

Business Start-ups and Productive Efficiency

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Abstract

This paper studies efficient allocation of resources in an economy in which potential entrepreneurs are heterogeneous initially regarding their wealth levels and whether they have ideas or not. An agent with an idea can start a business which generates random returns. I assume agents have private information about (1) their initial types, (2) the level of investment in their businesses, and (3) the realized returns. Returns being unobservable creates a novel motive for redistribution towards agents who are productive but lack resources to invest in their ideas. To analyze this motive in isolation I assume agents are risk-neutral and abstract away from equality and insurance considerations. The unobservability of wealth and productivity implies that the redistribution that poor agents with ideas get is limited by incentive-compatibility: the society should provide other agents with enough incentives so that they do not claim to be poor and productive. Efficient amount of redistribution from unproductive agents to productive ones arises from this trade-off. The paper then provides an implementation of this efficient allocation in an incomplete markets setup with very simple taxes on bond holdings: those who lend in the market are taxed and the proceeds are used to subsidize borrowers. Finally, I extend the model to allow for unobservable risk-free technology and negative consumption, and show that the redistribution result and its implementation are robust to these generalizations of the model.

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1 Introduction

Starting a business requires two main ingredients: a productive idea and resources to invest in that idea. Unfortunately, it is not necessarily the case that whoever has one of these ingredients also has the other one. Consequently, there is a potential mismatch among individuals in a society in terms of who holds productive resources and who can use them most efficiently. This paper explores how a society should cope with this mismatch in an environment in which wealth levels and ex-ante productive capabilities and investment returns of potential entrepreneurs are private information.

Individuals in the economy I study live for two periods and are risk-neutral. In period one, agents are heterogeneous with respect to wealth levels and whether they have ideas or not. Agents with ideas can create businesses which generate risky returns in the next period. In the absence of informational frictions, the "first-best" way of allocating resources involves two separate steps: (1) productive efficiency requires transferring resources to poor and productive agents initially to make sure that all productive agents can invest at the socially efficient level; (2) distributive efficiency then requires making transfers between agents so as to achieve the desired consumption distribution, which depends on the welfare criterion of the society.

Unfortunately, it is hardly the case that all relevant information about business start-ups are known publicly¹. First informational assumption I make is that ex-post returns to newly started businesses are unobservable by others². This assumption can be thought as an approximation to the fact that it is costly to obtain such information, especially in the case of start-ups. As a result of this friction, agents cannot write contracts with state-contingent repayment schedules, which in turn makes it impossible for agents who have ideas but do not possess sufficient initial resources (poor) to invest at the efficient level. If the society can transfer some of its resources to poor and productive members, it would relax these agents' budget constraints enabling them to produce at a level closer to the social optimal.

¹See Hubbard (1998) for a survey.

²This is a common assumption in financial contracting literature, even for established firms. Among others Diamond (1984), Gale and Hellwig (1985), Bolton and Scharfstein (1990) make the same assumption.

Consequently, the motivation for redistribution in this model is the need to finance the investment of poor and productive agents. In fact, due to risk-neutrality assumption, this is the only reason why redistribution is socially desirable.

The second informational assumption that I make is that agents' ex-ante types - wealth and productivity levels - are private information. Again, this assumption is a crude approximation to reality: when there are benefits at stake (like redistribution), people can pretend to be poor and have productive ideas, and it is costly to an outside party to monitor wealth or to understand whether people's ideas are productive or not. The unobservability of wealth and productivity, then, implies that the amount of redistribution to poor and productive agents is constrained by incentive compatibility: no agent should find it optimal to lie about their wealth and productivity.

The efficient level of redistribution arises from the trade-off between financing business start-ups of poor agents with ideas and doing this in a way that those who are going to lose as a result of the redistribution do not lie. It is important to note that this redistribution result is not an artifact of risk-neutrality; it survives even if agents have strictly concave utility functions. However, in that case, society would also have a taste for equality which would force a redistribution from the rich to the poor. Furthermore, since agents would be risk-averse, society would like to smooth their consumption across states and periods. The risk-neutrality assumption allows me to abstract from these additional distributive forces and focus solely on what productive efficiency dictates.

I decentralize the constrained efficient allocation in an incomplete markets setup where people trade risk-free bonds in a way observable to others. Given markets cannot provide redistribution on their own, an incomplete markets equilibrium under laissez-faire cannot attain constrained efficiency. In order to implement constrained efficiency, government should tax bond holdings to transfer resources from agents without ideas to ones with ideas.

The paper also assumes that investment decision of people who start businesses is private information. Whether investment is observable or not is not crucial for the redistribution result; it holds in either case. However, there are two reasons for making such an assump-

tion. First, even though this assumption is quite extreme, so is assuming the reverse, that investment is perfectly observable. Second, observability of investment is irrelevant for the need for redistribution towards poor and productive agents: as long as investment returns is unobservable, it is socially desirable to make the redistribution. However, when investment is observable, incentive compatibility conditions are relaxed since now agents who lie to be poor and productive have the additional burden of investing at the exactly same amount. In this sense, proving that redistribution is efficient is a stronger result when society cannot monitor investment.

Fernandez and Gali (1997) also analyzes an environment in which a society needs to overcome the mismatch among agents who have productive resources and those who can use them efficiently. In that paper, students are heterogenous with respect to their ability and wealth levels and schools are heterogenous in their quality. Authors compare the performances of markets and tournaments as allocative mechanisms. They find that under the presence of borrowing constraints tournaments dominate markets in terms of total output. The current paper is different from Fernandez and Gali (1997) in two respects. Firstly, there is no investment risk in their model: once a student is matched with a school, output is certain. Secondly and more importantly, I characterize constrained efficient allocation whereas they only compare two allocative mechanisms.³

This paper is also related to the literature on credit market failures initiated by the seminal paper of Stiglitz and Weiss (1981). This literature generally assumes that agents have private information about their probability of success when they start a project. They assume the output of a project is observable to others, and, hence agents can write contracts contingent upon output realizations. Thus, they analyze adverse selection credit economies and provide negative results on the existence and efficiency of equilibrium. My paper differs from this literature by assuming that outcomes of entrepreneurial projects are private information and by focusing on the characterization of efficient allocation.

Another strand of literature that is related to my paper is on optimal venture capital

³Heterogeneity in both students and schools makes solving for constrained efficient allocation hard in their environment.

contracts (Admati and Pfleiderer (1994), Schmidt (2003), Jovanovic and Szentes (2007)). In general, this literature considers the problem of calculating the optimal contract in a principal-agent relationship in which a venture capitalist monitors everything but an entrepreneur's effort. The main difference in the current paper is that I assume less transparency between agents by assuming that people cannot monitor each other's investment levels or output realizations. This rules out the existence of venture capital in my model.⁴ Consequently, I deal with the complementary problem of how a society should allocate productive resources in an environment in which agents are more opaque and hence less capable of allocating resources themselves.

The rest of the paper is organized as follows. Section 2 introduces the baseline model formally and analyzes the full information benchmark. Section 3 defines and solves for the constrained efficient allocation. In section 4, I provide an implementation of constrained efficient allocation in an incomplete markets setup. Section 5 studies an extension of the model in which all the agents have access to a risk-free technology and shows that redistribution result is robust. In section 6, I restudy the baseline model allowing for negative consumption and prove that the main result survives this extension as well. Finally, section 7 concludes.

2 Model

2.1 Environment

The economy is populated by a continuum of unit measure of agents who live for two periods. Agents are risk-neutral with the instantaneous utility function $u : \Re \rightarrow \Re$ defined as $u(c) = c$, for $c \geq 0$ and $u(c) = -\infty$, for $c < 0$.⁵ They are expected utility maximizers with

$$E_1\{u(c_1) + \beta u(c_2)\},$$

⁴I do not claim that venture capital does not exist in real life or it is not important. However, given that it requires some resources that are limited in supply (like time of experts) and, hence, serves a relatively small portion of business start-ups, an alternative less transparent relationship is also present.

⁵Allowing for negative consumption but setting its utility to negative infinity is a convenient way of securing non-negativity of consumption.

where c_t is period t consumption and $\beta \in (0, 1)$ is the discount factor.

