Entrepreneurship with borrowing constraints in a model of economic development

Rui Castro, Gian Luca Clementi, and Aubhik Khan*

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Very preliminary and incomplete

Abstract

We develop a quantitative dynamic general equilibrium model of economic development with limited enforceability of loan contracts. Our goal is to evaluate the extent to which a plausibly calibrated model, where countries vary in the extent to which contracts are enforceable, is able to explain cross-country differences in total factor productivity and income per capita.

In our model production is undertaken by entrepreneurs that vary in their ability. Entrepreneurs borrow to finance investment, and endogenous borrowing constraints arise in response to the limited enforceability of debt contracts. In this setting, entrepreneurial wealth serves as collateral and alleviates the borrowing constraint. As a result, all else equal, wealthier entrepreneur undertake more investment and operate larger firms. As entrepreneurial ability varies over time and across entrepreneurs, this misallocates capital across production units and reduces total factor productivity.

Another important channel through which borrowing constraints affect total factor productivity lies in the occupational choice response of individuals. By reducing the returns to entrepreneurial activity and thus the number of individuals who become entrepreneurs, more severe borrowing constraints lead to the concentration of production across fewer enterprises. Given decreasing returns to capital and labour within each firm, this reduces aggregate total factor productivity.

An extension of our basic model introduces sectoral production and endogenises the set of sectors active at any time. This allows us to address cross-country evidence on the higher relative price of investment goods, and the lower share of such goods in GDP, in poorer economies.

*email addresses: rui.castro@umontreal.ca, gclement@stern.nyu.edu and mail@aubhik-khan.net, respectively.

1 Introduction

A large body of empirical work has arisen suggesting that, not only is financial development coincident with economic development, but that there is a causal relation wherein the efficiency of financial markets and institutions affect the level of economic development.[1] At the same time, researchers have concluded that differences in total factor productivity are the leading determinant of cross-country variation in incomes per capita.[2] We address both sets of observations using a quantitative dynamic general equilibrium model where the limited enforceability of loan contracts affects not only the accumulation of capital, but also its allocation across productive enterprises. Allowing for occupational choices within households, we establish a connection between the severity of borrowing constraints and the decision to become an entrepreneur. In our model, the level of entrepreneurship is endogenous and less financially developed economies will have less entrepreneurship. Consequently, we link the efficiency of financial contracts to the scale of production itself and show that more severe borrowing constraints not only misallocate inputs across production locations, but also reduce the number of enterprises operated. Both mechanisms reduce total factor productivity as well as income per capita. Our goal is to develop a quantitative model of economic development and evaluate the extent to which cross-country differences in the level of financial development, proxied here by the severity of endogenous borrowing constraints, may explain differences in income per capita and total factor productivity across countries.

In our model economy individuals have uncertain lifetimes, and each period a constant measure of individuals is born. In the first period of life, individuals invest in human capital. At the end of this initial period of life, they draw a level of entrepreneurial ability. Entrepreneurs vary in their ability, which evolves stochastically through time and, for those that choose to operate an enterprise, determines total factor productivity at that firm. Production is distributed across the set of active entrepreneurs. There is occupational choice and individuals that choose not to operate an enterprise supply their labour as workers.

At the start of each period, after observing their current ability or total factor productivity, any entrepreneur may choose to shut down his enterprise and continue as a worker. Alternatively

[1] A review of the empirical evidence relating financial development and economic growth is contained in the survey by Levine (1999), see also Demirgtuc-Kunt and Levine (2001).

[2] See, for example, Prescott (1998) and Hall and Jones (1999).

he may allocate capital and labour towards current production. Capital is financed using the entrepreneur's own wealth and through loan contracts with competitive financial intermediaries. Entrepreneurs do not have to use their loans for production. Instead they may default, retaining a fraction of their capital stock, and continue thereafter as a worker. The fraction of capital that may be retained in default captures the enforceability of financial contracts. When this fraction is positive, the limited enforceability of debt contracts leads to endogenous borrowing constraints wherein any entrepreneur that seeks a loan first commits his own wealth, as both investment and collateral, into his firm.

