

Resuscitating Time-to-Build

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Abstract

A novel specification of the time-to-build (TTB) assumption is presented where firms invest in many projects that have complementarities, and the duration of the investment projects is uncertain. The model yields to a gradual (hump-shaped) response of investment to shocks, and it is shown to be equivalent, up to first-order linearization, to investment adjustment cost models where the cost of adjustment directly depends on the change in investment levels. The paper discusses how the new TTB specification is consistent with empirical features of investment decisions both at the aggregate and more disaggregated levels.

1 Introduction

Due to the time required to build (TTB) and plan (TTP) for investment, capital accumulation is often a lengthy process: should this feature be explicitly modeled when characterizing firms' investment decisions? This paper presents a novel specification of the TTB assumption and, in contrast to recent literature, it argues that the length of the investment process should be explicitly modeled as it represents one of its most critical features. The ideas of TTB and TTP are not new in economics as they can be traced back in time at least to the work of Kalecki [1935]. Although acknowledged by most economists, TTB has not been usually considered in either empirical and theoretical investment models (see Nickell [1978]). In their seminal contribution to quantitative macroeconomic modeling, instead, Kydland and Prescott [1982] (KP hereafter) argue that TTB is key to understanding post-World War II business cycle fluctuations in the United States. In the specification of TTB considered by KP, and almost universally by subsequent literature, firms invest in one type of good that requires, for its construction, a fixed amount of resources in each period up to the date of maturity.¹ Further, an investment project increases the stock of capital only at the end of the process when it matures. More recent studies have, however, downplayed the role of TTB. Rouwenhorst [1991], shows that KP's TTB formulation has little effect on the response of a real business cycle (RBC) model to productivity shocks, but for generating unrealistic cyclicalities in the response of the main macroeconomic aggregates. Similarly, Cogley and Nason [1995] show that KP's TTB formulation has little or no impact on the persistence of output growth. Following these works, relatively few recent macroeconomic studies

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¹As discussed below, the work of Edge [Forthcoming] and Casares [2006] are important exceptions to these modeling assumptions.

explicitly consider the length of the investment process. The common approach, instead, is to assume a one period delay in the capital accumulation process irrespective of whether the temporal unit of analysis is a quarter or a year.² Further, few empirical microeconomic models of investment have explicitly considered TTB.

The novel formulation of TTB presented in this paper departs from KP's in two ways: i) firms invest in many types of investment goods that have complementarities and ii), the duration of each investment project is uncertain. Because, as in KP, the scale of the ongoing projects is fixed, a firm's optimal investment decision is predetermined in part by the ones made in the past. Further, due to the complementarity between investment goods, the firm optimally decides not to fully counterbalance these earlier decisions when choosing the scale of the new projects that are about to start. The resulting optimal investment decision is inertial as previous decisions directly influence the current one.

The optimal investment decision under the TTB specification is first analyzed in partial equilibrium. The representative firm, calibrated on US manufacturing data, is subject to productivity and interest rate shocks. The dynamic responses of the model show that investment reacts only gradually after the realization of the shocks.

Two assumptions are crucial for the analytical tractability of the model. First, the duration of investment projects follows a Poisson process, so that the probability that a project matures is independent of when it started. Second, different investment goods are imperfect substitutes, but the capital stock of the firm is a homogeneous good that depreciates at a constant rate. Although these assumptions greatly simplify the analysis, neither is key to generate a hump-shaped response of investment to shocks. The key elements are, instead, i), a source of imperfect substitutability among the investment types and ii), the ex-post heterogeneity in the duration of the investment projects not necessarily due, though, to ex-ante uncertainty in project's duration. The paper discusses these points by comparing the novel TTB model to similar deterministic specifications that borrow from the ones proposed by Edge [Forthcoming] and Casares [2006].

The TTB specification is then compared to *capital* (e.g. Hayashi [1982]) and *investment* adjustment cost models (e.g. Christiano, Eichenbaum, and Evans [2005]). With capital adjustment costs, the firm faces costs that depend on the change in the capital stock, while with investment adjustment costs they directly depend on the change in investment levels. The paper shows that the TTB formulation is equivalent, up to first order linearization, to *investment* adjustment cost models. The model presented, thus, directly links investment adjustment cost models to TTB. Contrarily, the response of the TTB model differs from that of the capital adjustment cost model, where investment swiftly responds to the realization of the shocks. A vast empirical literature has tested the investment model with (convex) capital adjustment costs (see e.g. Chirinko [1993]), and the model is typically rejected in the data. The level of Tobin's Q is not a sufficient statistic for the investment decision, as predicted by the model, and the estimated adjustment costs appear to be unreasonably large. Further, investment appears to be more sluggish than what predicted by the model: past realizations of Tobin's Q enter significantly in the regression models, and the error terms are (highly) serially correlated. Finally, the current level of cash flows appears to be an important determinant of the investment decision, an observation often interpreted as indicating the existence of financial frictions (e.g. Hubbard [1998]). In this paper, I generate artificial data using the TTB model calibrated on US sectoral data, and then estimate the empirical model with convex adjustment costs. The three empirical findings discussed above are shown to be consistent with a misspecification of the empirical model that abstracts from the TTB technology underlying the data-generating process.

²A one period delay means that the current investment flow adds onto the capital stock only in the following period.

Finally the TTB model is embedded in an otherwise canonical RBC model. As for the partial equilibrium model, investment dynamic response to a technology shock is hump shaped. The model, thus, describes well the empirical evidence of aggregate investment on US data (Christiano, Eichenbaum, and Vigfusson [2004]). Further, due to the equivalence result with investment adjustment cost models and the findings of (Basu and Kimball [2005] and (Christiano, Eichenbaum, and Evans [2005], the TTB model also helps explain the dynamic response of aggregate investment to fiscal and monetary shocks. Finally, the paper shows that neither investment nor other aggregate variables display the cyclicalities highlighted by Rouwenhorst [1991] for KP's TTB formulation.

The remaining of the paper is organized as follows. The TTB model with uncertain duration is presented in the next Section, and then solved in Section 3. Section 4 compares the model with two alternative TTB formulations with deterministic duration. Section 5 relates the model to investment and capital adjustment cost models, while Section 6 discusses the implications of the model for empirical tests of capital adjustment cost models. Finally, Section 7 embeds the TTB model in an otherwise canonical RBC model.

2 The Model

Consider a firm that produces the good Y_t , using its capital stock, K_{t-1} , and a vector of variable factor inputs $\mathbf{x}_t$ in the production function $f(A_t, K_{t-1}, \mathbf{x}_t)$, where A_t is the level of productivity. Let $p_{x,t}$ be the price vector associated with the variable factor inputs. After maximizing out the variable factors, the date t flow of revenues net of variable factor cost are given by $\pi_t(K_{t-1}) \equiv \max_{\mathbf{x}_t} f(A_t, K_{t-1}, \mathbf{x}_t) - p'_{x,t} \mathbf{x}_t$. The function $\pi_t(\cdot)$ is assumed to be increasing and weakly concave.

To increase its capital stock, the firm invests in a fixed but large number of perfectly symmetric investment goods indexed by their type $j \in [0, 1]$. It takes time to build and plan for the construction of each investment good. The firm chooses the desired quantity of type- j investment *good* when it starts a type- j investment *project*. Each firm can only run one project per-investment good at a time, and the scale of the project cannot be modified once initiated. If a project matures at t a new scale may be chosen at $t + 1$. Let $\iota_t(j)$ denote the date t scale of the type- j project, $M_t \subseteq [0, 1]$ be the set of investment projects maturing at t , and NM_t its complement in $[0, 1]$. The scale of a project that has not matured is fixed, i.e., $\iota_t(j) = \iota_{t-1}(j)$ for $j \in NM_{t-1}$. This is a key element of the model as it implies that the investment decision at all dates t is in part predetermined due to the projects ($j \in NM_{t-1}$) initiated in the past and that have not yet matured. It is also important to note that my notation only indicates the scale of the investment project, $\iota_t(j)$, but omits to mark the date when the scale was actually chosen. Thus, it should be kept in mind that the scale of all non-matured projects was chosen in the past, and therefore it cannot include any information on the current realization of the shocks.

The duration of the investment projects is uncertain: the maturity of each project follows a Poisson-process with arrival rate θ . A project started at date t has a probability θ of being completed at the same date t . Uncompleted projects mature with constant probability θ at each of the subsequent dates, so that a firm expects its projects to mature at date $t + (1/\theta - 1)$.

An investment project increases the capital stock only when it matures. Accordingly let the variable

$$\iota_t^m(j) = \begin{cases} \iota_t(j) & \text{if } j \in M_t, \\ 0 & \text{otherwise,} \end{cases} \quad (2.1)$$

denote the level of type j investment that increases the capital stock at date t . The variable $\iota_t^m(j)$ is equal to

the scale of the project, $\iota_t(j)$, if it matures at date t ($j \in M_t$), and it is equal to zero otherwise. Investment goods are characterized by complementarities – i.e., the return to investing in each good is increasing with the availability of the others – and each $\iota_t^m(j)$ enters symmetrically in the date t investment *basket*

$$I_t \equiv \left(\int_0^1 \iota_t^m(j)^{1-1/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}, \quad (2.2)$$

where $\varepsilon > 1$, so that no single investment good is essential in the capital accumulation process.

The level of the investment basket I_t increases the firm's capital stock, which depreciates at the constant rate δ

$$K_t = (1 - \delta)K_{t-1} + I_t. \quad (2.3)$$

The installation of capital is costly: the firm pays adjustment costs $C(I_t, K_{t-1}) \equiv c(I_t/K_{t-1}) K_{t-1}$, where $c(\cdot)$ is increasing and convex and such that $c'(\delta) = c(\delta) = 0$ and $c''(\delta) = \psi$. Capital installation costs play no role in the TTB technology, and are only included in the partial equilibrium investment model.

Now consider the level of investment expenditure. An investment project started at t requires $\theta_{\iota_t}(j)$ units of Y_s at all dates $s \geq t$ up to maturity. The date t investment expenditure is equal to the sum of all projects' expenditure

$$E_t \equiv \theta \int_0^1 \iota_t(j) dj. \quad (2.4)$$

The firm uses on average one unit of Y for each unit of investment. Indeed, the projects last for an average of $1/\theta$ periods and the firm uses $\theta_{\iota_t}(j)$ units of Y_t per period. A time-to-plan formulation of the model, in which investment expenditure only occurs when the project matures, would yield the same expenditure function as in (2.4). Indeed, because the maturity of the project follows a Poisson-process, it would then follow that $E_t = \int_{j \in M_t} \iota_t(j) dj = \theta \int_0^1 \iota_t(j) dj$.

