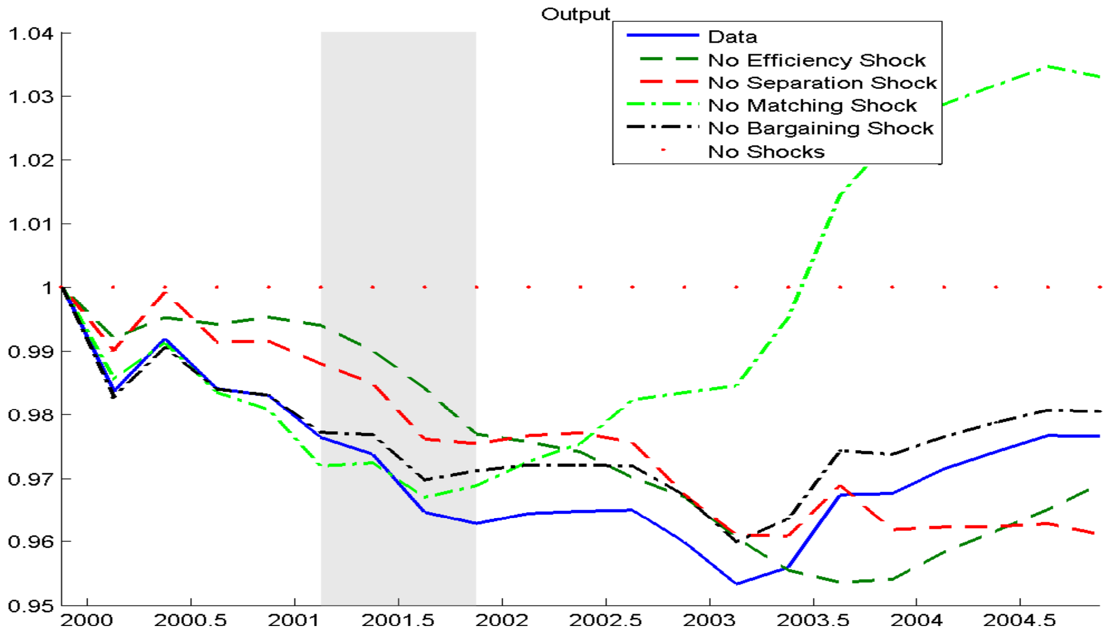

FIGURE 5. Output with all but one shock: TFP, separation, matching, bargaining

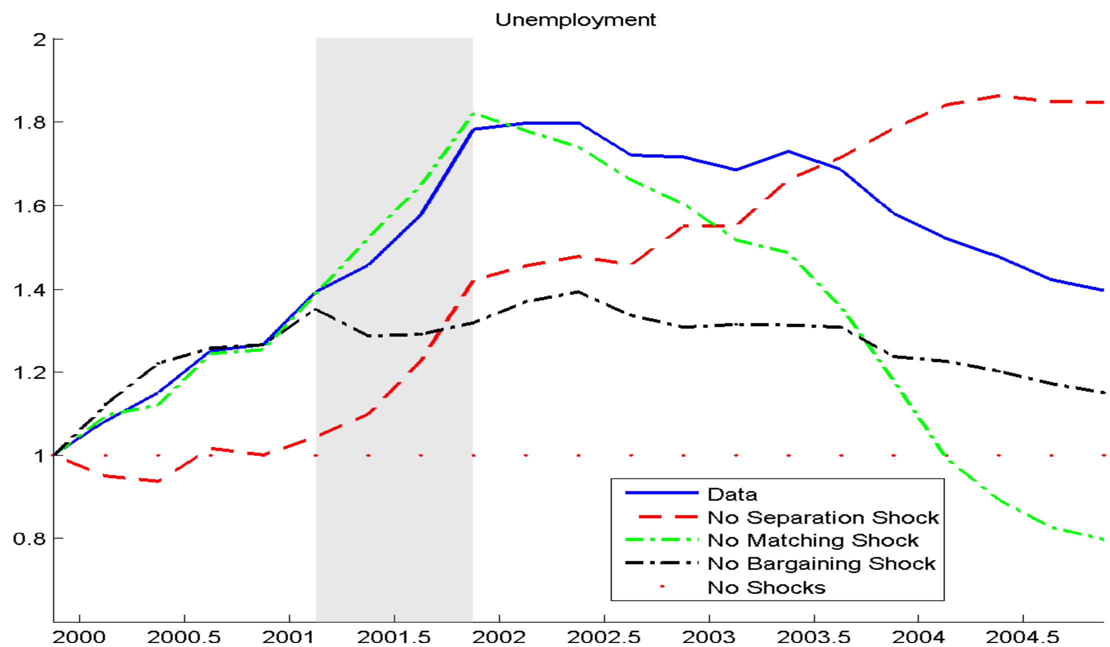

FIGURE 6. Unemployment with all but one shock: separation, matching, bargaining

As the number of workers seeking for jobs is high and the number of vacancies is low, the matching efficiency goes down, thus causing output to fall deeper and the recession to last longer. Figure 6 confirms that if there was no decline in matching efficiency the recovery from the recession would have been much faster. Hence, the so called "jobless recovery" is due mainly to matching shocks. We attribute this to some form of congestion, which still seeks an explanation. In terms of modeling it might be the case that our assumption of constant returns to scale is wrong, and when there are more parties searching for a match, the system gets congested, which would imply decreasing returns. It can also be some form of disorganization, when the least efficient and more specialized workers become desperate of finding a job and wait until better times, consistent with ideas of rest unemployment.

To measure the contribution of each shock we set all the other shocks to their steady-state values and simulate the model. We obtain paths of output, hours, unemployment and vacancies which would have taken place, if all the other distortions except one were absent. Table 5 reports the fractions of standard deviations over the whole period of output, hours, unemployment and vacancies, that can be explained by each one of the wedges. The contribution of the "labor wedge" is measured by hitting the economy by all three shocks: separation, bargaining and matching - at the same time.

Shock	TFP	Investment	Labor	Separation	Matching	Bargaining
Output	0.67	0.09	0.71	0.31	0.61	0.14
Hours	0.07	0.12	1.03	0.41	1.10	0.25
Unemployment	0.06	0.04	1.02	1.08	1.26	0.67
Vacancies	0.06	0.04	1.04	0.85	0.95	1.23

Table 5. Ratios of standard deviations explained by each shock over the whole period

Table 6 reports the same numbers from the original paper by Chari, Kehoe and McGrattan (2007, [5]). Comparing the second row of Tables 6 and 7 one can verify that the decompositions are comparable since the contributions of the TFP, Investment and Labor shocks are not that different.

Wedge	Efficiency	Investment	Labor
Output	0.73	0.31	0.59

Table 7. Ratios of standard deviations explained by each wedge
Source: Chari, Kehoe and McGrattan (2007)

