

# Housing Prices and Growth

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## **Abstract**

This paper develops a growth model with land, housing services, and other goods, coupled with a Markov regime-switching model of productivity growth. We show that such regime switches, if interpreted as changes in the relative productivity growth of non-housing goods and services, are a plausible candidate for explaining the large low-frequency changes in housing prices. We also argue that this interpretation is reasonable given the nature of the technology that produces housing services, and is in accord with the broad patterns in data on housing and land prices. We also explore the role of permanent changes in labor supply as an additional explanatory factor.

# 1 Housing Wealth, Rents, and Services

With the acceleration of housing prices since the mid-1990's in the United States, there has been increased attention given to the causes and effects of the enormous increase in housing wealth. In this paper we develop a growth framework to analyze the determinants of housing prices, and the role of various long-term factors in the relative value of housing. To start with some facts: Since 1995, the real quality-adjusted price of new houses has appreciated at an average rate of 2.2 percent annually. As Figure 1 indicates, similarly strong real appreciation took place in the 1960s and 1970s as well, followed by nearly 20 years of depreciation.

This paper develops a growth model with land, housing services, and other goods, coupled with a Markov regime-switching model of productivity growth as in Kahn and Rich (forthcoming). We show that such regime switches, if interpreted as changes in the relative productivity growth of non-housing goods and services, are a plausible candidate for explaining the large low-frequency changes in housing prices. We also argue that this interpretation is reasonable given the nature of the technology that produces housing services, and is in accord with the broad patterns in data on housing and land prices. We also explore the role of permanent changes in labor supply as an additional explanatory factor.

The real value of housing wealth, as measured by flow of funds data, has grown an average of 4.6 percent since 1952. This compares with 3.4 percent growth of private net worth excluding real estate, and 3.5 percent growth of personal consumption expenditures over the same time period. Figure 2a plots the ratio of nominal housing wealth to nominal consumption expenditure. This ratio nearly doubles between 1952 and 2005. Figure 2b plots the much more volatile ratio of housing wealth to total net worth. While the enormous volatility of non-real estate wealth hinders precise inferences about relative trends, this disparity is robust to different time periods, and is not just the result of the runup of the last 5-10 years in real estate wealth. The bottom line is that real estate has gone from 27 percent of net worth in 1952 to 42 percent by the end of 2005. Heathcote and Morris (2005), however, argue that there

are problems with the Flow of Funds data, particularly over long periods of time, and construct their own measures of housing wealth (though only going back to 1975) that exhibit a less clearcut trend.

What drives housing prices is a controversial topic. In one well-known study, Mankiw and Weil (1989) argued that population demographics were the prime determinant, and predicted (highly inaccurately, as it turned out) that prices would fall in the subsequent two decades with the maturation of the baby boom generation and resulting decline in the growth rate of the prime home-owning age group. More recently, Glaeser et al (2005) have argued that price increases since 1970 largely reflect artificial supply restrictions. We will take a different approach in examining the fundamental determinants of long-run trends in relative prices in a fully dynamic general equilibrium context. There it turns out that productivity growth (in particular, the relative productivity growth in the non-housing sectors of the economy) and labor supply are important drivers of housing prices in addition to population growth. Their importance is tied to their low-frequency movements: Productivity has exhibited periodic changes in trend that are well-represented by a regime-switching model (see Kahn and Rich, forthcoming). Labor supply has also exhibited low-frequency movements (see Figure 3), though its importance is more in offsetting the impact of underlying population trends.

One possible explanation for the relative increase in housing prices is a simple income effect, or non-homogeneity in preferences. As people get wealthier, they may prefer to have more of their consumption coming from housing services, the price of which will tend to rise because of its being relatively intensive in land, a fixed factor. The (nominal) share of housing services in GDP has gone from 7.5 percent in 1952 to over 10 percent in 2005. On the other hand, the share of housing services in consumer expenditures has only gone from 12.2 percent to 14.6 percent over the same period, and in fact has been without any meaningful trend since 1960 or so. A similar sort of explanation is that housing has a low elasticity of substitution with other goods, so that as its relative price rises (again because of land-intensity), it consumes

a higher share of income. Under plausible assumptions we can test both of these explanations by looking at cross-sectional disaggregated or microeconomic data, to see to what extent expenditures on housing services (relative to other expenditures on consumption) vary with the price of those services or with total expenditures. We do so in this paper using data from the Consumer Expenditure Survey. We can also examine aggregate data. The long-term behavior of aggregate expenditures on housing services suggests a unit income elasticity for such expenditures, but with somewhat inelastic demand. This is because housing services' expenditure share has no long-run trend, but is positively correlated with the relative price of housing services, at least as measured by NIPA. Figure 4 presents annual data going back to 1929 of the two series, which show a positive relationship for most of the sample, though recently they have diverged. This suggests a model with homothetic preferences but with an elasticity of substitution between housing and other consumption that may differ from one.

Another driver of housing prices could be differing technical progress trends in housing services versus other goods, as in Baumol (1967). The relatively large share of land and structures, two inputs usually thought to be less amenable to embodied technical progress, in the value of housing makes this story plausible. And indeed, we will argue that the timing of low-frequency changes in both housing prices and productivity suggests that this mechanism is important.

There is some evidence that in fact the increase in housing wealth does not stem from an increase in the value of houses per se, but rather from the increase in the value of the land upon which they are built. First, a price index that include the value of land, the Conventional Mortgage Home Price Index, has increased approximately 0.75% faster than indexes that do not, such as the Census's Composite Construction Cost index, on an annual basis. Davis and Heathcote (2004) compute a land price index based on this type of differential and find that land values have increased at an average annual rate of approximately 3.5% (inflation-adjusted) over the period 1975-2005. That price index may, however have an upward bias from not adequately

adjusting for quality changes.<sup>1</sup> Land price series available from the Bureau of Labor Statistics (see Figure 5) suggest behavior that is closer to the behavior of new home prices in Figure 1.

Finally, Figure 6 depicts the behavior of HP-filtered productivity growth (relative to a linear trend) over the postwar period. Clearly its pattern is similar to low-frequency movements in land and housing prices. Kahn and Rich (forthcoming) provide more detailed econometric estimates of a regime-switching model of the sort incorporated below into this paper, and find significant regime changes corresponding to the shifts depicted in the figure.

## 2 A Growth Model with Housing

This section presents a general equilibrium growth model that is capable of capturing the important stylized facts about housing and the economy. The model has two sectors, a “manufacturing” sector that produces non-housing related goods and services, as well as the capital (structures and durable goods) that go into housing services. A second sector uses capital, labor, and land to produce a flow of housing services. We focus on the rather special case of balanced aggregate growth, but then consider the behavior of the model under a regime-switching specification for productivity growth in the manufacturing sector. We assume a representative infinitely-lived agent who derives utility from two goods: a consumption good and housing services. Although the stochastic model that we will solve below will be in discrete time, for the sake of exposition we describe the model in its continuous time analogue.

Let  $\pi$  denote the fraction of time the representative agent works. There are  $N_t$  representative agents at time  $t$  supplying  $N_t\pi_t$  labor. We treat both  $\pi$  and  $N$  as exogenous, with  $N$  growing exponentially at constant rate  $\nu$ , and  $\pi$  constant for the

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<sup>1</sup>In addition, long-term evidence (from *Historical Statistics of the United States*) would seem to indicate that land prices have not grown as a proportion of real estate wealth. Other studies (e.g. Wheaton, 2006, and references contained therein) suggest that land prices largely just keep pace with inflation. Also, repeat-sales housing price indexes have an upward bias because they include price increases that may be attributable to improvements (additions, renovations, etc.).

sake of characterizing balanced growth paths. Later we will consider the impact of changing  $\pi$ . Variation in hours of work per capita (see Figure) has been an important contributor to low frequency changes in output per capita in the postwar U.S., and may also affect housing prices, as we shall see.