In this setting, when loan contracts are fully enforceable, the occupational choice is independent of individual wealth and solely determined by ability. However the limited enforceability of debt contracts introduces a positive correlation between entrepreneurial wealth and the size of the firm. Wealthier entrepreneurs will operate larger firms than poorer, but more productive, entrepreneurs. Hence borrowing constraints misallocate resources across enterprises, and this reduces aggregate total factor productivity. Moreover, they also reduce the returns to entrepreneurship, and thus the fraction of individuals engaged in this activity. As production in each firm is subject to decreasing returns to scale, the reduction in the extent of entrepreneurship that accompanies more severe borrowing constraints implies a further reduction in aggregate total factor productivity through an extensive margin. This effect is independent, though related, to the misallocation of resources given the scale of production, that is, given the number of entrepreneurs.

Finally, human capital accumulation is stimulated by financial development in that economies with milder borrowing constraints will have a larger number of enterprises. This increases the returns to labour, thereby increasing the return on human capital investment. Higher levels of human capital not only alleviate the loss of workers in an economy where more individuals have chosen to become entrepreneurs, they also amplify the effect of differences in the efficiency of financial intermediation. Economies with greater enforceability of loan contracts have more entrepreneurship and a higher levels of labour, measured in efficiency units.

We take as given cross-country differences in the enforceability of loan contracts, captured in our model by the fraction of capital that may be retained when in default. Nonetheless, borrowing constraints are endogenous in our model in that the size of a loan is determined by the borrower's willingness to repay. However the severity of borrowing constraints varies with the enforceability

of loan contracts. Our goal is to develop a quantitative model of economic development with entrepreneurship and use it to evaluate the extent to which cross-country differences in the level of financial development may hinder the allocation of inputs.[3] Preliminary results indicate that mild borrowing constraints in a plausibly calibrated model imply a sharp reduction in the scale of production, that is, in the fraction of individuals that operate enterprises. This, in turn, reduces output and the real interest rate. It also increases the concentration of production, thereby reducing total factor productivity.

Our work is related to Amaral and Quintin (2005), where there is also occupational choice in a model with varying entrepreneurial ability and endogenous borrowing constraints. The most important difference between our work and theirs is that we use a general equilibrium framework in that the real interest rate is endogenous and, moreover, changes with the extent to which loan contracts are enforceable. We find that this is an important channel through which financial markets affect the economy. Additionally, we allow for uncertain lifetimes and human capital accumulation. Long-lived entrepreneurship directly changes the nature of the borrowing constraint by altering the quantitative magnitudes involved in continuation and default. It also has an indirect effect in magnifying differences in wealth across entrepreneurs, and thus differences in the distribution of capital. Human capital accumulation propagates differences in the severity of borrowing constraints across countries into larger differences in income per capita. Importantly, since our setup is less stylized than Amaral and Quintin's (2005) it can not only be more easily calibrated, it can also linked to a broader set of development facts, such as cross-country differences in human capital accumulation.

We plan on extending our basic model in at least two ways. First, it contains several simplifying assumptions. Specifically, it assumes that all bequests are accidental and that, as a result, all young households have the same wealth. This limits the inequality of wealth in the economy and, in this sense, the effect of borrowing constraints. Additionally, we assume that once an entrepreneur shuts down, he must be a worker for the remainder of his life. In keeping with this assumption, we do not allow workers to become entrepreneurs. Again, this limits the effects of borrowing constraints by reducing the value of default. We plan to address each of these simplifications in revisions of

[3] Greenwood and Jovanovic (1996) and Khan (2001) develop theoretical models where both economic and financial development are endogenous.

the basic model.

The second extension to the basic model is aimed at addressing the development facts highlighted by Eaton and Kortum (2001) and Hsieh and Klenow (2006). More specifically, these authors have pointed out that poorer countries have higher relative investment prices, lower shares of equipment production, and a smaller variety of equipment goods being produced. Our idea is to introduce sectoral heterogeneity in our basic model in order to be able to address these facts. More specifically, we introduce sectoral differences in fixed costs of investment, and link such heterogeneity to the evidence provided by Rajan and Zingales (1998). Since this evidence supports higher fixed costs in equipment-producing sectors, this technological disadvantage translates into higher relative equipment prices and lower equipment production. When equipment is allowed to be traded internationally, poorer countries also produce a smaller range of equipment goods.

2 The model

There is a unit measure of individuals in the economy with uncertain lifetimes. After the first period of life, each individual is identified by its occupation, entrepreneurial productivity and wealth. Occupational choice involves the decision to allocate time to either the labour market or entrepreneurship. Entrepreneurial ability evolves stochastically over the lifetime, and determines total factor productivity. Production requires capital, labour and the entrepreneur's time. At the beginning of each period, after observing his current productivity, but before committing inputs to production, an entrepreneur may decide to shut down. Such decisions are permanent, and any exiting entrepreneur becomes a worker for the rest of his life. Furthermore, if a household initially chooses to become a worker, it forfeits its potential enterprise and may never become an entrepreneur.[4] It follows that entrepreneurial ability is irrelevant to a worker. All individuals value the homogenous consumption good each period, and may accumulate wealth.

