Network Competition with Local Network Externalities

Øystein Fjeldstad, Espen R. Moen, and Christian Riis

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Abstract

Local network externalities are present when the network externalities associated with entering a certain network depends not only on the total number of agents in the network, but on the identity of the agents in the network. We explore the consequences of local network externalities within a framework where two networks compete on the Hotelling circle. We first show that local network externalities, in contrast to global network externalities, do not sharpen competition. Then we show that the equilbrium suffers from composition inefficiency, due to a coordination failure. We also study the effects of the systematic difference between the marginal and the average consumer that the local network externalities create. The consequences are too little traffic within the network, and too much interconnection between the networks.

1 Introduction

Network externalities are present when an agent's utility of consumption of a good depends on the number of other agents that are consuming the good. As network effects are important in the new growth industries as telecommunication and information technology, the effects of network externalities on competition and on the efficiency of the market solution has achieved considerable attention lately. Examples include Michael L Katz and Carl Shapiro, 1985, W. Brian Arthur, 1989, Joseph Farrell and Garth Saloner, 1986, Michael L Katz and Carl Shapiro, 1992, Farrell and Garth Saloner, 1985).

In this literature, network externalities are only associated with the size of the network; the more customers that belong to a network, the more attractive is the network. Although network *structure* clearly impacts network externality effects, this topic is not much explored in the literature (Joseph Farrell and Paul Klemperer, 2003). Exceptions are Jeffrey Rohlf's (1974) and Beige (2001). In his early treatment of network externalities, Rohlfs points to what he identifies as "Community of interest groups" and "Few principle contacts" as factors that can increase the utility that customers obtain. Beige explores such effects in micro-structure coordination game.

In the present paper we introduce what we refer to as local network effects into a standard Hotelling model of competition between two networks offering differentiated products. Due to network effects, the agents' utility from choosing a given supplier depends on who else will choose the same supplier. In contrast with the existing literature we assume that the agents not only care about the size of the network, but also the identity of the agents in the network. More specifically, network externalities are stronger for agents that are close (in some well defined way) than for agents that are distant. We refer to this as local network externalities.

Let us give some examples. If the networks are communication goods with limited interconnection, an agent prefers (*cet par*) to be in the same network as the people with whom he communicates intensively. In the case of system goods (like operative systems), an agent would like to join the same system as other agents with which he exchanges files frequently. Furthermore, the network externalities may be indirect and associated for instance with the number and quality of applications available (Katz and Shapiro 1994). As an example, architects often use McIntosh. For an architect, the most interesting applications are the one that are related to his work. Therefore, when choosing between platforms an architect is primarily interested in the platform that attract more architects. If the other platform attracts a larger fractions of other professionals that may be of less importance.

The aim of the paper is twofold. First we explore how local network externalities influence competition between firms, and contrast our findings with the results with global network externalities. Second, we demonstrate that local network externalities create systematic differences between marginal and average customers within a network, and explore how this may create distortions in pricing and compatibility decisions made by the network owners.

It is well known that global network externalities may stiffen competition between firms (Gilbert 1992, Farrell and Saloner 1992, Foros and Hansen 2002, Shy 2001): by adding one more customer, a network becomes more attractive, and this tends to increase the elasticity of demand and lower equilibrium prices. With local network externalities this effect is weakened or even eliminated. At first glance this is rather surprising: In the presence of local network externalities, adding one more customer is very valuable for the marginal consumer, as the associated network externality is strong. However, at this point the previously marginal consumer is inframarginal. For the now marginal customers the network externality need not be stronger than it was for the "old" marginal consumer before the change. As a result, the local network externalities may not make demand more elastic, and therefore do not increase the competitive pressure in the same way as global network externalities.

Local externalities also create systematic differences between the average and the marginal customer which are absent with global network externalities. First we study the network owners' incentive to use two-part tariffs. A fixed fee is charged to connect to the network. The usage price is charged for use of the network (traffic minutes or number of applications used). Without compatibility, the marginal customer has lower usage than the average customer, as the average customer has fewer agents to communicate with within the network (or interested in fewer of the applications). The network owner thus has an incentive to increase usage costs and reduce the fixed fee. In a similar fashion as a monopolist involved in second order price discrimination the network owner distorts the usage of the network in order yo attract rents from the inframarginal customers. This contrasts the results obtained with global network externalities, in which case the communication intensity is the same for the inframarginal and the marginal consumers.

Then we study the network owner's incentives to invest in technology that enhances one-way compatibility (for instance the possibility of running applications written for the competing network). As observed by Farrel and Salloner (1992), the network owner with global network externalities chooses the socially optimal level of one-way compatibility. With local externalities this is no longer true. The marginal customer has a larger set of agents socially close to him than has the average customer, and therefore has a higher willingness to pay for one-way compatibility (like running the other network's applications). Hence, the network owner tends to overinvest in one-way compatibility. Similar effects rise if the owner of a communication network can discriminate between communication within the network and out of the network. Since the marginal customer communicates most intensely with agents outside the network, the network owner will have an incentive to subsidize communication out of the network and more than recoup the loss by increasing the fixed cost of being connected to the network.

There is considerable sociological empirical evidence that a variety of actor interactions follow semi-stable structured pre-established interrelationships with higher within group than between group exchange intensity (Mark Granovetter, 1973,1985, B. Uzzi, 1996, B. Wellman and S. Wortley, 1990) and recent work in economics has investigated the impact of such ex ante inter-actor linkages on economic exchange (Rachel E. Kranton and Deborah F. Minehart, 2001).

Local effects may be present in several important sectors and activities. Network externalities are obvious in some transaction services such as telephony or operation of stock exchanges (Nicholas Economides, 1993), and less apparent in other network industries e.g. banking and insurance (Nicholas Economides, 2001). In general there are strong returns to scale from network externalities in institutions (Douglass C. North, 1991) and hence the service provided by the corresponding transaction organizations that operate technologies and employ institutional instruments in order to facilitate efficient inter-actor exchange (Douglass C. North, 1991) and lower their customers' transaction costs (O. Williamson, 1975). Direct network externalities are not limited to transaction services, but they are of particular interest in such industries because the intrinsic value of the service typically is low (Michael L Katz and Carl Shapiro, 1985, Jeffrey Rohlfs, 1974). Although major welfare gains from transaction services result from the ability to extend trade beyond local contexts, local direct network effects are particularly plausible in the services provided by transaction organizations, (Douglass C. North and John Joseph Wallis, 1982) because a large number of exchanges are likely to take place between previously related parties (Mark Granovetter, 1995, 1985, B. Uzzi, 1996, D. J. Watts, 1999) even if their initial exchange was the result of the availability of transaction services. Most transaction services are provided with some form of interconnection between competitors, e.g. telecommunication firms allow cross firm calling and banks clear inter-bank transactions. Suppliers face a trade-off between increasing the utility of their services by interconnecting their networks and appropriating monopoly rents and they set prices for interconnection by competitors and for customers cross network transactions. Firms with larger networks depend less on other firms for interconnect and thus are less dependent on firms with smaller networks than vice versa (J. Laffont et al., 1998a, b).

The rest of the paper is organized as follows. Section 2 constructs basic model capturing local network externalities. Section 3 analyse the effect of local network externalities on competition intensity. Section 4 addresses the composition efficiency of the equilibrium. Section 5 studies the effects of the systematic difference between inframarginal and intramarginal consumers on pricing decisions and the allocation of resources. The last section concludes.

2 Modelling local network externalities

The starting point for our analysis is competition between two firms offering network or system goods. These goods have different technological characteristics (like PC and Macintosh). The consumers choose which of the two system goods to consume (if any). We represent the technology space in a standard way by a line, where the distance between the two platforms is normalized to one. Firm (or network) A is located at zero, and firm (network) B at 1. We let the metric y denote a distance measure from any given technology to technology platform A.

Consumers are distributed evenly on a circle with circumference equal to two.[1] Consumers join a network in order to communicate (broadly defined, this does not exclude system goods) with her "friends". Denote by z_i agent i's "social location" (around which the agent's friends are located), and assume that an agent with social index z_i has a set of friends distributed on z according to a symmetric and single peaked density function $g(z-z_i) := g_i(z)$ with support $[-\underline{z}, \underline{z}]$ and mean 0. The total number of friends a person with social index z_i has is $\int g(z - z_i)dz := \overline{g}$, where the integral is taken over the circle. The size of the support $[-\underline{z}, \underline{z}]$ is referred to as the density of friends. A higher density of friends means a smaller support (where technically we expand the density $g(z - z_i)$ proportionally such that the integral is constant equal to $\overline{g}$). In addition each agent has an ideal localization in technological space. Technology preferences are specified as follows,

$$y_i = |z_i + \varepsilon_i| \tag{1}$$

[1]The motivation behind letting agents be distributed on the circle is to avoid the asymmetry associated with consumers on the end of a line that only communicate in one direction. Each consumer has a set of other agents with which he or she communicate, hereafter called friends.

where ε_i is a random variable with mean zero and support on $[-\underline{\varepsilon}, \underline{\varepsilon}]$. We denote by $F(\varepsilon)$ and $f(\varepsilon)$ the CDF and the density of ε_i respectively.

Each individual is characterized by a pair (y_i, z_i).

Global network externalities arise in two cases; either each person friends are distributed evenly on the circle, that is $g_i(z) = \overline{g}/2$, or technological preference and social location are uncorrelated, in which case ε_i is uniformly distributed on $[-1, 1]$.

Consumers do not know the technology preferences of their friends. We assume the choice of network is uncoordinated, hence when choosing between networks an agent does not know which network his friends will join.

3 Equilibrium with uniform prices

In this section we derive the equilibrium of the model under the assumption that firms charge connection prices only. The timing of the model goes as follows:

1. The two firms A and B simultaneously and independently choose connection prices p_A and p_B, respectively. Firms are not able to price discriminate by setting different prices for agents with different locations at the technology circle.

2. The agents independently decide which network to assign to, given the prices and given their expectations about the number and social locations of customers within the networks. Agents cannot communicate with members of the opposite network. In equilibrium, we require that the expectations are rational.

The utility of agent of type y_i, z_i when joining network A can be written as

$$u^A(y_i, z_i) = \alpha - ty_i + \int g(z - z_i)H(z)dz - p_A \tag{2}$$

The constant α is a scaling parameter. The parameter t reflects the importance for the agents of being close to their preferred technology (intensity of technological preferences). $H(z)$ is the proportion of agents with social index z that join the A-network. Hence the number of friends than an agent with social index z_i can communicate with if she attends the A network is $\int g(z - z_i)H(z)dz$.

Analogously, joining network B yields utility

$$u^B(y_i, z_i) = \alpha - t(1 - y_i) + \int g(z - z_i)(1 - H(z))dz - p_B \tag{3}$$

A type (y_i, z_i) strictly prefers the A network if $u^A(y_i, z_i) > u^B(y_i, z_i)$, is indifferent if $u^A(y_i, z_i) = u^B(y_i, z_i)$ and strictly prefers the B network otherwise. Observe that if an agent (y_i, z_i) is indifferent between the two networks, then all agents with the same social index and technology preference $y < y_i$ strictly prefer A, whereas those with $y > y_i$ strictly prefer B. Hence the indifferent consumer is (if he exists) unique and referred to as $y(z_i)$. Furthermore if a consumer with social index z_i and technology preference y_i is indifferent between the two networks, a consumer with social location $-z_i$ and the same technology preference y_i is also indifferent, hence $y(z_i) = y(-z_i)$. Hence $H(z_i) \equiv F(y(z_i) - z_i) - F(-y(z_i) - z_i)$.

An equilibrium of the game can be described as follows:

$$\begin{gathered} i) \text{ If } u^A(y, z_i) > u^B(y, z_i) \text{ for all } y \epsilon [0, 1] \text{ then } H(z_i) = 1 \\ ii) \text{ If } u^A(y, z_i) < u^B(y, z_i) \text{ for all } y \epsilon [0, 1] \text{ then } H(z_i) = 0 \\ \text{Otherwise } H(z_i) \text{ is implicitly determined by} \\ iii) \; ty(z_i) - \int g(z - z_i)H(z)dz = \frac{p_B - p_A - \overline{g} + t}{2} \end{gathered} \tag{4}$$

The last condition follows from setting $u^A(y_i(z_i), z_i)$ equal to $u^B(y_i(z_i), z_i)$.

If firms charge equal prices the sign of the right hand side of the last condition in (4) depends on the size of "transport cost" t relatively to the network effect $\overline{g}$. Note that if $\overline{g} > t$ (and p_A equals p_B), then $H(z) = 0$ all z is an equilibrium candidate (as is $H(z) = 1$). To avoid such corner solutions, we assume that $t > \overline{g}$. The condition $t > \overline{g}$ has a correspondence in the literature on global network externalities - an equilibrium with two active networks requires that the transport cost dominates the network externality - otherwise the market "tips" and a monopoly is established (as may occur in the present model).

The model is characterized by global network externalities if friends are uniformly distributed on the circle *or* the technology preference is independent of social location. In that case consumers choose network according to technological preference (the proportion of friends is the same everywhere) thus $H(z) = 0.5$ for all z (given that network prices are equal). On the other extreme, if technology preferences and social locations are perfectly correlated (that is if the F-distribution is degenerate), each agent joins the network which is closest to the agent's position (social as technological) on the circle. Then, with symmetric prices it follows that $H(z) = 1$ for $z > 0.5$ and $H(z) = 0$ otherwise.

If social index and technology preferences are imperfectly correlated, in equilibrium people that share the same social index may still join different network according to variations in technological preferences. As shown below, this creates an externality which is not captured in the traditional literature on global externalities.

Differentiating (4 iii) yields

$$ty'(z_i) = -\int g'(z - z_i)H(z)dz < 0$$

Since $g(z - z_i)$ is single peaked and symmetric and $H(z)$ is decreasing, it follows that $y(z_i)$ is decreasing.

If density of friends is sufficiently low, a person with a strong technological preference for the B-network will join this network even in the situation where the majority of his friends belongs to the A-network. Hence $H(z)$ is in that case strictly between 0 and 1 for all z. Considering a higher density of friends, at some level all agents with social index at 0 is best off joining the A-network, also those with strong preference for B. Correspondingly all agents with social index 1 join the B-network. In that case $H(z) = 1$ for z around 0 and $H(z) = 0$ for z around 1. In this situation there exist a "first" location with an indifferent consumer (closest to A) and a "last" location with an indifferent agent (closest B) denoted z_0 and z_1 respectively. For all $z_0 < z < z_1$ we have that $0 < H(z) < 1$. Due to symmetry $0 < H(z) < 1$ if $-z_0 < z < -z_1$.

Consider stage one price game. Firm A chooses p_A so as to maximize profit π_A given by $p_A \int H(z)dz$. First order condition for profit maximum can thus be given by

$$p_A \frac{d \int H(z)dz}{dp_A} + \int H(z)dz = 0$$

Consider the first term. Decreasing p_A shifts $H(z)$ upwards, as the A network expands on both sides of the market. We can now demonstrate that the effect of network externalities on price competition depends crucially on the measure of density of friends. To be precise; the intensity of price competition is decreasing in the measure of density of friends, and at a sufficiently large density, network externalities have some effect on intensity in price competition. The critical density of friends at which the effect from network externalities disappears is described as follows:

Consider the "first" marginal consumer with social location z_0. She has her friends symmetrically distributed around z_0, below z_0 everybody is in the A-network, and above z_0 there are friends in either network, some in A and some in B. On the other side of the market, due to symmetry, we find type $-z_0$, the "first" marginal consumer on the "negative" side. If $2z_0 > \underline{z}$ the first marginal consumer on the "positive" side of the market does not communicate with the first marginal consumer on the "negative" side of the market (and certainly not with marginal consumers located even further away). This situation is referred to as pure local network externalities PLNE. Under PLNE $F(-y(z) - z) = 0$ all z where $y(z)$ exists. Hence under PLNE we can write (4 iii) as follows:

$$ty(z_i) - \int g(z - z_i)F(y(z) - z)dz = \frac{p_B - p_A - \overline{g} + t}{2}$$

subtracting tz_i on both sides yields

$$t[y(z_i) - z_i] - \int g(z - z_i)F(y(z) - z)dz = \frac{p_B - p_A - \overline{g} + t}{2} - tz_i \qquad (5)$$

Due to symmetry, the left hand side of (5) is invariant with respect to monotone transformations satisfying $\Delta y(z) = \Delta z$ all z. Consider now a decrease in p_A. Since the left hand side of (5) is constant, we can differentiate the right hand side which yields:

$$\frac{dy(z_i)}{dp_A} = \frac{dz_i}{dp_A} = -\frac{1}{2t}$$

Since an increase in p_A yields a pure location shift in $H(z)$, we find that (using Δ as shift parameter)

$$\frac{d\int H(z)dz}{dp_A} = \frac{\partial \int F(y(z) - z - \Delta)dz}{\partial \Delta}\frac{d\Delta}{dp_A} = \int f(y(z) - z)dz\frac{1}{2t} = \frac{1}{2t}$$

Since $\int H(z)dz = 0.5$ in a symmetric equilibrium (and according to Proposition 1 is unique), the profit maximizing price is

$$p_A = t$$

As is well known, global network externalities tend to increase competition and thus reduce equilibrium prices. However, our first proposition shows that this is no longer true with pure local network externalities

Proposition 1 *Suppose the network externalities are pure local (PLNE). Then the network externalities have no effect on equilibrium prices.*

As mentioned above, the additional value for the marginal customer of increasing the network's market share is positive and proportional to $\overline{g} > 0$. Still, this will not influence the pricing decision of the firm.

To gain intuition for the proposition, first note that global network externalities tend to increase price competition, because they increase the price elasticity of demand. Reducing the price will then increase the size of the network, and this will make the network even more attractive. This mechanism does not hold with local externalities. A reduction in price will increase the network size, and this has a substantial effect on the utility of the agents that previously were marginal. However, these agents are now inframarginal. The utility of joining the network for the marginal agents is unchanged (taking into account that the identities of the marginal consumers also change).

Consider briefly the situation where density of friends is so small that expanding market on one side affects the network value of marginal consumers on the other side of the market. The last consumer affected has social location $z_i = \underline{z}/2$. Hence all consumers between $\underline{z}/2$ and z_0, including the marginal consumers, communicate with some marginal consumers on the other side of the market. Hence enlarging the network, increases some marginal customers' willingness til pay for access. In this situation the network externalities intensify price competition, however with a more moderate effect than under global externalities (in which case all marginal consumer appreciate a larger network).

4 Composition efficiency

With local network externalities network composition has welfare implications, as it affects the potential number of communication links (unless networks are perfectly compatible). In a symmetric equilibrium with *two* incompatible networks, the number of communication links is *minimized* if $H(z) = 0.5$ for all z, in which case each consumer can communicate with exactly half of her friends. The number of communication links is *maximized* if $H(z)$ equals 1 up to a certain z value and then jumps to zero.[2] However, in that case some of the marginal consumers bear a high cost associated with a strong technological preference for the other network than they are attached to. Hence there is a trade-off between the social benefit of increasing the number of communication links and total transport costs.

We distinguish between optimal network *size,* with measure $\int H(z)dz$, and optimal network *composition*, reflected by the profile $H(z)$.We say that the profile $H(z)$ satisfies *composition efficiency* if it coincides with the social planner's optimal profile taking the two networks market shares as given. By *scale efficiency* we refer to socially efficient distributions of market shares, holding the composition profile $H(z)$ fixed. Our first result in this section regards an important difference between efficiency with global externalities and with pure local externalities (proofs in appendix)

Proposition 2 *The equilibrium with global network externalities is composition efficient, but not scale efficient. The equilibrium with pure local network externalities is scale efficient, but not composition efficient. With pure local network externalities the social efficient composition profile $H^W(z)$ is steeper than the equilibrium profile $H^*(z)$.*

Let us now provide intuition to the second result, that the efficient profile $H^w(z)$ is steeper than the equilibrium profile $H^*(z)$, hence the equilibrium does not satisfy composition efficiency. Consider hypothetically moving some marginal customers of type z_i from the A network to the B network. As these consumers are indifferent with respect to which network to join, the movement has no direct welfare effect. But it exerts a positive externality on customers in the B network, and a negative externality on remaining customers in the A network. The positive externality is of measure $\int g(z - z_i)[1 - H(z)]dz$ and the negative externality of measure $\int g(z - z_i)H(z)dz$. Since the equilibrium $H(z)$ profile is symmetric, and the $g(.)$ function peaks at z_i, it follows that the positive externality dominates the negative if $H(z_i) < 0.5$. In words, when a consumer located at $z_i < 0.5$ (hence has her majority of friends in the A-network)

[2] Clearly network composition is highest if all consumers are attached to the same network. However, in that case some consumers bear a very high cost associated with a strong technological preference for the other network (actually, since the transport cost dominates the network externality - the consumers with very strong preference for one of the networks, will attach to that network no matter how many of her friends who also join the same network).

joins the B-network (due to technological preference), she exerts a negative net externality since her majority of friends suffer. With global network externalities, composition efficiency is reached. Technically we have $\int g(z-z_i)[1-H(z)]dz = \overline{g}\int[1-H(z)]dz = \overline{g}/2 = \overline{g}\int H(z)dz$ in a symmetric equilibrium. Intuitively, composition efficiency is with global externalities not a matter of concern due to anonymity.

Consider the the first result regarding scale efficiency. Increasing A's market share move marginal customers from the B network to the A network. As these customers are indifferent between the two networks, the direct welfare effect of the movement is of second order. However, the movement has a first order effect on the welfare of *inframarginal* consumers as the number of communication links increase for inframarginal consumers in the A network, who benefit from the movement, whereas inframarginal consumers in the B network suffer. These two effects cancel out if the number of inframarginal customers affected by the movement in the two networks are equal. This is trivially satisfied if the two networks are of equal size (symmetry). With global externalities optimizing the sizes of the two networks yields asymmetric market shares. E.g. moving marginal customers from the smaller to the larger network exert a net positive externality since the number of customers that benefit from the movement (all customers in the large network) exceeds the number of customers that suffer. With local network externalities, scale efficiency holds: if consumers located at 0 and 1 do not communicate with the marginal consumers, the number of affected customers are equal in the networks, yielding no net externality. However, if the customers located at 0 or 1 do communicate with some of the marginal customers, there will be a weak net externality. Recall that pure local network externalities requires that the marginal consumers on each side of the market do not communicate, that does not rule out inframarginal consumers communicating with marginal consumers on both sides.

5 Endogenous consumer heterogeneity

It is well known that differences between marginal and average customers may give rise to distortions. This was first explored in Spence (1975). He studied a monopolist's choice of product quality level, and showed that this will depend on the marginal consumer's preferences for quality. A social planner's choice of quality, by contrast, depends on the average preference for quality among the consumers. If marginal and average valuation differ, the quality level chosen by the monopolist is not socially optimal.

Local network externalities give rise to a certain structure on the difference between the marginal and the average customers in a network. The marginal customers have a larger number of friends outside the network than the average customer, and this will influence both their calling patterns and preferences for interconnection. We will explore the consequences of these differences with the help of three examples.

5.1 Two-part tariffs

In this subsection we analyse the firms' incentives to price discriminate by offering non-linear contracts, and show how this may lead to distortions. We focus on two-part tariffs.

In order to analyse the effect of a two-part tariff we must explicitly incorporate network usage as a choice variable for consumers. We let x denote the intensity (or duration) of the exchange. The utility attained from one contact with intensity x is denoted $\omega(x)$ and the marginal cost is denoted c. For simplicity, we assume that only calls paid by the customer give rise to utility.

Let p_k and q_k, $k = A, B$, denote the fixed part (connection fee) and the variable part (traffic fee) of the two-part tariff respectively. The firms set p_k and q_k simultaneously and independently. The consumers then choose network and communication intensity.

The net surplus $v(q_A)$ of a call for a customer attached to the A-network is given by

$$v(q_A) = \underset{x}{Max}[\omega(x) - q_A x]$$

We denote the optimal traffic intensity by $x(q_A) \equiv -v'(q_A)$. The net surplus for a consumer (z_i, y_i) of joining the A network is then

$$u^A(y_i, z_i) = \alpha - ty_i + v(q_A) \int g(z - z_i)H(z)dz - p_A \tag{6}$$

and similarly, the net benefit of joining the B network is

$$u^B(y_i, z_i) = \alpha - t(1 - y_i) + v(q_B) \int g(z - z_i)[1 - H(z)]dz - p_B \tag{7}$$

As in section 2, if all z_i types prefer the A network then $H(z_i) = 1$, and if all prefer the B network $H(z_i) = 0$. Otherwise, z_i types are divided in two groups. Those with technology preference $y_i < y(z_i)$ who prefer the A network, and $y_i > y(z_i)$ who prefer the B network. Consumers with technology preference $y(z_i)$ are indifferent, that is $u^A(y(z_i), z_i) = u^B(y(z_i), z_i)$ or

$$ty(z_i) - \frac{v(q_A) + v(q_B)}{2} \int g(z - z_i)H(z)dz = \frac{p_B - p_A - v(q_B)\overline{g} + t}{2} \quad (8)$$

The profit of firm A is given by

$$\pi_A = p_A \int H(z)dz + x(q_A)(q_A - c) \int g(z - z_i)H(z)dz$$

which is maximized with respect to p_A and q_A.

It is convenient to solve firm As profit maximization problem by distinguishing between *optimal scale* and *optimal composition*. Observe that with pure local network externalities changing the connection fee p_A yields pure location (scale) shifts in the $H(z)$ function - analogous to "income effects" in consumer theory. Reducing the usage price however, yields location (scale) shift (since the attractiveness of the network increases) as well as composition shift (since communication intensity is attached a higher weight in the indirect utility function, hence technology preference relatively less) - analogous to income and substitution effects. Our procedure is then first to characterize the optimal usage price (hence subscriber composition) holding network size fixed (requires an adjustment in connection fee), and secondly characterize the optimal network size (determine the optimal connection fee).

Regarding the network size, due to symmetry, the equilibrium function $H(z)$ is symmetric (more precisely anti-symmetric) around $z = 0.5$. Changing usage price q_A holding network size fixed requires that (8) is unchanged for $y(z_i) = z_i = 0.5$. Inserting in (8) and differentiate yields $dp = 0.5v'(q)\overline{g}dq = -0.5x(q)\overline{g}dq$.

Maximizing π_A with respect to q_A, given $dp = -0.5x(q)\overline{g}dq$, yields the following first order condition:

$$[1-\gamma(0.5)]x(q_A)+(q_A-c)x'(q_A)+\frac{x(q_A)(q_A - c) \int g(z - z_i)\frac{dH(z)dz}{dq_A}}{\int g(z - z_i)H(z)dz} = 0, \quad (9)$$

where $\gamma(0.5) := 0.5\overline{g}/\int g(z-z_i)H(z)dz$. The variable γ thus measures the average number of communication links for the marginal customers relative to the average number of communication links for all customers in the network. Due to symmetry, the marginal consumers communicate with half of their friends on average[3]. Note that $\gamma \in (1/2, 1]$: if the network externality is global the communication intensity is type invariant, hence $\gamma = 1$. If externalities are local, the lowest feasible γ is 1/2, corresponding to a situation with an infinitely large network. The marginal consumer has half of the persons he would like to communicate with inside the network, while for the average consumer all the agents that he wants to communicate with is within his network.

The last term is the composition effect of usage prices. A lower usage price increases the network value for all consumers but at most for those consumers who have many friends in their network. Since the weight attached to usage in the utility function depends on number of friends, reducing usage price increases utility more for consumers close to z_0 (where $H(z)$ is large) than for consumers close to z_1.This tilts the $H(z)$ profile which becomes steeper (proofs are given in appendix). Hence an increase in usage price yields a flatter $H(z)$ function, which deteriorate composition efficiency.

With global externalities $\gamma = 1$. Furthermore, the composition effect vanish due to anonymity. Hence it follows from (9) that $q_A = c$. With local externalities, $\gamma < 1$, hence the first term is positive. Regarding the composition effect, market share decreases below 0.5 and increases above 0.5 (with zero net effect). Since the denominator is decreasing in z_i last term is negative as long as q_A exceeds c. Since the last term is zero if q_A equals c (and positive if q_A is lower than c) the first term of (9) dominates and it follows that $q_A > c$ in equilibriums.

Proposition 3 *With local network externalities, the firms set the traffic price q_k, $k = A, B$ above marginal cost. Thus, the traffic price exceeds the price level that induces a static first best level of traffic represented by marginal cost pricing .*

With global network externalities, profit is maximized by setting the traffic price q so as to maximize consumer surplus, and then extract this surplus

[3]Clearly, marginal consumers located close to zero communicates with a mojority of friends if they join the A network, as marginal consumers located close to 1 have most of their friends in the B network. On average, however, the number of friends in the A network is 0.5.

through a higher fixed fee p. With local network externalities, the marginal customer has a lower level of exchange than the inframarginal. Hence, the network owner is better off increasing the usage price slightly and compensate the marginal customer by reducing the fixed fee. The firm thus trades off efficiency for the "low-type" consumers and rent extraction for the "high-type" consumers. It can be show that this result does not depend on the particular specification of two part tariffs. With an optimal general contract, increasing marginal price for marginal customers increases the cost of "mimicking" inferior types, and hence enables to firm to extract more information rent. Also, the effect is amplified with a larger market share, hence a smaller network tends to price internal traffic more in accordance with static efficiency. Finally, the effect is weakened by the composition effect, since a higher usage price yields a less efficient composition which reduces the traffic volume. Since the network has a positive margin on traffic, this is costly for the network.

Note the analogy to the first order condition for an ordinary monopoly; the network owner price internal traffic as if he had some degree of market power, where the degree of market power is captured by the relative deviation between the marginal and average intensity of exchange. With global network externalities, symmetry between consumers prevails (hence $\gamma = 1$), which means that the network adopts marginal cost pricing. Note that $\gamma > 1/2$. Denoting the inverse demand elasticities by ε, the equilibrium usage price (assuming constant elasticity) is (ignoring composition effect)

$$q_A = \frac{1}{1 - \varepsilon \left[1 - \gamma(\overline{y})\right]} c \tag{10}$$

Note that as $\gamma(\overline{y})$ decreases in network size, $\gamma'(\overline{y}) < 0$. Thus, large network will have a larger tendency than small networks to overprice traffic.

Consider the socially efficient usage price. In the proposition we referred to static first best usage price as marginal cost pricing. Consider now the constrained efficient usage price, represented by the usage price that maximizes net welfare. Then the following holds:

Proposition 4 *The constrained efficient usage price is below marginal cost under pure local network externalities.*

The proposition is a collolary to Proposition 2. Since the socially optimal $H(z)$ profile is steeper than the equilibrium profile, and a lower usage prize induce consumers to transfer to the network at which they have their majority of friends, a reduction is usage price increases the degree of composition efficiency. Hence it is constrained efficient to subsidize the usage price. In optimum, the usage price trades off the loss associated with static inefficiency in that the communication intensity exceeds the marginal cost level of communication, with the marginal gain from a better composition of networks. Hence, the networks in equilibrium (compared to static first best) adjust the usage price in opposite direction of what the planner would prefer.

5.2 Compatibility

In this section we discuss the agents' incentives to undertake investments in order to make the networks compatible. We focus on the situation with one-way compatibility. Thus, network A may give its members (inferior) access to network B by undertaking an investment C. Let $\theta_A \leq 1$ denote the degree at which the agents in network A can utilize network B, and write the cost of compatibility as $C(\theta_A)$.

The timing of the game is as follows: Firms first set prices p_i, $i = A, B$ and the degree of compatibility θ_i, $i = A, B$, independently and simultaneously. Then consumers choose which platform to assign to. The utility of a customer in network A is then given by

$$u^A(y_i, z_i) = \alpha - ty_i + \int g(z-z_i)H(z)dz + \theta_A \int g(z-z_i)(1-H(z))dz - p_A \quad (11)$$

with a similar expression for u^B. As before $H(z) = 1$ for consumers located close to 0 and $H(z) = 0$ close to one. In between there are indifferent consumers with technological preference $y(z_i)$, such that $u^A(y(z_i), z_i) = u^B(y(z_i), z_i)$, or equivalently

$$ty(z_i) - \left(1 - \frac{\theta_A + \theta_B}{2}\right) \int g(z - z_i)H(z)dz = \frac{p_B - p_A - (1 - \theta_A)\overline{g} + t}{2}$$

Network A's net profit equals

$$\pi_A = p_A \int H(z)dz - C(\theta_A) \quad (12)$$

As with two-part tariff case, it is convenient to distinguish between composition effect and the scale effect when solving the network's optimization problem in fixed fee p_A and degree of compatibility θ_A. Increasing p_A for a given θ_A shifts the $H(z)$ profile to the right with no effects on its slope. The optimal degree of compatibility can then be characterized by maximizing (12) w.r.t. θ_A and simultaneously adjusting p_A such that the average customer is indifferent (who has half of her friends in the network). Analogously with the former section, we have the required adjustment $dp = (\overline{g}/2)d\theta$.

Simple manipulations then give the following first order condition for compatibility

$$\frac{\overline{g}}{2} - C'(\theta_A) = 0 \tag{13}$$

Increased compatibility is valuable to the consumers and allow the network to raise access price without making the marginal customers (on average) worse off - this is captured by the first term of (13). In optimum the network's marginal gain from compatibility exactly balances the marginal cost of compatibility.

The socially efficient degree of compatibility (contingent on equal market shares), by contrast, maximizes welfare given by

$$W = \int_0^1 \int_{-y(z_i)}^{y(z_i)} u^A(y_i, z_i) f(y_i - z_i) dy_i dz_i + \int_0^1 \int_{y(z_i)}^{-y(z_i)} u^B(y_i, z_i) f(y_i - z_i) dy_i dz_i - C(\theta_A)$$

The socially efficient solution is given by the first order condition (recall that the effects on the end points in the integration can be ignored according to the envelope theorem),

$$2 \int \int g(z - z_i) H(z_i)(1 - H(z)) dz dz_i + \frac{\partial W}{\partial (H(z))} \frac{dH(z)}{d\theta_A} - C'(\theta_A) = 0 \tag{14}$$

The first term in (14) is the average customer value of compatibility. The second term captures the composition effect of a higher degree of compatibility.

If the network externality is global, a comparison of (13) and (14) shows that the market solution is optimal: Average consumer value coincides with marginal consumers value due to anonymity, and the composition effect is zero. With local externalities both terms matter. The intuition for the result is clear: With local network externalities, the marginal consumers value compatibility higher than the average consumer, since the marginal consumer communicates more with the agents in the other network than does the average consumer. Since the firms compete for the marginal consumer, it is his/her preferences that governs the choice of compatibility, and therefore too much resources are spent on making the systems compatible compared with the socially optimal level.

Consider then the composition effect. Increasing θ_A has a partial negative effect on composition efficiency since it attracts customers that communicate intensively with the other network (that is types $z_i > 0.5$) and punish customers with most of their friends in the A-network (types $z_i < 0.5$). Hence the $H(z)$ profile becomes flatter. This yields the following result.

Proposition 5 *Suppose the network externalities are local. Then the firms have too strong incentives to make the networks (one-way) compatible.*

5.3 Interconnection fees

In this section we generalize the model by introducing internetwork exchange. We maintain the linear price structure. We assume that the networks can discriminate between internal and external traffic, and we denote by q_A^T be the unit price charged by customers in network A for communication with the customers in network B. We assume that the termination price is exogenously given (for instance set by the regulator) and equal to c. Our main finding is that as marginal consumers communicate more intensively with agents outside the network, it is optimal for the firms to set the price for external traffic *below* marginal cost.

The game structure corresponds to the previous section on two-part tariffs. In stage one of the game the networks independently and simultaneously determine their inter- and infranetwork prices q_i and q_i^T and the fixed parts of the tariffs p_i, $i = A, B$, whereas customers independently choose which network to join in stage 2. The utility of customer y if he joins network A is

$$u^A(y_i, z_i) = \alpha - ty_i + v(q_A) \int g(z-z_i)H(z)dz + v(q_A^T) \int g(z-z_i)(1-H(z))dz - p_A$$

with corresponding expressions for the utility obtained if he becomes a member of the B network.

Network A's profit can be written

$$\begin{aligned} \pi_A &= p_A \int H(z)dz + x(q_A)(q_A - c)\int g(z-z_i)H(z)dz \\ &\quad + x(q_A^T)(q_A^T - c)\int g(z-z_i)(1-H(z))dz \end{aligned} \tag{15}$$

As in previous section we can characterize the optimal inter- and intranet usage prices by maximizing (15) with respect to q_A and q_A^T conditioned on scale, $u^A(0.5)$ constant. The optimal value of q_A is still given by (10). Maximizing (15) with respect to q_A^T gives the analogous expression

$$[1-\gamma(0.5)]x(q_A^T) + (q_A^T - c)x'(q_A^T) + \frac{x(q_A^T)(q_A^T - c)\int g(z-z_i)\frac{dH(z)dz}{dq_A^T}}{\int g(z-z_i)H(z)dz} = 0, \tag{16}$$

where $\gamma^T(0.5)$ is given by

$$\gamma^T(\overline{y}) := \frac{0.5\overline{g}}{\int g(z-z_i)(1-H(z))dz} > 1$$

(recall from the section on compatibility that $N^c(\overline{y},\overline{y})$ is the number of calls out of the network by the marginal subscriber). Thus, $\gamma^T(\overline{y})$ is the number of intranet transactions for the marginal customer relative to that of the average customer, and with local externalities this fraction is greater than unity. It thus follows immediately that the firm subsidizes traffic out of the network. Solving (16) gives

$$q_A^T = \frac{1}{1-\varepsilon\left[1-\gamma^T(1-\overline{y})\right]}c$$

We have thus shown the following:

Proposition 6 *With local network externalities, the firms set the price for traffic out of the net, q_k, $k = A, B$ below marginal cost. Thus, the traffic price is below the price level that induces a first best level of traffic out of the net.*

Observe that network composition is not a concern in this section due to full compatibility. Hence the socially efficient usage prices are represented by marginal cost pricing.

6 Concluding remarks

This paper contributes to the small but growing literature on local externalities. We demonstrate the local externalities do not stiffen competition. Furthermore, due to the coordination problem that arises with non-anonymity, the equilibrium suffers from composition efficiency. Finally we derive the effects of local externalities on price setting when networks charge two parts tariffs.

7 Appendix

Composition efficiency

Social welfare is (recall that the "positive" and the "negative" sides of the circle are identical)

$$\begin{aligned} W &= \int_0^1 [\int_{-y(z_i)}^{y(z_i)} [\int g(z-z_i)H(z)dz - ty_i]f(y_i - z_i)dy_i \qquad (17) \\ &+ \int_{y(z_i)}^{-y(z_i)} [\int g(z-z_i)(1-H(z))dz - t(1-y_i)]f(y_i - z_i)dy_i]dz_i \end{aligned}$$

Consider next composition efficiency. Denote by $H^W(z)$ the efficient customer composition, the division of customers across networks that maximizes W. Consider a small expansion of the A-network in a small interval $[z', z'+k]$, and a small contraction in the interval $[z'', z''+k]$ that is $\Delta y(z) = \delta$ if $z\varepsilon[z', z'+k]$, $\Delta y(z) = -\delta$ if $z\varepsilon[z'', z''+k]$ and $\Delta y(z) = 0$ otherwise. Due to the envelope theorem the effects on W through the changes in the integration end points $y(z_i)$ cancels out (the marginal consumers are indifferent between the networks, that is trades off the communication level against transport costs). Hence, the net change in W by an increase in network A's market share (with the market equilibrium as starting point) is, where $\widetilde{f}(z)$ denotes

$(f(y(z)-z)+f(-y(z)-z)$

$$\begin{aligned}
\Delta W &= \int_0^1 [\int_{-y(z_i)}^{y(z_i)} [\int_{[z',z'+k]} g(z-z_i)\widetilde{f}(z)dz]f(y_i-z_i)dy_i \\
&\quad - \int_{y(z_i)}^{-y(z_i)} [\int_{[z',z'+k]} g(z-z_i)\widetilde{f}(z)dz]f(y_i-z_i)dy_i]dz_i \\
&\quad - \int_0^1 [\int_{-y(z_i)}^{y(z_i)} [\int_{[z'',z''+k]} g(z-z_i)\widetilde{f}(z)dz]f(y_i-z_i)dy_i \\
&\quad - \int_{y(z_i)}^{-y(z_i)} [\int_{[z'',z''+k]} g(z-z_i)\widetilde{f}(z)dz]f(y_i-z_i)dy_i]dz_i \\
&= \int_0^1 [\int_{[z',z'+k]} g(z-z_i)\widetilde{f}(z)dz](2H(z_i)-1)dz_i \\
&\quad - \int_0^1 [\int_{[z'',z''+k]} g(z-z_i)\widetilde{f}(z)dz](2H(z_i)-1)dz_i \\
&= \int_{[z',z'+k]} \left[\int g(z_i-z)H(z_i)dz_i - \int g(z_i-z)[1-H(z_i)]dz_i\right]\widetilde{f}(z)dz \\
&\quad - \int_{[z'',z''+k]} \left[\int g(z_i-z)H(z_i)dz_i - \int g(z_i-z)[1-H(z_i)]dz_i\right]\widetilde{f}(z)dz
\end{aligned}$$

Hence

$$\begin{aligned}
&\Delta W \ > 0 \text{ if } \int g(z_i-z)H(z_i)dz_i > \int g(z_i-z)[1-H(z_i)]dz_i \quad (18) \\
&\text{all } z\varepsilon[z',z'+k] \\
&\text{and} \int g(z_i-z)H(z_i)dz_i < \int g(z_i-z)[1-H(z_i)]dz_i \\
&\text{all } z\varepsilon[z'',z''+k]
\end{aligned}$$

that is, reallocating some customer types from the B network to the A network, increases social efficiency if these types number of communication links in the A network exceeds the number of communication links of these types in the B network.

Compatibility

Firm A chooses θ_A and p_A so as to maximize $2p_A\overline{y} - C(\theta_A)$ given that $u^A(\overline{y}) = \alpha - t\overline{y} + N(\overline{y},\overline{y}) + \theta_A N^c(\overline{y},\overline{y}) - p_A = k$ for some constant k. From this constraint it follows that $\frac{dp_A}{d\theta_a} = N^c(\overline{y}, \overline{y})$. The first order condition for profit maximization is thus that $C'(\theta_A) = 2\overline{y}N^c(\overline{y}, \overline{y})$, as stated in the text.

Interconnection fees

Consider the configuration where all $|y| \leq |\overline{y}|$ belong to network A and all $|y| > |\overline{y}|$ belong to network B. Given q_A and q_A^T customer y prefers the A network if and only if

$$t\,|1-y| - t\,|y| \geq \left[v(q_A^T) - v(q_A)\right]\left[N(y,\overline{y}) - N^T(y,\overline{y})\right]$$

The assumed configuration is an equilibrium if and only if (**??**) holds for all y. Observe that the LHS as well as the RHS of (**??**) are strictly decreasing in y and approach zero as y approach $\overline{y}$. Since the LHS is linear in y, and the slope of the RHS is $\left[v(q_A^T) - v(q_A)\right]2(m(\overline{y}-y) - m(\overline{y}-y)) \geq -2\left[v(q_A^T) - v(q_A)\right]m(\overline{y}-y)$ (with strict equality when y approach $\overline{y}$) and where the latter expression is strictly concave in y , it follows that no customer is better off switching to the other network if the LHS is steeper than the RHS at $y = \overline{y}$. This yields the condition (note that neither q_A nor q_A^T depend on t) $t \geq \left[v(q_A^T) - v(q_A)\right]m(0)$

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