Let  $H$  denote the aggregate housing services produced per unit of time, and  $C$  the aggregate consumption good. We assume that  $C$  and homogeneous capital  $K$  are produced in a manufacturing sector  $m$  with capital  $K_m$ , labor  $N_m$ , and land  $L_m$ . Housing services are produced in a second sector that also combines capital, labor, and land (in different proportions from the manufacturing sector). Thus residential structures are treated as just another kind of capital input, albeit in a different production function. The stocks of capital and land in this sector would correspond to residential real estate, though the capital could include durable goods associated with housing services (e.g. appliances). Labor in this sector would be partly non-market household labor, and partly service sector labor (particularly for apartment buildings). We will assume (mainly for convenience) that capital's share is the same in both sectors, but labor's share is higher in manufacturing (implying of course that land's share is higher in the housing sector).

Let  $c$  and  $h$  denote per capita quantities of  $C$  and  $H$ , while  $k$ ,  $\ell$ ,  $k_i$ ,  $\ell_h$  refer to per worker quantities in sector  $i$  (e.g.  $k_h \equiv K_h/N_h$ ,  $k \equiv K/N$ , i.e. no subscript refers to aggregates), while  $n_i \equiv N_i/N$ , ( $i = m, h$ ). We assume the economy solves the following planner's problem:

$$\max U = \int_0^\infty e^{-\rho t} \ln \left( [\omega_c c^{(\epsilon-1)/\epsilon} + \omega_h h^{(\epsilon-1)/\epsilon}]^{\epsilon/(\epsilon-1)} \right) dt \quad (1)$$

subject to resource constraints

$$c + \dot{k}\pi + (\nu + \delta)k\pi = A_m k_m^\alpha \ell_m^{\beta_m} n_m \pi \quad (2)$$

$$h = A_h k_h^\alpha \ell_h^{\beta_h} n_h \pi \quad (3)$$

$$k_m n_m + k_h n_h = k \quad (4)$$

$$\ell_m n_m + \ell_h n_h = \ell \quad (5)$$

$$n_m + n_h = 1. \quad (6)$$

Total land  $L$  is assumed fixed, so  $\dot{\ell}/\ell = -\nu$ , and  $\dot{\ell}_i/\ell_i = -(\nu + \dot{n}_i/n_i)$ ,  $i = m, h$ . Average technological progress in sector  $i$ , i.e. the average growth rate of  $A_{it}$ , is denoted  $\gamma_i$  ( $i = m, h$ ). that is,  $E\{A_{it}/A_{it-1}\} = 1 + \gamma_i$ . We assume  $\pi$  is the same in the two sectors.

It is worth mentioning that technical progress in the  $h$  sector is unrelated to technological progress in construction. (In fact, home construction occurs in the  $m$  sector in this model.) Rather, it refers to an increase in the housing services from given stocks of  $K_h$ ,  $L_h$ , and labor inputs  $\pi N_h$ . What this means in practice depends on exactly what the term “housing services” encompasses, and on how one measures  $K_h$ . In the model it is assumed for simplicity to be indistinguishable from  $K_m$  other than by its allocation to the  $h$  sector. In particular, it is assumed to have the same price as  $K_m$  and  $C$ . In principle it would include both residential structures and housing service-related consumer durables (home appliances).  $L_h$  would include both non-market and market labor involved in household production—time devoted to housework, food preparation, home and yard maintenance, and the like.

The model obviously abstracts from a number of potentially important factors. First and foremost, the housing and construction sectors are heavily affected by government intervention, both via distortionary taxation and regulations. In particular, much land in the U.S. (and in most other countries as well) is neither residential nor commercial, and is either owned or heavily restricted in its use by the government. Second, there is tremendous heterogeneity in land and housing values. Land near navigable bodies of water, or ports, or along coastlines is much more valuable than land that does not have these features. Obviously this model will have nothing directly to say about the cross-sectional distribution of land values or housing prices (though many of the factors that affect them over time undoubtedly come into play in the cross-section as well). Nonetheless if all of these factors remain relatively

constant over time, then ignoring them in a model such as this should not be too great a sin.

Based on the modified Hamiltonian

$$\begin{aligned} \mathcal{H} = & \left[ \ln \phi(c, h) + \mu_m (A_m k_m^\alpha \ell_m^{\beta_m} \pi^{1-\alpha} n_m - c - k(\nu + \delta)) \right. \\ & \left. + \mu_h (A_h k_h^\alpha \ell_h^{\beta_h} \pi^{1-\alpha} n_h - h) \right] e^{-\rho t}, \end{aligned} \quad (7)$$

where  $\phi(c, h) \equiv [\omega_c c^{(\epsilon-1)/\epsilon} + \omega_h h^{(\epsilon-1)/\epsilon}]^{\epsilon/(\epsilon-1)}$ , the first-order conditions are:

$$\omega_c \phi^{-(\epsilon-1)/\epsilon} c^{-1/\epsilon} = \mu_m \quad (8)$$

$$\omega_h \phi^{-(\epsilon-1)/\epsilon} h^{-1/\epsilon} = \mu_h \quad (9)$$

$$\mu_m \beta_m A_m k_m^\alpha \ell_m^{\beta_m-1} \pi^{1-\alpha} = \mu_h \beta_h A_h k_h^\alpha \ell_h^{\beta_h-1} \pi^{1-\alpha-\beta_h} \quad (10)$$

$$\mu_m A_m k_m^{\alpha-1} \ell_m^{\beta_m} \pi^{1-\alpha} = \mu_h A_h k_h^{\alpha-1} \ell_h^{\beta_h} \pi^{1-\alpha-\beta_h} \quad (11)$$

$$\mu_m A_m k_m^\alpha \ell_m^{\beta_m} \pi^{1-\alpha} = \mu_h A_h k_h^\alpha \ell_h^{\beta_h} \pi^{1-\alpha-\beta_h} \quad (12)$$

$$\alpha A_m k_m^{\alpha-1} \ell_m^{\beta_m} \pi^{1-\alpha} = \rho + \delta + \nu - \dot{\mu}_m / \mu_m. \quad (13)$$

$\mu_m$  and  $\mu_h$  are shadow prices on the resource constraints (2) and (3). These can be shown to imply that

$$k_m = k_h = k \quad (14)$$

$$\frac{\ell_m}{\ell_h} = \frac{\beta_m}{\beta_h}, \quad (15)$$

The price of  $h$  in terms of  $c$  is

$$p \equiv \frac{\mu_h}{\mu_m} = \frac{A_m \ell_m^{\beta_m} \pi^{-\beta_m}}{A_h \ell_h^{\beta_h} \pi^{-\beta_h}} = \frac{A_m}{A_h} \left( \frac{\beta_m}{\beta_h} \right)^{\beta_m} \left( \frac{\ell_h}{\pi} \right)^{-(\beta_h - \beta_m)} \quad (16)$$

We will see that while (15) implies that while  $\ell_m$  and  $\ell_h$  have the same growth rate, that rate is not straightforward to compute, and is not necessarily constant. In the Cobb-Douglass case with  $\epsilon = 1$ , both will shrink exponentially at rate  $\nu$ , but

otherwise their rate of change depends on how expenditure shares evolve. For now we let  $\dot{\ell}_m/\ell_m = \dot{\ell}_h/\ell_h \equiv -(\lambda + \nu)$ , and will determine  $\lambda_t$  later. Here we just note that  $\lambda_t$  will generally be positive if expenditure shares on housing are growing, negative if they are shrinking. In particular, if  $p$  is constant, then  $\lambda = 0$ .

So we have, under certainty, and assuming  $\pi$  is constant:

$$\dot{p}/p = \gamma_m - \gamma_h + (\beta_h - \beta_m)(\lambda + \nu). \quad (17)$$

where  $\lambda$  is in fact a function of  $\dot{p}/p$ , one that equals zero when  $\dot{p}/p = 0$ . If  $\epsilon < 1$ , the behavior of  $\lambda$  reinforces price acceleration. The mechanism is that price appreciation causes labor and expenditures to shift toward the housing sector, which further drives up the price of land and the price of housing services. Thus growth in the price of housing services reflects both relative productivity growth in manufacturing, and the increasing scarcity of land.