The firm's manager acts in the interest of the shareholders and maximizes at each date t the value of future dividends discounted by the gross rate R_{s+1} between all date pairs s and $s+1$ for all $s \geq t$. Thus the firm solves

$$\max \mathbb{E}_t \left\{ \sum_{r=0}^{\infty} \left(\prod_{s=1}^r R_{t+s}^{-1} \right) D_{t+r} \right\}, \quad (2.5)$$

with respect to K_t and $\{\iota(j)\}_{j \in M_{t+r}}$, subject to the law of motion of the capital stock (2.3) and the investment technology described above. The dividend at date $t+r$ is

$$D_{t+r} = \pi_{t+r}(K_{t-1+r}) - E_{t+r} - C(I_{t+r}, K_{t+r-1}). \quad (2.6)$$

At the beginning of each period t , the firm observes the level of productivity, A_t , the vector price of the variable factor inputs, the interest rate R_t and the set of projects that have matured in the previous period $j \in M_{t-1}$. Given the stock of K_{t-1} , the firm then decides the level of production, Y_t , and the corresponding vector of variable inputs $\mathbf{x}_t$. Then the firm invests. This decision, as previously discussed, is only in part under its control. Investment in the projects that are still under way, $j \in NM_{t-1}$, cannot be modified, while the firm decides the scale for the investment types whose projects have matured in the previous period, $j \in M_{t-1}$. The set, M_t , of date t projects that mature is then realized, and, thanks to these projects, the capital stock, K_t is increased. The scale of these projects, $j \in M_t$, are under control of the firm in the following period, while the scale of all remaining projects, $j \in NM_t$, remains fixed.

3 Solution of the Model

When deciding upon the scale of a new investment project, the firm needs to consider its effects on the investment baskets and expenditures at all future dates, due to the uncertain duration of the projects. The optimal investment decision can be characterized using two different solution strategies.

A direct approach is to choose $\iota_t(j)$ for $j \in M_{t-1}$ by maximizing (2.5). The first order condition that characterizes the optimal decision involves values of E_{t+s} and I_{t+s} for all $s \geq 0$. Following the literature on staggered pricing decisions (e.g. Yun [1996]) the first order condition can be linearized, and after some algebra, expressed as a second order difference equation.

A simpler solution method explicitly accounts for the large number of investment projects. It is important to note that despite the uncertain duration of each investment project, no uncertainty in the overall investment decision exists. This follows from the constant fraction of projects that matures at each date, and from the fact that all projects have equal probability of maturing. Further, it is possible to show that the levels of E_t and I_t depend on earlier investment decisions only through E_{t-1} and I_{t-1} . As it is shown below, this greatly simplifies the solution of the investment problem by dividing it into an intratemporal and an intertemporal decision. After characterizing the optimal investment decision, the remaining of the Section makes additional assumptions on the functional forms of the model. Finally, the model is calibrated on US manufacturing data and then simulated.

3.1 Optimal Investment Decision

The optimal investment decision is solved by first characterizing the date t intratemporal expenditure allocation across the investment goods $j \in M_{t-1}$. The intertemporal decision is then expressed and solved solely in terms of E_t , I_t and K_t , without directly considering the scales $\iota_t(j)$'s.

Consider the intratemporal investment decision. Because the maturity of the projects follows a Poisson process, all projects have equal probability of maturing independently of their starting date. Thereby, the date t averages of the quantities $\iota_t(j)$ and $\iota_t(j)^{1-1/\varepsilon}$ among the projects that mature at the end of the period, $j \in M_t$, and those that do not mature, $j \in NM_t$, are the same. Furthermore, because of the large number of projects, the exact fraction of projects that mature is θ , and the remaining $1 - \theta$ projects do not mature. It follows from this discussion that

$$\frac{\int_{j \in M_t} x_t(j) dj}{\theta} = \frac{\int_{j \in NM_t} x_t(j) dj}{1 - \theta} = \int_0^1 x_t(j) dj \text{ for each } x_t(j) = \{\iota_t(j), \iota_t(j)^{1-1/\varepsilon}\}. \quad (3.1)$$

where the expression after the second equality is the average scale over all j 's. Using (3.1), (2.2) becomes

$$I_t = \left(\theta \int_0^1 \iota_t(j)^{1-1/\varepsilon} dj \right)^{\varepsilon/(\varepsilon-1)}. \quad (3.2)$$

It now follows the crucial step of the intratemporal investment problem, which is to express E_t and I_t in terms of their respective lagged values and the date t scale of the projects that just matured. First note that I_t can be rewritten as

$$\begin{aligned}
I_t &= \left(\theta \left(\int_{j \in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + \int_{j \in NM_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj \right) \right)^{\varepsilon/(\varepsilon-1)} = \\
&= \left(\theta \left(\int_{j \in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + (1-\theta) \left(\int_{j \in NM_{t-1}} (\iota_{t-1}(j))^{1-1/\varepsilon} dj \right) / (1-\theta) \right) \right)^{\varepsilon/(\varepsilon-1)} = \\
&= \left(\theta \int_{j \in M_{t-1}} \iota_t(j)^{1-1/\varepsilon} dj + (1-\theta) (I_{t-1})^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}. \tag{3.3}
\end{aligned}$$

The expression after the first equality simply follows from rewriting the integral in (3.2). The second equality makes use of the fact that the scale of the investment project is fixed, or $\iota_t(j) = \iota_{t-1}(j)$, for those projects that did not mature, $j \in NM_{t-1}$. The third equality follows from (3.2) lagged by one period and (3.1). Using analogous steps, (2.4) can be rewritten as

$$E_t = \theta \int_{j \in M_{t-1}} \iota_t(j) dj + (1-\theta) E_{t-1}. \tag{3.4}$$

The firm only controls the current expenditure for the fraction θ of projects that had matured, while the expenditure in the remaining fraction $1-\theta$ is predetermined.

So long as $\varepsilon < \infty$ the firm chooses the same scale for all maturing investment projects, because they enter symmetrically into I_t and all have the same expected cost of one unit of Y . This result follows from a simple expenditure minimization of (3.4) subject to (3.3) with respect to $\{\iota_t(j)\}_{j \in M_{t-1}}$. Let ι_t be the optimal scale that is chosen by the firm at date t , then $\iota_t(j) = \iota_t$ for all $j \in M_{t-1}$.³ Using this result in (3.3) and (3.4), and then substituting ι_t from (3.3) into (3.4) yields

$$E_t = \Omega(I_t, I_{t-1}) + (1-\theta)E_{t-1}, \tag{3.5}$$

where

$$\Omega(I_t, I_{t-1}) \equiv \theta^{\frac{2}{1-\varepsilon}} \left(I_t^{1-\frac{1}{\varepsilon}} - (1-\theta) I_{t-1}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}.$$

The function $\Omega(I_t, I_{t-1})$ is increasing in its first argument and decreasing in the second. Indeed, a larger gap between the current and the lagged level of I_t corresponds to a bigger level of ι_t . Condition (3.5) is the only additional condition that needs to be included in an otherwise standard intertemporal maximization problem.

The Bellman-Jacobi equation associated with the intertemporal problem is

$$V_t(K_{t-1}, E_{t-1}, I_{t-1}) = \max_{\{K_t, I_t, E_t\}} \pi_t(K_{t-1}) - E_t - C(I_t, K_{t-1}) + \mathbb{E}_t R_{t+1}^{-1} V_{t+1}(K_t, E_t, I_t), \tag{3.6}$$

subject to the constraint (3.5) and the law of motion of the capital stock (2.3). In the general case in which $\varepsilon < \infty$ and $\theta < 1$, the value function, as well as the optimal investment rules, are function of E_{t-1} and I_{t-1} . As it will be discussed below, this is a crucial result of the model. The value function is also time dependent due to the temporal dependence of prices (p_{xt} , R_t), and of the productivity shock (A_t).

Let q_t and $-\mu_t$ be the date t shadow values of K_t and E_t respectively. The first order conditions associated

³Because the $\iota_t(j)$'s affect future quantities only through E_t and I_t , the optimal choice of $\iota_t(j)$ in the static minimization problem is equivalent to that following from the intertemporal program.

with (3.6) are

$$\begin{aligned}
(K_t) \quad q_t &= \mathbb{E}_t R_{t+1}^{-1} V_{1,t+1}(K_t, E_t, I_t), \\
(E_t) \quad \mu_t &= 1 - \mathbb{E}_t R_{t+1}^{-1} V_{3,t+1}(K_t, E_t, I_t), \\
(I_t) \quad \mathbb{E}_t R_{t+1}^{-1} V_{2,t+1}(K_t, E_t, I_t) + q_t &= C_{1,t}(I_t, K_{t-1}) + \mu_t \Omega_1(I_t, I_{t-1}),
\end{aligned} \tag{3.7}$$

where a numeric subscript indicates the argument with respect to which a derivative is taken. The first order condition with respect to K_t equalizes the shadow value of capital, q_t , to the partial change in the discounted value function (or marginal Q) which, using the envelope condition with respect to K_{t-1} , is

$$V_{1t}(K_{t-1}, E_{t-1}, I_{t-1}) = \pi_{1,t} + (1 - \delta)q_t - C_{2,t}(I_t, K_{t-1}).$$

The first order condition with respect to E_t equates E_t 's shadow *cost*, $\mu_t > 0$, to the sum of the cost of one additional unit of E_t and the increase of $(1 - \theta)$ in E_{t+1} evaluated at E_{t+1} 's shadow *cost*. Indeed using the envelope condition for E_{t-1} one obtains that

$$V_{2t}(K_{t-1}, E_{t-1}, I_{t-1}) = -(1 - \theta)\mu_t.$$

The first order condition with respect to I_t equates the marginal benefit and the cost of an additional unit of I_t . The marginal benefit is equal to the marginal increase in the capital stock evaluated at q_t and the discounted marginal reduction in E_{t+1} evaluated at μ_{t+1} . Indeed from the envelope condition for I_{t-1} follows that

$$V_{3t}(K_{t-1}, E_{t-1}, I_{t-1}) = -\mu_t \Omega_2(I_t, I_{t-1}).$$

The marginal cost of an additional unit of I_t is equal to the sum of the marginal increase in the installation cost and in the investment expenditure evaluated at μ_t .

Substituting the partial derivatives of the value function from the envelope conditions above into (3.7) one obtains

$$\begin{aligned}
(K_t) \quad q_t &= \mathbb{E}_t R_{t+1}^{-1} (\pi_{1,t+1} + (1 - \delta)q_{t+1} - C_{2,t+1}(I_{t+1}, K_t)), \\
(E_t) \quad \mu_t &= 1 + (1 - \theta)\mathbb{E}_t R_{t+1}^{-1} \mu_{t+1}, \\
(I_t) \quad q_t - \mathbb{E}_t R_{t+1}^{-1} \mu_{t+1} \Omega_2(I_{t+1}, I_t) &= C_{1,t}(I_t, K_{t-1}) + \mu_t \Omega_1(I_t, I_{t-1}).
\end{aligned} \tag{3.8}$$

These three conditions fully characterize the optimal investment decision.

