A Model of Cross-Section of Equity Returns and Firm Dynamics

(Prelimary[1])

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Abstract

We put forward a general equilibrium model that links the cross-section variation of expected returns to firms' life cycle dynamics. In the model all assets have the same exposure to short-run consumption risks, but differ in their exposure to long-run consumption risks (Bansal and Yaron (2004)). An econometrician who uses *conditional* CAPM regression to predict asset returns will obtain higher α for assets with higher exposure to long-run risks. Growth options have *lower* exposure to long-run risks than assets in place because the cost of exercising the growth options has high exposure to long-run risks and therefore provides a hedge against the risks of assets place. Small firms have *higher* exposure to long-run risks because they have higher operating leverage.

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Introduction

We put forward a general equilibrium model that links the cross-section of expected returns to firms' life cycle dynamics. The model specifies exogenously the aggregate consumption process, but focus on the general equilibrium in the public equity sector, in which claims of firms are publicly traded. We use the framework of Bansal and Yaron (2004) to introduce two distinct sources of risks in the economy: long-run and short-run consumption risks. In the model, assets have identical exposure to the short-run consumption risks, but differential exposure to the long-run consumption risks. The *conditional* CAPM fails, because it is a one-factor model and cannot account for assets' differential exposure to long-run risks. In the range of empirically plausible parameter values, assets have higher conditional α if it has higher exposure to the long-run consumption risks.

Our model generates an endogenous mechanisms through which growth options have lower exposure to long-run risks. Growth firms are options on assets in place. In order to exercise the option, growth firms need to set up an establishment. The cost of establishment itself is exposed to long-run consumption risks and becomes a hedge against assets in place. The cost of establishment is determined by the zero-profit condition of a marginal firm who purchase the establishment. In good times, the cross-section distribution of the firms' productivity has a fat right tail, that is, there are more highly productive firms in the economy. Because the establishments are inelastically supplied, this drives up the price of establishments. In bad times, firms productivity is low, ans so is the demand for the establishments, therefore the price of establishments drops. This implies ownerships of establishments are risky. Growth firms are a long position in assets in place, and a short position in the establishments. The high risk exposure of establishments acts as a hedge against assets in place, making growth firms less risky. The key to this argument is the cost of setting up new firms is highly pro-cyclical, which can be empirically tested.

Small firms have higher exposure to long-run risks in the model. In the model, firms' size is determined by its productivity. More productive firms generate higher cash flow. All firms need to pay a fixed operating cost in order to receive the cash flow. Firms choose to exit if the productivity is too low. Small firms have lower productivity and are closer to the shut-down point. This implies the operating leverage of small firms is high, and thus small firms have higher exposure to long-run risks.

The idea of long-run risks follows Bansal and Yaron (2004). Bansal, Dittmar, and Lundblad (2002) and Bansal, Dittmar, and Lundblad (2005) argues that differential exposure to long-run consumption risks can account for a significant portion of cross-section variation of risk premia. My model rationalize this observation in a general equilibrium setting.

A sizable literature addresses the issue of cross-section variation of expected returns in partial or general equilibrium models. For example, Berk, Green, and Naik (1999), Carlson, Fisher, and Giammarino (2004),Cooper (2006), Gala (2005), Gomes, Kogan, and Zhang (2003), Gourio (2006), Panageas and Yu (2006), Zhang (2005), among others. The key difference between my model and the above is that the above models typically have only one source of risks. Therefore, although they do generate a higher expected return for value firms, and/or small firms, conditional CAPM still holds in these models. There are two independent sources of risks in my model, long-run consumption risks and short-run consumption risks. Firms have identical exposure to short-run consumption risks but different exposure to long-run consumption risks in the model. If both long-run risks and short-run risks are observable to econometricians, a two-factor model perfectly predicts expected return. However, a one-factor model like conditional CAPM can never capture both sources of risks. In reality, firms might have different exposure to both short-run consumption risks and long-run consumption risks. However, in long-run risk models, most of the risk premium comes from exposure to long-run consumption risks. Therefore in order to generate a quantitatively important cross-section dispersion of expected returns, differential exposure to long-run risks is of first-order importance, and this is the key dimension captured by our model.

Like this paper, Berk, Green, and Naik (1999), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004) also use real options model to study the cross-section of equity returns. However, these models typically imply growth options are more risky than assets in place. The intuition is that options are replicated by a long position in asset in place and a short position in the risk-free strike asset. Therefore, options are leveraged positions on existing assets, and hence more risky. My model shows, in general equilibrium, the strike asset may no longer be risk-free. In fact, it may have higher exposure to long-run risks, and be a hedge against assets in place. In Carlson, Fisher, and Giammarino (2004), book-to-market ratio and size uniquely indentify a firm's conditional β and are thus proxies for risk. In my model, book-to-market ratio and size proxy for risk by uniquely identifying a firm's exposure to long-run risk.

My model admits closed form solutions of the cross-section distribution of firms' characteristics like in Luttmer (2007). In Luttmer (2007), consumption is determined endogenously in an economy without aggregate risk. Absence of aggregate risk implies the cross-section distribution of asset return is trivial: all assets have the same expected return. The focus in Luttmer (2007) is on the size distribution of firms. My model takes aggregate consumption as given and focus on the public equity sector. It could be imbedded in a general equilibrium that endogenize consumption, however, at the expense of analytical tractability of the model. My model has interesting predictions on the cross-section distribution of equity returns, which is explored in Ai (2007).

The paper is organized as follows. Section I sets up the model and defines an appropriate notion of equilibrium. Section II discusses the solution to firms' maximization problems and characterizes the cross-section distribution of firms. Section III provides an analysis of the failure of conditional CAPM and provides conditions under which conditional CAPM will under or over measure the risk exposure of assets. Section IV concludes.

I Set-up of the Model

A Preference

Consider an infinite-horizon economy with a representative consumer. The aggregate consumption is assumed to follow the process:

$$dC_t = C_t \left[\theta_t dt + \sigma\left(\theta_t\right) dB_t\right] \tag{1}$$

where $\{B_t\}_{t\geq 0}$ is a one-dimensional standard Brownian motion with respect to some filtered probablility space $\left(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P\right)$. $\{\theta_t\}_{t\geq 0}$ is a two-state Markov process (with respect to the filtration $\{\mathcal{F}_t\}_{t\geq 0}$) with state space $\Theta = \{\theta_H, \theta_L\}$, where $\theta_H > \theta_L$. The intensity matrix of the continuous-time Markov chain is:

$$\begin{bmatrix} -\lambda_H, & \lambda_H \\ \lambda_L, & -\lambda_L \end{bmatrix}$$

The Markov chain $\{\theta_t\}_{t\geq 0}$ is assumed to be persistent to capture the idea of long-run risks as in Bansal and Yaron (2004). We also allow the diffusion coefficient of consumption growth to depend on the state variable θ. This provides a parsimonous way of allowing for stochastic volatility. It is clear from equation (1), the consumption growth rate of the economy is determined by both θ and the Brownian motion $\{B_t\}_{t\geq 0}$. Fluctuations in θ can be interpreted as business cycles. When $\theta = \theta_H$, the expected growth rate of consumption is high, the economy is in a boom. When $\theta = \theta_L$, the expected consumption growth is low, consequently, the economy is in a recession. In (1), we also allow the volatility of consumption growth to depend on the state variable θ.

It is notationally convenient to represent the Markov chain as a compound Poisson process. In particular, let $\{N_{H,t}\}_{t\geq 0}$ be a Poisson process with intensity λ_H, and $\{N_{L,t}\}_{t\cdot 0}$ be a Poisson process with intensity λ_L. let $I_{\{x\}}$ be the indicator function, that is,

$$I_{\{x\}}(y) = \begin{cases} 1 & if\ y = x \\ 0 & if\ y \neq x \end{cases}$$

Then $\{\theta_t\}$ can be represented as the following compound Poisson process:

$$d\theta_t = \Delta\eta\left(\theta_t^-\right)^T dN_t \tag{2}$$

where

$$\Delta = \theta_H - \theta_L$$

and $\eta(\theta)$, and $N(t)$ are vector notations:

$$\eta(\theta) = \left[-I_{\{\theta_H\}}(\theta),\ I_{\{\theta_L\}}(\theta)\right]^T \tag{3}$$

$$N(t) = [N_{Ht},\ N_{Lt}]^T \tag{4}$$

Here we use the convention that $\{\theta_t\}$ is right-continuous with left limits, and use the notation

$$\theta_t^- = \lim_{s\to t, s<t} \theta_s$$

The representative consumer's intertemporal preference is represented by the Kreps and Porteus (1978) utility with constant relative risk aversion parameter γ and constant intertemporal elasticity of substitution parameter ψ. The preference specification follows Duffie and Epstein (1992a) and Duffie and Epstein (1992b), which is a continuous time version of the recursive preference considered in Kreps and Porteus (1978), Epstein and Zin (1989) and Weil (1989). Since the aggregate consumption process contains a persistent component $\{\theta_t\}_{t\geq 0}$, the economy is a continuous time version of the Bansal and Yaron (2004) economy. We use $\{V_t\}_{t\geq 0}$ to denote the utility process of the representative agent. The representative consumer's preference is specified by a pair of aggregators $(f, \mathcal{A})$ such that:

$$dV_t = [-f(C_t, V_t) - \frac{1}{2}\mathcal{A}(V_t)||\sigma_V(t)||^2]dt + \sigma_V(t)dB_t \tag{5}$$

We adopt the convenient normalization $\mathcal{A}(v) = 0$ (Duffie and Epstein (1992b)), so that the aggregator $f(C,V)$ is:

$$f(C,V)=\frac{\beta}{1-1/\psi}\frac{C^{1-1/\psi}-((1-\gamma)V)^{\frac{1-1/\psi}{1-\gamma}}}{((1-\gamma)V)^{\frac{1-1/\psi}{1-\gamma}-1}} \tag{6}$$

In this paper, we focus on the case $\gamma,\psi\neq 1$. Extension to the limiting cases $\gamma=1$ and $\psi=1$ are straightforward. The date t utility of an agent can be written as:

$$V_t=E_t\left[\int_t^{\infty} f\left(C_s,V_s\right)ds\right] \tag{7}$$

We use $\{\pi_t\}_{t\geq 0}$ to denote the state price process of the economy. It can be shown that $\{\pi_t\}$ is a Levy process of the form:

$$d\pi_t=\pi_t\left[-\bar{r}\left(\theta_t^-\right)dt-\gamma\sigma\left(\theta\right)dB_t-\eta_\pi\left(\theta_t^-\right)^T dN_t\right]$$

where

$$\bar{r}\left(\theta\right)=\beta+\frac{1}{\psi}\theta-\frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)\sigma\left(\theta\right)^2$$

and

$$\eta_\pi\left(\theta\right)=\left[\left(1-\hat{\delta}^{-1}\right)I_{\{H\}}\left(\theta\right),\left(1-\hat{\delta}\right)I_{\{L\}}\left(\theta\right)\right] \tag{8}$$

where $\hat{\delta}$ is a constant given in appendix I.

B Life Cycle Dynamics of Firms

There are two sectors in the economy, the public equity sector and the private equity sector. Claims to firms in the public equity sector are traded on the stock market. The output of the firms in both sectors sum up to aggregate consumption. We explicitly model the production decisions of firms in the public equity sector, and assume that the difference between aggregate consumption and output of firms in the public equity sector is produced by the private equity sector.

Firms are born continuously. At calendar time t, a firm enters the public equity sector by obtaining a blueprint at competitively determined price, denoted $f_{I,t}$. In the class of equilibria we consider below, the price of drawing the blue print measured in date t consumption numeraire is of the form:

$$f_{I,t}=f_I\left(\theta_t\right)C_t$$

A blue print uniquely identifies a productivity process of the firm that owns the blueprint. Firms are indexed by i. We denote the calendar time at which firm i draws its blueprint t^i. The initial productivity of a blueprint is drawn from a continuous density $u(\cdot)$ with support $\left[0, \overline{Z}\right]$. Once a blue print is drawn, the productivity associated with the blue print follows a stochastic process specified by:

$$dZ_t^i = Z_t^i \left[\mu_z(\theta_t)\, dt + \sigma_z(\theta_t)\, dB_t^i\right], \qquad t \geq t^i \tag{9}$$

where $\left\{B_t^i\right\}_{t\geq 0}$ is a standard Brown motion independent among blueprints, and independent of $\{B_t\}_{t\geq 0}$. The drift and diffusion coefficient of the productivity process are identical among all firms and are assumed to depend on the long-run risk state variable θ. We also assume $\mu_z(\theta_H) \geq \mu_z(\theta_L)$ to capture the idea that the dividend of firms in the publicly traded sector is more sensitive to long-run risk than aggregate consumption. After birth, firms die exogenously at a Poisson rate κ. Conditioning on survival, a firm needs to pay an operating cost of $o_I C_t$ in order to keep the blueprint alive. Firms can exit voluntarily at any time. The decision to exit is irreversible. Once exit, the blueprint evaporates, and the firm receives nothing. At any point in time, the total measure of blueprints flow into the economy is of fixed supply, and is assumed to be a function of θ_t. We denote the rate of measure of blueprints injected into the economy to be $m_I(\theta_t)$.

Conditioning on survival, a firm can set up a plant by purchasing an establishment at competitively determined price $f_{II,t}$. In the class of equilibria we consider below, $f_{II,t}$ is of the form

$$f_{II,t} = f_{II}(\theta_t)\, C_t$$

At any point in time, the total measure of new establishment injected into the economy is of fixed supply, and is denoted $m_{II}(\theta_t)$.

A blueprint will generate a cash flow only after the plant is set up. The cash flow of firm i at calendar time $t \geq t^i$ is equal to $Z_t^i C_t$. Firms also have to pay an operating cost of $o_{II} C_t$ in order to receive the cash flow. Firms have identical fixed operating cost, but differs in their individual productivity parameter Z_t^i. A firm can choose to exit voluntarily. The decision to exit is irreversible. Once exit, the firm can no longer receive any cash flow. Only a fraction $n \in (0.1)$ of the establishment can be recovered. A firm is said to be in the first stage (or stage I) of its life cycle if it has already obtained a blueprint, but has not obtained an establishment yet. Firms in the first stages of their life cycles do not have any cash flow. A firm enters the second stage (or stage II) of its life cycle by purchasing an establishment and begin to generate cash flows.

C Stationary Public Equity Sector Equilibrium

For a firm in stage I of its life cycle, it has an option to set up an establishment and begin to generate cash flows, and an option to give up the blueprint and exit. These decisions depend on the firm's productivity level Z_t^i. In the class of equilibria we consider, the decision rules for firms at the first stage of their life cycle are characterized by two threshold levels $\left[\hat{Z}_I(\theta),\ Z_I^*(\theta)\right]$. Firms at stage I of their life cycles choose to set up a plant and enter into stage II of their life cycles if its productivity level exceeds the threshold level $\hat{Z}_I$, which depend on θ, that is,

$$Z_t^i \geq \hat{Z}_I(\theta_t)$$

Firms choose to exit the public equity sector during the first stage of their life cycle if its productivity level is sufficiently low:

$$Z_t^i \leq Z_I^*(\theta)$$

A firm in the second stage of its life cycle has the option to exit and recover a fraction n of the establishment at the prevailing market price of establishments. Again, in the class of equilibria we consider, the decision rule to exit is also a threshold level $Z_{II}^*(\theta_t)$ that depends only on the state variable θ. Firms choose to exit when

$$Z_t^i \leq Z_{II}^*(\theta_t)$$

Figure 1 depicts the flow of firms at calendar time t. At time t, a cohort of blueprints were drawn from the distribution $u(\cdot)$ with support $\left[0, \overline{Z}\right]$. The total measure of these firms is $m_I(\theta_t)$. Among them, group [1]'s productivity is high enough ($\geq \hat{Z}_I(\theta_t)$) so that they obtain establishments and enter into stage II directly. Group [2] enters into the first stage. Some of them will enter into the second stage in the future if their productivity becomes high enough, while others end up exiting the public equity sector during stage I if their productivity hits the lower threshold $Z_I^*(\theta_\tau)$ first. Firms in group [3] exit the public equity sector directly because their productivity is too low, and the prospect of their future cash flow is not worth the operating cost $o_I C_t$.

At time t, certain measure of firms in stage I of their life cycles die prematurely, that is, they exit the public equity sector without generating any cash flow because their productivity falls below the lower threshold level $Z_I^*(\theta_t)$. This is group [5] in figure 1. Group [4] in Figure 1 are the firms in stage I whose productivity is high enough to set up an establishment and enter into stage II. Group [6] are stage-II firms whose productivity is too

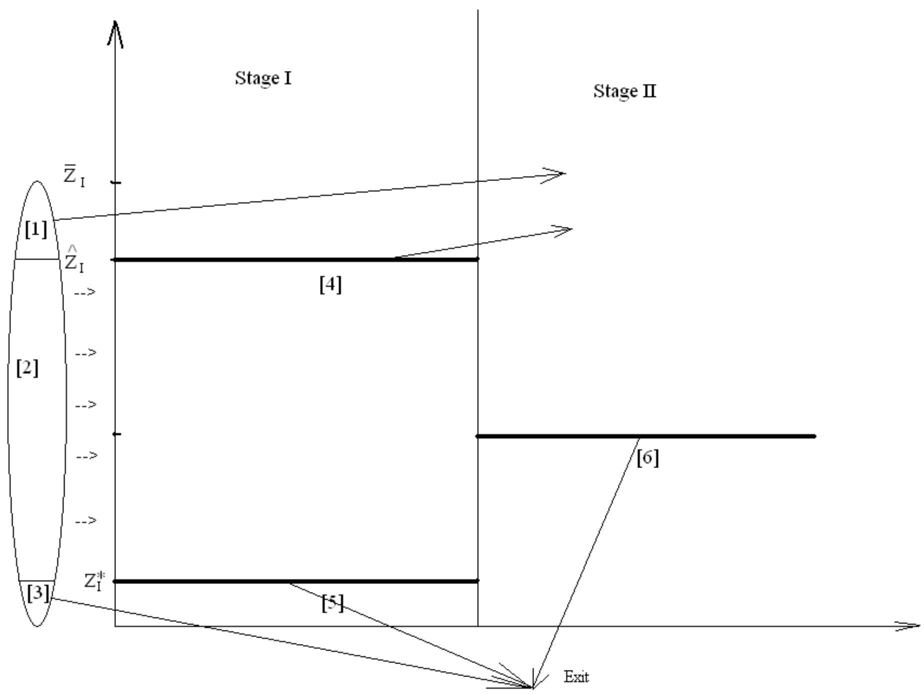

Figure 1: Life Cycle Dynamics of Firms

low to keep operating, so that they exit the public equity sector during the second stage of their life cycle.