## 2.1 Balanced Growth

Aggregate output per capita  $y$  (in terms of manufactured goods) is  $A_m k^\alpha \ell_m^{\beta_m} \pi^{1-\alpha} n_m + p A_h k^\alpha \ell_h^{\beta_h} \pi^{1-\alpha-\beta_h} n_h$ , or (after substituting for  $p$  and simplifying)

$$y = A_m k^\alpha \ell_m^{\beta_m} \pi^{1-\alpha}. \quad (18)$$

We then have

$$\dot{k} = A_m k^\alpha \ell_m^{\beta_m} \pi^{1-\alpha} - (c + ph) - k(\nu + \delta)$$

Let total expenditure  $c + ph$  be denoted by  $s$ . It also turns out that  $\mu_m = s^{-1}$ , hence  $\dot{\mu}_m/\mu_m = -\dot{s}/s$ , and we have

$$\dot{s}/s = \alpha A_m k^{\alpha-1} \ell_m^{\beta_m} \pi^{1-\alpha} - (\rho + \delta + \nu) \quad (19)$$

$$\dot{k}/k = A_m k^{\alpha-1} \ell_m^{\beta_m} \pi^{1-\alpha} - s/k - (\nu + \delta) \quad (20)$$

We can define aggregate balanced growth under certainty as an equilibrium path in which  $s$  and  $k$  both grow at a constant rate, and in which the interest rate (i.e. the marginal product of capital) is also constant. Clearly we need  $A_m \alpha k^{\alpha-1} \ell_m^{\beta_m}$  to be constant. Given the discussion in the previous subsection, it is clear that strictly speaking, balanced growth can only occur in the knife-edge cases of  $\epsilon = 1$  or  $\gamma_m - \gamma_h + (\beta_h - \beta_m) \nu = 0$ . For the sake of exposition we will assume the second condition holds in the long run, that is:

**Assumption A1:**  $\gamma_m = \gamma_h - (\beta_h - \beta_m) \nu$ .

We then have the result that along a balanced growth path,

$$\dot{k}/k = (\gamma_m - \beta_m \nu) / (1 - \alpha). \quad (21)$$

Note that this is also the growth rate of output per hour in the  $m$  sector. For balanced growth we also need  $s$  and  $k$  to grow at the same rate. This implies

$$s = (1 - \alpha) y + \rho k \quad (22)$$

on the balanced growth path.

Thus the aggregate economy, valued in terms of manufactured goods, behaves exactly as the standard neoclassical growth model, albeit only if A1 holds so that there is no long-term trend in  $p$ .<sup>2</sup> How does sectoral labor allocation evolve over time? Following Ngai-Pissarides (2004), let  $\sigma_h$  denote the share of expenditure on housing services  $h$  relative to total expenditure on goods  $s$ , and  $\sigma_c = 1 - \sigma_h$  the share of  $s$  spent on  $c$ :

$$\sigma_h \equiv \frac{ph}{s} = \left( \frac{\omega_h}{\omega_c} \right)^\epsilon p^{-(\epsilon-1)} \left[ 1 + \left( \frac{\omega_h}{\omega_c} \right)^\epsilon p^{-(\epsilon-1)} \right] \quad (23)$$

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<sup>2</sup>Note that if factor shares in the two sectors were identical, balanced growth would obtain for the aggregate economy even if  $p$  were growing over time. See Ngai-Pissarides (2004).

Then  $ph = A_mk^\alpha\pi^{1-\alpha}n_h = yn_h = ph = \sigma_h s$ , and we have

$$n_h = \sigma_h \frac{s}{y} \quad (24)$$

$$n_m = 1 - \frac{s}{y} + \sigma_c \frac{s}{y} \quad (25)$$

So while there is balanced growth in the aggregate, over time we have, assuming  $\epsilon < 1$ ,  $\sigma_h \rightarrow 1$ ,  $\sigma_m \rightarrow 0$ . Consequently, in the long-run  $n_h \rightarrow s/y$ ,  $n_m \rightarrow 1 - s/y$ , and eventually all but an infinitesimal amount of labor is going toward producing housing services or structures. This is presumably not a realistic implication, but the model can still be a reasonable description of behavior over a very long time period.

## 2.2 Stochastic Growth

To consider a stochastic, dynamic version of the model we will switch to discrete time notation. We will also suppress the labor supply term  $\pi$  for now by setting it equal to one. The resource constraints and first-order conditions are shown in an Appendix. Specifically, we will assume (admittedly rather artificially) that the mean growth rates in the model satisfy  $(1 + \gamma_m) = (1 + \gamma_h)(1 + \nu)^{-\beta_h}$  so that  $p_t$  has zero drift. We then suppose that the growth rate of  $A_h$  is fixed at  $\gamma_h$ , but that of  $A_m$  follows a Markov regime-switching process:

$$A_{mt}/A_{mt-1} = (1 + \tilde{\gamma}_{mt}) \eta_t / \eta_{t-1}$$

where

$$\tilde{\gamma}_{mt} = \begin{cases} \gamma_m^1 & \text{if } \xi_t = 1 \\ \gamma_m^0 & \text{if } \xi_t = 0 \end{cases} \quad (26)$$

$\eta_t$  is a transitory disturbance, and  $\xi_t$  is a state variable with Markov transition matrix  $Q$ , where  $Q[i, j] = \Pr(\xi_t = j | \xi_{t-1} = i)$ . Since the columns of  $Q$  must sum to one, we

write  $Q$  as

$$Q = \begin{bmatrix} q_1 & 1 - q_0 \\ 1 - q_1 & q_0 \end{bmatrix}.$$

If the diagonal elements of  $Q$  are close to one, the growth states will be highly persistent, and a shift from one state to the other will carry with it a sizeable adjustment in the long-term level of  $A_m$ . Since the stationary distribution of  $\{\xi^1, \xi^0\}$  is  $\{(1 - q_0) / (2 - q_1 - q_0), (1 - q_1) / (2 - q_1 - q_0)\}$ , we require that

$$\frac{1 - q_0}{2 - q_1 - q_0} (1 + \gamma_m^1) + \frac{1 - q_1}{2 - q_1 - q_0} (1 + \gamma_m^0) = (1 + \gamma_h) (1 + \nu)^{-\beta_h} \quad (27)$$

so that there is no trend in  $p$ . For concreteness we will call  $\xi^1$  the “high-growth” regime, and  $\xi^0$  the “low-growth” regime, i.e. we assume  $\gamma_m^1 > \gamma_m^0$ .

Following Hamilton (1994) and Farmer et al (2006), we can describe the above Markov chain as an AR(1) process. First, define

$$z_t = \begin{bmatrix} z_{1t} \\ z_{0t} \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \text{if } \xi_t = 1 \\ \begin{bmatrix} 0 \\ 1 \end{bmatrix} & \text{if } \xi_t = 0 \end{cases}. \quad (28)$$

Then  $z_t = Qz_{t-1} + v_t$ , where  $v_t = z_t - E_{t-1}z_t$ , so that  $E\{v_t|\xi_{t-1}\} = 0$ . Thus, for example, if  $\xi_{t-1} = 1$ , we have

$$v_t = \begin{bmatrix} v_{1t} \\ v_{0t} \end{bmatrix} = \begin{cases} \begin{bmatrix} 1 - q_1 \\ -(1 - q_1) \end{bmatrix} & \text{if } \xi_t = 1 \quad (q_1) \\ \begin{bmatrix} -q_1 \\ q_1 \end{bmatrix} & \text{if } \xi_t = 0 \quad (1 - q_1) \end{cases} \quad (29)$$

and if  $\xi_{t-1} = 0$ .

$$v_t = \begin{bmatrix} v_{1t} \\ v_{0t} \end{bmatrix} \begin{cases} \begin{bmatrix} q_0 \\ -q_0 \end{bmatrix} & \text{if } \xi_t = 1 \quad (1 - q_0) \\ \begin{bmatrix} -(1 - q_0) \\ 1 - q_0 \end{bmatrix} & \text{if } \xi_t = 0 \quad (q_0) \end{cases} \quad (30)$$

where the terms in parenthesis are conditional probabilities. Note that while  $E(v_t|\xi_{t-1}) = 0$ ,  $v_t$  is not identically distributed over time, as conditional distribution depends on  $\xi_{t-1}$ .