At the beginning of period one, some agents are born with ideas, $i = 1$, and some without ideas, $i = 0$. Let $I = \{0, 1\}$. The fraction of agents born with (without) an idea is η_1 (η_0). In order to produce an agent should have an idea. Agents are also born with different levels of initial endowment of the only consumption good, $w \in W = \{p, r\}$. Fraction ζ_w are born with initial wealth level w . There is no endowment in period two. So, in our economy, there are four types of agents in period one: $\{(p, 0), (p, 1), (r, 0), (r, 1)\}$.

Agent of type (w, i) operates the following production technology:

$$y = i\theta k^\alpha, \alpha \in (0, 1),$$

where k is the amount invested in period one, θ is the random return on capital and y is the random output produced in period two. θ is drawn from the set $\Theta = \{\theta_l, \theta_h\}$, where $\theta_l < \theta_h$. The probability distribution of returns μ is i.i.d. over agents. Agent gets to learn the realization of return after the investment is made. Hence, agents face idiosyncratic investment risk. i is in the production function to denote that only agents with ideas can start businesses.

The information structure and timing of events are as follows: An agent's type, his realized return and his investment choice are private information. The rest of the data of the economy is public information. In period one, after learning his initial endowment and whether he has an idea or not, an agent chooses his levels of consumption and investment. Then, in period two, θ is realized, production takes place and the agent consumes.⁶

We know that one way to think about resource allocation is to consider a benevolent social planner who is choosing allocations for agents. Since consumption-investment choice is unobservable, the planner cannot choose allocations directly. Instead, each period he makes transfers between agents based on their reports of their private histories. This way the planner manipulates agents' actions. In addition, there is no outside party which means

⁶Whether θ is realized in period one or two is immaterial for the results; the important thing is it is realized after investment decision is made.

the planner cannot save or borrow resources through time.

An allocation in this economy is a vector $(c, k, \tau) \equiv (c_1, c_2, k_1, \tau_1, \tau_2)$, where

$$\begin{aligned} c_1 &: W \times I \rightarrow \mathfrak{R} \\ k_1 &: W \times I \rightarrow \mathfrak{R}_+ \\ c_2 &: W \times I \times \Theta \rightarrow \mathfrak{R} \\ \tau_1 &: W \times I \rightarrow \mathfrak{R} \\ \tau_2 &: W \times I \times \Theta \rightarrow \mathfrak{R}. \end{aligned}$$

In the above, $c_1(w, i)$ and $k_1(w, i)$ refer to period one levels of consumption and investment of the agent who has initial wealth w and idea i . Similarly, $c_2(w, i, \theta)$ is the consumption level of the agent of type (w, i) who has a realized return θ in period two⁷. $\tau_1(w, i)$ and $\tau_2(w, i, \theta)$ are the levels of transfers received by corresponding types.

Feasibility. We say that an allocation (c, k, τ) is *feasible* if

$$\begin{aligned} \sum_{w,i} \zeta_w \eta_i \tau_1(w, i) &\leq 0, \\ \sum_{w,i} \sum_{\theta} \zeta_w \eta_i \mu_{\theta} \tau_2(w, i, \theta) &\leq 0, \end{aligned} \tag{1}$$

and for every $(w, i) \in W \times I$

$$\begin{aligned} c_1(w, i) + k_1(w, i) &\leq w + \tau_1(w, i), \\ c_2(w, i, \theta) &\leq i\theta k_1(w, i)^{\alpha} + \tau_2(w, i, \theta), \end{aligned} \tag{2}$$

$$k_1(w, i) \geq 0. \tag{3}$$

Here, (1) is the *aggregate feasibility* condition which says that planner should balance its budget every period. (2) is *individual feasibility* and stands for the fact that allocation should be affordable by each agent. (3) is just the non-negativity constraint on investment.

⁷Since an agent with no idea cannot produce, her period two consumption is independent of θ . So $c_2(w, 0, \theta_l) = c_2(w, 0, \theta_h)$

Incentive-compatibility. Using the terminology of mechanism design literature, there are two sources of private information in our model. Firstly, there is hidden information: an agent's initial type and period two investment return are unobservable to the planner. Secondly, agents are involved in hidden action: their consumption and investment levels are hidden. Hence, they can deviate from an allocation recommended by the planner in two ways: they can lie about their private information and/or they can choose an investment level that is different from what the planner recommended. I invoke a powerful revelation principle introduced by Myerson (1982) and define an allocation to be incentive compatible as follows.

First we need to define reporting and investment strategies. Let $\tilde{\sigma}(w, i) = (\tilde{\sigma}_1(w, i), \tilde{\sigma}_2(w, i, \theta))$, where $\tilde{\sigma}_1(w, i) \in W \times I$ and $\tilde{\sigma}_2(w, i, \theta) : \Theta \rightarrow \Theta$, be agent (w, i) 's reporting strategy. Also, let us define $\tilde{k}_1(w, i) \in \mathbb{R}_+$ as agent's risky investment strategy. Then, $(\tilde{\sigma}(w, i), \tilde{k}_1(w, i))$ is a complete strategy of agent (w, i) .⁸ Then, let $(\tilde{\sigma}, \tilde{k}_1) \in \Gamma$ be a strategy profile of agents where Γ is the set of all strategy profiles.

Given τ , for any (w, i) , the utility of following a strategy $(\tilde{\sigma}, \tilde{k}_1)$ is:

$$V_{w,i}(\tilde{\sigma}, \tilde{k}_1; \tau) \equiv u(w + \tau_1(\tilde{\sigma}_1(w, i)) - \tilde{k}_1(w, i)) + \beta \sum_{\theta} \mu_{\theta} u(i\theta \tilde{k}_1(\omega_i)^{\alpha} + \tau_2(\tilde{\sigma}_2(w, i, \theta)))$$

We say that an allocation (c, k, τ) is *incentive-compatible* if

$$V_{w,i}(\sigma, k_1; \tau) \geq V_{w,i}(\tilde{\sigma}, \tilde{k}_1; \tau), \text{ for all } (w, i) \in W \times I, \text{ for all } (\tilde{\sigma}, \tilde{k}_1) \in \Gamma, \quad (4)$$

where σ denotes truth-telling reporting strategy profile. An allocation that is feasible and incentive-compatible is called *incentive-feasible*.

⁸Myerson (1982) calls this *participation strategy*. Also, note that we do not need to add consumption as a part of the strategy since it is implied by the choice of σ and $\tilde{k}_1$.

2.2 First-Best (FB) benchmark

Before analyzing constrained efficient allocation, let us see what the society can achieve under perfect information. Under the Utilitarian objective, the first-best allocation is the solution to the following problem:

FB Problem.

$$\max_{c,k} \sum_{w,i} \zeta_w \eta_i \left\{ u(c_1(w,i)) + \beta \sum_{\theta} \mu_{\theta} u(c_2(w,i,\theta)) \right\}$$

s.t.

$$\begin{aligned} \sum_{w,i} \zeta_w \eta_i [c_1(w,i) + k_1(w,i)] &\leq \sum_w \zeta_w w, \\ \sum_{w,i} \zeta_w \eta_i \left\{ \sum_{\theta} \mu_{\theta} c_2(w,i,\theta) \right\} &\leq \sum_{w,i} \zeta_w \eta_i \sum_{\theta} \mu_{\theta} i \theta k_1(w,i)^{\alpha}, \\ k_1(w,i) &\geq 0, \text{ for all } (w,i) \in W \times I. \end{aligned}$$

Assuming that total initial wealth in period one is large enough, first-order optimality condition for investment of agents with an idea reads:

$$1/\beta = \alpha k_1(w,1)^{\alpha-1} \sum_{\theta} \mu_{\theta} \theta.$$

Left hand side of the equality is the marginal social cost of investing an additional unit in terms of period two utility. Right hand side is the marginal social benefit of investment in same units. This condition defines

$$k^{fb} = \left\{ \beta \alpha \sum_i \mu_i \theta_i \right\}^{\frac{1}{1-\alpha}}$$

as the FB level of investment provided that the following holds:

Assumption 1. *Total resources in the economy in period one is sufficient to finance k^{fb}*

investment for each $(w, 1)$ agent, or

$$\eta_1 k^{fb} \leq \sum_w \zeta_w w.$$

Assumption 1 formally states that cumulative initial wealth is sufficiently large.⁹

Lemma 1. *Suppose Assumption 1 holds. Then,*

1. *FB level of investment for agents with ideas is equal to k^{fb} , irrespective of their wealth. FB level of investment for agents without ideas is zero.*
2. *As long as it provides non-negative consumption to all agents and uses all output, distribution of individual consumption does not matter.*

Looking at the objective function in the FB problem, one can see that Utilitarian welfare with equal weights and risk-neutrality together imply that society has no preference for how total consumption should be distributed, as long as no one gets negative consumption. The society is only concerned about right agents making right amounts of investment. Therefore, the above problem actually defines productive efficiency: how to maximize production subject to feasibility. In the next section, we analyze the same problem but this time under private information. As a result, we will be maximizing production subject to incentive-feasibility.

3 Constrained-efficient Allocation

In analyzing the benchmark case the only assumption made was about total initial wealth. However, with private information, as we will see, the comparison of p, r and k^{fb} becomes important. First assumption about this comparison is the following:

⁹If Assumption 1 does not hold, then FB level of investment will be a corner solution: $\frac{\sum_w \zeta_w w}{\eta_1}$, and all the results of the paper go unchanged.

Assumption 2. $p < k^{fb} < r$.

The crucial part of this assumption is $p < k^{fb}$, which says that initial wealth of the poor is not large enough to cover FB level of investment. Thus, a poor agent with an idea cannot operate her idea at the most efficient level on her own. If, to the contrary, $p \geq k^{fb}$ is the case, the economy reaches FB without agents interacting at all. Obviously, this case is neither interesting nor realistic. The second part of the assumption, that $k^{fb} < r$, is not necessary for the results. In fact, the redistribution result would still hold even if we did not have the rich types in the model at all. However, including rich types such that $k^{fb} < r$ allows us to see the perverse effects of wealth inequality. Getting rid of the wealth inequality completely would have huge positive welfare effects on its own. However, since the society has no way of screening agents' wealth levels, this turns out not to be possible.

The remainder of this section first defines and then characterizes constrained efficient allocation.

Definition 1. *An allocation (c^*, k^*, τ^*) is called constrained-efficient if it solves the following social planner's problem:*

$$\max_{c, k, \tau} \sum_{w, i} \zeta_w \eta_i \left\{ u(c_1(w, i)) + \beta \sum_{\theta} \mu_{\theta} u(c_2(w, i, \theta)) \right\}$$

subject to (1), (2), (3) and (4).

As in the benchmark case, objective function clearly shows that society does not care about how consumption is going to be distributed among individuals. Consequently, the above problem is one of constrained productive efficiency: maximizing total output subject to incentive-feasibility. This implies there can be many constrained efficient allocations all of which have the same investment allocation and hence same total production and welfare, but different consumption allocations. Nonetheless, it should also be noted that incentive compatibility arising from private information does put some discipline on the distribution of consumption across agents compared to the first best benchmark.

3.1 Characterizing constrained (productive) efficient allocation

First make the following observation which simplifies the analysis. If transfer levels depend on period two announcements of agents, then any agent will report the type that brings the highest level of transfers. Therefore, any transfer mechanism in which a transfer level depends on period two shock cannot be incentive-compatible. Consequently, without loss of generality, we will be restricting attention to contracts in which transfers are functions of period one announcements only.

Now I make the second assumption comparing p and k^{fb} .

Assumption 3. $\frac{k^{fb}-p}{\beta} > \theta_l k^{fb\alpha}$.

To see why I make this assumption, suppose it does not hold. Observe that in order to invest at the FB level the poor agent with an idea needs at least $k^{fb} - p$ additional resources in period one. Also observe that the most this agent can pay back in period two in low state is $\theta_l k^{fb\alpha}$. Assumption 3 does not hold, then, means even in the worst contingency in period two, poor and productive agent would be able to pay back the amount he borrowed in period one to finance FB level of investment. Obviously, in this case, the fact that entrepreneurial returns are private information would not have a bite. Therefore, no redistribution is necessary for constrained efficiency. The society can implement first best outcome by just making sure that contracts are perfectly enforced (by saying anyone who does not obey a previously signed contract is punished very harshly). Agents, then, sign state non-contingent contracts that set interest rate of $1/\beta$ and optimal investment would be attained. However, that even in the worst case an entrepreneur can pay back his debt is highly unrealistic, especially for businesses that are newly forming.¹⁰

The proposition below formally shows that when Assumption 3 does not hold FB allocation is trivially reached without any redistribution between agents.

¹⁰The existence of a huge literature on financial contracting which studies optimal contracts when entrepreneurial revenues are private information is another proof that Assumption 3 is appropriate.

Proposition 1. *Suppose that $\frac{k^{fb}-p}{\beta} \leq \theta_l k^{fb\alpha}$. Then, constrained efficiency is attained without any redistribution between agents. Furthermore, FB welfare is attained in the constrained efficient allocation, i.e. for all $w \in W$, $k_1^*(w, 1) = k^{fb}$.*

Proof. We need to show that we can find an allocation that is incentive-feasible and attains FB investment levels. Set $\tau_1^*(p, 1) = k^{fb} - p$ and $\tau_2^*(p, 1) = -\frac{k^{fb}-p}{\beta}$. Then, in period one, poor agent with idea will face the budget $c_1 + k_1 \leq k^{fb}$ and choose $k_1(p, 1) = k^{fb}$. In period two in the low return state, her consumption will be $c_2 l = \theta_l k^{fb\alpha} - \frac{k^{fb}-p}{\beta} \geq 0$ by assumption. This clearly implies that $c_2(p, 1, \theta_h) \geq 0$, too.

Then choose $\tau_1^*(w, i)$ for $(w, i) \neq (p, 1)$ such that:

$$\sum_{(w,i) \neq (p,1)} \zeta_w \eta_i \tau_1^*(w, i) = -(k^{fb} - p) \zeta_p \eta_1, \quad (5)$$

$$\tau_1^*(r, 1) \geq k^{fb} - r, 0 \leq \tau_1^*(w, 0) \leq w, \quad (6)$$

and

$$\tau_2^*(w, i) = -\frac{\tau_1^*(w, i)}{\beta}. \quad (7)$$

By assumption 1, such a τ^* exists. Observe that NPV of transfers of any agent is equal to zero. Also observe that conditions (5) and (7) guarantee that transfers sum up to zero in period one and two, respectively. Thus, aggregate feasibility is satisfied.

We also need to show that non-negative consumption is feasible for all agents in any period and any state. For $(p, 1)$, we already have done so. Condition (6) guarantees that $(r, 1)$ agent can choose investment equal to k^{fb} and still consume a non-negative amount in period one. Similarly, (6) guarantees that period one consumption of agents who do not have an idea is also non-negative. Finally, period two consumption of agents $(w, i) \neq (p, 1)$ is non-negative again by (6).

The only thing left is to check that given τ^* agents will tell the truth about their types, but this is straightforward given that NPV of transfers of any type is equal to zero. $\square$

From now on I analyze the more interesting case in which Assumption 3 holds: agents

with ideas have sufficiently uncertain returns. Proposition 2 below shows that constrained efficiency requires transferring a positive NPV of resources from unproductive agents to productive ones in this case. In what follows I am also going to make Assumption 4 below. This assumption is not substantial in the sense that it is not necessary for the redistribution result. However, to attain FB allocation we need NPV of redistribution to be at least $\delta(k^{fb}) = \frac{k^{fb}-p}{\beta} - \theta_l k^{fb\alpha} > 0$, and this requires Assumption 4.¹¹ The reason why I want to have FB allocation attained in the constrained efficiency problem in the baseline model is that it will be easier to compare the results of the baseline model with the results of the extended models I introduce later on.

Assumption 4.

- a. $\eta_0 \sum_w \zeta_w w > \eta_1 [(k^{fb} - p)\zeta_p + \delta(k^{fb})\zeta_r]$.
- b. $\frac{\eta_1 \delta(k^{fb})}{\eta_0} \leq p$.

Proposition 2. *Suppose Assumptions 3 and 4 hold. Then constrained efficiency requires the society to redistribute $\delta(k^{fb}) = k^{fb} - p - \beta\theta_l k^{fb\alpha}$ from agents without ideas to agents with ideas. Moreover, FB welfare is attained in the constrained efficient allocation, i.e. for all $w \in W$, $k_1^*(w, 1) = k^{fb}$.*

Proof. Set $\tau_1^*(p, 1) = k^{fb} - p$ and $\tau_2^*(p, 1) = -\theta_l k^{fb\alpha}$. Then, in period one, poor agent with idea will face the budget $c_1 + k_1 \leq k^{fb}$ and choose $k_1(p, 1) = k^{fb}$. In period two in the low return state, her consumption will be $c_2(p, 1, \theta_l) = \theta_l k^{fb\alpha} - \frac{k^{fb}-p}{\beta} = 0$ by assumption. This clearly implies that $c_2(p, 1, \theta_h) \geq 0$, too. Observe that $\tau_1^*(p, 1) + \beta\tau_2^*(p, 1) = \delta(k^{fb}) > 0$.

Set $\tau_2^*(r, 1) = \delta(k^{fb}) + \epsilon$ and $\tau_2^*(r, 1) = -\frac{\epsilon}{\beta}$, where $\epsilon > 0$ can be arbitrarily close to zero. Then, in period one, rich agent with idea will face the budget $c_1 + k_1 \leq r + \delta(k^{fb}) + \epsilon$ and choose $k_1(r, 1) = k^{fb}$. In period two, her consumption will be $c_{2\theta} = \theta k^{fb\alpha} - \frac{\epsilon}{\beta} \geq 0$ since ϵ is arbitrarily small. Note that $\tau_1^*(r, 1) + \beta\tau_2^*(r, 1) = \delta(k^{fb}) > 0$, too.

¹¹Without Assumption 4, the amount of redistribution will be strictly less than $\delta(k^{fb})$, but will still be strictly positive. For example, if Assumption 4.b does not hold, redistribution will be equal to $\frac{\eta_0}{\eta_1} p$.

Then choose $\tau_1^*(w, 0)$ such that:

$$\eta_0(\zeta_r \tau_1^*(r, 0) + \zeta_p \tau_1^*(p, 0)) = -\eta_1[(k^{fb} - p)\zeta_p + \delta(k^{fb})\zeta_r], \quad (8)$$

$$0 \leq \tau_1^*(w, 0) \leq w, \quad (9)$$

and

$$\tau_2^*(w, 0) = -\frac{\frac{\eta_1}{\eta_0}\delta(k^{fb}) + \tau_1^*(w, 0)}{\beta}. \quad (10)$$

By Assumption 4 part a, conditions (8) and (9) are compatible. Also, note that $\tau_1^*(w, 0) + \beta\tau_2^*(w, 0) = -\frac{\eta_1\delta(k^{fb})}{\eta_0}$, for all $w \in W$. This is because giving $(p, 0)$ greater NPV of transfers is not incentive compatible.

By Assumption 4 part b, condition (9) guarantees that $(w, 0)$ agents can afford non-negative consumption in period one whereas condition (10) implies that period two consumption is non-negative.

Thus, the allocation (c^*, k^*, τ^*) that results is feasible, has non-negative consumption for all agents at all periods and states, and allows agents with ideas to invest FB level.

The only thing left is to check that given τ^* agents will tell the truth about their types. An agent with an idea weakly prefers her transfers to anyone else since her transfer has NPV of $\delta(k^{fb})$ which is weakly higher than NPV of anyone else's transfer. Also, her transfer allows her to invest k^{fb} without consuming a negative amount. Agents of $(w, 0)$ do not lie to be $(w, 1)$ since this brings strictly negative consumption in period two. They do not lie to be each other since their transfers have the same NPV. $\square$

The intuition for this result is as follows. Given that the planner has to provide poor and productive agents with NPV of transfers equal to $\delta(k^{fb})$, incentive-compatibility implies rich and productive agents should receive same NPV of transfers. Aggregate feasibility then implies unproductive agents should get transfers with negative NPV. This transfer system is incentive-compatible only if period two transfers of productive agents are both negative. Only in that case unproductive agents cannot pretend to be productive since that would imply negative transfers and, hence, negative consumption in period two. Such a transfer

system implies period one transfers of rich and productive agent should be strictly greater than $\delta(k^{fb})$.

One thing that Proposition 2 claims is that the society attains FB allocation, even under the informational constraints. This result is an artifact of risk-neutrality and hence will vanish if more general utility functions are assumed. Also, as we will see in the extensions sections 5 and 6, this result crucially depends on unproductive agents' inability to store resources from period one to two or the fact that we do not allow for negative consumption. However, the main result of proposition 2, that productive efficiency requires redistribution from unproductive agents to productive ones, is robust to any of these perturbations of the model.

4 Implementation

In this section, we are going to first look at what the market mechanism can do cope with the mismatch in the society in terms of who holds productive resources and who can use them efficiently. It is shown that in the case in which constrained efficiency requires redistribution, the market system alone cannot attain constrained efficiency. Then, we provide a simple tax system that implements constrained allocation by carrying out the required redistribution.

The physical and informational environment is the same as described in section 2. The main difference is that there is an incomplete markets structure that allows agents to competitively trade risk-free bonds in period one. Bonds pay back a gross return R in period two that is determined in equilibrium. Individual trades in the bonds market is public information and there is full enforcement, meaning no one can die without paying back their debt.

There is a government who can tax individuals based on their observable characteristics. Since bond holdings is the only observable, a tax system is defined as a function of bond holdings. Formally, a tax system is a functions $T : \mathfrak{R} \rightarrow \mathfrak{R}$.

Taking the tax system T and the interest rate R as given, an agent of type (w, i) solves the following problem:

Agent's problem.

$$\max_{c,k,b} u(c_1) + \beta \sum_{\theta} \mu_{\theta} u(c_{2\theta}) \quad (11)$$

s.t.

$$c_1 + k_1 + b_1 \leq w - T(b_1),$$

$$c_{2\theta} \leq i\theta k_1^{\alpha} + Rb_1,$$

$$k_1 \geq 0.$$

Now we are ready to define incomplete markets equilibrium with taxes.

Definition 2. *Given T , an IM equilibrium is individual choices*

$(c_1(w, i), c_2(w, i, \theta), k_1(w, i), b_1(w, i))_{w \in W, i \in I, \theta \in \Theta}$ and interest rate R s.t.

1. *Given R , for each agent, $(c_1(w, i), c_2(w, i, \theta), k_1(w, i), b_1(w, i))$ solves (11),*
2. *Bond market clears: $\sum_w \sum_i \zeta_w \eta_i b_1(w, i) = 0$,*
3. *Gov't budget balances: $\sum_w \sum_i \zeta_w \eta_i T(b_1(w, i)) = 0$.*

We say that an allocation (c, k, τ) is *implementable by taxes on bond holdings $T(b)$* if, given $T(b)$, (c, k, b) with some interest rate R constitute an IM equilibrium.

4.1 Incomplete markets under laissez-faire

Before talking about implementation, let us first analyze what happens under no government intervention, i.e. $T(b_1) = 0$, for all b_1 .

Consider first the case in which no redistribution is needed to attain constrained efficiency. Proposition 3 below shows that market without any government intervention can support constrained efficient allocation in this case.

Proposition 3. *Suppose that $\frac{k^{fb}-p}{\beta} \leq \theta_l k^{fb\alpha}$. Then, laissez-faire IM equilibrium attains constrained efficient welfare level.*

Proof. It is easy to show that in any IM equilibrium $R = 1/\beta$. Looking at agents problem (11), bond demand of $(w, 0)$ agents will be bounded only by agents preference for non-negative consumption:

$$0 \leq b_1(w, 0) \leq w. \quad (12)$$

For agents with ideas, problem (11) gives that agent would like to set $k_1 = k^{fb}$. This means $b_1(w, 1) \leq w - k^{fb}$. Output in low state in period two implies that $b_1(w, 1) \geq -\beta\theta_l k^{fb\alpha}$.

One can show that $b_1(w, i) = -\tau_1^*(w, i)$ as defined in proposition 1 actually satisfies the above equilibrium restrictions on bond demand by each agent. Thus, these bond holdings satisfy individual optimality. Then, market clearing follows from the fact that τ^* is feasible. $\square$

In the more interesting case in which efficiency requires redistribution market mechanism cannot attain constrained efficiency under laissez-faire.

Proposition 4. *Suppose Assumptions 3 and 4 hold. Then constrained efficient welfare level cannot be attained in the equilibrium of IM under laissez-faire.*

Proof. Suppose for contradiction that it can be achieved. Then, $b_1(p, 1) = p - k^{fb}$. Since $R = 1/\beta$, $c_2(p, 1, \theta_l) = \theta_l k^{fb\alpha} + \frac{p-k^{fb}}{\beta} < 0$ by assumption. But this cannot be an optimal choice for the agent since agent could do better just by setting $b_1(p, 1) = 0$. Thus, we have a contradiction. $\square$

4.2 Optimal Taxes

Previous subsection showed markets, on their own, do not provide redistribution, which is required for constrained efficiency. This points to a rationale for government intervention which can ensure the needed redistribution through taxes on bond holdings.

In that regard, define

$$T(b_1) = \begin{cases} -\delta(k^{fb}), & \text{if } b_1 < 0; \\ \frac{\eta_1}{\eta_0}\delta(k^{fb}), & \text{if } b_1 \geq 0, \end{cases} \quad (13)$$

Proposition 5. *Suppose Assumptions 3 and 4 hold. Then, incomplete markets equilibrium with tax system defined in (13) implements constrained efficient allocation.*

Proof. I am going to construct an IM equilibrium where $R = 1/\beta$ and agents with ideas in period one invest at the socially optimal amount in their ideas.

Under the specified taxes, an agent faces the following problem:

Agent's problem with taxes.

$$\max_{c,k,b} u(c_1) + \beta \sum_{\theta} \mu_{\theta} u(c_{2\theta})$$

s.t.

$$c_1 + k_1 + b_1 \leq \begin{cases} w + \delta(k^{fb}), & \text{if } b_1 < 0; \\ w - \frac{\eta_1}{\eta_0}\delta(k^{fb}), & \text{if } b_1 \geq 0. \end{cases}$$

$$c_{2\theta} \leq i\theta k_1^{\alpha} + Rb_1,$$

$$k_1 \geq 0.$$

First, consider agents who have an idea in period one. When a poor agent with an idea chooses $b_1 < 0$, then she has to set $b_1(p, 1) = p - k^{fb} + \delta(k^{fb})$ in order to invest k^{fb} and have non-negative consumption. That implies in period two in low state $c_2(p, 1, \theta_l) = \theta_l k^{fb\alpha} + \frac{p - k^{fb} + \delta(k^{fb})}{\beta} = \theta_l k^{fb\alpha} + \frac{p - k^{fb} + k^{fb} - p - \beta\theta_l k^{fb\alpha}}{\beta} = 0$. This implies $c_2(p, 1, \theta_h) > 0$. Hence, this tax system enables the poor agent with idea to afford investing at socially efficient level without consuming negative amount. Thus, she chooses to do it.

It is clear that the rich agent with idea can afford k^{fb} . Therefore, the utility of $(w, 1)$

type when she chooses $b_1 < 0$ is:

$$\begin{aligned} w + \delta(k^{fb}) - k^{fb} - b_1(w, 1) + \beta \sum_{\theta} \mu_{\theta} [\theta k^{fb\alpha} + b_1(w, 1)/\beta] \\ = w + \delta(k^{fb}) - k^{fb} + \beta \sum_{\theta} \mu_{\theta} \theta k^{fb\alpha} \end{aligned}$$

On the other hand, if an agent with an idea chooses $b_1 \geq 0$, then, letting her optimal choices be $\tilde{k}_1(w, 1), \tilde{b}_1(w, 1)$, her utility would be:

$$\begin{aligned} w - \frac{\eta_1}{\eta_0} \delta(k^{fb}) - \tilde{k}_1(w, 1) - \tilde{b}_1(w, 1) + \beta \sum_{\theta} \mu_{\theta} [\theta \tilde{k}_1(w, 1)^{\alpha} + \tilde{b}_1(w, 1)/\beta] \\ = w - \frac{\eta_1}{\eta_0} \delta(k^{fb}) - \tilde{k}_1(w, 1) + \beta \sum_{\theta} \mu_{\theta} \theta \tilde{k}_1(w, 1)^{\alpha} \end{aligned}$$

Given that k^{fb} maximizes the function $-k + \beta \sum_{\theta} \mu_{\theta} \theta k^{\alpha}$, clearly utility under $b_1 < 0$ is higher than that under $b_1 \geq 0$. Hence, we showed agents with an idea choose to borrow. Specifically, $b_1(p, 1) = p - k^{fb} + \delta(k^{fb})$ and demand for bond holdings of rich and productive agents is $-\beta \theta_l k^{fb\alpha} \leq b_1(r, 1) < 0$.

Now consider agents who do not have an idea in period one. If they choose $b_1 < 0$, then in period two they have to consume strictly negative amount; thus they have to choose $b_1 \geq 0$. So, their demand for bond holdings is indeterminate with $0 \leq b_1(w, 0) \leq w - \frac{\eta_1}{\eta_0} \delta(k^{fb})$.

Market clearing condition implies, we should have

$$\begin{aligned} \eta_1 [(k^{fb} - p - \delta(k^{fb}))\zeta_p + \epsilon\zeta_r] &= \eta_0 \sum_w \zeta_w b_1(w, 1) \\ \Rightarrow \eta_1 (k^{fb} - p - \delta(k^{fb}))\zeta_p &< \eta_0 \sum_w \zeta_w [w - \frac{\eta_1}{\eta_0}] \\ \Rightarrow \eta_1 [(k^{fb} - p)\zeta_p + \delta(k^{fb})\zeta_r] &< \eta_0 \sum_w \zeta_w w, \end{aligned}$$

which is satisfied by Assumption 3.

Finally, we need to show that government budget holds:

$$\sum_{w,i} \zeta_w \eta_i T(b_1(w, i)) = \eta_1 \delta(k^{fb}) + \eta_0 \left(-\frac{\eta_1}{\eta_0} \delta(k^{fb})\right) = 0.$$

□

Government subsidizes those who borrow in period one. Unproductive agents would like to borrow in period one just to get the subsidy but then they have to pay back their debt in period two since default is not allowed. However, that would imply negative consumption for unproductive individuals. As a result, only productive agents borrow and get the subsidy $\delta(k^{fb})$. This, in turn, allows poor productive agents to invest at the socially efficient level.

5 Unobservable risk-free technology

This section considers a physical environment that is the same as in the baseline model with one modification: any agent can now operate a risk-free technology that pays x units in period two for each unit invested in period one. An agent's level of investment in the risk-free technology is publicly unobservable.

The purpose of this extension is to show that our main result that under private information productive efficiency requires redistribution from the agents without ideas to the ones with ideas does not crucially depend on our assumption that the former is completely unproductive. Introduction of risk-free saving for everyone may prevent the society from accomplishing the amount of redistribution required to attain FB welfare; however, as long as the risk-free rate is strictly below β^{-1} , there is still redistribution in the constrained efficient allocation.

Assumption 5. $x < 1/\beta$.

This assumption says that it is costly to use the hidden risk-free technology. It is obvious that, under Assumption 5, using risk-free technology is suboptimal from society's point of

view. Therefore, FB solution in this new environment is the same as the FB solution in the baseline model.

5.1 Constrained efficiency

We now want to analyze informationally constrained efficient allocation, which is denoted by (c^*, k^*, τ^*) , as in the baseline model. This allocation solves the following social planner's problem:

$$\max_{c, k, s, \tau} \sum_{w, i} \zeta_w \eta_i \left\{ u(c_1(w, i)) + \beta \sum_{\theta} \mu_{\theta} u(c_2(w, i, \theta)) \right\}$$

subject to

$$\begin{aligned} \sum_{w, i} \zeta_w \eta_i \tau_1(w, i) &\leq 0, \\ \sum_{w, i} \sum_{\theta} \zeta_w \eta_i \mu_{\theta} \tau_2(w, i, \theta) &\leq 0, \\ c_1(w, i) + k_1(w, i) + s_1(w, i) &\leq w + \tau_1(w, i), \\ c_2(w, i, \theta) &\leq i\theta k_1(w, i)^{\alpha} + x s_1(w, i) + \tau_2(w, i, \theta), \\ k_1(w, i), s_1(w, i) &\geq 0 \text{ for all } (w, i) \in W \times I, \end{aligned}$$

and subject to

$$V_{w, i}(\sigma^*, k_1, s_1; \tau) \geq V_{w, i}(\sigma, \tilde{k}_1, \tilde{s}_1; \tau) \text{ for all } (w, i) \in W \times I, (\sigma, \tilde{k}_1, \tilde{s}_1) \in \Gamma.$$

Here $s_1(w, i)$ is the risk-free investment of type (w, i) and function V and the set Γ are redefined to incorporate risk-free saving in the obvious way.

The following proposition shows that if the risk-free rate of return is sufficiently low, then the redistribution needed to achieve FB, $\delta(k^{fb})$, is still incentive-compatible. Before introducing the proposition, I make one more assumption. Assumption 4.b below strengthens Assumption 4.a introduced in section 3 and is only for simplicity. It is straightforward to show all the results below would go through without this assumption.

Assumption 4.b. $k^{fb} - p < \frac{\sum_w \zeta_w w - \zeta_p \eta_1 (k^{fb} - p)}{\zeta_r \eta_1}$.

Proposition 6. *FB welfare is attained in the constrained efficient allocation if and only if*

$$x \leq \frac{\theta_l k^{fb\alpha}}{k^{fb} - p + \frac{\eta_1}{\eta_0} \delta(k^{fb})}. \quad (14)$$

More importantly, in order to attain FB, the society redistributes $\delta(k^{fb})$ units from unproductive to productive agents.

Before we begin the proof, let us denote NPV of transfers to agent type (w, i) under a transfer system τ by $\Delta_{w,i}$.

Proof. Suppose FB is attained. Then, $\tau_1^*(p, 1) = k^{fb} - p$ and $\tau_2^*(p, 1) = -\theta_l k^{fb\alpha}$. By incentive-compatibility, $\Delta_{w,1}^* = k^{fb} - p - \beta \theta_l k^{fb\alpha} = \delta(k^{fb})$, independent of w .

When agent $(w, 0)$ announces to be (w', i) , she solves:

$$\max_{s_1} u(c_1) + \beta u(c_2) \quad (15)$$

s.t.

$$c_1 + s_1 \leq w + \tau_1^*(w', i) \quad (16)$$

$$c_2 \leq x s_1 + \tau_2^*(w', i). \quad (17)$$

Thus, for $(w, 0)$ not to lie to be productive, we need:

$$u(w + \tau_1^*(w, 0) - s_1) + \beta u(x s_1 + \tau_2^*(w, 0)) \geq u(w + \tau_1^*(w', 1) - s'_1) + \beta u(x s'_1 + \tau_2^*(w', 1)), \quad (18)$$

where s_1, s'_1 are optimal savings levels for the agent under truth-telling and lying, respectively.

This implies we need, for all $w' \in W$,

$$w + \Delta_{w,0}^* \geq \begin{cases} w + \delta(k^{fb}) - \frac{\tau_1^*(w',1) - \delta(k^{fb})}{\beta x} + \beta x \frac{\tau_1^*(w',1) - \delta(k^{fb})}{\beta x}, & \text{if } x(\tau_1^*(w',1) + w) \geq -\tau_2^*(w',1); \\ -\infty, & \text{if else.} \end{cases} \quad (19)$$

Here, LHS is utility of truth-telling whereas RHS is utility of lying to be $(w', 1)$. LHS already takes into account the fact that transfers are such that when agents tell the truth, they do not have to use risk-free technology. Hence, they do not use it. RHS already has the fact that $(w, 0)$ has to set $s'_1 \geq \frac{-\tau_2^*(w',1)}{x} = \frac{\tau_1^*(w',1) - \delta(k^{fb})}{\beta x}$ to have non-negative consumption in period two.

Assume for contradiction that $x > \frac{\theta_l k^{fb\alpha}}{k^{fb} - p + \frac{\eta_1}{\eta_0} \delta(k^{fb})}$. Then, it follows that $x > \frac{\theta_l k^{fb\alpha}}{k^{fb}}$. Hence, $x(\tau_1^*(w',1) + w) > \frac{\tau_1^*(w',1) - \delta(k^{fb})}{\beta} = -\tau_2^*(w',1)$, for some $w' \in W$. Thus, both unproductive agents can lie without consuming negative amount which means first line of (19) must hold.

That implies

$$x \leq \frac{\theta_l k^{fb\alpha}}{k^{fb} - p - \Delta_{w,0}^*}, \quad (20)$$

for all $w \in W$.

Then, $x > \frac{\theta_l k^{fb\alpha}}{k^{fb} - p + \frac{\eta_1}{\eta_0} \delta(k^{fb})}$, together with (20) imply that $\frac{\eta_1}{\eta_0} \delta(k^{fb}) > -\Delta_{w,0}^*$, for all $w \in W$, which contradicts with $\frac{\eta_1}{\eta_0} \delta(k^{fb}) = \sum_w \zeta_w \Delta_{w,0}^*$.

Second, assume that (14) holds. We want to show that constrained efficiency attains FB. Set transfers such that $\Delta_{w,0}^* = -\frac{\eta_1}{\eta_0} \delta(k^{fb})$. This guarantees that neither of unproductive agents lie to be productive. Since NPV of their transfers are the same, they do not lie to be each other either.

□

The intuition for this result is very simple. Under the transfer mechanism that attains FB, unproductive agents get transfers with NPV equal to $-\frac{\eta_1}{\eta_0} \delta(k^{fb})$, whereas productive agents get NPV of transfers equaling to $\delta(k^{fb})$. Therefore, from unproductive types' perspective, the benefit of lying is equal to $(1 + \frac{\eta_1}{\eta_0}) \delta(k^{fb})$. On the other hand, the disadvantage of doing so is that in period two one has to get negative transfer. This is a disadvantage for an

unproductive agent because this means she has to save through risk-free production, which has an inferior return. The proposition says that there is a threshold level of x such that below this level the disadvantage of lying outweighs the advantage and unproductive agents tell the truth.

Now we want to analyze constrained efficient allocation in case FB cannot be attained.

Proposition 7. *Suppose $x > \frac{\theta_l k^{fb\alpha}}{k^{fb} - p + \frac{\eta_1}{\eta_0} \delta(k^{fb})}$. Then in the constrained efficient allocation $k_1^*(r, 1) = k^{fb}$ whereas $k_1^*(p, 1) = \bar{k}$ is given by:*

$$x = \frac{\theta_l \bar{k}^\alpha}{\bar{k} - p + \frac{\eta_1}{\eta_0} \delta(\bar{k})}, \quad (21)$$

where $\delta(k) = k - p - \beta \theta_l k^\alpha$. More importantly, society redistributes $\delta(\bar{k})$ units of resources from the agents without ideas to the ones with ideas.

Proof. It is easy to show that in the efficient allocation $c_1^*(p, 1) = 0$ and hence $k_1^*(p, 1) = \tau_1^*(p, 1) + p$. By a similar logic, one can show $\tau_2^*(p, 1) = -\theta_l k_1^*(p, 1)^\alpha$.

It is also easy to see that $\Delta_{p,1}^* = \Delta_{r,1}^*$. If $\Delta_{p,1}^* > \Delta_{r,1}^*$, then $(r, 1)$ lies and gets the NPV with higher transfers; therefore this cannot be true. On the other hand, if $\Delta_{p,1}^* < \Delta_{r,1}^*$, then one can propose a new transfer system $\tilde{\tau}$ same as τ^* , except for $\tilde{\tau}_1(p, 1) = \tau_1^*(p, 1) + \epsilon$ and $\tilde{\tau}_1(r, 1) = \tau_1^*(r, 1) - \frac{\zeta_p}{\zeta_r} \epsilon$. Clearly, this transfer mechanism is a part of a feasible allocation. This new allocation is also incentive compatible: $(r, 1)$ does not lie to be $(p, 1)$ since $\epsilon > 0$ is small and unproductive agents do not lie to be $(p, 1)$ since with original transfers they were not lying to be $(r, 1)$ and the NPV of transfers of $(p, 1)$ is still lower than that of $(r, 1)$ under original transfer mechanism. But the allocation that is attained by this transfer mechanism has strictly greater aggregate utility. The reason is $(p, 1)$ agent's utility increases strictly more than ϵ with the new allocation since she was investment constrained under τ^* .

Then, for unproductive agents not to lie to be productive, we need

$$u(w + \tau_1^*(w, 0) - s_1) + \beta u(x s_1 + \tau_2^*(w, 0)) \geq u(w + \tau_1^*(w', 1) - s'_1) + \beta u(x s'_1 + \tau_2^*(w', 1)). \quad (22)$$

Let $k_1^*(p, 1) = k$. Then, (22) implies we need, for all $w' \in W$,

$$w + \Delta_{w,0}^* \geq \begin{cases} w + \delta(k) - \frac{\tau_1^*(w',1) - \delta(k)}{\beta x} + \beta x \frac{\tau_1^*(w',1) - \delta(k)}{\beta x}, & \text{if } x(\tau_1^*(w', 1) + w) \geq -\tau_2^*(w', 1); \\ -\infty, & \text{if else.} \end{cases} \quad (23)$$

Again LHS is utility of truth-telling whereas RHS is utility of lying to be $(w', 1)$. LHS already takes into account the fact that transfers are such that when agents tell the truth, they do not have to use risk-free technology. Hence, they do not use it. RHS already has the fact that $(w, 0)$ has to set $s'_1 \geq \frac{-\tau_2^*(w', 1)}{x} = \frac{\tau_1^*(w', 1) - \delta(k)}{\beta x}$ to have non-negative consumption in period two.

Rearranging (23) implies we need, for any $w \in W$:

$$x < \frac{\theta_l k^\alpha}{k - p + w}, \text{ or } x \leq \frac{\theta_l k^\alpha}{k - p - \Delta_{w,0}^*}. \quad (24)$$

Also feasibility implies:

$$\sum_w \zeta_w \Delta_{w,0}^* = -\frac{\eta_1}{\eta_0} \delta(k). \quad (25)$$

Suppose for contradiction that k is such that $x > \frac{\theta_l k^\alpha}{k - p + \frac{\eta_1}{\eta_0} \delta(k)}$. Due to feasibility, for some $w \in W$, $\Delta_{w,0}^* \leq -\frac{\eta_1}{\eta_0} \delta(k)$, which implies $x > \frac{\theta_l k^\alpha}{k - p - \Delta_{w,0}^*}$. Also, since $\frac{\eta_1}{\eta_0} \delta(k) < \frac{\eta_1}{\eta_0} \delta(k^{fb}) \leq p < r$, $x > \frac{1}{\beta} \frac{A(k) - \delta(k)}{A(k) + \frac{\eta_1}{\eta_0} \delta(k)} > \frac{1}{\beta} \frac{A(k) - \delta(k)}{A(k) + w}$, which implies (24) does not hold, a contradiction.

Now suppose for contradiction that k is such that $x < \frac{\theta_l k^\alpha}{k - p + \frac{\eta_1}{\eta_0} \delta(k)}$. Then, define a new transfer system τ' which is identical to the efficient one, τ^* , except for $\tau'_1(w, 1) = \tau_1^*(w, 1) + \epsilon$ and $\tau'_1(w, 0) = \tau_1^*(w, 0) - \frac{\eta_1}{\eta_0} \epsilon$, $\epsilon > 0$. This increases k to $k' = k + \epsilon$ and hence decreasing $\frac{\theta_l k^\alpha}{k - p + \frac{\eta_1}{\eta_0} \delta(k)}$. However, for ϵ small, $x < \frac{\theta_l k'^\alpha}{k' - p + \frac{\eta_1}{\eta_0} \delta(k')}$ still holds. Thus, this new allocation is incentive-compatible. It is clearly feasible. Finally, it strictly increases total welfare since the increase in $(p, 1)$'s utility is strictly greater than ϵ . Then, τ cannot be efficient, a contradiction. □

5.2 Implementation

First, note that in the incomplete markets equilibrium under laissez-faire no one uses the risk-free technology since $x < \beta^{-1}$. Then, it is obvious that IM equilibrium under laissez-faire is identical to that in the baseline model. IM equilibrium cannot make any redistribution on its own and hence cannot attain constrained efficient level of welfare.

In what follows, we propose a simple tax system to implement an allocation that attains constrained efficient welfare as IM equilibrium. In that regard, define

$$T(b_1) = \begin{cases} -\delta(k_1^*(p, 1)), & \text{if } b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha; \\ \frac{\eta_1}{\eta_0} \delta(k_1^*(p, 1)), & \text{if else.} \end{cases} \quad (26)$$

Proposition 8. *Incomplete markets equilibrium with tax system defined in (26) attains constrained efficient level of welfare.*

Proof. I am going to construct an IM equilibrium where $R = 1/\beta$ and agents with ideas in period one invest at the constrained efficient levels in their ideas.

Under the specified taxes, an agent faces the following problem:

Agent's problem with taxes.

$$\max_{c, k, b, s} u(c_1) + \beta \sum_{\theta} \mu_{\theta} u(c_{2\theta}) \quad (27)$$

s.t.

$$c_1 + k_1 + b_1 + s_1 \leq \begin{cases} w + \delta(k_1^*(p, 1)), & \text{if } b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha; \\ w - \frac{\eta_1}{\eta_0} \delta(k_1^*(p, 1)), & \text{if else,} \end{cases}$$

$$c_{2\theta} \leq i\theta k_1^\alpha + Rb_1 + xs_1,$$

$$k_1 \geq 0.$$

First, consider an agent who have an idea in period one. If a poor agent with an idea chooses $b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha$, she chooses $k_1 = k_1^*(p, 1)$ and $b_1 = p - k_1^*(p, 1) + \delta(k_1^*(p, 1)) =$

$-\beta\theta_l k_1^*(p, 1)^\alpha$. Suppose for contradiction that this is not true. Then, there is (k'_1, b'_1, s'_1) , where $(k'_1, s'_1) \neq (k_1^*(p, 1), 0)$, that gives strictly greater utility to the agent. $s'_1 = 0$ follows immediately from the fact that the return to bonds is strictly greater than the risk-free return. Now modify the efficient transfer mechanism such that $\tau'_1(p, 1) = k'_1 - p$ and $\tau'_2(p, 1) = -\frac{k'_1 - p - \delta(k_1^*(p, 1))}{\beta}$. Also, redefine the transfers of other agents such that aggregate feasibility holds. It is easy to show that individual consumption levels are going to be non-negative. The only thing left to check is incentive compatibility. That holds since the amount of transfers from unproductive agents to productive agents is still $\delta(k_1^*(p, 1))$. Therefore, this new transfer system is incentive-feasible and provides strictly greater utility, a contradiction.

Similarly, one can show that when $b_1(r, 1) \leq -\beta\theta_l k_1^*(p, 1)^\alpha$, $(r, 1)$ agent chooses to invest at the constrained efficient level, k^{fb} .

Therefore, the utility of $(w, 1)$ type when she chooses $b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha$, is:

$$w + \delta(k_1^*(p, 1)) - k_1^*(w, 1) - b_1(w, 1) + \beta \sum_{\theta} \mu_{\theta} [\theta k_1^*(w, 1)^\alpha + b_1(w, 1)/\beta] \quad (28)$$

$$= w + \delta(k_1^*(p, 1)) - k_1^*(w, 1) + \beta \sum_{\theta} \mu_{\theta} \theta k_1^*(w, 1)^\alpha. \quad (29)$$

On the other hand, if an agent with an idea chooses $b_1 > -\beta\theta_l k_1^*(p, 1)^\alpha$, then, letting her optimal choices be $\tilde{k}_1(w, 1), \tilde{b}_1(w, 1)$, her utility would be:

$$w - \frac{\eta_1}{\eta_0} \delta(k_1^*(p, 1)) - \tilde{k}_1(w, 1) - \tilde{b}_1(w, 1) + \beta \sum_{\theta} \mu_{\theta} [\theta \tilde{k}_1(w, 1)^\alpha + \tilde{b}_1(w, 1)/\beta] \quad (30)$$

$$= w - \frac{\eta_1}{\eta_0} \delta(k_1^*(p, 1)) - \tilde{k}_1(w, 1) + \beta \sum_{\theta} \mu_{\theta} \theta \tilde{k}_1(w, 1)^\alpha. \quad (31)$$

Given that k^{fb} maximizes the function $-k + \beta \sum_{\theta} \mu_{\theta} \theta k^\alpha$ and $k_1^*(p, 1) \leq k^{fb}$, clearly utility under $b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha$ is higher than that under $b_1 > -\beta\theta_l k_1^*(p, 1)^\alpha$ since $\tilde{k}_1(w, 1) < k_1^*(p, 1)$. Hence, we showed agents with an idea choose efficient allocation.

Now consider agents who do not have an idea in period one. If they choose $b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha$, then $c_2(w, 0) \leq -\theta_l k_1^*(p, 1)^\alpha + x s_1(w, 0)$. To keep consumption non-negative, $s_1(w, 0) \geq \frac{\theta_l k_1^*(p, 1)^\alpha}{x}$. Since $x < \beta^{-1}$, these agents will invest as few as possible in risk-free

technology. This implies they choose $b_1 = -\beta\theta_l k_1^*(p, 1)^\alpha$ and $s_1(w, 0) = \frac{\theta_l k_1^*(p, 1)^\alpha}{x}$. The utility then is $w + \delta(k_1^*(p, 1)) + \beta\theta_l k_1^*(p, 1)^\alpha - \frac{\theta_l k_1^*(p, 1)^\alpha}{x}$.

When an unproductive agent chooses $b_1 > -\beta\theta_l k_1^*(p, 1)^\alpha$, she sets $s_1 = 0$ and chooses constrained efficient allocation. The utility she gets is $w - \frac{\eta_1}{\eta_0} \delta(k_1^*(p, 1))$.

We need to show that, for unproductive agents, utility under $b_1 > -\beta\theta_l k_1^*(p, 1)^\alpha$ is greater than utility under $b_1 \leq -\beta\theta_l k_1^*(p, 1)^\alpha$. The difference is equal to

$$w - \frac{\eta_1}{\eta_0} \delta(k_1^*(p, 1)) - [w + \delta(k_1^*(p, 1)) + \beta\theta_l k_1^*(p, 1)^\alpha - \frac{\theta_l k_1^*(p, 1)^\alpha}{x}] \quad (32)$$

$$= \frac{-\delta(k_1^*(p, 1))}{\eta_0} - \theta_l k_1^*(p, 1)^\alpha (\beta - 1/x) \quad (33)$$

$$\geq \frac{-\delta(k_1^*(p, 1))}{\eta_0} - \theta_l k_1^*(p, 1)^\alpha (\beta - \beta \frac{k_1^*(p, 1) - p - \delta(k_1^*(p, 1))}{\beta\theta_l k_1^*(p, 1)^\alpha}) \quad (34)$$

$$\geq 0, \quad (35)$$

where last inequality follows from (14) and (21).

Market clearing and government budget balance conditions are immediate. $\square$

6 Allowing for negative consumption

Suppose utility function is of the form:

$$u(c) = \begin{cases} c, & \text{if } c \geq 0; \\ \lambda c, & c < 0, \end{cases} \quad (36)$$

where $\lambda > 1$ is a constant.

The purpose of this extension is to show that our main redistribution result does not crucially hinge upon non-negativity restriction on consumption. We show that, as long as $\lambda > 1$, meaning marginal disutility of decreasing consumption when $c < 0$ is strictly greater than marginal disutility of decreasing consumption when $c \geq 0$, constrained efficient allocation involves redistribution from unproductive to productive agents.

Given Assumption 1, which says $\eta_1 k^{fb} \leq \sum_w \zeta_w w$, FB analysis is the same as it is in the benchmark case: both productive agents invest at the socially efficient level, k^{fb} , and consumption distribution is such that aggregate feasibility constraints in FB problem hold with equality and no agent consumes a negative amount in any period and any state.

6.1 Constrained efficiency

This subsection characterizes efficient allocation under private information. Since the only change in the physical environment compared to the baseline model is the utility function, the definition of constrained efficiency remains the same as in section 3. We first provide a proposition that gives a threshold level $\underline{\lambda}$ such that FB *welfare* is attained in the private information environment if and only if $\lambda \geq \underline{\lambda}$. Then, we go on to prove that the constrained efficient allocation features redistribution to productive types, as long as $\lambda > 1$.

Proposition 9. *Constrained efficiency attains FB if and only if*

$$\lambda \geq \underline{\lambda} = \frac{k^{fb} - p + \frac{\eta_1}{\eta_0} \delta(k^{fb})}{k^{fb} - p - \delta(k^{fb})}. \quad (37)$$

More importantly, in the constrained efficient allocation, δ units are redistributed from unproductive to productive agents.

Proof. Suppose FB welfare is attained. This requires $\tau_1^*(p, 1) \geq k^{fb} - p$ and $\tau_2^*(p, 1) \geq -\theta_l k^{fb\alpha}$. Hence, NPV of transfers to poor and productive agent has to equal at least $\delta(k^{fb})$. For $(r, 1)$ not to lie to be $(p, 1)$, NPV of transfers to $(r, 1)$ should also be at least $\delta(k^{fb})$. For unproductive agents not to lie to be $(r, 1)$, $\tau_1^*(r, 1) > 0$ and $\tau_2^*(r, 1) < 0$.

For an unproductive agent $(w, 0)$ not to lie to be a productive agent $(w', 1)$, we need:

$$u(w + \tau_1^*(w, 0)) + \beta u(\tau_2^*(w, 0)) \geq u(w + \tau_1^*(w', 1)) + \beta u(\tau_2^*(w', 1)). \quad (38)$$

Given that no one gets to consume a negative amount in the constrained efficient allocation,

(38) implies we need, for all $w, w' \in W$,

$$w + \Delta_{w,0}^* \geq w + \tau_1^*(w', 1) + \beta\lambda\tau_2^*(w', 1). \quad (39)$$

Feasibility requires

$$\sum_w \zeta_w \Delta_{w,0}^* \leq -\frac{\eta_1}{\eta_0} \delta(k^{fb}). \quad (40)$$

It is easy to show that (39) and (40) together imply that in order to set productive agents' NPV of transfers to its highest possible amount in an incentive compatible way, the planner has to set $\Delta_{r,0}^* = \Delta_{p,0}^*$, which then implies $\Delta_{w,0}^* \leq -\frac{\eta_1}{\eta_0} \delta(k^{fb})$, for all $w \in W$. (39) then implies, for all $w' \in W$, the following should hold:

$$-\frac{\eta_1}{\eta_0} \delta(k^{fb}) \geq \delta(k^{fb}) + \beta(\lambda - 1)\tau_2^*(w', 1) \quad (41)$$

$$\Rightarrow \lambda \geq 1 - \delta(k^{fb}) \frac{1 + \eta_1/\eta_0}{\beta\tau_2^*(w', 1)} \quad (42)$$

$$\Rightarrow \lambda \geq 1 - \delta(k^{fb}) \frac{1 + \eta_1/\eta_0}{\delta(k^{fb}) - \tau_1^*(w', 1)}. \quad (43)$$

Rearranging (41) and plugging in $\tau_1^*(w', 1)$, we get $\lambda \geq \frac{k^{fb} - p + \frac{\eta_1}{\eta_0} \delta(k^{fb})}{k^{fb} - p - \delta(k^{fb})}$.

It is straightforward to prove the reverse direction of the proposition. $\square$

Consequently, if λ is large enough, the amount of redistribution needed to achieve FB investment level, δ , can be reached even in the case with private information. The intuition is as follows. From unproductive agents perspective, the benefit of lying to be productive is getting $(1 + \frac{\eta_1}{\eta_0})\delta$ additional transfer whereas the cost comes from consuming a negative amount in period two. When λ is sufficiently high, the cost outweighs the benefit and hence deters unproductive agents from reporting to be productive.

Now we consider the constrained efficient allocation when $\lambda < \underline{\lambda}$. The social planner still transfers strictly positive NPV of resources to poor and productive agents; however now the amount of redistribution is smaller than δ due to incentive-compatibility.

The social planner's goal is still to make poor and productive agent invest as close to

FB level as possible and do this without making her consume a huge negative amount. This pushes for a redistribution from other agents to $(p, 1)$ and incentive compatibility constraints push in the reverse direction. The constrained efficient allocation arises from this trade-off. The next corollary, which follows directly from Proposition 9, states this formally.

Corollary 1. *In the constrained efficiency allocation, there is strictly positive redistribution from unproductive agents to productive ones.*

7 Conclusion

In this paper I analyze efficient allocation of resources in an economy in which potential entrepreneurs have private information about (1) their wealth and productivity levels, (2) their investment levels, and (3) the returns to their investment. Returns being unobservable creates a novel motive for redistribution towards agents who are productive but lack resources to invest in their ideas. The unobservability of wealth and productivity, on the other hand, implies that this redistribution has to be incentive-compatible: the society should provide other agents with enough incentives so that they do not claim to be poor and productive. Efficient amount of redistribution from unproductive agents to productive ones arises from this trade-off. The paper also provides an implementation of this efficient allocation in an incomplete markets setup with very simple taxes on bond holdings.

The way I modeled informational frictions does not allow agents to monitor each others' private information at all. An important question is what happens if I allow agents to monitor some relevant information at some cost. Extending the analysis in this direction may be an interesting step for future work.

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