We consider a general equilibrium in the public equity sector by taking the aggregate supply of blueprints and establishments as given. The notion of equilibrium we use is stationary public equity sector equilibrium with instantaneous transition, whose precise definition is given below:

Definition 1 *: Stationary Public Equity Sector Equilibrium with Instantaneous Transition*

A SPESE with instantaneous transition consists of:

1) Price of the firms: $\left\{V_I\left(\theta_t, Z_t^i, C_t\right),\ V_{II}\left(\theta_t, Z_t^i, C_t\right)\right\}_{i,t}$

Here we use $V_I\left(\theta_t, Z_t^i, C_t\right)$ to denote the value of a firm i at the first stage of its life cycle. we use $V_{II}\left(\theta_t, Z_t^i, C_t\right)$ to denote the value of a firm i at the second stage of its life cycle.

2) Optimal decision rules: $\left\{\hat{Z}_I\left(\theta_t\right),\ Z_I^*\left(\theta_t\right);\ Z_{II}^*\left(\theta_t\right)\right\}_{i,t}$.

$\hat{Z}_I\left(\theta_t\right)$ is the threshold level above which firms in stage I will enter into second stage of their life cycle. $Z_I^*\left(\theta_t\right)$ is the threshold level below which firms will exit the public equity sector from the first stage of their life cycle, and $Z_{II}^*\left(\theta_t\right)$ is the threshold level below which firms will exit the public equity sector from the second stage of their life cycle.

3) Stationary measures of the cross-section distribution of firms's productivity: $\left\{\Phi_I\left(\cdot;\theta_t\right),\ \Phi_{II}\left(\cdot;\theta_t\right)\right\}$ with support $\left[Z_I^*\left(\theta_t\right),\ \hat{Z}_I\left(\theta_t\right)\right]$, and $\left[Z_{II}^*\left(\theta_t\right),\ \infty\right]$, respectively.

We focus on equilibria in which the cross-section distribution of firms' characteristics is stationary (after appropriate normalization). This distribution will depend on the state variable θ, however. When there is a regime switch in θ, the cross-section distribution of firms will only gradually converge to the new stationary distribution. Here we are assuming that the convergence happens instantaneously in order for the analytical tractability of the model, hence the qualifier "with instantaneous transition". However, since the Markov chain $\{\theta_t\}$ is very persistent, the transition period is short relatively to the length of each of the regimes. Numerical results show this is indeed a good approximation.

Note, $Z_I^*\left(\theta_t\right)$ and $\hat{Z}_I\left(\theta_t\right)$ are the absorbing barriers of the stationary measure $\Phi_I\left(\cdot;\theta_t\right)$ and $Z_{II}^*\left(\theta_t\right)$ is the absorbing barrier of the stationary measure $\Phi_{II}\left(\cdot;\theta_t\right)$. We use the notation $m_{EXIT}\left(\Phi, Z\right)$ to denote the rate of outflow of the stationary measure Φ at the absorbing barrier Z.

A SPESE with instantaneous transition is a collection of prices and quantities listed above that satisfies the following equilibrium conditions:

1) Share holder value maximization for firms in the second stage of their life cycle. $\forall\theta, Z^i, C$, the threshold level $Z_{II}^*(\theta)$ solves the the following optimal stopping problem:

$$V_{II}\left(\theta, Z^i, C\right) = \max_{\tau} E\left[\int_0^{\tau} e^{-\kappa t}\frac{\pi_t}{\pi_0}\left(Z_t^i - o_{II}\right)C_t dt + e^{-\kappa\tau}\frac{\pi_\tau}{\pi_0} n f_{II}\left(\theta_\tau\right)C_\tau \middle| \theta, Z^i, C\right] \quad (10)$$

2) Share holder value maximization for firms in the first stage of life cycle. $\forall\theta, Z^i, C$, the threshold level $\left[\hat{Z}_I(\theta),\ Z_I^*(\theta)\right]$ solves the the following optimal stopping problem:

$$V_I\left(\theta, Z^i, C\right) = \max_{\tau_1,\tau_2} E\left[\int_0^{\min\{\tau_1,\tau_2\}} -e^{-\kappa t}\frac{\pi_t}{\pi_0} o_I C_t dt + e^{-\kappa\tau_1}\frac{\pi_{\tau_1}}{\pi_0}\left\{V_{II}\left(\theta_{\tau_1}, Z_{\tau_1}^i, C_{\tau_1}\right) - f_{II}\left(\theta_{\tau_1}\right)C_{\tau_1}\right\} \middle| \theta, Z^i, C\right] \quad (11)$$

3) Market Clearing for blueprints. The price of drawing a blueprint is equal to the expected value of the blueprint.

$$\forall\theta,\quad f_I(\theta)\,C = \int_{Z_I^*(\theta)}^{\hat{Z}_I(\theta)} V_I(\theta, z, C)\,u(z)\,dz + \int_{\hat{Z}_I(\theta)}^{\overline{Z}} \left[V_{II}(\theta, z, C) - \ f_{II}(\theta)\,C\right]u(z)\,dz \quad (12)$$

4) Market Clearing for establishments. Total demand of establishments equals the total supply of establishments:

$$\forall\theta,\ m_I(\theta)\int_{\hat{Z}_I(\theta)}^{\overline{Z}} u(z)\,dz + m_{EXIT}\left[\Phi_I(\theta), \hat{Z}_I(\theta)\right] = m_{II}(\theta) + n \times m_{EXIT}\left[\Phi_{II}(\theta), Z_{II}^*(\theta)\right] \quad (13)$$

5) Stationarity Condition.

Total entry into the public equity sector equals total exit out of the public equity sector:

$$\forall\theta,\quad m_I(\theta) = m_I(\theta)\int_0^{Z_I^*(\theta)} u(z)\,dz + m_{EXIT}\left[\Phi_I(\theta), Z_I^*(\theta)\right] + m_{EXIT}\left[\Phi_{II}(\theta), Z_{II}^*(\theta)\right] \quad (14)$$

Total entry into the first stage equals total exit out of the first stage:

$$m_I(\theta)\int_{Z_I^*(\theta)}^{\hat{Z}_I(\theta)} u(z)\,dz = m_{EXIT}\left[\Phi_I(\theta), \hat{Z}_I(\theta)\right] + m_{EXIT}\left[\Phi_I(\theta), Z_I^*(\theta)\right] \quad (15)$$

Total entry into the second stage equals total exit out of the second stage:

$$m_I(\theta)\int_{\hat{Z}_I(\theta)}^{\overline{Z}} u(z)\,dz + m_{EXIT}\left[\Phi_I(\theta), \hat{Z}_I(\theta)\right] = m_{EXIT}\left[\Phi_{II}(\theta), Z_{II}^*(\theta)\right] \quad (16)$$

II The Cross-Section Distribution of Expected Returns

A Firms' Optimal Stopping Problems

The aggregate consumption of this economy grows without bound, therefore we need to make some assumptions on the primitive parameters of the model so that the representative agent's utility is always finite. We also need assumptions on the firms' cash flow process so that the value of firm at any time of their life cycle is always finite. The following assumptions will guarantee the finiteness of agents' utility and firms' value:

Assumption 1:

$$\left[\beta+\frac{1}{2}\gamma\left(1-\frac{1}{\psi}\right)\sigma_C^2\left(\theta_H\right)-\left(1-\frac{1}{\psi}\right)\theta_H+\lambda_H\right]\left[\beta+\frac{1}{2}\gamma\left(1-\frac{1}{\psi}\right)\sigma_C^2\left(\theta_L\right)-\left(1-\frac{1}{\psi}\right)\theta_L+\lambda_L\right]>\lambda_H\lambda_L \tag{17}$$

Assumption 2:

$$\left[\kappa+\bar{r}\left(\theta_H\right)-\theta_H+\gamma\sigma\left(\theta_H\right)^2-\mu_z\left(\theta_H\right)+\lambda_H\right]\left[\kappa+\bar{r}\left(\theta_L\right)-\theta_L+\gamma\sigma\left(\theta_L\right)^2-\mu_z\left(\theta_L\right)+\lambda_L\right]>\lambda_H\lambda_L \tag{18}$$

where $\bar{r}\left(\theta\right)$ are given by equation (48) in the appendix.

Proposition 1 *Under assumption 1, the life-time utility of the representative agent is finite. Under assumption 2, firms' values are finite.*

We will assume assumption 1 and 2 are true in the rest of the paper. We first consider the optimization problem of firms in the second stage of their life cycle. It is easy to see that the value function $V_{II}\left(\theta,Z^i,C\right)$ must be linearly homogeneous in C, since both the objective function and the constraints are. Therefore we use the notation:

$$\begin{bmatrix} V_{II}\left(\theta_H,Z^i,C\right) \\ V_{II}\left(\theta_L,Z^i,C\right) \end{bmatrix}=\begin{bmatrix} G_{II}\left(Z,\theta_H\right)C \\ G_{II}\left(Z,\theta_L\right)C \end{bmatrix} \tag{19}$$

For notational convenience, we use $G_{II}\left(Z\right)$ to denote the vector $\left[G_{II}\left(Z,\theta_H\right),G_{II}\left(Z,\theta_L\right)\right]^T$. The solution to the optimal stopping problem for firms in the second stage of their life cycles is characterized by the following proposition:

Proposition 2 *The value function of the optimization problem for firms in the second stage* *of their life cycles of the form in (17), where*

$$G_{II}(Z) = aZ - bo_2 + \sum_{i=1}^{2} e_i K_i Z^{\alpha_i} \tag{20}$$

where

$$a = \begin{bmatrix} a_H \\ a_L \end{bmatrix}, \; b_{II} = \begin{bmatrix} b_H \\ b_L \end{bmatrix}, \; e_i = \begin{bmatrix} e_{i,H} \\ e_{i,L} \end{bmatrix}, i = 1, 2 \tag{21}$$

are vectors of constants given in appendix II.

The optimal stopping time τ is given by:

$$\tau = \inf \left\{ t : \theta_t = \theta_H, \text{ and } Z_t^i < Z_{II}^*(\theta_H) \right\} \cup \left\{ \theta_t = \theta_L, \text{ and } Z_t^i < Z_{II}^*(\theta_L) \right\}$$

where $Z_{II}^*(\theta_H)$, $Z_{II}^*(\theta_L)$ along with the two constants K_1, and K_2 are jointly determined by the two value matching conditions:

$$\begin{bmatrix} G_{II}(Z_{II}^*(\theta_H), \theta_H) \\ G_{II}(Z_{II}^*(\theta_L), \theta_L) \end{bmatrix} = \begin{bmatrix} nf_{II}(\theta_H) \\ nf_{II}(\theta_L) \end{bmatrix}$$

and the smooth pasting conditions:

$$\begin{bmatrix} G'_{II}(Z_{II}^*(\theta_H), \theta_H) \\ G'_{II}(Z_{II}^*(\theta_L), \theta_L) \end{bmatrix} = 0$$

The intuition of the above proposition is that the optimal stopping problem of the firms are characterized by a pair of optimal exit boundaries, $Z_{II}^*(\theta_H)$ and $Z_{II}^*(\theta_L)$. Firms in the second stage of their life cycles exit the public equity sector optimally if its productivity hits $Z_{II}^*(\theta_H)$ from above in state θ_H, or hits $Z_{II}^*(\theta_L)$ from above in state θ_L, whichever happens earlier.

We turn now to the firm's maximization problem in the first stage of their life cycle. By the same reason, the value function is also linearly homogenous in C, we denote:

$$\begin{bmatrix} V_I\left(\theta_H, Z^i, C\right) \\ V_I\left(\theta_L, Z^i, C\right) \end{bmatrix} = \begin{bmatrix} G_I\left(Z^i, \theta_H\right) C \\ G_I\left(Z^i, \theta_L\right) C \end{bmatrix} \tag{22}$$

The solution to the optimal stopping problem for firms in the first stage of their life cycles is characterized by the following proposition:

Proposition 3 *The value function of the optimization problem for firms in the second stage of their life cycles of the form in (22), where*

$$G_I(Z) = -bo_I + \sum_{i=1}^{4} e_i L_i Z^{\alpha_i} \tag{23}$$

where $a, b, [e_i]_{i=1,2,3,4}$ are vectors of constants given in appendix II.

The optimal stopping time τ_1 and τ_2 are given by:

$$\tau_1 = \inf\left\{t : \theta_t = \theta_H,\ \text{and } Z_t^i < Z_I^*(\theta_H)\right\} \cup \left\{\theta_t = \theta_L,\ \text{and } Z_t^i < Z_I^*(\theta_L)\right\}$$

and

$$\tau_2 = \inf\left\{t : \theta_t = \theta_H,\ \text{and } Z_t^i > \hat{Z}_I(\theta_H)\right\} \cup \left\{\theta_t = \theta_L,\ \text{and } Z_t^i > \hat{Z}_I(\theta_L)\right\}$$

where $Z_I^*(\theta_H)$, $Z_I^*(\theta_L)$ $\hat{Z}_I(\theta_H)$, $\hat{Z}_I(\theta_L)$ along with the four constants $[L_i]_{i=1,2,3,4}$ are jointly determined by the value matching and smooth pasting conditions:

$$\begin{bmatrix} G_I(Z_I^*(\theta_H), \theta_H) \\ G_I(Z_I^*(\theta_L), \theta_L) \end{bmatrix} = 0; \quad \begin{bmatrix} G_I'(Z_I^*(\theta_H), \theta_H) \\ G_I'(Z_I^*(\theta_L), \theta_L) \end{bmatrix} = 0$$

$$\begin{aligned} \begin{bmatrix} G_I\left(\hat{Z}_{II}(\theta_H), \theta_H\right) \\ G_I\left(\hat{Z}_{II}(\theta_L), \theta_L\right) \end{bmatrix} &= \begin{bmatrix} G_{II}\left(\hat{Z}_{II}(\theta_H), \theta_H\right) - f_{II}(\theta_H) \\ G_{II}\left(\hat{Z}_{II}(\theta_L), \theta_L\right) - f_{II}(\theta_L) \end{bmatrix} \\ \begin{bmatrix} G_I'\left(\hat{Z}_{II}(\theta_H), \theta_H\right) \\ G_I'\left(\hat{Z}_{II}(\theta_L), \theta_L\right) \end{bmatrix} &= \begin{bmatrix} G_{II}'\left(\hat{Z}_{II}(\theta_H), \theta_H\right) \\ G_{II}'\left(\hat{Z}_{II}(\theta_L), \theta_L\right) \end{bmatrix} \end{aligned}$$

The above proposition implies that the firm's optimization problem can also be solved in close form as given in (23). The value of a firm in stage I is proportional to aggregate consumption and contains two components. The first term in equation (23) is the present value of the operating cost. The second term in (23) is the value of the option to set up an establishment. Firms in the first stage of life cycle need to pay an operating cost in order to keep the blueprint alive. In this stage, firms will stay in the public equity sector as long as the value of the option is higher than the present value of the operating cost. Once the a firm's productivity is high enough, that is, Z_t^i hits the option exercise boundary $\hat{Z}_I(\theta)$ from below, it will set up an establishment and begin to generate the cash flow. However, if the firm's productivity is too low, that is, it hits the optimal exit boundary $Z_I^*(\theta)$ from above, the value of the option to set up an establishment is too low to cover the operating cost, the firm will choose to exit the public equity sector.

B The Stationary Cross-Section Distribution

At any point in time, the productivity of firms in the first stage their life cycles lies in the interval $\left[Z_I^*(\theta), \hat{Z}_I(\theta)\right]$. The productivity of firms in the second stage of their life cycles is with the open interval $[Z_{II}^*(\theta), \infty)$. If the economy stays in the state θ for long enough, the cross-section distribution of firms in stage I and stage II will both converge to a stationary distribution. Since the Markov chain $\{\theta_t\}_{t\geq 0}$ is set up to model long-run risks, it is indeed very persistent. This means the time of transition to the stationary cross-section distribution is short relative to the average length of each regime. Here we assume the transition happens instantaneously, so that the equilibrium can be characterized in closed form.

To derive the cross-section distribution of firm's productivity, we need to specify the density from which the initial distribution of blueprint are drawn. To facilitate an explicit solution of the cross-section distribution of firms' productivity, we assume the initial distribution is a uniform distribution.

Assumption 3:

$$u(Z) = \frac{1}{\bar{Z}} Z, \quad Z \in \left[0, \bar{Z}\right] \tag{24}$$

We are interested in equilibria in which the stationary distribution of firms in both stages exists. Note for firms in the second stage of their life cycle, the productivity can grow arbitrarily high. To ensure the existence of stationary distributions, we need the growth rate of firm's productivity to be low enough or the probability of the exogenous death is high enough so that there will not be masses of firms accumulate at ∞. The following assumption guarantees the existence of stationary measures for firms in both stages:

Assumption 4:

$$\forall\theta, \quad \mu_z(\theta) < \kappa \tag{25}$$

At any point in time, the total measure of firm that enter into the economy is $m_I(\theta)$, and their initial productivity are uniformly distributed on the interval $\left[0, \bar{Z}\right]$. Among them, a measure $\frac{\bar{Z}-\hat{Z}_I(\theta)}{\bar{Z}} m_I(\theta)$ of firms has productivity higher than the option exercising barrier $\hat{Z}_I(\theta)$, and enters into the second stage of their life cycle immediately after birth. This is group [1] in Figure 1. A measure $\frac{\hat{Z}_I(\theta)-Z_I^*(\theta)}{\bar{Z}} m_I(\theta)$ of firms has initial productivity between the exit boundary $Z_I^*(\theta)$ and the option exercise boundary $\hat{Z}_I(\theta)$. They will stay in the first stage until their productivity hit one of the boundaries and either enter into the second stage or exit the public equity sector. This is group [2] firms depicted in Figure 1. Finally, a measure $\frac{\hat{Z}_I(\theta)-Z_I^*(\theta)}{\bar{Z}}m_I(\theta)$ of firms has initial productivity lower than the exit boundary, so they exit the public equity sector directly. This is group [3] depicted in Figure 1.

Newly born firms enter into stage I at the rate of $\frac{\hat{Z}_I(\theta)-Z_I^*(\theta)}{\bar{Z}}m_I(\theta)$. At the same time, some measure of firms exit stage I at the boundary $\hat{Z}_I(\theta)$. This is group [4] depicted in Figure 1. Also, some measure of firms die prematurely and exit stage I at the boundary $Z_I^*(\theta)$. This is group [5] in Figure 1. Sationarity of the cross-section distribution of firms' productivity in stage I requires the total measure of group [4] and group [5] equals the total measure of entry firms, as required in (15). Therefore $Z_I^*(\theta)$ and $\hat{Z}_I(\theta)$ are the absorbing barriers of the productivity of firms in stage I, and the support of the distribution of the productivity of firms in stage I, denoted $\Phi_I(\theta)$ is $\left[Z_I^*(\theta), \hat{Z}_I(\theta)\right]$. It is more convenient to characterize the density of $logZ$, we denote the density of $\log Z$ by $\phi(\cdot;\theta)$. This density can actually be solved in closed form, and is characterized by the following proposition.

Proposition 4 *For firms in the first stage, the stationary measure of $logZ$ is:*

$$\phi_I(z;\theta) = \frac{m_I(\theta)}{\left(\kappa + \mu_z(\theta) - \sigma(\theta)^2\right)\bar{Z}}\left[e^z + \xi_1(\theta)e^{\alpha_1(\theta)z} + \zeta_2(\theta)e^{\alpha_2(\theta)z}\right] \tag{26}$$

where

$$\alpha_1(\theta) > 1,\ \alpha_2(\theta) < 0,\ \zeta_1(\theta), \zeta_2(\theta) < 0 \tag{27}$$

are constants given in appendix III.

The exit rate of firms at the absorbing barriers $Z_I^*(\theta), \hat{Z}_I(\theta)$ are:

$$m_{EXIT}[\Phi_I(\theta), Z] = \frac{1}{2}\sigma(\theta)^2\left|\phi_I'(\ln Z;\theta)\right| \tag{28}$$

where $Z = Z_I^*(\theta),\ \hat{Z}_I(\theta)$, respectively.

We now turn to the stationary distribution of firms in stage II. There are two sources of entrance of firms into stage II: The new born firms enter into stage II directly if their productivity is higher than the threshold $\hat{Z}_I(\theta)$. This is group [1] in Figure 1. The rate of entrance from this source is $\frac{\bar{Z}-\hat{Z}_I(\theta)}{\bar{Z}}m_I(\theta)$. Another source of entrance is the firms in stage I whose productivity hits the absorbing barrier $\hat{Z}_I(\theta)$. The rate of entrance from this channel is given in (28). Firms in stage II will exit the public equity sector if its productivity hits the exit barrier $Z_{II}^*(\theta)$. The rate of exit is denoted $m[\Phi_{II}(\theta), Z_{II}^*]$. Stationarity requires the rate of entrance is the same as the rate of exit as in (16). The stationary distribution of the firms in stage II, denoted $\Phi_{II}(\theta)$, is a continuous measure with support $[Z^*_{II}(\theta), \infty)$. Again, it is more convenient to work with the density of $\log Z^i$. We denote the density of $logZ^i$ for firms in the second stage as $\phi_{II}(z;\theta)$. This density is characterized by the following proposition:

Proposition 5 *For firms in the first stage, the stationary measure of $logZ^i$ is:*

$$\phi_{II}(z;\theta) = \begin{cases} m_I(\theta) \times \zeta_A(\theta) \times e^{\alpha_2(\theta)z} & z \in \left[\ln \bar{Z}, \infty\right) \\ m_I(\theta) \times \left[\zeta_{C,0}(\theta) e^{z} + \zeta_{C,1}(\theta) e^{\alpha_1(\theta)z} + \zeta_{C,2}(\theta) e^{\alpha_2(\theta)z}\right] & z \in \left[\ln \hat{Z}_I(\theta), \ln \bar{Z}\right) \\ m_I(\theta) \times \left[\zeta_{B,1}(\theta) e^{\alpha_1(\theta)z} + \zeta_{B,2}(\theta) e^{\alpha_2(\theta)z}\right] & z \in \left[\ln Z^*_{II}(\theta), \ln \hat{Z}_I(\theta)\right) \end{cases} \tag{29}$$

where $\alpha_1(\theta)$ and $\alpha_2(\theta)$ are the same as in (27), and $\zeta_A(\theta)$, $\left[\zeta_{B,i}(\theta)\right]_{i=1,2}$, $\left[\zeta_{C,i}(\theta)\right]_{i=0,1,2}$ are constants given in appendix III.

The rate of firms that exit stage II is time-invariant and given by:

$$m_{EXIT}\left[\Phi_{II}(\theta), Z^*_{II}(\theta)\right] = \frac{1}{2}\sigma^2 \left|\phi'_{II}\left(\ln\left(Z^*_{II}\ (\theta)\right);\theta\right)\right| \tag{30}$$

Note given the price of establishment, $f_{II}(\theta)$, the optimal entry and exit barriers for firms in both stages are given in proposition 2 and proposition 3. Given these barriers, the rate of entry and exit of stage I and II are determined by proposition 4 and proposition 5. Imposing the market clearing condition for establishments (13) determines the equilibrium price of establishments $f_{II}(\theta)$ and closes the model.

III Failure of CAPM

In the economy constructed above an econometrician who use CAPM to predict expected stock returns will fail even if he or she has infinite amount of data, and is able to estimate conditional α and β perfectly. A conditional CAPM is the regression:

$$R^i_{t,t+\Delta} - r_{t,t+\Delta} = \alpha_{t,\Delta} + \left(R^M_{t,t+\Delta} - r_{t,t+\Delta}\right)\beta_{t,\Delta} + \varepsilon_{t,t+\Delta} \tag{31}$$

In the above specification, $R^i_{t,t+\Delta}$ is the return of asset i during the time interval $[t, t+\Delta]$, $r_{t,t+\Delta}$ is the return of the risk-free asset during the same time interval. $R^M_{t,t+\Delta}$ is the return to the market portfolio. We use the notation $\alpha^i_{t,\Delta}$, $\beta^i_{t,\Delta}$ to allow these quantities to be time-varying, and possibly depend on i. Of course, under the null hypothesis of the CAPM regression, $\forall i, t,\ \alpha^i_{t,\Delta} = 0$. In the economy we constructed, the null hypothesis is false. Suppose the researcher is equipped with infinite amount of data, and is able to recover the true value $\alpha^i_{t,\Delta}$, $\beta^i_{t,\Delta}$ perfectly, and use the model (31) to predict expected returns of the assets in the economy.

If the econometrician is able to recover the true value of $\alpha^i_{t,\Delta}$, $\beta^i_{t,\Delta}$, we know

$$\beta^i_{t,\Delta} = \frac{Cov_t\left(R^i_{t,t+\Delta}, R^M_{t,t+\Delta}\right)}{Var_t\left(R^M_{t,t+\Delta}\right)} \tag{32}$$

and

$$\alpha^i_{t,\Delta} = \left(E_t\left[R^i_{t,t+\Delta}\right] - r_{t,t+\Delta}\right) - \beta_{t,\Delta}\left(E_t\left[R^M_{t,t+\Delta}\right] - r_{t,t+\Delta}\right) \tag{33}$$

We use $\{\pi_t\}_{t\geq 0}$ to denote the true pricing kernel of the economy. Therefore the theoretical risk-premium of asset i is given by:

$$E_t\left[R^i_{t,t+\Delta}\right] - r_{t,t+\Delta} = -Cov_t\left(\pi_{t,t+\Delta}, R^i_{t,t+\Delta}\right) \tag{34}$$

Combining equation (32)-(34), we can express the true value of $\alpha_{t,\Delta}$ as a function of the covariances of the pricing kernel and the returns of assets. This is summarized in the following proposition:

Proposition 6 *The true value of $\alpha^i_{t,\Delta}$ in the conditional CAPM (31) is given by:*

$$\alpha^i_{t,\Delta} = -Cov_t\left(R^i_{t,t+\Delta}, \pi_{t,t+\Delta}\right) - \frac{E_t\left[R^M_{t,t+\Delta}\right] - r_{t,t+\Delta}}{Var_t\left(R^M_{t,t+\Delta}\right)} Cov_t\left(R^i_{t,t+\Delta}, R^M_{t,t+\Delta}\right) \tag{35}$$

Since we are dealing with continuous time models, it is more convenient to define:

$$\alpha^i_t = \lim_{\Delta \to 0} \frac{1}{\Delta} \alpha^i_{t,\Delta}$$

and the discuss the continuous time limit of the measured α in (35). In the economy we considered in this paper, the value of any firm is of the form:

$$V^i_t = G\left(Z^i, \theta_t\right) C_t$$

The value of the market portfolio is of the form:

$$V_t^M = s(\theta_t) C_t$$

for some function $s(\theta_t)$ given in appendix IV. As shown in appendix IV, the return to asset i is a Levy process of the form:

$$dR_t^i = R_t^i \left[\mu_t^i dt + \sigma(\theta_t) dB_t^i + \sigma B_t + \eta^i(\theta_t) dN(t)\right] \quad (36)$$

where

$$\eta^i(\theta_t) = \left[\frac{G(Z^i, \theta_L)}{G(Z^i, \theta_H)} - 1, \frac{G(Z^i, \theta_H)}{G(Z^i, \theta_L)} - 1\right]^T$$

The return to the market is of the form:

$$dR_t^M = R_t^M \left[\mu_t^M dt + \sigma B_t + \eta^M(\theta_t) dN(t)\right]$$

where

$$\eta^i(\theta_t) = \left[\frac{s(\theta_L)}{s(\theta_H)} - 1, \frac{s(\theta_H)}{s(\theta_L)} - 1\right]^T$$

Using the above notation, taking the continuous time limit of (35), and denote

$$\omega_t = \lim_{\Delta \to 0} \frac{E_t\left[R_{t,t+\Delta}^M\right] - r_{t,t+\Delta}}{Var_t\left(R_{t,t+\Delta}^M\right)}$$

that is, ω_t is the Sharpe ratio of the market divided by the volatility of market return. We have:

$$\alpha_t^i = (\gamma - \omega_t)\sigma^2 + \sum_{j=H,L} \lambda_j \eta_j^i(\theta)\left[\eta_{\pi,j}(\theta_t) - \omega_t \eta_{M,j}(\theta_t)\right] \quad (37)$$

Equation (37) is the key to understand the failure of CAPM in the model. CAPM in this economy fails because there are two independent sources of risks in the economy, short-run risks, modeled by the Brownian motion $\{B_t\}$, and the long-run risks, modeled by the compound Poisson process $\{N_t\}$. It is also clear from equation (36), although all assets in this economy have the same exposure to the short-run risks $\{B_t\}$, they have different exposure to the long-run risks. Asset's exposure to long-run risks depends on the shape of the G function, which is determined by which stage the firm is in, and the individual productivity, Z_t^i. To capture the differential exposure to long-run risks, we need a two factor model. There is no way a single factor model can correctly predict expected stock return in this economy.

Equation (37) also reveals what kind of asset in the economy will have a higher measured α. Measured α differs across assets because of their differential exposure to the long-run risks, represented by the second term in (37). $\eta^i(\theta)$ measures the sensitivity of the return of asset i to the long-run risks. $\eta_\pi(\theta)$ and $\eta^M(\theta)$ measures the sensitivity of the pricing kernel and the sensitivity of the return to market to the long-run risks, respectively. Equation (37) implies that provided $\eta_\pi(\theta) - \omega_t \eta_M(\theta) > 0$, $\forall\theta$, then assets with higher exposure to long-run risks will have a higher measured α in the conditional CAPM regression. The following proposition make the statement precise:

Proposition 7 *Suppose*

$$1 - \hat{\delta} > \omega_t \left(1 - \frac{s(\theta_L)}{s(\theta_H)}\right) \tag{38}$$

then

$$\frac{G\left(Z^i, \theta_H\right)}{G\left(Z^i, \theta_L\right)} > \frac{G\left(Z^j, \theta_H\right)}{G\left(Z^j, \theta_L\right)} \tag{39}$$

implies $\alpha_t^i > \alpha_t^j$.

Note $1 - \hat{\delta}$ is the jump size of the pricing kernel when θ_t jumps from θ_H to θ_L, and $1 - \frac{s(\theta_L)}{s(\theta_H)}$ is the jump size of the return to market when θ_t jumps from θ_H to θ_L. Condition (38) in fact the empirically plausible by a back-of-the-envelope calculation. Assume a market risk-premium of 6%, and a volatility of market return of 20%, then ω_t is roughly 1.5. In calibrated long-run risks models, the pricing kernel is typical much more sensitive to the long-run risks than the return to market. For example, using a first order approximation of both side of the inequality, condition (38) can be greatly simplified to:

$$\left(\gamma - \frac{1}{\psi}\right) > \omega_t \left(1 - \frac{1}{\psi}\right) \tag{40}$$

This approximation is in fact accurate if θ_t is assumed to be a diffusion process. If we use Bansal and Yaron (2004)'s choice of the parameters $\gamma = 10$ and $\psi = 1.5$, (40) is obviously true. For any reasonable calibrated long-run risk models, (38) and (40) will be true, which means an econometrician who use conditional CAPM regressions to measure expected returns of assets will obtain a higher α for assets whose exposure to long-run risks is higher, in the sense of equation (39).

IV Quantitative Implication of the Model

To be added.

V Conclusion

We put forward a general equilibrium model that links the cross-section variation of expected returns to firm's life cycle dynamics. In the model all assets have the same exposure to short-run consumption risks, but differ in their exposure to long-run consumption risks (Bansal and Yaron (2004)). An econometrician who uses *conditional* CAPM regression to predict asset returns will obtain higher α for assets with higher exposure to long-run risks. Growth options have *lower* exposure to long-run risks than asset in place because the cost of exercising the growth option has high exposure to long-run risk and therefore provides a hedge against the risk of assets place. Small firms have *higher* exposure to long-run because they have higher operating leverage.

Appendix I: Pricing Kernel of the Economy

We first characterize the value function of the representative agent. The HJB for recursive utility is Duffie and Epstein (1992b):

$$\bar{f}\left(C_t, V\left(\theta_t, C_t\right)\right) + \mathcal{L}\left[V\left(\theta_t, C_t\right)\right] = 0$$

where $\bar{f}$ is the normalized aggregator with $\mathcal{A} = 0$. By homogeneity, $V\left(\theta, C\right) = \frac{1}{1-\gamma} H\left(\theta\right) C^{1-\gamma}$, we have

$$\begin{aligned} \bar{f}\left(C, V\left(\theta, C\right)\right) &= \frac{\beta}{1-1/\psi} \frac{C^{1-1/\psi} - \left(\left(1-\gamma\right)V\right)^{\frac{1-1/\psi}{1-\gamma}}}{\left(\left(1-\gamma\right)V\right)^{\frac{1-1/\psi}{1-\gamma}-1}} \\ &= \frac{\beta}{1-1/\psi} C^{1-\gamma}\left[H\left(\theta\right)^{-\frac{1-1/\psi}{1-\gamma}} - 1\right] H\left(\theta\right) \end{aligned} \tag{41}$$

Proof. Using generalized Ito's formula,

$$\begin{aligned} \mathcal{L}\left[V\left(\theta, C\right)\right] &= H\left(\theta\right) C^{1-\gamma}\theta - \frac{1}{2}\gamma C^{1-\gamma} H\left(\theta\right)\sigma_C^2\left(\theta\right) + \\ & \frac{1}{1-\gamma} C^{1-\gamma} \sum_{j=H,L} \lambda_j I_j\left(\theta\right)\left[H\left(\theta_j^C\right) - H\left(\theta_j\right)\right] \end{aligned} \tag{42}$$

Combine (41) and (42), we have:

$$\frac{\beta}{1-1/\psi}\left[H(\theta)^{-\frac{1-1/\psi}{1-\gamma}}-1\right]+\theta-\frac{1}{2}\gamma\sigma_C^2(\theta)+\frac{1}{1-\gamma}\sum_{j=H,L}\lambda_j I_j(\theta)\left[\frac{H\left(\theta_j^C\right)}{H(\theta_j)}-1\right]=0 \quad (43)$$

Denote

$$\phi=\frac{H(\theta_H)}{H(\theta_L)}$$

We have

$$\begin{aligned}\frac{\beta}{1-1/\psi}H(\theta_H)^{-\frac{1-1/\psi}{1-\gamma}} &= -\left\{-\frac{\beta}{1-1/\psi}+\theta_H-\frac{1}{2}\gamma\sigma_C^2(\theta_H)+\frac{1}{1-\gamma}\lambda_H\left(\phi^{-1}-1\right)\right\} \quad (44)\\ \frac{\beta}{1-1/\psi}H(\theta_L)^{-\frac{1-1/\psi}{1-\gamma}} &= -\left\{-\frac{\beta}{1-1/\psi}+\theta_L-\frac{1}{2}\gamma\sigma_C^2(\theta_L)+\frac{1}{1-\gamma}\lambda_L(\phi-1)\right\}\end{aligned}$$

Therefore, ϕ is the solution to

$$\phi^{-\frac{1-1/\psi}{1-\gamma}}=\frac{-\frac{\beta}{1-1/\psi}+\theta_H-\frac{1}{2}\gamma\sigma_C^2(\theta_H)+\frac{1}{1-\gamma}\lambda_H\left(\phi^{-1}-1\right)}{-\frac{\beta}{1-1/\psi}+\theta_L-\frac{1}{2}\gamma\sigma_C^2(\theta_L)+\frac{1}{1-\gamma}\lambda_L(\phi-1)} \quad (45)$$

or

$$\phi^{-\frac{1-1/\psi}{1-\gamma}}=\frac{\beta-\left(1-\frac{1}{\psi}\right)\theta_H+\frac{1}{2}\gamma\left(1-\frac{1}{\psi}\right)\sigma_C^2(\theta_H)-\frac{1-1/\psi}{1-\gamma}\lambda_H\left(\phi^{-1}-1\right)}{\beta-\left(1-\frac{1}{\psi}\right)\theta_L+\frac{1}{2}\gamma\left(1-\frac{1}{\psi}\right)\sigma_C^2(\theta_L)-\frac{1-1/\psi}{1-\gamma}\lambda_L(\phi-1)} \quad (46)$$

Using (44), this in turn implies

$$\begin{aligned}H(\theta_H) &= \left\{\frac{1}{\beta}\left[\beta+\frac{1}{2}\gamma\left(1-\frac{1}{\psi}\right)\sigma_C^2-\left(1-\frac{1}{\psi}\right)\theta_H-\frac{1-\frac{1}{\psi}}{1-\gamma}\lambda_H\left(\phi^{-1}-1\right)\right]\right\}^{-\frac{1-\gamma}{1-1/\psi}} \quad (47)\\ H(\theta_L) &= \left\{\frac{1}{\beta}\left[\beta+\frac{1}{2}\gamma\left(1-\frac{1}{\psi}\right)\sigma_C^2-\left(1-\frac{1}{\psi}\right)\theta_L-\frac{1-\frac{1}{\psi}}{1-\gamma}\lambda_L(\phi-1)\right]\right\}^{-\frac{1-\gamma}{1-1/\psi}}\end{aligned}$$

One can show that under assumption 1 (17), (45) has a unique solution on the open interval $(0,1)$. Therefore (46) and (47) fully characterize the value function of the representative agent. The state price process of this economy is given by Duffie and Epstein (1992b):

$$\frac{d\pi_t}{\pi_t}=\frac{d\bar{f}_C(C_t,V_t)}{\bar{f}_C(C_t,V_t)}+\bar{f}_V(C_t,V_t)\,dt$$

Using Ito's formula, one can show $\{\pi_t\}$ is a Levy process of the form:

$$d\pi_t = \pi_t \left[-\bar{r}(\theta_t)\, dt - \gamma\sigma_C(\theta)\, dB_t - \eta_\pi\left(\theta_t^-\right)^T dN_t \right]$$

where

$$\bar{r}(\theta) = \beta + \frac{1}{\psi}\theta - \frac{1}{2}\gamma\left(1+\frac{1}{\psi}\right)\sigma^2(\theta) + \frac{\frac{1}{\psi}-\gamma}{1-\gamma}\left[\lambda_H I_H(\theta)\left(\phi^{-1}-1\right) + \lambda_L I_L(\theta)(\phi-1)\right] \quad (48)$$

and

$$\eta_\pi(\theta) = \left[\left(1-\hat{\phi}^{-1}\right) I_{\{H\}}(\theta), \left(1-\hat{\phi}\right) I_{\{L\}}(\theta)\right] \quad (49)$$

where

$$\hat{\phi} = \phi^{\frac{1/\psi-\gamma}{1-\gamma}}, \text{ and } \phi = \frac{H(\theta_H)}{H(\theta_L)}$$

■

Proposition 8 *The risk-free interest rate of this economy is given by:*

$$r(\theta) = \bar{r}(\theta) + \lambda_1 I_{\{H\}}(\theta)\left[1-\phi^{-\frac{1/\psi-\gamma}{1-\gamma}} + \frac{1/\psi-\gamma}{1-\gamma}\left(\phi^{-1}-1\right)\right] + \lambda_2 I_{\{L\}}(\theta)\left[1-\phi^{\frac{1/\psi-\gamma}{1-\gamma}} + \frac{1/\psi-\gamma}{1-\gamma}(\phi-1)\right]$$

Additionally, if $\gamma > 1 > \frac{1}{\psi}$, then $\phi < 1$ and $\hat{\phi} < 1$.

Appendix II: Firms' Optimal Stopping Problems

Lemma 1 *General Solutions to the Optimal Stopping Problem: Consider the following general formulation of the optimal stopping problem considered in this paper:*

$$\begin{aligned}
V\left(\theta_t, Z_t^i, C_t\right) &= \max_\tau E\left[\int_t^\tau e^{-\kappa(s-t)}\frac{\pi_s}{\pi_t} D\left(Z_s^i, \theta_s\right) C_s ds + e^{-\kappa(\tau-t)}\frac{\pi_\tau}{\pi_t} B\left(Z_\tau^i, \theta_\tau\right) C_\tau \middle| \theta_t, Z_t^i, C_t\right] \quad (50)\\
dZ_t^i &= Z_t^i\left[\mu_z(\theta_t)\, dt + \sigma_z(\theta_t)\, dB_t^i\right]\\
dC_t &= C_t\left[\theta_t dt + \sigma_C(\theta_t)\, dB_{Ct}\right]\\
d\pi_t &= \pi_t\left[-\bar{r}(\theta_t)\, dt - \gamma\sigma_C(\theta_t)\, dB_{Ct} - \eta_\pi\left(\theta_t^-\right)^T dN_t\right]\\
d\theta_t &= \Delta \times \eta\left(\theta_t^-\right)^T dN_t
\end{aligned}$$

Then

$$V(\theta, Z, C) = G(Z, \theta)\, C$$

where $G(Z,\theta)$ is the solution to the following couple ODE:

$$D(Z,\theta_H) - G(Z,\theta_H)\beta_H + G'(Z,\theta_H) Z\mu_H + \frac{1}{2}G''(Z,\theta_H) Z^2\sigma_H^2 \\ +\lambda_H \left[\hat{\phi}^{-1} G(Z,\theta_L) - G(Z,\theta_H)\right] = 0 \tag{51}$$

and

$$D(Z,\theta_L) - G(Z,\theta_L)\beta_L + G'(Z,\theta_L) Z\mu_L + \frac{1}{2}G''(Z,\theta_L) Z^2\sigma_L^2 \\ +\lambda_L \left[\hat{\phi} G(Z,\theta_H) - G(Z,\theta_L)\right] = 0 \tag{52}$$

We denote

$$\begin{aligned} \beta_H &= \kappa + \bar{r}(\theta_H) - \theta_H + \gamma\sigma_C^2(\theta_H) \\ \beta_L &= \kappa + \bar{r}(\theta_L) - \theta_L + \gamma\sigma_C^2(\theta_L) \end{aligned}$$

and

$$\mu_j = \mu_z(\theta_j), \quad \sigma_j = \sigma_z(\theta_j), \ j = H, L$$

Proof. Multiply both sides of the equation 1 by $e^{-t}\pi_t$, suppress the superscript i of Z_t^i for notational convenience, we have:

$$e^{-\kappa t}\pi_t V(Z_t,\theta_t,C_t) + \int_0^t e^{-\kappa s}\pi_s D(Z_t,\theta_s) C_s ds$$

is a martingale. Therefore,

$$e^{-\kappa t}\pi_t D(Z_t,\theta_t) C_t + \mathcal{L}\left\{e^{-\kappa t}\pi_t V(Z_t,\theta_t,C_t)\right\} = 0 \tag{53}$$

where $\mathcal{L}$ is the generator associated with the vector Markov process $\{Z_t, C_t, \pi_t, \theta_t\}$. Using homogeneity, $V(\theta_t, Z_t, C_t) = G(Z_t,\theta_t) C_t$, we have

$$\mathcal{L}\left\{e^{-\kappa t}\pi_t V(Z_t,\theta_t,C_t)\right\} = -\kappa e^{-\kappa t}\pi_t C_t G(Z_t,\theta_t) + e^{-\kappa t}\mathcal{L}\left\{\pi_t C_t G(Z_t,\theta_t)\right\} \tag{54}$$

Using generalized Ito's formula (**?**)), we have:

$$
\begin{aligned}
\frac{1}{\pi_t C_t}\mathcal{L}\left\{\pi_t C_t G\left(Z_t,\theta_t\right)\right\} &= G\left(Z_t,\theta_t\right)\left[-\bar{r}\left(\theta_t\right)+\theta_t-\gamma\sigma_C^2\left(\theta_t\right)\right] \\
&\quad + G'\left(Z_t,\theta_t\right)Z_t\mu_z\left(\theta_t\right)+\frac{1}{2}G''\left(Z_t,\theta_t\right)Z_t^2\sigma_z^2\left(\theta_t\right) \\
&\quad + \sum_{j=H,L}\lambda_j\left[\left[1-\eta_{\pi,j}\left(\theta_t\right)\right]G\left(Z_t,\theta_t+\Delta\eta_j\left(\theta_t\right)\right)-G\left(Z_t,\theta_t\right)\right]
\end{aligned}
$$

Therefore equation (53) is written as:

$$
\begin{gathered}
D\left(Z_t,\theta_t\right)+G\left(Z_t,\theta_t\right)\left[-\kappa-\bar{r}\left(\theta_t\right)+\theta_t-\gamma\sigma_C^2\left(\theta_t\right)\right] \\
+G'\left(Z_t,\theta_t\right)Z_t^i\mu_z\left(\theta_t\right)+\frac{1}{2}G''\left(Z_t,\theta_t\right)Z_t^2\sigma^2\left(\theta_t\right) \\
+I_{\{H\}}\left(\theta_t\right)\lambda_H\left[\hat{\phi}^{-1}G\left(Z_t,\theta_L\right)-G\left(Z_t,\theta_H\right)\right] \\
+I_{\{L\}}\left(\theta_t\right)\lambda_L\left[\hat{\phi}G\left(Z_t,\theta_H\right)-G\left(Z_t,\theta_L\right)\right]=0
\end{gathered}
$$

This is the same as (51) and (52). ■

Lemma 2 *The general solution to the homogeneous part of (51) and (52) is of the following form:*

$$
G\left(Z,\theta_H\right)=\sum_{i=1}^{4}K_i e_{i,H}Z^{\alpha_i},\quad G\left(Z,\theta_L\right)=\sum_{i=1}^{4}K_i e_{i,L}Z^{\alpha_i}
$$

where $\{K_i\}_{i=1,2,3,4}$ *are constants.* $\{\alpha_i\}_{i=1,2,3,4}$ *are the eigenvalues of the following quadratic eigenvalue problem:*

$$
\frac{1}{2}Me\alpha^2+Ne\alpha+Le=0
$$

where

$$
\begin{aligned}
M &= \begin{bmatrix}\sigma_H^2 & 0\\ 0 & \sigma_L^2\end{bmatrix},\ N=\begin{bmatrix}\mu_H-\frac{1}{2}\sigma_H^2 & 0\\ 0 & \mu_L-\frac{1}{2}\sigma_L^2\end{bmatrix} \\
L &= \begin{bmatrix}-\left(\lambda_H+\beta_H\right) & \lambda_H\hat{\phi}^{-1}\\ \lambda_L\hat{\phi} & -\left(\lambda_L+\beta_L\right)\end{bmatrix}
\end{aligned}
$$

and

$$
e_i=\left[e_{i,H},e_{i,L}\right]
$$

are the normalized eigenvectors associated with $\{\alpha_i\}_{i=1,2,3,4}$.

Furthermore, without loss of generality, $\{\alpha_i\}_{i=1,2,3,4}$ *can be ordered so that* $\alpha_1 < \alpha_2 < 0 < 1 < \alpha_3 < \alpha_4$*.*

Proof. To be added.

Lemma 3 *Consider the following two special cases.*

1) Suppose $D(Z,\theta) = Z$*,* $\tau = \infty$*, then*

$$G(Z,\theta_H) = a_H Z,\ G(Z,\theta_L) = a_L Z$$

where

$$\begin{aligned} a_H &= \frac{\beta_L - \mu_L + \lambda_L + \lambda_H \hat{\phi}^{-1}}{(\beta_H - \mu_H + \lambda_H)(\beta_L - \mu_L + \lambda_L) - \lambda_H \lambda_L} \\ a_L &= \frac{\beta_H - \mu_H + \lambda_H + \lambda_L \hat{\phi}}{(\beta_H - \mu_H + \lambda_H)(\beta_L - \mu_L + \lambda_L) - \lambda_H \lambda_L} \end{aligned}$$

2) Suppose $D(Z,\theta) = Z$*,* $\tau = \infty$*, then*

$$G(Z,\theta_H) = b_H,\ G(Z,\theta_L) = b_L$$

where

$$\begin{aligned} b_H &= \frac{\beta_L + \lambda_L + \lambda_H \hat{\phi}^{-1}}{(\beta_H + \lambda_H)(\beta_L + \lambda_L) - \lambda_H \lambda_L} \\ b_L &= \frac{\beta_H + \lambda_H + \lambda_L \hat{\phi}}{(\beta_H + \lambda_H)(\beta_L + \lambda_L) - \lambda_H \lambda_L} \end{aligned}$$

■

Appendix III: The Cross-Section Distribution of Firms' Characteristics

For notational convenience, we supress the dependence of the stationary density on the stage of the firm, and the state variable θ, and use $\phi(z)$ to denote the density of the stationary distribution of $z = \log Z$. For stage I firms, $\phi(z)$ must satisfy the following forward equation **?**)

$$g(z) - \kappa\phi(z) - \left[\mu(\theta) - \frac{1}{2}\sigma^2(\theta)\right]\phi'(z) + \frac{1}{2}\sigma^2(\theta)\phi''(z) = 0 \tag{55}$$

where $g(z)$ is the entry density and is given by:

$$g(z) = \frac{m_I(\theta)}{\bar{Z}} e^z,\ z \in \left[\ln Z_I^*(\theta), \ln \hat{Z}_I(\theta)\right],\ 0\ otherwise$$

The general solution to the above is given by;

$$\phi(z) = \frac{m_I(\theta)}{\left(\kappa + \mu_z(\theta) - \sigma(\theta)^2\right) \bar{Z}} \left[e^z + e^z + \xi_1(\theta) e^{\alpha_1(\theta) z} + \zeta_2(\theta) e^{\alpha_2(\theta) z}\right]$$

where

$$\alpha_1(\theta) = \left(\frac{\mu(\theta)}{\sigma^2(\theta)} - \frac{1}{2}\right) + \sqrt{\left(\frac{\mu(\theta)}{\sigma^2(\theta)} - \frac{1}{2}\right)^2 + \frac{2\kappa}{\sigma^2(\theta)}} > 1$$

$$\alpha_2(\theta) = \left(\frac{\mu(\theta)}{\sigma^2(\theta)} - \frac{1}{2}\right) - \sqrt{\left(\frac{\mu(\theta)}{\sigma^2(\theta)} - \frac{1}{2}\right)^2 + \frac{2\kappa}{\sigma^2(\theta)}} < 0$$

are the two solutions to the characteristic quadratic of equation (55). The constants $\xi_1(\theta)$ and $\xi_2(\theta)$ are determined by the boundary conditions of $\phi(z)$ at the absorbing barriers $\ln Z_I^*(\theta)$ and $\ln \hat{Z}_I(\theta)$:

$$\phi(\ln Z_I^*(\theta)) = \phi\left(\ln \hat{Z}_I(\theta)\right) = 0$$

Hence,

$$\xi_1(\theta) = \frac{\hat{Z}_I(\theta) \left[Z_I^*(\theta)\right]^{\alpha_2(\theta)} - Z_I^*(\theta) \left[\hat{Z}_I(\theta)\right]^{\alpha_2(\theta)}}{\left[Z_I^*(\theta)\right]^{\alpha_1(\theta)} \left[\hat{Z}_I(\theta)\right]^{\alpha_2(\theta)} - \left[Z_I^*(\theta)\right]^{\alpha_2(\theta)} \left[\hat{Z}_I(\theta)\right]^{\alpha_1(\theta)}}$$

$$\xi_2(\theta) = \frac{\hat{Z}_I(\theta)^{\alpha_1(\theta)} Z_I^*(\theta) - \left[Z_I^*(\theta)\right]^{\alpha_1(\theta)} \hat{Z}_I(\theta)}{\left[Z_I^*(\theta)\right]^{\alpha_1(\theta)} \left[\hat{Z}_I(\theta)\right]^{\alpha_2(\theta)} - \left[Z_I^*(\theta)\right]^{\alpha_2(\theta)} \left[\hat{Z}_I(\theta)\right]^{\alpha_1(\theta)}}$$

To solve for the stationary distribution of stage II firms, we first consider the stationary distribution associated with the uniform entry density over the interval $\left[\hat{Z}_I(\theta),\ \bar{Z}\right]$, denoted $\phi^U(z)$. Again, $\phi(z)$ has to satisfy equation (55), where

$$g(z) = \frac{m_I(\theta)}{\bar{Z}} e^z,\ z \in \left[\ln \hat{Z}_I(\theta),\ \ln \bar{Z}\right],\ 0\ otherwise$$

Therefore $\phi(z)$ is given by:

In region $A = \left[\ln \bar{Z},\ \infty\right)$,

$$\phi^U(z) = m_I(\theta) D(\theta) c_{A,2}(\theta) e^{\alpha_2(\theta) z}$$

where

$$D(\theta) = \frac{1}{\left(\bar{Z} - \hat{Z}_I(\theta)\right)\left[\kappa + \mu_z(\theta) - \sigma^2(\theta)\right]}$$

and

$$\begin{aligned} c_{A,2}(\theta) &= \frac{1-\alpha_1(\theta)}{\alpha_1(\theta)-\alpha_2(\theta)}\left\{\hat{Z}_I(\theta)^{1-\alpha_2(\theta)} - \bar{Z}^{1-\alpha_2(\theta)}\right\} \\ &\quad - \frac{1-\alpha_1(\theta)}{\alpha_1(\theta)-\alpha_2(\theta)} E(\theta)\left\{\hat{Z}_I(\theta)^{1-\alpha_1(\theta)} - \bar{Z}^{1-\alpha_1(\theta)}\right\} \end{aligned}$$

and

$$E(\theta) = \left[Z_{II}^*(\theta)\right]^{\alpha_1(\theta)-\alpha_2(\theta)}$$

In region $B = \left[\ln Z_{II}^*(\theta),\ \ln \hat{Z}_I(\theta)\right)$,

$$\phi^U(z) = m_I(\theta) D(\theta)\left[c_{B,1}(\theta) e^{\alpha_1(\theta)z} + c_{B,2}(\theta) e^{\alpha_2(\theta)z}\right]$$

where

$$c_{B,1}(\theta) = \frac{1-\alpha_2(\theta)}{\alpha_1(\theta)-\alpha_2(\theta)}\left\{\hat{Z}_I(\theta)^{1-\alpha_1(\theta)} - \bar{Z}^{1-\alpha_1(\theta)}\right\}$$

$$c_{B,1}(\theta) = -E(\theta) c_{B,1}(\theta)$$

In region $C = \left[\ln \hat{Z}_{II}(\theta),\ \ln \bar{Z}(\theta)\right)$,

$$\phi^U(z) = m_I(\theta) D(\theta)\left[e^z + c_{C,1}(\theta) e^{\alpha_1(\theta)z} + c_{C,2}(\theta) e^{\alpha_2(\theta)z}\right]$$

where

$$c_{C,1}(\theta) = \frac{\alpha_2(\theta)-1}{\alpha_1(\theta)-\alpha_2(\theta)}\bar{Z}^{1-\alpha_1(\theta)}$$

$$c_{C,2}(\theta) = \frac{1-\alpha_1(\theta)}{\alpha_1(\theta)-\alpha_2(\theta)}\hat{Z}_I(\theta)^{1-\alpha_2(\theta)} - \frac{1-\alpha_2(\theta)}{\alpha_1(\theta)-\alpha_2(\theta)} E(\theta)\left\{\hat{Z}_I(\theta)^{1-\alpha_1(\theta)} - \bar{Z}^{1-\alpha_1(\theta)}\right\}$$

Next, we solve for the stationary distribution associated with entry from stage I firms, denoted $\phi^P(z)$. By a similiar arguement,

In region $A \cup C = \left[\ln \hat{Z}_I(\theta),\ \infty\right)$,

$$\phi^P = m_{EXIT}\left[\Phi_I(\theta), Z\right] \times A(\theta) e^{\alpha_2(\theta)z}$$

where

$$A(\theta) = \frac{1}{\sqrt{\left[\mu_z(\theta) - \frac{1}{2}\sigma^2(\theta)\right]^2 + 2\kappa\sigma^2(\theta)}} \left[\left[\hat{Z}_I(\theta)\right]^{-\alpha_2(\theta)} - E(\theta)\left[\hat{Z}_I(\theta)\right]^{-\alpha_1(\theta)}\right]$$

In region $B = \left[\ln Z^*_{II}(\theta),\ \ln \hat{Z}_I(\theta)\right)$,

$$\phi^P(z) = m_{EXIT}\left[\Phi_I(\theta), Z\right] \times \left[B_1(\theta) e^{\alpha_1(\theta)z} + B_2(\theta) e^{\alpha_2(\theta)z}\right]$$

where

$$B_1(\theta) = \frac{1}{\sqrt{\left[\mu_z(\theta) - \frac{1}{2}\sigma^2(\theta)\right]^2 + 2\kappa\sigma^2(\theta)}} \left[\hat{Z}_I(\theta)\right]^{-\alpha_1(\theta)}$$

$$B_2(\theta) = -B_1(\theta) E(\theta)$$

Appendix IV: Analysis of CAPM

To be added.

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