Business Cycle Dynamics under Rational Inattention*

Bartosz Maćkowiak
European Central Bank and CEPR

Mirko Wiederholt
Northwestern University

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Abstract

This paper studies a dynamic stochastic general equilibrium model with rational inattention. Decisionmakers have limited attention and choose the optimal allocation of their attention. We study the implications of rational inattention for business cycle dynamics. For example, we study how rational inattention affects impulse responses of prices and quantities to monetary policy shocks, aggregate technology shocks and micro-level shocks.

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1 Introduction

This paper studies a dynamic stochastic general equilibrium model with rational inattention. We model the idea that agents cannot attend perfectly to all available information. Following Sims (2003), we model limited attention as a constraint on information flow. We let decisionmakers choose the optimal allocation of attention. For example, decisionmakers decide how to allocate their attention across their different decision problems. Furthermore, for each decision problem decisionmakers decide how to attend to the different factors that determine the optimal decision.

The economy consists of households, firms and a government. Households consume a variety of goods, hold nominal government bonds and supply differentiated types of labor. Firms hire labor and produce differentiated goods. The central bank sets the nominal interest rate according to a Taylor rule. There are no adjustment costs. Every period, households take consumption and wage setting decisions, and firms take input and price setting decisions. We compute the impulse responses of prices and quantities to monetary policy shocks, aggregate technology shocks and micro-level shocks under both perfect information and rational inattention.

Some of the main findings are: First, the responses of aggregate variables to a monetary policy shock under rational inattention (with a low marginal value of additional attention) are similar to the responses in a Calvo model with an average price duration of 7.5 months. Second, prices respond more quickly to aggregate technology shocks than to monetary policy shocks. Furthermore, prices respond almost perfectly to micro-level shocks. The reason is the optimal allocation of attention. Third, losses in profits due to deviations of the actual price from the static profit-maximizing price are an order of magnitude smaller than in a Calvo model that generates similar real effects of monetary policy shocks.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 presents the solution under perfect information. Section 4 presents the solution under rational inattention. Section 5 concludes.

2 Model

2.1 Households

There are J households. Households supply differentiated types of labor, consume a variety of goods and can hold nominal government bonds.

Each household seeks to maximize the expected discounted sum of period utility. The discount factor is $\beta \in (0,1)$. The period utility function is

$$U\left(C_{jt}, L_{j1t}, \ldots, L_{jNt}\right) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \varphi \sum_{n=1}^{N} e^{-\chi_{jnt}} \frac{L_{jnt}^{1+\psi}}{1+\psi}, \tag{1}$$

where C_{jt} is composite consumption by household j in period t, L_{jnt} is supply of household j's nth type of labor in period t, and χ_{jnt} is a preference shock affecting the disutility of supplying the nth type of labor. We introduce preference shocks in order to generate variation in relative wage rates. We assume that each household supplies N types of labor in order to allow for a certain degree of risk sharing within the household. The parameter $\gamma > 0$ is the coefficient of relative risk aversion, the parameter $\varphi > 0$ affects the disutility of supplying labor and the parameter $\psi > 0$ is the inverse of the Frisch elasticity of labor supply. Composite consumption is given by the usual Dixit-Stiglitz aggregator

$$C_{jt} = \left(\sum_{i=1}^{I} C_{ijt}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \tag{2}$$

where C_{ijt} is consumption of good i by household j in period t. We assume that the elasticity of substitution between different goods exceeds one, $\theta > 1$.

Households can save by holding nominal government bonds. The flow budget constraint of household j in period t is

$$\sum_{i=1}^{I} P_{it} C_{ijt} + B_{jt} = R_{t-1} B_{jt-1} + (1+\tau_w) \sum_{n=1}^{N} W_{jnt} L_{jnt} + \frac{D_t}{J} - \frac{T_t}{J}, \tag{3}$$

where P_{it} is the price of good i in period t, B_{jt} are bond holdings by household j between period t and period $t+1$, R_{t-1} is the nominal interest rate on bond holdings between period $t-1$ and period t, τ_w is a wage subsidy, W_{jnt} is the nominal wage rate for household j's nth type of labor, (D_t/J) is a pro-rata share of nominal aggregate profits and (T_t/J) is a pro-rata share of nominal lump-sum taxes. We assume that all J households have the same initial bond holdings. We assume a natural debt limit.

Every period each household chooses a consumption vector, $(C_{1jt}, \ldots, C_{Ijt})$, and a vector of nominal wage rates, $(W_{j1t}, \ldots, W_{jNt})$.[1] Each household commits to supply any quantity of labor at the chosen nominal wage rates.

We will solve the household problem under two alternative assumptions. Initially, we will assume that households have perfect information. Later, we will assume that households have limited attention, that is, households take their decisions subject to a constraint on information flow.

2.2 Firms

There are I firms in the economy. Firms hire labor in order to produce differentiated goods.

The technology of firm i is given by

$$Y_{it} = e^{a_t} e^{a_{it}} L_{it}^{\alpha}, \tag{4}$$

where Y_{it} is output, $(e^{a_t} e^{a_{it}})$ is total factor productivity and L_{it} is composite labor input of firm i in period t. Total factor productivity has an aggregate component, e^{a_t}, and a firm-specific component, $e^{a_{it}}$. The parameter $\alpha \in (0, 1]$ is the elasticity of output with respect to composite labor. Composite labor is given by the following constant elasticity aggregator

$$L_{it} = \left(\sum_{j=1}^{J} \sum_{n=1}^{N} L_{ijnt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{5}$$

where L_{ijnt} is firm i's input of type jn labor in period t. Recall that type jn labor is household j's nth type of labor. We assume that the elasticity of substitution between different types of labor exceeds one, $\eta > 1$.

The nominal profits of firm i in period t equal

$$(1 + \tau_p) P_{it} Y_{it} - \sum_{j=1}^{J} \sum_{n=1}^{N} W_{jnt} L_{ijnt}, \tag{6}$$

[1] Bond holdings then follow from the labor demand function derived below and the flow budget constraint (3).

where τ_p is a production subsidy.

Every period each firm chooses a labor mix and a price, P_{it}. Each firm commits to supply any quantity of the good at the chosen price.

We will solve the firm problem under two alternative assumptions. First, we will assume that decisionmakers in firms have perfect information. Afterwards, we will assume that decisionmakers in firms have limited attention, that is, they take their decisions subject to a constraint on information flow.

2.3 Government

There is a monetary authority and a fiscal authority. Let $\Pi_t = (P_t/P_{t-1})$ denote inflation where P_t is a price index that will be defined later. Let $Y_t = \sum_{i=1}^{I} Y_{it}$ denote aggregate output. The central bank sets the nominal interest rate according to the rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho_R} e^{\varepsilon_t^R}, \tag{7}$$

where R, Π and Y are the values of the nominal interest rate, inflation and output in the non-stochastic steady state, and ε_t^R is a monetary policy shock. The policy parameters satisfy $\rho_R \in [0,1)$, $\phi_\pi > 1$ and $\phi_y \geq 0$.

The government budget constraint in period t reads

$$T_t + (B_t - B_{t-1}) = (R_{t-1} - 1)\, B_{t-1} + \tau_w \left(\sum_{j=1}^{J}\sum_{n=1}^{N} W_{jnt} L_{jnt}\right) + \tau_p \left(\sum_{i=1}^{I} P_{it} Y_{it}\right). \tag{8}$$

The government has to finance interest on nominal government bonds, the wage subsidy and the production subsidy. The government can collect taxes or issue new government bonds.

2.4 Shocks

There are four types of shocks in the economy: monetary policy shocks, aggregate productivity shocks, firm-specific productivity shocks and labor-specific preference shocks. We assume that, for all i and jn, the processes $\{\varepsilon_t^R\}$, $\{a_t\}$, $\{a_{it}\}$ and $\{\chi_{jnt}\}$ are independent. Furthermore, we assume that all the firm-specific productivity processes, $\{a_{it}\}$, are independent across firms and all the labor-specific preference shocks, $\{\chi_{jnt}\}$, are independent across types of labor. Finally, we assume that all these processes are stationary Gaussian processes with mean zero. In the following, we denote the period t innovation to a_t, a_{it} and χ_{jnt} by ε_t^A, ε_{it}^I and ε_{jnt}^χ, respectively.

3 Solution under perfect information

In this section we derive the equilibrium under perfect information, that is, we assume that in period t all households and all firms know all variables up to and including period t. We will show that under perfect information the classical dichotomy holds. Monetary policy has no real effects. Quantities and relative prices depend only on aggregate productivity (a_t), firm-specific productivity (a_{it}) and labor-specific disutility of work (χ_{jnt}).