Now let  $\zeta_t \equiv N_t^{\beta_h - \beta_m} A_{mt} / A_{ht}$  and  $\tilde{\ell}_{mt} = \ell_{mt} N_t$ . As defined,  $z_t$  will have a unit root with zero drift, and  $\tilde{\ell}_{mt}$  will be nonlinearly (and inversely, assuming  $\epsilon < 1$ ) related to  $\zeta_t$ . That is, as  $\zeta_t$  wanders from any givent starting point,  $\tilde{\ell}_{mt}$  will wander in the opposite direction. We then have

$$p_t = (\beta_m / \beta_h)^{\beta_h} \tilde{\ell}_{mt}^{-(\beta_h - \beta_m)} \zeta_t. \quad (31)$$

Note that the comovements of  $\tilde{\ell}_{mt}$  and  $\zeta_t$  reinforce the impact of  $z_t$  on  $p_t$ . That is, the elasticity of  $p$  with respect to  $\zeta$  generally exceeds one. Finally, define  $w_t = (1 + \nu)^{\beta_h - \beta_m} (1 + \tilde{\gamma}_{mt}) / (1 + \gamma_h)$ , so that  $E(w_t) = 1$ .

## 2.3 Solving the Model

Since a number of the endogenous variables exhibit trend growth, it will be helpful to redefine them as follows:

$$\begin{aligned} \tilde{k}_t &= k_t / \left( \ell_{mt}^{\beta_m} A_{mt} \right)^{1/(1-\alpha)} \\ \tilde{\mu}_{mt} &= \mu_{mt} \left( \ell_{mt}^{\beta_m} A_{mt} \right)^{1/(1-\alpha)} \\ \tilde{\mu}_{ht} &= \mu_{ht} \left( \ell_{mt}^{\beta_m} A_{mt} \right)^{1/(1-\alpha)} \end{aligned}$$

$$\begin{aligned}
\tilde{x}_t &= \left[ \left( \frac{\ell_{mt}}{\ell_{mt-1}} \right)^{\beta_m} \frac{A_{mt}}{A_{mt-1}} \right]^{-1/(1-\alpha)} \\
\tilde{h}_t &= h_t / \left( \ell_{mt}^{\beta_m} A_{mt} \right)^{1/(1-\alpha)} \\
\tilde{c}_t &= c_t / \left( \ell_{mt}^{\beta_m} A_{mt} \right)^{1/(1-\alpha)}
\end{aligned}$$

Given (14)-(16) we can eliminate  $k_m$ ,  $k_h$ ,  $\mu_h$  and  $\ell_h$  and equations (4), (10), and (11) from the equilibrium conditions. Adding  $p_t$  to the system, we then get a system of eight equations for the eight unknowns  $\tilde{k}_t, \tilde{x}_t, \tilde{h}_t, \tilde{c}_t, \tilde{\ell}_{mt}, n_{ht}, p_t, \tilde{\mu}_{mt}$ :

$$(1 + \nu) \tilde{k}_t = \left( \tilde{k}_{t-1} \tilde{x}_t \right)^\alpha - \tilde{\mu}_{mt}^{-1} + (1 - \delta) \tilde{k}_{t-1} \tilde{x}_t. \quad (32)$$

$$p_t \tilde{h}_t = \left( \tilde{k}_{t-1} \tilde{x}_t \right)^\alpha n_{ht} \quad (33)$$

$$\tilde{\ell}_{mt} = \bar{L} (1 + [(\beta_h - \beta_m) / \beta_m] n_{ht})^{-1} \quad (34)$$

are the resource constraints, and the first-order conditions are

$$\omega_c \tilde{\phi}^{-(\epsilon-1)/\epsilon} \tilde{c}_t^{-1/\epsilon} = \tilde{\mu}_{mt} \quad (35)$$

$$\tilde{h}_t / \tilde{c}_t = (\omega_h / \omega_c)^\epsilon p_t^{-\epsilon} \quad (36)$$

$$(1 + \nu) \tilde{\mu}_{mt} = \theta E_t \left\{ (\tilde{\mu}_{mt+1} \tilde{x}_{t+1}) \left[ \alpha \left( \tilde{k}_t \tilde{x}_{t+1} \right)^{\alpha-1} + 1 - \delta \right] \right\} \quad (37)$$

$$p_t = \left( \frac{\tilde{\ell}_{mt}}{N_t} \right)^{-(\beta_h - \beta_m)} \frac{A_{mt}}{A_{ht}} \left( \frac{\beta_m}{\beta_h} \right)^{\beta_h} \quad (38)$$

$$\tilde{x}_t = \left[ \left( \frac{\tilde{\ell}_{mt}}{\tilde{\ell}_{mt-1} (1 + \nu)} \right)^{\beta_m} (1 + \tilde{\gamma}_{mt}) \eta_t / \eta_{t-1} \right]^{-1/(1-\alpha)} \quad (39)$$

where  $\theta = (1 + \rho)^{-1}$  is the consumer's time preference factor. As redefined, all eight variables are constant in the steady state (i.e. under A1 so that  $p$  is constant).

Unfortunately, unless  $\epsilon = 1$ , the solution to this problem off of the steady state is not straightforward. The difficulty is that as  $p$  grows (for example, in the high-growth regime),  $\hat{\ell}_m$  diminishes, but not at a constant rate, which means that  $\tilde{x}_t$  depends on

both the regime state  $\xi_{t-1}$  and on  $A_{mt}/A_{ht}$ . The reason for the latter dependency is that  $\tilde{\ell}_{mt}/\tilde{\ell}_{mt-1}$  depends on  $n_{ht}/n_{ht-1}$ , which cannot be constant because  $n_{ht}$  is bounded between 0 and 1.

We will approximate the solution to the model as follows: We will linearize the model around the steady state that is defined for  $\dot{p} = 0$ , given some initial value for  $p$ . While the sectoral variables in the model are locally nonstationary (because of the unit root in  $z_t$ ) the linearized aggregate economy, represented by the variables  $\tilde{k}$ ,  $\tilde{\mu}_m$ , and  $\tilde{x}$ , will be well-behaved around its steady state, since those variables return to their steady state values after a permanent change in  $p$ . The remaining variables in the system,  $\tilde{h}_t, \tilde{c}_t, \tilde{\ell}_{mt}, n_{ht}, p_t$ , enter the system in terms of changes.