Table 7 reports the same fractions of standard deviations as Table 5, but averaged over a selection of recession periods. It demonstrates that during recessions the labor wedge and TFP play a slightly more important role in business cycles that in normal times, while the contribution of investment shocks are negligible both in recessions and overall.

Shock	TFP	Investment	Labor	Separation	Matching	Bargaining
Output	0.76	0.08	0.72	0.34	0.61	0.16
Hours	0.05	0.05	1.02	0.28	0.82	0.31
Unemployment	0.04	0.02	0.99	0.48	0.59	0.60
Vacancies	0.05	0.03	1.01	0.60	0.68	1.41

Table 7. Ratios of standard deviations explained by each shock averaged over 5 recessions (70,75,82,91,01)

These results address the debate between Fujita and Ramey (2007, [9]) and Shimer (2005, [20]) on whether job destruction or job creation is more important for fluctuations in unemployment and output. We find that although shocks to job creation are more important for the behavior of output and unemployment, shocks to job destruction cannot be ignored. Changes in the separation rate account for a significant fraction of fluctuations and explain the initial increase in unemployment and decrease in output which start the recession. Thus, even though their contribution is relatively smaller, without job destruction shocks recessions might not be there in the first place. Appendix G demonstrates that these results are not a feature of the last recession only, but are robust across the last four recessions.

V. Conclusion

Motivated by the fact that variations in the labor wedge account for a large fraction of business cycle fluctuations, we develop a theory of the labor wedge and quantitatively analyze it using a model that features time-varying search and matching frictions in the spirit of Mortensen and Pissarides (1994, [18]), Shimer (2005, [19]) and Hall (2005, [12]). Using a methodology similar to that of Chari, Kehoe and McGrattan (2007, [5]) we identify the sources of fluctuations and assess the importance of job creation and job destruction shocks for business cycles.

To do so we introduce exogenous shocks to the separation rate and the matching efficiency that determine how many jobs are destroyed and created every period, and incorporate an exogenous process, governing fluctuations in the bargaining power, general enough to mimic the behavior of most known mechanisms generating time-varying bargaining power. This extension allows us to decompose the labor wedge into three different shocks: the *separation shock*, the *matching shock* and the *bargaining shock*.

We find that the labor wedge is mainly driven by matching shocks. However, both job creation and job destruction shocks play an important role in output and unemployment fluctuations. This last result has direct implications for the debate between Fujita and Ramey (2007, [9]) and Shimer (2005, [20]) on whether job destruction or job creation is more important for fluctuations in unemployment and output.

We find that job destruction and job creation shocks play a role at different points in time. In a recession, separation shocks account for the initial increase in unemployment, while matching shocks are responsible for the slow recovery.

Our results provide a potential solution to Shimer's puzzle. In our model the bargaining power of the workers increases in recessions because their reservation value falls and the wage does not. This mechanism allows us to match the observed volatility of unemployment, vacancies and wages. Hence, introducing variations in the reservation value of workers is a feature worth exploring in search and matching models.

Finally, given that matching shocks are the main source of inefficiency in the labor market and explain most of the volatility of output and unemployment, we strongly encourage the use of tools that can help smooth the matching process, such as online job sites like careerbuilder.com and monster.com. Also, taking into account the finding that separation shocks tend to start recessions, identifying the causes of layoffs might help avoid them.

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VI. Appendix.

VI.1. **Appendix A. Identification.** In this section we show how given data on allocations: output, investment, consumption, employment, vacancies and unemployment one can solve for the wedges. Let us first rewrite the equations of the model given the parametric assumptions and functional forms used in the paper:

$$X_t = K_{t+1} - (1-\delta_K) K_t \tag{76}$$

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{77}$$

$$C_t + \frac{X_t}{T_t} + G_t = Y_t \tag{78}$$

$$L_t = (1-\delta_{Lt}) L_{t-1} + B_t U_t^{\theta} V_t^{1-\theta} \tag{79}$$

$$\frac{1}{T_t} = \beta E_t \frac{C_t}{C_{t+1}} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + \frac{(1-\delta_K)}{T_{t+1}} \right) \tag{80}$$

$$\mu_t = w_t - C_t \chi (L_t + U_t + V_t)^{\gamma} + \beta E_t \frac{C_t}{C_{t+1}} \mu_{t+1} (1-\delta_{Lt+1}) \tag{81}$$

$$\eta_t = (1-\alpha) \frac{Y_t}{L_t} - w_t + \beta E_t \frac{C_t}{C_{t+1}} \eta_{t+1} (1-\delta_{Lt+1}) \tag{82}$$

$$C_t \chi (L_t + U_t + V_t)^{\gamma} = \eta_t B_t \left(\frac{U_t}{V_t} \right)^{\theta} \tag{83}$$

$$C_t \chi (L_t + U_t + V_t)^{\gamma} = \mu_t B_t \left(\frac{V_t}{U_t} \right)^{1-\theta} \tag{84}$$

$$\frac{\eta_t}{\mu_t} = \frac{1-\phi_t}{\phi_t} \tag{85}$$

Now we shall describe a mechanism to recover the wedges given parameters and functional forms. Given data on consumption C_t (or government spending G_t), output Y_t, investment X_t, employment L_t+V_t, number of vacancies V_t and the unemployment rate $\frac{U_t}{L_t+V_t+U_t}$, one can uniquely recover the time path for the variables of interest L_t,V_t,U_t. Then equation (76) uniquely pins down the path for capital given the initial level K_0, equation (77) pins down the efficiency wedge A_t, equation (78) pins down consumption or government spending, equation (80) can be solved forward to obtain the path for the investment wedge as in Chari, Kehoe and McGrattan (2007).

From equations (83) and (84) it follows that $\eta_t U_t = \mu_t V_t$. Then summing up equations (81) and (82) one obtains:

$$-C_t \chi (L_t + U_t + V_t)^{\gamma} + (1-\alpha) \frac{Y_t}{L_t} = \mu_t \left(1 + \frac{V_t}{U_t} \right) - \beta E_t \frac{C_t}{C_{t+1}} \mu_{t+1} \left(1 + \frac{V_{t+1}}{U_{t+1}} \right) (1-\delta_{Lt+1}) \tag{86}$$

Using equation (83) the lagrange multiplier μ_t can be expressed as a function of the matching shock B_t:

$$\mu_t = \frac{\chi (L_t + U_t + V_t)^\gamma}{B_t \left(\frac{V_t}{U_t}\right)^{1-\theta}} \tag{87}$$

Also the separation rate is connected to the matching shock through the labor accumulation equation (79):

$$(1 - \delta_{Lt+1}) = \frac{L_{t+1} - B_{t+1} U_{t+1}^{\theta} V_{t+1}^{1-\theta}}{L_t} \tag{88}$$

Then substituting equations (87) and (88) into equation (86) we obtain:

$$\left(\frac{1 + \frac{V_t}{U_t}}{\left(\frac{V_t}{U_t}\right)^{1-\theta}} \frac{1}{B_t} - 1 \right) L_t = \frac{(1-\alpha) Y_t}{C_t \chi (L_t + U_t + V_t)^\gamma} +$$

$$+\beta E_t \frac{C_t}{C_{t+1}} \frac{1 + \frac{V_{t+1}}{U_{t+1}}}{\left(\frac{V_{t+1}}{U_{t+1}}\right)^{1-\theta}} \left(\frac{L_{t+1} + U_{t+1} + V_{t+1}}{L_t + U_t + V_t} \right)^\gamma \left[\frac{L_{t+1}}{B_{t+1}} - U_{t+1}^{\theta} V_{t+1}^{1-\theta} \right] \tag{89}$$

Equation (VI.1) provides a forward-looking equation for the matching shock B_{t+1} as a function of B_t. Solving this equation recursively given some initial value B_0 we can recover the whole path for the matching shock. Then equation (88) allows us to back up the separation rate, equations (84) and (83) allow us to calculate the lagrange multipliers μ_t and η_t. Then from equation (85) we can compute the bargaining wedge ϕ_t.

Altogether equations (76-85) describe a one-to-one mapping between the data and the underlying wedges. However the algorithm described here is hard to implement directly for two reasons. First, the equations are forward looking and can only be solved under certain assumptions about expectation formation. Second, many of the parameters of the model are unknown and cannot be simply calibrated from microeconomic data. That is the reason why we postulate stochastic processes for the wedges, linearize the model around a steady-state to compute an approximate solution and use Kalman filter to recover the underlying processes for the wedges.

VI.2. **Appendix B. Computing the Steady-State.**

0) Choose a value of L_{ss}
1) $z_{ss} = (a_{ss} \tau_{ss}^{\alpha})^{\frac{1}{1-\alpha}}$
2) Denote $\varphi = \left(\left(\frac{z_{ss}}{\beta} + \frac{1-\delta_K}{\tau_{ss}} \right) / \alpha a_{ss} \right)^{-\frac{1}{1-\alpha}}$
3) $k_{ss} = \varphi L_{ss}$
4) $y_{ss} = a_{ss} \varphi^{\alpha} L_{ss}$
5) $c_{ss} = \left[(1 - g_{ss}) a_{ss} \varphi^{\alpha} - \left(z_{ss} - \frac{(1-\delta_K)}{\tau_{ss}} \right) \varphi \right] L_{ss}$
6) $B_{ss} = \frac{1}{\omega_{ss}} \left(\frac{\phi_{ss}}{1-\phi_{ss}} \right)^{1-\theta}$
7) $U_{ss} = \omega_{ss} \delta_L L_{ss}$

8) $V_{ss} = \frac{1-\phi_{ss}}{\phi_{ss}} U_{ss}$
9) $\xi = \frac{y_{ss}}{L_{ss}c_{ss}} \frac{1-\alpha}{\left(1+\frac{\omega_{ss}}{\phi_{ss}}\left(1-\frac{\beta}{z_{ss}}(1-\delta_L)\right)\right)}$
10) We have assumed a normalization $\chi = \frac{\xi}{(L_{ss}+U_{ss}+V_{ss})^{\gamma}}$
11) $\kappa_{ss} = \xi c_{ss}$
12) $m_{ss} = 1 - \delta_L$
13) $\Gamma_{ss} = \xi c_{ss} \frac{\omega_{ss}}{\phi_{ss}}$
14) $w_{ss} = \phi_{ss}(1-\alpha)\frac{y_{ss}}{L_{ss}} - (1-\phi_{ss})\kappa_{ss}$
15) $j_{ss} = \frac{\beta}{z_{ss}}$
16) $q_{ss} = \kappa_{ss}$

VI.3. **Appendix C. Expressing the Labor Wedge in Terms of the Three Wedges.** Combining equations (33) and (34) and taking into account the functional forms we get:

$$\frac{1-\phi_t}{\phi_t} = \frac{V_t}{U_t} \tag{90}$$

$$\Gamma_t = -\frac{U'_{U_t}U_t + U'_{V_t}V_t}{U'_{C_t}M_t} = MRS_t \frac{V_t + U_t}{M_t} \tag{91}$$

Combining equation (90) with equation (41) it follows, that:

$$\frac{V_t + U_t}{M_t} = \frac{U_t \frac{1-\phi_t}{\phi_t} + Ut}{B_t U_t \left(\frac{V_t}{U_t}\right)^{1-\theta}} = \frac{1}{\phi_t} \frac{1}{B_t \left(\frac{1-\phi_t}{\phi_t}\right)^{1-\theta}} = \frac{1}{B_t \phi_t^{\theta} (1-\phi_t)^{1-\theta}} \tag{92}$$

Substituting these into equation (32) moving everything except the marginal product to one side we get:

$$MP_t = \left(1 + \frac{1}{B_t \phi_t^{\theta}(1-\phi_t)^{1-\theta}} - \beta L^{-1} \frac{U'_{C_t}}{U'_{C_{t-1}}} \frac{(1-\delta_{Lt})}{B_t \phi_t^{\theta}(1-\phi_t)^{1-\theta}}\right) MRS_t \tag{93}$$

where we define the inverse lag operator as follows

$$L^{-1}X_t = E\left(X_{t+1}|\Omega_t\right)$$

Equation (93) shows how the separation, bargaining and matching wedges together form connection between the marginal product of labor and the marginal rate of substitution between leisure and consumption: the labor wedge.

VI.4. **Appendix D. Prior and Posterior Distributions of Parameters.**

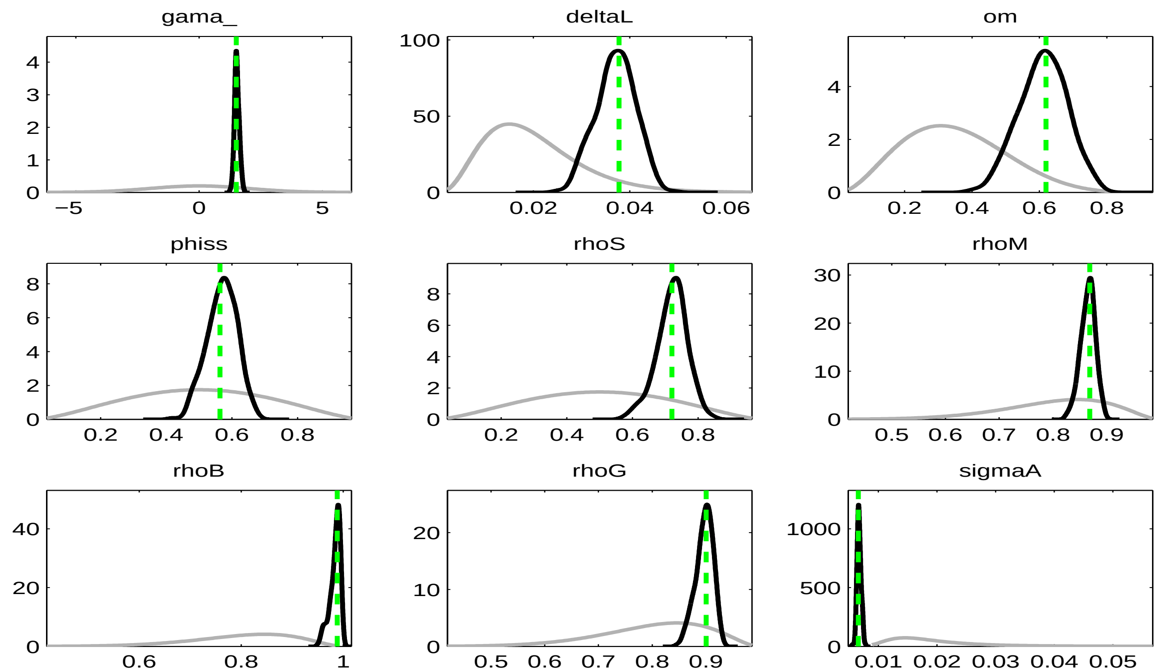

FIGURE 7. Prior (grey) and posterior (black) distributions of parameters

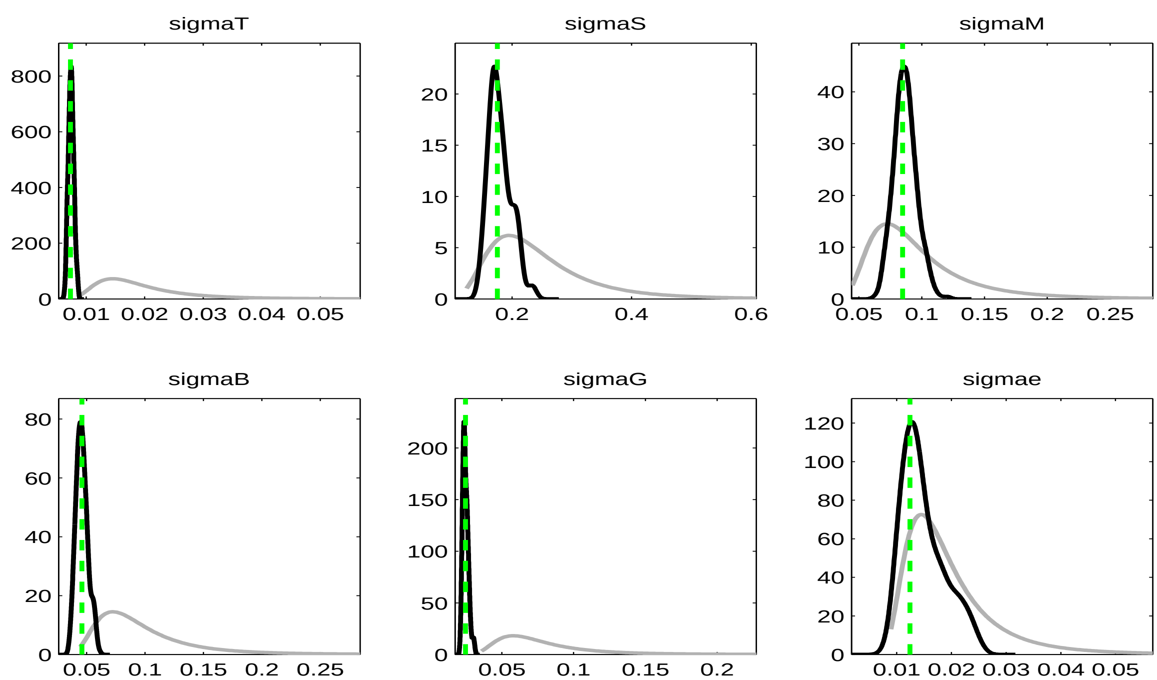

FIGURE 8. Prior (grey) and posterior (black) distributions of parameters

VI.5. **Appendix E. Comparison of the Labor Wedge with Standard Estimates.**

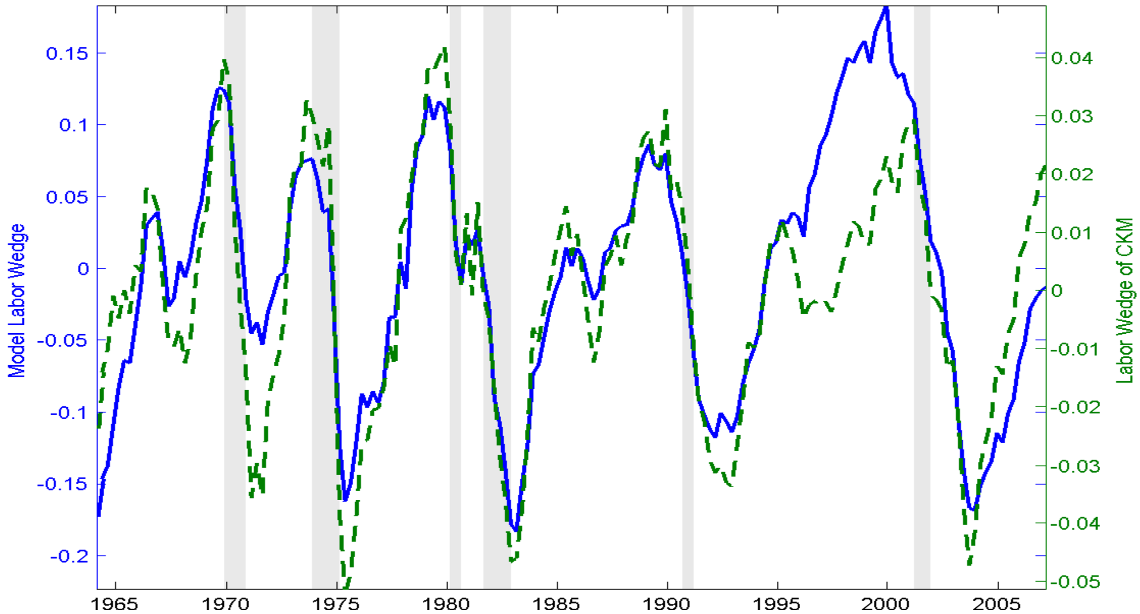

FIGURE 9. Comparison of the Labor Wedge to the Estimate of CKM

VI.6. **Appendix F. Full Decompositions and Correlation Structure.**

Shock	TFP	Investment	Government	Labor
Output	0.76	0.08	0.05	0.71
Consumption	0.71	0.87	0.66	0.48
Investment	0.68	0.47	0.48	0.42
Hours	0.05	0.05	0.06	1.02
Unemployment	0.04	0.02	0.03	0.99
Vacancies	0.05	0.03	0.03	1.01

