An Empirical Model of Intra-Household Allocations and the Marriage Market*

Eugene Choo
University of Calgary

Shannon Seitz
Boston College and
Queen's University

Aloysius Siow
University of Toronto

December 11, 2007

Abstract

We develop an empirical collective matching model with endogenous marriage formation, participation, and labor supply decisions. The sharing rule in our collective matching model arises endogenously as a transfer that clears the marriage market. With information on at least two independent marriage markets, incorporating matching in the collective model allows us to identify the sharing rule from observations on marriage decisions, regardless of the labor supply decisions of wives and husbands. For couples in which both partners work, introducing the marriage market in the collective model generates a set of over-identifying restrictions that allow us to test whether the sharing rule that rationalizes labor supplies is consistent with the sharing rule that clears the marriage market. We estimate the sharing rule using 2000 Census data and implement a semi-parametric test of the over-identifying restrictions.

*We would like to thank Karim Chalak, Peter Gottschalk, Arthur Lewbel, seminar participants at Duke, New York University, the University of Connecticut, and the 2007 Analytical Labor Economics Conference at the University of Chicago for many helpful comments. Seitz gratefully acknowledges the support of the Social Sciences and Humanities Research Council of Canada.

1 Introduction

The unitary model of the household, long the standard for analyzing household behavior, has been replaced by the collective model of Chiappori (1988, 1992) in recent years. The reason for the success of the collective model is twofold. First, the collective model is appealing on theoretical grounds. The unitary model assumes household members act as if they share a common set of preferences. In contrast, men and women retain distinct and potentially conflicting individual preferences following marriage in the collective model. The second reason for the success of the collective model is that a large body of empirical evidence consistently finds the restrictions implied by the unitary model are rejected while those implied by the collective model are not.[1]

Within the collective model, the resource allocation process is summarized by the sharing rule, a lump sum transfer of non-labor income within the household, taken as given by couples when household allocations are chosen. Under a minimal set of assumptions it is possible to uncover preferences as well as the sharing rule, up to an additive constant, from observations on labor supplies.[2] This is an extremely useful result and allows the researcher to gain valuable insight into the inner workings of the household. However, the basic collective model restricts attention to currently married couples and is silent on other important questions such as: Why do some marriages form and not others? Where does the sharing rule come from? As stated by Lundberg and Pollak (1996):

> *Models that analyze bargaining within existing marriages can give only an incomplete picture of the determinants of the well-being of men and women. The marriage market is an important determinant of distribution between men and women. At a minimum, the marriage market determines who marries and who marries whom.*

The above quotation suggests that empirical models of intra-household allocations in *existing* marriages are well developed. The next question to be

[1] A partial list includes Lundberg (1988), Thomas (1990), Fortin and Lacroix (1997), Chiappori, Fortin, and Lacroix (2002), and Duflo (2003).

[2] The standard identifying assumptions in the collective model are that household allocations are Pareto efficient and preferences are egoistic or caring. For further details, see Browning and Chiappori (1998) and Chiappori and Ekelund (2002).

addressed in this research program is clear: how do we empirically investigate marriage matching with intra-household allocations? Our paper represents one step in this direction.

The goal of this paper is to develop an empirical framework for analyzing both marriage matching and the intra-household allocation of resources. A primary criteria in this development is that the empirical framework deliberately minimizes restrictions on observed behavior. To this end, we take as our starting point Chiappori's (1992) collective model of intra-household allocations. Standard assumptions, Pareto efficiency and egoistic preferences, from the collective framework are maintained and no additional assumptions are imposed on the intra-household problem for married couples. We then embed the collective model in the marriage matching model of Choo and Siow (2006; hereafter CS). Within CS, utility is transferable and equilibrium transfers clear the marriage market. The model of CS produces a nonparametric matching function that is consistent with any observed marriage matching pattern in a single marriage market. Together, the collective model and the model of CS produce a flexible empirical model of marriage and household labor supply.

From the collective matching model, we derive a sharing rule that is the equilibrium outcome of marriage matching. In contrast to the standard collective framework, our theory on marriage matching produces an exact functional form for the sharing rule. Analogous to Chiappori, Fortin, and Lacroix (2002; hereafter CFL), the sharing rule in our framework depends on 'distribution factors,' factors that affect household allocations only through the sharing rule (they do not directly affect preferences or the budget set). As in CFL, the distribution factors in our sharing rule are characteristics that determine one's outside options in the marriage market, including the stocks of men and women with the same characteristics as the husband and wife, respectively, in the market. We thus provide a formal justification for intra-household transfers that depend on marriage market conditions. Our equilibrium sharing rule is also a function of the wages and population supplies of all other types of men and women that are available in other types of matches. Thus, our model generates a sharing rule with many distribution factors.

Our framework is very flexible. We allow preferences and marital production technologies to vary depending on the participation status of the wife and on the characteristics of both spouses. Our framework also provides a convenient way to model marriage decisions in combination with discrete and/or continuous labor supply decisions for married men and women.[3] In our application, it is assumed husbands choose continuous hours and wives either choose continuous hours or not to work. A natural interpretation of this set-up is that individuals choose whether to enter 'specialized' (non-working wife) or 'non-specialized' (working wife) marriages. Our framework can be readily extended to cases where one or both spouses are not working. As we demonstrate below, identification of the sharing rule does not rely upon on variation in labor supplies within married households in our framework.

We establish identification of all of the structural parameters analytically. Identification in our framework is therefore completely transparent. We show:

1. Identification of the collective model is exactly the same as in CFL for couples in non-specialized marriages. In this instance, the sharing rule can be identified up to an additive constant from observations on labor supply. What is new in our framework is that the sharing rule can also be identified independently from the marriage decision. As a result, our framework generates over-identifying restrictions that allow us to test whether the sharing rule arising from the marriage market is consistent with the sharing rule that determines labor supplies within married households. The goal of our empirical analysis will be to test these restrictions.

2. With information on at least two independent marriage markets, one can non-parametrically identify the sharing rule from observations on marriage decisions, regardless of the labor supply decisions of wives and husbands. The introduction of information on marriage decisions is crucial for identification in the case where one or both spouses are not working, as information on behavior within the household is limited when there is no variation in household labor supply.[4]

We are able to identify all of the structural parameters of interest. As a result, our model is both flexible and at the same time amenable to policy analysis.

We estimate the collective matching model for non-specialized couples using data from the 2000 United States Census. After estimating the model,

[3]Blundell, Chiappori, Magnac and Meghir (2007), Vermeulen (2006), and Lise and Seitz (2007) present collective models in which one or both spouses do not work full time.

[4]Our result can be generalized to any discrete choice of hours, including zero hours, part-time and full-time work.

we test the restrictions on labor supplies and marriage decisions implied by the collective matching model. As pointed out by Blundell, Browning, and Crawford (2003) in a related context, a rejection of the collective framework using parametric tests could arise due to a failure of the collective model itself or to functional form misspecification of preferences. To this end, we formulate a semi-parametric test of our collective model which involves comparing the partial derivatives of the sharing rule from labor supplies to those from marriage decisions.[5] The results indicate **coming soon...**

The remainder of the paper is organized as follows. We survey the related literature in Section 2. In Section 3, we describe our benchmark version of the collective model and Section 4 describes the marriage market and the equilibrium. In Section 5, we establish conditions under which the structural parameters of the model (preference parameters and the sharing rule) are identified. In Section 6 we show how the restrictions of our model can be tested nonparametrically. We nonparametrically estimate our model using data from the 2000 Census and present the estimation results in Section 7. Section 8 concludes.

2 Related Literature

We are indebted to a large literature. The study of intra-household allocations began with, among others, Becker (1973, 1994; summarized in 1991), the bargaining models of Manser and Brown (1980) and McElroy and Horney (1981), McElroy (1990) and the collective model of Chiappori (1988, 1992). A related literature, encompassing a diverse set of models, has studied the link between marriage market conditions and marriage rates. Becker (1973) was the first to consider the relationship between sex ratios and marriage rates. Brien (1997) tests the ability of several measures of marriage market conditions to explain racial differences in marriage rates. The link between sex ratios and household outcomes was also extended to the labor supply decision. Grossbard-Schectman (1984) constructs a model where more favorable conditions in the marriage market improve the bargaining position of

[5] It should be emphasized that our identification results are still parametric in the sense that some assumptions on the functional form of preferences are imposed, namely that preferences are egoistic. Chiappori (1988) develops non-parametric identification results for the collective model. In recent work, Vermeulen et al. (2007) extend the results of Chiappori to a collective framework with externalities and public goods.

individuals within marriage. One implication of Grossbard-Schectman and related models that has been tested extensively in the literature is that, for example, an improvement in marriage market conditions for women translates into a greater allocation of household resources towards women, which has a direct income effect on labor supply. Tests of this hypothesis have received support in the literature (see among others, Becker, 1981; Grossbard-Schectman, 1984, 1993; Grossbard-Schectman and Granger, 1998; Chiappori, Fortin, and Lacroix, 2002; Seitz, 2004; Grossbard and Amuedo-Dorantes, 2007). Our empirical work considers the link between the sex ratio and both marriage and labor supply decisions in a general version of the collective model with matching.

Several important predecessors of our work integrate the collective model and the marriage market (Becker and Murphy, 2000; Browning, Chiappori and Weiss, 2003) and extend the integrated model to consider pre-marital investments (Chiappori, Iyigun, and Weiss, 2006; Iyigun and Walsh, 2007). In these integrated collective models and in our work the sharing rule arises endogenously in the marriage market. Our paper differs from this recent work in focus. Our goal is to develop an empirical framework that minimizes *a priori* restrictions on marriage matching and labor supply patterns. In this respect, our empirical framework can be used to test some of the qualitative predictions of the existing integrated models.

Our treatment of discrete labor supply choices within the collective model, while different in formulation, was influenced by the work of Blundell, Chiappori, Magnac, and Meghir (2007). Blundell, et al. (2007) establish identification of the collective model in the case where the labor supply decision of one spouse is discrete and of the other spouse is continuous. In contrast to Blundell et al. (2007), information on marriage behavior can be used to identify the sharing rule in our framework, even in the case where both household members are not working.