We linearize this system as follows, letting variables without adornment indicate steady state values (or, in the case of  $n_h$  or  $c/h$ , some reasonably central value), and "ˆ" over a variable denote logarithmic deviation from the steady state:

$$\begin{aligned}
(1 + \nu) \hat{\mu}_{mt} &= x\theta E_t \left\{ [\alpha (kx)^{\alpha-1} + (1 - \delta)] \hat{\mu}_{mt+1} \right. \\
&\quad \left. + [\alpha^2 (kx)^{\alpha-1} + (1 - \delta)] \hat{x}_{t+1} - \alpha (1 - \alpha) (kx)^{\alpha-1} \hat{k}_t \right\} \\
(1 + \nu) \hat{k}_t &= [\alpha k^{\alpha-1} x^\alpha + x (1 - \delta)] (\hat{k}_{t-1} + \hat{x}_t) - (k\mu_m)^{-1} \hat{\mu}_{mt} \\
\hat{x}_t &= -(1 - \alpha)^{-1} (\Delta \hat{\ell}_{mt} + \hat{w}_t + \Delta \hat{\eta}_t) \\
\Delta \hat{\ell}_{mt} &= -[bn_n / (1 + bn_h)] \Delta \hat{n}_{ht} \\
\Delta \hat{n}_{ht} &= \Delta p_t + \Delta \hat{h}_t - \alpha (\Delta \hat{k}_{t-1} + \Delta \hat{x}_t) \\
\Delta \hat{\mu}_{mt} &= \frac{(1 - \epsilon)}{\epsilon} \Delta \hat{\phi} - (1/\epsilon) \Delta \hat{c}_t \\
\Delta \hat{h}_t - \Delta \hat{c}_t &= -\epsilon \Delta \hat{p}_t \\
\Delta \hat{p}_t &= -(\beta_h - \beta_m) \Delta \hat{\ell}_{mt} + \hat{w}_t + \Delta \hat{\eta}_t \\
\hat{w}_t &= \gamma_m^1 z_{1t} + \gamma_m^0 z_{0t}
\end{aligned}$$

where  $b = (\beta_h - \beta_m) / \beta_m$  and

$$\Delta \hat{\phi}_t = \frac{\omega_c c^{(\epsilon-1)/\epsilon}}{[\omega_c c^{(\epsilon-1)/\epsilon} + \omega_h h^{(\epsilon-1)/\epsilon}]} \Delta \hat{c}_t + \frac{\omega_h h^{(\epsilon-1)/\epsilon}}{[\omega_c c^{(\epsilon-1)/\epsilon} + \omega_h h^{(\epsilon-1)/\epsilon}]} \Delta \hat{h}_t$$

$$\equiv \psi \Delta \hat{c}_t + (1 - \psi) \Delta \hat{h}_t.$$

This system is of the form

$$0 = AX_t + BX_{t-1} + CY_t + DZ_t \quad (40)$$

$$0 = E_t \{FX_{t+1} + GX_t + HX_{t-1} + JY_{t+1} + KY_t + LZ_{t+1} + MZ_t\} \quad (41)$$

$$Z_{t+1} = NZ_t + \Theta_{t+1} \quad (42)$$

where

$$X_t = \begin{bmatrix} \hat{k}_t & \hat{k}_{t-1} & \hat{x}_t & \hat{\mu}_{mt} \end{bmatrix}' \quad (43)$$

$$Y_t = \begin{bmatrix} \Delta \hat{\ell}_{mt} & \Delta \hat{n}_{ht} & \Delta \hat{h}_t & \Delta \hat{c}_t & \Delta \hat{p}_t & \hat{w}_t \end{bmatrix}' \quad (44)$$

$$Z_t = \begin{bmatrix} z_{1t} & z_{0t} & \hat{\eta}_t & \hat{\eta}_{t-1} \end{bmatrix}' \quad (45)$$

and

$$A = \begin{bmatrix} 1 + \nu & 0 & -[\alpha k^{\alpha-1} x^\alpha + x(1 - \delta)] & (k\mu_m)^{-1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \alpha & \alpha & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -[\alpha k^{\alpha-1} x^\alpha + x(1-\delta)] & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\alpha & -\alpha & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \quad (46)$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\alpha)^{-1} & 0 & 0 & 0 & 0 & (1-\alpha)^{-1} \\ 1 & \frac{bn_h}{1+bn_h} & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & -(1-\psi)\frac{1-\epsilon}{\epsilon} & \frac{1-\psi(1-\epsilon)}{\epsilon} & 0 & 0 \\ 0 & 0 & 1 & -1 & \epsilon & 0 \\ \beta_h - \beta_m & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (47)$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & (1-\alpha)^{-1} & -(1-\alpha)^{-1} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ -\gamma_m^1 & -\gamma_m^0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
F &= \begin{bmatrix} 0 & 0 & -\theta[\alpha^2 k^{\alpha-1} x^\alpha + x(1-\delta)] & -\theta[\alpha k^{\alpha-1} x^\alpha + x(1-\delta)] \end{bmatrix}, \\
G &= \begin{bmatrix} \theta\alpha(1-\alpha)k^{\alpha-1}x^\alpha & 0 & 0 & 1+\nu \end{bmatrix}, \\
H &= \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, J = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} = K = L = M
\end{aligned}$$

$$N = \begin{bmatrix} q_1 & 1-q_0 & 0 & 0 \\ 1-q_1 & q_0 & 0 & 0 \\ 0 & 0 & \chi & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

and

$$\Theta_{t+1} = \begin{bmatrix} v_{1t} \\ v_{2t} \\ u_t \\ 0 \end{bmatrix}$$

where  $v_{1t}$  and  $v_{2t}$  are as defined earlier. We can then use the method of undetermined coefficients outlined by Uhlig (1997) to find the solution of the model.

## 2.4 Microeconomic Evidence

We examined data from the Consumer Expenditure Survey (CEX) to gauge the extent of housing service expenditure share variability as a function of total expenditures and of the relative price of housing services. To do so we construct rent (or owner's equivalent rent) relative to other expenditures, and match this up with data on housing prices by region, total expenditures, and demographic controls. If we find that the relative expenditure shares for housing services increase in response to changes in the relative price of those services, this would suggest an elasticity parameter ( $\epsilon$ ) less than one. [Results to be added]

## 2.5 Calibration

Most of the parameters take on standard values:  $\alpha = 0.33$ ,  $\nu = 0.01$ ,  $\delta = 0.05$  (a compromise for structures and equipment). The parameters  $\beta_h$  and  $\beta_m$  should reflect the shares of land in the cost of housing services and non-housing output respectively. We set  $\beta_h = 0.5$  and  $\beta_m = 0.05$ . The aggregate behavior of housing services expenditures and prices suggests  $\epsilon < 1$ ; we set  $\epsilon = 0.8$ . Since housing services represent about 20 percent of overall consumer expenditures, we set  $\omega_h = 0.2$ ,  $\omega_c = 0.8$ . We set the time preference rate  $\rho$  equal to 0.1. Finally, we choose the parameters of the regime-switching process for productivity to correspond roughly to the results in Kahn and Rich (forthcoming):  $\gamma_m^1 = 0.029$ ,  $\gamma_m^0 = 0.013$ ,  $q_1 = 0.99$ ,  $q_0 = 0.983$ . Thus high growth regimes are slightly more persistent than low-growth, and implied the overall mean growth rate of  $A_m$ ,  $\gamma_m$ , is 2.31 percent.

To begin, we compute a steady state under the assumption of constant  $\zeta$  (recall  $\zeta_t \equiv N_t^{\beta_h - \beta_m} A_{mt} / A_{ht}$ ). from

$$(1 + \nu)k = (kx)^\alpha - \mu_m^{-1} + (1 - \delta)kx. \quad (48)$$

$$ph = (kx)^\alpha n_h \quad (49)$$

$$\ell_m = \bar{L} (1 + [(\beta_h - \beta_m) / \beta_m] n_h)^{-1} \quad (50)$$

$$\omega_c \phi^{-(\epsilon-1)/\epsilon} c^{-1/\epsilon} = \mu_m \quad (51)$$

$$h/c = (\omega_h / \omega_c)^\epsilon p^{-\epsilon} \quad (52)$$

$$(1 + \nu)(1 + \rho) = x(\alpha(kx)^{\alpha-1} + 1 - \delta) \quad (53)$$

$$p = \ell_m^{-(\beta_h - \beta_m)} N^{(\beta_h - \beta_m)} \frac{A_m}{A_h} \left( \frac{\beta_m}{\beta_h} \right)^{\beta_h} \quad (54)$$

$$x = \left[ (1 + \nu)^{-\beta_m} (1 + \gamma_m) \right]^{-1/(1-\alpha)} \quad (55)$$

Equation (53) implies that steady state  $k$ , which we denote by  $k^*$ , is

$$k^* = \left[ \frac{\alpha (1 + \nu)^{-\beta_m} (1 + \gamma_m)}{(1 + \nu) (1 + \rho) \left[ (1 + \nu)^{-\beta_m} (1 + \gamma_m) \right]^{1/(1-\alpha)} - (1 - \delta)} \right]^{1/(1-\alpha)}$$

The parameter choices above imply a steady-state real interest rate of 5.68 percent.