Table F1. Ratios of standard deviations explained by each shock averaged over 5 recessions (70,75,82,91,01)

Shock	TFP	Separation	Matching	Bargaining
Output	0.76	0.34	0.62	0.16
Consumption	0.71	0.36	0.56	0.19
Investment	0.68	0.48	0.52	0.06
Hours	0.05	0.28	0.82	0.31
Unemployment	0.04	0.48	0.57	0.60
Vacancies	0.05	0.60	0.68	1.41

Table F2. Ratios of standard deviations explained by each shock averaged over 5 recessions (70,75,82,91,01)

Shock	TFP	Investment	Government	Labor
Output	0.67	0.09	0.05	0.70
Consumption	0.68	0.86	0.65	0.51
Investment	0.57	0.43	0.40	0.41
Hours	0.07	0.12	0.08	1.02
Unemployment	0.06	0.04	0.04	1.00
Vacancies	0.06	0.04	0.04	1.00

Table F3. Ratios of standard deviations explained by each shock over the whole period (1964-2007)

Shock	TFP	Separation	Matching	Bargaining
Output	0.67	0.32	0.61	0.14
Consumption	0.68	0.38	0.63	0.07
Investment	0.57	0.38	0.51	0.05
Hours	0.07	0.41	1.10	0.25
Unemployment	0.06	1.08	1.26	0.67
Vacancies	0.06	0.85	0.95	1.23

Table F4. Ratios of standard deviations explained by each shock over the whole period (1964-2007)

	Correlation of X with Y at lag k				
Shocks (X,Y)	-2	-1	0	1	2
TFP, Investment	-0.70	-0.68	-0.61	-0.58	-0.54
TFP, Government	0.08	0.03	-0.01	-0.04	-0.06
Investment, Government	-0.14	-0.16	-0.15	-0.13	-0.10
TFP, Separation	-0.44	-0.45	-0.44	-0.39	-0.29
TFP, Matching	-0.39	-0.31	-0.21	-0.12	-0.00
TFP, Bargaining	0.26	0.11	-0.02	-0.15	-0.26
Separation, Bargaining	0.89	0.85	0.72	0.57	0.43
Separation, Matching	-0.77	-0.68	-0.55	-0.39	-0.25
Bargaining, Matching	-0.72	-0.83	-0.89	-0.90	-0.85

Table F5. Cross Correlations of Shocks and Their Lags

VI.7. **Appendix G. Decompositions of Unemployment in 4 recessions and of Hours and Vacancies in Recession 2001.**

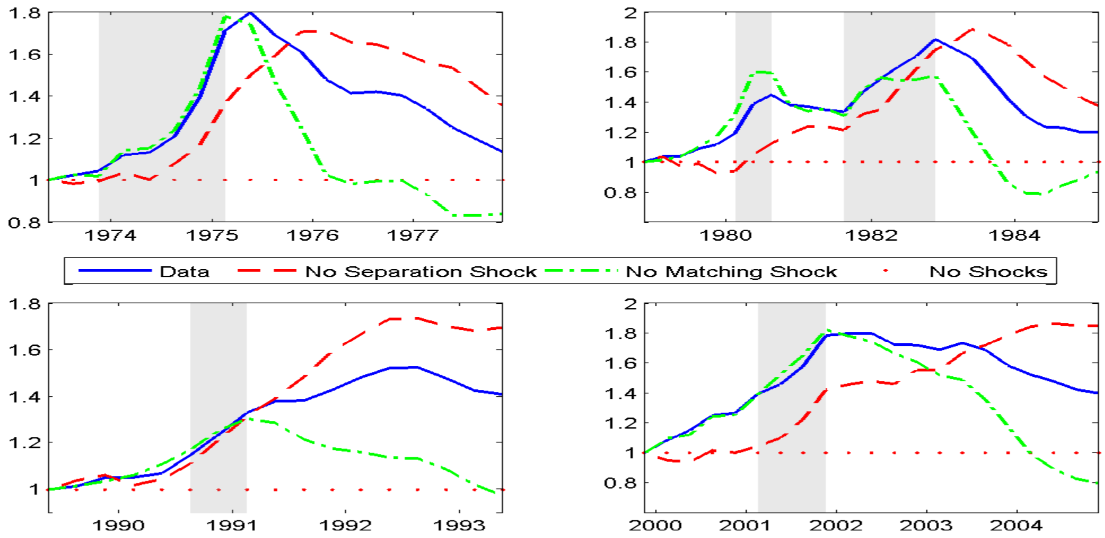

FIGURE 10. Effects of separation and matching shocks on unemployment in the last four recessions

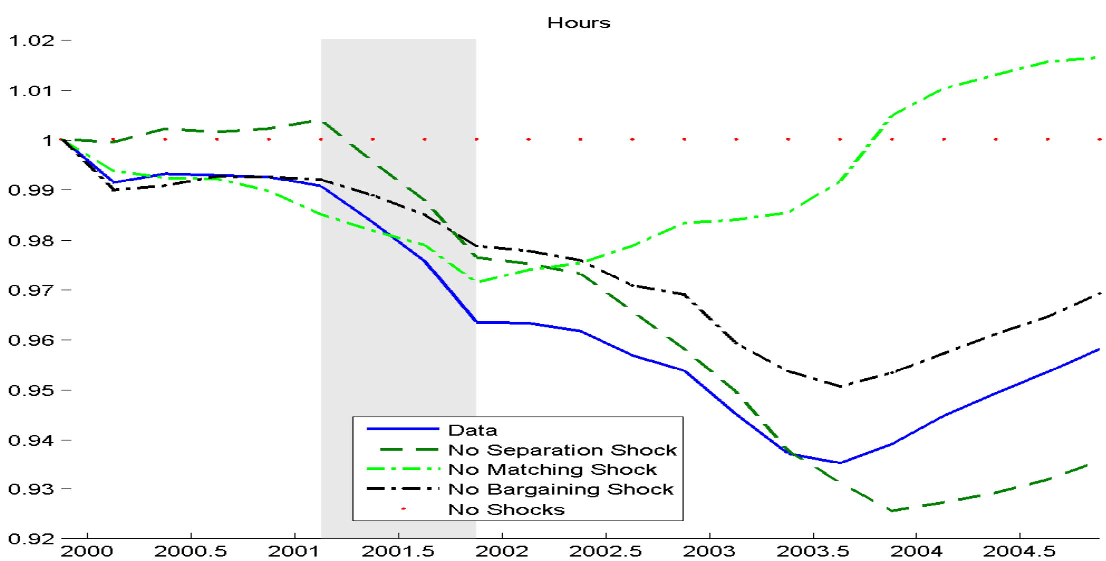

FIGURE 11. Hours with all but one shock: separation, matching, bargaining

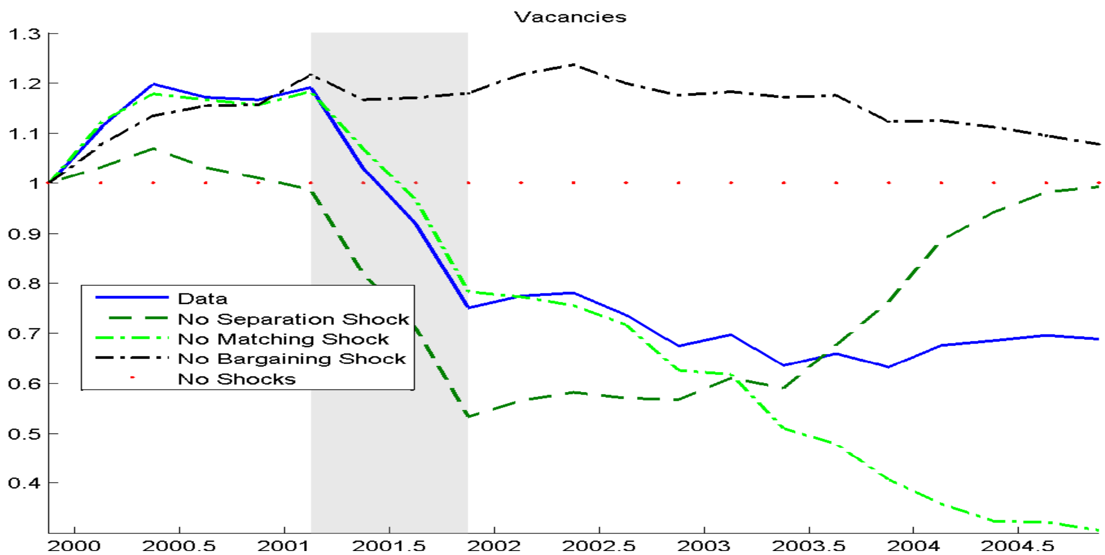

FIGURE 12. Vacancies with all but one shock: separation, matching, bargaining