3.2 Parametrization of $\pi_t(K_{t-1})$ and Steady State

The functional form of the cash flow function $\pi_t(\cdot)$ is specified in the first part of the section. Using this specification into the first order conditions (3.8), the model is then solved in steady state.

It is common in the investment literature to assume that firms are perfectly competitive and operate a constant return to scale production function. These assumptions simplify the empirical test of investment models with adjustment costs, as marginal Q , a crucial determinant of the investment decision, which is unobserved to econometricians, is equal to average Q , which is, instead, easier to measure for firms with publicly traded shares (Hayashi [1982]).

In this section I consider, instead, a monopolistically competitive firm that operates an increasing returns to scale production function due to overhead costs. There are two advantages of this model over the competitive model with constant return to scale. The first is that this model fits well with the empirical evidence on

US sectoral level data, which shows that firms tend to set prices above marginal costs and economic profits are close to zero (e.g. Hall [1988]).⁴ The second, is that the resulting cash flow function $\pi_t(\cdot)$ is strictly concave, so that the scale of the firm, as measured by the capital stock, is pinned down in the steady state of the model. This is particularly useful as the model is simulated in the next section by linearizing the first order conditions around the non-stochastic steady state.⁵

The production function is assumed to be of the form

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} - \phi, \quad (3.9)$$

with $0 < \alpha < 1$ and $\phi > 0$. The only variable factor of production is labor, L_t , and the firm takes as given the wage rate, w_t . The parameter ϕ is an overhead fixed cost such that economic profits are equal to zero in steady state. The firm faces an isoelastic demand function

$$p_t = z_t Y_t^{-\eta},$$

where $0 < \eta < 1$ is the inverse of the elasticity of demand, and z_t is a demand shifter variable. Solving (3.9) for L_t , the cash flow function can be rewritten in terms of K_{t-1} as

$$\pi_t(K_{t-1}) = \max_{Y_t} z_t Y_t^{1-\eta} - w_t \left(\frac{Y_t + \phi}{A_t K_t^\alpha} \right)^{\frac{1}{1-\alpha}}. \quad (3.10)$$

The optimal choice of Y_t yields that the firm sets the price of Y_t as a constant markup over its marginal cost

$$p_t = \frac{1}{1-\eta} MC_t, \quad (3.11)$$

where the marginal cost is

$$MC_t = w_t (1-\alpha)^{-1} ((Y_t + \phi)^\alpha (A_t K_t)^{-1})^{1/(1-\alpha)}.$$

Using the envelope theorem, the derivative of the cash-flow function is then

$$\pi'_t(K_{t-1}) = \alpha MC_t \left(\frac{Y_t + \phi}{K_t} \right). \quad (3.12)$$

The next step is to compute the value of economic profits in steady state. The value of ϕ is then chosen so as to guarantee that economic profits are zero in the steady state. I further normalize the steady state values of the productivity shock, the wage rate and the demand shifter to one, or $A_{ss} = 1$, $w_{ss} = 1$ and $z_{ss} = 1$. In the remaining the subscript ss denotes the steady state level for each variable. Note that the shadow rental rate on capital is given by (3.12) and the total variable cost is equal to $w_t L_t = (1-\alpha)(Y_t + \phi)MC_t$. Thus using (3.11), economic profits in steady state are

$$\pi_{ss}(K_{ss}) - \pi'_{ss}(K_{ss})K_{ss} = p_{ss}(Y_{ss} - (1-\eta)(Y_{ss} + \phi)). \quad (3.13)$$

⁴Aside from the empirical evidence, the assumption of zero economic profits in steady state, implies that even in the presence of imperfect competition cost and revenue based factor shares are equal. This simplifies the calculation of the production function elasticity with respect to the production factors (see Hall [1988]).

⁵Under perfect competition and a constant returns to scale production function the scale of the firm would not be determined in steady state

Equating (3.13) to zero, the overhead cost is

$$\phi = \frac{\eta}{1-\eta} Y_{ss}. \quad (3.14)$$

Evaluating (E_t) of (3.8) in steady state, the shadow cost of investment expenditure is equal to the discounted value of future expenditures

$$\mu_{ss} = \frac{1}{(1 - R^{-1}(1 - \theta))}. \quad (3.15)$$

From (I_t) of (3.8) and (3.15) the shadow value of capital is

$$q_{ss} = \theta^{-\frac{1}{\varepsilon-1}}. \quad (3.16)$$

The shadow value is greater than one in steady state due to the following reason. Investment projects increase the capital stock only when they mature. For given E_{ss} the size of I_{ss} falls with θ , due to the lower fraction θ of projects maturing in each period *and* the complementarity amongst investment goods.⁶ The higher shadow value reduces the value of K_{ss} in steady state. Using (3.10), (3.14), (3.16) into (I_t) of (3.8) evaluated at steady state, it follows that

$$K_{ss} = \Lambda \left(\frac{\alpha \theta^{\frac{1}{\varepsilon-1}}}{R - (1 - \delta)} \right)^{\frac{\alpha + \eta(1-\alpha)}{\eta}},$$

where $\Lambda \equiv \left((1 - \eta)^{1-\eta} (1 - \alpha)^{(1-\alpha)(1-\eta)} \right)^{1/\eta}$. From (3.11) evaluated in steady state then

$$Y_{ss} = \left((1 - \eta)^{\frac{1}{1-\alpha}} (1 - \alpha) \right)^{\frac{1-\alpha}{\alpha + \eta(1-\alpha)}} K_{ss}^{\frac{\alpha}{\alpha + \eta(1-\alpha)}},$$

which used into (3.14) yields the value of ϕ .

3.3 Calibration and Numerical Solution

The properties of the model described so far are discussed in this Section by means of the impulse responses of the model to unexpected innovations in the shocks. For brevity attention is restricted to interest rate and productivity shocks, while the wage rate, w_t , and the demand shifter, z_t , are kept constant at their steady state levels. The productivity and interest rate shocks evolve according to first order autoregressive processes

$$\begin{aligned} \log A_t &= \rho_A \log A_{t-1} + \varepsilon_t^A \\ R_t &= (1 - \rho_R)R + \rho_R R_{t-1} + \varepsilon_t^R \end{aligned} \quad (3.17)$$

The solution of the model is computed by loglinearizing the first order conditions around the non stochastic steady state. The resulting system of expectational difference equations is then solved using Anderson and Moore [1985] algorithms.

The parameters of the model are calibrated on post World War II US data at yearly frequencies, and are

⁶It is possible to eliminate the steady state inefficiency, by assuming that each investment project costs $\theta^{\frac{\varepsilon}{\varepsilon-1}}$ per period rather than θ .

Name	Variable	Value
Gross Interest Rate in s.s.	R_{ss}	1.08
Depreciation Rate	δ	.08
Demand Elasticity	η^{-1}	4
Capital Share in Production	α	.27
Capital Adjustment Cost Parameter	ψ	2
Persistency of $\log A_t$	ρ_A	.78
Persistency of R_t	ρ_R	.31
Wage Rate	w_t	1
Demand Shifter Variable	z_t	1

Table 1: Partial Equilibrium Model: Calibration of Parameter Values

summarized in Table 1.

The gross interest rate R_t is the rate of return to the firm's share and debt-holders. Thus, the value of R_t is constructed as a weighted average of the ex-post real returns on the S&P 500 index and the Moody's Baa Corporate Bond Yield during the years 1950-2000.⁷ The weight on the equity return is the median share of equity over total asset (3/4) of Compustat firms over 1960-2000 (Welch [2004]). The series A_t , adjusted to account for the monopolistically competitive setting described above, is from the NBER-CES Manufacturing Industry Database (Bartelsman, Becker, and Gray [2000]). The database reports data on the entire US manufacturing sector at the 4-digit SIC code over the years 1958-1996.⁸ The capital share in production, α , is computed as the average in the database, while the depreciation rate is set to 8%. The elasticity of labor demand, η^{-1} implies a markup over marginal cost of 33% (Woodford and Rotemberg [1999]). The adjustment cost parameter is from Cummins, Hassett, and Oliner [2003].⁹

The impulse response functions to unexpected 1% innovations in the interest rate and the productivity shocks are reported in Figures 1 through 4. The Figures display the responses of investment, the scale of new investment projects ($j \in M_{t-1}$), the shadow value of capital, the capital stock and labor input.¹⁰ All responses are expressed as percentage deviations from steady state values, and the term investment refers to both E_t and I_t , as the two variables are equal when expressed as deviations close to steady state.

Empirical evidence on the investment process in the US finds an approximate duration of 2 years for

⁷The index and dividend of the S&P500 are from Shiller Robert [2000], while the S&P 500 index and the Moody's Baa Corporate Bond Yield is from publication H.15 of the Board of Governors of the F.R.S.. Inflation is computed using the Consumer Price Index.

⁸The (unadjusted) TFP series is the 5-factor TFP annual growth rate, computed as the difference between real sales' growth rate and the sum of production inputs' growth rate weighted by the respective production function elasticities. The elasticities are computed from the share of each factor's expenditure over total revenues. With imperfect competition and overhead costs, the revenue based shares are unbiased estimates of the true elasticities so long as economic profits are zero (Hall [1988]). In the TFP calculation, however, the weight on the sales growth depends on the markup. With labor being the only variable production factor, for example: $\dot{A}_t = (1 - \eta)\dot{Y}_t - \alpha\dot{K}_t - (1 - \alpha)\dot{L}_t$, where a dotted variable denotes the logarithmic growth rate. Starting with the TFP growth rate computed under perfect competition, $\dot{A}_t^{comp}$, the growth rate of TFP is then simply constructed as $\dot{A}_t = \dot{A}_t^{comp} - \eta\dot{Y}_t$. In the theoretical model TFP is constant in steady state. The TFP index is thereby detrended in each sector. The value of ρ^a and the volatility in the innovation (used in the simulations of the next sections) is estimated through a linear AR(1) model for $\log A_t$ by pooling all the available data (over time and across sector).

⁹The value is taken from Table 3, second panel. Cummins, Hassett, and Oliner [2003] assume a quadratic adjustment cost function. Close to steady state ψ is approximately equal to the sensitivity of the investment to capital ratio with respect to marginal Q , when there is no TTB. The calibration below, uses the same value of ψ in the TTB model so that the models can be easily compared. As shown in Section 6, however, the empirically estimates of ψ would in general differ when TTB is present.