3.1 Equations characterizing equilibrium

Cost minimization implies that the demand for type jn labor in period t is given by

$$L_{jnt} = \left(\frac{W_{jnt}}{W_t}\right)^{-\eta} L_t, \tag{9}$$

where W_t is the following wage index

$$W_t = \left(\sum_{j=1}^{J}\sum_{n=1}^{N} W_{jnt}^{1-\eta}\right)^{\frac{1}{1-\eta}}, \tag{10}$$

and L_t is the aggregate composite labor input

$$L_t = \sum_{i=1}^{I} L_{it}. \tag{11}$$

The problem of household j is to choose a contingent plan for the consumption vector and for the vector of nominal wage rates so as to maximize

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{jt}^{1-\gamma}-1}{1-\gamma} - \varphi\sum_{n=1}^{N} e^{-\chi_{jnt}}\frac{L_{jnt}^{1+\psi}}{1+\psi}\right)\right], \tag{12}$$

subject to the consumption aggregator (2), the flow budget constraint (3), the natural debt limit and the labor demand function (9). The first-order conditions for the household problem are:

$$C_{jt}^{-\gamma} = E_t\left[\beta\frac{R_t}{\Pi_{t+1}}C_{jt+1}^{-\gamma}\right], \tag{13}$$

for all i

$$\frac{C_{ijt}}{C_{jt}} = \left(\frac{P_{it}}{P_t}\right)^{-\theta}, \tag{14}$$

and for all n

$$\frac{W_{jnt}}{P_t} = \frac{1}{1+\tau_w}\frac{\eta}{\eta-1}\varphi e^{-\chi_{jnt}}\left[\left(\frac{W_{jnt}}{W_t}\right)^{-\eta} L_t\right]^{\psi} C_{jt}^{\gamma}, \tag{15}$$

where E_t is the expectation operator conditioned on information in period t and P_t is the following price index

$$P_t = \left(\sum_{i=1}^{I} P_{it}^{1-\theta}\right)^{\frac{1}{1-\theta}}. \tag{16}$$

Equation (13) is the consumption Euler equation. Equation (14) characterizes the optimal consumption basket. Equation (15) characterizes the optimal wage setting behavior. Throughout the paper we will assume that the government sets the wage subsidy τ_w so as to correct the distortion arising from households' market power on the labor market. Here this implies that

$$1+\tau_w = \frac{\eta}{\eta-1}. \tag{17}$$

Multiplying equation (14) by C_{jt} and summing over all households yields the demand for good i in period t

$$C_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t, \tag{18}$$

where C_t is aggregate composite consumption

$$C_t = \sum_{j=1}^{J} C_{jt}. \tag{19}$$

The problem of firm i under perfect information is to choose a price and a labor mix so as to maximize profits (6) subject to the technology (4)-(5), the requirement that output has to equal demand and the demand function (18). The firm problem is a static decision problem, because there are no adjustment costs and the demand function (18) is static. The first-order conditions for the firm problem are:

$$P_{it} = \frac{1}{1+\tau_p}\frac{\theta}{\theta-1}W_t\frac{1}{\alpha}\frac{\left[\left(\frac{P_{it}}{P_t}\right)^{-\theta} C_t\right]^{\frac{1}{\alpha}-1}}{\left(e^{a_t}e^{a_{it}}\right)^{\frac{1}{\alpha}}}, \tag{20}$$

and for all jn

$$\frac{L_{ijnt}}{L_{it}} = \left(\frac{W_{jnt}}{W_t}\right)^{-\eta}, \tag{21}$$

where W_t is the wage index (10). Equation (20) characterizes the profit-maximizing price. Throughout the paper we will assume that the government sets the production subsidy τ_p so as to correct the distortion arising from firms' market power on the goods market. This now implies that

$$1 + \tau_p = \frac{\theta}{\theta - 1}. \tag{22}$$

Equation (21) characterizes the profit-maximizing labor mix. Multiplying equation (21) by L_{it} and summing over all firms yields the labor demand function (9).

3.2 Non-stochastic steady state

We call the following situation a non-stochastic steady state: there are no shocks; all equations characterizing equilibrium are satisfied; and quantities, relative prices, the nominal interest rate and inflation are constant over time. Here we report some relationships in the non-stochastic steady state that we will use below.

Equation (20) implies that in the non-stochastic steady state all firms set the same price. Thus households choose a consumption basket with equal weights, implying that all firms produce the same amount and have the same composite labor input. It follows from the price index (16), the consumption aggregator (2), the definition of aggregate output and the definition of aggregate composite labor input (11) that

$$\left(\frac{P_i}{P}\right)^{1-\theta} = \left(\frac{C_{ij}}{C_j}\right)^{\frac{\theta-1}{\theta}} = \frac{Y_i}{Y} = \frac{L_i}{L} = \frac{1}{I}, \tag{23}$$

where (P_i/P) denotes the value of (P_{it}/P_t) in the non-stochastic steady state etc.

Since all households face the same decision problem, all households choose the same composite consumption. It follows from the definition of aggregate composite consumption (19) that

$$\frac{C_j}{C} = \frac{1}{J}. \tag{24}$$

Furthermore, equation (15) implies that in the non-stochastic steady state all households set the same wage rate for all different types of labor. Thus firms choose a labor mix with equal weights. It follows from the wage index (10) and the labor aggregator (5) that

$$\left(\frac{W_{jn}}{W}\right)^{1-\eta} = \left(\frac{L_{ijn}}{L_i}\right)^{\frac{\eta-1}{\eta}} = \frac{1}{JN}, \tag{25}$$

where (W_{jn}/W) denotes the value of (W_{jnt}/W_t) in the non-stochastic steady state etc.

3.3 Log-linearization

In this subsection, we log-linearize the equations characterizing equilibrium. Afterwards we report the log-linear equilibrium dynamics under perfect information. In the following, $\tilde{P}_{it} = (P_{it}/P_t)$ denotes the relative price of good i, $\tilde{W}_{jnt} = (W_{jnt}/P_t)$ denotes the real wage rate for type jn labor and $\tilde{W}_t = (W_t/P_t)$ denotes the real wage index. Furthermore, small letters denote log-deviations from the non-stochastic steady state.

Log-linearizing the households' first-order conditions yields

$$c_{jt} = E_t\left[-\frac{1}{\gamma}\left(r_t - \pi_{t+1}\right) + c_{jt+1}\right], \tag{26}$$

$$c_{ijt} - c_{jt} = -\theta\tilde{p}_{it}, \tag{27}$$

and

$$\tilde{w}_{jnt} = -\frac{1}{1+\eta\psi}\chi_{jnt} + \frac{\eta\psi}{1+\eta\psi}\tilde{w}_t + \frac{\psi}{1+\eta\psi}l_t + \frac{\gamma}{1+\eta\psi}c_{jt}. \tag{28}$$

Furthermore, dividing the definition of the price index (16) by P_t, log-linearizing and using (23) yields

$$\sum_{i=1}^{I}\tilde{p}_{it} = 0. \tag{29}$$

Log-linearizing both the demand function (18) and the definition of aggregate composite consumption (19) and using (24) yields

$$c_{it} = -\theta\tilde{p}_{it} + c_t, \tag{30}$$

and

$$c_t = \frac{1}{J}\sum_{j=1}^{J}c_{jt}. \tag{31}$$

Log-linearizing the firms' first-order conditions yields

$$\tilde{p}_{it} = \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right), \tag{32}$$

and

$$l_{ijnt} - l_{it} = -\eta\left(\tilde{w}_{jnt} - \tilde{w}_t\right). \tag{33}$$

Dividing the definition of the wage index (10) by W_t, log-linearizing and using (25) yields

$$\tilde{w}_t = \frac{1}{JN}\sum_{j=1}^{J}\sum_{n=1}^{N}\tilde{w}_{jnt}. \tag{34}$$

Log-linearizing the production function (4) as well as the labor aggregator (5) and using (25) yields

$$y_{it} = a_t + a_{it} + \alpha l_{it}, \tag{35}$$

and

$$l_{it} = \frac{1}{JN}\sum_{j=1}^{J}\sum_{n=1}^{N} l_{ijnt}. \tag{36}$$

Log-linearizing the labor demand function (9) as well as the definition of aggregate composite labor input (11) and using (23) yields

$$l_{jnt} = -\eta\left(\tilde{w}_{jnt} - \tilde{w}_t\right) + l_t, \tag{37}$$

and

$$l_t = \frac{1}{I}\sum_{i=1}^{I} l_{it}. \tag{38}$$

Log-linearizing the monetary policy rule (7) yields

$$r_t = \rho_R r_{t-1} + (1-\rho_R)\left(\phi_\pi \pi_t + \phi_y y_t\right) + \varepsilon_t^R. \tag{39}$$

Finally, log-linearizing the definition of aggregate output and using (23) yields

$$y_t = \frac{1}{I}\sum_{i=1}^{I} y_{it}. \tag{40}$$

3.4 Log-linearized solution

Assume that I and N are sufficiently large so that[2]

$$\frac{1}{I}\sum_{i=1}^{I} a_{it} = 0, \tag{41}$$

[2] Up to this point $N = 1$ is a special case of the model. Now we are making the assumption that N is sufficiently large so that households can insure against labor-specific preference shocks within the household. This assumption implies that all households have the same consumption level.

and

$$\frac{1}{N}\sum_{n=1}^{N}\chi_{jnt}=0. \tag{42}$$

Then the log-linearized aggregate dynamics under perfect information are given by

$$\begin{aligned}
y_t &= c_t=\frac{1+\psi}{1-\alpha+\alpha\gamma+\psi}a_t, &(43)\\
l_t &= \frac{1-\gamma}{1-\alpha+\alpha\gamma+\psi}a_t, &(44)\\
\tilde{w}_t &= \frac{\gamma+\psi}{1-\alpha+\alpha\gamma+\psi}a_t, &(45)\\
r_t-E_t\left[\pi_{t+1}\right] &= \gamma\frac{1+\psi}{1-\alpha+\alpha\gamma+\psi}E_t\left[a_{t+1}-a_t\right]. &(46)
\end{aligned}$$

The proof is in Appendix A. Under perfect information aggregate output, aggregate employment, the real wage index and the real interest rate depend only on aggregate productivity. Monetary policy has no real effects. The nominal interest rate and inflation follow from the monetary policy rule (39) and the real interest rate (46). Since $(1-\rho_R)\phi_\pi>0$ and $(1-\rho_R)\phi_\pi+\rho_R>1$, the equilibrium paths of the nominal interest rate and inflation are locally determinate.[3]

Substituting the solution (43) and (45) into the price setting equation (32) yields

$$\tilde{p}_{it}=-\frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}a_{it}, \tag{47}$$

which from (27) implies that

$$c_{ijt}-c_{jt}=\frac{\theta\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}a_{it}. \tag{48}$$

The relative price of good i and relative consumption of good i depend only on the firm-specific component of the productivity of firm i.

Since all households face the same decision problem and labor-specific preference shocks average out within the household, all households choose the same composite consumption. Thus $c_{jt}=c_t$. Substituting $c_{jt}=c_t$ and the solution (43)-(45) into the wage setting equation (28) yields

$$\tilde{w}_{jnt}-\tilde{w}_t=-\frac{1}{1+\eta\psi}\chi_{jnt}, \tag{49}$$

[3] See Woodford (2003), chapter 2, Proposition 2.8.

which from (33) implies that

$$l_{ijnt} - l_{it} = \frac{\eta}{1 + \eta\psi}\chi_{jnt}. \tag{50}$$

The relative wage rate for type jn labor and the relative input of type jn labor depend only on the labor-specific disutility of work.