Our static model is restrictive. We assume spouses have access to binding marital agreements and there is no divorce. There is an active empirical literature studying dynamic intra-household allocations and marital behavior. Seitz (2004) constructs and estimates a dynamic model in which the sex ratio, marriage, and employment decisions are jointly determined. She finds that variation in the ratio of single men to single women across race can explain much of the black-white differences in marriage and employment in the US. Brien, Lillard, and Stern (2006) estimate a dynamic model of cohabitation, and divorce where individuals learn about the quality of their match over time. Del Boca and Flinn (2006a, 2006b) estimate models of household labor supply where the household members can choose to interact in either a cooperative or a noncooperative fashion. Mazzocco and Yamaguchi (2006) study savings, marriage, and labor supply decisions in a collective framework, in which an individual's weight in the household's allocation process depends on the outside options of each spouse, in this case, divorce. Choo and Siow (2005) estimate a dynamic nonparametric matching model. Our focus here is on developing and estimating an empirical model of intrahousehold allocations and matching that imposes a minimal set of assumptions. Incorporating dynamics in our framework in the spirit of Choo and Siow (2005) is an important extension left to future work.

3 The Collective Model

Consider a society that is composed of many segmented marriage markets. For expositional ease, assume there is one type of man and one type of woman within each society. We extend our analysis to multiple types in the empirical analysis described in Section 6. All men and women within the same market have the same ex-ante opportunities and preferences. Let m be the number of men and f be the number of women within a market.

Decisions are made within the model in two stages. In the first stage, individuals choose whether to marry and whether the wife works within the marriage. We refer to marriages in which the wife does not work in the labor market as *specialized* marriages (s) and marriages in which both spouses work as *non-specialized* marriages (n). The index k, $k \in \{n, s\}$ describes whether a married couple is non-specialized or specialized.[6] In the second stage, labor supply decisions for working spouses and consumption allocations are chosen. For simplicity, all men and all unmarried women are assumed to have positive hours of work.

3.1 Preferences

Let C and c be the private consumption of women and men, respectively, and H and h denote their respective labor supplies. Preferences are described by:

$$U_k(C, 1 - H) + \Gamma_k + \varepsilon_k$$

[6] It is straightforward to extend our model to the case where neither partner works.

for married women and

$$u_k(c, 1-h) + \gamma_k + \epsilon_k$$

for married men. For both spouses, the first term is defined over consumption and leisure and affects the intrahousehold allocation. The last two terms, as in CS, affect marriage behavior but do not directly influence the intrahousehold allocation.

The parameters Γ_k and γ_k capture invariant gains to a marriage of type k for wives and husbands, respectively and are assumed to be separable from consumption and leisure.[7] Idiosyncratic type I extreme value preference shocks, ε_k and ϵ_k, are realized before marriage decisions are made. It is assumed that the preference shocks do not depend on the specific identity of the spouse.

3.2 Intrahousehold allocations

We begin by considering the second stage intrahousehold decision process. This is the familiar setting of the collective model, first developed by Chiappori (1988, 1992). We consider in turn the intra-household allocation problem for singles, non-specialized and specialized couples.

3.2.1 Singles

The problem facing a single woman is:

$$\max_{\{C,H\}} U_0(C, 1-H) + \Gamma_0 + \varepsilon_0 \tag{S}$$

subject to the budget constraint

$$C \leq W_0 H + A_0$$

and likewise for a single man:

$$\max_{\{c,h\}} u_0(c, 1-h) + \gamma_0 + \epsilon_0$$

subject to the budget constraint

$$c \leq w_0 h + a_0$$

[7] As in CS, invariant gains to marriage allow the model to fit the observed marriage matching patterns in the data.

3.2.2 Non-specialized couples

Consider a husband and wife in a couple where both partners work. Total non-labor family income, denoted A, and wages for the husband w and wife W are realized before marriage decisions are made. The social planner's problem for the household is:

$$\max_{\{c,C,h,H\}} U_n(C,L) + \Gamma_n + \varepsilon_n + \omega_n \Big[u_n(c,l) + \gamma_n + \epsilon_n \Big] \tag{PN}$$

subject to the family budget constraint:

$$c + C \leq A + WH + wh$$

where the Pareto weight on the husband's utility in non-specialized marriages is ω_n.

A major insight of Chiappori (1988) is that, if household decisions are Pareto efficient, the above program can be decentralized subject to a lump sum transfer or *sharing rule*, denoted $\tau_n(W, w, A, \mathbf{R})$. At present, we conjecture that the sharing rule is of a form similar to that in CFL: transfers depend on wages, nonlabor income, and a set of 'distribution factors' $\mathbf{R}$, which contains labor and marriage market conditions for all the possible matches an individual could have entered. The decentralized problem for the wife in the second stage is:

$$\max_{\{C,L\}} U_n(C,L)$$
$$\text{subject to } C \leq WH + \tau_n(W, w, A, \mathbf{R})$$

and the problem facing husbands in the second stage is:

$$\max_{\{c,l\}} u_n(c,l)$$
$$\text{subject to } c \leq wh + A - \tau_n(W, w, A, \mathbf{R})$$

The sharing rule in the decentralized problem, and the Pareto weights in the social planner's problem, are treated as pre-determined at the point consumption and leisure allocations are chosen. The large literature on collective models is, with few exceptions, agnostic regarding the origins of the sharing rule. A central focus of our paper is to derive a sharing rule from marriage market clearing. In Section 4.1, we show that marriage market clearing generates a sharing rule of precisely the form presented here.

3.2.3 Specialized couples

For households in which the wife does not work, the social planner's problem is:

$$\max_{\{c,C,h\}} U_s(C,1) + \Gamma_s + \varepsilon_s + \omega_s \Big[u_s(c,l) + \gamma_s + \epsilon_s\Big] \tag{PS}$$

subject to the family budget constraint:

$$c + C \leq A + w(1-h)$$

where the Pareto weight on the husband's utility in specialized marriages is ω_s. The decentralized problem for the husband is

$$\max_{\{c,l\}} u_s(c,l,\epsilon_g)$$
$$\text{subject to } c \leq wh + A - \tau_s(w,A,\mathbf{R})$$

and the wife simply receives $U_s(C,1)$, where $C = \tau_s(w,A,\mathbf{R})$. Since the wife is not working, the Pareto weight in specialized marriages only depends on the husband's wage, non-labor income, and the distribution factors.[8]

3.3 The marriage decision

In the first period, agents decide whether to marry and whether to specialize. Once the idiosyncratic gains from marriage, ε_k and ϵ_k, are realized, individuals choose the household structure that maximizes utility. Individuals have three alternatives: remain single, enter a specialized marriage, or enter a non-specialized marriage. For women, the indirect utility from remaining single is:

$$V_0(\varepsilon_0) = Q_0[W_0, Y_0] + \Gamma_0 + \varepsilon_0$$

from entering a specialized marriage is:

$$V_s(\varepsilon_s) = Q_s[Y_s] + \Gamma_s + \varepsilon_s$$

and from entering a non-specialized marriage is:

$$V_n(\varepsilon_n) = Q_n[W, Y_n] + \Gamma_n + \varepsilon_n$$

[8]Although individual types have been suppressed here for convenience, the sharing rule will in general depend on characteristics of both spouses and also on the society in which the couple resides.

where

$$Y_s = \tau_s(w, A, \mathbf{R})$$
$$Y_n = \tau_n(W, w, A, \mathbf{R})$$

The functions $Q_0[W_0, Y_0]$, $Q_s[Y_s]$, and $Q_n[W, Y_n]$ are the indirect utilities resulting from the second stage consumption and labor supply decisions. Given the realizations of ε_0, ε_n, and ε_s, she will choose the marital choice which maximizes her utility. The utility from her optimal choice will satisfy:

$$V^*(\varepsilon_0, \varepsilon_s, \varepsilon_n) = \max[V_0(\varepsilon_0), V_s(\varepsilon_s), V_n(\varepsilon_n)] \quad (1)$$

The problem facing men in the first stage is analogous to that of women. Given the realizations of ϵ_0, ϵ_n, and ϵ_s, he will choose the marital choice which maximizes utility. The indirect utility from remaining single is:

$$v_0(\epsilon_0) = q_0[w_0, y_0] + \gamma_0 + \epsilon_0$$

from entering a specialized marriage is:

$$v_s(\epsilon_s) = q_s[w, y_s] + \gamma_s + \epsilon_s$$

and from entering a non-specialized marriage is:

$$v_n(\epsilon_n) = q_n[w, y_n] + \gamma_n + \epsilon_n$$

where

$$y_s = A - \tau_s(w, A, \mathbf{R})$$
$$y_n = A - \tau_n(W, w, A, \mathbf{R})$$

and the $q_0[w_0, y_0]$, $q_s[w, y_s]$, and $q_n[w, y_n]$ are the indirect utilities from the household's problem in the second stage. Given the realizations of ϵ_0, ϵ_n, and ϵ_s, he will choose the marital choice which maximizes his utility. The utility from his optimal choice will therefore satisfy:

$$v^*(\epsilon_0, \epsilon_s, \epsilon_n) = \max[v_0(\epsilon_0), v_s(\epsilon_s), v_n(\epsilon_n)] \quad (2)$$

4 The Marriage Market

In this section, we construct supply and demand conditions in the marriage market and define an equilibrium for this market. Our model of the marriage market closely follows CS. Assume that there are many women and men in the marriage market, each woman is solving (1) and each man is solving (2). Under the assumption ϵ_k and ϵ_k are i.i.d. extreme value random variables, McFadden (1974) shows that, within a market, the number of marriages relative to the number of females can be expressed as the probability women prefer entering a type k marriage relative to all other alternatives, including remaining single:

$$\frac{\Pi_k}{f} = \frac{\exp\Big(\Gamma_k + Q_k[\cdot]\Big)}{\sum_{l\in\{0,s,n\}} \exp\Big(\Gamma_l + Q_l[\cdot]\Big)} \tag{3}$$

where Π_k is the number of marriages of type k. Similarly, for every man

$$\frac{\pi_k}{m} = \frac{\exp\Big(\gamma_k + q_k[\cdot]\Big)}{\sum_{l\in\{0,s,n\}} \exp\Big(\gamma_l + q_l[\cdot]\Big)} \tag{4}$$

where π_k is the number of marriages of type k.

Equations (3) and (4) imply the following supply and demand equations:

$$\ln \Pi_k - \ln \Pi_0 = (\Gamma_k - \Gamma_0) + Q_k[\cdot] - Q_0[W_0, Y_0] \tag{5}$$

where Π_0 is the number of females who choose to remain unmarried, and

$$\ln \pi_k - \ln \pi_0 = (\gamma_k - \gamma_0) + q_k[w, y_k] - q_0[w_0, y_0] \tag{6}$$

and π_0 is the number of males who choose to remain unmarried. CS call the left hand side of (5) the net gains to a k type marriage relative to remaining unmarried for women and the left hand side of (6) the net gains to a k type marriage relative to remaining unmarried for men.

Marriage market clearing requires the supply of wives to be equal to the demand for wives for each type of marriage:

$$\Pi_k = \pi_k = \Pi^*_k \ \forall\, k. \tag{7}$$

The following feasibility constraints ensure that the stocks of married and single agents of each gender and type do not exceed the aggregate stocks of agents of each gender in the market:

$$f = \Pi_0 + \sum_k \Pi_k^* \tag{8}$$

$$m = \pi_0 + \sum_k \Pi_k^*. \tag{9}$$

We can now define a rational expectations equilibrium. There are two parts to the equilibrium, corresponding to the two stages at which decisions are made by the agents. The first corresponds to decisions made in the marriage market; the second to the intra-household allocation. In equilibrium, agents make marital status decisions optimally, the sharing rules clear each marriage market, and conditional on the sharing rules, agents choose consumption and labor supply optimally. Formally:

Definition 1 *A rational expectations equilibrium consists of a distribution of males and females across marital status and type of marriage* $\{\hat{\Pi}_0\ ,\ \hat{\pi}_0,\ \hat{\Pi^*}_k\}$, *a set of decision rules for marriage* $\{\widehat{V}^*(\varepsilon_{0G}, \varepsilon_{sG}, \varepsilon_{nG}),\ \widehat{v}^*(\varepsilon_{0g}, \varepsilon_{sg}, \varepsilon_{ng})\}$, *a set of decision rules for spousal consumption and leisure* $\{\hat{C}_0,\ \hat{C}_n,\ \hat{c}_0,\ \hat{c}_k,\ \hat{L}_0,\ \hat{L}_n,\ \hat{l}_0,\ \hat{l}_k\}$, *exogenous marriage and labor market conditions* $W, w, A, \mathbf{R}$ *and a set of transfers* $\{\widehat{\tau}_k(\cdot)\}$ *such that:*

1. *The decision rules* $\{\widehat{V}^*(\cdot),\ \widehat{v}^*(\cdot)\}$ *solve (1) and (2);*
2. *All marriage markets clear implying (7), (8), (9) hold;*
3. *For a type* n *marriage, the decision rules* $\{\hat{C}_n,\ \hat{c}_n,\ \hat{L}_n,\ \hat{l}_n\}$ *solve (PN);*
4. *For a type* s *marriage, the decision rules* $\{\hat{c}_s,\ \hat{l}_s\}$ *solve (PS).*

Theorem 2 *A rational expectations equilibrium exists.*

Sketch of proof: We have already demonstrated *1*, *3*, and *4*. We next need to show that there is a set of transfers, $\{\widehat{\tau}_n, \widehat{\tau}_s\}$ which clears the marriage market. Let τ be the vector of transfers. For every type of marriage k in the marriage market, define the excess demand function for marriages by men:

$$E_k(\tau) = \pi_k(\tau) - \Pi_k(\tau). \tag{10}$$

The demand and supply functions (3) and (4) satisfy the weak gross substitute property for every marriage market. So the excess demand functions also satisfy the weak gross substitute property.[9]

The equilibrium stocks of marriages of each type, as well as the stocks of singles of each type, will depend on wages and non-labor incomes, as well as labor and marriage market conditions across all alternatives, summarized by $\mathbf{R}$, and are denoted:

$$\Pi_k^*(W, w, A, \mathbf{R})$$
$$\Pi_0^*(W, w, A, \mathbf{R})$$
$$\pi_0^*(W, w, A, \mathbf{R})$$

Remark: In monogamous marriage markets, where different types of spouses are substitutes for each other (since an individual can at most marry one type), the weak gross substitute property is generic. Thus existence of marriage market equilibrium is more general than our specific random utility model for spousal choice, which we use for empirical convenience. Kelso and Crawford (1982) were the first to use the gross substitute property to demonstrate existence in matching models.

4.1 Derivation of the sharing rule

In this section, we show that sharing rules arise endogenously as the transfers that clear the marriage market. Recall, the demand and supply conditions can be expressed as:

$$\ln \Pi_k - \ln \Pi_0 = (\Gamma_k - \Gamma_0) + Q_k[\cdot] - Q_0[W_0, Y_0]$$

and

$$\ln \pi_k - \ln \pi_0 = (\gamma_k - \gamma_0) + q_k[\cdot] - q_0[w_0, y_0]$$

respectively. Marriage market clearing requires $\Pi_k = \pi_k$ which implies

$$\begin{aligned} \ln T = \ln \tilde{\Gamma_n} - \ln \tilde{\gamma_n} + Q_n[W, \tau_n(W, w, A, \mathbf{R})] \\ - Q_0[W_0, Y_0] - q_n[w, A - \tau_n(W, w, A, \mathbf{R})] + q_0[w_0, y_0] \end{aligned} \quad (11)$$

[9] Mas-Colell, Winston and Green (1995: p. 646, exercise 17.F.16^C) provide a proof of existence of market equilibrium when the excess demand functions satisfy the weak gross substitute property. For convenience, we reproduce their proof in our context in Appendix B. The proof does not rely on Walras Law or that excess demand is homogenous of degree zero in τ, neither of which are satisfied by our model.

for non-specialized couples, and

$$\ln T = \ln \tilde{\Gamma}_s - \ln \tilde{\gamma}_s + Q_s[\tau_s(w, A, \mathbf{R})] \\ - Q_0[W_0, Y_0] - q_s[w, A - \tau_s(w, A, \mathbf{R})] + q_0[w_0, y_0] \tag{12}$$

for specialized couples, where $\ln \tilde{\Gamma}_k = \Gamma_k - \Gamma_0$ and $\ln \tilde{\gamma}_k = \gamma_k - \gamma_0$. Equations (11) and (12) can be solved for τ_n and τ_s as a function of wages, non-labor incomes and market tightness. The resulting sharing rules are used in the second stage household problem as outlined in Section 3.

5 Identification

In this section, we demonstrate how information on spousal labor supplies and on marriage decisions can be used to identify the preferences of individual household members as well as the sharing rule.

5.1 Non-specialized couples

5.1.1 Labor supply

Consider couples in which both partners work strictly positive hours. Assume that the unrestricted labor supplies for husbands and wives, $H_n(W, w, A, \mathbf{R})$ and $h_n(W, w, A, \mathbf{R})$ are continuously differentiable. The Marshallian labor supply functions associated with the collective framework are related to the reduced form according to:

$$H_n(W, w, A, \mathbf{R}) = \tilde{H}_n[W, \tau_n(W, w, A, \mathbf{R})] \tag{13}$$

$$h_n(W, w, A, \mathbf{R}) = \tilde{h}_n[w, A - \tau_n(W, w, A, \mathbf{R})] \tag{14}$$

This is exactly the setting of CFL. As in CFL, it is straightforward to show that the partial derivatives of the sharing rule can be recovered from labor supplies. For completeness, we reproduce the identification results of CFL here. Suppose for simplicity that $\mathbf{R}$ has one element, r. Define $B_1 = \frac{h_{nW}}{h_{nA}}$, $D_1 = \frac{h_{nr}}{h_{nr}}$, $E_1 = \frac{H_{nw}}{H_{nA}}$, and $F_1 = \frac{H_{nr}}{H_{nA}}$. The following proposition outlines the necessary and sufficient conditions for identification of the sharing rule.

Proposition 3 *Take any point such that $H_{nA} \cdot h_{nA} \neq 0$. Then: (i) If there exists exactly one distribution factor r such that $D_1 \neq F_1$, the following* *conditions are necessary for any pair $(H(\cdot), h(\cdot))$ to be solutions of (PN) for some sharing rule $\tau_n(W, w, A, r)$:*

$$\frac{\partial}{\partial r}\Big(\frac{D_1}{D_1 - F_1}\Big) = \frac{\partial}{\partial A}\Big(\frac{D_1 F_1}{D_1 - F_1}\Big)$$

$$\frac{\partial}{\partial W}\Big(\frac{D_1}{D_1 - F_1}\Big) = \frac{\partial}{\partial A}\Big(\frac{B_1 F_1}{D_1 - F_1}\Big)$$

$$\frac{\partial}{\partial w}\Big(\frac{D_1}{D_1 - F_1}\Big) = \frac{\partial}{\partial A}\Big(\frac{D_1 E_1}{D_1 - F_1}\Big)$$

$$\frac{\partial}{\partial W}\Big(\frac{D_1 F_1}{D_1 - F_1}\Big) = \frac{\partial}{\partial r}\Big(\frac{B_1 F_1}{D_1 - F_1}\Big)$$

$$\frac{\partial}{\partial w}\Big(\frac{D_1 F_1}{D_1 - F_1}\Big) = \frac{\partial}{\partial r}\Big(\frac{D_1 E_1}{D_1 - F_1}\Big)$$

$$\frac{\partial}{\partial w}\Big(\frac{B_1 F_1}{D_1 - F_1}\Big) = \frac{\partial}{\partial W}\Big(\frac{D_1 E_1}{D_1 - F_1}\Big)$$

$$H_{nW} - H_{nA}\Big(H_n + \frac{B_1 F_1}{D_1 - F_1}\Big)\Big(\frac{D_1 - F_1}{D_1}\Big) \geq 0$$

$$h_{nw} - h_{nA}\Big(h_n - \frac{D_1 E_1}{D_1 - F_1}\Big)\Big(-\frac{D_1 - F_1}{F_1}\Big) \geq 0$$

(ii) Under the assumption that the conditions in (i) hold, the sharing rule is defined up to an additive constant κ. The partial derivatives of the sharing rule are given by:

$$\tau_{nA} = \frac{D_1}{D_1 - F_1}$$

$$\tau_{nr} = \frac{D_1 F_1}{D_1 - F_1}$$

$$\tau_{nW} = \frac{B_1 F_1}{D_1 - F_1}$$

$$\tau_{nw} = \frac{D_1 E_1}{D_1 - F_1}$$

Proof: See Proposition 3 and the Appendix in CFL. Identification from labor supplies is identical to that in CFL in the case where both partners work.[10]