¹⁰The shadow value of capital is equal to the discounted value of marginal Q as shown in (3.7). The values of marginal and average Q differ in the model. Although most of the literature uses average Q as a proxy of marginal Q , other papers construct direct measures of marginal Q (Abel and Blanchard [1986], Gilchrist and Himmelberg [1998] and Cummins, Hassett, and Oliner [2003]).

projects involving structures, and lower values for investment in equipment.¹¹ Section 5 discusses how one can parametrize the elasticity of substitution, ε , using empirical results in the investment adjustment cost literature.

First consider the response of the model to an interest rate shock. Due to the positive serial autocorrelation, a higher realization of R_t implies higher expected rates also in the future. These, in turn, reduce the firm's weight on future dividends so that the shadow value of capital q_t falls. The incentives to invest are reduced and so are the capital stock and future levels of production.

The response of investment depends on the values of ε and θ . Consider the comparative static with respect to θ displayed in Figure 1, where ε is equal to 2. With a higher expected duration of the investment project (lower θ), investment responds less and only gradually to the shock. Two separate channels are in action. First, loglinearizing (2.2) yields to

$$\hat{I}_t = \theta \hat{\iota}_t + (1 - \theta) \hat{I}_{t-1}. \quad (3.18)$$

In each period, the fraction of projects under control of the firm falls with a longer duration, and thus investment is increasingly inertial, i.e. it depends relatively more on earlier choices of I_t for given ι_t . Moreover as θ falls, the scale of the new investment projects, ι_t , responds less to the current realization of the shocks. To see why this is so, consider the first order condition for the optimal scale of the investment project. After some algebra it follows that the first order condition can be written as

$$\mathbb{E}_t \sum_{r=0}^{\infty} \left(\prod_{s=1}^r R_{t+s}^{-1} (1 - \theta) \right) \left\{ q_{t+r} \left(\frac{I_{t+r}}{\iota_t(j)} \right)^{1/\varepsilon} - 1 - C_1(I_{t+r}, K_{t+r-1}) \right\} = 0.$$

The firm equates the expected marginal increase in the basket, $(I_{t+r}/\iota_j(t))^{1/\varepsilon}$, weighted by shadow value of capital, q_{t+r} , to the expected marginal cost of the investment project, inclusive of the installation cost of capital $(1 - C_1(I_{t+r}, K_{t+r-1}))$. As θ falls, the firm puts additional weight in the future trade off between costs and benefits. Since the current innovation is dissipated as time passes, the firm optimally chooses to respond less to the current realization of the shock as it carries fewer information relevant to evaluate the trade off.

The degree of complementarity among investment projects, ε , also determines the response of investment. Figure 3 displays the responses for different values of ε , holding θ fixed at $2/3$. From (3.18) the degree of investment inertia depends on how ι_t adjusts to the shocks for a given θ . The average scale of the investment projects chosen in the past, which is fully summarized in the values of E_{t-1} and I_{t-1} , affects the marginal return to current investment due to the complementarity between the investment goods. With a high degree of complementarity, low ε , the firm does not have large incentives to reduce the scale of the new projects. Indeed, due to the relatively large scale of the other projects, the return to investment remains high. On the contrary, the current choice of the investment projects is hardly affected by earlier investment choices for high levels of ε . Indeed as shown in Figure 3, for $\varepsilon = 100$, the firm fully re-balances I_t by over adjusting the scale of the new investment projects, and the inertial response of I_t completely disappears.

Now consider the response of the model to a productivity shock. Due to the positive autocorrelation, the expected marginal product of capital increases after a positive innovation, and thus the firm has incentives to invest more. The higher productivity also raises the demand for labor. Due to the higher level of the

¹¹Empirical work on TTB is almost exclusive to US data. Three different levels of aggregation are considered: project level (Mayer and Sonenblum [1955]), firm level (Koeva [2001]) or aggregate level (e.g. Altug [1989], Oliner, Rudebusch, and Sichel [1995] and ?).

factor inputs and productivity, output increases. As shown in Figures 2 and 4, the response of investment once again depends on the values of θ and ε . As for the interest rate shock, investment response is gradual and dampened with longer duration and higher complementarity. As for the interest rate shock, indeed, with lower values of θ , and given ι_t , investment is increasingly inertial from (3.18). Further, ι_t responds less to the realization of the shock as it carries fewer information about the future returns. The adjustment of ι_t also crucially depends on the value of ε : as the degree of complementarity falls, the firm over adjusts the scale of the new investment projects, so as to offset the portion of investment that is predetermined. In this case, investment responds swiftly to the shocks and it is no longer inertial.

4 TTB with Deterministic Duration

This Section compares the TTB model with uncertain duration, with two analogous models where the duration of the investment projects is deterministic. The comparison serves to highlight the elements of the model that are key to the gradual response of investment. As discussed in Section 3.3, the hump-shaped response of investment follows from the firm commitment to past investment decisions for the uncompleted projects, and the imperfect substitutability of the investment goods.

In the model presented so far, some investment projects last longer than others ex-post, but, because the maturity of the project follows a Poisson process, all projects are perfectly homogeneous before the uncertainty is resolved. This implies, for example, that projects started many periods in advance have the same probability to mature as the ones that have just begun. Further, due to the uncertain maturity and the assumption that the investment projects increase the scale of the investment basket, I_t , only when the projects mature, only a fraction θ of the investment goods increase the scale of I_t at each date. The Poisson assumption and the impossibility for the firm to postpone the use of the investment goods, are not crucial for investment to respond gradually. The crucial element, as shown below, is the ex-post heterogeneity in the projects' duration and the imperfect substitutability among the investment goods.

In the first model presented in this Section, which is similar to the one considered by Edge [Forthcoming], a firm invests in different projects that have complementarities in I_t similarly to the model previously discussed. The duration of each project is, however, certain and it differs across the investment goods. As shown from the impulse responses, investment responds gradually to shocks also in this model.¹²

In the second model presented, analogous to the one proposed by Casares [2006], the duration of the projects is also heterogeneous ex-ante. In this model, however, the capital stock is a composite of different capital types that are imperfect substitutes. The stock of each capital type is increased when the corresponding investment project matures. The response of investment expenditure in this model is also hump-shaped, a result that highlights the fact that the key element for a hump-shaped response is a mechanism that induces the firm not to fully compensate the overall investment decision through the projects under control. The specific mechanism, whether a complementarity between investment goods or capital goods, for example, is not a crucial element.

I will now describe the first model with complementarity between investment goods and deterministic duration of the projects. The model is a generalization of KP's, and the notation borrows from theirs. The firm invests in $j = 1, \dots, J$ investment goods, and each investment project lasts for $N_j \geq 1$ periods. Let $S_{t,j,n}$ denote the scale of the project of type j at date t which is n stages away from completion. The scale of the

¹²This model continues to consider a single homogeneous capital good. Edge [Forthcoming], instead, considers many investment *and* capital goods both characterized by imperfect substitutability.

investment project is chosen once and for all at its initiation (i.e., at N_j stages from completion), and it is fixed thereafter

$$S_{t+1,j,n} = S_{t,j,n+1} \text{ for all } j = 1, \dots, J \text{ and } n = 1, \dots, N_j. \quad (4.1)$$

At each point in time the expenditure for each type j of investment is equal to sum on all J expenditures at all stages of completion

$$E_{t,j} = \sum_{n=1}^{N_j} \omega_{j,n} S_{t,j,n}. \quad (4.2)$$

Following KP it is assumed that $\omega_{j,n} = \omega_j$ for all n . The value of the ω_j 's is chosen below so that the firm's discounted expenditure on each project j is independent of its duration N_j . The total investment expenditure is

$$E_t = \sum_{j=1}^J a_j E_{t,j}, \quad (4.3)$$

where the a_j 's weigh each expenditure's importance in steady state, as discussed below. The weights are such that $0 < a_j < 1$ and $\sum_{j=1}^J a_j = 1$. Only completed projects add on the capital stock, and similarly to Section 2 the investment basket is

$$I_t = \left(\sum_{j=1}^J a_j \iota_{t,j}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)}, \quad (4.4)$$

where

$$\iota_{t,j} = S_{t,j,1}. \quad (4.5)$$

The capital stock K_t depreciates at the constant rate δ as in (2.3). Maximizing out the variable production factors, the intertemporal investment decision of the firm is to maximize (2.5), where D_{t+r} is given by (2.6), subject to the investment technology (4.1)-(4.5). Let q_t be the date t shadow value of capital. The first order conditions that characterize the optimal investment decisions are then

$$\begin{aligned} & \mathbb{E}_t \left\{ \phi_j \left(\sum_{n=0}^{N_j-1} \left(\prod_{s=1}^n R_{t+s}^{-1} \right) \right) \right\} = \\ & = \mathbb{E}_t \left\{ \left(\prod_{n=1}^{N_j} R_{t+n}^{-1} \right) (q_{t+N_j-1} - C_1(I_{t+N_j-1}, K_{t+N_j-2})) \left(\frac{I_{t+N_j-1}}{\iota_{t+N_j-1}} \right)^{\frac{1}{\varepsilon}} \right\}, \end{aligned} \quad (4.6)$$

for all $j = 1, \dots, J$ and equation (K_t) of (3.8). The optimal choice of $S_{t,j,1}$ equates the discounted marginal investment expenditure, shown in the first line of (4.6), to the marginal benefit. The benefit is equal to the discounted marginal increase in the investment basket $N_j - 1$ from when decision is taken, weighted by the shadow value of capital q_{t+N_j-1} net of the marginal increase in the installation cost $C_1(\cdot, \cdot)$. The interpretation of the first order condition with respect to K_t follows that of Section 3.3. The values of ω_j 's are such that the steady state is symmetric $\iota_{ss,j} = \iota_{ss,j}$ and the shadow cost of capital is equal to one. From (4.6) evaluated at steady state values it follows that

$$\omega_j = \frac{1 - R^{1+N_j}}{1 - R}, \quad (4.7)$$

for all $j = 1, \dots, J$. The steady state level of the capital stock and of the other variables are the same as in

Section 3.3 when $\theta = 1$. The model is solved by log-linearization around the non-stochastic steady state. The parameters of the model are the same as in Table 1. The average duration of the investment project is one and a half year, thus $J = 2$, $N_j = j$ and $a_j = 1/2$. The elasticity of substitution in the investment basket, ε , is equal to 2. The response of the model to a one-percent innovation in the interest rate shock is shown in Figure 6, which also displays the response of the uncertain duration model calibrated with analogous parameters (θ equal to $2/3$). The response of investment expenditure after an interest rate shock is gradual as for the model with uncertain duration. Indeed, the key element to obtain a gradual investment response is the ex-post heterogeneity in the duration of the projects. Although not crucial for the gradual response, the uncertain duration affects the response of investment, which tends to be smoother in this model for the following reason. The response of the project's scale in the deterministic model falls with longer duration. The same was true for the model with uncertain duration. But due the uncertain duration, it was possible for some projects to last more than two periods. Thus the scale of the project with uncertain duration responds less than in the deterministic models. This also results in a smaller response of investment expenditure. Further, total expenditure with uncertain duration displays a smooth pattern over time, while in the deterministic model, the response of expenditure is delayed only for the first two years, after which the model response is as in the model with no TTB. In the model with deterministic duration the expenditure averages over the investment decisions of the preceding two periods. In the uncertain duration model, instead, investment expenditure summarizes decisions taken at all previous dates, albeit with smaller weights on earlier decisions, due to the random duration of the projects.

Now consider the second specification of the deterministic model, which differs from the one just presented in the source of imperfect substitutability among the investment types. The capital stock of the firm is made of J capital goods

$$K_t = \left(\sum_{j=1}^J a_j k_{t,j}^{1-1/\varepsilon} \right)^{\varepsilon/(\varepsilon-1)},$$

where the weights a_j 's are defined as before. Each type of capital good j depreciates at the constant rate δ and investment projects increase the capital stock only when they are completed, thus

$$k_{t,j} = (1 - \delta)k_{t-1,j} + \iota_{t,j},$$

where $\iota_{j,t} = S_{t,j,1}$. It is assumed that the firm pays installation costs for each type of capital, thus the dividend flow at date t is

$$D_t = \pi_t(K_{t-1}) - E_t - \sum_{j=1}^J a_j C(S_{t,j,1}, k_{t-1,j}),$$

where E_t is given by (4.3). Let $a_j q_{t,j}$ be the shadow value of the type j capital stock. The first order conditions, obtained by maximizing (2.5) subject to the investment technology, are then

$$\begin{aligned} (S_{t,j,1}) \quad & \mathbb{E}_t \left\{ \phi_j \left(\sum_{n=0}^{N_j-1} \left(\prod_{s=1}^n R_{t+s}^{-1} \right) \right) \right\} = \\ & = \mathbb{E}_t \left\{ \left(\prod_{n=1}^{N_j} R_{t+n}^{-1} \right) (q_{t+N_j-1} - C_1(\iota_{t+N_j-1,j}, k_{t+N_j-2,j})) \right\}, \\ (k_{j,t}) \quad & q_{t,j} = \mathbb{E}_t R_{t+1}^{-1} (\pi_{1,t+1} \left(\frac{K_t}{k_{t,j}} \right)^{\frac{1}{\varepsilon}} + (1 - \delta) q_{t+1,j} - C_{2,t+1}(\iota_{t+1,j}, k_{t,j})), \end{aligned}$$

for all $j = 1, \dots, J$. The economic interpretation of these optimality conditions is analogous to what discussed above in this Section. Further, as before, the values of the ω_j 's are equal to (4.7), so that the steady state

is symmetric: $k_{ss,j} = k_{ss}$ and $\iota_{ss,j} = \iota_{ss}$. All remaining parameters are chosen as before. The impulse responses to a one percent innovation in the interest rate shock are shown in Figure 5. Once again, the delayed response of investment expenditure lasts for the first two years after the shock. The overall shape of the response is hump-shaped, although investment now peaks after the shock.¹³ Once again, investment expenditure is in part predetermined by previous decisions, and the firm does not adjust it completely after that the shock hits due to the imperfect substitutability among the investment types.

The previous discussion suggests that an interesting alternative to the model with uncertain duration would be one where the capital stock of the firm were made of heterogeneous capital types, as in the model just presented. But with a random maturity, the level of the each capital type would be uncertain in steady state, and thus it wouldn't be possible to solve the model by linearization.¹⁴ The results just presented for the deterministic models, however, highlight that the response of investment in this alternative formulation would also be hump-shaped, which is the crucial implication of the TTB model of Section 2. Indeed as long as investment in some projects is partially predetermined and there is imperfect substitutability among the different types of investment, the firm optimally decides to respond only gradually to shocks.

The next Section compares the TTB model with uncertain maturity to capital and investment adjustment cost models. The main result of the Section is an equivalence result, up to first linearization, of the TTB and investment adjustment cost models.

5 Time-to-Build and Adjustment Cost Models

This section compares the TTB model of Section 2 with two investment adjustment cost models. The first is a *capital* adjustment cost model (Lucas [1967], Treadway [1969], Uzawa [1969]) in which firms faces costs of adjustment that depend on the change in the capital stock. An example of this adjustment function is the installation cost function $C(I_t, K_{t-1}) = c(I_t/K_{t-1})K_{t-1}$, included in the model of Section 2. The second model is one where the cost of investment adjustment depends on the difference between the current and lagged *investment* flows (e.g. $S(I_t, I_{t-1})$). Christiano, Eichenbaum, and Evans [2005] show that investment adjustment cost models capture well the dynamic response of aggregate investment to shocks.

The main result of this Section is that, up to first order linearization around the steady state and for appropriate choice of the parameter values, the TTB and investment adjustment cost models share the same steady state and local dynamics in its neighborhood. Instead, as discussed in the next paragraph, investment dynamics with capital adjustment costs differs as this model of investment lacks the inertial response of investment that characterize the other two models.

Consider the investment model with capital adjustment costs, i.e. $\theta = 1$ in the model of Section 2. With capital adjustment costs, the firm responds less to shocks than without these costs of adjustment. Absent any cost of capital adjustment, indeed, the firm fully responds to the shocks and q_t is always equal to one. With capital adjustment costs, instead, firms adjusts its investment decision and, thus, q_t may diverge from one (see Figures 2 to 3). Thus, similarly to the TTB model, capital adjustment costs reduce investment response and volatility. The two models differ, however, in the shape of the dynamic response. Because the shocks considered are temporary, the firm has the highest benefits and costs to increase investment when the

¹³The parameters of the investment technology (ε and the a_j 's) were kept constant to the ones of Figure 6 to facilitate the comparison. Note, however, that the parameters have a different meaning in the two formulations and the investment expenditure can peak in the second period for alternative choices of the parameter values.

¹⁴Further the analysis in this model is also complicated by the fact that one cannot express the intertemporal decision problem in terms of "aggregated" quantities as in Section 2.

shock hits in both models. But while with the TTB, the short run adjustment is very costly and induces the firm to respond gradually, in the capital adjustment cost model the cost is not sufficiently large to induce a gradual response. This is due to the fact that, for standard parametrization of the depreciation rate, the level of investment is only a small fraction of the capital stock in a neighborhood of the steady state.

The adjustment costs around steady state, instead, are large when they directly depend on the change in investment flow. This alternative adjustment cost specification has been proposed by Christiano, Eichenbaum, and Evans [2005] to describe the hump-shaped response of aggregate investment to monetary policy shocks. In this model, the firm solves at each date t

$$\max \mathbb{E}_t \left\{ \sum_{r=0}^{\infty} \left(\prod_{s=1}^r R_{t+s}^{-1} \right) ((\pi_{t+r}(K_{t+r-1}) - \tau (I_{t+r} - S(I_{t+r}, I_{t+r-1}))) \right\}, \quad (5.1)$$

with respect to K_t and I_t , subject to the law of motion of K_t (2.3).¹⁵ The adjustment cost function has the form

$$S(I_{t+r}, I_{t+r-1}) \equiv s \left(\frac{I_{t+r}}{I_{t+r-1}} \right) I_{t+r-1}, \text{ where } s(1) = s'(1) = 0 \text{ and } s''(1) = \chi, \quad (5.2)$$

and τ is a fixed parameter that measures the shadow value of capital in steady state. Because the adjustment cost depends on the change in the investment levels rather than in the capital stock, investment's response is in general hump-shaped as in the TTB model. In particular, an equivalence exists between the two models as the following Proposition shows

Proposition 1 (Equivalence between Investment Adjustment Costs and TTB) *Consider a firm that solves (5.1) subject to (5.2) and (2.3), and one that solves (3.6) with $\psi = 0$ subject to (3.5) and (2.3). If $\tau = \theta^{\frac{1}{1-\varepsilon}}$ and $\chi = \frac{(1-\theta)}{\theta \varepsilon (1-R^{-1}(1-\theta))}$ then the two models share the same steady state and local dynamics in its neighborhood.*

Proof.

In Appendix. ■

Although the literature has shown that investment adjustment cost models help explain the response of aggregate investment to shocks, it has not provided yet a foundation for the existence of such costs. The results of Proposition 1, instead, directly link these costs to TTB, which thus can be thought as a microfoundation for the investment adjustment cost models. It also follows from the equivalence result that given values of the project duration, θ , and of the second derivative of the adjustment cost function, χ , it is always possible to obtain a value for the parameter ε . For example, starting with a value of χ equal to .36, one obtains that, for $\theta = 2/3$, $\varepsilon = 2$, which is the benchmark parameter value in this paper. To account for the higher long run elasticity of investment, the value of χ is smaller than what was estimated by Christiano, Eichenbaum, and Evans [2005] on quarterly data.¹⁶ This is obviously only an approximation and the value of χ should be directly estimated on annual data.¹⁷

¹⁵The formulation of (5.1) follows the investment literature in that it includes the adjustment cost function in the dividend, and thus is expressed in units of output. Christiano, Eichenbaum, and Evans [2005], instead, include the adjustment cost in the law of motion of (K_t) so that (2.3) becomes

$$K_t = (1 - \delta)K_{t-1} + I_t - S(I_t, I_{t-1}).$$

In this formulation the cost is paid in units of capital. The discussion that follows does not depend on the units of measurement of the adjustment cost function.

¹⁶Christiano, Eichenbaum, and Evans [2005], Table 2 on page 17, find $\chi = .9$.

¹⁷An alternative strategy to relate quarterly to annual estimates is to assume that the elasticity of substitution ε is time-independent and thus obtain its value using interest rates and duration at annual frequencies. However ε in general is also time-dependent as the ability for a firm to substitute one good for another depends on the time available for it to adjust

6 TTB and Empirical Models with Capital Adjustment Costs

In recent years, a vast empirical literature has tested the investment model with convex *capital* adjustment costs. The literature finds only weak support for the model (see Chirinko [1993] for a review) as discussed below. The objective of this section is to present implications of the TTB model with uncertainty that are relevant to the empirical investment literature. In order to do so, I will first generate artificial data by simulating the TTB model calibrated on US sectoral data. I will then run the same regression models considered in the literature, and show that some of the empirical failures discussed in the literature are consistent with a misspecification error of the empirical model which omits to include the TTB technology underlying the data-generating process. Consider the benchmark model of the empirical literature. The model follows from the first order conditions presented in Section 3.1, when the capital adjustment cost function takes the quadratic form

$$c\left(\frac{I_t}{K_{t-1}}\right) \equiv \frac{\psi}{2} \left(\frac{I_t}{K_{t-1}} - \delta - e_t\right)^2, \quad (6.1)$$

and $\theta = 1$. The variable e_t is a shock to the adjustment cost function assumed to be serially uncorrelated. Using (6.1) into (I_t) of (3.8) yields the linear regression model

$$\frac{I_t}{K_{t-1}} = \beta_0 + \beta_1 Q_t + e_t, \quad (6.2)$$

The model (6.2) implies that Q_t is a sufficient statistic for the investment decision, and that the capital adjustment cost parameter ψ can be estimated as $\psi = 1/\beta_1$. This simple characterization of the investment decision has been rejected in the data along different dimensions. First, the estimated costs of capital adjustment are unreasonably large (e.g. Summers, Bosworth, Tobin, and White [1981]). Further, the value of Q_t is hardly a sufficient statistic for the investment decision. Indeed, lagged values of Q_t enter significantly in the regression model, and this evidence, taken together with the high serial correlation of the error term, e_t , is interpreted (e.g. Oliner, Rudebusch, and Sichel [1995]) as indicating that the investment decision is significantly more inertial than what is predicted by model (6.2). Further, the literature on financial frictions augments the set of controls in 6.2 to also include the ratio of a firm's cash flow scaled by the capital stock, CF_t/K_{t-1} (for a review of this literature see Hubbard [1998]). The value of a firm's cash flow (revenues less taxes and expenses, excluding investment), is used to approximate the change in firm's net worth. Firms with higher net worth have more internal funds, and thus, tend to invest when financial frictions exist, due to the lower costs of financing and non-binding financing constraints. The empirical literature finds that cash flows are highly significant when included in the regression model, and tend to be economically more important than Q_t in explaining the investment decision.

As shown next, the evidence described above is consistent with investment decisions underlying the data that include TTB and an empirical model that does not. To show this, I first generate data on investment decisions using the model calibrated on US sectoral data reported in Table 1, and then regress the model (6.2) on the artificially generated data. The adjustment cost shock e_t is assumed to be serially uncorrelated and has a one percent standard deviation.¹⁸ Each simulation is composed of 5,000 observations, a number comparable to empirical studies at the firm (e.g. Cummins, Hassett, and Oliner [2003]) and sectoral level

¹⁸The standard deviation of the capital adjustment cost is not calibrated on empirical data. The value of the volatility, however, is not crucial in the results and discussion that follows. The main effect of a higher volatility in e_t is a lower fit of the regression in terms of R^2 and standard errors of the estimated coefficients.

(Bartelsman, Becker, and Gray [2000]). I consider two data generating processes. The value of ε is equal to 2 in both models. The value of θ is equal to one in the first model (no TTB) and to 2/3 in the second (average duration of one and a half year). While the true error term is serially uncorrelated the empirical model allows for first order serial correlation: $e_t = \rho e_{t-1} + \varepsilon_t^e$. The parameter of the model and ρ are estimated using the Cochrane-Orcutt estimation procedure. An important issue in empirical work is the measure of marginal Q_t . The vast majority of the empirical literature approximates the value of marginal Q with average Q , which is easily measured for publicly traded companies. Indeed as shown by Hayashi [1982], average and marginal Q are equal when the firm is perfectly competitive *and* operates a constant return to scale technology. Both assumptions are not likely to hold empirically, and Abel and Blanchard [1986] and Cummins, Hasset, and Oliner [2003] among others, construct empirical measures for marginal Q . The model presented in the previous sections implies that average and marginal Q differ. This is not just due to the monopolistically competitive framework but also because of the TTB technology. Because, the objective here is to focus on the implications of the TTB technology, the analysis abstracts from measurement errors on Q_t and assumes that the econometrician can directly observe it. The results of the simulations are presented in the Table 3. All numbers are averages over 100 simulations. The Table has six columns: the value of θ in the data generating process in the first three columns is equal to 1 and in the last three is equal to 2/3. For each of the two data generating processes, I consider three regression models. The first model only includes the contemporaneous value of Q_t . The second model augments (6.2) with four lags of Q_t , while the third model includes CF_t/K_{t-1} .

Consider the parameter estimates obtained from the data generating process without *TTB*. As shown in the first three columns, the coefficient on Q_t is always statistically significant and close to .5, so that ψ is roughly equal to 2 as in the data generating process. In model (2), only the second lag is statistically significant, but the magnitude of the coefficient is approximately equal to zero. The coefficients on all other lags are smaller and none of them is statistically significant at conventional levels. In model (3), cash flows scaled by the capital stock is not statistically significant. As shown in the bottom part of the Table the estimate of ρ is roughly zero as in the data generating process, and the Durbin Watson statistic indicates the lack of serial correlation in the model.

Now consider the regression models when the data generating process includes *TTB* of one and a half year. Although the coefficient on Q_t is always statistically significant in the three specifications, its magnitude is less than half than the true value. As discussed in Section 3.3, investment expenditure responds less to the realization of the shocks as *TTB* increases, as a lower fraction of the investment projects is under the control of the firm. As shown in Figures 1 to 4, the value of Q is hardly affected by the longer duration of *TTB*, and thus the volatility of investment expenditure, E_t , with respect to Q_t falls. Because the empirical model (6.2) does not include *TTB*, however, the lower volatility of E_t is erroneously accounted by the empirical model as higher installation costs of capital. As discussed in Section 3.3, investment responds gradually to shocks with *TTB*, while the value of Q_t immediately responds to the productivity and interest rate shocks. The investment model without *TTB* cannot account for the inertial response in investment. As a result the residuals of the regression model are serially correlated. Indeed from the bottom panel of Table 3, the estimated values of ρ vary from .7 to .8, and, aside from model (5), the Durbin Watson statistic indicates that the residuals have persistence of order greater than one. Moreover, in the regression model (5), all lagged values of Q_t are statistically significant, and the magnitude of the regression coefficients are comparable to that of Q_t . Finally in the regression model (6), the cash-flows to capital stock ratio is statistically significant, and is economically more important than Q_t as an explanatory variable for investment. Indeed

a one standard deviation increase in the cash flow ratio (equal to .12) raises the investment to capital ratio by almost twice the amount following a one standard deviation in the cash-flow to capital ratio (equal to .07). In the model presented in this paper no financial frictions are present, and cash flows help explaining the investment decision only due to the misspecification error.

The results presented above hint that a promising venue to explain the empirical puzzles just described could be to estimate the TTB model with uncertainty. I will now discuss strategies to estimate the parameters. First note that, due to the unobservability of I_t , it is only possible to estimate a linear approximation to the model. For example, by linearizing the first order conditions (3.8) around the steady state values of $\frac{E_t}{E_{t-1}}$ and $\frac{E_t}{K_{t-1}}$, one obtains that

$$\frac{E_t}{E_{t-1}} = \beta_0 + \beta_1 Q_t + \beta_2 \frac{E_{t+1}}{E_t} + \beta_3 \frac{E_t}{K_{t-1}} + \varepsilon_t, \quad (6.3)$$

where ε_t is an expectational error term, which is orthogonal to the information set available at date t , i.e. $\mathbb{E}_t(\varepsilon_t) = 0$. The value of the structural parameter can be reconstructed from the estimated values of the parameter β 's by noting that: $\beta_2 = R^{-1}$, $\beta_1 = \frac{\theta\varepsilon(1-R^{-1}(1-\theta))}{(1-\theta)}\theta^{\frac{1}{\varepsilon-1}}$ and $\beta_3 = -\theta^{\frac{1}{\varepsilon-1}}\beta_1\psi$. Using the orthogonality condition of ε_t , the model in 6.3 can be estimated using a linear-GMM by instrumenting the right hand side variables with variables that belong to the date t information set. The goodness of the linear approximation remains an open question, even more so at higher levels of aggregation (e.g. sectoral or aggregate investment) where the investment series tend to be non-stationary. Further the correct estimates of the structural parameters crucially depend on the quality of the empirical approximation to marginal Q_t .

Although an empirical test of the *TTB* model is beyond the scope of this work, the results on simulated data presented in this Section underscore how the *TTB* model has the potential to help better understand the empirical behavior of firms' investment decisions.

7 Real Business Cycle Model

This section embeds the TTB model of Section 2 in an otherwise canonical real business cycle model. The response of the model to an unexpected innovation in the TFP shock is then compared to that of a model with no time-to-build and with KP's TTB specification.

The representative household is infinitely lived. At each date t he decides how much to consume, C_t , work, L_t , and how many stocks, S_t , to hold so as to maximize the discounted value of utility flows

$$\max_{\{C_t, L_t, S_t\}} \mathbb{E}_t \sum_{r=0}^{\infty} \beta^r U(C_{t+r}, 1 - L_{t+r}),$$

where the utility function is specified as

$$U(C_t, 1 - L_t) = \log C_t + v \log(1 - L_t),$$

and $0 < \beta < 1$ is a discount factor, which measures household's rate of impatience. At every date, the household is subject to the flow budget constraint

$$C_t + S_t V_t \leq W_t L_t + S_{t-1} (V_t + D_t),$$

Name	Variable	Value
Gross Interest Rate in s.s.	R_{ss}	$(1.08)^{.25}$
Depreciation Rate	δ	.02
Demand Elasticity	η^{-1}	∞
Capital Share in Production	α	.36
Capital Adjustment Cost Parameter	ψ	0
Labor weight in utility	v	such that $L_{ss} = .28$
Persistency of log A_t	ρ_A	.95

Table 2: RBC Model: Calibration of Parameter Values

where $W_t L_t$ is his labor income, while V_t and D_t are the stock price and dividend of the representative firm. Finally, S_t is the level of stock holdings.

The production sector of the economy is composed of large number (measure one) of firms that produce Y_t , using the production function (3.9). Date t prices are expressed in terms of Y_t , and the prices of the investment good and of C_t are equal to one. For brevity, I consider the case in which the demand function is perfectly elastic, so that the representative firm's markup is one and the overhead cost (3.14) is zero.

The first order conditions of the household's maximization problem yield

$$\begin{aligned} (L_t) \quad & U_{2,t} = w_t U_{1,t} \\ (S_t) \quad & U_{c,t} S_t = \mathbb{E}_t \beta U_{c,t+1} (S_{t+1} + D_{t+1}). \end{aligned} \tag{7.1}$$

Integrating (S_t) of (7.1) forward and using the transversality condition $\lim_{r \rightarrow \infty} \beta^r U_{c,t+r} S_{t+r}$ yields that the ex-post realization of the firm's discount factor is

$$R_{t+1}^{-1} = \beta \frac{U_{c,t+1}}{U_{c,t}}.$$

From (3.12) it also follows that with perfect competition,

$$\pi'_t(K_{t-1}) = \alpha A_t K_{t-1}^{\alpha-1} L_t^{1-\alpha},$$

in equilibrium. The dividend of the representative firm is equal to a constant fraction α of Y_t in equilibrium. The first order conditions that characterize the firm's optimal decisions are given by (3.8) along with the inverse demand for labor services in equilibrium $w_t = (1 - \alpha)Y_t/L_t$. Finally market clearing in the financial market yields to the aggregate resource constraint $C_t + E_t = Y_t$.

Given the initial levels of K , E and I , and a sequence of TFP shocks, $\{A_t\}$, an equilibrium is defined as a state-contingent sequence of prices and quantities such that a) firms and households solve their respective maximization problem, b) goods, labor and the financial markets clear.

Now consider the TTB model of Kydland and Prescott [1982]. They consider an investment technology with TTB but where the duration of the investment project is certain. Further, they assume that the representative firm makes only one type of investment. Their model is easily obtained by considering the case of $J = 1$ in either models with deterministic maturity presented in Section 4. I follow their original analysis and assume that the time unit is a quarter, that the investment projects last for four quarters, and that investment expenditure is equally split over the four quarters, or $\omega_n = 1/4$ for all n . I also assume that the duration of the project in the random maturity TTB model is one year, or $\theta = 1/4$. The remaining parameter values common to the two models are reported in Table 2.

The parameters are calibrated on post-World War II US data (e.g. Prescott [1986]). The impulse response functions to a one percent innovation in the productivity shock are shown Figure 7. The Figure reports the response of the uncertain TTB (TTB) and the Kydland and Prescott [1982] (TTB KP) model, and of a canonical RBC model with no TTB (no TTB). First consider the response of the canonical RBC model. With the exception of aggregate consumption, all variable peak when the productivity shock hits and then decay exponentially to pre-shock levels. The response of aggregate consumption is, instead, hump-shaped due to households' intertemporal smoothing motive. Overall the response of all variables in the KP model follow those of the canonical RBC model. The main difference between the two models, are the deterministic cycles in the responses of the KP model, as first highlighted by Rouwenhorst [1991]. The representative firm increases the scale of the new projects when the technology shock hits the economy. In the three quarters after the shock, however, the firm reduces the scale of new projects, because it has to live up to the previous commitments. In the fourth quarter after the shock, the projects with a relatively large scale mature raising the capital stock, households' incentives to consume and of firms to invest. The pattern then repeats itself, creating the cyclicalities in the impulse response functions. As noted by Rouwenhorst [1991], however, these cyclicalities are small compared to the overall response of the model, which closely follows the canonical RBC model. Thus Rouwenhorst [1991] challenges the central role of TTB posed by KP.

Now consider the TTB model with uncertain duration. As in the partial equilibrium analysis, investment expenditure responds only gradually to the higher productivity levels. The representative firm only controls the scale of the investment projects that have just matured, and it optimally decides not to perfectly adjust the overall expenditure through the projects under control due to the complementarities. The gradual investment response is in stark contrast with the swift responses in the canonical RBC model and in KP's, and describes well the empirical response investment to shocks. Indeed the response of investment to productivity shocks (Christiano, Eichenbaum, and Vigfusson [2004]) and to monetary policy shocks (Christiano, Eichenbaum, and Evans [1998]) is hump-shaped. For monetary policy shocks, Christiano, Eichenbaum, and Evans [2005] find that a model with investment adjustment costs capture well the response of investment, thus from the results of Section 5, so does the TTB technology proposed in this paper.

Due to the lower response of investment expenditure, the response of aggregate consumption is amplified in a closed economy. Further due to the initial spike in aggregate consumption, households substitute labor for leisure and thus labor can fall after a positive productivity shock.¹⁹ Lucca[2006] exploits the higher response of consumption due TTB to explain the higher volatility of aggregate consumption in low versus high income countries. For high income economies, it is possible to eliminate the initial spike in consumption and the fall in hours worked using preferences that display habit-persistence (see ?] and Christiano, Eichenbaum, and Evans [2005]).

8 Conclusions

This paper presented a TTB model with a novel set of implications for the theory of investment. The model departs from the one considered by KP in two ways: firms invest in many investment projects that have complementarities, and the duration of each investment project is uncertain. The main implication of the model is that investment responds only gradually to shocks. The gradual response follows from the impossibility to modify the scale of the projects that have not matured, so that investment decisions are

¹⁹For the empirical response of labor to technology shocks see the discussion in Gali [1999], Basu, Fernald, and Kimball [2004] and Christiano, Eichenbaum, and Vigfusson [2004].

partly predetermined , and the unwillingness of the firm to compensate earlier commitments by adjusting the scale of new projects, due to the imperfect substitutability between investment goods.

As shown in the paper, the model is analitically very tractable, and it is equivalent, up to first order linearization, to investment adjustment cost models (Christiano, Eichenbaum, and Evans [2005]), recently emphasized in the macroeconomic literature for capturing well the empirical response of aggregate investment to shocks. Further, numerical simulations in this paper show that the TTB model may also help explain some of the empirical shortcomings of the investment model with capital adjustment costs.

Future work should attempt to estimate the model's parameters on data at different levels of aggregation: for example, at the firm, sectoral and aggregate levels. Aside from gauging the empirical magnitudes of the parameters, this analysis would help understand whether the sluggish response of aggregate investment is actually due to TTB at the firm level. A failure to uncover TTB at more disaggregated level would suggest alternative interpretations of the model at higher levels of aggregation.

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A Proof of Proposition 1

First consider the investment problem of a firm that faces investment adjustment costs. The firm solves (5.1) subject to (5.2) and (2.3). The first order conditions of the maximization problem are

$$\begin{aligned} (K_t) \quad q_t &= \mathbb{E}_t R_{t+1}^{-1} (\pi_{1,t+1} + (1 - \delta) q_{t+1}), \\ (I_t) \quad q_t &= \tau (1 + S_1(I_t, I_{t-1}) + \mathbb{E}_t R_{t+1}^{-1} \mu_{t+1} S_2(I_{t+1}, I_t)). \end{aligned} \quad (\text{A.1})$$

From (A.1) the steady state level of capital solves

$$\pi'(K_{ss}) = \tau(R - (1 - \delta)). \quad (\text{A.2})$$

For each variable X_t , let the corresponding hatted variable denote a percentage deviation from steady state, $\hat{X}_t = (X_t - X_{ss})/X_{ss}$. Equation (I_t) of (A.1) can be log-linearized in

$$\tau \hat{q}_t = \chi \left(\Delta \hat{I}_t - \mathbb{E}_t R_{t+1}^{-1} \Delta \hat{I}_{t+1} \right), \quad (\text{A.3})$$

where $\Delta \hat{I}_t = \hat{I}_t - \hat{I}_{t-1}$.

Now consider the TTB model. The first order conditions that characterize the investment decision are (3.8), from which the capital stock in steady state solves

$$\pi'(K_{ss}) = \theta^{\frac{1}{1-\varepsilon}} (R - (1 - \delta)) \quad (\text{A.4})$$

By loglinearizing (E) of (3.8) it follows that

$$\hat{\mu}_t = R^{-1} (1 - \theta) \mathbb{E}_t \left(\hat{\mu}_{t+1} - \hat{R}_{t+1} \right). \quad (\text{A.5})$$

By loglinearizing (I) of (3.8) and substituting $\hat{\mu}_t$ from (A.5) it follows that

$$\hat{q}_t = \frac{(1 - \theta)}{\theta \varepsilon (1 - R^{-1}(1 - \theta))} \left(\Delta \hat{I}_t - \mathbb{E}_t R_{t+1}^{-1} \Delta \hat{I}_{t+1} \right). \quad (\text{A.6})$$

From (A.2) and (A.4), the capital stock in steady state is the same in the two models when

$$\tau = \theta^{\frac{1}{1-\varepsilon}}, \quad (\text{A.7})$$

holds. It then easily follows that all other quantities are also equal in steady state. When (A.7) and $\chi = \frac{(1-\theta)}{\theta \varepsilon (1 - R^{-1}(1 - \theta))}$ hold, the loglinearized first order condition (A.3) and (A.6) are also equal. Further note that the first order condition with respect to (K) is the same in both models, absent capital adjustment costs in the TTB model ($\phi = 0$). In the investment adjustment cost model, investment expenditure is equal to τI_t , so that the log-deviations of the basket and expenditure are equal. This is also the case in the TTB model. By loglinearizing (2.2) and (3.5) it follows that

$$\hat{E}_t = \int_0^1 \hat{i}_t(j) dj = \hat{I}_t.$$

Thus the claim of the Proposition follows

Figure 1: IRFs with $\varepsilon = 2$: Interest Rate Shock

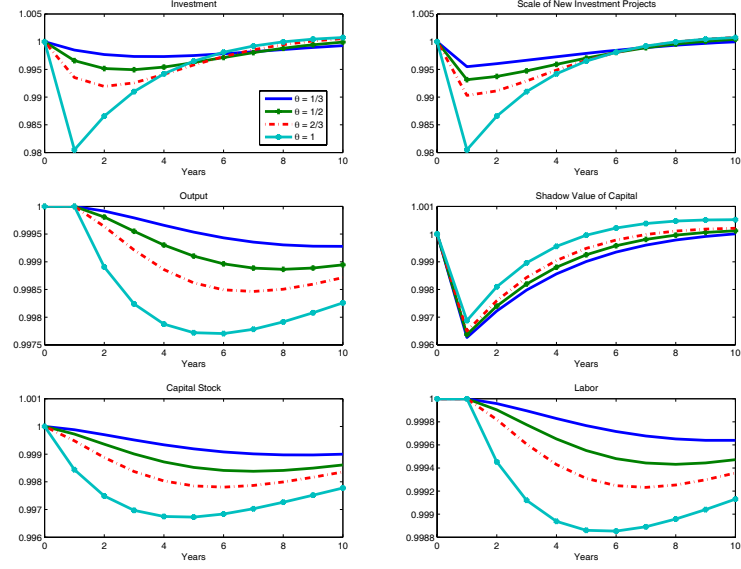


Figure 2: IRFs with $\varepsilon = 2$: Technology Shock

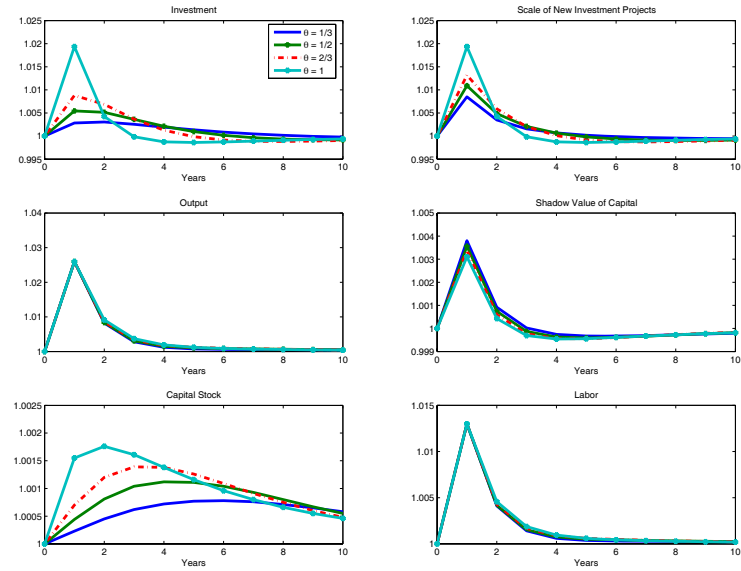


Figure 3: IRFs with $\theta = 2/3$: Interest Rate Shock

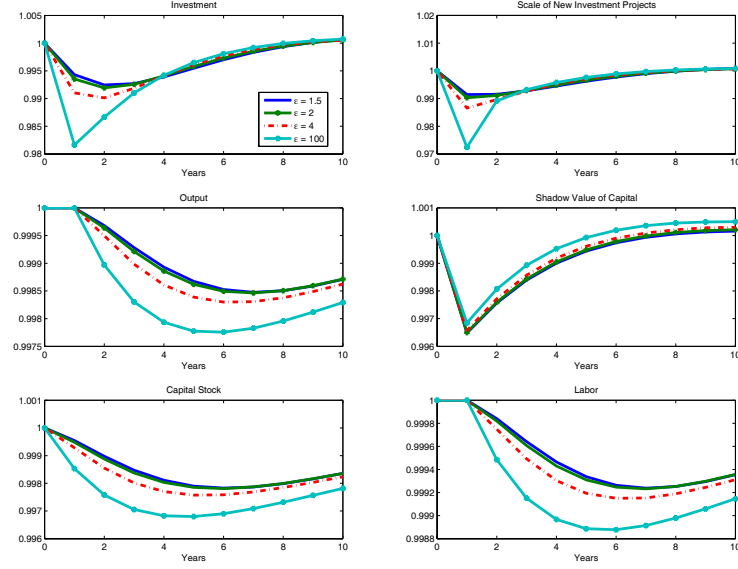


Figure 4: IRFs with $\theta = 2/3$: Technology Shock

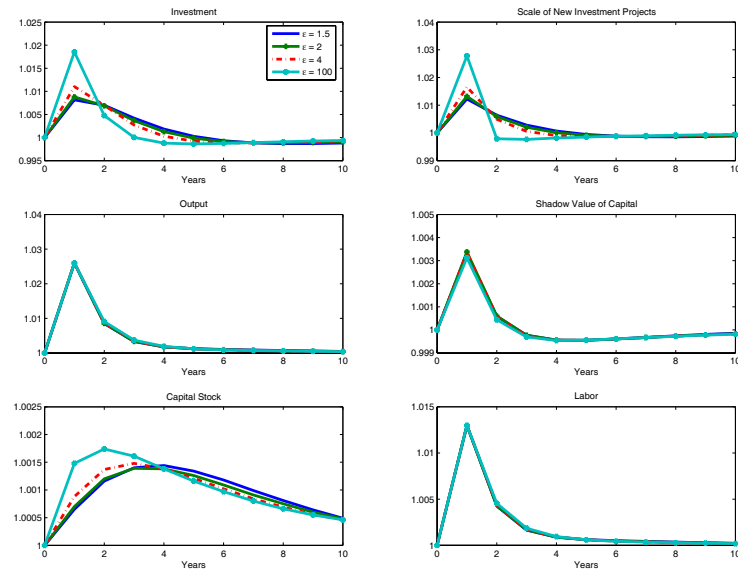


Figure 5: IRFs: Comparison with Deterministic TTB Model with Multiple Cap. Types; Interest Rate Shock

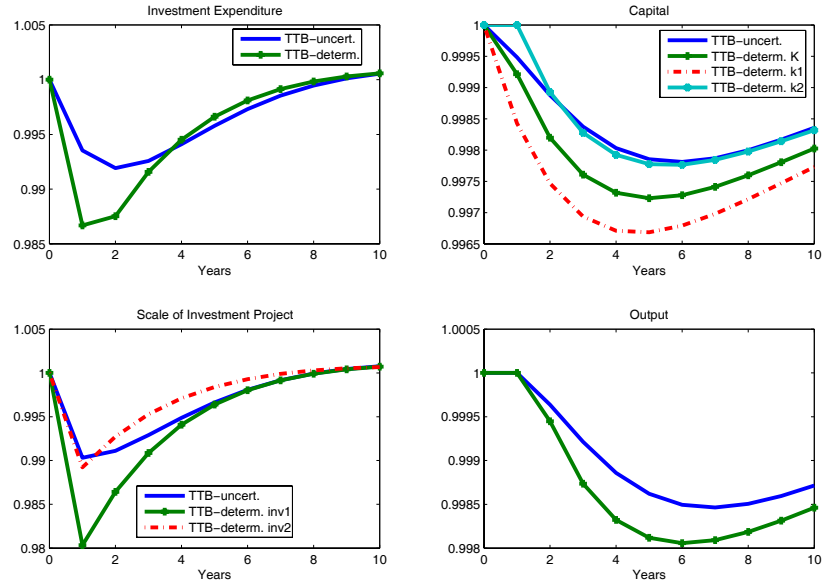


Figure 6: IRFs: Comparison with Deterministic TTB Model with Multiple Inv. Types; Interest Rate Shock

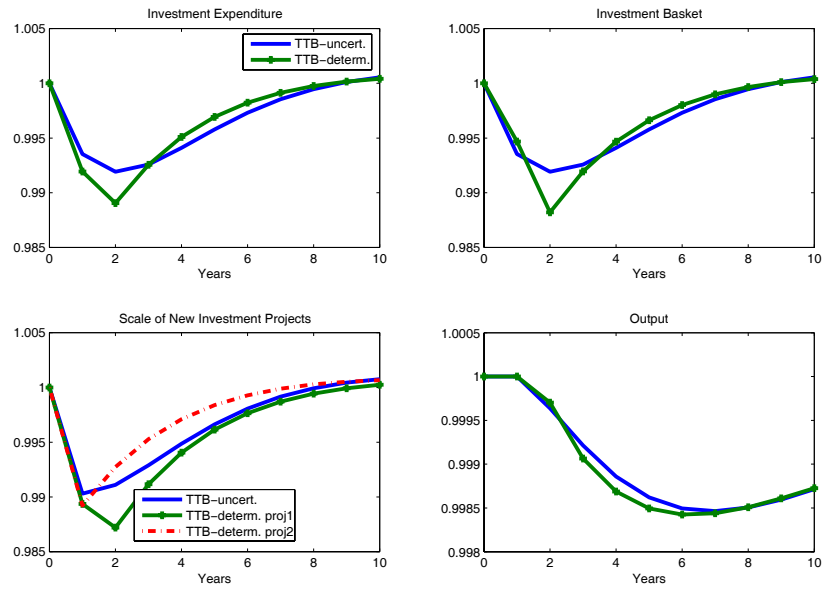


Figure 7: IRFs: RBC Model; Technology Shock

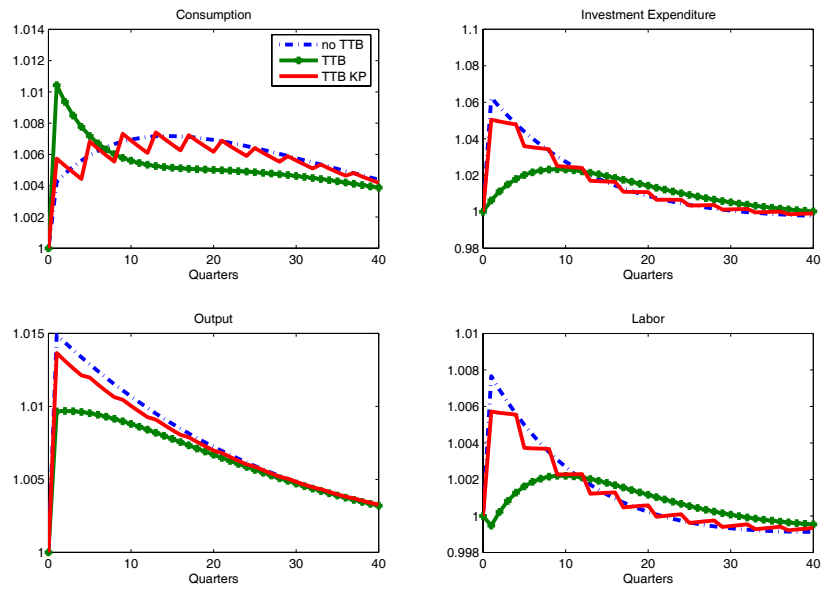


Table 3: Regression Models on Artificially Generated Data

Dependent Variable: $E(t)/K(t-1)$						
Value of θ in data generating process	1			2/3		
Model	(1)	(2)	(3)	(4)	(5)	(6)
Q(t)	.480 (.003)	.478 (.003)	.418 (.006)	.165 (.001)	.202 (.001)	.066 (.005)
Q(t-1)		.006 (.003)			.105 (.001)	
Q(t-2)		-.001 (.003)			.051 (.001)	
Q(t-3)		-.001 (.003)			.023 (.001)	
Q(t-4)		-.001 (.003)			.008 (.001)	
CF(t)/K(t-1)			.004 (.006)			.064 (.005)
Durbin Watson Statistic	2	2	2	1.36	2.01	1.37
ρ	.002	.001	.013	.745	.793	.773
Adj. R^2	.828	.83	.833	.732	.953	.758

Notes: Linear regression models estimated on artificially generated data calibrated with parameters of Table 1 and $\sigma(\varepsilon) = .01$. Each simulation is made of 5,000 observations, and the numbers reported are averages over 100 simulations. The value of ε is 2, and θ is equal to 1 or 2/3. All regression models are estimated using the Cochrane-Orcutt estimation procedure. The parameter ρ is the first order serial correlation in the error term. Standard errors of the estimated coefficients are reported in parenthesis below each coefficient.