In summary, in this model monetary policy has no real effects under perfect information. Under perfect information fluctuations in quantities and in relative prices are driven by aggregate productivity shocks, firm-specific productivity shocks and labor-specific preference shocks. Next we will solve the model assuming that decisionmakers have limited attention.

4 Case 1: Firms rational inattention, households perfect information

In this section, we assume that decisionmakers in firms have limited attention. For the moment, we continue to assume that households have perfect information in order to isolate the role of limited attention on the side of firms.

4.1 Firms' objective

We assume that firm i chooses the allocation of attention so as to maximize the expected discounted sum of profits. Nominal profits are given by (6), technology is given by (4)-(5), and the demand function is given by (18) because households have perfect information. Substituting the technology (4)-(5) and the demand function (18) into the expression for nominal profits (6) and dividing by P_t yields the real profit function. Computing a log-quadratic approximation of the real profit function around the non-stochastic steady state yields the following expression for the expected discounted sum of losses in profits due to suboptimal behavior[4]

$$E\left[\sum_{t=0}^{\infty} \beta^t \frac{1}{2}\left(x_t - x_t^*\right)' H \left(x_t - x_t^*\right)\right]. \tag{51}$$

[4] In this expression losses in profits have a negative sign.

Here x_t is the following vector of dimension JN

$$x_t = \begin{pmatrix} \tilde{p}_{it} \\ \hat{l}_{i11t} \\ \vdots \\ \hat{l}_{iJ(N-1)t} \end{pmatrix}, \tag{52}$$

where $\tilde{p}_{it}$ is the relative price of good i and $\hat{l}_{ijnt} = l_{ijnt} - l_{it}$ is the relative input of type jn labor at firm i. The optimal behavior under perfect information, x_t^*, is given by equations (32), (33) and (34). The matrix H is of dimension $(JN) \times (JN)$ and equals

$$H = Y_i \alpha \frac{\theta - 1}{\theta} \begin{bmatrix} \frac{\theta(\theta-1)}{\alpha} - \frac{\theta^2}{\alpha^2} & 0 & \cdots & \cdots & 0 \\ 0 & -\frac{2}{\eta JN} & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} \\ \vdots & -\frac{1}{\eta JN} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & -\frac{1}{\eta JN} \\ 0 & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} & -\frac{2}{\eta JN} \end{bmatrix}. \tag{53}$$

The derivation is in Appendix B. After the log-quadratic approximation of the profit function, losses in profits due to suboptimal behavior depend only on the deviation from the optimal behavior. Furthermore, the optimal behavior is given by the usual log-linear first-order conditions. In addition, the H matrix contains all the information (up to second order) about how costly different types of mistakes are. The H matrix is the matrix of second derivatives of the real profit function with respect to x_t and x_t' evaluated at the non-stochastic steady state. The diagonal elements of the H matrix contain information about the cost of a mistake in a single variable. The off-diagonal elements of the H matrix contain information about how a mistake in one variable affects the cost of a mistake in another variable.

4.2 Firms' attention problem

We now formalize the idea that decisionmakers in firms have limited attention. Following Sims (2003), we model decisionmakers' limited attention as a bound on information flow. In particular, we place a bound on the information flow between the stochastic process for the optimal behavior and the stochastic process for the actual behavior. In other words, the optimal behavior cannot contain too much information about the actual behavior, and vice versa. This implies that the actual behavior will differ from the optimal behavior. Agents will make mistakes. We assume that decisionmakers choose the allocation of attention (the use of the total information flow) so as to maximize the expected discounted sum of profits. Formally, the attention problem of firm i reads:

$$\max_{B(L),C(L)} E\left[\sum_{t=0}^{\infty}\beta^t \frac{1}{2}\left(x_t - x_t^*\right)' H \left(x_t - x_t^*\right)\right], \tag{54}$$

subject to a) an equation linking a variable in the objective and a choice variable

$$\tilde{p}_{it} - \tilde{p}_{it}^* = p_{it} - p_{it}^*, \tag{55}$$

b) the equations characterizing the optimal behavior under perfect information

$$p_{it}^* = \underbrace{A_{p1}(L)\,\varepsilon_t^A}_{p_{it}^{A*}} + \underbrace{A_{p2}(L)\,\varepsilon_t^R}_{p_{it}^{R*}} + \underbrace{A_{p3}(L)\,\varepsilon_{it}^I}_{p_{it}^{I*}} \tag{56}$$

$$\hat{l}_{ijnt}^* = A_l(L)\,\varepsilon_{jnt}^{\chi}, \tag{57}$$

c) the equations characterizing the actual behavior

$$p_{it} = \underbrace{B_{p1}(L)\,\varepsilon_t^A + C_{p1}(L)\,\nu_{it}^A}_{p_{it}^A} + \underbrace{B_{p2}(L)\,\varepsilon_t^R + C_{p2}(L)\,\nu_{it}^R}_{p_{it}^R} + \underbrace{B_{p3}(L)\,\varepsilon_{it}^I + C_{p3}(L)\,\nu_{it}^I}_{p_{it}^I} \tag{58}$$

$$\hat{l}_{ijnt} = B_l(L)\,\varepsilon_{jnt}^{\chi} + C_l(L)\,\nu_{ijnt}^{\chi}, \tag{59}$$

and d) the information flow constraint

$$\mathcal{I}\left(\left\{p_{it}^{A*}, p_{it}^{R*}, p_{it}^{I*}, \hat{l}_{i11t}^*, \ldots, \hat{l}_{iJ(N-1)t}^*\right\}; \left\{p_{it}^A, p_{it}^R, p_{it}^I, \hat{l}_{i11t}, \ldots, \hat{l}_{iJ(N-1)t}\right\}\right) \leq \kappa. \tag{60}$$

Here ν_{it}^A, ν_{it}^R, ν_{it}^I and ν_{ijnt}^{χ} follow Gaussian white noise processes that are mutually independent and independent of all other shocks in the economy. Equations (56) and (57) characterize the optimal behavior. $A_{p1}(L)$, $A_{p2}(L)$, $A_{p3}(L)$ and $A_l(L)$ are infinite-order lag polynomials. Equations (58) and (59) specify the actual behavior. Choosing the process for the actual behavior is formalized as choosing the lag polynomials $B_{p1}(L)$, $C_{p1}(L)$, $B_{p2}(L)$, $C_{p2}(L)$, $B_{p3}(L)$, $C_{p3}(L)$, $B_l(L)$ and $C_l(L)$. If the decisionmaker had unlimited attention, the actual behavior would equal the optimal behavior. Formally, $B_{p1}(L) = A_{p1}(L)$ and

$C_{p1}(L) = 0$, etc. The constraint on information flow (60) implies that this is not possible. The operator $\mathcal{I}$ measures the information flow between stochastic processes.[5] The information flow constraint states that the information flow between the optimal behavior and the actual behavior cannot exceed the parameter κ. In other words, the optimal behavior cannot contain too much information about the actual behavior, and vice versa. Finally, equation (55) gives the relationship between the mistake in the dollar price of good i, $p_{it} - p_{it}^*$, and the mistake in the relative price of good i, $\tilde{p}_{it} - \tilde{p}_{it}^*$. The firm chooses the dollar price of good i, while real profits depend on the relative price of good i.

4.3 Computing the equilibrium

We use an iterative procedure to solve for the equilibrium of the model. First, we make a guess concerning the process for the profit-maximizing price (56) and the process for the profit-maximizing labor mix (57). Second, we solve the firms' attention problem (54)-(55). Third, we aggregate the individual prices to obtain the aggregate price level

$$p_t = \frac{1}{I}\sum_{i=1}^{I} p_{it}. \tag{61}$$

Fourth, we compute the aggregate dynamics implied by the price level dynamics. The following equations have to be satisfied in equilibrium:

$$r_t = \rho_R r_{t-1} + (1-\rho_R)\left[\phi_\pi (p_t - p_{t-1}) + \phi_y y_t\right] + \varepsilon_t^R, \tag{62}$$

$$c_t = E_t\left[-\frac{1}{\gamma}(r_t - p_{t+1} + p_t) + c_{t+1}\right], \tag{63}$$

$$\tilde{w}_t = \psi l_t + \gamma c_t, \tag{64}$$

$$y_t = c_t, \tag{65}$$

$$y_t = a_t + \alpha l_t, \tag{66}$$

$$a_t = \rho_A a_{t-1} + \varepsilon_t^A. \tag{67}$$

The first equation is the Taylor rule. The second equation is the consumption Euler equation. The third equation follows from optimal wage setting by households. See Appendix

[5] For a definition of the operator $\mathcal{I}$, see equations (1)-(4) in Maćkowiak and Wiederholt (2007).

C. The fourth equation follows from the requirement that output equals demand. The fifth equation follows from the production function and the sixth equation is the process for aggregate productivity. We employ a standard solution method for linear rational expectations models to solve the system of equations containing the price level dynamics and these six equations. We obtain the law of motion for $(r_t, c_t, y_t, l_t, \tilde{w}_t)$ implied by the price level dynamics. Fifth, we compute the law of motion for the profit-maximizing price from

$$p_{it}^* = p_t + \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right). \tag{68}$$

If the process for the profit-maximizing price differs from our guess, we update our guess. We iterate until we reach a fixed point. Finally, we compute the equilibrium relative wages and the equilibrium relative labor inputs. This is explained in Appendix C.

4.4 Benchmark parameter values and solution

In this section we report the numerical solution of the model for the following parameter values. We set $\alpha = 2/3$, $\beta = 0.99$, $\gamma = 1$, $\psi = 1$, $\theta = 4$, and $\eta = 4$.