## 2.6 Model Simulations

The solution of the model takes the form

$$\begin{aligned} X_t &= \Gamma X_{t-1} + \Phi Z_t \\ Y_t &= \Lambda X_{t-1} + \Psi Z_t. \end{aligned}$$

The key to doing interesting simulations is to take the peculiar error structure of the disturbance process into account. Even though the conditional expectation of the errors in the  $z_t$  process (the regime states) is zero, actual realizations of zero are not possible, and in fact given the values of  $q_1$  and  $q_0$ , a small error (of absolute value  $1 - q_0$  or  $1 - q_1$ ) that leaves  $z$  unchanged is highly likely in any given time period. So rather than consider a one-time shock to  $v$ , it makes sense to consider a single large shock (a regime-switch) followed by a sequence of identical small shocks that leave the regime unchanged for an extended period of time.

Figure 7 gives an example of this type of simulation. The economy is in the low growth regime in periods 1 to 5, and then switches to the high growth regime, where it remains. The chart plots the behavior of  $k$  and  $p$  (logarithmically, but including growth trends), along with  $A_m$ . Remember that  $p$  is the price of housing services, essentially a rental price, not the asset price of a house and its land. Of course the two will move together in steady state, but interest rate changes will imply different dynamics in response to shocks. Later drafts of this paper will examine the behavior of housing prices, interpreted as the price of the capital and land allocated

to producing housing services. Observe that  $k$  initially dips down in response to the regime shift. This is because faster growth raises the equilibrium interest rate. But  $p$  responds smoothly, drifting down during the low-growth regime, then accelerating after the regime shift.

## 2.7 Changes in Labor Supply

Next we will examine the impact of permanent changes in labor supply. In keeping with the paper's emphasis on frequency behavior, for now we will limit the analysis to comparisons of balanced growth paths. [to be added]

## 3 Conclusions

[To be added]

## 4 Appendix

The equilibrium conditions in discrete time are as follows:

Resource constraints:

$$\begin{aligned}
c_t + (1 + \nu) k_t - k_{t-1} (1 - \delta) &= A_{mt} k_{mt-1}^\alpha \ell_{mt}^{\beta_m} \pi_t^{1-\alpha-\beta_m} n_{mt} \\
h_t &= A_{ht} k_{ht-1}^\alpha \ell_{ht}^{\beta_h} \pi_t^{1-\alpha-\beta_h} n_{ht} \\
k_{mt-1} n_{mt} + k_{ht-1} n_{ht} &= k_{t-1} \\
\ell_{mt} n_{mt} + \ell_{ht} n_{ht} &= \ell_t \\
n_{mt} + n_{ht} &= 1.
\end{aligned}$$

First-order conditions:

$$\begin{aligned}
\omega_c \phi^{-(\epsilon-1)/\epsilon} c_t^{-1/\epsilon} &= \mu_{mt} \\
\omega_h \phi^{-(\epsilon-1)/\epsilon} h_t^{-1/\epsilon} &= \mu_{ht} \\
\mu_{mt} \beta_m A_{mt} k_{mt-1}^\alpha \ell_{mt}^{\beta_m-1} \pi_t^{1-\alpha-\beta_m} &= \mu_{ht} \beta_h A_{ht} k_{ht-1}^\alpha \ell_{ht}^{\beta_h-1} \pi_t^{1-\alpha-\beta_h} \\
\mu_{mt+1} A_{mt+1} k_{mt}^{\alpha-1} \ell_{mt+1}^{\beta_m} \pi_t^{1-\alpha-\beta_m} &= \mu_{ht+1} A_{ht+1} k_{ht}^{\alpha-1} \ell_{ht+1}^{\beta_h} \pi_t^{1-\alpha-\beta_h} \\
\mu_{mt} A_{mt} k_{mt-1}^\alpha \ell_{mt}^{\beta_m} \pi_t^{1-\alpha-\beta_m} &= \mu_{ht} A_{ht} k_{ht-1}^\alpha \ell_{ht}^{\beta_h} \pi_t^{1-\alpha-\beta_h} \\
E_t \left\{ \mu_{mt+1} \left[ A_{mt+1} \alpha k_{mt}^{\alpha-1} \ell_{mt+1}^{\beta_m} \pi_t^{1-\alpha-\beta_m} + 1 - \delta \right] \right\} \theta &= \mu_{mt} (1 + \nu).
\end{aligned}$$

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Figure 1: Real Price of New Homes (Quality-Adjusted)

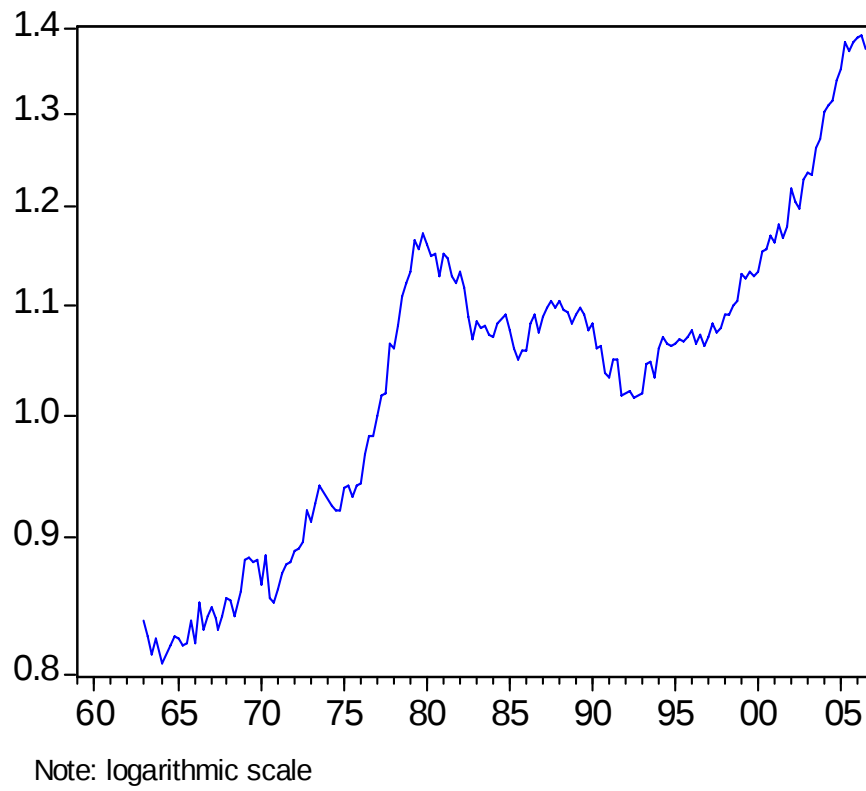


Figure 1:

Figure 2a: Ratio of Housing Wealth to Consumption

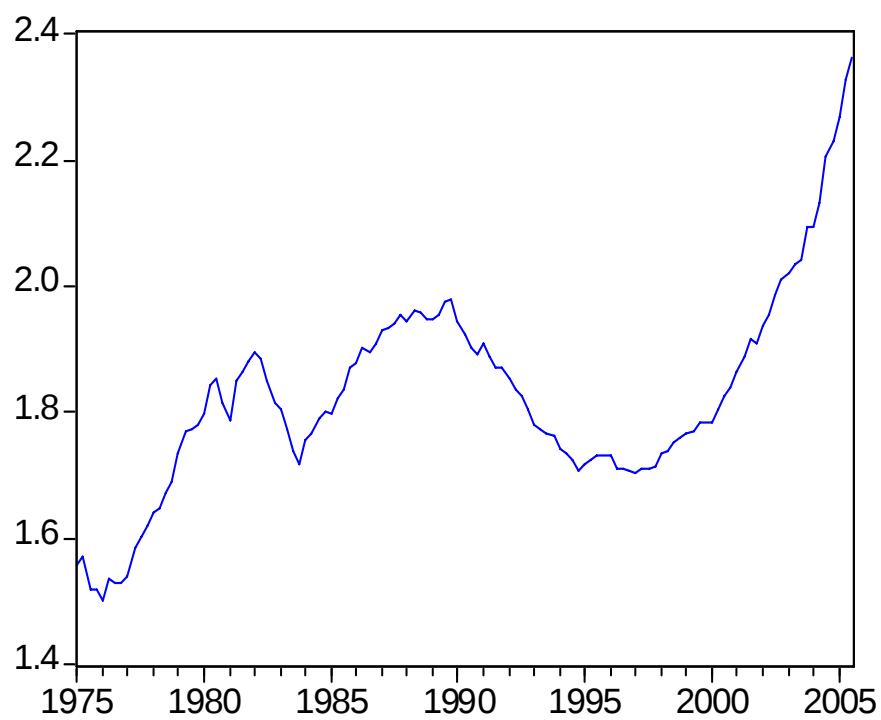


Figure 2b: Ratio of Housing Wealth to Total Net Worth

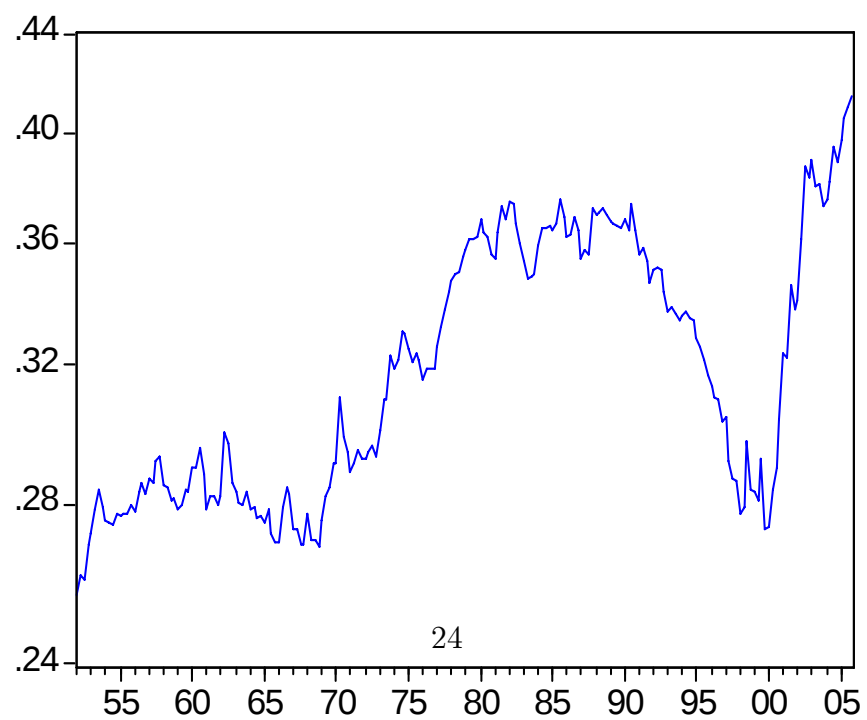
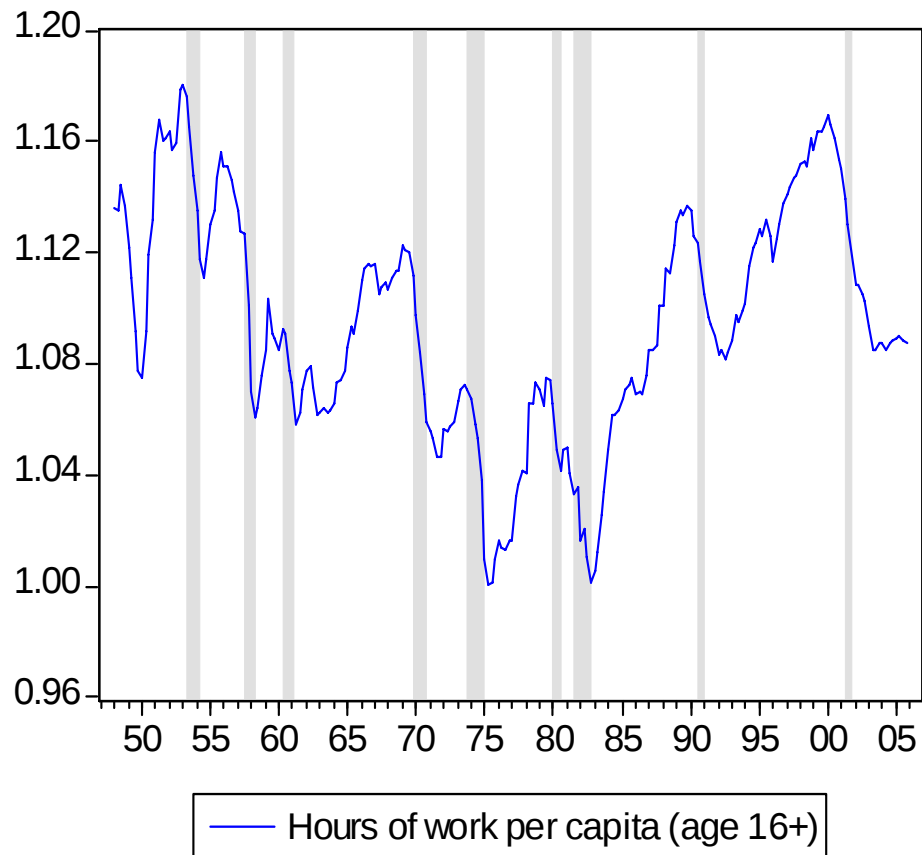


Figure 3: Trends in Per Capita Hours



Note: Shaded areas are NBER-defined recessions

Figure 3:

Figure 4: Housing Services: Expenditures and Prices

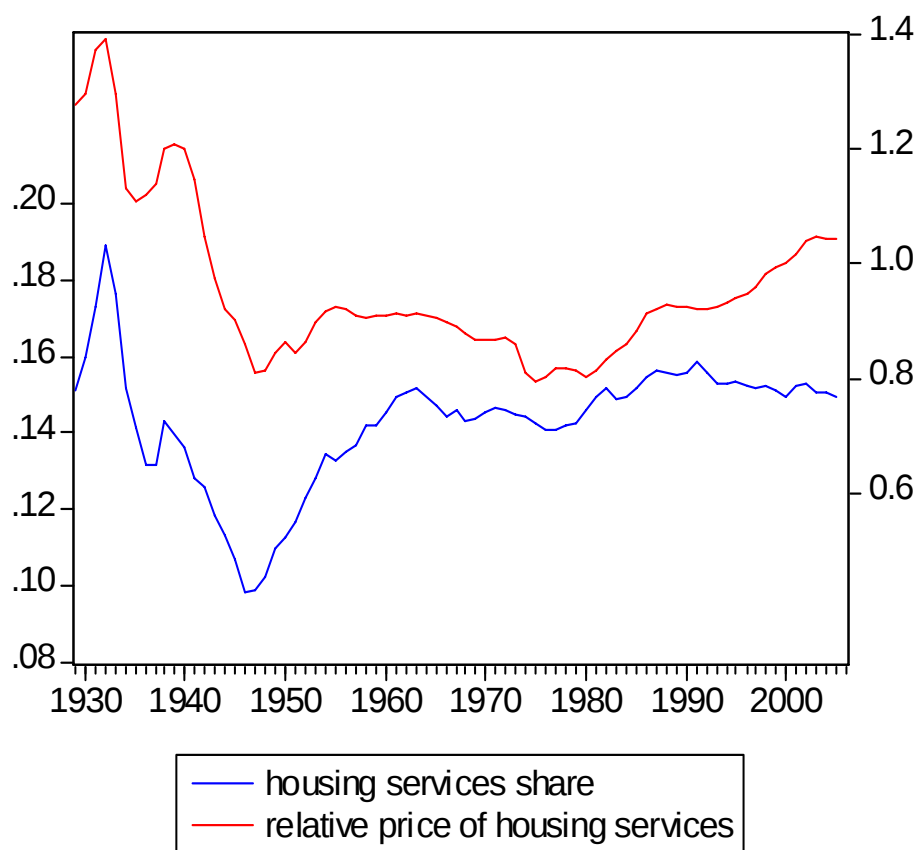
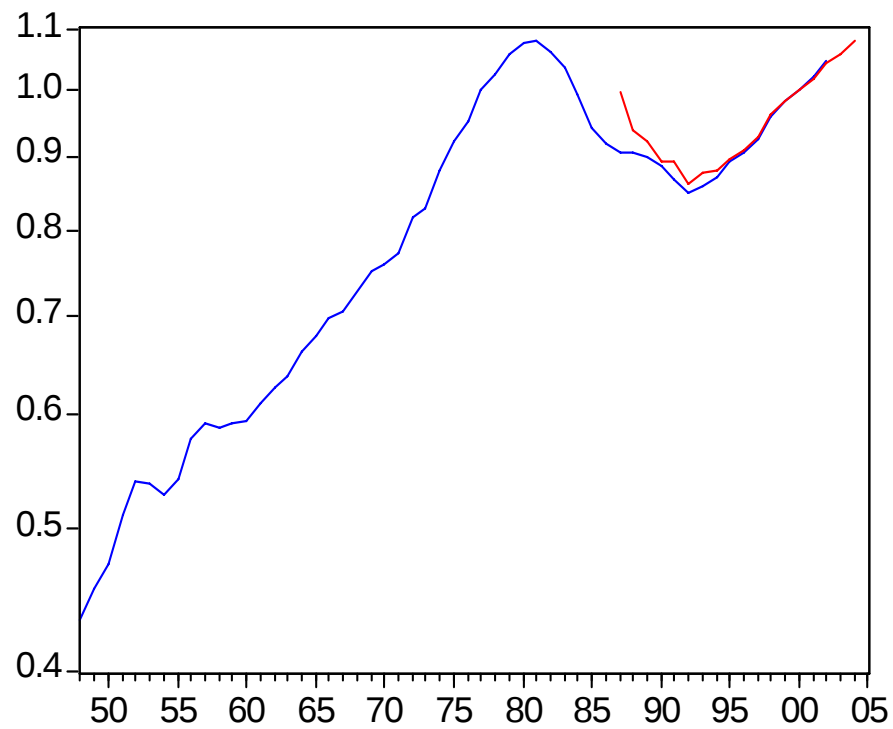


Figure 4:

Figure 5: Real Price of Land



Source: BLS

Note: Logarithmic scale. The two series are from different vintages of BLS data.

Figure 5:

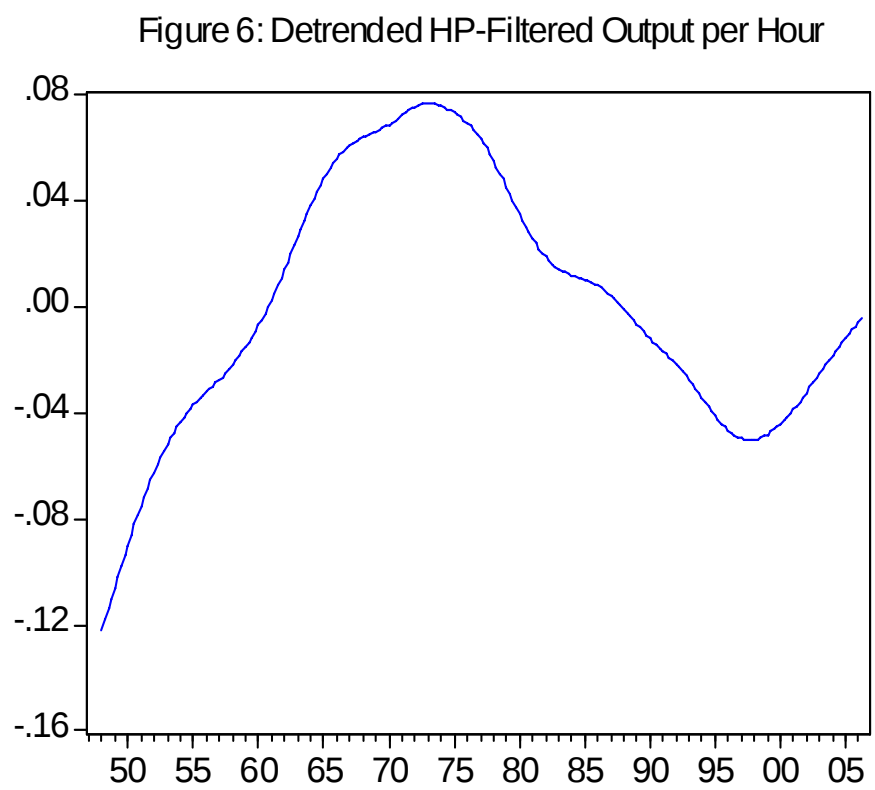


Figure 6:

Figure 7: Model Simulation of Regime Switch

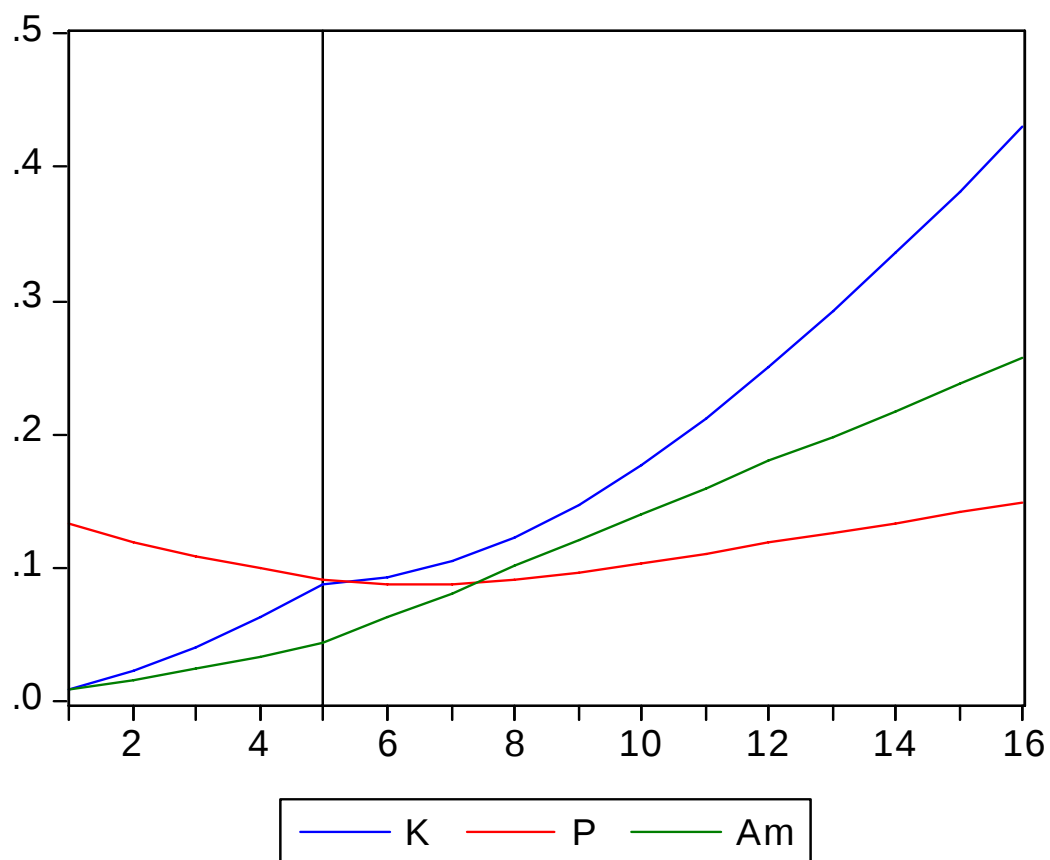


Figure 7: