

On-the-Job Search, Minimum Wages, and Labor Market Outcomes in an Equilibrium Bargaining Framework¹

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Abstract

We look at the impact of a binding minimum wage on labor market equilibrium outcomes and welfare distributions in a model of matching and bargaining in the presence of on-the-job search. We show that in any employment spell, which is defined as a sequence of jobs at different firms in which there are no intervening spells of unemployment, only at the onset of the first job can an employee be paid the minimum wage. The model is estimated using the method of simulated maximum likelihood estimation on a recently-collected sample taken from the Survey of Income and Program Participation. Even though individuals will be paid the minimum wage for a small proportion of their labor market careers, we find the efficient level of the minimum wage to be high, even when allowing for endogenous contact rates as in Pissarides (2000). This result stems from the low estimates of worker bargaining power and the relative search efficiency of unemployed and employed individuals.

1 Introduction

In this paper we extend the minimum wage analysis of Flinn (2005) to include on-the-job search. His continuous-time model was set in a stationary environment in which meetings between unemployed searchers and firms with vacancies occurred at a constant, though endogenously-determined rate. When a contact was made, an i.i.d. draw from a match distribution $G(\theta)$ was taken, and the parties used a Nash bargaining rule to determine whether an employment relationship should be established, and, if so, at what wage payment to the worker. A government-mandated minimum wage, m , appeared as a side-constraint to the bargaining problem, in that all bargained wages had to be no less than m . Using the matching function formulation of Pissarides (2000), the contact rate was made endogenous. Using point sample data taken from the Current Population Survey in conjunction with a firm profit rate measure, he was able to estimate the primitive parameters of the model and, using an aggregate welfare criterion, was able to determine an optimal minimum wage for the population. Using 1996 CPS data, when the federal minimum wage was \$4.25 per hour, he found that the optimal minimum wage was approximately double that amount if the contact rate was fixed, while allowing the contact rate to be endogenous resulted in an optimal minimum wage of \$3.35 an hour.

The addition of on-the-job search to the bargaining model is a critical extension. From descriptive evidence, we know that in the U.S. labor market there are a large number of job-to-job transitions that don't involve an intervening spell of unemployment. By ignoring this fact, there exists the potential for a significant degree of model misspecification leading to inconsistent estimates of model parameters and misleading policy implications drawn from those estimates. The addition of on-the-job search is likely to be particularly relevant for purposes of investigating minimum wage effects. The work of Leighton and Mincer (1981), and more recently by Acemoglu and Pischke (2002), investigate the potential impacts of minimum wage laws on life-cycle wage profiles through reductions in general human capital investment. While we do not consider human capital investment in our model, the potential for minimum wage impacts on the shape of lifetime wage profiles exists due to effects on the bargaining environment. In the models of Postel-Vinay and Robin (2002) and Dey and Flinn (2005), employees were able to extract sizable portions of the match specific surplus at a particular job only when they had significant changes to their opportunity costs associated with remaining at the firm. These shocks occurred when they came into contact with another employer. This feature is shared by our model as well. While only efficient

moves will occur in equilibrium - that is, individuals will always accept the job at which their match value is highest - individuals benefit from a competing offer which gives them a stronger bargaining position when renegotiating the wage contract. By reducing the set of match values that result in an employment contract, the minimum wage typically results in a lower equilibrium employment rate, but the fact that individuals generally spend a larger proportion of their labor market career in the unemployed state is not necessarily the most significant welfare cost. An additional consequence is that employed individuals will get obtain fewer offers from the "viable" part of the match distribution, thereby reducing the level of interfirm competition for labor services. By delaying entry into employment, the minimum wage delays the start of the firm competition process during which significant wage gains occur. By estimating the primitive parameters of the process, we are able to quantify these costs. Our analysis of minimum wages within the bargaining with on-the-job search framework leads to some testable implications. We show that within any employment spell, which is defined as a sequence of jobs with different firms with no intervening spell of unemployment, only at the first job in the sequence can the employee be paid the minimum wage. Moreover, the minimum wage may only be paid to the employee at the beginning of their first job in the employment spell, since the arrival of alternative, viable employment opportunities with other firms will result in a renegotiated wage payment larger than the minimum. This implies that individuals will spend only a small amount of employment time at jobs paying the minimum wage if the contact rate of employees with other firms with vacancies is sufficiently large.

In introducing on-the-job search the econometric framework employed in Flinn (2005) had to be extended. The point sample CPS data he used are no longer sufficient for determining the parameters characterizing the more complicated employment processes modeled in this paper. We utilize event history data taken from the Survey of Income and Program Participation (SIPP), with the data coming primarily from the period 1997-2000. We show that all of the primitive parameters of the model are identified using our event history data once we fix the average labor share (from other data sources) and the discount rate. Since we are working with an on-the-job search model, it is necessary for us to introduce measurement error into the model to account for the job-to-job transitions with wage reductions that are observed in the data. We cannot utilize classical measurement error, however, since the model implies that a positive proportion of wages will be massed at the (binding) minimum wage, while the addition of i.i.d. measurement error to all observations would result in the implication that no wages would be observed at the minimum wage - a counterfactual result. We therefore implement a measurement error model in which classical

measurement error is assumed to be present in a proportion τ of observations, and we estimate the parameter τ and the variance of the measurement error shock.

Perhaps the biggest substantive extension of the paper is the incorporation of employed searchers in the matching function. Most treatments of this problem assume that employed and unemployed searchers devote equal amounts of time to search, so that the total number of searchers equals the number of labor market participants (see, e.g., Pissarides (2000)). Estimates of contact rates of unemployed and employed searchers in partial equilibrium analyses consistently indicate that the contact rate of the unemployed is at least three times larger than that of the employed, making this assumption unattractive. We define the searching population as equal to $U + \nu E$, where U is the size of the unemployed population, E is the size of the employed population, and ν is a technological parameter partially characterizing the search technology. We derive a consistent estimator of ν , and find a value of 0.13. This estimate has important implications for the welfare analyses we conduct.

The studies of Dey and Flinn (2005) and Cahuc, Postel-Vinay, and Robin (2005) have shown that the addition of on-the-job search to the worker-firm bargaining problem results in much lower implied values of the worker's bargaining power. Essentially, the worker's bargaining power mainly derives from her ability to receive outside offers that enable her to extract significant portions of the match value, at least over the course of the job spell. Our estimates follow this pattern as well. Where Flinn (2005) estimated a worker bargaining power parameter of 0.40 in the case of no on-the-job search, our estimates with on-the-job search (and a different data set and econometric framework) put the value around 0.19. This low value implies that the match value distribution has very little mass at values of less than \$10 per hour, for example. Thus even a high minimum wage rate of \$8 per hour, say, implies that only a small proportion of viable match opportunities would be lost. Thus high minimum wages do not lead to significant delays in the interfirm competition for labor, which is responsible for workers getting a large share of the match value over time. This does not imply that a high minimum wage will have beneficial welfare effects in general equilibrium, however, since firms could reduce their supply of vacancies, thus impeding the contact rate process to a significant degree. Given our estimates, this is not the case, primarily as a result of the estimate of ν we referred to above. As minimum wages are increased, some jobs are terminated, but they are the low match value jobs. These individuals switch from employed search to unemployed search, thus increasing the effective supply of searchers. This increase in supply is met by an increase in the supply of vacancies, thus actually increasing contact rates over some range of m . This effect

accounts for the high welfare-maximizing minimum wage of \$8.18 we derive. While it is generally thought the general equilibrium effects mitigate the value of high minimum wages, we have found that this not need be the case, and have identified a rather interesting mechanism that subverts conventional wisdom.

The remainder of this paper proceeds as follows. In Section 2 we derive the model. In Section 3, we present the analysis of the effects on labor market outcomes of minimum wages with OTJ search. Section 4 contains a discussion of the data used to estimate the equilibrium model while Section 5 develops the econometric methodology. Section 6 presents the empirical results and Section 7 presents some simulations. In Section 8 we conclude.

2 Model

In this section we describe the behavioral model of labor market search with matching and bargaining in which the interactions between applicants and firms are constrained by the presence of a minimum wage. The minimum wage, m , is set by the government and is assumed to apply to all potential matches. We assume that the only compensation provided by the firm is the wage. Thus there are no other forms of compensation the firm can adjust so as to “undo” the minimum wage payment requirement.

2.1 A Model with Exogenous Contact Rates

The model assumes labor market stationarity and is formulated in continuous time. We assume that there exists an invariant, technologically-determined distribution of worker-firm productivity levels which is given by $G(\theta)$. When a potential employee and a firm meet the productive value of the match θ is immediately observed by both the applicant and the firm. At this point a division of the match value is proposed using a Nash bargaining framework. The searcher’s instantaneous discount rate is given by $\rho > 0$. The rate of (exogenous) termination of employment contracts is $\eta \geq 0$.

We denote the current labor market state of an unemployed individual by $\theta^*(m)$, which is the value of θ required for an unemployed searcher and a firm to initiate an employment contract. While unemployed individuals search, their instantaneous utility is given by b , which can assume positive or negative values. Unemployed workers meet firms at the exogenous rate λ_n , at which point the productive value of the match θ is immediately observed by both the applicant and the

firm. If both the firm and the worker accept the match, then they split the it using a Nash bargaining framework and determine a wage $w(\theta, \theta^*(m))$. It is assumed that labor is the only factor of production and if an individual and a firm meet, but the firm "passes" on the applicant, then the firm receives a value of 0. This is the firm's disagreement value in the Nash bargaining framework. Likewise, the disagreement value for the searcher is the value of continued search denoted by $V_n(m)$.

While employed, workers meet firms at the exogenous rate λ_e which is independent of the employed worker's current match value. For simplicity, we assume that OTJ search is costless. Letting w represent the worker's wage, we denote the current labor market state of an employed individual by (θ, w) and any potential new state by (θ', w') . We now consider the rent division problem facing a currently employed agent who encounters a new potential employer.

Let there be a currently employed individual with wage w and match value $\theta > \theta^*(m)$, who meets a new potential employer with match value θ' . We assume that the potential match value will only be reported to the current employer if the employee has an incentive to do so. One situation in which this will be the case is when $\theta' > \theta$. When this occurs, we assume that a bargaining process for the individual's services begins between the current and potential employers and stops when one of the firms' surplus reaches zero. This will clearly be the current employer when $\theta' > \theta$. Let the maximal value of the match θ to the worker be given by $Q(\theta)$. Then the objective function for the Nash bargaining problem when $\theta' > \theta$ is:

$$S(\theta', w', \theta) = \{V_e(\theta', w') - Q(\theta)\}^\alpha \times \{V_f(\theta', w') - 0\}^{1-\alpha}$$

where $V_f(\theta', w')$ denotes the new firm's value of the match, assuming that each firm's threat point is zero, and α represents the bargaining power parameter of the worker.

The firms value of the current employment contract is defined as follows. Consider an infinitesimally small period of time Δt . Over this "period", the firm earns a profit of $(\theta - w) \Delta t$, which is discounted back to the present with the "infinitesimal" discount factor $(1 + \rho \Delta t)^{-1}$. With probability $\eta \Delta t$, the match is exogenously terminated and the firm earns no profit. With probability $\lambda_e \Delta t$, the worker receives a job offer from an alternative firm. If he reports this offer to his current firm, his wage might be renegotiated. With probability $(1 - \lambda_e \Delta t - \eta \Delta t)$, the worker does not receive another job offer and he is not exogenously dismissed over the "period" Δt . In this case the status quo is maintained. Finally, the term $o(\Delta t)$ represents the probability that two or more events occur over the "period" Δt , and has the property that $\lim_{\Delta t \rightarrow 0} \left(\frac{o(\Delta t)}{\Delta t} \right) = 0$. We denote the

value to the firm as:

$$\begin{aligned}
V_f(\theta, w) = & \left(\frac{(\theta - w)\Delta t}{1 + \rho\Delta t} \right) + \left(\frac{\eta\Delta t}{1 + \rho\Delta t} \times 0 \right) + \left(\frac{\lambda_e\Delta t}{1 + \rho\Delta t} \int_{\hat{\theta}(w)}^{\theta} V_f(\theta, w(\theta, \tilde{\theta})) dG(\tilde{\theta}) \right) + \\
& + \left(\frac{\lambda_e\Delta t G(\hat{\theta}(w))}{1 + \rho\Delta t} \times V_f(\theta, w) \right) + \left(\frac{(1 - \lambda_e\Delta t - \eta\Delta t)}{1 + \rho\Delta t} \times V_f(\theta, w) \right) \\
& + \left(\frac{o(\Delta t)}{1 + \rho\Delta t} \right),
\end{aligned}$$

where $V_f(\theta, w(\theta, \tilde{\theta}))$ represents the equilibrium value to a firm of the productive match θ when the worker's next best option has a match $\tilde{\theta}$. The function $\hat{\theta}(w)$ is defined as the maximum value of θ for which the contract (w) would leave the firm with no profit. This value is implicitly defined by $V_f(\hat{\theta}(w), w) = 0$. Any encounter with a potential firm in which the match value is less than $\hat{\theta}(w)$ will not be reported by the employee. In this case, the firm's value remains $V_f(\theta, w)$. Any new contract with a match value greater than $\hat{\theta}(w)$ will be reported to the current firm and will result in either a renegotiation of the current contract or a separation. A separation occurs if the new match value $\theta' > \theta$, in which case the worker quits his current firm to work at the new firm with match value θ' . After rearranging terms and taking limits as $\Delta t \rightarrow 0$, we have

$$V_f(\theta, w) = \left(\rho + \eta + \lambda_e \tilde{G}(\hat{\theta}) \right)^{-1} \times \left\{ \theta - w + \lambda_e \int_{\hat{\theta}(w)}^{\theta} V_f(\theta, w(\theta, \tilde{\theta})) dG(\tilde{\theta}) \right\}.$$

The worker's value of being employed is defined as follows. Consider an infinitesimally small period of time Δt . Over this "period", the worker earns a wage $w\Delta t$, which is discounted back to the present with the "infinitesimal" discount factor $(1 + \rho\Delta t)^{-1}$. With probability $\eta\Delta t$, the match is exogenously terminated and the worker returns to unemployment, the value of which is $V_n(m)$. With probability $\lambda_e\Delta t$, the worker receives a job offer from an alternative firm. Reporting the offer to his current firm will result either in the renegotiation of his wage at the current firm or in his separation from the current firm to work at the alternative firm. With probability $(1 - \lambda_e\Delta t - \eta\Delta t)$, the worker does not receive another job offer and he is not exogenously dismissed over the "period" Δt . In this case the status quo is maintained. Once again, the term $o(\Delta t)$ represents the probability that two or more events occur over the "period" Δt , and has the property that $\lim_{\Delta t \rightarrow 0} \left(\frac{o(\Delta t)}{\Delta t} \right) = 0$. For the employee, the value of employment at a current match value θ and wage w is given by

$$\begin{aligned}
V_e(\theta, w) = & \left(\frac{w\Delta t}{1 + \rho\Delta t} \right) + \left(\frac{\eta\Delta t}{1 + \rho\Delta t} \times V_n(m) \right) + \left(\frac{\lambda_e\Delta t}{1 + \rho\Delta t} \int_{\hat{\theta}(w)}^{\theta} V_e(\theta, w(\theta, \tilde{\theta})) dG(\tilde{\theta}) \right) + \\
& + \left(\frac{\lambda_e\Delta t}{1 + \rho\Delta t} \int_{\theta} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta)) dG(\tilde{\theta}) \right) + \left(\frac{\lambda_e\Delta t G(\hat{\theta}(w))}{1 + \rho\Delta t} \times V_e(\theta, w) \right) + \\
& + \left(\frac{(1 - \lambda_e\Delta t - \eta\Delta t)}{1 + \rho\Delta t} \times V_e(\theta, w) \right) + \left(\frac{o(\Delta t)}{1 + \rho\Delta t} \right),
\end{aligned}$$

where $V_e(\theta, w(\theta, \tilde{\theta}))$ is the equilibrium value of employment to a worker with match value θ when his next best option has a match value of $\tilde{\theta}$. Note that when an employee encounters a firm with a new match value $\tilde{\theta}$ which is lower than his current match value but is capable of being used to increase the value of his current employment contract (i.e., $\theta > \tilde{\theta} > \hat{\theta}(w)$), his new value of employment at the current firm becomes $V_e(\theta, w(\theta, \tilde{\theta}))$. Instead, when the match value at the newly-contacted firm exceeds that of the current firm, the employee changes employers. The value of employment at the new firm is given by $V_e(\tilde{\theta}, w(\tilde{\theta}, \theta))$. Thus, the match value at the current firm becomes the determinant of the threat point faced by the new firm and plays a role in the determination of the new wage. Finally, when the match value at the new firm is less than $\hat{\theta}(w)$, the contact is not reported to the current firm since it would not result in any new improvement in the current contract. Because of this selective reporting, the value of employment contracts must be monotonically increasing both within and across consecutive job spells. Declines can only be observed following a transition into the unemployed state.

After rearranging terms and taking limits, we have

$$\begin{aligned}
V_e(\theta, w) = & \left(\rho + \eta + \lambda_e \tilde{G}(\hat{\theta}(w)) \right)^{-1} \times \{ w + \eta V_n(m) + \\
& + \lambda_e \int_{\hat{\theta}(w)}^{\theta} V_e(\theta, w(\theta, \tilde{\theta})) dG(\tilde{\theta}) + \lambda_e \int_{\theta} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta)) dG(\tilde{\theta}) \}.
\end{aligned}$$

With a new match value of $\theta' > \theta$, the surplus attained by the individual at the new match value with respect to the value she could attain at the old match value after extracting all the surplus associated with it is

$$V_e(\theta', w(\theta', \theta)) - Q(\theta)$$

where $Q(\theta) = V_e(\theta, w^*(\theta))$ is the value of employment to the employee if he receives the total surplus of the match θ . In this case, the equilibrium wage function is $w^*(\theta) = w(\theta, \theta)$. Then,

$$Q(\theta) = \left(\rho + \eta + \lambda_e \tilde{G}(\theta) \right)^{-1} \times \{w^*(\theta) + \eta V_n + \lambda_e \int_{\theta} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta)) dG(\tilde{\theta})\},$$

where we have used the fact that $\hat{\theta}(w^*(\theta)) = \theta$.

The model is closed after specifying the value of nonemployment $V_n(m)$. Up until this point, we have assumed that $\theta^*(m)$ is the value of θ required for an unemployed searcher and a firm to initiate an employment contract. In the absence of a minimum wage (i.e. with $m = 0$), $\theta^*(m)$ is the reservation match value and the minimal acceptable wage. All match values greater than this value will result in employment. With a minimum wage, however, this is the "implicit" reservation match value. The minimal acceptable wage and match value is, instead, the imposed minimum value m .

We only consider the case in which $m \geq \theta^*(m)$. When $m < \theta^*(m)$, the minimum wage constraint is nonbinding since all matches greater than $\theta^*(m)$ result in employment at wages greater than the minimum. In addition, all acceptable matches to a firm must be greater than or equal to m . Any match values less than m would result in negative profits. To summarize, we have $\theta > m > \theta^*(m)$, for all θ among employed individuals.

The searcher's value of being unemployed is defined as follows. Consider an infinitesimally small period of time Δt . Over this "period", the searcher earns an instantaneous utility of $b\Delta t$, which is discounted back to the present with the "infinitesimal" discount factor $(1 + \rho\Delta t)^{-1}$. With probability $\lambda_n\Delta t$, the searcher receives a job offer and draws a match value θ . If the offer is acceptable, he becomes employed. With probability $(1 - \lambda_n\Delta t)$, the searcher does not receive a job offer over the "period" Δt , in which case he remains unemployed and continues to search. The term $o(\Delta t)$ represents the probability that two or more job offers arrive over the "period" Δt , and has the property that $\lim_{\Delta t \rightarrow 0} \left(\frac{o(\Delta t)}{\Delta t} \right) = 0$. For the unemployed searcher, the value of unemployment is given by

$$\begin{aligned} V_n(m) = & \left(\frac{b\Delta t}{1 + \rho\Delta t} \right) + \left(\frac{\lambda_n\Delta t}{1 + \rho\Delta t} \int_m V_e(\theta, w(\theta, \tilde{\theta})) dG(\tilde{\theta}) \right) + \\ & + \left(\frac{\lambda_n\Delta t G(m)}{1 + \rho\Delta t} \times V_n(m) \right) + \left(\frac{(1 - \lambda_n\Delta t)}{1 + \rho\Delta t} \times V_n(m) \right) + \left(\frac{o(\Delta t)}{1 + \rho\Delta t} \right) \end{aligned}$$

Taking the limit as $\Delta t \rightarrow 0$, we have

$$V_n(m) = \left(\rho + \lambda_n \tilde{G}(m) \right)^{-1} \times \{b + \lambda_n \int_m V_e(\tilde{\theta}, w(\tilde{\theta}, \theta^*(m))) dG(\tilde{\theta})\}.$$

When an employed agent meets a new potential employer, the solution to the Nash bargaining problem is given by

$$w(\theta', \theta) = \arg \max_{w \geq m} S(\theta', w, \theta)$$

where $S(\theta, w, \theta^*(m)) = \{V_e(\theta, w) - Q(\theta)\}^\alpha \times \{V_f(\theta, w) - 0\}^{1-\alpha}$. When an unemployed agent meets a new potential employer, the solution to the Nash bargaining problem is given by

$$w(\theta, \theta^*(m)) = \arg \max_{w \geq m} S_n(\theta, w, \theta^*(m))$$

where $S_n(\theta, w, \theta^*(m)) = \{V_e(\theta, w) - V_n\}^\alpha \times \{V_f(\theta, w) - 0\}^{1-\alpha}$. Regardless if the searcher is employed, the minimum wage acts as a side-constraint in the Nash bargaining problem. In the next section we show that individuals can be paid the minimum wage only at their first job in an employment spell and only until they receive an alternative offer with which they can renegotiate their current employment contract.

2.2 A Model with Endogenous Contact Rates

As in Pissarides (2000), we now explicitly model the firm's decisions to create vacancies for searchers. Job search models in the macroeconomic literature typically assume a constant returns to scale matching technology $M(S, V) = Vq(\kappa)$ where $\kappa = \frac{S}{V}$, S is the size (measure) of the set of searchers and V is the size of the set of vacancies. In a model with on-the-job search, searchers can be unemployed or employed. Most treatments of this problem assume that employed and unemployed searchers devote equal amounts of time to search, so that the total number of searchers equals the number of labor market participants (see, e.g., Pissarides (2000)). Estimates of contact rates of unemployed and employed searchers in partial equilibrium analyses consistently indicate that the contact rate of the unemployed is at least three times larger than that of the employed, making this assumption unattractive. We define the searching population as equal to $U + vE$, where U is the size of the unemployed population, E is the size of the employed population, and v is a technological parameter partially characterizing the search technology. Defining the searching population in this way, the contact rate per vacancy is given by $\frac{M(S, V)}{V} = q(\kappa)$ and the contact rate per effective searcher is $\frac{M(S, V)}{S} = \frac{q(\kappa)}{\kappa}$. The contact rate per unemployed (employed) searcher is the probability that the searcher is unemployed (employed) multiplied by the contact rate per

effective searcher:

$$\begin{aligned}\lambda_n &= \frac{U}{S} \frac{M(S, V)}{S} \\ \lambda_e &= \frac{vE}{S} \frac{M(S, V)}{S}.\end{aligned}$$

We will make a functional form assumption regarding $q(\kappa)$.

Assuming that there exists a population of potential firm entrants with an outside option value of 0, firms create vacancies until the point that expected profits are zero. Assuming the cost of creating a vacancy is given by $\psi > 0$, the expected value of creating a vacancy is given by

$$\rho V_v = -\psi + \frac{M(S, V)}{V} (J(m, \theta^*(m)) - V_v)$$

where V_v is the value of a vacancy, $\frac{M(S, V)}{V}$ is the rate at which a firm fills a vacancy, and $J(m, \theta^*(m))$ is the expected value of a filled vacancy,

$$J(m, \theta^*(m)) = \frac{\lambda_e}{\lambda_n + \lambda_e} \int_m \int_\theta V_f(\theta', \theta) \frac{g(\theta')}{\tilde{G}(\theta)} g_{ss}(\theta) d\theta' d\theta + \frac{\lambda_n}{\lambda_n + \lambda_e} \int_m V_f(\theta, \theta^*(m)) \frac{g(\theta)}{\tilde{G}(m)} d\theta$$

where $g_{ss}(\theta)$ is the steady state distribution of match values and $V_f(\theta', \theta)$ is the value to the firm of employing a worker whose productivity value is θ' and whose threat point is θ .

Assuming a free entry condition (FEC), $V_v = 0$ and we have

$$0 = -\psi + \frac{M(S, V)}{V} (J(m, \theta^*(m))).$$

This equation can be used to solve for the flow cost of creating a vacancy, ψ , given the equilibrium number of vacancies V , the expected value of a filled vacancy $J(m, \theta^*(m))$, and the size of the set of searchers S .

We note that with exogenous contact rates, the steady state unemployment rate is defined by

$$U = \frac{\eta}{\eta + \lambda_n \tilde{G}(m)}$$

whereas with endogenous contact rates, the steady state unemployment rate is defined by

$$U = \frac{\eta}{\eta + \frac{U}{S} \frac{M(S, V)}{S} \tilde{G}(m)}.$$

As we discuss in the econometric section of this paper, to solve for v we use the endogenous contact rates λ_n and λ_e and the steady state unemployment and employment rates U and E .

With endogenous contact rates, a labor market equilibrium in the presence of a minimum wage is characterized by the quadruplet $(U, v, V, \theta^*(m))$ which is solely a function of the primitive parameters $(\rho, b, \eta, \alpha, \psi)$ and the parameters of the match distribution $G(\theta)$.

3 Analysis of Model

In this section we determine when workers receive the minimum wage and analyze how the existence of a minimum wage affects labor market outcomes conditional on whether workers are allowed to search while employed.

In Flinn (2005) there is no on-the-job search ($\lambda_e = 0$). The wage equation that is derived from the Nash bargaining framework is

$$w(\theta, \theta^*(m)) = \alpha\theta + (1 - \alpha)\theta^*(m)$$

where θ is the worker's current match value, $\theta^*(m)$ denotes the match value associated with unemployment, and α is the worker's bargaining power parameter. Examining the wage equation, we find that under this division of the match, the worker would receive a wage of m when $\theta = \hat{\theta}_m(m, \theta^*(m))$, where

$$\hat{\theta}_m = \frac{m - (1 - \alpha)\theta^*(m)}{\alpha}$$

For notational convenience we will write $\hat{\theta}_m(m, \theta^*(m))$ as $\hat{\theta}_m$. It is assumed that $\hat{\theta}_m > m$ in order to avoid the uninteresting case in which all "feasible" matches (i.e. those greater than m) generate wage offers at least as large as m . According to the wage equation, when $\theta \in [m, \hat{\theta}_m]$, the wage is less than m . However, when confronted with the choice of giving some of its surplus to the worker versus a return of 0, the firm pays the wage of m for all $\theta \in [m, \hat{\theta}_m]$. Wages for $\theta \geq \hat{\theta}_m$ are determined according to the wage equation $w(\theta, \theta^*(m))$.

Figure 1 depicts the no OTJ search case. Here, the wage function maps a single productivity value θ to a wage for each $\theta > \theta^*(m)$. The line $w = \theta$ depicts the wage at which the firm breaks even for each productivity value θ . The firm earns a profit of $(\theta - w)$ for a worker with match value θ , depicted as the vertical distance between the line $w = \theta$ and the wage function $w(\theta)$. Once we introduce a minimum wage into the graph, we see that some θ (particularly $\theta \in [\theta^*(m), m]$) are no longer available to the worker. This is the standard (negative) employment effect. Additionally, for all current match values $\theta \in [m, \hat{\theta}_m]$, the firm wants to pay the worker the wage $w(\theta) < m$, but cannot do so under the minimum wage law. Therefore, it pays the worker $w = m$ which lowers its profits from $(\theta - w(\theta)) \geq 0$ to $(\theta - m) \geq 0$ for each $\theta \in [m, \hat{\theta}_m]$. For all $\theta > \hat{\theta}_m$, the firm pays the worker $w(\theta) > m$ and earns a profit of $(\theta - w(\theta)) > 0$.

When we extend Flinn (2005) to allow workers to search on-the-job, the wage that is derived from the Nash bargaining framework is represented by the function $w(\theta', \theta)$. Extending the original

model to allow for on-the-job search produces two immediate implications regarding when workers receive the minimum wage. We prove both results below. We show that within any employment spell, only at the first job in the sequence can the employee be paid the minimum wage. Moreover, the minimum wage may only be paid to the employee at the beginning of their first job in the employment spell, since the arrival of alternative, viable employment opportunities with other firms will result in a renegotiated wage payment larger than the minimum. This implies that individuals will spend only a small amount of employment time at jobs paying the minimum wage if the contact rate of employees with other firms with vacancies is sufficiently large.

Since workers can be paid the minimum wage only at their first job, any minimum wage earner must use $\theta^*(m)$ as their best outside option, which is unemployment. The definition of the match value $\hat{\theta}_m$ when allowing for OTJ search is identical to its definition without OTJ search. That is, $\hat{\theta}_m$ is the maximum match value with which the worker receives the minimum wage while using $\theta^*(m)$ as a match value that represents their best outside option. We can re-write the value of nonemployment that we derived in the previous section without having to re-write the value of employment. The value of nonemployment is

$$V_n(m) = \left(\rho + \lambda_n \tilde{G}(\theta^*(m)) \right)^{-1} \times \left\{ b + \lambda_n \int_{\theta^*(m)}^{\tilde{\theta}^{(m)}} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta^*(m)) = m) dG(\tilde{\theta}) \right. \\ \left. + \lambda_n \int_{\tilde{\theta}^{(m)}} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta^*(m)) > m) dG(\tilde{\theta}) \right\}$$

We now formally state and prove each proposition:

Proposition 1 *A worker can receive the minimum wage only at his first job.*

Proof. Let $\theta^*(m)$ be the reservation match value when there exists a minimum wage; let θ^1 such that $\theta^1 \in [m, \hat{\theta}_m]$ be the match value that the worker draws upon exiting unemployment; and let θ^2 such that $\theta^2 > \theta^1$ be the first match value the worker draws while employed at his first job. Note: the match values θ^1 and θ^2 are *not* the first two match values in a discretized support of the $G(\theta)$ distribution, but are the first and second match values drawn chronologically. The initial wage at the first job is $w(\theta^1, \theta^*(m))$. A draw $\theta^2 > \theta^1$ causes the worker to change firms in order to receive the wage $w(\theta^2, \theta^1)$. We know that $m \leq \theta^1 = w(\theta^1, \theta^1) < w(\theta^2, \theta^1)$, where the first inequality follows from the requirement that all accepted match values $\theta \geq m$ once a minimum wage is set; the equality follows from the fact that the worker receives all the rents from the match when his best outside option is denoted by his current match value; and the second inequality follows from the fact that

the wage function is monotonically increasing in its first argument. Thus, $m < w(\theta^2, \theta^1)$ implies that the wage at his second job will always be greater than the minimum wage. ■

Proposition 2 *A worker can receive the minimum wage at his first job only until the first alternative job offer arrives with which he can renegotiate his current contract.*

Proof. The wage function $w(\theta', \theta)$ is monotonically increasing in its first and second argument: $w_1(\theta', \theta) > 0$ and $w_2(\theta', \theta) > 0$ (where w_1 and w_2 denote the partial derivatives of the wage function with respect to its first and second arguments, respectively) by the nature of the Nash bargaining framework.

If an unemployed individual draws $\theta^1 \in [m, \hat{\theta}_m]$, then he will accept the offer and receive a wage $w(\theta^1, \theta^*(m)) = m$. If he receives a second draw θ^2 such that $\hat{\theta}_m \geq \theta^2 > \theta^1$ then he will leave the current firm and receive a wage $w(\theta^2, \theta^1) > m$ at his second job by proposition 1. Alternatively, consider the situation in which the individual draws a match value θ^2 , such that $m < \theta^2 < \hat{\theta}_m$, coming out of unemployment in which case his wage is $w(\theta^2, \theta^*(m)) = m$. Here the superscript 2 refers to the second draw from the previous example, but is the first draw in this example. If he subsequently draws a match value θ^1 (the first match value from the previous example) such that $m < \theta^1 < \theta^2$, he renegotiates his contract at his current firm to $w(\theta^2, \theta^1)$, which we have already seen is greater than the minimum wage. Thus coming out of unemployment, the worker can receive the minimum wage. When another job offer arrives, the worker renegotiates his contract and receives a wage greater than the minimum wage. ■

When there exists OTJ search, the wage equation is a function that maps elements from the cross-product of identical subsets of the support of the productivity distribution to a wage. The domain of the wage function is the set $[\theta^*(m), \infty) \times [\theta^*(m), \infty)$. Figure 2 depicts the OTJ search case. There are three axes. The x axis consists of the match value θ' at the current firm. The y axis consists of the match values θ with which the worker threatens. And the z axis is the wage function $w(\theta', \theta)$. The wage function is a 3-dimensional non-linear surface in the space $[\theta^*(m), \infty) \times [\theta^*(m), \infty) \times [\theta^*(m), \infty)$.

Introducing a minimum wage into this 3-dimensional graph requires inserting a plane at $w(\theta', \theta) = m$. There are two immediate results of introducing a minimum wage when individuals are allowed to search while employed. The first is the standard (negative) employment effect. Similar to the no OTJ search case, we see that some θ (particularly $\theta \in [\theta^*(m), m)$) on the x axis are no longer available to the worker. This effect causes unemployed searchers to wait longer to receive accept-

able offers. The second result is what we call the (negative) renegotiation effect. A minimum wage limits the amount of match values with which employed individuals can renegotiate their current wage. Specifically, the match values $\theta \in [\theta^*(m), m)$ are no longer available to the worker. This second effect implies that employed individuals must wait longer to receive an alternative job offer which serves as a credible threat to their current employer and with which they will renegotiate their current wage.

4 Data

The data used to estimate the model contain information on individuals from the 1996 panel of the Survey of Income and Program Participation (SIPP). The main objective of the SIPP is to provide accurate and comprehensive information about and the principal determinants of the income and program participation of individual households in the United States. The SIPP collects monthly information regarding individual's labor market activity including earnings, average hours worked, and whether the individual changed jobs during the month.

Although the target sample size for the SIPP is quite large, our sample size has been greatly reduced by several restrictions which we will now discuss. We only consider individuals ages 16-24 who neither participate in the armed services nor in any welfare program (e.g. AFDC, Food Stamps, WIC) during the sample period.

The minimum wage changed from \$4.75 to \$5.15 on September 1, 1997. Although the survey interviews individuals every four months for up to twelve times from 1996 to 2000, we use data only from October 1997 to February 2000 in order to avoid minimum wage changes within the sample period. We found that discontinuities in respondent's employment histories were increasingly present as individuals near the end of the panel. Because our econometric specification relies heavily on identifying transitions between different labor market states, it is essential that individuals have complete labor market histories if we are to include them in our estimation procedure. Thus, all individuals with discontinuities in their labor history were removed from the sample. This restriction reduced our sample size significantly.

Table 1 contains descriptive statistics generated from the data. We will discuss the construction of labor market histories in the next section, however, at this point we note that we used data from any individual who experienced at least one unemployment spell in the sample period (i.e. from October 1997 to February 2000). Each individual could contribute a complete (or right-censored)

unemployment spell as well as a subsequent first job duration. Wages at the first job and the reason the first job ended (if it was not right-censored) were also used.

We define the sample window as the number of weeks a respondent remains in the SIPP panel from October 1997 until the end of the panel. Because panel data suffers from sample attrition as the length of the panel increases, it was more likely that individuals had shorter sample windows in the second half of the panel than in the first half. Although the maximum possible sample window was 27.54 months, many individuals left the panel before this time. This is observed in the average and standard deviation of the sample window for two groups of individuals—those that did and did not experience an unemployment spell at some point in the panel.

Out of the 1974 individuals who are unemployed at some point in the sample period, 78.3% of them transition to a job before leaving the panel. These individuals leave the unemployment state within approximately three months on average. Note, however, that the 21.7% of the remaining individuals who never exit the unemployment state during the remainder of their sample window have an average unemployment duration of about 10 months. Because this duration is larger than we had anticipated, we would like to take a moment to discuss it.

The lengthy average duration of unemployment can be partly due to the fact that we do not condition on schooling status when defining search behavior. We have chosen to use a fairly young sample, with individuals ages 16-24, since most minimum wage earners are disproportionately young (e.g. see Table 3 which shows the distribution of ages among minimum wage earners in our sample). With such a young sample, however, many individuals are likely to be enrolled in school, limiting the amount of time devoted to job search and to employment. Because schooling decisions and their effect on employment are not central to our analysis, we chose not to exploit this type of heterogeneity. While it is possible that individuals enrolled in school have longer unemployment durations since they devote less time to search, it is also likely that some of these individuals only work during the summer and remain unemployed throughout the academic year. This, coupled with the fact that individuals have shorter sample windows in the latter half of the SIPP panel, could also contribute to long right-censored unemployment durations.

While long average unemployment durations could be attributed to including people in the sample who are out of the labor force, we chose to exclude any individual who responded that they were out of the labor force (i.e. "not working, not looking for work, and not on layoff") at *any* point in their sample window. In doing so, we have extrapolated the results of Flinn and Heckman (1983) from cross-sectional data to panel data. Flinn and Heckman (1983) concluded

that unemployment and out of the labor force are different labor states, but supported their analysis using cross-sectional data. Our view is that it is easier to examine the difference between being out of the labor force and unemployed in an analysis using cross-sectional data than using panel data. In a cross-sectional data set, individuals can easily be categorized as being in one of these two states by their response to a question asking if they searched for work in the past several weeks. With panel data individuals are asked a similar question, but they are asked it each time they are interviewed. As a result, one can observe a sequence of responses in which the individual is unemployed for several periods, then transitions to being OLF, then re-enters the unemployed state, etc. Incorporating transitions from the being out of the labor force to being unemployed and estimating the parameters that govern the environment in which these transitions are made might be a fruitful area of research, but remains one which we chose not to explore.

The mean accepted wage for those individuals who transitioned from unemployment to employment is \$6.80¹. About 8.74% of these individuals receive the minimum wage at their first job. Right-censored first job spells last about one year, while first job spells both that end with a transition to unemployment and that end with a transition to another job last about 5 months on average. Conditional on having a second job in an employment spell, the average wage at the second job is greater than the average wage at the first job by about \$0.40. A closer inspection of wages across subsequent job spells shows that about 75% of wages at a second job are greater than or equal to wages at a first job. Recall that the nature of the wage bargaining process between individuals and firms in our model predicts that wages have to be at least as large at the destination job as at the current job in order for the individual to leave the current job. We will discuss the introduction of measurement error in the next section to reconcile the existence of wage decreases across subsequent job spells.

5 Econometric Specification

In this section we discuss the econometric specification used to estimate the parameters of the model developed above. We estimate the model using standard maximum likelihood methods. In appendix A, we illustrate how we approximate the value functions in the model in order to define the equilibrium equations and to estimate the parameters of the model.

¹As we will discuss in the next section, we chose to include individuals with observed wages below the minimum. We view these wages as being measured with error.

In order to derive the likelihood function, we define labor market cycles as sequences of labor market states beginning with an unemployment spell and ending with the last job prior to the following unemployment spell for a given individual. We truncate the cycles after the first job spell not due to data limitations, but because it enables us to write the likelihood function in a tractable way. Depending on the states an individual enters during his duration in the panel, we use his unemployment duration, the length of his first job spell, and the initial wage at his first job. Although we do not use wage or duration data from any subsequent job spell, we use the reason the first job spell ends when defining an individual's likelihood contribution.

In job search models without minimum wages, measurement error is usually introduced in order to make the econometric model and the data consistent with one another. It is used to account for the job-to-job transitions with wage reductions that are observed in the data. Our on-the-job search model predicts that individuals will only change jobs when they receive a job offer and draw a match value that is larger than their current match value. As a result, wages must increase when an individual undergoes a job-to-job transition. About 25 percent of the individuals with job-to-job transitions accept a lower wage at their second job. Data for these individuals in our sample will produce a likelihood value of 0 at all points in the parameter space. Adding measurement error to the model makes these observations possible at all points in the parameter space.

In the presence of a minimum wage, introducing classical measurement error would let us observe wages that are less than m , depending of course on the distribution of the error term. Unfortunately, it would prevent us from observing a probability mass at the minimum wage. As a result, classical measurement error is not introduced to all wages. Instead, we assume that a proportion τ of observed wages are measured with error and a proportion $(1 - \tau)$ are not measured with error. We can set bounds on τ by noting that all minimum-wage earners do not have wages measured with error. Thus, the proportion of people who do not earn the minimum wage serves as an upper-bound for τ . Also, all wages less than the minimum are definitely measured with error. Thus, the proportion of workers who have wages less than the minimum serves as a lower-bound for τ . For those wages that are measured with error, we assume that the observed wages $\tilde{w}$ are related to true wages w by $\tilde{w} = w \exp(\varepsilon)$ where ε is an independently and identically continuously distributed random variable. Our econometric specification will posit that ε is normally distributed $N(\mu_\varepsilon, \sigma_\varepsilon)$ with mean $\mu_\varepsilon = 0$, so that $\ln \tilde{w} = \ln w + \varepsilon$ and $E(\ln \tilde{w}) = \ln w$. We estimate the parameters τ and σ_ε in our econometric procedure.

As is often the case when attempting to estimate dynamic models, we face difficult initial

conditions problems. In our framework, and common to most stationary search models, entry into the unemployment state essentially "resets" the process. While we will utilize all cases in the data in estimating the model, our focus will be on those cases that contain an unemployment spell. The likelihood function is written in terms of the employment cycles referred to above, so that only those cases that contain an unemployment spell are "directly" utilized. Let Ψ take the value of 1 if a sample member experiences an unemployment spell at some point during their observation period and let it equal 0 when this is not the case. The likelihood of this event is a function of the length of the sample period, which we will denote by T . We denote the likelihood $P(\Psi = 1|T)$ by $\omega(T)$. It is assumed that the length of the sample window is independently distributed with respect to all of the outcomes determined within the model. In deriving the likelihood of this event, we refer the reader to Dey and Flinn (2005).

The likelihood function consists of five main types of contributions, with several "subtypes" within each type. For the sample cases in which $\Psi = 1$ the data utilized in our estimation procedure is given by $\{t_u, t_1, \tilde{w}_1\}$, where t_u is the observed duration of unemployment, t_1 is the observed duration of employment at an individual's first job, and $\tilde{w}_1$ is the observed wage at an individual's first job. The types (and subtypes) of contributions are:

- (1) A right-censored unemployment spell.
- (2) A completed unemployment spell followed by a right-censored one-job employment spell.
 - (2a) A completed unemployment spell followed by a right-censored one-job employment spell in which $\tilde{w}_1 = m$.
 - (2b) A completed unemployment spell followed by a right-censored one-job employment spell in which $\tilde{w}_1 > m$.
 - (2c) A completed unemployment spell followed by a right-censored one-job employment spell in which $\tilde{w}_1 < m$.
- (3) A completed unemployment spell followed by a one-job employment spell in which the job ends in termination.
 - (3a) A completed unemployment spell followed by a one-job employment spell in which $\tilde{w}_1 = m$ and the job ends in termination.

- (3b) A completed unemployment spell followed by a one-job employment spell in which $\tilde{w}_1 > m$ and the job ends in termination.
- (3c) A completed unemployment spell followed by a one-job employment spell in which $\tilde{w}_1 < m$ and the job ends in termination.
- (4) A completed unemployment spell followed by a first job spell which is immediately followed by a second job.
 - (4a) A completed unemployment spell followed by a first job spell in which $\tilde{w}_1 = m$ and which is immediately followed by a second job.
 - (4b) A completed unemployment spell followed by a first job spell in which $\tilde{w}_1 > m$ and which is immediately followed by a second job.
 - (4c) A completed unemployment spell followed by a first job spell in which $\tilde{w}_1 < m$ and which is immediately followed by a second job.
- 1. An employment spell for individuals in our sample who do not experience an unemployment spell within their sample window.

Each of these cases is individually considered below.

Case 1: If the cycle consists only of a right-censored unemployment spell of duration t_u , then the likelihood contribution is given by

$$L_1(t_u, \Psi = 1|T) = \omega(T) \Pr(\tilde{t}_u > t_u) = \omega(T) \exp(-\lambda_n \tilde{G}(m) t_u)$$

where $\tilde{t}_u$ is the unobserved (uncensored) duration time and t_u is the observed (censored) duration time.

Case 2: If the cycle consists of a completed unemployment spell followed by a right-censored one-job employment spell, then the variables in this case are $(t_u, t_1, \tilde{w}_1)$. Since unemployment durations follow a negative exponential distribution with parameter $\lambda_n \tilde{G}(m)$, the likelihood of a completed unemployment duration of t_u is just $f_u(t_u) = \lambda_n \tilde{G}(m) \exp(-\lambda_n \tilde{G}(m) t_u)$.

Case2a: The likelihood of a right-censored one-job employment spell in which $\tilde{w}_1 = m$ is derived as follows. Recalling that $\hat{\theta}_m$ is the largest productivity value for which the individual can receive the minimum wage, the probability that the individual will receive the minimum wage at

his or her first job is equal to $Pr(\tilde{w}_1 = m) = \left(\frac{G(\hat{\theta}_m) - G(m)}{\tilde{G}(m)} \right)$. The joint density of right-censored first job durations and wages given $\tilde{w}_1 = m$ is equal to

$$\int_m^{\hat{\theta}_m} \left(\exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) \left(\frac{g(\theta)}{G(\hat{\theta}_m) - G(m)} \right) d\theta$$

where the first term in the integral is the probability that the first job duration will be right-censored, $Pr(\tilde{t}_1 > t_1) = \left(\exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right)$ where $\tilde{t}_1$ is the uncensored (unobserved) employment duration and t_1 is the censored (observed) employment duration. Note that the combined hazard rate of leaving this first job (due to termination or finding another job) is $(\eta + \lambda_e \tilde{G}(\theta))$ where θ is the match value at the current job. The second term in the integral is the density of match values given that the individual receives the minimum wage for this match. We integrate over all match values for which this is the case to derive the equation above. The joint wage-duration density of a right-censored one-job employment spell in which $\tilde{w}_1 = m$ is therefore:

$$\begin{aligned} k_{2a}(m, t_1) &= \left(\frac{G(\hat{\theta}_m) - G(m)}{\tilde{G}(m)} \right) \times \\ &\quad \times \int_m^{\hat{\theta}_m} \left(\exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) \left(\frac{g(\theta)}{G(\hat{\theta}_m) - G(m)} \right) d\theta \\ &= \frac{1}{\tilde{G}(m)} * \int_m^{\hat{\theta}_m} \left(\exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) g(\theta) d\theta \end{aligned}$$

Since we assume that true wages are measured correctly when observed wages are equal to the minimum wage, we have to weight this likelihood by the probability of these wages being measured correctly $(1 - \tau)$. We denote the weighted density by $\tilde{k}_{2a}(m, t_1)$.

$$\tilde{k}_{2a}(m, t_1) = k_{2a}(m, t_1)(1 - \tau)$$

Multiplying the this density by the likelihood of the completed unemployment spell duration, we get

$$L_{2a}(m, t_u, t_1, \Psi = 1|T) = \omega(T) \tilde{k}_{2a}(m, \tilde{t}_1) * f_u(t_u).$$

Case 2b: We will now derive the likelihood of a right-censored one-job employment spell in which $\tilde{w}_1 > m$. We know the probability that the individual will receive a true wage that is greater the minimum wage at his first job is equal to $Pr(w_1 > m) = \left(\frac{\tilde{G}(\hat{\theta}_m)}{\tilde{G}(m)} \right)$. An individual in the unemployment state who accepts a job offer with match value θ will be paid a wage equal

to $w(\theta, \theta^*(m))$, where w is the equilibrium wage function and $\theta^*(m)$ is the implicit reservation wage². Given the match value with which the individual threatens at his current job $\theta^*(m)$, this wage function is monotonically increasing in its first argument θ as long as $\theta > \hat{\theta}_m$, a condition that is satisfied if the individual receives a wage greater than the minimum at his first job. As a result, the mapping $w(\theta, \theta^*(m))$ is 1-1 for all $\theta > \hat{\theta}_m$ and we can invert wage function to define the function $\tilde{\theta}(w, \theta^*(m))$ for all wages $w > m$. The distribution of wages for individuals entering the current job from the unemployment state and receiving a wage greater than the minimum wage is

$$\left(\frac{\delta \tilde{\theta}(w, \theta^*(m))}{\delta w} * \frac{g(\tilde{\theta}(w, \theta^*(m)))}{\tilde{G}(\hat{\theta}_m)} \right).$$

The density of a right-censored first job employment duration is $\exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))t_1)$ where $(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))$ is the combined hazard rate at which the individual leaves the first job (either due to termination or finding another job). The joint wage-duration density of a right-censored one-job employment spell in which $\tilde{w}_1 > m$ is therefore

$$\tilde{k}_{2b}(\tilde{w}_1, t_1) = (1 - \tau)k_{2b}(\tilde{w}_1, t_1) + (\tau) \left[P(w_1 = m) \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) + \int_m \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) k_{2b}(w_1, t_1) dw_1 \right]$$

where

$$\begin{aligned} k_{2b}(w_1, t_1) &= \left(\frac{\tilde{G}(\hat{\theta}_m)}{\tilde{G}(m)} \right) \left(\frac{\delta \tilde{\theta}(w_1, \theta^*(m))}{\delta w_1} \frac{g(\tilde{\theta}(w_1, \theta^*(m)))}{\tilde{G}(\hat{\theta}_m)} \right) * \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w_1, \theta^*(m))))t_1) \\ &= \left(\frac{1}{\tilde{G}(m)} \right) \left(\frac{\delta \tilde{\theta}(w_1, \theta^*(m))}{\delta w_1} g(\tilde{\theta}(w_1, \theta^*(m))) \right) * \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w_1, \theta^*(m))))t_1) \end{aligned}$$

The first term in equation $\tilde{k}_{2b}(\tilde{w}_1, t_1)$ is the joint wage-duration density of a right-censored one-job employment spell in which $\tilde{w}_1 > m$, conditional on there being no measurement error, multiplied by the probability that there is no measurement error. The term enclosed by brackets is the density that is derived from a mixture-distribution that accounts for the existence of measurement error in the true wage w_1 . It has a mass-point at the minimum wage and a continuous distribution for all true wages greater than the minimum wage..The first term enclosed in brackets is the probability that the individual's true wage is equal to the minimum wage, multiplied by the probability density function for the measurement error distribution. The second term enclosed in brackets is the joint density for durations and true wages multiplied by the probability density function for the measurement error distribution, integrated over all true wages greater than the minimum wage.

²See Appendix A for approximation of $w(\theta, \theta^*(m))$ and the derivation of the "implicit" reservation match value equation $\theta^*(m)$.

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{2b}(\tilde{w}_1, t_u, t_1, \Psi = 1|T) = \omega(T)\tilde{k}_{2b}(\tilde{w}_1, t_1) * f_u(t_u).$$

Case 2c: The likelihood of a right-censored one-job employment spell in which $\tilde{w}_1 < m$ is derived as follows. The model predicts that all true wages must be greater than or equal to the minimum wage ($w_1 \geq m$). Therefore, any observed wage below the minimum must correspond to a true wage that is measured with error. The joint wage-duration density of observed wages $\tilde{w}_1 < m$ and right-censored durations t_1 is:

$$\tilde{k}_{2c}(\tilde{w}_1, t_1) = (\tau) \left[P(w_1 = m) \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) + \int_m \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) k_{2b}(w_1, t_1) dw_1 \right]$$

where the terms enclosed in brackets are identical to those enclosed in the brackets in $\tilde{k}_{2b}(\tilde{w}_1, t_1)$ in the previous case (2b).

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{2c}(\tilde{w}_1, t_u, t_1, \Psi = 1|T) = \omega(T)\tilde{k}_{2c}(\tilde{w}_1, t_1) * f_u(t_u).$$

Case 3: If the cycle consists of a completed unemployment spell followed by a completed employment spell in which the first job ends in termination, then the variables in the likelihood function are $(t_u, t_1, \tilde{w}_1)$. As in case 2, the likelihood of a completed unemployment duration of t_u is

$$f_u(t_u) = \lambda_n \tilde{G}(m) \exp(-\lambda_n \tilde{G}(m) t_u)$$

Case 3a: The likelihood of an employment spell in which the first job ends in termination and for which $\tilde{w}_1 = m$ is derived as follows. The probability that the individual will receive the minimum wage at his first job is equal to $Pr(\tilde{w}_1 = m) = \left(\frac{G(\hat{\theta}_m) - G(m)}{\tilde{G}(m)} \right)$. The joint density of completed first-job durations that end in termination and wages given $\tilde{w}_1 = m$ is equal to

$$\int_m^{\hat{\theta}_m} \left(\eta \exp(-(\eta + \lambda_e \tilde{G}(\theta)) t_1) \right) \left(\frac{g(\theta)}{G(\hat{\theta}_m) - G(m)} \right) d\theta$$

where the first term in the integral is the product of the density of a first job duration which ends in termination, $f(t_1 | \theta, m) = \eta \exp(-\eta t_1)$, and the probability that an individual has not

found a better job by time t_1 given that he initially drew a match value θ such that $\theta \in [m, \hat{\theta}_m]$, $\exp(-(\lambda_e \tilde{G}(\theta))t_1)$. This product is expressed more concisely as

$$\eta \exp(-\eta t_1) \times \exp(-(\lambda_e \tilde{G}(\theta))t_1) = \eta \exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1)$$

The joint wage-duration density of an employment spell in which the first job ends in termination and for which $\tilde{w}_1 = m$ is therefore:

$$\begin{aligned} K_{3a}(m, t_1) &= \left(\frac{G(\hat{\theta}_m) - G(m)}{\tilde{G}(m)} \right) \times \\ &\quad \times \int_m^{\hat{\theta}_m} \left(\eta \exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) \left(\frac{g(\theta)}{G(\hat{\theta}_m) - G(m)} \right) d\theta \\ &= \frac{1}{\tilde{G}(m)} \int_m^{\hat{\theta}_m} \left(\eta \exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) g(\theta) d\theta \end{aligned}$$

Since we assume that true wages are measured correctly when observed wages are equal to the minimum wage, we have to weight this likelihood by the probability of being measured correctly $(1 - \tau)$. We denote the weighted density by $\tilde{k}_{3a}(m, \tilde{t}_1)$.

$$\tilde{k}_{3a}(m, \tilde{t}_1) = k_{3a}(m, \tilde{t}_1)(1 - \tau)$$

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{3a}(m, t_u, \tilde{t}_1, \Psi = 1|T) = \omega(T) \tilde{k}_{3a}(m, \tilde{t}_1) * f_u(t_u).$$

Case3b: The likelihood of an employment spell in which the first job ends in termination and for which $\tilde{w}_1 > m$ is derived as follows. We know the probability that the individual will receive a true wage that is greater the minimum wage at his first job is equal to $Pr(w_1 > m) = \left(\frac{\tilde{G}(\hat{\theta}_m)}{\tilde{G}(m)} \right)$. Additionally, as in Case 2b, the distribution of wages for individuals entering the current job from the unemployment state and receiving a wage greater than the minimum wage is

$$\left(\frac{\delta \tilde{\theta}(w, \theta^*(m))}{\delta w} * \frac{g(\tilde{\theta}(w, \theta^*(m)))}{\tilde{G}(\hat{\theta}_m)} \right).$$

Recall from Case 3a that the density of an employment spell in which the first job ends in termination is $\eta \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))t_1)$ where $(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))$ is the combined hazard rate at which the individual leaves the first job (either due to termination or finding another

job). The joint wage-duration density of an employment spell in which the first job ends in termination and for which $\tilde{w}_1 > m$ is therefore

$$\tilde{k}_{3b}(\tilde{w}_1, t_1) = (1 - \tau)k_{3b}(\tilde{w}_1, t_1) + (\tau) \left[P(w_1 = m) \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) + \int_m \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) k_{3b}(w_1, t_1) dw_1 \right]$$

where

$$\begin{aligned} k_{3b}(w_1, t_1) &= \left(\frac{\tilde{G}(\hat{\theta}_m)}{\tilde{G}(m)} \right) \left(\frac{\frac{\delta \tilde{\theta}(w_1, \theta^*(m))}{\delta w_1} g(\tilde{\theta}(w_1, \theta^*(m)))}{\tilde{G}(\hat{\theta}_m)} \right) \left(\eta \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m)))) t_1) \right) \\ &= \left(\frac{1}{\tilde{G}(m)} \right) \left(\frac{\frac{\delta \tilde{\theta}(w_1, \theta^*(m))}{\delta w_1} g(\tilde{\theta}(w_1, \theta^*(m)))}{\tilde{G}(m)} \right) \left(\eta \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m)))) t_1) \right) \end{aligned}$$

Both $\tilde{k}_{3b}(\tilde{w}_1, t_1)$ and $k_{3b}(w_1, t_1)$ are almost identical to $\tilde{k}_{2b}(\tilde{w}_1, t_1)$ and $k_{2b}(w_1, t_1)$ except for the fact that the employment duration densities differ. In case 3b, it is $\eta \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m)))) t_1)$ while in case 2b it is $\exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m)))) t_1)$.

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{3b}(\tilde{w}_1, t_u, t_1, \Psi = 1|T) = \omega(T) \tilde{k}_{3b}(\tilde{w}_1, t_1) * f_u(t_u).$$

Case 3c: The likelihood of an employment spell in which the first job ends in termination and for which $\tilde{w}_1 < m$ is derived as follows. The model predicts that all true wages must be greater than or equal to the minimum wage ($w_1 \geq m$). Therefore, any observed wage below the minimum must correspond to a true wage that is measured with error. The joint wage-duration density of observed wages $\tilde{w}_1 < m$ and employment durations t_1 is:

$$\tilde{k}_{3c}(\tilde{w}_1, t_1) = (\tau) \left[P(w_1 = m) \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) + \int_m \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) k_{3b}(w_1, t_1) dw_1 \right]$$

where the terms enclosed in brackets are identical to those enclosed in the brackets in $\tilde{k}_{3b}(\tilde{w}_1, t_1)$ in the previous section (case 3b).

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{3c}(\tilde{w}_1, t_u, t_1, \Psi = 1|T) = \omega(T) \tilde{k}_{3c}(\tilde{w}_1, t_1) * f_u(t_u).$$

Case 4: If the cycle consists of a completed unemployment spell followed by a completed employment spell in which the first job ends with a transition into a second job, then the variables in the likelihood function are $(t_u, t_1, \tilde{w}_1, \tilde{w}_2)$. As in cases 2 and 3, the likelihood of a completed unemployment duration of t_u is

$$f_u(t_u) = \lambda_n \tilde{G}(m) \exp(-\lambda_n \tilde{G}(m) t_u)$$

Case 4a: The likelihood of an employment spell in which the first job ends with a transition into a second job and for which $\tilde{w}_1 = m$ is derived as follows. The probability that the individual will receive the minimum wage at his first job is equal to $Pr(\tilde{w}_1 = m) = \left(\frac{G(\hat{\theta}_m) - G(m)}{\tilde{G}(m)} \right)$. The joint density of completed first-job durations that end in termination and wages given $\tilde{w}_1 = m$ is equal to

$$\int_m^{\hat{\theta}_m} \left(\lambda_e \tilde{G}(\theta) \exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) \left(\frac{g(\theta)}{G(\hat{\theta}_m) - G(m)} \right) d\theta$$

where the first term in the integral is the product of the density of a first job duration which ends with a transition to a second job given that the individual initially drew a match value θ at his first job such that $\theta \in [m, \hat{\theta}_m]$, $f(t_1 | \theta, m) = \lambda_e \tilde{G}(\theta) \exp(-\lambda_e \tilde{G}(\theta)t_1)$, and the probability that an individual has not been terminated from his first job by time t_1 , $\exp(-\eta t_1)$. This product is expressed more concisely as

$$\exp(-\eta t_1) \times \lambda_e \tilde{G}(\theta) \exp(-(\lambda_e \tilde{G}(\theta))t_1) = \lambda_e \tilde{G}(\theta) \exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1).$$

The joint wage-duration density of an employment spell in which the first job ends with a transition into a second job and for which $\tilde{w}_1 = m$ is therefore:

$$\begin{aligned} K_{4a}(m, t_1) &= \left(\frac{G(\hat{\theta}_m) - G(m)}{\tilde{G}(m)} \right) \times \\ &\quad \times \int_m^{\hat{\theta}_m} \left(\lambda_e \tilde{G}(\theta) \exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) \left(\frac{g(\theta)}{G(\hat{\theta}_m) - G(m)} \right) d\theta \\ &= \frac{1}{\tilde{G}(m)} \int_m^{\hat{\theta}_m} \left(\lambda_e \tilde{G}(\theta) \exp(-(\eta + \lambda_e \tilde{G}(\theta))t_1) \right) g(\theta) d\theta \end{aligned}$$

Since we assume that true wages are measured correctly when observed wages are equal to the minimum wage, we have to weight this likelihood by the probability of being measured correctly $(1 - \tau)$. We denote the weighted density by $\tilde{k}_{4a}(m, \tilde{t}_1)$.

$$\tilde{k}_{4a}(m, \tilde{t}_1) = k_{4a}(m, \tilde{t}_1)(1 - \tau)$$

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{4a}(m, t_u, \tilde{t}_1, \Psi = 1|T) = \omega(T) \tilde{k}_{4a}(m, \tilde{t}_1) * f_u(t_u).$$

Case4b: The likelihood of an employment spell in which the first job ends with a transition into a second job and for which $\tilde{w}_1 > m$ is derived as follows. We know the probability that the individual will receive a true wage that is greater the minimum wage at his first job is equal to $Pr(w_1 > m) = \left(\frac{\tilde{G}(\hat{\theta}_m)}{\tilde{G}(m)}\right)$. Additionally, as in Cases 2b and 3b, the distribution of wages for individuals entering the current job from the unemployment state and receiving a wage greater than the minimum wage is

$$\left(\frac{\delta\tilde{\theta}(w, \theta^*(m))}{\delta w} * \frac{g(\tilde{\theta}(w, \theta^*(m)))}{\tilde{G}(\hat{\theta}_m)} \right).$$

Recall from Case 4a that the density of an employment spell in which the first job ends with a transition into a second job is $\lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))) \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))t_1)$, the joint wage-duration density of an employment spell in which the first job ends with a transition into a second job and for which $\tilde{w}_1 > m$ is therefore

$$\tilde{k}_{4b}(\tilde{w}_1, t_1) = (1 - \tau)k_{4b}(\tilde{w}_1, t_1) + (\tau) \left[P(w_1 = m) \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) + \int_m \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) k_{4b}(w_1, t_1) dw_1 \right]$$

where

$$\begin{aligned} k_{4b}(w_1, t_1) &= \left(\frac{\tilde{G}(\hat{\theta}_m)}{\tilde{G}(m)} \right) \left(\frac{\delta\tilde{\theta}(w_1, \theta^*(m))}{\delta w_1} \frac{g(\tilde{\theta}(w_1, \theta^*(m)))}{\tilde{G}(\hat{\theta}_m)} \right) \left(\lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))) \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))t_1) \right) \\ &= \left(\frac{1}{\tilde{G}(m)} \right) \left(\frac{\delta\tilde{\theta}(w_1, \theta^*(m))}{\delta w_1} g(\tilde{\theta}(w_1, \theta^*(m))) \right) \left(\lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))) \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))t_1) \right) \end{aligned}$$

Both $\tilde{k}_{4b}(\tilde{w}_1, t_1)$ and $k_{4b}(w_1, t_1)$ are almost identical to $\tilde{k}_{3b}(\tilde{w}_1, t_1)$ and $k_{3b}(w_1, t_1)$ except for the fact that the employment duration densities differ. In case 4b, it is $\lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))) \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))t_1)$ while in case 3b it is $\eta \exp(-(\eta + \lambda_e \tilde{G}(\tilde{\theta}(w, \theta^*(m))))t_1)$.

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{4b}(\tilde{w}_1, t_u, t_1, \Psi = 1|T) = \omega(T) \tilde{k}_{4b}(\tilde{w}_1, t_1) * f_u(t_u).$$

Case 4c: The likelihood of an employment spell in which the first job ends with a transition into a second job and for which $\tilde{w}_1 < m$ is derived as follows. The model predicts that all true wages must be greater than or equal to the minimum wage ($w_1 \geq m$). Therefore, any observed wage below the minimum must correspond to a true wage that is measured with error. The joint wage-duration density of observed wages $\tilde{w}_1 < m$ and employment durations t_1 is:

$$\tilde{k}_{4c}(\tilde{w}_1, t_1) = (\tau) \left[P(w_1 = m) \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) + \int_m \frac{1}{w_1} f\left(\frac{\tilde{w}_1}{w_1}\right) k_{4b}(w_1, t_1) dw_1 \right]$$

where the terms enclosed in brackets are identical to those enclosed in the brackets in $\tilde{k}_{4b}(\tilde{w}_1, t_1)$ in the previous section (case 4b).

Multiplying the above density by the likelihood of the completed unemployment spell duration, we get

$$L_{4c}(\tilde{w}_1, t_u, t_1, \Psi = 1|T) = \omega(T)\tilde{k}_{4c}(\tilde{w}_1, t_1) * f_u(t_u).$$

Case 5: We derive the likelihood contribution of those individuals in our sample who do not experience an unemployment spell. Using their sample window T , we construct this probability

$$p(\Psi = 0|T) = 1 - \omega(T)$$

Let the set of individuals whose likelihood contribution was defined in case j be denoted by S_j . Notice that some j 's have subscripts a, b , or c to denote the subtypes found in the derivation of the various components of the likelihood function. For example, S_{2b} denotes the set of individuals with an unemployment spell followed by a right-censored first job spell in which they earn a wage greater than the minimum.

Given the results from Cases 1 through 5, the log likelihood function is given by

$$\begin{aligned} \ln L = & \sum_{i \in S_1} \ln L_1(t_{u,i}, \Psi_i=1|T_i) + \\ & + \sum_{i \in S_{2a}} \ln L_{2a}(m, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) + \sum_{i \in S_{2b}} \ln L_{2b}(\tilde{w}_{1,i}, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) + \sum_{i \in S_{2c}} \ln L_{2c}(\tilde{w}_{1,i}, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) \\ & + \sum_{i \in S_{3a}} \ln L_{3a}(m, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) + \sum_{i \in S_{3b}} \ln L_{3b}(\tilde{w}_{1,i}, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) + \sum_{i \in S_{3c}} \ln L_{3c}(\tilde{w}_{1,i}, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) \\ & + \sum_{i \in S_{4a}} \ln L_{4a}(m, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) + \sum_{i \in S_{4b}} \ln L_{4b}(\tilde{w}_{1,i}, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) + \sum_{i \in S_{4c}} \ln L_{4c}(\tilde{w}_{1,i}, t_{u,i}, t_{1,i}, \Psi_i=1|T_i) \\ & + \sum_{i \in S_5} p(\Psi_i = 0|T_i) \end{aligned}$$

Our model is characterized in terms of the parameter vector

$$\Upsilon = (\lambda_n, \lambda_e, \eta, G, \sigma_e, \theta^*(m), \tau, \alpha, b, \rho)',$$

where $G(\theta)$ denotes a set of parameters that characterize the match distribution. Following Flinn (2005), we estimate all parameters of the model conditional on a value of the bargaining

power parameter³, α . In order to estimate this parameter, we use several estimates of the average wage share of revenues across all firms in the market. We refer to these as the "actual" labor shares in our description of the procedure below.

We estimate the model for each actual labor share. To do so, we simulate the model, obtain the wages and match values of all individuals who are employed in the steady state, and calculate the implied labor share as the ratio of the steady state average wage to the steady state average productivity value. If the implied labor share differs (according to some distance metric) from the labor share that we are attempting to match, we update the value of α and re-estimate the model. We repeat this procedure until the implied labor share converges to the actual labor share. The value of the bargaining power parameter at which this convergence occurs is our estimate of α .

Because of the "non-smooth" nature of the likelihood function, we cannot use gradient methods to estimate the function and have chosen to use the Nelder-Mead Simplex Algorithm to perform a directed-search optimization procedure. To compute the standard errors of the estimates, we have utilized bootstrap methods. We found the bootstrap approach attractive due to the nature of the approximations used in solving the decision rules (see Appendix A). Although solving the model is computationally intensive, it was feasible to reestimate the model 50 times. Our bootstrap estimates of the standard errors are based on these replications.

5.1 Estimation of Demand Side Parameters

In this section we discuss the identification and estimation of demand side parameters, namely the size of the set of vacancies V , the flow cost of creating a vacancy ψ , and the technological parameter v characterizing the search technology. (### Insert here the extrapolation of Flinn (2005) propositions using cross-sectional data to using event history data) As detailed in Flinn (2005), we are able to estimate of the size of the set of vacancies V only if we assume that the matching technology $M(S, V)$ has no unknown parameters and is monotone in $\frac{V}{S}$. Identification of these parameters builds on the consistent estimators of the remaining parameters of the model $(b, \rho, \lambda_n, \lambda_e, \eta, \mu_\theta, \sigma_\theta, \sigma_\epsilon, \theta^*(m), \tau, \alpha)$. For example, using the estimates of the arrival rates of job offers to unemployed and employed searchers (i.e. λ_n and λ_e) from the model with exogenous contact rates, we can obtain the estimates of v , ψ , and V .

³Given a parameter estimates for $(\lambda_n, \lambda_e, \eta, G, \sigma_\epsilon, \theta^*(m), \tau, \alpha)$, we assume a value of ρ and use our equilibrium equations to back out an estimate of b .

To obtain an estimate of v we recall that

$$\begin{aligned}\lambda_n &= \frac{U}{S} \frac{M(S, V)}{S} \\ \lambda_e &= \frac{vE}{S} \frac{M(S, V)}{S}.\end{aligned}$$

By forming the ratio $\frac{\lambda_n}{\lambda_e}$, we have

$$\frac{\lambda_n}{\lambda_e} = \frac{\frac{U}{S} \frac{M(S, V)}{S}}{\frac{vE}{S} \frac{M(S, V)}{S}} \Rightarrow \hat{v} = \frac{\lambda_e}{\lambda_n} \frac{U}{E}$$

Note that the estimate of the technological parameter is not a function of the specification of the matching function $M(S, V)$. Given the estimate $\hat{v}$, and given U and E , we can use either contact rate equation to solve for the vacancy rate. We will use the equation for the contact rate to an unemployed searcher to back out an estimate $\hat{V}$ for the value to the firm of creating a vacancy:

$$\lambda_n = \frac{U}{U + \hat{v}E} \frac{M(U + \hat{v}E, \hat{V})}{U + \hat{v}E}$$

Finally, given our estimates $\hat{v}$ and $\hat{V}$, we impose a free entry condition for firms that causes the value of creating a vacancy to be driven to zero, $V_v = 0$. We obtain an estimate of the cost of a vacancy to a firm $\hat{\psi}$.

$$\hat{\psi} = \frac{M(S, \hat{V})}{\hat{V}} J(m, \theta^*(m))$$

where $S = U + \hat{v}E$.

6 Results

This section presents the estimation results based on the econometric model discussed in the previous section. It is well-known that with the type of data available to us identification of primitive parameters requires that parametric assumptions be made regarding the distribution $G(\theta)$ (see Flinn and Heckman, 1982). We assume that the productivity distribution $G(\theta)$ is lognormal with scale parameter μ_θ and shape parameter σ_θ . We fix the annual discount rate, ρ , at 0.05.

6.1 Model Estimates

Table 4 contains model estimates from the SIPP sample. The estimates of the model are dependent on the average labor share of the firm's revenue. Out of the set of parameters estimated in our model, the bargaining power parameter and the parameters that characterize the distribution of

match values are the most dependent on this demand side information. Several authors have estimated the average labor share of firm revenue in the U.S. Blanchard, Olivier and Justin Wolfers (2000) find a value of 0.67, Krusell et al. (2000) estimate an average labor share of 0.69, and Pissarides (2000) finds a value of (###insert here). Because there is no single value of the ratio of average wages paid to firm revenues, we estimate the model using average labor shares of firm revenue of 0.65, 0.70, and 0.75. Changing the average labor share results in virtually no change in the sampling distributions associated with λ_n , λ_e , and η . This is to be expected, as the bargaining power parameter is primarily identified using the value of the labor share of firm revenue and the bargaining power parameter, in turn, affects the mapping between the distribution of observed wages and the matching distribution. As a result, there should be a minimal dependence between the parameters that affect the duration of unemployment and employment spells and the value of the labor share of firm revenues.

We first report the estimates using a labor share of 0.70. We then compare these estimates with the estimates obtained using labor shares of 0.65 and 0.75. The estimates in the second column in Table 4 indicate that the average time between contacts for unemployed searchers is $\frac{1}{0.195}$, slightly over 5 months. When a job offer arrives to an unemployed searcher, the probability he accepts it is $\tilde{G}(m) = 0.998$. Therefore, the estimated length of an unemployment spell is $\frac{1}{\lambda_n \tilde{G}(m)} = 5.137$ months. Once employed, an individual receives a alternative job offer from a competing firm approximately every $\frac{1}{0.074} = 13.5$ months. Given that an employed searcher has received an alternative job offer, the probability that the he accepts the job offer depends on his current wage. For example, if an individual is employed at a wage of \$7.50, \$10.00, or \$12.50, the probability that a job offer is acceptable is 0.95, 0.74, and 0.45, respectively. The estimated exogenous dissolution rate of jobs is 0.070, implying that workers are exogenously terminated from their jobs every 14.3 months on average. We acknowledge that this is a short average tenure among jobs that end with a transition from employment to unemployment. This may be due to several characteristics (e.g. age and schooling status) of the individuals in our sample for which did not condition upon in our econometric specification. We estimated the model using a sample comprised of young workers who traditionally have high rates of labor market turnover. Additionally, some individuals who are enrolled in school might work only for several months during the summer, after which they transition from employment to unemployment.

When the average labor share of firm revenue is 0.70, the average ln match draw in the population is 2.486 with a standard deviation of 0.288 (the implied mean and standard deviation of

the match draw θ in levels are 12.52 and 3.68). Although the standard deviation of match values is small, we must recall that $\ln$ observed wages are a product of $\ln$ true wages and an error term. The standard deviation of the true wage distribution is identified from a mixture of the standard deviations of the observed wage distribution and the measurement error distribution. We find that the standard deviation parameter of the classical measurement error (normal) distribution is 0.417 and the proportion of individuals of workers who have wages that are measured with error is estimated to be 0.322. Finally, the "implicit" reservation wage is estimated to be \$4.247. In our estimation procedure, we constrained this value always to be less than the minimum wage of \$5.15.

Varying the average labor share from 0.65 to 0.75 serves as a useful demonstration of the ways in which the observed wage distribution is mapped into the matching distribution. As the labor share increases from 0.65 to 0.70 to 0.75, we find that the estimate of the bargaining power parameter increases from 0.152 to 0.187 to 0.229. In order for workers to receive more of the firm's revenue, they must possess a more favorable bargaining position. As the bargaining power parameter increases, the wage function becomes more steeply sloped with respect to the worker's current match value given the match value that represents the workers best outside option. Any set of true wages is mapped into a set of match values with smaller variance and a lower mean. When the labor share increases from 0.65 to 0.75 and the bargaining power parameter increases from 0.152 to 0.229, we find that the mean and standard deviation of the match value distribution decrease from 13.26 to 11.03 and 4.93 to 2.88, respectively. As the match value distribution shifts to the left and becomes less variable, the "implicit" reservation match value $\theta^*(m)$ decreases from 4.519 to 4.199.

The estimate of the bargaining power parameter is of particular importance in determining the effect of an increase in the minimum wage on labor market outcomes. In Flinn (2005), in which workers were not allowed to search on the job, the estimate of the bargaining power parameter was 0.424 with a standard deviation of 0.007. In the current model in which employed workers receive alternative offers of employment, the value of the bargaining power parameter is significantly lower than 0.424. We attribute this result to the idea that ability of workers to receive alternative job offers from competing firms and to renegotiate their wage at their current job, serves as a form of bargaining power. Thus, we expect that in equilibrium models of on-the-job search in which the arrival rate of job offers for employed searchers is lower, the value of the bargaining power parameter will be higher. We will return to a discussion of the implications of this result on labor market welfare in the following section.

The estimates that we have discussed thus far pertain to our model with exogenous contact rates. We also estimate a model in which the contact rates between searchers and firms are endogenously determined. Whether the contact rates are determined exogenously or endogenously will affect the policy impacts from changes in the minimum wage. The demand-side parameters that we estimate are the technological parameter v in the definition of an effective searcher S , the cost to a firm of creating a vacancy ψ , and the size of the set of vacancies V . As we illustrated in the econometrics section, these parameters are identified from the estimates of the remaining parameters of the model. As a result, we have a set of estimates for v , ψ , and V , for each value of the labor share that we used in the estimation procedure. These results are contained in Table 5. In all three cases, we assumed the CRS form of the matching distribution

$$M(S, V) = V \left(1 - \exp \left(\frac{-S}{V} \right) \right).$$

For all three labor shares, the estimated vacancy rate V was approximately 0.102. The estimated flow cost of a vacancy is 67.05, 57.18, and 44.03 when the labor share is 0.65, 0.70, and 0.75.

For all three labor shares the technological parameter v was approximately 0.138. The standard error of the estimate of v was less than 0.030 under all three labor shares. We conclude that the estimate of v is statistically significantly different than the value of 1, as it is typically used in most treatments of on-the-job search models with endogenous contact rates (see, e.g., Pissarides (2000)). Thus, we can decidedly reject the notion that unemployed and employed individuals possess the same search technology. In fact, our estimate of v reveals that unemployed searchers are much more efficient in searching than their employed counterparts. This can be due to the fact that our sample consists of younger workers, some of whom may be enrolled in school. Spending time in school and simultaneously being employed at a job may restrict the time with which these individuals can search for better jobs. Additionally, we expect that the types of jobs at which the individuals in our sample work are jobs that are characterized by a lack of job opportunity networking possibilities which typically enhance search efficiency of older, employed individuals. It appears that in our sample, employed individuals cannot obtain better information about potential job opportunities than unemployed individuals.

7 Policy Experiments

In this section we use the model estimates obtained under different model specifications to determine the welfare impacts of the minimum wage. We plot the steady state unemployment rate as a function of the minimum wage. We plot the average steady state values associated with the state of unemployment and employment as well as an aggregate welfare measure of the two. We also use the model estimates to determine the effects of the minimum wage on age-earnings profiles of all workers.

7.1 Steady State Proportions and Average Welfare Measures

Figures 3a and 3b plot the changes in the steady state proportions of unemployed and employed individuals as the minimum wage is increased in the models in which contact rates are exogenously and endogenously determined. We observe that the steady state proportion of unemployed individuals increases in the minimum wage. This is the standard (negative) employment effect.

Figures 4a and 4b plots the changes in the average values of occupying the states of unemployment and employment. An aggregate welfare measure is also plotted. It is constructed by forming a weighted sum of the average steady state values of unemployment and employment where the respective weights are the steady state proportions of each group. By plotting the changes in these values we calculate the optimal minimum wage in the models with exogenous and endogenous contact rates. Figure 4a depicts the results from the model with exogenous contact rates and Figure 4b depicts the results from the model with endogenous contact rates. Figures 4a and 4b plot results from simulations using estimates obtained using a labor share of 0.70 only. Results from simulations of both models using estimates obtained by using labor shares of 0.65, 0.70, and 0.75 are found in Table 6.

In both figures, the average welfare values in the unemployed and employed states are single-peaked in the minimum wage m . In the model with exogenous contact rates, the value of m that maximizes the welfare of all labor market participants is \$10.78. In the model with endogenous contact rates, the value of m that maximizes the welfare of all labor market participants is \$8.18. Flinn (2005) analyzed models with exogenous and endogenous contact rates in which workers were not allowed to search for alternative job opportunities. In the model with exogenous contact rates, he found the value of m that maximizes the welfare of all labor market participants was \$8.66. In the model with endogenous contact rates, the value of m that maximizes the welfare of all labor

market participants is \$3.36.

The welfare-maximizing minimum wage is much greater in each model with on-the-job search than without on-the-job search. We attribute this to the fact that the value of the bargaining power parameter is lower in the model without on-the-job search. This low value implies that there is very little mass at lower values (e.g. \$10 per hour) of the match distribution. Thus even a high minimum wage rate of \$8 per hour, say, implies that only a small proportion of viable match opportunities would be lost. Thus high minimum wages do not lead to significant delays in the interfirm competition for labor, which is responsible for workers getting a large share of the match value over time. This effect accounts for the high welfare-maximizing minimum wages we derive in the model with exogenous contact rates. This does not imply that a high minimum wage will have beneficial welfare effects in general equilibrium, however, since firms could reduce their supply of vacancies, thus impeding the contact rate process to a significant degree. Given our estimates, this is not the case, primarily as a result of the estimate of the technological parameter $v = 0.13$ in the individual's search technology. As minimum wages are increased, some jobs are terminated, but they are the low match value jobs. These individuals who lose their jobs switch from employed search to unemployed search, thus increasing the effective supply of searchers. This increase in supply is met by an increase in the supply of vacancies, thus actually increasing contact rates over some range of m . This effect accounts for the high welfare-maximizing minimum wages we derive in the model with endogenous contact rates. While it is generally thought the general equilibrium effects mitigate the value of high minimum wages, we have found that this not need be the case, and have identified a rather interesting mechanism that subverts conventional wisdom.

7.2 Minimum Wage Effects on Age-Earnings Profiles

The work of Leighton and Mincer (1981), and more recently by Acemoglu and Pischke (2002), investigate the potential impacts of minimum wage laws on life-cycle wage profiles through reductions in general human capital investment. While we do not consider human capital investment in our model, the potential for minimum wage impacts on the shape of lifetime wage profiles exists due to effects on the bargaining environment. In our model, the minimum wage makes less viable matches available to the searcher while unemployed (e.g. the standard (negative) employment effect) and while employed (e.g. the (negative) renegotiation effect). By delaying entry into employment, the employment effect impacts workers' wage profiles by delaying the start of the firm competition process during which significant wage gains occur. The renegotiation effect also impacts workers'

wage profiles. As the minimum wage increases, employed individuals receive fewer offers from the "viable" part of the match distribution. This reduces the level of interfirm competition for labor services and reduces the rate of wage growth at each job.

To analyze how increasing the minimum wage affects individuals' age-earnings profiles, we simulate the model using our parameter estimates and various minimum wages. We use the model in which all contact rates are exogenously determined. For each value of the minimum wage, we resolve the model (e.g. a change in m produces a change in the "implicit" reservation match value $\theta^*(m)$ which affects the wage function). Figure 5 plots the average wage as a function of the time elapsed in individuals' labor market careers for various minimum wage values. All individuals begin the simulation in unemployment. The labor market career of any individual can consist of many labor market cycles, defined as sequences of labor market states beginning with an unemployment spell and ending with the last job prior to the following unemployment spell for a given individual. Therefore, throughout their labor market career, individuals can become employed at a job, renegotiate their wage at their current job, change jobs to receive a higher wage, and become exogenously terminated and return to unemployment.

We observe in Figure 5 that individuals' age-earnings profiles are increasing in age. Most wage growth occurs early in their labor market careers. At any point in the labor market career, the sample in our simulation consists of individuals who have been employed for a long time, individuals who have been employed for a short time, and individuals who are unemployed. Thus, the average wage at any point in time is a mixture of the wages of individuals who have been employed for various durations of time. Those individuals with long tenures will have renegotiated their wage and changed jobs enough times so that they earn higher wages, but face slower rates of wage growth. Contrarily, those individuals with short tenures have relatively lower wages, but face faster rates of wage growth.

We observe two effects of increasing the minimum wage on these age-earnings profiles. The first is that average wages are higher at any point in time in the labor market career. Higher minimum wages delay entry into employment, but guarantee better job offers once employed. The second is that wage profiles flatten out earlier in individuals' labor market careers when minimum wages are higher. Individuals receive better offers once employed, but these offers arrive less frequently when the minimum wage is higher. The decrease in the level of interfirm competition once employed reduces the rate of wage growth at each job.

8 Conclusion

In a matching model with binding minimum wages, increases in the minimum wage reduce the set of matches that lead to employment contracts. Nonetheless, based on estimates of the parameters describing our stationary labor market setting, we find that relatively dramatic increases in the minimum wage from its year 2000 level of \$5.15 an hour would have increased aggregate welfare. The reasons behind the welfare gain we estimate, while stemming from the particular features of the modeling framework we employ, are illuminating and potentially important to the minimum wage policy debate.

A number of analysts, including Dey and Flinn (2005) and Cahuc et al (2005), have found that allowing for direct firm competition for workers in a bargaining setting, which takes place only when employees can search on-the-job, results in low estimates of worker bargaining power. Essentially, workers can capture a substantial proportion of match surplus by having a large degree of bargaining power, which is determined by general labor market conditions, by having specific firms ‘bid’ against one another for their services based on knowledge of the individual’s potential productivity at both firms, or a combination of the two forces. Under rather standard assumptions on the search technology, these studies find that the direct firm competition for employee services is the major factor accounting for the division of match surplus in the steady state and the growth in wages over employment spells. Estimated values of workers’ generic bargaining power, α , are low as a result.

Results of Hosios (1990) indicate that for labor market efficiency, the share of match surplus allotted to a labor market participant (searchers or firms) should be set equal to their marginal contribution to the creation of match opportunities. While we allow for vacancy creation on the part of firms, we do not look explicitly at the labor market participation decision of agents on the supply side of the market. Nonetheless, we do find that substantially higher minimum wages raise aggregate welfare, and the reasons for this finding can be interpreted through the filter of the Hosios condition. The low estimated bargaining power parameter implies that workers capture only a small portion of the match surplus when accepting a job while unemployed. This implies that the most of the mass of the match distribution is located above \$10 an hour, say. Thus high minimum wages don’t result in many missed employment creation opportunities. Though the main input into the creation of match possibilities is firm vacancy creation, because the bargaining power of workers is so low, firms are essentially being overcompensated for their efforts on this front. Though

setting a higher minimum wage doesn't induce more individuals to enter the market since we have fixed the size of the participating population, turning some previously employed individuals into unemployed searchers increases the effective size of the searching population, since unemployed individuals devote more effort to search than to employees. As a result, high minimum wages are optimal in this setting whether we consider contact rates to be exogenous or endogenous. This is a marked departure from the policy analysis reported in Flinn (2005), where allowing for endogeneity of contact rates in a model without on-the-job search produced the implication that the minimum wage should be even lower than its current level. While there are many caveats to this finding, it does serve to illustrate that a binding minimum wage, when properly viewed as an incentive mechanism and not just a redistributive tool, can have beneficial impacts on the aggregate economy.

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9 Appendix A: Approximation of Decision Rules

In this appendix, we derive the approximations of the decisions rules that characterize the equilibrium of the Nash bargaining model with OTJ search. To solve this model computationally, one must know the equilibrium wage functions $w(\theta', \theta^*(m))$, the and the critical match values $\theta^*(m)$ and $\hat{\theta}_m$. Approximating the system of value functions lightens the computational burden associated with this task.

Taking the first order Taylor series approximations to $V_f(\theta', w(\theta', \theta))$ and $V_e(\theta', w(\theta', \theta))$ with respect to w (around $w = \theta$), we have

$$\begin{aligned} V_f(\theta', w(\theta', \theta)) &\approx V_f(\theta', w(\theta', \theta))|_{w=\theta} + (w - \theta) \frac{\delta V_f(\theta', w(\theta', \theta))}{\delta w} \Big|_{w=\theta} \\ V_e(\theta', w(\theta', \theta)) &\approx V_e(\theta', w(\theta', \theta))|_{w=\theta} + (w - \theta) \frac{\delta V_e(\theta', w(\theta', \theta))}{\delta w} \Big|_{w=\theta} \end{aligned}$$

After applying Leibniz's rule, the derivatives of the above equations evaluated at $w = \theta$ can be shown to equal

$$\begin{aligned} \frac{\delta V_f(\theta', w(\theta', \theta))}{\delta w} \Big|_{w=\theta} &= \frac{-1}{\rho + \eta + \lambda_e \tilde{G}(\theta)} \\ \frac{\delta V_e(\theta', w(\theta', \theta))}{\delta w} \Big|_{w=\theta} &= \frac{1}{\rho + \eta + \lambda_e \tilde{G}(\theta)} \end{aligned}$$

Substituting these derivatives back into the Taylor series approximations,

$$\begin{aligned} V_f(\theta', w(\theta', \theta)) &\approx V_f(\theta', w(\theta', \theta))|_{w=\theta} + \frac{-(w - \theta)}{\rho + \eta + \lambda_e \tilde{G}(\theta)} \\ V_e(\theta', w(\theta', \theta)) &\approx V_e(\theta', w(\theta', \theta))|_{w=\theta} + \frac{(w - \theta)}{\rho + \eta + \lambda_e \tilde{G}(\theta)} \end{aligned}$$

where

$$V_f(\theta', w(\theta', \theta))|_{w=\theta} = \left(\rho + \eta + \lambda_e \tilde{G}(\theta) \right)^{-1} \times \left\{ (\theta' - \theta) + \lambda_e \int_{\theta}^{\theta'} V_f(\theta', w(\theta', \tilde{\theta})) dG(\tilde{\theta}) \right\}$$

For ease of notation, we rename the value to the firm of receiving all of the rents $\bar{V}_f(\theta', \theta) \equiv V_f(\theta', w(\theta', \theta))|_{w=\theta}$. Similarly, we rename the value to the worker of the firm receiving all the rents as $Q(\theta) = V_e(\theta', w(\theta', \theta))|_{w=\theta}$. Therefore, when a worker changes from a firm at which he or she had a productivity value θ to a firm at which he or she has a productivity value $\theta' > \theta$, the value $Q(\theta)$ is the value to the worker from receiving none of the rents at the new firm. Alternatively, $Q(\theta)$ can be considered the value to the worker of staying at the old firm, but receiving all of the rents at the old firm.

Using our new notation, we define the two value functions as

$$\begin{aligned} V_f(\theta', w(\theta', \theta)) &\approx \bar{V}_f(\theta', \theta) + \frac{-(w - \theta)}{\rho + \eta + \lambda_e \tilde{G}(\theta)} \\ V_e(\theta', w(\theta', \theta)) &\approx Q(\theta) + \frac{(w - \theta)}{\rho + \eta + \lambda_e \tilde{G}(\theta)} \end{aligned}$$

The first order condition for maximization of the Nash bargaining problem, $S(w, \theta', \theta)$, is given by

$$\alpha \left(\bar{V}_f(\theta', \theta) + \frac{-(w - \theta)}{\rho + \eta + \lambda_e \tilde{G}(\theta)} \right) = (1 - \alpha) \left(Q(\theta) + \frac{(w - \theta)}{\rho + \eta + \lambda_e \tilde{G}(\theta)} - Q(\theta) \right).$$

Solving the first order condition for wages yields the equilibrium wage function for this pair of match values. Note, however, that we are temporarily specifying the wage function for any pair of match values (θ', θ) . We will later define the equilibrium wage function as a piece-wise function incorporating the minimum wage. For the present analysis, one can think of the pair of match values (θ', θ) as possessing the property that either $\theta' > \theta > \hat{\theta}_m$ or $\theta' > \hat{\theta}_m > \theta > m$, so that this equilibrium wage function characterizes wages greater than the minimum wage.

$$w(\theta', \theta) = \theta + \alpha \bar{V}_f(\theta', \theta) (\rho + \eta + \lambda_e \tilde{G}(\theta))$$

We note the following properties of the wage function:

$$\begin{aligned} \alpha &= 0, \lambda_e > 0 \implies w(\theta', \theta) = \theta \\ \alpha &> 0, \lambda_e = 0 \implies w(\theta', \theta) = \alpha \theta' + (1 - \alpha) \theta \end{aligned}$$

Substituting the equilibrium wage function into the approximations to the value functions, we get

$$\begin{aligned} V_f(\theta', w(\theta', \theta)) &\approx (1 - \alpha) \bar{V}_f(\theta', \theta) \\ V_e(\theta', w(\theta', \theta)) &\approx Q(\theta) + \alpha \bar{V}_f(\theta', \theta). \end{aligned}$$

Using these approximations, we can rewrite the value functions

$$\begin{aligned} \bar{V}_f(\theta', \theta) &= \left(\rho + \eta + \lambda_e \tilde{G}(\theta) \right)^{-1} \times \left\{ (\theta' - \theta) + \lambda_e \int_{\theta}^{\theta'} V_f(\theta', w(\theta', \tilde{\theta})) dG(\tilde{\theta}) \right\} \\ &= \left(\rho + \eta + \lambda_e \tilde{G}(\theta) \right)^{-1} \times \left\{ (\theta' - \theta) + \lambda_e \int_{\theta}^{\theta'} (1 - \alpha) \bar{V}_f(\theta', \tilde{\theta}) dG(\tilde{\theta}) \right\} \\ &= \left(\rho + \eta + \lambda_e \tilde{G}(\theta) \right)^{-1} \times \left\{ (\theta' - \theta) + (1 - \alpha) \lambda_e \int_{\theta}^{\theta'} \bar{V}_f(\theta', \tilde{\theta}) dG(\tilde{\theta}) \right\} \end{aligned}$$

and

$$\begin{aligned}
Q(\theta) &= \left(\rho + \eta + \lambda_e \tilde{G}(\theta) \right)^{-1} \times \left\{ \theta + \eta V_n + \lambda_e \int_{\tilde{\theta}} V_e(\tilde{\theta}, w(\tilde{\theta}, \theta)) dG(\tilde{\theta}) \right\} \\
&= \left(\rho + \eta + \lambda_e \tilde{G}(\theta) \right)^{-1} \times \left\{ \theta + \eta V_n + \lambda_e \int_{\tilde{\theta}} \left[Q(\theta) + \alpha \bar{V}_f(\tilde{\theta}, \theta) \right] dG(\tilde{\theta}) \right\} \\
&= (\rho + \eta)^{-1} \times \left\{ \theta + \eta V_n + \alpha \lambda_e \int_{\tilde{\theta}} \bar{V}_f(\tilde{\theta}, \theta) dG(\tilde{\theta}) \right\}
\end{aligned}$$

We will ease the computational burden of solving this model by analyzing only bargaining out of the unemployment state. We define the function $x(\theta) = Q(\theta) - V_n(m)$:

$$x(\theta) = \frac{\theta - \rho V_n(m) + \alpha \lambda_e \int_{\tilde{\theta}} \bar{V}_f(\tilde{\theta}, \theta) dG(\tilde{\theta})}{\rho + \eta}$$

To compute one of our critical match values, the reservation match value associated with being in the nonemployment state $\theta^*(m)$, we solve the fixed point equation

$$x(\theta^*(m)) = 0.$$

This equation characterizes the individual's indifference between unemployed search and employment at $(\theta^*(m), \theta^*(m))$. The fixed point equation is

$$\theta^*(m) = \rho V_n(m) - \alpha \lambda_e \int_{\theta^*(m)} \bar{V}_f(\tilde{\theta}, \theta^*(m)) dG(\tilde{\theta}).$$

Using the approximations to the value functions, the value of nonemployment can be written as

$$\rho V_n(m) = b + \lambda_n \int_m^{\hat{\theta}_m} \left[\frac{m - \theta^*(m)}{\rho + \eta + \lambda_e \tilde{G}(\theta^*(m))} \right] dG(\tilde{\theta}) + \alpha \lambda_n \int_{\hat{\theta}_m} \bar{V}_f(\tilde{\theta}, \theta^*(m)) dG(\tilde{\theta})$$

which enables us to write our first fixed point equation as

$$\theta^*(m) = b + \lambda_n \int_m^{\hat{\theta}_m} \left[\frac{m - \theta^*(m)}{\rho + \eta + \lambda_e \tilde{G}(\theta^*(m))} \right] dG(\tilde{\theta}) + \alpha \lambda_n \int_{\hat{\theta}_m} \bar{V}_f(\tilde{\theta}, \theta^*(m)) dG(\tilde{\theta}) - \alpha \lambda_e \int_{\theta^*(m)} \bar{V}_f(\tilde{\theta}, \theta^*(m)) dG(\tilde{\theta}).$$

We note the following properties of the fixed point equation. When $\alpha > 0$ and $\lambda_e = 0$ the fixed point equation reduces to

$$\theta^*(m) = b + \lambda_n \int_m^{\hat{\theta}_m} \left[\frac{m - \theta^*(m)}{\rho + \eta} \right] dG(\tilde{\theta}) + \alpha \lambda_n \int_{\hat{\theta}_m} \frac{(\tilde{\theta} - \theta^*(m))}{(\rho + \eta)} dG(\tilde{\theta})$$

which is the fixed point equation used in Flinn (2005) to estimate a model of minimum wages without OTJ search.

Finally, by definition of $\hat{\theta}_m$ we can solve the equation $m = w(\hat{\theta}_m, \theta^*(m))$ for $\hat{\theta}_m$ in order to get our fourth equilibrium equation. We can summarize the equilibrium solely in terms of the parameters of the model:

$$\left[\begin{array}{l} w(\theta', \theta^*(m)) = \left\{ \begin{array}{ll} m, & \text{if } \theta' \in (m, \hat{\theta}_m] \\ \theta^*(m) + \alpha \bar{V}_f(\theta', \theta^*(m)) \left(\rho + \eta + \lambda_e \tilde{G}(\theta^*(m)) \right), & \text{if } \theta' > \hat{\theta}_m. \end{array} \right. \\ \theta^*(m) = b + \lambda_n \int_m^{\hat{\theta}_m} \left[\frac{m - \theta^*(m)}{\rho + \eta + \lambda_e \tilde{G}(\theta^*(m))} \right] dG(\tilde{\theta}) + \\ + \alpha \lambda_n \int_{\hat{\theta}_m} \bar{V}_f(\tilde{\theta}, \theta^*(m)) dG(\tilde{\theta}) - \\ \alpha \lambda_e \int_{\theta^*(m)} \bar{V}_f(\tilde{\theta}, \theta^*(m)) dG(\tilde{\theta}).. \\ \hat{\theta}_m : m = w(\hat{\theta}_m, \theta^*(m)) = \theta^*(m) + \alpha \bar{V}_f(\hat{\theta}_m, \theta^*(m)) \left(\rho + \eta + \lambda_e \tilde{G}(\theta^*(m)) \right). \end{array} \right]$$

Table 1: Summary Statistics		
Sample	Number	Sample Window
Full sample	4039	18.18 (9.64)
Without a non-employment spell during sample window	2065	20.11 (10.11)
With a non-employment spell during sample window	1974	16.17 (8.69)

From unemployment	Number	Spell duration	Initial Wage	Accepted Wage
Right censored	429	10.15 (8.89)	–	–
To a job	1545	3.20 (3.54)	–	6.80 (2.27)

From a job	Number	Spell duration	Initial Wage	Accepted Wage
Right censored	420	11.82 (7.68)	7.05 (2.52)	–
To unemployment	753	5.13 (4.59)	6.75 (2.17)	–
To a job	372	5.06 (4.04)	6.64 (2.14)	7.04 (2.41)

$\Pr(w_1 = m \mid \text{have at least 1 job})$	0.0874
$\Pr(w_2 = m \mid w_1 = m \text{ and have a j2j transition})$	0.0000
$\Pr(w_2 = m \mid w_1 = m \text{ and have a j2j transition})$	0.0094
$\Pr(w_2 \geq w_1 \mid \text{and have a j2j transition})$	0.753
$\Pr(w_2 < w_1 \mid \text{and have a j2j transition})$	0.247

Note: Based on the 1996 Survey of Income and Program Participation. The sample includes individuals aged 16-24. See text for further selection criteria. Wages are measured in dollars per hour and reported durations are in months. Standard deviations are in parentheses. The sample window measures the length of time (in months) an individual responds to the survey.

Table 2: Decomposition by Gender, Race, and Age

Gender:	
Proportion Males in Sample	0.559
Proportion Females in Sample	0.441
Race:	
Proportion White	0.847
Proportion Black/Hispanic	0.153
Age	Relative. Frequency
16	0.0703
17	0.0946
18	0.0988
19	0.1094
20	0.1312
21	0.1139
22	0.1206
23	0.1248
24	0.1367
Total:	1.0000

Note: This table uses the full sample, including those individuals who do not experience an unemployment spell in their sample window.

Table 3: Age Decomposition for Minimum Wage Earners	
Age	Proportion of MW earners at first job
16	0.1926
17	0.2148
18	0.1333
19	0.0815
20	0.1333
21	0.0740
22	0.0740
23	0.0519
24	0.0444
Total:	1.0000

Table 4: Maximum Likelihood Estimates				
LS	0.65	0.70	0.75	0.90
m	5.15	5.15	5.15	5.15
λ_n	0.196 (0.007)	0.195 (0.008)	0.196 (0.009)	0.207 (0.016)
λ_e	0.075 (0.004)	0.074 (0.005)	0.075 (0.004)	0.076 (0.007)
η	0.071 (0.003)	0.070 (0.003)	0.070 (0.002)	0.070 (0.003)
μ_θ	2.520 (0.050)	2.486 (0.075)	2.368 (0.071)	1.993 (0.041)
σ_θ	0.360 (0.024)	0.288 (0.046)	0.257 (0.047)	0.215 (0.029)
σ_e	0.417 (0.026)	0.419 (0.021)	0.419 (0.021)	0.417 (0.024)
$\theta^*(m)$	4.519 (0.080)	4.247 (0.169)	4.199 (0.203)	4.348 (0.119)
τ	0.315 (0.028)	0.322 (0.027)	0.327 (0.027)	0.322 (0.025)
α	0.152 (0.006)	0.187 (0.008)	0.229 (0.006)	0.443 (0.004)
$E(\theta)$	13.26	12.52	11.03	7.51
$Sd(\theta)$	4.93	3.68	2.88	1.63

Table 5: Point Estimates of Demand-Side Parameters			
	Labor Share		
Characteristic	0.65	0.70	0.75
Technological Parameter	0.1381	0.1374	0.1381
	(0.012)	(0.013)	(0.029)
Vacancy Rate	0.1021	0.1013	0.1019
Flow Vacancy Cost	67.05	57.18	44.03

Table 6: Simulated Welfare Measures						
Outcome	Labor Share					
	0.65	0.65	0.70	0.70	0.75	0.75
	Exo	Endo	Exo	Endo	Exo	Endo
Value of Unemployment	\$11.88	\$9.66	\$10.78	\$8.18	\$9.47	\$7.88
Value of Employment	\$12.07	\$9.66	\$10.95	\$8.18	\$9.47	\$7.76
Aggregate Labor Market Welfare	\$11.85	\$9.66	\$10.78	\$8.18	\$9.34	\$7.76

Note: Elements in this table are minimum wages that maximize the corresponding outcome (a welfare measure for individuals in a certain labor market state).

"Exo" refers to exogenous contact rates; "Endo" refers to endogenous contact rates.

Aggregate labor market welfare is a weighted average of the value of unemployment and the value of employment where the weights correspond to the steady state probabilities of being in each state. A CRS matching function is used when there are endogenous contact rates

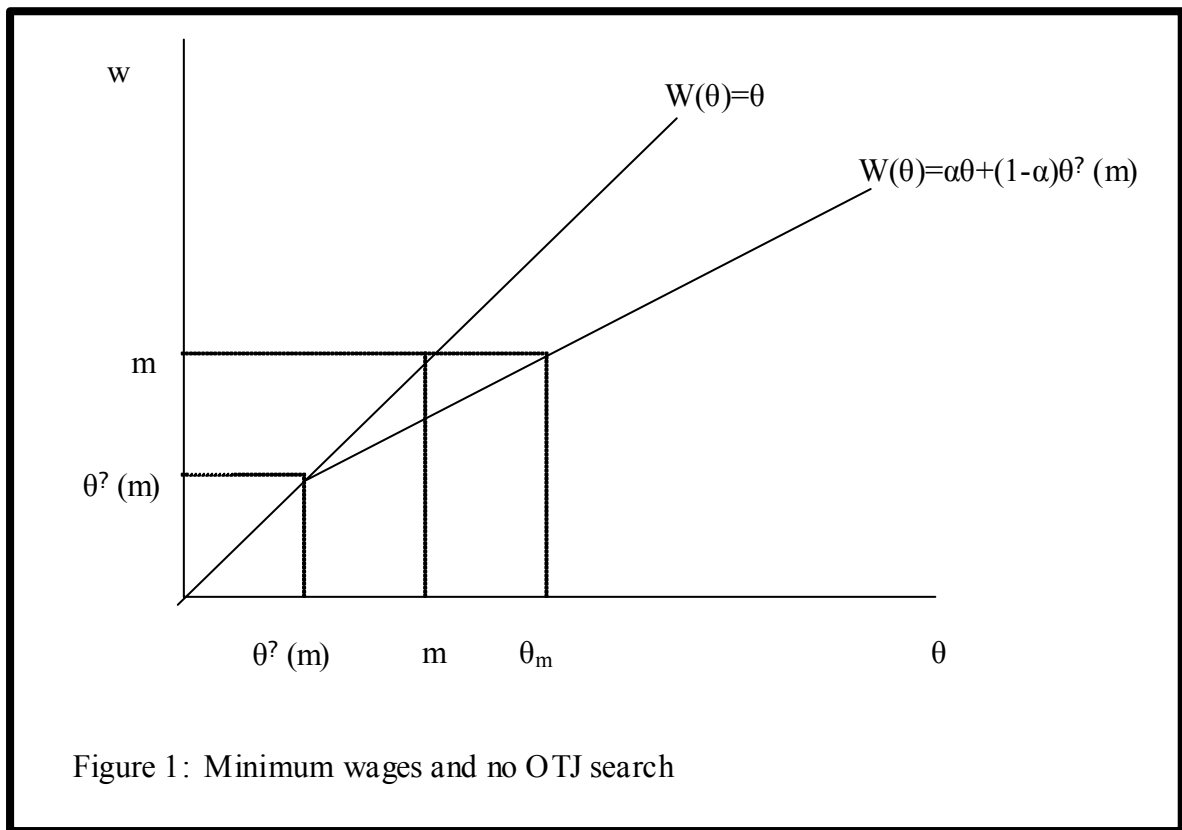


Figure 2 Wage Function in OTJ Search

(TO BE INSERTED HERE)

Figure 3.a
Steady State Proportions
Exogenous Contact Rates

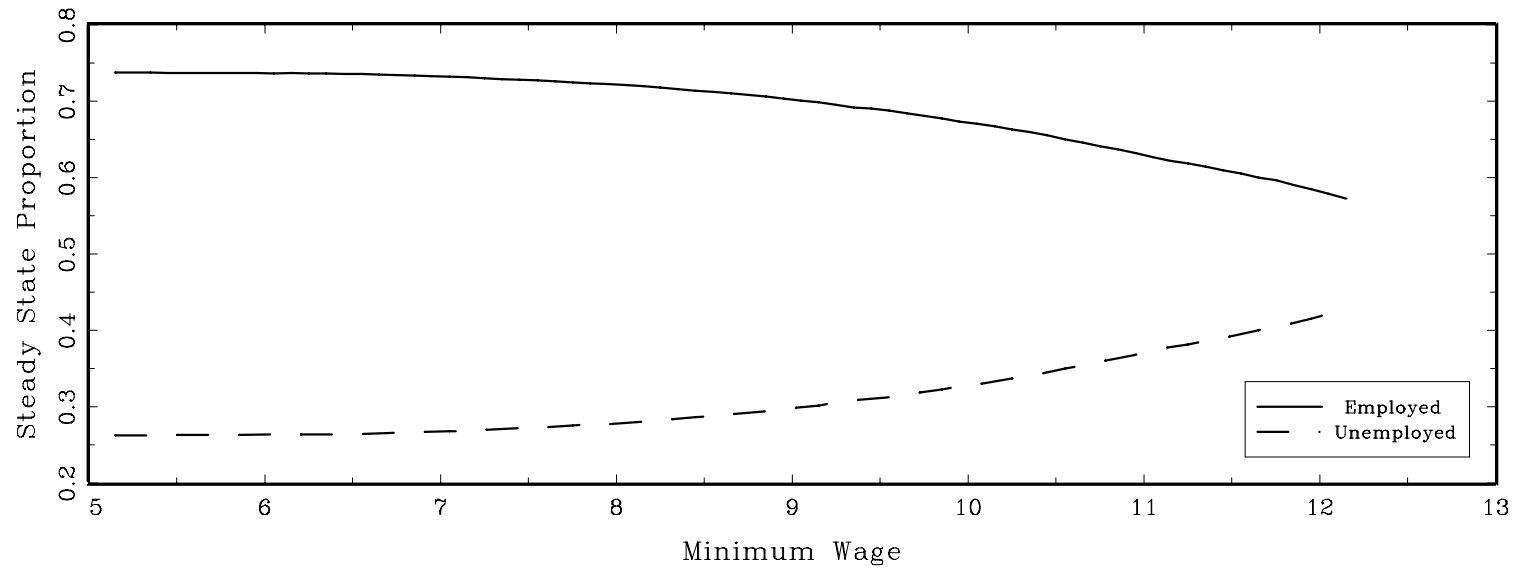


Figure 3.b
Steady State Proportions
Endogenous Contact Rates

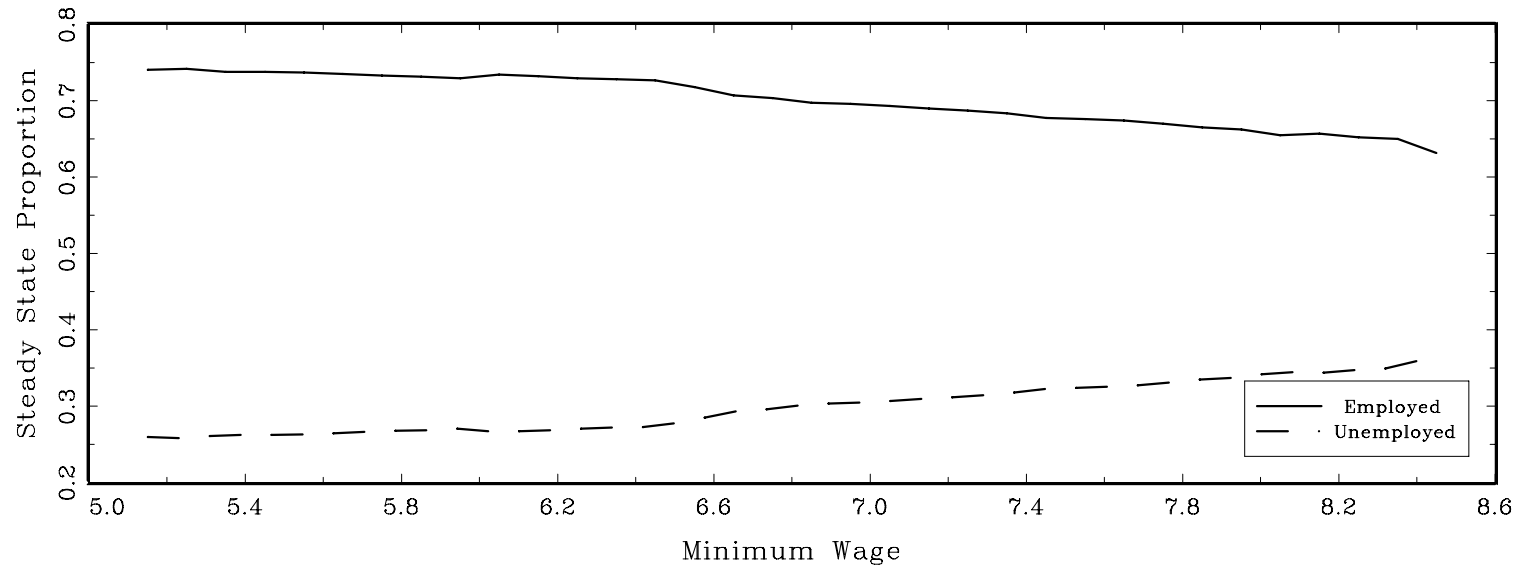


Figure 4.a
Average Welfare
Exogenous Contact Rates

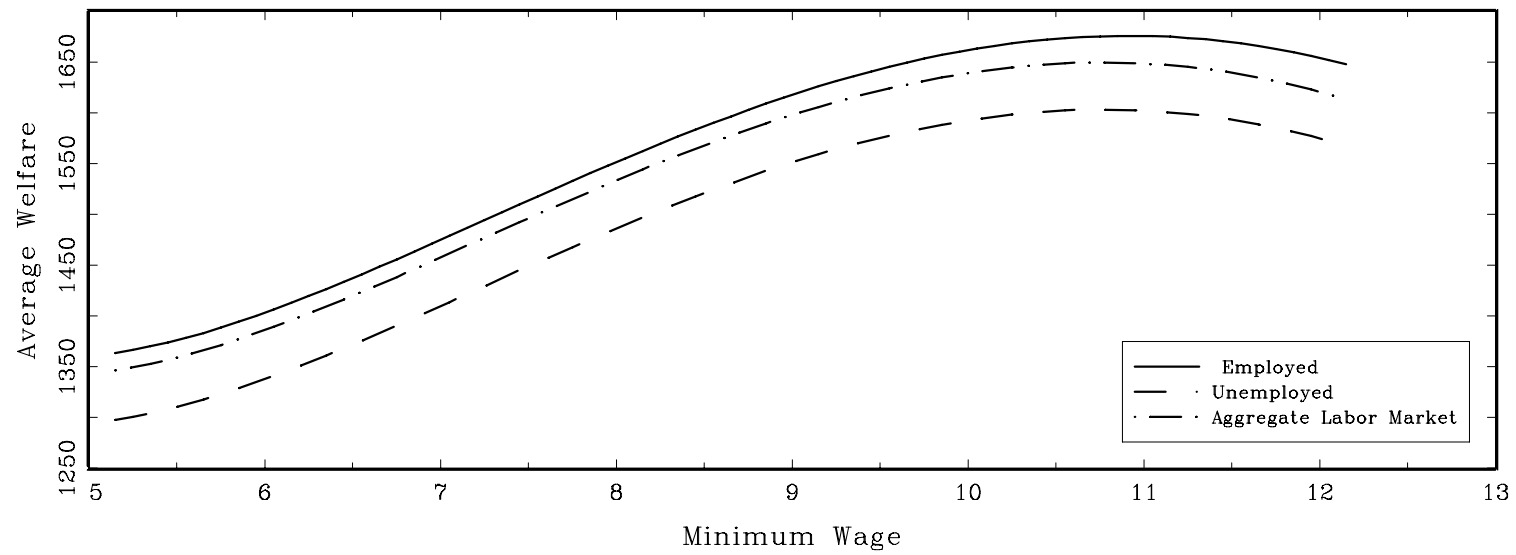


Figure 4.b
Average Welfare
Endogenous Contact Rates

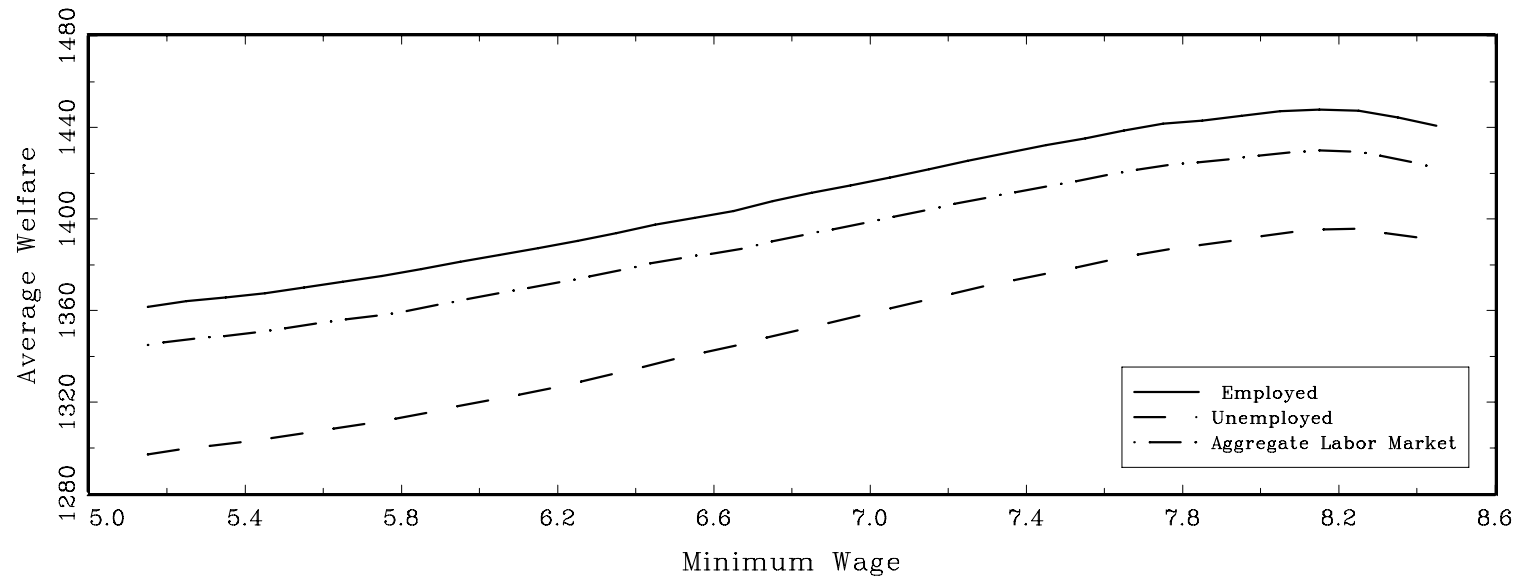


Figure 5
Wage Profile
Exogenous Contact Rates