To set the parameters governing the process for aggregate technology, equation (67), we consider quarterly U.S. data from 1960 Q1 to 2006 Q4. We first compute a time series for aggregate technology, a_t, using equation (66) and measures of y_t and l_t. We use the log of real output per person, detrended with a linear trend, as a measure of y_t. We use the log of hours worked per person, demeaned, as a measure of l_t.[6] We then fit equation (67) to the time series for a_t obtaining $\rho_A = 0.96$ and a standard deviation of the innovation equal to 0.0085. In the benchmark economy we set $\rho_A = 0.95$ and we set the standard deviation of ε_t^A equal to 0.0085.

To set the parameters of the Taylor rule we consider quarterly U.S. data on the Federal Funds rate, inflation and real GDP from 1960 Q1 to 2006 Q4.[7] We fit the Taylor rule

[6] We use data for the non-farm business sector. The source of the data is the website of the Federal Reserve Bank of St.Louis.

[7] We compute a time series for four-quarter inflation rate from the price index for personal consumption expenditures excluding food and energy. We compute a time series for percentage deviations of real GDP from potential real GDP. The sources of the data are the websites of the Federal Reserve Bank of St.Louis and the Congressional Budget Office.

(62) to the data obtaining $\rho_R = 0.89$, $\phi_\pi = 1.53$, $\phi_y = 0.33$, and a standard deviation of the innovation equal to 0.0021. In the benchmark economy we set $\rho_R = 0.9$, $\phi_\pi = 1.5$, $\phi_y = 0.33$, and the standard deviation of ε_t^R equal to 0.0021.

We assume that firm-specific productivity shocks follow a first-order autoregressive process. Recent papers calibrate the autocorrelation of firm-specific productivity to be about two-thirds in monthly data, e.g., Klenow and Willis (2007) use 0.68, Midrigan (2006) uses 0.5, and Nakamura and Steinsson (2007) use 0.66. Since $(2/3)^3$ equals about 0.3, we set the autocorrelation of firm-specific productivity in our quarterly model equal to 0.3. We then choose the standard deviation of the innovation to firm-specific productivity such that the average absolute size of price changes in our model equals 9.7 percent under perfect information. The value 9.7 percent is the average absolute size of price changes excluding sales reported in Klenow and Kryvtsov (2007). This yields a standard deviation of the innovation to firm-specific productivity equal to 0.22.

We assume that labor-specific preference shocks follow a white noise process. This assumption implies that solving for relative wages and relative labor inputs is straightforward. See Appendix C. To set the standard deviation of labor-specific preference shocks we proceed as follows. Autor, Katz and Kearney (2005) report the variance of log hourly wages of men in the U.S. between 1975 and 2003. The average variance of log hourly wages of men in this period was 0.32. We choose the variance of χ_{jnt} such that the variance of $\tilde{w}_{jnt}$ in our model equals 0.32 under perfect information. This yields a standard deviation of labor-specific preference shocks equal to 2.83. See equation (49) and recall that $\psi = 1$ and $\eta = 4$. We set the number of different types of labor that a firm hires to $JN = 100$.

We compute the solution of the model by fixing the marginal value of information flow. The total information flow, κ, is then determined within the model. The idea is the following. As long as the marginal value of information flow is high, decisionmakers have an incentive to increase information flow in order to take better decision. In contrast, when the marginal value of information flow is low, decisionmakers have little incentive to increase the information flow. We set the marginal value of information flow equal to 0.25 percent of steady state output.

We first report the optimal allocation of attention at the rational inattention fixed point.

The total information flow at the solution equals 133 bits. The decisionmaker in a firm allocates: 0.76 bits of information flow (his/her attention) to tracking aggregate technology, 0.41 bits of information flow to tracking monetary policy, 2.46 bits of information flow to tracking firm-specific productivity, and 1.31 bits of information flow to tracking each relative wage rate. The expected per period loss in profits due to imperfect tracking of aggregate technology equals 0.12 percent of steady state output. The expected per period loss in profits due to imperfect tracking of monetary policy equals 0.07 percent of steady state output. The expected per period loss in profits due to imperfect tracking of firm-specific productivity equals 0.18 percent of steady state output. Together these numbers imply that the total expected per period loss in profits due to suboptimal price setting equals 0.37 percent of steady state output. We think that this is a reasonable number.

Figures 1 and 2 show impulse responses of the price level, inflation, output, and the nominal interest rate at the rational inattention fixed point (green lines with circles). For comparison, the figures also include impulse responses of the same variables at the perfect information equilibrium (blue lines with points). All impulse responses are drawn such that an impulse response equal to one means "a one percent deviation from the non-stochastic steady state". All impulse responses are to shocks of one standard deviation. Time is measured in quarters along horizontal axes.

Consider Figure 1. The price level shows a dampened and delayed response to a monetary policy shock compared with the case of perfect information. The response of inflation to a monetary policy shock is persistent. Output falls after a positive innovation in the Taylor rule and the decline in output is persistent. The nominal interest rate increases on impact and converges slowly to zero. The impulse responses to a monetary policy shock under rational inattention differ markedly from the impulse responses to a monetary policy shock under perfect information. Under perfect information the price level adjusts fully on impact to a monetary policy shock, there are no real effects, and the nominal interest rate fails to change.

Consider Figure 2. The price level and inflation show a dampened response to an aggregate technology shock compared with the case of perfect information. The output gap is negative for a few quarters after the shock. Output and the nominal interest rate show hump-shaped impulse responses to an aggregate technology shock. Note that under rational inattention the response of the price level to an aggregate technology shock is less dampened and less delayed than the response of the price level to a monetary policy shock. The reason is the optimal allocation of attention. Decisionmakers allocate about twice as much attention to aggregate technology than to monetary policy. Therefore, prices respond more strongly and more quickly to aggregate technology shocks than to monetary policy shocks. As a result, the output gap is negative for only 5 quarters after an aggregate technology shock, while the output gap is negative for more than 10 quarters after a monetary policy shock.[8]

Figure 3 shows the impulse response of an individual price to a firm-specific productivity shock. Note that decisionmakers respond almost perfectly to firm-specific productivity shocks. The reason is again the optimal allocation of attention.

4.5 Comparison to the Calvo model

For comparison, we solved the Calvo model for the same parameter values and assuming that prices change every 2.5 quarters on average.[9] Figures 4 and 5 reproduce the impulse responses given in Figures 1 and 2. In addition, Figures 4 and 5 show the impulse responses in the Calvo model (red lines with crosses).

The impulse responses to a monetary policy shock in the benchmark economy are very similar to the impulse responses to a monetary policy shock in the Calvo model assuming that prices in the Calvo model change every 2.5 quarters on average. In contrast, the impulse responses to an aggregate technology shock are quite different in the two models. Note that inflation responds to a monetary policy shock by the same amount on impact in the benchmark economy and in the Calvo model, whereas inflation responds to an aggregate technology shock twice more strongly on impact in the benchmark economy than in the

[8] See also Paciello (2007). Paciello solves the white noise case of a similar model analytically. Here white noise case means that: (i) all exogenous processes are white noise processes, (ii) there is no lagged interest rate in the Taylor rule, and (iii) the price level instead of the inflation rate appears in the Taylor rule. He then studies the differential response of prices to monetary policy shocks and to aggregate technology shocks.

[9] Klenow and Kryvtsov (2007) find that the median price duration excluding sale-related price changes is 7.2 months, or about 2.5 quarters.

Calvo model. The reason is that decisionmakers in the benchmark economy allocate more attention to aggregate technology than to monetary policy.

Firms in the benchmark economy and firms in the Calvo model experience losses in profits due to deviations of the actual price from the static profit-maximizing price. The expected per period loss in profits due to suboptimal response to firm-specific productivity shocks in the Calvo model is twenty five times larger than in the benchmark economy. The reason is that prices in the benchmark economy respond almost perfectly to firm-specific productivity shocks. Furthermore, the expected per period loss in profits due to suboptimal response to aggregate shocks in the Calvo model is twelve times larger than in the benchmark economy. One reason is that prices in the benchmark economy respond fairly well to aggregate technology shocks. Another reason is that in the benchmark economy deviations of the actual price from the static profit-maximizing price are less likely to be extreme than in the Calvo model.

4.6 Varying parameter values

Figure 6 compares the benchmark economy to an economy with a higher degree of real rigidity.[10] We set $\psi = 0$ implying that the coefficient on aggregate output in the equation for the profit-maximizing price falls from 1 to 0.5 (after substituting in the wage equation). Real effects of monetary policy shocks become larger and more persistent. The output gap is negative for about 20 quarters after a monetary policy shock. At the same time, profit losses due to imperfect tracking of aggregate conditions decrease substantially. The expected per period loss in profits due to imperfect tracking of aggregate technology falls by 25 percent. The expected per period loss in profits due to imperfect tracking of monetary policy falls by 60 percent.

Figure 7 compares the benchmark economy to an economy with larger monetary policy shocks. We increase the standard deviation of monetary policy shocks by roughly a factor of two, to 0.004. This matches the standard deviation of monetary policy shocks estimated

[10] Ball and Romer (1990) refer to the sensitivity of the profit-maximizing price to aggregate output as the degree of real rigidity, where a low sensitivity of the profit-maximizing price to aggregate output corresponds to a high degree of real rigidity.

by Justiniano and Primiceri (2006) for the high inflation episode in the 1970s.[11] At the rational inattention fixed point with larger monetary policy shocks firms allocate 0.84 bits to monetary policy, an increase by 100 percent compared with the benchmark economy. Since firms allocate more attention to monetary policy, a monetary policy shock of a given size has smaller real effects and larger inflationary effects.[12] The Phillips curve becomes steeper. Despite this, the expected per period loss in profits due to imperfect tracking of monetary policy almost doubles, and the variance of output due to monetary policy shocks increases. This is because the average size of monetary policy shocks has increased.

5 Conclusion

We have introduced rational inattention on the side of firms into a dynamic stochastic general equilibrium model. The impulse responses under rational inattention have several properties of empirical impulse response functions. The next step is to add rational inattention on the side of households.

[11] Justiniano and Primiceri (2006) estimate a DSGE model that allows for time variation in the size of shocks.

[12] In Figure 7 one must divide an impulse response in the economy with larger monetary policy shocks by roughly one-half to obtain an impulse response to a shock of the same size as in the benchmark economy.

A Solution under perfect information

First, $y_{it} = c_{it}$ and equations (29), (30) and (40) imply that

$$y_t = c_t.$$

Second, computing the average of the production function (35) over all i and using (38), (40) and (41) yields

$$y_t = a_t + \alpha l_t.$$

Third, computing the average of the price setting equation (32) over all i and using (29), (41) and $l_t = \frac{1}{\alpha}(c_t - a_t)$ yields

$$\tilde{w}_t = c_t - l_t.$$

The real wage index equals output per labor input. Fourth, computing the average of the wage setting equation (28) over all jn and using (31), (34) and (42) yields

$$\tilde{w}_t = \psi l_t + \gamma c_t.$$

The real wage index equals the marginal rate of substitution of consumption for leisure. When we solve the last four equations for y_t, c_t, l_t and $\tilde{w}_t$, we arrive at equations (43)-(45). Finally, computing the average of the Euler equation (26) over all j and using (31) yields

$$c_t = E_t\left[-\frac{1}{\gamma}(r_t - \pi_{t+1}) + c_{t+1}\right].$$

Substituting the solution for c_t into the last equation yields equation (46).

B Derivation of the firms' objective

The nominal profits of firm i in period t equal

$$(1+\tau_p) P_{it} Y_{it} - L_{it}\left(\sum_{j=1}^{J}\sum_{n=1}^{N} W_{jnt}\hat{L}_{ijnt}\right),$$

where $\hat{L}_{ijnt} = (L_{ijnt}/L_{it})$. The term in brackets is the wage bill per unit of composite labor. The production function (4) implies that

$$L_{it} = \left(\frac{Y_{it}}{e^{a_t + a_{it}}}\right)^{\frac{1}{\alpha}}.$$

The labor aggregator (5) implies that

$$1=\sum_{j=1}^{J}\sum_{n=1}^{N}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}},$$

or equivalently

$$\hat{L}_{iJNt}=\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}.$$

Furthermore, since households have perfect information, the demand function equals

$$C_{it}=\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t.$$

Substituting the production function, the labor aggregator and the demand function into the expression for nominal profits yields the profit function

$$\left(1+\tau_p\right)P_{it}\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t-\left(\frac{\left(\frac{P_{it}}{P_t}\right)^{-\theta}C_t}{e^{a_t+a_{it}}}\right)^{\frac{1}{\alpha}}\left[\sum_{jn\neq JN}W_{jnt}\hat{L}_{ijnt}+W_{JNt}\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right].$$

Dividing by P_t yields the real profit function

$$\left(1+\tau_p\right)\tilde{P}_{it}^{1-\theta}C_t-\left(\frac{\tilde{P}_{it}^{-\theta}C_t}{e^{a_t+a_{it}}}\right)^{\frac{1}{\alpha}}\left[\sum_{jn\neq JN}\tilde{W}_{jnt}\hat{L}_{ijnt}+\tilde{W}_{JNt}\left(1-\sum_{jn\neq JN}\hat{L}_{ijnt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}\right], \quad (69)$$

where $\tilde{P}_{it}=(P_{it}/P_t)$ and $\tilde{W}_{jnt}=(W_{jnt}/P_t)$. One can express the real profit function in terms of log-deviations from the non-stochastic steady state

$$\begin{aligned}&\left(1+\tau_p\right)\tilde{P}_iC_ie^{(1-\theta)\tilde{p}_{it}+c_t}\\&-L_ie^{\frac{1}{\alpha}(-\theta\tilde{p}_{it}+c_t-a_t-a_{it})}\tilde{W}\frac{1}{JN}\left[\sum_{jn\neq JN}e^{\tilde{w}_{jnt}+\hat{l}_{ijnt}}+e^{\tilde{w}_{JNt}}\left(JN-\sum_{jn\neq JN}e^{\frac{\eta-1}{\eta}\hat{l}_{ijnt}}\right)^{\frac{\eta}{\eta-1}}\right]\end{aligned} \quad (70)$$

Here we have used equation (25).

We assume that firm i chooses the allocation of attention so as to maximize the expected discounted sum of profits

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{jt}}{C_{j0}}\right)^{-\gamma}F\left(\tilde{P}_{it},C_t,e^{a_t},e^{a_{it}},\tilde{W}_{11t},\ldots,\tilde{W}_{JNt},\hat{L}_{i11t},\ldots,\hat{L}_{iJ(N-1)t}\right)\right], \quad (71)$$

where $\beta^t \left(\frac{C_{jt}}{C_{j0}}\right)^{-\gamma}$ is the stochastic discount factor and F is the real profit function (69). Two remarks concerning firm i's objective may be helpful. First, C_{jt} is composite consumption by household j in period t. Since in equilibrium all households have the same composite consumption, the stochastic discount factor does not depend on j. Second, E is the unconditional expectation operator. Here we are using the assumption that firms choose the allocation of attention before receiving any information. This assumption slightly simplifies the computation of the equilibrium. The assumption can be relaxed.

One can express the objective (71) in terms of log-deviations from the non-stochastic steady state

$$E\left[\sum_{t=0}^{\infty}\beta^t e^{-\gamma(c_{jt}-c_{j0})} f\left(\tilde{p}_{it}, c_t, a_t, a_{it}, \tilde{w}_{11t}, \ldots, \tilde{w}_{JNt}, \hat{l}_{i11t}, \ldots, \hat{l}_{iJ(N-1)t}\right)\right], \tag{72}$$

where f is the real profit function (70).

Next we compute a second-order Taylor approximation around the non-stochastic steady state of the term inside the expectation operator of (72). Afterwards, we deduct from the quadratic objective the value of the quadratic objective at the profit-maximizing behavior $\left\{\tilde{p}_{it}^*, \hat{l}_{i11t}^*, \ldots, \hat{l}_{iJ(N-1)t}^*\right\}_{t=0}^{\infty}$. This yields the following expression for (minus) the expected discounted sum of losses in profits due to suboptimal behavior:

$$E\left[\sum_{t=0}^{\infty}\beta^t \frac{1}{2}\left(x_t - x_t^*\right)' H \left(x_t - x_t^*\right)\right],$$

where

$$\underset{(JN\times 1)}{x_t} = \begin{pmatrix} \tilde{p}_{it} \\ \hat{l}_{i11t} \\ \vdots \\ \hat{l}_{iJ(N-1)t} \end{pmatrix},$$

$$\underset{(JN\times JN)}{H} = \tilde{W} L_i \begin{bmatrix} \frac{\theta(\theta-1)}{\alpha} - \frac{\theta^2}{\alpha^2} & 0 & \cdots & \cdots & 0 \\ 0 & -\frac{2}{\eta JN} & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} \\ \vdots & -\frac{1}{\eta JN} & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & -\frac{1}{\eta JN} \\ 0 & -\frac{1}{\eta JN} & \cdots & -\frac{1}{\eta JN} & -\frac{2}{\eta JN} \end{bmatrix},$$

and x_t^* is given by the following two equations:

$$\tilde{p}_{it}^* = \frac{1}{1+\theta\frac{1-\alpha}{\alpha}}\tilde{w}_t + \frac{\frac{1-\alpha}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}c_t - \frac{\frac{1}{\alpha}}{1+\theta\frac{1-\alpha}{\alpha}}\left(a_t + a_{it}\right),$$

and

$$\hat{l}_{ijnt}^* = -\eta\left(\tilde{w}_{jnt} - \tilde{w}_t\right).$$

Here $\tilde{w}_t = \frac{1}{JN}\sum_{j=1}^{J}\sum_{n=1}^{N}\tilde{w}_{jnt}$. Note that deducting from the quadratic objective the value of the quadratic objective at the profit-maximizing behavior is simply a monotone transformation of the quadratic objective. This transformation simplifies the quadratic objective without affecting the solution to the optimization problem.

C Firms rational inattention, households perfect information: solving for relative wages

In this subsection, we solve for the relative wage rate for type jn labor and the relative input of type jn labor at firm i. Solving for relative wage rates is more complicated than solving for relative prices, because firms' inattention lowers the wage elasticity of labor demand, which affects households' wage setting behavior. In order to make the derivation as clear as possible, we assume that all the χ_{jnt} follow a common white noise process.

Let $\hat{w}_{jnt} = \tilde{w}_{jnt} - \tilde{w}_t$ denote the relative wage rate for type jn labor. We guess that in equilibrium

$$\hat{w}_{jnt} = A\chi_{jnt}, \tag{73}$$

where A is an unknown coefficient. Let $\hat{l}_{ijnt} = l_{ijnt} - l_{it}$ denote the relative input of type jn labor at firm i. The profit-maximizing relative input of type jn labor at firm i is given by equation (33):

$$\hat{l}_{ijnt}^* = -\eta\hat{w}_{jnt}. \tag{74}$$

Since χ_{jnt} follows a Gaussian white noise process, both $\hat{w}_{jnt}$ and $\hat{l}_{ijnt}^*$ follow Gaussian white noise processes. Tracking an optimal decision that follows a Gaussian white noise process with an information flow equal to κ_χ yields the following decision under rational inattention when the aim is to minimize the mean squared error

$$\hat{l}_{ijnt}^{RI} = \left(1 - \frac{1}{2^{2\kappa_\chi}}\right)\hat{l}_{ijnt}^* + \sqrt{\frac{1}{2^{2\kappa_\chi}} - \frac{1}{2^{4\kappa_\chi}}}\sqrt{Var\left(\hat{l}_{ijnt}^*\right)}\nu_{ijnt}^{\chi}, \tag{75}$$

where ν_{ijnt}^{χ} follows an independent Gaussian white noise process with unit variance. See, for example, Proposition 3 in Maćkowiak and Wiederholt (2007). Using equation (74) to substitute for $\hat{l}_{ijnt}^{*}$ in equation (75), we arrive at

$$\hat{l}_{ijnt}^{RI} = -\eta \left(1 - \frac{1}{2^{2\kappa_\chi}}\right) \left(\hat{w}_{jnt} - \sqrt{\frac{1}{2^{2\kappa_\chi} - 1}} \sqrt{Var\left(\hat{w}_{jnt}\right)} \nu_{ijnt}^{\chi}\right). \tag{76}$$

Now it is easy to verify that the signal

$$s_{ijnt} = \hat{w}_{jnt} - \sqrt{\frac{1}{2^{2\kappa_\chi} - 1}} \sqrt{Var\left(\hat{w}_{jnt}\right)} \nu_{ijnt}^{\chi} \tag{77}$$

has the property

$$\hat{l}_{ijnt}^{RI} = E\left[\hat{l}_{ijnt}^{*} | s_{ijnt}, s_{ijnt-1}, \ldots\right].$$

Hence, one can interpret the decision under rational inattention as being due to the fact that firms pay limited attention to the relative wage rate for type jn labor. Furthermore, comparing (74) to (76) one can see that firms' limited attention lowers the wage elasticity of labor demand from η to $\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)$.

Computing the average of (76) over all firms and using the fact that noise is idiosyncratic yields

$$l_{jnt} - l_t = -\eta \left(1 - \frac{1}{2^{2\kappa_\chi}}\right) \hat{w}_{jnt}, \tag{78}$$

where $l_{jnt} = \frac{1}{I}\sum_{i=1}^{I} l_{ijnt}$ and $l_t = \frac{1}{I}\sum_{i=1}^{I} l_{it}$. Exponentiating both sides of equation (78), multiplying by L_{jn} and using the fact that $L_{jn} = \left(\frac{W_{jn}}{W}\right)^{-\eta} L$ yields

$$L_{jnt} = \left(\frac{W_{jn}}{W}\right)^{-\frac{\eta}{2^{2\kappa_\chi}}} \left(\frac{W_{jnt}}{W_t}\right)^{-\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)} L_t. \tag{79}$$

The first term on the RHS is due to the fact that in the non-stochastic steady state the wage elasticity of labor demand equals η rather than $\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)$.

When firms have limited attention, the household problem is to maximize (12) subject to (2), (3) and the new labor demand function (79). The optimality conditions for consumption are still equations (13) and (14). So long as $\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right) > 1$, the optimal wage rate for type jn labor is given by

$$\frac{W_{jnt}}{P_t} = \frac{1}{1 + \tau_w} \frac{\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right)}{\eta\left(1 - \frac{1}{2^{2\kappa_\chi}}\right) - 1} \varphi e^{-\chi_{jnt}} L_{jnt}^{\psi} C_{jt}^{\gamma}.$$

We continue to assume that the government sets the wage subsidy τ_w so as to correct the distortion arising from households' market power on the labor market. This now implies that

$$1+\tau_w = \frac{\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)}{\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)-1}.$$

Therefore the wage setting equation reduces to

$$\frac{W_{jnt}}{P_t} = \varphi e^{-\chi_{jnt}} L_{jnt}^{\psi} C_{jt}^{\gamma}. \tag{80}$$

Taking logs on both sides of (80) and using the fact that (80) with $\chi_{jnt}=0$ also holds in the non-stochastic steady state yields

$$\tilde{w}_{jnt} = -\chi_{jnt} + \psi l_{jnt} + \gamma c_{jt}.$$

Using the labor demand function (78) and $\hat{w}_{jnt} = \tilde{w}_{jnt} - \tilde{w}_t$ to substitute for l_{jnt} in the last equation yields

$$\tilde{w}_{jnt} = -\chi_{jnt} + \psi\left[-\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)(\tilde{w}_{jnt}-\tilde{w}_t) + l_t\right] + \gamma c_{jt}. \tag{81}$$

Computing the average of (81) over all types of labor and using (34), (42) and $c_{jt} = c_t$ yields the following expression for the real wage index

$$\tilde{w}_t = \psi l_t + \gamma c_t. \tag{82}$$

Computing the difference between (81) and (82) and using again $c_{jt} = c_t$ yields the following expression for the relative wage rate for type jn labor

$$\tilde{w}_{jnt} - \tilde{w}_t = -\frac{1}{1+\eta\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\psi}\chi_{jnt}. \tag{83}$$

Comparing (73) and (83) shows that the guess (73) was correct. Firm i's profit-maximizing labor mix then follows from equation (74) and firm i's actual labor mix under rational inattention follows from equation (76).

We still need to solve for the equilibrium attention allocated to the profit-maximizing relative input of type jn labor. Equations (74), (76) and (83) imply that the mean squared error in the relative input of type jn labor equals

$$E\left[\left(\hat{l}_{ijnt}^{*} - \hat{l}_{ijnt}^{RI}\right)^2\right] = \frac{1}{2^{2\kappa_\chi}} \frac{\eta^2}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^2}\sigma_\chi^2.$$

The derivative of the mean squared error with respect to κ_χ equals

$$\frac{\partial E\left[\left(\hat{l}^{*}_{ijnt}-\hat{l}^{RI}_{ijnt}\right)^{2}\right]}{\partial \kappa_\chi}=-2\ln\left(2\right)\frac{1}{2^{2\kappa_\chi}}\frac{1+\left(1+\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^{3}}\eta^{2}\sigma_\chi^{2}.$$

It follows from the objective (54) that the marginal value of paying attention to the profit-maximizing relative input of type jn labor equals

$$\lambda_\chi=\frac{1}{1-\beta}\frac{1}{\eta JN}2\ln\left(2\right)\frac{1}{2^{2\kappa_\chi}}\frac{1+\left(1+\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi}{\left[1+\left(1-\frac{1}{2^{2\kappa_\chi}}\right)\eta\psi\right]^{3}}\eta^{2}\sigma_\chi^{2}. \tag{84}$$

By equating the marginal value of attention across different activities we obtain the equilibrium κ_χ.

D Solving the Calvo model

If we assume that firms and households have perfect information but firms face a Calvo friction, we obtain the following version of the New Keynesian Phillips curve

$$\pi_t=\frac{\left(1-\lambda\right)\left(1-\lambda\beta\right)}{\lambda}\frac{\frac{\psi}{\alpha}+\gamma+\frac{1-\alpha}{\alpha}}{1+\frac{1-\alpha}{\alpha}\theta}\left(c_t-c_t^f\right)+\beta E_t\left[\pi_{t+1}\right], \tag{85}$$

where $(1-\lambda)$ is the fraction of goods prices that change every period and c_t^f is the flexible price solution given by equation (43). The aggregate dynamics are obtained by solving the system containing equations (62)-(67) and equation (85). The solution of the Calvo model reported in Figures 3-4 sets $\lambda = 0.6$.

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Figure 1: Impulse responses, benchmark economy

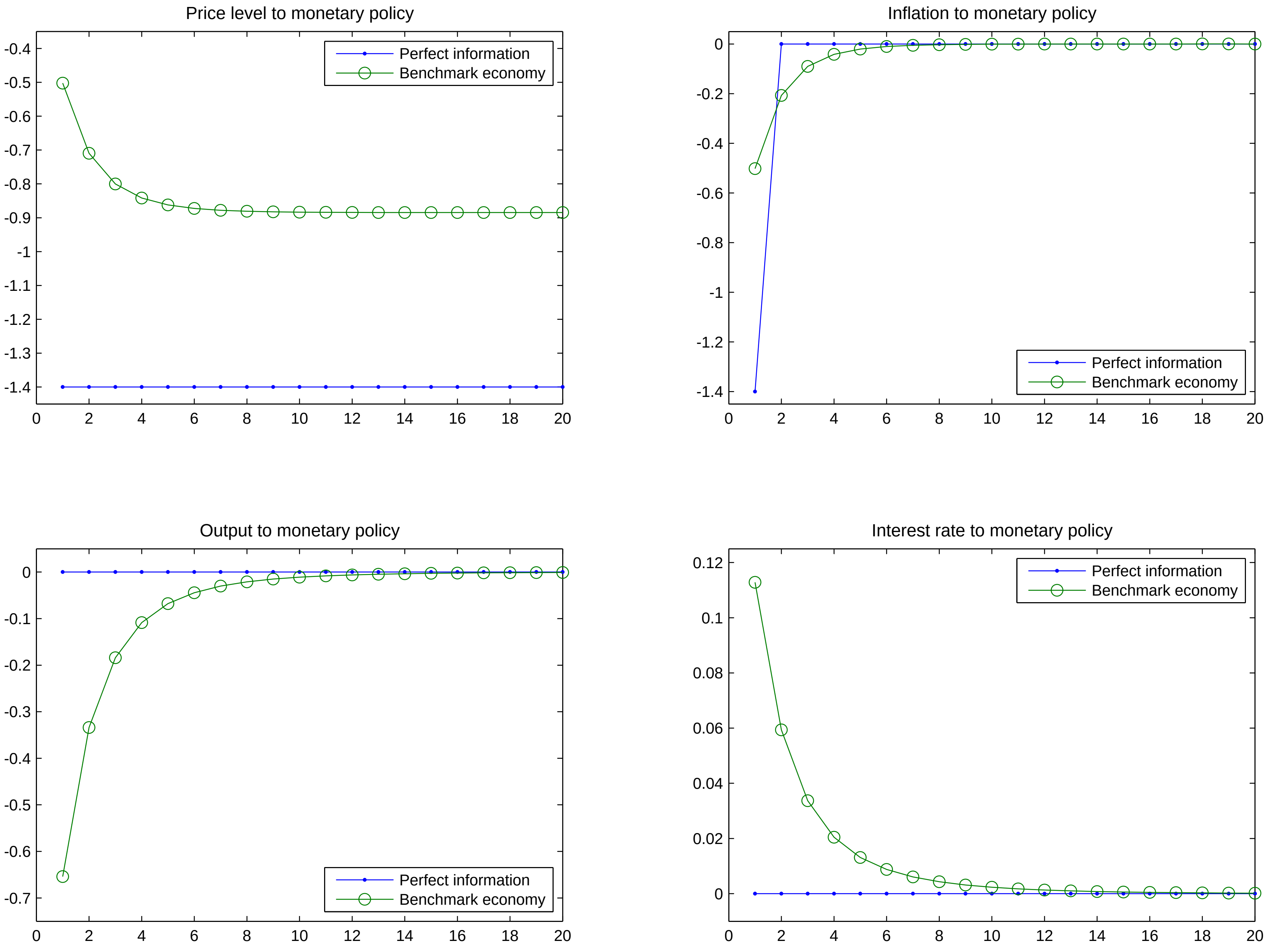

Figure 2: Impulse responses, benchmark economy

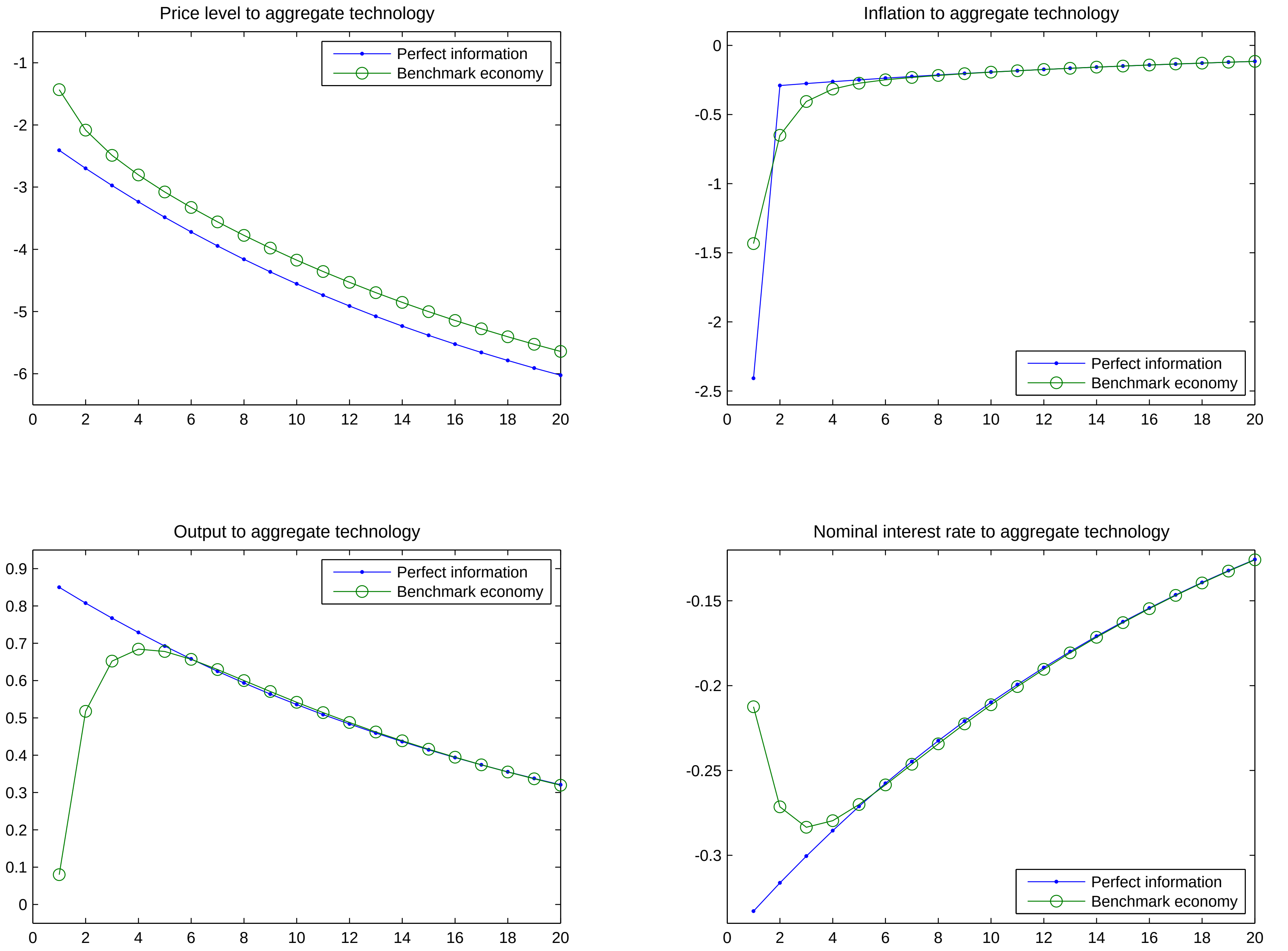

Figure 3: Impulse response of an individual price to a firm-specific productivity shock

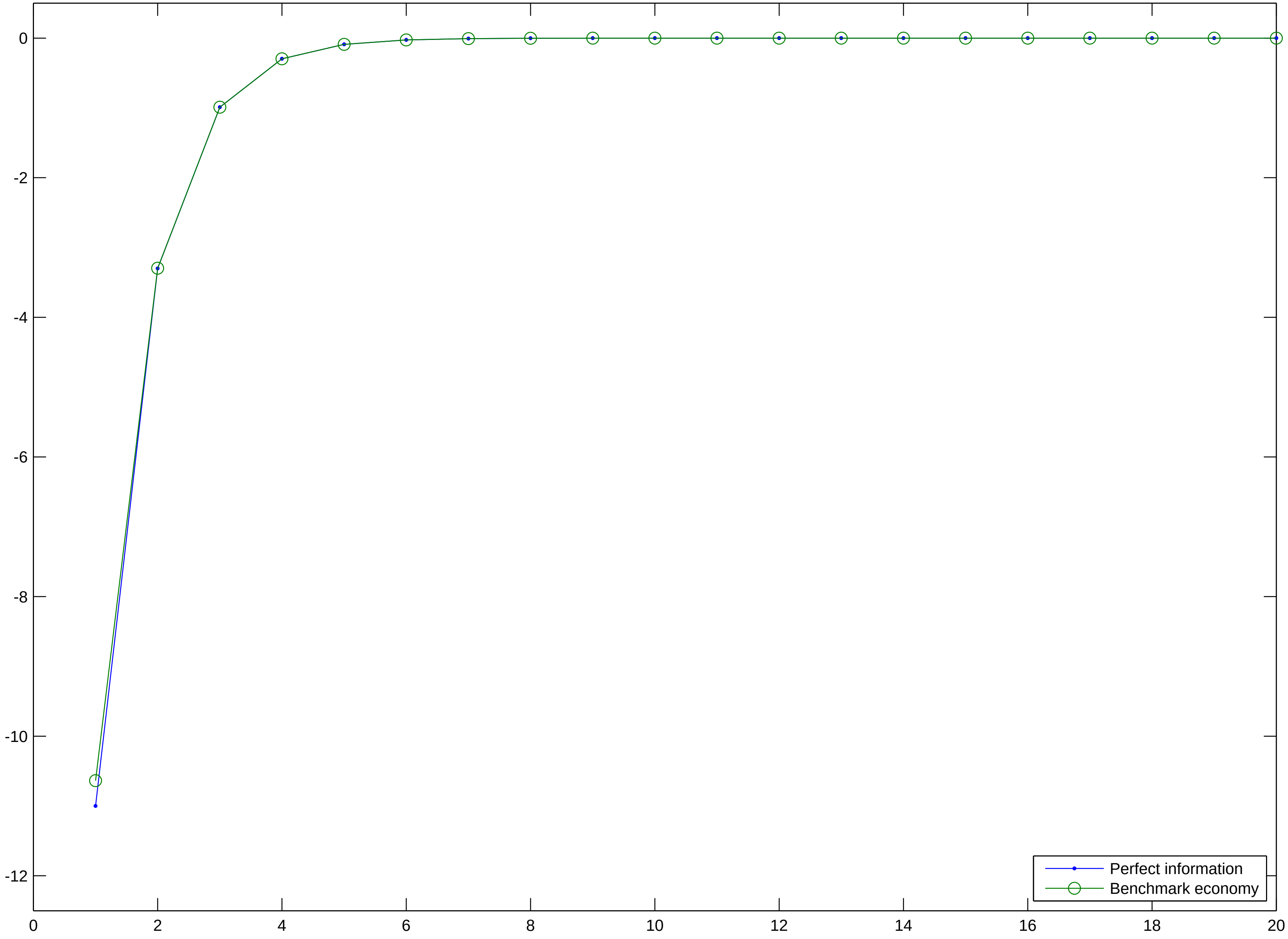

Figure 4: Impulse responses, benchmark economy and the Calvo model

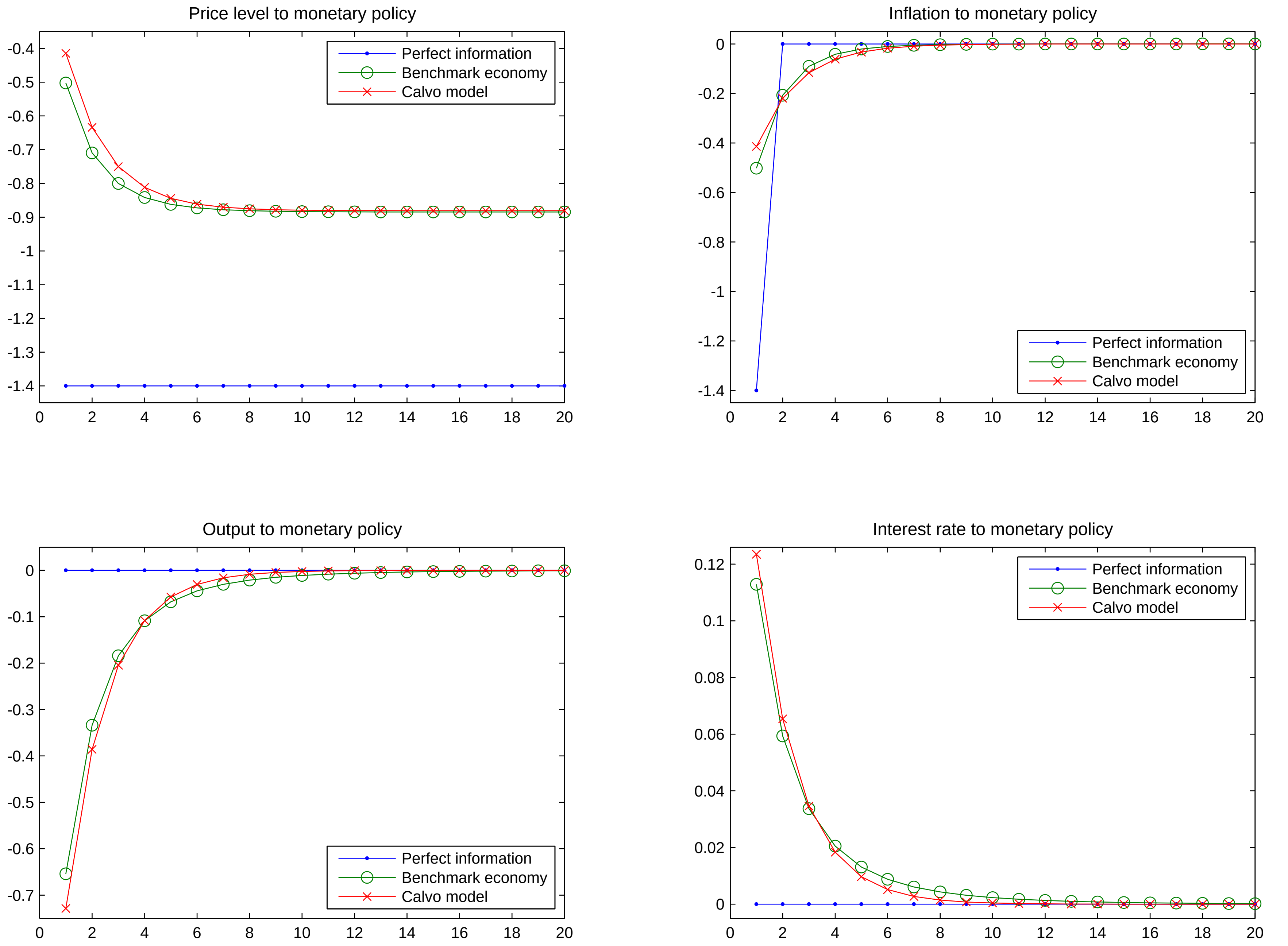

Figure 5: Impulse responses, benchmark economy and the Calvo model

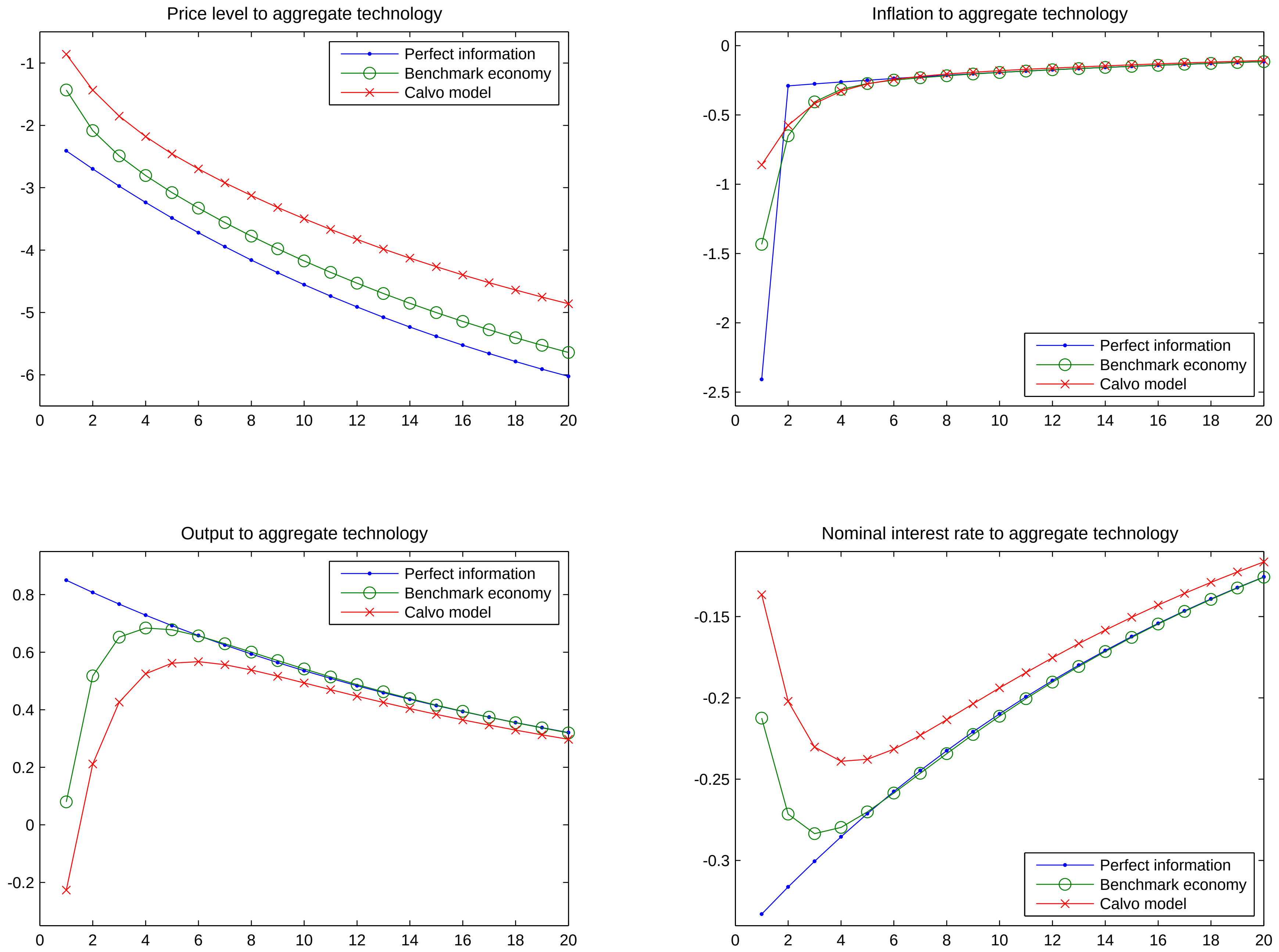

Figure 6: Impulse responses, benchmark economy and an economy with more real rigidity

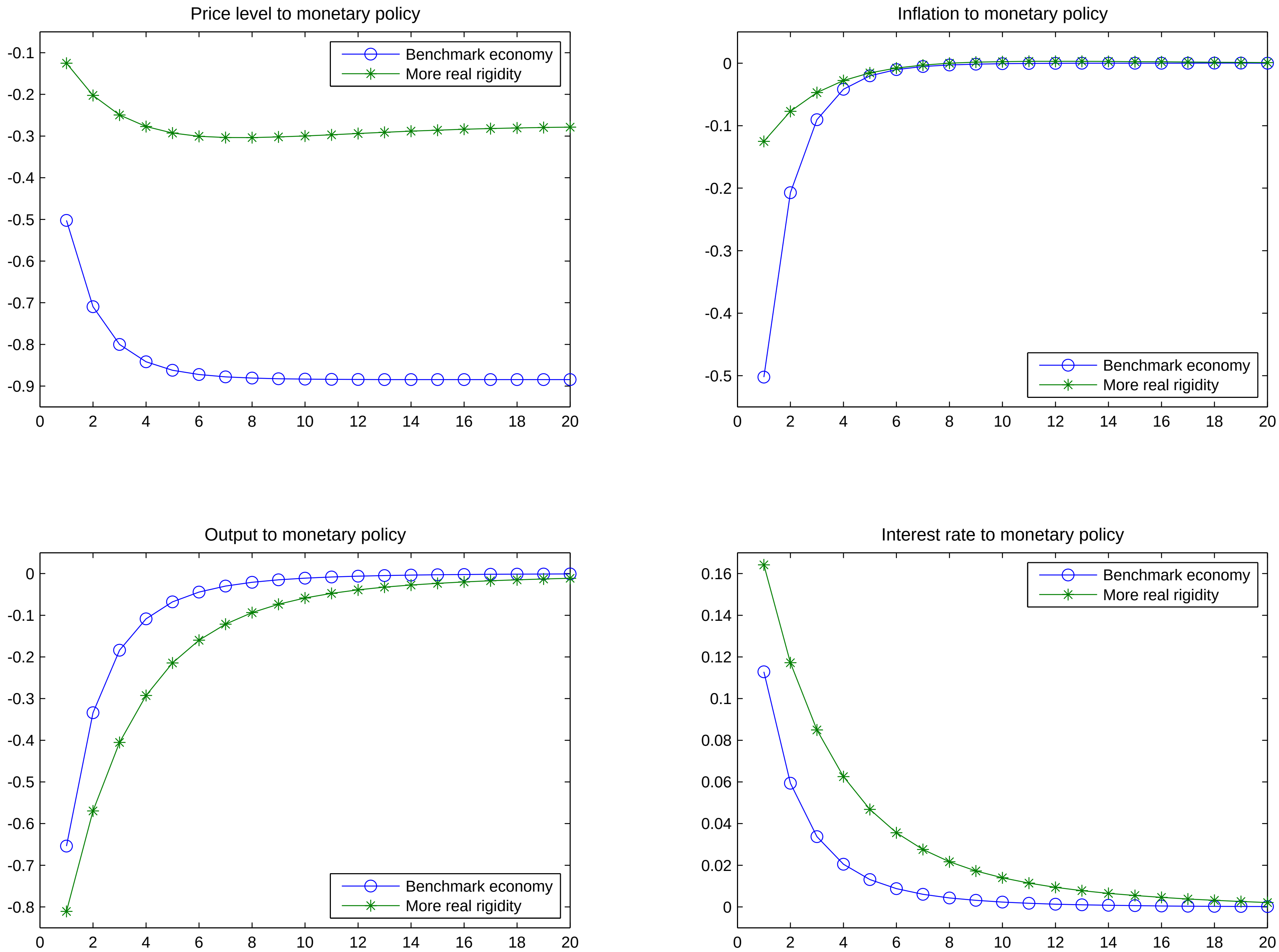

Figure 7: Impulse responses, benchmark economy and an economy with larger monetary policy shocks