Individuals born at the same time are initially identical. They have a common level of ability, normalized to 1, and wealth is determined by accidental bequests from those that died last period.[5]

[4] In this section we describe the basic model, and indicate extensions that are in progress. One such extension allows for bi-directional occupational mobility. This allows for workers to accumulate wealth and become an entrepreneur.

[5] Extensions of this basic model will allow for intentional bequests. This will propagate inequality over time thereby amplifying the effects of borrowing constraints.

New households use their wealth to finance consumption and human capital accumulation. There are no human capital decisions after the first period of life.

In the first period of their lives, given their initial wealth, a_0, individuals must choose consumption, savings and human capital, (c, a, h). At the end of the period, after having determined their current decision rules, they draw an initial level of entrepreneurial ability, θ, which is a Markov Chain, $\Pr\left\{\theta = \theta_i, \theta' = \theta_j\right\} = \pi^{\theta}_{ij}$, $i, j = 1, \ldots, N_\theta$. The initial draw is from the invariant distribution implied by $\left(\pi^{\theta}_{ij}\right)$, denoted $\left(\pi^{\theta}_{0j}\right)$. Observing their initial entrepreneurial ability, individuals must decide whether to become an entrepreneur, or a worker. As already mentioned, if an entrepreneur ever decides to shut down his enterprise, he becomes a worker for the remainder of his life.

While entrepreneurs bear risk, driven by shocks to their ability, there is no uncertainty within the period and occupational choices occur after the observation of the current shock. It then follows that after observing his current level of ability, an entrepreneur will investment his savings, a, into his enterprise, and choose some level of borrowing, $k - a$. At this point the entrepreneur may default, retain λk fraction of his capital stock, and work as a labourer for the remainder of his life.[6] Should he choose to not default, then he will hire l efficiency units of labour giving total production, $y = \theta h^{\alpha_h}\left(k^{\alpha} l^{1-a}\right)^{\nu}$. Thereafter the loan is repaid at the rental rate $(r + \delta)$ and the entrepreneur allocates his income across consumption and savings.

Workers with human capital h, earn labour income wh. They also have wealth a which earns a real rate of return r. Each period a worker with (h, a) solves a familiar consumption and savings decision. There are no other decisions for a worker and their entrepreneurial ability, θ, is irrelevant.

Individuals die with probability $(1 - \pi^s)$, each period after the first. Thus all individuals have a certain initial period of existence, thereafter there is a constant probability of death.

Let the aggregate state of the economy be S, which, at any time, involves a distribution of entrepreneurs over productivity levels and human capital, and the measure of workers with each level of human capital. When describing individual problems we assume $S' = \Gamma(S)$, a law of motion which we will solve later on. The real interest rate, r, and real wage, w, are both functions of this aggregate state.

[6] Extensions to the basic model allow for occupational choice after default. This amplifies the effect of the borrowing constraint by increasing the option value of default.

2.1 Individual decisions

2.1.1 Human capital accumulation

In the initial period of life all individuals are allocated a_0. This is funded by accidental bequests from all households that died in the previous period and the number of new households is equal to $(1-\pi^s)$ where π^s is the number of households that died in the previous period. Each new households must decide how much human capital to purchase at relative price p^h and how much to save, a'. Next, they realise their initial entrepreneurial productivity parameter, θ, and choose whether to become an entrepreneur or worker.

$$\begin{aligned} V^0(a_0;S) &= \max_{c,h,a',x^0} \left(U(c)+\beta V^1\left(a',h;S'\right)\right) && (1)\\ &\text{subject to} \\ c+p^h h+a' &\leq (1+r)\,a_0 && (2)\\ V^1\left(a',h;S'\right) &= \sum_{i=1}^{N_\theta} \pi^\theta_{0,i} \max\left\{V^e\left(\theta_i,a',h;S'\right),V^w\left(a',h;S'\right)\right\} && (3) \end{aligned}$$

The expected discounted lifetime utility represented by V implies the optimal choice of occupation at the beginning of the next period, and is defined over the expected value of this choice in (3). Let $c^0(a_0)$ describe the optimal choice of consumption, $h^0(a_0)$ the optimal choice of human capital and the optimal level of savings for these households. All decision rules are also functions of the aggregate state, but this argument is suppressed for readability.