5.1.2 Marriage

The collective matching model also imposes restrictions on marriage decisions. Recall, the indirect utilities for wives and husbands in non-specialized marriages are:

$$\begin{aligned} V_n[\varepsilon_{nG}] &= Q_n[W, \tau_n(W, w, A, \mathbf{R})] + \Gamma_n + \varepsilon_{nG} \\ v_n[\varepsilon_{ng}] &= q_n[w, A - \tau_n(W, w, A, \mathbf{R})] + \gamma_n + \varepsilon_{ng} \end{aligned}$$

respectively, and the indirect utilities for single men and women are:

$$\begin{aligned} V_0[\varepsilon_{0G}] &= Q_0[W_0, Y_0] + \Gamma_0 + \varepsilon_{0G} \\ v_0[\varepsilon_{0g}] &= q_0[w_0, y_0] + \gamma_0 + \varepsilon_{0g} \end{aligned}$$

Denote the log odds ratio for choosing to be in a non-specialized marriage $\hat{P}_n$ relative to being single being single $\hat{P}_0$, respectively for women and $\hat{p}_n$ and $\hat{p}_0$ for men. Under the extreme value assumption for the ε's we can express this log odds ratio as:

$$\frac{\hat{P}_n}{\hat{P}_0} = \frac{\exp(Q_n[W, \tau_n(W, w, A, \mathbf{R})] + \Gamma_n)}{\exp(Q_0[W_0, Y_0] + \Gamma_0)} \tag{15}$$

for women and

$$\frac{\hat{p}_n}{\hat{p}_0} = \frac{\exp(q_n[w, A - \tau_n(W, w, A, \mathbf{R})] + \gamma_n)}{\exp(q_0[w_0, y_0] + \gamma_0)} \tag{16}$$

[10] We can also uncover the partial derivatives of the Marshallian labor supply functions as:

$$\begin{aligned} \tilde{H}_{nW} =& H_{nW} + H_{nA}\Big(\frac{D_1 - C_1 - B_1 C_1}{C_1}\Big) \\ \tilde{H}_{nY} =& -H_{nA}\Big(\frac{D_1 - C_1}{D_1}\Big) \\ \tilde{h}_{nw} =& h_{nw} + h_{nA}\Big(\frac{D_1 - C_1 - D_1 A_1}{C_1}\Big) \\ \tilde{h}_{ny} =& -h_{na}\Big(\frac{D_1 - C_1}{C_1}\Big) \end{aligned}$$

Denote $P_k = \ln \hat{P}_k - \ln \hat{P}_0$ and $p_k = \ln \hat{p}_k - \ln \hat{p}_0$. Define $B_2 = \frac{P_{nr}}{P_{nA}}$, $D_2 = \frac{p_{nW}}{p_{nA}}$, $E_2 = \frac{P_{nw}}{P_{nA}}$, and $F_2 = \frac{p_{nr}}{p_{nA}}$.[11]

The following proposition shows that the structure of the marriage choice probabilities imposes testable restrictions on marriage behavior that allow us to uncover the partial derivatives of the sharing rule without using information on labor supplies:

Proposition 4 *Take any point such that $P_{nA} \cdot p_{nA} \neq 0$. Then, the following results hold: (i) If there exists exactly one distribution factor such that $B_2 \neq F_2$, the following conditions are necessary for any pair (P_n, p_n) to be consistent with marriage market clearing for some sharing rule $\tau_n(W, w, A, \mathbf{R})$:*

$$\frac{\partial}{\partial r}\Big(\frac{F_2}{F_2 - B_2}\Big) = \frac{\partial}{\partial A}\Big(\frac{B_2 F_2}{F_2 - B_2}\Big)$$

$$\frac{\partial}{\partial W}\Big(\frac{F_2}{F_2 - B_2}\Big) = \frac{\partial}{\partial A}\Big(\frac{B_2 D_2}{F_2 - B_2}\Big)$$

$$\frac{\partial}{\partial w}\Big(\frac{F_2}{F_2 - B_2}\Big) = \frac{\partial}{\partial A}\Big(\frac{F_2 E_2}{F_2 - B_2}\Big)$$

$$\frac{\partial}{\partial W}\Big(\frac{F_2 B_2}{F_2 - B_2}\Big) = \frac{\partial}{\partial r}\Big(\frac{B_2 D_2}{F_2 - B_2}\Big)$$

$$\frac{\partial}{\partial w}\Big(\frac{F_2 B_2}{F_2 - B_2}\Big) = \frac{\partial}{\partial r}\Big(\frac{F_2 E_2}{F_2 - B_2}\Big)$$

$$\frac{\partial}{\partial w}\Big(\frac{B_2 D_2}{F_2 - B_2}\Big) = \frac{\partial}{\partial W}\Big(\frac{E_2 F_2}{F_2 - B_2}\Big)$$

(ii) Under the assumption that the conditions in (i) hold, the sharing rule is defined up to an additive constant ρ. The partial derivatives of the sharing rule are given by:

$$\tau_{nA} = \frac{F_2}{F_2 - B_2}$$

$$\tau_{nr} = \frac{B_2 F_2}{F_2 - B_2}$$

[11] It is helpful to express the choice probabilities in this form, as the marriage choice only depends on the characteristics of the current match and the value of remaining single, not on all other potential matches.

$$\tau_{nW} = \frac{B_2 D_2}{F_2 - B_2}$$

$$\tau_{nw} = \frac{E_2 F_2}{F_2 - B_2}$$

Proof: See Appendix C.1.[12] Thus, information on marriage decisions allows us to identify the partial derivatives of the sharing rule independently of the partial derivatives identified from labor supplies. This provides us with the following set of over-identifying restrictions

$$\begin{aligned}
\tau_{nA} &= \frac{D_1}{D_1 - F_1} = \frac{F_2}{F_2 - B_2} \\
\tau_{nr} &= \frac{D_1 F_1}{D_1 - F_1} = \frac{B_2 F_2}{F_2 - B_2} \\
\tau_{nW} &= \frac{B_1 F_1}{D_1 - F_1} = \frac{B_2 D_2}{F_2 - B_2} \\
\tau_{nw} &= \frac{D_1 E_1}{D_1 - F_1} = \frac{E_2 F_2}{F_2 - B_2}
\end{aligned} \tag{IN}$$

that allow us to test whether the sharing rule that is consistent with family labor supplies is a market clearing transfer.

[12]From the marriage decisions of the husband and wife, it is also straightforward to show that we can uncover the partial derivatives of the indirect utilities:

$$\begin{aligned}
q_{ny} &= -\Big(\frac{P_{nA} p_{nr} - P_{nr} p_{nA}}{P_{nr}}\Big) \\
Q_{nY} &= \frac{P_{nA} p_{nr} - P_{nr} p_{nA}}{p_{nr}} \\
Q_{nW} &= P_{nW} - \frac{P_{nr} p_{nW}}{p_{nr}} \\
q_{nw} &= p_{nw} - \frac{P_{nw} p_{nr}}{P_{nr}}
\end{aligned}$$

5.2 Specialized couples

5.2.1 Labor supply

Consider couples in which only the husband works in the labor market. The Marshallian labor supply function for husbands in specialized marriages is:

$$h_s(w, A, \mathbf{R}) = \tilde{h}_s(w, A - \tau_s(w, A, \mathbf{R})).$$

Denote non-labor incomes for the wife and husband, respectively, as:

$$\begin{aligned} Y_s &= \tau_s(w, A, \mathbf{R}) \\ y_s &= A - \tau_s(w, A, \mathbf{R}) \end{aligned}$$

The partial derivatives of the labor supply function are

$$\begin{aligned} h_{sw} &= \tilde{h}_{sw} - \tilde{h}_{sy}\tau_{sw} \\ h_{sA} &= \tilde{h}_{sy}(1 - \tau_{sA}) \\ h_{sr} &= -\tilde{h}_{sy}\tau_{sr} \end{aligned}$$

The above system has three equations and five unknowns. It is clear that if we only observe variation in the husband's labor supply it is not possible to uncover preferences and the sharing rule.

5.2.2 Marriage

The indirect utilities for wives and husbands in specialized marriages are:

$$\begin{aligned} V_s[\varepsilon_{sG}] &= Q_s[\tau_s(w, A, \mathbf{R})] + \Gamma_s + \varepsilon_{sG} \\ v_s[\varepsilon_{sg}] &= q_s[w, A - \tau_s(w, A, \mathbf{R})] + \gamma_s + \varepsilon_{sg} \end{aligned}$$

Denote the probability of choosing to be in a specialized marriage $\hat{P}_s$ for women and $\hat{p}_s$ for men. The ratio of choice probabilities for specialized marriage relative to remaining single is:

$$\frac{\hat{P}_s}{\hat{P}_0} = \frac{\exp(Q_s[\tau_s(w, A, \mathbf{R})] + \Gamma_s)}{\exp(Q_0[W_0, Y_0] + \Gamma_0)}$$

for women and

$$\frac{\hat{p}_s}{\hat{p}_0} = \frac{\exp(q_s[w, A - \tau_s(w, A, \mathbf{R})] + \gamma_s)}{\exp(q_0[w_0, y_0] + \gamma_0)}$$

for men. As is the case for non-specialized marriages, the following proposition shows that it is straightforward to derive the partial derivatives of the sharing rule from marriage decisions. Define $B_3 = \frac{P_{sr}}{P_{sA}}$, $D_3 = \frac{p_{sr}}{p_{sA}}$, and $E_3 = \frac{P_{sw}}{P_{sA}}$. Then

Proposition 5 *Take any point such that $P_{nA} \cdot p_{nA} \neq 0$. Then, the following results hold: (i) If there exists exactly one distribution factor such that $D_3 \neq B_3$, the following conditions are necessary for any pair (P_s, p_s) to be consistent with marriage market clearing for some sharing rule $\tau_s(w, A, \mathbf{R})$:*

$$\begin{aligned} \tau_{sA} &= \frac{D_3}{D_3 - B_3} \\ \tau_{sr} &= \frac{B_3 D_3}{D_3 - B_3} \\ \tau_{sw} &= \frac{D_3 E_3}{D_3 - B_3} \end{aligned} \tag{IS}$$

Thus, it is possible to identify the sharing rule, up to an additive constant, for specialized couples if we observe changes in marriage behavior in response to changes in wages, non-labor incomes, or market tightness.[13] A conclusion that immediately follows from the above is that it is always possible to identify the sharing rule from marriage decisions in the absence of data on labor supply.

6 Estimation and Model Tests

In this section, we outline our strategy for estimating and testing the collective matching model. Our primary interest is in testing whether the sharing rule that clears the marriage market is consistent with the sharing rule that rationalizes the labor supply decisions. Thus we are interested in testing the restrictions (IN) for nonspecialized couples.

We start by describing the data used to estimate our model. We next describe our estimation strategy and and how we deal with special estimation issues raised by our data.

[13] As is the case for non-specialized couples, the indirect utilities and the sharing rules can both be identified up to an additive constant. Once the partial derivatives of the sharing rule are known, it is straightforward to recover the partial derivatives of the Marshallian labor supply function for husbands $\tilde{h}_y$ and $\tilde{h}_w$.

6.1 Data

We use data from the 2000 US Census to estimate our model. For the purposes of our empirical analysis, we allow for the presence of many segregated marriage markets and many discrete types of men and women, where i denotes the male's type and j the female's types. The type of each individual is defined by the combination of race, education, and age. There are four race categories (black, white, hispanic, other), and three education types (less than high school, high school graduate and/or some college, college graduate). As a starting point, we consider individuals aged 21 to 60, grouped into five year age categories (six age categories). There are 72 potential types of women and men in the marriage market. As in CFL, we further assume that each state constitutes a separate marriage market r, where marriage markets are defined at the state level. Our unit of observation is (ijr) and there are a total of $259,200$ possible cells.

Our goal is to test the over-identifying restrictions implied by the collective matching model for nonspecialized couples. To this end, we limit our sample to couples in which both spouses work strictly positive hours. We also restrict our sample to couples without children. Our measure of labor supply is annual hours of work. Wages are measured by average hourly earnings, constructed by dividing total labor income by annual hours. Nonlabor income is measured as the sum several income sources including social security income, supplementary security income, welfare, interest, dividend and rental income, and other income. Wages, hours of work and nonlabor income are subsequently aggregated up to the cell level using the household sampling weights.[14]

The marital choice for women is defined as the log of the ratio of the number ijr marriages to the number of jr single women and likewise for men. Finally, market tightness for cell ijr is defined as the ratio of single men of type i divided by single women of type j in market r. Many types of matches, for example across ethnic or education categories are relatively uncommon. We eliminate cells with less than five observations and observations for whom nonlabor income is negative. Finally we trim the top and bottom 3% of the wage, nonlabor income and sex ratio distributions to eliminate outliers. Our final sample is contains $*$ cells.

Table 1 contains descriptive statistics for our sample of households. On

[14]Note that measurement error in earnings and hours at the individual level is reduced when the data are aggregated up to the cell level.

average, married men work more hours per year than married women, and men have higher wages. The average level of nonlabor income in the sample is slightly over \$9,000 and approximately 70% of households have children. The value for market tightness suggests that there is a slight shortage of single men: there are 98 single men for every 100 single women.

Statistics on the ethnic, education, and age composition of marriages are presented inTables 2, 3, and 4, respectively. As is well known from previous studies, the vast majority of marriages (99%) are between men and women of the same race, with marriages between white men and white women comprising 90% of all marriages. With respect to other characteristics, individuals are also very likely to match within own-education categories (74% of marriages) and and own-age group (52% of marriages) but not to the same extent as for race, especially regarding age as women are more likely to marry slightly older spouses.

6.2 Econometric Specification

Our primary interest is in testing whether the sharing rule that clears the marriage market is consistent with the sharing rule that rationalizes the labor supply decisions. Our tests of the collective matching model do not rely upon any particular functional form for preferences, thus it is desirable to impose as few parametric assumptions on preferences as possible in our empirical analysis. The goal of this section is to outline a strategy for obtaining nonparametric estimates of our collective matching model. The reduced form labor supply and marriage equations of interest are:

$$H^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{H}_{ij}[W^r_{ij}, \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij})] \tag{17}$$

$$h^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{h}_{ij}[w^r_{ij}, A^r_{ij} - \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij})] \tag{18}$$

and

$$P^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{P}_{ij}[W^r_{ij}, \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}), W^r_{0j}, A^r_{0j}] \tag{19}$$

$$p^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{p}_{ij}[w^r_{ij}, A^r_{ij} - \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}), w^r_{i0}, A^r_{i0}] \tag{20}$$

respectively. The vector $\mathbf{R}^r_{ij}$ is a vector of distribution factors that includes the match specific sex ratio S^r_{ij} and the wages and other incomes of single men w^r_{i0}, A^r_{i0} and wages and other incomes of single women W^r_{0j}, A^r_{0j} of the same type as each spouse. Thus $\mathbf{R}^r_{ij} = \{S^r_{ij}, w^r_{i0}, W^r_{0j}, A^r_{i0}, A^r_{0j}\}$.[15]

We introduce subscripts for the match type (ij) and the state of residence r to make two points. The first is that our framework (and our empirical analysis) is general enough to accommodate heterogeneity in preferences and sharing rules by match type. The second point is that an important identifying assumption that preferences and invariant net gains can vary across different types of matches, but not across locations. Thus, the partial derivatives of the labor supplies, the marriage log odds ratios (and therefore the sharing rules) will be identified off cross-state variation in hours worked and marriage rates within match type. Estimation entails two stages:

Stage 1: Obtain nonparametric estimates of the partial derivatives of the reduced form labor supply $(\hat{H}_{nx}, \hat{h}_{nx})$ and marriage matching equations $(\hat{P}_{nx}, \hat{p}_{nx})$.

We estimate a semiparametric partially linear version of the model:

$$H^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{H}[W^r_{ij}, \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij})] + \alpha^H_{ij} + e^{Hr}_{ij} \tag{21}$$

$$h^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{h}[w^r_{ij}, A^r_{ij} - \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij})] + \alpha^h_{ij} + e^{hr}_{ij} \tag{22}$$

and

$$P^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{P}[W^r_{ij}, \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}), W^r_{0j}, A^r_{0j}] + \alpha^P_{ij} + e^{Pr}_{ij}$$
$$p^r_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}) = \tilde{p}[w^r_{ij}, A^r_{ij} - \tau_{ij}(W^r_{ij}, w^r_{ij}, A^r_{ij}, \mathbf{R}^r_{ij}), w^r_{i0}, A^r_{i0}] + \alpha^p_{ij} + e^{pr}_{ij}.$$

Our econometric specification differs from the most general version of our model in two respects. First, we restrict the match type ij so that it only affects the labor supplies and marriage log odds ratio through an intercept. This allows differences in match-specific characteristics to affect individual labor supply and marriage market decisions, albeit in a limited way. Second, we introduce *iid* cell-specific error terms (e^{Hr}_{ij}, e^{hr}_{ij}, e^{Pr}_{ij}, e^{pr}_{ij}) that are independent of wages, other incomes, and the sex ratio.

[15] We experimented with several alternative specifications, including a specification with a set of sex ratios that proxy an individual's close substitutes in marriage: the ratio of men of the same age and race as the husband to women of the same age and race as the wife for an ij couple of type, the ratio of men of the same age and education as the husband to women of the same age and education as the wife, and the ratio of men of the same race and education of the husband to women of the same race and education as the wife. The results for this specification can be found in Appendix D.2. The results from other specifications are available from the authors upon request.

We estimate the model using a combination of Robinson (1988) and Speckman (1988) estimator and local linear estimation (Ruppert and Wand, 1994). We illustrate our estimation procedure with the female labor supply equation.[16]

Stage 2: Construct nonparametric estimates of the partial derivatives of the sharing rule from the first stage estimates and test the equality of the model restrictions.

Recall, the test of the collective model we are interested in examining is a test of the restrictions (IN), a test of whether the sharing rule that is consistent with market clearing is also consistent with household labor supplies. The set of restrictions we intend to test are

$$\tau_{nA} = \frac{h_{nS}H_{nA}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nS}P_{nA}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

$$\tau_{nS} = \frac{h_{nS}H_{nS}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nS}P_{nS}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

$$\tau_{nW} = \frac{h_{nW}H_{nS}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nW}P_{nS}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

$$\tau_{nw} = \frac{h_{nS}H_{nw}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nS}P_{nw}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

$$\tau_{nw_0} = \frac{h_{nS}H_{nw_0}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nS}P_{nw_0}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

$$\tau_{nW_0} = \frac{H_{nS}h_{nW_0}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nW_0}P_{nS}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

$$\tau_{nA_{i0}} = \frac{h_{nS}H_{nA_{i0}}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nS}P_{nA_{i0}}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

$$\tau_{nA_{0j}} = \frac{H_{nS}h_{nA_{0j}}}{h_{nS}H_{nA} - H_{nS}h_{nA}} = \frac{p_{nA_{0j}}P_{nS}}{p_{nS}P_{nA} - P_{nS}p_{nA}}$$

[16] See Racine and Li (2007) for a full description of both methods and Appendix **??** for a full description of estimation for our particular model.

Step 1: Take the conditional expectation of H_n with respect to the non-parametric part of the model. Let $\mathbf{s}$ denote a vector containing W, w, A, T. Then

$$E[H_n|\mathbf{s}] = E[\mathbf{X}|\mathbf{s}]'\beta_H + H_n(\mathbf{s}) \tag{23}$$

and we difference from the labor supply equation to obtain:

$$H_n - E[H_n|\mathbf{s}] = [\mathbf{X} - E[\mathbf{X}|\mathbf{s}]]'\beta_H + \nu_H. \tag{24}$$

Step 2: Estimate $E[H_n|\mathbf{s}]$ and $E[\mathbf{X}|\mathbf{s}]$ by local linear regression. The local linear estimator of the conditional mean function at $\mathbf{s} = (W, w, A, T)$ is $\hat{\alpha}$ and the derivative at this point is $\hat{\rho}$. The local linear estimator for $E[H_n|\mathbf{s}]$ is:

$$\min_{\alpha,\rho} \sum_i \Big\{ H_{ni} - \alpha - \rho'(\mathbf{S}_i - \mathbf{s}) \Big\}^2 \prod_{h=1}^{4} K\Big(\frac{S_{ih} - s_h}{b_h}\Big) \tag{25}$$

where $K(\cdot)$ is a one-dimensional kernel function, the bandwidth for the i^{th} covariate is b_i and the kernel weight function is a product kernel. From standard weighted least squares theory, the local linear estimator for the conditional mean is

$$\hat{H}_n(\mathbf{s}) = e_1(\mathbf{S}_s'\mathbf{W}_s\mathbf{S}_s)^{-1}\mathbf{S}_s'\mathbf{W}_s\mathbf{H_n} \tag{26}$$

where e_1 is a $5+1$ vector with 1 for the first element and 0 everywhere else, $\mathbf{H_n} = (H_{n1}, H_{n2}, ...H_{nN})'$ where N is the number of observations,

$$\mathbf{W}_s = diag\Big\{ \prod_{i=1}^{4} K\Big(\frac{S_{i1} - s_{i1}}{b_i}\Big) \ \dots \ \prod_{i=1}^{4} K\Big(\frac{S_{iN} - s_{iN}}{b_i}\Big) \Big\} \tag{27}$$

and

$$S_s = \begin{pmatrix} 1 & (\mathbf{S}_1 - \mathbf{s})' \\ \vdots & \vdots \\ 1 & (\mathbf{S}_N - \mathbf{s})' \end{pmatrix} \tag{28}$$

To get an estimate of the derivative of $H_n(\mathbf{S})$ wrt to the $k-$th derivative,

$$\frac{\partial \hat{H_n}(\mathbf{s})}{\partial s_k} = e_{k+1}\big(\mathbf{S}_s'\mathbf{W}_s\mathbf{S}_s\big)^{-1}\mathbf{X}_s'\mathbf{W}_s\mathbf{H}_n \tag{29}$$

Currently the $i-$th covariate bandwidth is set according to a rule of thumb of $c\sigma_i/n^{d+4}$ where σ_i is the standard deviation of the $i-$th covariate and n is the sample size. Refer to Ruppert and Wand (1994) for expressions of the bias and variance on the derivative estimate.

Step 3: Estimate

$$H_n - \hat{E}[H_n|\mathbf{s}] = [\mathbf{X} - \hat{E}[\mathbf{X}|\mathbf{s}]]'\beta_H + \nu_H.$$

using OLS to obtain $\hat{\beta}_H$.

Step 4: We can obtain an estimate of $H_n(W, w, A, T)$ as

$$H_n(W, w, A, T) = \hat{E}[H_n|\mathbf{s}] - \hat{E}[\mathbf{X}|\mathbf{s}]'\beta_H \tag{30}$$

7 Results

Coming soon.

The actual values of the parameter estimates for the sharing rule are in Appendix D.1.

8 Conclusion

In this paper, we develop and estimate an empirical collective matching model. Our model integrating the collective model of Chiappori (1992) and the matching model of Choo and Siow (2006). We establish that two models fit together without modification and can be analyzed independently, as done in previous studies. The sharing rule in our collective matching model arises endogenously as a transfer that clears the marriage market. In addition to marital matching between different types of individuals, the model can incorporate participation decisions of one or both spouses. With information on at least two independent marriage markets, incorporating matching in the collective model allows us to identify the sharing rule from observations on marriage decisions, regardless of the labor supply decisions of wives and husbands. Introducing the marriage market in the collective model generates over-identifying restrictions that allow us to test whether the sharing rule that rationalizes labor supplies is consistent with the sharing rule that clears the marriage market.

Table 1: **Household Characteristics**

	Husbands	Wives
Annual hours	2262.16	1726.86
Average hourly wage	25.11	17.61
Nonlabor income		9,333.18
Fraction of households with children		0.69
Market tightness		0.69
Cells		13,990

Table 2: **Ethnic Distribution of Marriages**

		Husbands				
		White	**Black**	**Hispanic**	**Other**	**Total**
	White	89.84	0.10	0.38	0.09	90.41
	Black	0.01	6.65	0.00	0.00	6.66
Wives	**Hispanic**	0.56	0.02	1.66	0.02	2.26
	Other	0.17	0.01	0.02	0.48	0.68
	Total	90.57	6.78	2.06	0.59	100.00

Table 3: **Education Distribution of Marriages**

		Husbands			
		< High School	**High School**	**College**	**Total**
	< High School	0.97	1.08	0.01	2.07
Wives	**High School**	2.41	51.87	11.97	66.24
	College	0.07	10.54	21.08	31.69
	Total	3.45	63.50	33.05	100.00

Table 4: **Age Distribution of Marriages**

		Husbands						
		31-35	**36-40**	**41-45**	**46-50**	**51-55**	**56-60**	**Total**
	31-35	9.03	6.23	1.51	0.35	0.07	0.00	17.19
	36-40	2.09	10.43	7.02	1.80	0.48	0.08	21.90
	41-45	0.38	2.28	10.61	6.85	1.75	0.38	22.25
Wives	**46-50**	0.06	0.45	1.98	9.71	6.10	1.19	19.48
	51-55	0.00	0.06	0.29	1.40	7.79	4.33	13.87
	56-60	0.00	0.00	0.03	0.12	0.74	4.42	5.32
	Total	11.56	19.44	21.44	20.22	16.93	10.41	100.00

Table 5: **Semi-parametric labor supply estimates**

	Wives			Husbands		
	Ages 21-60	Ages 21-30	< college	Ages 21-60	Ages 21-30	< college
Other income	-4.9081	-31.1007	-60.0377	-3.6523	-16.0376	-53.7831
Sex ratio	25.3886	-27.0555	125.5543	2.2124	33.2901	155.8961
Husband's wage	-73.8226	395.1950	-982.1317	-368.9239	-485.0592	-2122.3611
Wife's wage	20.2379	519.6946	2284.0385	58.7218	320.9968	2064.8422
Single male wage	96.5217	364.5089	1727.7524	134.8009	1065.5490	4085.0492
Single male income	12.3881	20.7512	-67.4978	-7.9733	-28.4873	-92.5767
Single female wage	574.4265	567.7983	1210.4147	-230.4554	678.6645	118.7158
Single female income	-15.8679	-72.2159	-159.8792	0.2793	-38.1971	-282.3505
Observations	5206	666	261	5206	666	261

Table 6: **Semi-parametric marriage log odds estimates**

	Wives			Husbands		
	Ages 21-60	Ages 21-30	$<$ college	Ages 21-60	Ages 21-30	$<$ college
Other income	0.1630	0.2259	0.2126	0.1633	0.2051	0.2112
Sex ratio	1.0165	0.3330	0.3087	-0.2750	-0.5060	-0.5265
Husband's wage	3.7942	0.3779	1.4482	3.6635	0.7458	1.4447
Wife's wage	1.0894	-0.1074	-5.7151	0.8258	-0.4208	-5.7086
Single male wage	-6.4758	-13.1600	-24.1142	-7.2044	-13.1300	-19.5708
Single male income	-0.0665	-0.1886	-0.4117	-0.0733	-0.1344	-0.3715
Single female wage	-7.1048	-11.2079	-9.8085	-6.4652	-10.7614	-11.2377
Single female income	-0.1306	-0.4795	-0.1153	-0.1203	-0.4510	-0.0112
Observations	5206	666	261	5206	666	261

Table 7: **Change in τ with respect to a 10% increase in variable (as a percentage of other income in parentheses)**

	Labor Supply			Marriage		
	Ages 21-60	Ages 21-30	< college	Ages 21-60	Ages 21-30	< college
Other income	78.49	43.99	33.24	63.71	46.42	29.75
	[3.38]	[5.50]	[6.83]	[5.80]	[2.75]	[6.11]
Sex ratio	-22.86	5.97	-25.26	1.56	1.16	21.57
	[-0.99]	[0.75]	[-5.19]	[0.15]	[0.07]	[4.43]
Husband's wage	-31.85	-12.57	46.25	126.95	-100.13	107.19
	[-1.37]	[-1.57]	[9.50]	[-12.52]	[5.47]	[22.02]
Wife's wage	-54.79	47.91	-130.32	8.13	28.71	64.69
	[-2.36]	[5.99]	[-26.78]	[3.59]	[0.35]	[13.29]
Single male wage	81.05	-46.82	-89.09	-91.24	-392.45	-291.60
	[3.49]	[-5.85]	[-18.30]	[-49.06]	[-3.93]	[-59.91]
Single male income	-41.03	-20.99	35.20	-32.52	-27.13	-17.35
	[-1.77]	[-2.62]	[7.23]	[-3.39]	[-1.40]	[-3.56]
Single female wage	-143.30	18.89	33.03	163.23	137.05	65.30
	[-6.18]	[2.36]	[6.79]	[17.13]	[7.04]	[13.42]
Single female income	1605.90	-1.97	-1.16	2063.52	723.08	327.35
	[1.37]	[-0.25]	[-0.24]	[90.39]	[88.97]	[67.35]
Observations	5206	666	261	5206	666	261

References

[1] Becker, Gary (1973). “A Theory of Marriage: Part I,” *Journal of Political Economy,* 81(4), 813-46.

[2] Becker, Gary (1974). “A Theory of Marriage: Part II,” *Journal of Political Economy,* 82(2), S11-S26, Part II.

[3] Becker, Gary (1991). *A Treatise on the Family,* Harvard University Press, Second Edition.

[4] Becker, Gary and Kevin Murphy (2000). *Social Economics.* Cambridge, MA: Harvard University Press.

[5] Blundell, Richard, Browning, Martin, and Ian Crawford (2003). “Nonparametric Engel Curves and Revealed Preference,” *Econometrica,* 71(1), 205-240.

[6] Blundell, Richard, Chiappori, Pierre-Andre, Magnac, Theirry, and Costas Meghir (2007). “Collective Labour Supply: Heterogeneity and Non-Participation,” *Review of Economic Studies,* 74(2), 417-445.

[7] Blundell, Richard, Chiappori, Pierre-Andre, and Costas Meghir (2006). “Collective Labor Supply with Children,” *Journal of Political Economy,* 113(6), 1277-1306.

[8] Brien, Michael (1997). “Racial Differences in Marriage and the Role of Marriage Markets,” *Journal of Human Resources,* 32(4), 741-778.

[9] Brien, Michael, Lillard, Lee, and Steven Stern (2006) “Cohabitation, Marriage, and Divorce in a Model of Match Quality,” *International Economic Review,* 47(2), 451-494.

[10] Browning, Martin and Pierre-Andre Chiappori (1998). “Efficient Intra-Household Allocations: A General Characterization and Empirical Tests,” *Econometrica,* 66(6), 124-1278.

[11] Browning, Martin, Chiappori, Pierre-Andre, and Yoram Weiss (2003). “A Simple Matching Model of the Marriage Market,” University of Chicago, unpublished manuscript.

[12] Cherchye, Laurens, De Rock, Bram, and Frederick Vermeulen (2007). "The Collective Model of Household Consumption: A Nonparametric Characterization," *Econometrica,* 75(2), 553-574.

[13] Chiappori, Pierre-Andre (1988). "Rational Household Labor Supply," *Econometrica* 56(1), 63-90.

[14] Chiappori, Pierre-Andre (1992). "Collective Labor Supply and Welfare," *Journal of Political Economy*, 100(3), 437-467.

[15] Chiappori, Pierre-Andre and Ivar Ekelund (2006). "The Micro Economics of Efficient Group Behavior: Identification," Columbia University working paper.

[16] Chiappori, Pierre-Andre, Fortin, Bernard, and Guy Lacroix (2002). "Household Labor Supply, Sharing Rule, and the Marriage Market," *Journal of Political Economy*, 110(1), 37-72.

[17] Chiappori, Pierre-Andre, Iyigun, Murat, and Yoram Weiss (2006). "Investment Schooling and the Marriage Market," IZA Working Paper No. 2454, November.

[18] Choo, Eugene and Aloysius Siow (2005). "Lifecycle Marriage Matching: Theory and Evidence," University of Toronto working paper.

[19] Choo, Eugene and Aloysius Siow (2006). "Who Marries Whom and Why," *Journal of Political Economy*, 114(1), 175-201.

[20] Del Boca, Daniela and Christopher Flinn (2006a) "Endogenous Household Interaction," NYU working paper.

[21] Del Boca, Daniela and Christopher Flinn (2006b) "Household Time Allocation and Modes of Behavior: A Theory of Sorts," NYU working paper.

[22] Duflo, Esther (2003). "Grandmothers and Granddaughters: Old Age Pension and Intra-household Allocation in South Africa," *World Bank Economic Review,* 17(1), 1-25.

[23] Fortin, Bernard and Guy Lacroix (1997). "A Test of the Unitary and Collective Models of Household Labour Supply," *Economic Journal,* 107(443), 933-955.

[24] Grossbard, Shoshana and Shoshana Grossbard and Catalina Amuedo-Dorantes (2007). "Cohort Level Sex Ratio Effects on Women's Labor Force Participation," *Review of Economics of the Household,* 5(3), 249-278.

[25] Grossbard-Schectman, Amyra (1984). " A Theory of Allocation of Time in Markets for Labour and Marriage." *The Economic Journal* 94(376), 863-82.

[26] Grossbard-Schectman, Shoshana (1993). *On the Economics of Marriage: A Theory of Marriage, Labor, and Divorce*, Boulder, Colorado: Westview Press.

[27] Grossbard-Schectman, Shoshana and Clive Granger (1998). "Womens Jobs and Marriage, Baby-Boom versus Baby-Bust," *Population,* 53, 731-52, September 1998.

[28] Iyigun, Murat and Randall Walsh (2007). "Building the Family Nest: Premarital Investments, Marriage Markets, and Spousal Allocations," *Review of Economic Studies,* 74(2), 507-535.

[29] Kelso Alexander and Vincent Crawford (1982). "Job Matching, Coalition Formation, and Gross Substitutes," *Econometrica,* 50, 1483-1504.

[30] Li, Qi and Jeffrey Racine (2007). *Nonparametric Econometrics,* Princeton University Press.

[31] Lise, Jeremy and Shannon Seitz (2007). "Consumption Inequality and Intra-Household Allocations," Queen's University, unpublished manuscript.

[32] Lundberg, Shelly (1988). "Labor Supply of Husbands and Wives: A Simulataneous Equations Approach," *The Review of Economics and Statistics,* 70(2), 224-235.

[33] Lundberg, Shelly, and Robert Pollak (1996). "Bargaining and Distribution in Marriage," *Journal of Economic Perspectives*, 10(4), 139-158.

[34] Manser, Marilyn, and Murray Brown (1980). "Marriage and Household Decision-Making: A Bargaining Analysis." *International Economic Review*, 21(1), 31-44.

[35] Mas-Colell, Andreu, Green, Jerry and Michael Whinston (1995). *Microeconomic Theory.* Oxford University Press.

[36] Mazzocco, Maurizio (2004). "Saving, Risk Sharing and Preferences for Risk," *American Economic Review*, 94(4), 1169-1182.

[37] Mazzocco, Maurizio, and Shintaro Yamaguchi (2006). "Labor Supply, Wealth Dynamics, and Marriage Decisions," University of Wisconsin-Madison, unpublished manuscript.

[38] McElroy, Marjorie (1990). "The Empirical Content of Nash-Bargained Household Behavior," *The Journal of Human Resources,* 25(4), 559-583.

[39] McElroy, Marjorie and Mary-Jean Horney (1981). "Nash-Bargained Household Decisions: Toward a Generalization of the Theory of Demand." *International Economic Review*, 22(2), 333-349.

[40] McFadden, Daniel (1974). "Conditional Logit Analysis of Qualitative Choice Behavior." In *Frontiers in Econometrics,* edited by Paul Zarembka. New York: Academic Press.

[41] Robinson, P. (1988). "Root N-Consistent Semiparametric Regression," *Econometrica,* 56, 931-954.

[42] Ruppert, D. and M. Wand (1994). "Multivariate Locally Weighted Least Squares Regression," *The Annals of Statistics,* 22(3), 1346-1370.

[43] Seitz, Shannon (2004). "Accounting for Racial Differences in Marriage and Employment," Queen's University working paper.

[44] Speckman, P. (1988) "Kernel Smoothing in Partial Linear Models," *Journal of the Royal Statistical Society,* Series B, 50, 413-446.

[45] Thomas, Duncan (1990). "Intra-Household Resource Allocation: An Inferential Approach," *The Journal of Human Resources,* 25(4), 635-664.

[46] Vermeulen, Frederick (2006). "A Collective Model for Female Labour Supply with Nonparticipation and Taxation," *Journal of Population Economics*, 19(1), 99-118.

A Proof of Proposition 1

Abstracting from gG, the social planner solves:

$$\max_{\{C,c,H,h\}} \mathbf{E}(\widehat{Q}(C, 1-H, K|Z) + p\mathbf{E}(\widehat{q}(c, 1-h, K)|Z)$$

subject to, for each state S,

$$c + C + K \leq A + W(1-H) + w(1-h) \tag{31}$$

Let Z^* be the value of Z evaluated at the optimum. The first order conditions with respect to c, C, H, h, K and the multiplier λ for each state S are:

$$\widehat{Q}^*_c = \lambda \tag{32}$$
$$p\widehat{q}^*_c = \lambda \tag{33}$$
$$\widehat{Q}^*_{1-H} = \lambda W \tag{34}$$
$$p\widehat{q}^*_{1-h} = \lambda w \tag{35}$$
$$\widehat{Q}^*_K + p\widehat{q}^*_K = \lambda \tag{36}$$

Using the first order conditions, as p changes, for each state S,

$$\frac{\partial \widehat{Q}^*}{p\partial p} = \frac{\lambda}{p}(C^*_p + WL^*_p + K^*_p) - \widehat{q}^*_K K^*_p \tag{37}$$
$$\frac{\partial \widehat{q}^*}{\partial p} = \frac{\lambda}{p}(c^*_p + wl^*_p) + \widehat{q}^*_K K^*_p \tag{38}$$
$$\frac{\partial \widehat{Q}^*}{p\partial p} + \frac{\partial \widehat{q}^*}{\partial p} = \frac{\lambda}{p}(c^*_p + wl^*_p + K^*_p + C^*_p + WL^*_p)) \tag{39}$$

Since the budget constraint has to hold for every S,

$$c^*_p + C^*_p + K^*_p + wl^*_p + WL^*_p = 0$$
$$\frac{\partial \widehat{Q}^*}{\partial p} = -p\frac{\partial \widehat{q}^*}{\partial p} \tag{40}$$

Since (40) holds for every state S, (**??**) obtains.

B Proof of Existence of Equilibrium

In the proof, we need:

$$E_{ij}(\underline{p}) > 0 \text{ as } \underline{p} \to \infty \qquad \text{(Condition A1)}$$

$$E_{ij}(\underline{p}) < 0 \text{ as } \underline{p} \to 0 \qquad \text{(Condition A2)}$$

That is, the utility functions q and Q must be such that as $\underline{p}$ approaches 0, men will not want to marry. And as $\underline{p}$ approaches ∞, women will not want to marry.

Let $\beta_{ij} = (1 + p_{ij})^{-1}$ where $\beta_{ij} \in [0, 1]$ is the utility weight of the wife in an $\{i, j\}$ marriage and $(1 - \beta_{ij})$ is the utility weight of the husband.

We know:

$$\frac{\partial \underline{\mu}_{ij}}{\partial p_{ij}} > 0 \tag{41}$$

$$\frac{\partial \underline{\mu}_{ij}}{\partial p_{ik}} < 0, \; k \neq j \tag{42}$$

$$\frac{\partial \underline{\mu}_{kl}(\beta)}{\partial p_{ij}} = 0; \; k \neq i, l \neq j \tag{43}$$

$$\frac{\partial \overline{\mu}_{ij}}{\partial p_{ij}} < 0 \tag{44}$$

$$\frac{\partial \overline{\mu}_{ij}}{\partial p_{kj}} > 0, \; k \neq i \tag{45}$$

$$\frac{\partial \overline{\mu}_{kl}(\beta)}{\partial p_{ij}} = 0; \; k \neq i, l \neq j \tag{46}$$

Let β be a matrix with typical element β_{ij} and the IxJ matrix function $E(\beta)$ be:

$$E(\beta) = \underline{\mu}(\beta) - \overline{\mu}(\beta) \tag{47}$$

An element of $E(\beta)$, $E_{ij}(\beta)$, is the excess demand for j type wives by i type men given β.

An equilibrium exists if there is a β^* such that $E(\beta^*) = 0$.

Assume that there exists a function $f(\beta) = \alpha E(\beta) + \beta$, $\alpha > 0$ which maps $[0, 1]^{I*J} \to [0, 1]^{I*J}$ and is non-decreasing in β. Tarsky's fixed point theorem says if a function $f(\beta)$ maps $[0, k]^N \to [0, k]^N$, $k > 0$, and is non-decreasing in β, there exists $\beta^* \in [0, k]^N$ such that $\beta^* = f(\beta^*)$. Let $f(\beta) = \alpha E(\beta) + \beta$,

$k = 1$ and $N = I * J$, and apply Tarsky's theorem to get $\beta^* = \alpha E(\beta^*) + \beta^* \Rightarrow E(\beta^*) = 0$.

Thus the proof of existence reduces to showing $f(\beta)$ which has the required properties.

We know from (41) to (46) that:

$$\frac{\partial E_{ij}(\beta)}{\partial \beta_{ij}} < 0 \tag{48}$$

$$\frac{\partial E_{ik}(\beta)}{\partial \beta_{ij}} > 0 \tag{49}$$

$$\frac{\partial E_{kj}(\beta)}{\partial \beta_{ij}} > 0 \tag{50}$$

$$\frac{\partial E_{kl}(\beta)}{\partial \beta_{ij}} = 0;\ k \neq i, l \neq j \tag{51}$$

(48) to (51) imply that $E(\beta)$ satisfies the Weak Gross Substitutability (WGS) assumption.

We now show that the WGS property of $E(\beta)$ implies that we can construct $f(\beta)$, such that $f(\beta)$ maps $[0,1]^{I*J} \to [0,1]^{I*J}$ and is non-decreasing in β. The proof follows the solution to exercise 17.F.16^C of Mas-Colell, Whinston and Green given in their solution manual. (N.B. Unlike them, we do not start with Gross Substitution, we begin from WGS, but it turns out to be sufficient for Tarsky's conditions)

For notational convenience, now onwards we'll treat the matrix function $E(\beta)$, as a vector function.

Let $N = I * J$ and 1_N be a $N \times 1$ vector of ones. $E(\beta) : [0,1]^N \to R^N$ is continuously differentiable and satisfies $E(0_N) >> 0_N$ and $E(1_N) << 0_N$ (Conditions A1 and A2).

For every $\beta \in [0,1]^N$ and any n, if $\beta_n = 0$, then $E_n(\beta) > 0$.

For every $\beta \in [0,1]^N$ and any n, if $\beta_n = 1$, then $E_n(\beta) < 0$.

If $\beta = \{0_N, 1_N\}$, the facts follow from Conditions A1 and A2. Otherwise, they are due to Conditions A1 and A2, and (48) to (51), i.e. WGS.

For each n, define $C_n = \{\beta \in [0,1]^N : E_n(\beta) \geq 0\}$ and $D_n = \{\beta \in [0,1]^N : E_n(\beta) \leq 0\}$.

Then $C_n \subset \{\beta \in [0,1]^N : \beta_n < 1\}$ and $D_n \subset \{\beta \in [0,1]^N : \beta_n > 0\}$.

Then by continuity, the following two minima, $_{ij}((1-\beta_n)/E_n(\beta) : \beta \in C_n)$ and $_{ij}(-\beta_n/E_n(\beta) : \beta \in D_n)$, exist and are positive. Let $\underline{\beta}_n > 0$ be smaller than those two minima. Then, for all $\alpha \in (0, \underline{\beta}_n)$ and any $\beta \in [0,1]^N$, we have $0 \leq \alpha E_n(\beta) + \beta_n \leq 1$.

For each n, define $L_n =_{ij} \{|\partial E_n(\beta)/\partial \beta_n| : \beta \in [0,1]^N\}$. Then, for all $\alpha \in (0, 1/L_n)$,

$$\frac{\partial(\alpha E_n(\beta) + \beta_n)}{\partial \beta_n} = \alpha \frac{\partial E_n(\beta)}{\partial \beta_n} + 1 \geq -\alpha L_n + 1 > 0$$
$$\frac{\partial(\alpha E_n(\beta) + \beta_n)}{\partial \beta_m} = \alpha \frac{\partial E_n(\beta)}{\partial \beta_m} \geq 0; n \neq m, \text{ follows from (48) to (51).}$$

Now let $K = {}_{ij}\{\underline{\beta}_1, .., \underline{\beta}_N, 1/L_1, .., 1/L_N\}$, choose $\alpha \in (0, K)$, then $f(\beta) = \alpha E(\beta) + \beta \in [0,1]^N$ and $\partial f(\beta)/\partial \beta_n \geq 0$ for every $\beta \in [0,1]^N$, and any n. Hence Tarsky's conditions are satisfied.

C Identification

C.1 Marriage, non-specialized couples

Denote the probability of choosing to be in a non-specialized marriage P_n and the probability of being single P_0, respectively for women and p_n and p_0 for men. Under the extreme value assumption for we can express the ratio of the choice probabilities for non-specialized marriage relative to remaining single as:

$$\frac{P_n}{P_0} = \frac{\exp(\tilde{Q}_n[W, Y_n] + \Gamma_n)}{\exp(\tilde{Q}_0[W_0, Y_0] + \Gamma_0)}$$

for women and

$$\frac{p_n}{p_0} = \frac{\exp(\tilde{q}_n[w, y_n] + \gamma_n)}{\exp(\tilde{q}_0[w_0, y_0] + \gamma_0)}$$

Denote $\hat{P}_n = \ln P_n - \ln P_0$ and $\hat{p}_n = \ln p_n - \ln p_0$.

Differentiating with respect to wages, household non-labor income and μ

yields:

$$\begin{aligned}
\hat{P}_{nA} &= \tilde{Q}_{nY}\tau_{nA} \\
\hat{P}_{nT} &= \tilde{Q}_{nY}\tau_{nT} \\
\hat{P}_{nW} &= \tilde{Q}_{nW} + \hat{Q}_{nY}\tau_{nW} \\
\hat{P}_{nw} &= \tilde{Q}_{nY}\tau_{nw}
\end{aligned}$$

for women and

$$\begin{aligned}
\hat{p}_{nA} &= \tilde{q}_{ny}(1 - \tau_{nA}) \\
\hat{p}_{nT} &= -\tilde{q}_{ny}\tau_{nT} \\
\hat{p}_{nw} &= \tilde{q}_{nw} - \tilde{q}_{ny}\tau_{nw} \\
\hat{p}_{nW} &= -\tilde{q}_{ny}\tau_{nW}
\end{aligned}$$

C.2 Marriage, specialized couples

Denote the probability of choosing to be in a specialized marriage P_s and the probability of being single P_0, respectively for women and p_s and p_0 for men. Under the extreme value assumption for we can express the ratio of the choice probabilities for specialized marriage relative to remaining single as:

$$\frac{\hat{P}_s}{\hat{P}_0} = \frac{\exp(\tilde{Q}_s[Y_s] + \Gamma_s)}{\exp(\tilde{Q}_0[W_0, Y_0] + \Gamma_0)}$$

for women and

$$\frac{\hat{p}_s}{\hat{p}_0} = \frac{\exp(\tilde{q}_s[w, y_s] + \gamma_s)}{\exp(\tilde{q}_0[w_0, y_0] + \gamma_0)}$$

D Additional estimation results

D.1 Parameter estimates for sharing rule

Table 8: **Sharing rule estimates and test statistics**

	Benchmark		Ages 21-30		< college	
	Labor supply	Marriage	Labor supply	Marriage	Labor supply	Marriage
Other income	0.3384	0.2847	0.5499	0.5802	0.6829	0.6112
Sex ratio	-0.3109	0.0212	0.0585	0.0114	-0.2549	0.2177
Husband's wage	-1.8088	7.2092	-0.9678	-7.7086	4.0498	9.3865
Wife's wage	-4.3555	0.6457	4.3597	2.6123	-14.1348	7.0168
Single male wage	5.2056	-5.8601	-3.9509	-33.1178	-8.6243	-28.2282
Single male income	-0.2719	-0.2155	-0.3730	-0.4821	0.9540	-0.4702
Single female wage	-11.1777	12.7327	1.8344	13.3063	3.8586	7.6280
Single female income	0.1978	12.8496	-0.0339	12.4498	-0.0211	5.9870
Observations	5206	666	261	5206	666	261

D.2 Estimation results for specification where the sharing rule includes measures of close substitutes.

Table 9: **Semi-parametric labor supply estimates**

	Wives			Husbands		
	Benchmark	Ages 21-30	< college	Benchmark	Ages 21-30	< college
Other income		19.3357			15.53866	
Sex ratio		-75.6033			-199.8190	
Husband's wage		1303.4035			-1229.5691	
Wife's wage		181.0737			-372.7425	
Single male wage		-771.1945			-208.8846	
Single male income		437.7424			322.2468	
Single female wage		-27.4113			274.2810	
Single female income		2561.4964			1405.4254	
Sex ratio by age and race		71.6046			-22.7786	
Sex ratio by age and education		195.0148			1104.4226	
Sex ratio by race and education		-3.9183			-366.0257	
Observations	5206	666		5206	666	

Table 10: **Semi-parametric marriage log odds estimates**

	Wives			Husbands		
	Benchmark	Ages 21-30	< college	Benchmark	Ages 21-30	< college
Other income		0.1685			0.0689	
Sex ratio		1.4213			0.3450	
Husband's wage		4.4192			0.6624	
Wife's wage		0.9748			5.3799	
Single male wage		-0.7691			1.0238	
Single male income		0.3656			-0.6740	
Single female wage		0.9292			0.4177	
Single female income		-1.3677			-18.1905	
Sex ratio by age and race		-0.2247			-0.3793	
Sex ratio by age and education		-13.7379			1.9999	
Sex ratio by race and education		-0.0255			0.1017	
Observations	5206	666		5206	666	

Table 11: **Sharing rule estimates and test statistics**

	Benchmark		Ages 21-30		< college	
	Labor supply	Marriage	Labor supply	Marriage	Labor supply	Marriage
Other income			0.4114	0.0890		
Sex ratio			-0.2871	0.2153		
Husband's wage			3.9071	3.0725		
Wife's wage			-3.9949	0.3563		
Single male wage			-0.0444	-0.5383		
Single male income			-0.8649	-7.8728		
Single female wage			-0.0069	0.4244		
Single female income			-0.0186	-0.4522		
Sex ratio by age and race			-0.0599	-0.4137		
Sex ratio by age and education			-1.0051	0.2233		
Sex ratio by race and education			-3.3346	-0.2906		
Observations	5206	666		5206	666	