Let $x^0(a_0,\theta)=1$ if a young household with wealth a_0 and entrepreneurial productivity θ decides to operate a firm in the next period, it is 0 otherwise.

$$x^0(a_0,\theta)=1 \text{ iff } V^e\left(\theta,A^0(a_0),h^0(a_0);S'\right)\geq V^w\left(A^0(a_0),h^0(a_0);S'\right) \quad (4)$$

There will be $\pi^\theta_{0,j}$ such households. Finally, the relative price of human capital is, in the current model, will be simply assumed to be determined by a constant marginal cost production technology that converts p^h units of output into 1 unit of human capital.

2.1.2 Entrepreneurs and Workers

At the start of any period, an entrepreneur with current productivity θ, human capital, h, and wealth a must decide whether to continue operating his enterprise.

$$V(\theta, a, h; S) = \max\left\{V^e(\theta, a, h; S), V^w(a, h; S)\right\}$$

The indicator function $x(\theta, a, h) = 1$ if the entrepreneur decides to continue in his current occupation.

$$x(\theta, a, h) = 1 \text{ iff } V^e(\theta, a, h; S) \geq V^w(a, h; S) \tag{5}$$

If he continues in his existing occupation, he must choose a level of borrowing, $k - a$. Thereafter, the entrepreneur may default, retain λk and become a worker for the rest of his life, or hire l efficiency units of labour and organise production.

The optimal contract is effectively a static one, lending at the economy-wide rental rate r, subject to an endogenous borrowing constraint. Borrowing occurs after the current realization of entrepreneurial ability, θ, and repayment is prior to the resolution of any future uncertainty. In this context, as in Cagetti and DiNardi (2006), limited enforceability implies a non-state contingent repayment at the economywide rental rate $(r + \delta)$ while imposing an individual rationality constraint on the borrower.[7]

A continuing entrepreneur's problem is described below.

[7] If capital were predetermined within the firm, then the contract would become state-contingent as in Marcet and Marimon (1992) and Cooley, Marimon and Quadrini (2004). Both of these particular papers assume risk-neutral entrepreneurs that consume nothing until their own lifetime utility entitlement within the contract has risen, because of binding enforcement constraints in past periods, to such a level that the unconstrained level of investment is undertaken. See also Albuquerque and Hopenhayn (2004).

$$V^e(\theta, a, h; S) = \max_{k,l,c,a',x} \left(U(c) + \beta \pi^s \sum_{j=1}^{N_\theta} \pi_{ij}^\theta V\left(\theta', a', h; S'\right) \right) \quad (6)$$

subject to

$$y = \theta h^{\alpha_h} \left(k^\alpha l^{1-\alpha}\right)^\nu \quad (7)$$

$$c + a' \leq y - wl - (r + \delta)(k - a) \quad (8)$$

$$U(c) + \beta \pi^s \sum_{j=1}^{N_\theta} \pi_{ij} V\left(\theta', a', h; S'\right) \geq V^w(\lambda k, h; S) \quad (9)$$

Equation (9) ensures that the entrepreneur finds it worthwhile, given his optimal plan (k, l, c, a'), to repay his loan $k - a$ instead of defaulting. Default allows him to retain λk, but prevents him from ever operating another enterprise and V^w is the value function for a worker. Let $k(\theta, a, h)$ and $l(\theta, a, h)$ define the capital and labour allocated to an entrepreneur of type (θ, a, h). Further, let $A^e(\theta, a, h)$ be the decision rule for a' and $c^e(\theta, a, h)$ the decision rule for c.

Workers solve the following problem.

$$V^w(a, h; S) = \max\left(U(c) + \beta \pi^s V^w\left(a', h; S'\right)\right) \quad (10)$$

subject to

$$c + a' \leq (1 + r) a + wh \quad (11)$$

$$a \geq 0. \quad (12)$$

Let $A^w(\theta, a, h)$ be the decision rule for a' and $c^w(\theta, a, h)$ be the decision rule for consumption for a worker of type (θ, a, h). It is worth mentioning that workers may wish to borrow, depending on how the economy is developing over time. However we assume that they are unable to do so.

2.2 Equilibrium

The aggregate state vector, S, includes a distribution of entrepreneurs over (θ, h, a), μ^e, and workers over (a, h), μ^w. While workers continue to draw entrepreneurial ability, θ, they cannot return to entrepreneurship. Therefore their ability is irrelevant to the aggregate state of the economy.

Aggregate output is the sum of production across locations.

$$Y = \int \left(\theta h^{\alpha_h}\right) \left(k\left[\theta, a, h\right]^{\alpha} l\left[\theta, a, h\right]^{1-\alpha}\right)^{\nu} d\mu^{e}\left(\theta, a, h\right) \tag{13}$$

Aggregate capital for next period is the sum of savings across all individuals, and aggregate consumption is defined similarly.

$$K' = \int A^{e}\left(\theta, a, h\right) d\mu^{e}\left(\theta, a, h\right) + \int A^{w}\left(a, h\right) d\mu^{w}\left(a, h\right) + A^{0}\left(a_0\right)\left(1 - \pi^{s}\right) \tag{14}$$

$$C = \int c^{e}\left(\theta, a, h\right) d\mu^{e}\left(\theta, a, h\right) + \int c^{w}\left(a, h\right) d\mu^{w}\left(a, h\right) + c^{0}\left(a_0\right)\left(1 - \pi^{s}\right) \tag{15}$$

The goods market clears when the sum of expenditures, whether consumption, investment or human capital accumulation, is less than aggregate production.

$$C + \left(K' - \left(1 - \delta\right) K\right) + p^{s} h^{0}\left(a_0\right)\left(1 - \pi^{s}\right) \leq Y \tag{16}$$

Labour market clearing requires that the demand for employment, in efficiency units, across firms is equal to the total stock of worker hours, which is the right hand side of the following constraint.

$$\int l\left[\theta, a, h\right] d\mu^{e}\left(\theta, a, h\right) = \int h d\mu^{w}\left(a, h\right) \equiv L \tag{17}$$

The allocation of aggregate capital is restricted by the supply, the aggregate capital stock being given by the right hand side of the following constraint.

$$\int l\left[\theta, a, h\right] d\mu^{e}\left(\theta, a, h\right) = \int a d\mu^{e}\left(\theta, a, h\right) + \int a d\mu^{w}\left(a, h\right) + a_0\left(1 - \pi^{s}\right) \equiv K \tag{18}$$

Let $x_H\left(h\right)$ be an indicator function that equals 1 only if $h \in H$, and 0 otherwise; x_Θ and x_A are defined analogously. Remember that if an entrepreneur of type $\left(\theta_j, a, h\right)$ chooses to continue operation, then $x\left(\theta_j, a, h\right) = 1$. The evolution of the aggregate state, in particular μ^e and μ^w is described below.

$$
\begin{aligned}
\mu^{e\prime}(\Theta, H, A) &= \pi^s \sum_{j=1}^{N_\theta} x_\Theta(\theta_j) \sum_{i=1}^{N_\theta} \pi_{ij}^\theta \int_{\{h\in H,\ A^e(\theta,a,h)\in A\}} x(\theta_j, a, h)\, d\mu^e(\theta_i, d[h\times a]) \\
&\quad + x_H\left(h^0(a_0)\right) x_A\left(A^0(a_0)\right) \sum_{j=1}^{N_\theta} x_\Theta(\theta_j)\, \pi_{0,j}^\theta x^0(a_0, \theta_j)\,(1-\pi^s) \\
\mu^{w\prime}(H, A) &= \pi^s \int_{\{h\in H,\ A^w(a,h)\in A\}} d\mu^w(a, h) \\
&\quad + x_H\left(h^0(a_0)\right) x_A\left(A^0(a_0)\right) \sum_{j=1}^{N_\theta} \pi_j^0 \left[1 - x^0(a_0, \theta_j)\right](1-\pi^s) \\
&\quad + \pi^s \sum_{j=1}^{N_\theta} x_\Theta(\theta_j) \sum_{i=1}^{N_\theta} \pi_{ij}^\theta \int_{\{h\in H,\ A^e(\theta,a,h)\in A\}} [1 - x(\theta_j, a, h)]\, d\mu^e(\theta_i, d[h\times a])
\end{aligned}
$$

The initial wealth of the young, a_0, is also part of the aggregate state vector, and it evolves according to the following equation.

$$
a_0^{'} = (1-\pi^s)\left(\int A^e(\theta, a, h)\, d\mu^e(\theta, a, h) + \int A^w(a, h)\, d\mu^w(a, h)\right).
$$

The aggregate state vector is $S = (\mu^e, \mu^w, a_0)$.

Definition 1 *A recursive equilibrium is a set of functions*

$$
\left((V^e, V^w), (k, l, c^e, A^e, x), (c^w, A^w), \left(c^0, A^0, h^0, x^0\right), \Gamma\right)
$$

such that V^e solves the entrepreneur's problem (6) and (k, l, c^e, A^e, x) attains the supremum, V^w solves the worker's problem (10) and (c^w, A^w) attains the maximum, $\left(c^0, A^0, h^0, x^0\right)$ solves the young individuals' problem described by (1); the goods and labour markets clear, and the evolution of the aggregate state is consistent with individual decision rules.

3 Preliminary Results

3.1 The full enforcement benchmark

Before presenting our numerical results, we derive some analytical results for the full enforcement economy, described by the above model when 9 is suppressed. This allows us to illustrate, analytically, some properties of the model whereby the scale of production affects aggregate output and

total factor productivity. Furthermore, the full enforcement economy is a useful benchmark against which we contrast our results for the economy with borrowing constraints.

We begin by deriving some results that characterize the entrepreneur's capital and labour demand. In the absence of any borrowing constraint, an entrepreneur described by (θ, a, h) maximises profits from the operation of his firm in the current period independently of his wealth, a, by solving

$$\max_{k,l} \theta h^{\alpha_h} \left(k^{\alpha\nu} l^{(1-\alpha)\nu}\right) - wl - (r+\delta)\, k,$$

where w and r both be functions of the aggregate state, S, and production is $y = \theta h^{\alpha_h} \left(k^{\alpha\nu} l^{(1-\alpha)\nu}\right)$. The first-order conditions for (k, l) are listed below are

$$\alpha\nu \frac{y}{k} = r + \delta \tag{19}$$

and

$$(1-\alpha)\,\nu \frac{y}{l} = w. \tag{20}$$

They imply that $l = \frac{1-\alpha}{\alpha} \frac{r+\delta}{w} k$ and the entrepreneur's earnings from operating his firm are $(1-\nu)\, y$.

The solution for capital and labour at the firm is given by the following functions.

$$k = \left[\alpha\nu\theta h^{\alpha_h} \left(\frac{1-\alpha}{\alpha}\right)^{\nu-\alpha\nu} (r+\delta)^{\nu-\alpha\nu-1}\, w^{-\nu+\alpha\nu}\right]^{\frac{1}{1-\nu}} \tag{21}$$

$$l = \left[\alpha\nu\theta h^{\alpha_h} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha\nu} (r+\delta)^{-\alpha\nu}\, w^{-1+\alpha\nu}\right]^{\frac{1}{1-\nu}} \tag{22}$$

These, in turn, imply that production is

$$y = \left[\alpha\nu\theta h^{\alpha_h} \left(\frac{1-\alpha}{\alpha}\right)^{\nu(1-\alpha)} (r+\delta)^{-\alpha\nu}\, w^{-\nu(1-\alpha)}\right]^{\frac{1}{1-\nu}}$$

and that the entrepreneur's income from operating his firm is

$$(1-\nu)\,\theta^{\frac{1}{1-\nu}} h^{\frac{\alpha_h}{1-\nu}} \left[\alpha\nu \left(\frac{1-\alpha}{\alpha}\right)^{\nu(1-\alpha)} (r+\delta)^{-\alpha\nu}\, w^{-\nu(1-\alpha)}\right]^{\frac{1}{1-\nu}}. \tag{23}$$

3.1.1 Aggregation

In the absence of borrowing constraints, entrepreneurs production decisions are divorced of his or her wealth, as indicated by (21) and (22). In such instances there is a simple aggregation of production, given the total number of enterprises. This allows us to illustrate the effect of entrepreneurship on aggregate output.

Define

$$\Delta = \left[\alpha\nu h^{\alpha_h} \left(\frac{1-\alpha}{\alpha} \right)^{-\alpha\nu} (r+\delta)^{-\alpha\nu} w^{-\nu+\alpha\nu} \right]^{\frac{1}{1-\nu}},$$

so that we may write (22) as $l = \theta^{\frac{1}{1-\nu}} \Delta \left[w^{\nu-1} \frac{1-\alpha}{\alpha} \right]^{\frac{1}{1-\nu}}$ and (21) as $k = \theta^{\frac{1}{1-\nu}} \Delta \left[(r+\delta)^{\nu-1} \left(\frac{1-\alpha}{\alpha} \right)^{\nu} \right]^{\frac{1}{1-\nu}}$. Now let E represent the total number of firms operated in the current period, $E = \int d\mu^e$. This implies that the number of firms with current productivity θ_i is then $\pi^{\theta}_{0,i} E$.

Summing over firms, we derive the aggregate quantities of labour and capital.

$$\begin{aligned} L &= \left(\sum_{i=1}^{N_\theta} \pi^{\theta}_{0,i} \theta_i^{\frac{1}{1-\nu}} \right) \Delta \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{1-\nu}} \frac{1}{w} E \\ K &= \left(\sum_{i=1}^{N_\theta} \pi^{\theta}_{0,i} \theta_i^{\frac{1}{1-\nu}} \right) \Delta \left(\frac{1-\alpha}{\alpha} \right)^{\frac{\nu}{1-\nu}} \frac{1}{r+\delta} E \end{aligned}$$

Using the notation $\Theta = \left(\sum_{i=1}^{N_\theta} \pi^{\theta}_{0,i} \theta_i^{\frac{1}{1-\nu}} \right)$, we find that the allocation of aggregate labour and capital is given by

$$l = \frac{\theta^{\frac{1}{1-\nu}}}{\Theta E} L \tag{24}$$

$$k = \frac{\theta^{\frac{1}{1-\nu}}}{\Theta E} K. \tag{25}$$

This, in turn, gives $y = \frac{\theta^{\frac{1}{1-\nu}} h^{\alpha_h} K^{\alpha\nu} L^{(1-a)\nu}}{(\Theta E)^{\nu}}$. Since aggregate production is $Y = \sum_{i=1}^{N_\theta} \pi^{\theta}_{0,i} E y_i$,

$$\begin{aligned} Y &= \frac{\left(\sum_{i=1}^{N_\theta} \pi^{\theta}_{0,i} \theta_i^{\frac{1}{1-\nu}} \right) E h^{\alpha_h} K^{\alpha\nu} L^{(1-a)\nu}}{(\Theta E)^{\nu}} \\ &= \Theta^{1-\nu} E^{1-\nu} h^{\alpha_h} K^{\alpha\nu} L^{(1-a)\nu}. \end{aligned} \tag{26}$$

Aggregate production is then increasing in the measure of entrepreneurs, E, given decreasing returns to scale at each production location. This result, for the full enforcement economy, relies on the optimal allocation of inputs across locations implied by (24) and (25). Nonetheless, it reflects the same extensive margin through which entrepreneurship affects total factor productivity in the model with endogenous borrowing constraints. Given the aggregate stock of capital and labour, increases in the number of enterprises, which we refer to as the scale of production, leads to a rise in gross domestic product. The derivation of aggregate production also allows to develop a share expression for output at each firm, as a function of its productivity θ_i,

$$y_i = \frac{\theta_i^{\frac{1}{1-\nu}}}{\Theta E} Y. \tag{27}$$

Finally, using (24), (25) and 27 in (19) and (20), we have

$$\alpha\nu\frac{Y}{K} = r + \delta \text{ and } (1-\alpha)\,\nu\frac{Y}{L} = w.$$

Thus, in the full enforcement economy where inputs are allocated solely on the basis of their marginal products and independently of entrepreneurial wealth, the real interest rate and the real wage follow the conventional marginal product relation with aggregate output

3.2 Stationary equilibrium

We report some very preliminary results on the effect of borrowing constraints for a version of the model without a human capital investment decision. The results here are for the stationary equilibria of the model with borrowing constraints which is contrasted with the full enforcement model describe above. At present we only report the effect of borrowing constraints on the extent of entrepreneurship, the distribution of wealth, and real interest rates and wages. However these preliminary results still indicate the strong effects of even a mild form of borrowing constraint on the scale of production in the model. Thus they illustrate the potential of the model to explain relatively large differences in incomes per capital and total factor productivity as a function of differences in the extent to which contracts are enforceable. Future versions will introduce human capital accumulation and evaluate the effects of limited enforceability on aggregate output and total factor productivity.

In parameterizing the model, we set the length of a period to 5 years, and the subjective discount factor $\beta = 0.843$, implying that a complete markets model would yield a 4 percent real rate of interest per annum. Entrepreneurs and workers work for an average of 9 periods, or 45 years, which implies $\pi^s = 0.8899$.

We assume returns to scale at the firm level of 0.85, consistent with the value reported by Atkeson, Khan and Ohanian (1996). Given this value for ν, we set $\alpha = 0.36$ thus determining capital's share of output. Given our abstraction from human capital in the simple example solved here, we set $\alpha_h = 0.01$ and $h = 1$. Next, $\delta = 0.2661$ again implying a 6 percent depreciation rate of capital per year. The variability of the stochastic process for entrepreneurial ability could be calibrated to match data on the distribution of wealth, as in Cagetti and DiNardi (2006). However, for the purposes of our current example, we simply choose an annual persistence of 0.9, which implies a persistence per period in the model of 0.59. The standard deviations of innovations, over these 5-yearly periods, is set to 0.1, and the shock process is discretised this using a 25 state Markov Chain.

We contrast a full enforcement economy, where markets remain incomplete, for example individuals do not have access to contingent claims against shocks to their ability, or access to annuities, against an economy where entrepreneurs may retain $\lambda = 0.5$ fraction of their capital stock in the event of default. The endogenous borrowing constraint prevents such default by limiting the amount lent to any entrepreneur as a function of his wealth, a, productivity, θ. The value of default is endogenous in that λ affects the equilibrium real interest rate, and wage rate, and thus the expected lifetime utility of workers.

The stationary equilibrium of the full enforcement economy has a real wage of 0.333 and an annual real interest rate of 6 percent. Entrepreneurship is the occupational choice of 11.6 percent of the population. The unconditional mean of entrepreneurial ability is 1.0, and the occupational choice for young individuals is characterized by a threshold ability level of 1.055. Those with θ_i higher than the threshold become entrepreneurs, the remainder choose to work for a real wage. This threshold does not change with wealth, but entrepreneurs who subsequently experience ability shocks that take them below the threshold shut down their enterprise and become workers. Overall there is considerable income, and thus consumption, volatility for entrepreneurs, but none for workers. This leads entrepreneurs to have higher savings rates, to buffer against shocks to their

earnings. Thus the mean level of assets held by workers is 0.265 while that of entrepreneurs is 0.4102. There is also inequality of wealth amongst workers, originating from differences in wealth across entrepreneurs when they shut down and join the labour force. The coefficient of variation of asset holdings are 4.9128 and 1.5717 for entrepreneurs and workers, respectively.

The model with endogenous borrowing constraints has a real wage of 0.30, quite close to that of the full enforcement economy, but an annual real interest rate of 4.9 percent, more than a full percentage point lower. This reflects the misallocation of capital across production units due to the borrowing constraint. While the borrowing constraint we have assumed is relatively mild, in that only half of the capital stock may be retained when an entrepreneur defaults and such an action prevents any further entrepreneurial activity, over 77 percent of entrepreneurs are constrained in their borrowing. The reduced returns to entrepreneurship halve the number of entrepreneurs, which falls to 6.42 percent of the population. It is this sharp drop in entrepreneurial activity in response to even mild borrowing constraints that promises potentially large productivity losses. In this model, borrowing constraints not only misallocate inputs across production units, but actually reduce the number of production units themselves. In this simple example, we see the number of locations falls by almost half.

Examining the assets of workers and entrepreneurs in the economy with endogenous borrowing constraints, the mean wealth of entrepreneurs is 1.3182 while that of workers is 0.2579. The higher wealth of entrepreneurs in the economy with borrowing constraints is a result of the incentive to accumulate wealth which serves as collateral in production loans. There is also higher inequality in this economy, with the coefficient of variation of entrepreneurial wealth rising to 5.4739 while that for workers is 2.2315. Figure 1 shows the distribution of entrepreneurs in the economy over ability levels and wealth.

4 Further work

The preliminary results reported above involved several simplifications. The first is also made in the model section. We did not allow for intentional bequests. Introducing this element into the model will perpetuate inequality over time. In a model with borrowing constraints where differences in entrepreneurial wealth affect the allocation of capital and labour across production locations, such intergenerational linkages, once introduced, will drive larger differences with the

full enforcement economy. The second simplifying assumption made here was to eliminate human capital accumulation. This implies that efficiency units of labour fall with a fall in the number of workers, and thus provides a countervailing force against large changes in entrepreneurship, given that this affects the number of workers through the occupational choice constraint.

A more substantive extension involves the introduction of sectoral production, as discussed in the introduction. This will allow us to link borrowing constraints to the measure of intermediate capital goods produced in the economy, and introduce an additional extensive margin through which the enforceability of contracts affects not only total factor productivity as well as income per capita, but also the share of investment goods produced in the economy and their relative price.

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Figure 1: the distribution of wealth of entrepreneurs

