

Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt.*

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Abstract

This paper proposes a politico-economic theory of debt and government expenditure. Agents have preferences over a private and a government-provided public good, financed through labor taxation. Subsequent generations of voters choose taxation, government expenditure and debt accumulation through repeated elections. Debt raises a conflict of interest between young and old voters as well as between current and future generations. We characterize the Markov Perfect Equilibrium of the dynamic voting game. If taxes do not distort labor supply, the economy progressively depletes its resources through debt accumulation, leaving future generations “enslaved”. However, if tax distortions are sufficiently large, the economy converges to a stationary debt level which is bounded away from the endogenous debt limit. The key factor that discipline the current fiscal policy is the concern of young voters that future public good provision will be diminished as the debt increases. The steady-state and dynamics of debt depend on the voters’ taste for public consumption. As such taste increases, the economy accumulates less debt.

We test the implications of the theory in the presence of political shocks affecting the taste for public consumption. Government debt is mean reverting and left-wing governments are predicted to accumulate less debt. Data from the US and from a panel of 21 OECD countries confirm the main predictions of the theory.

JEL No D72, E62, H41, H62, H63.

Keywords: Government debt, Fiscal policy, Fiscal discipline, Intergenerational conflict, Markov equilibrium, Political economy, Repeated voting.

***PRELIMINARY AND INCOMPLETE.** Appendix, conclusions and references are missing. We would like to thank Andreas Müller for research assistance.

1 Introduction

There are large differences in fiscal policies and government debt across countries and across time. For instance, the population-weighted average debt-GDP ratio was 62% for the major OECD countries in 2004, up from 22% in 1970.¹ In countries like Belgium, Greece, Italy and Japan, the debt-GDP ratios have exceeded 100% for most recent years, while on the opposite tail of the distribution, those of Australia, Ireland, Korea and Norway have been less than 30%.² Budgetary policies are subject to major political conflict. In spite of this, there is still a limited theoretical understanding of the politico-economic forces determining public debt.

Public debt breaks the link between taxation and expenditure, allowing governments to shift the fiscal burden to future generations. In a world where Ricardian equivalence does not hold, this raises a conflict of interest between current and future generation. As future generations are naturally underrepresented in democratic decision making, there is a politico-economic force pushing towards debt accumulation. A fundamental question is, then: what prevents the current generations from passing the entire bill for current spending to the future generations?

Financial markets could be part of the explanation; markets must believe that government liabilities will be honored, and public debt may increase local interest rates. Yet, debt remains significantly below levels threatening solvency in industrialized countries. Moreover, despite the large cross-country heterogeneity in debt-GDP ratios, interest rates respond little to the size of debt.³ In this paper, we abstract from effects working through changes in interest rates, and explore a complementary explanation, focusing on the dynamic game between successive generations of voters who care about public good provision. We construct a theory where fiscal policy is set through repeated elections, so that current governments cannot bind future governments' choice of taxation, debt, and public-good provision. The theory shows that the inter-generational political conflict combined with lack of commitment can endogenously discipline fiscal policy, even in a world where agents have no concern for future generations.

¹The countries include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Great Britain, Greece, Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland, and the United States. Source: OECD, central government debt.

²When including their pension trust fund, the Norwegian government has a net financial wealth (negative debt) of about 130% of GDP.

³For instance, the interest rate is almost uniform within the Euro area, although the debt ratios are very different across member countries. In the same vein, Japan has been the OECD country with the highest debt-to-GDP ratio and the lowest interest rate in the last decade.

We model a small open economy populated by two-period-lived agents who work when young and consume a private and a government-provided public good both periods of life. The government can issue debt up to the natural borrowing constraint and is committed to repay its debt. Every period agents vote on public-good provision, distortive labor taxation, and debt accumulation. The intergenerational conflict plays out as follows. The old voters wish to maximize current public good consumption, and thus support the maximum attainable deficit. Young voters, however, are more averse to debt, because they care about both current and future public good provision. In particular, they anticipate that future governments inheriting a large debt will cut spending on public goods. The political process, represented as a probabilistic-voting model à la Lindbeck and Weibull (1987), generates a compromise between these two desired policies.[?]

The forward-looking voting behavior of the young is key. When deciding debt policy today, they think strategically about how the debt left to the future generation affects tomorrow's political incentives for public-good provision. A large inherited debt can trigger three different adjustments: higher taxes, lower expenditure, and further debt expansion. The more future governments respond by cutting expenditure, the stronger discipline the young voters will impose on today's fiscal policy. Conversely, the more current young voters expect higher debt to lead to future increases in taxes (or debt), the less they will oppose a large deficit today. Thus, the nature of future governments' response determines how strict a discipline young voters want to impose on debt accumulation.

In our model, the expectations about the conduct of future governments are built into the dynamic voting equilibrium. In particular, we focus on Markov equilibria where the strategies of current voters can be conditioned only on pay-off-relevant state variables. In our model, the only such state variable is the debt level, which greatly simplifies the analysis. Along the equilibrium path, the expectations about the conduct of future governments depend crucially on the extent of tax distortions. Intuitively, the more distortionary taxation, the less future governments will be tempted to increase taxes, and the more they will cut public good provision in response to a larger inherited debt. Therefore, the fiscal discipline becomes stronger as taxes are more distortionary, i.e., the more concave is the Laffer curve.⁴

We show that, in the absence of labor supply distortions, the economy would deplete its resources through a progressive debt accumulation that would “enslave” future generations.

⁴International tax competition provides a simple example. Suppose that at some level of taxation, labor supply became infinitely elastic due to international tax competition. Then, future governments could not increase taxes beyond that level, and any marginal adjustment to a larger debt must be in the form of a reduction in expenditure. This strengthens the fiscal discipline as the tax competition kicks in.

Namely, future generations would be forced to work to service the outstanding debt, while their consumption, both private and public, would diminish to zero. Instead, if tax distortions are sufficiently large, the economy converges to an “interior” debt level which is bounded away from the endogenous debt limit. In this steady-state, both private consumption and public good provision are positive. In other words, labor market distortions provide future generations with a credible threat that prevents fiscal abuse by their rotten parents.

This endogenous discipline hinges on the lack of commitment. In fact, in a Ramsey problem when the first generation of voters can commit the entire future fiscal policy, debt is systematically larger than under repeated voting.⁵ This shows an interesting property of our theory. On the one hand, the lack of commitment reduces the welfare of the first generation of voters compared with the Ramsey allocation. On the other hand, future generations are better off in the political equilibrium than under Ramsey. In this sense, our time inconsistency has a *benign* nature; it redistributes resources from earlier to later generations.⁶

Our political equilibrium features a determinate debt level. If we introduce an unexpected fiscal shock, such as a war, this will be financed partly by a short-term increase in debt, and partly by an increase in taxation and a reduction in (non-military) public-good provision. When the war is finished, debt, taxes, and public goods revert back smoothly to their steady state levels. This prediction contrasts with the tax-smoothing implication of Barro (1979). He shows that if the distortionary costs of taxation are convex, governments should use debt to absorb fiscal shocks, and spread the tax burden evenly over future periods. Thus, debt should not be mean-reverting; after the war, there is no reason to reduce debt unless new shocks occur. Interestingly, the same result holds in our model under commitment. The data support the prediction of our politico-economic theory. Bohn (1998) shows that a short-lived increase in US government expenditures implies an increase in debt with a subsequent reversion in debt. In our empirical section we show that this stylized fact holds up for a panel data set of OECD countries. Moreover, as noted by Barro (1986), non-military spending is crowded out during wars in the US— exactly as our model predicts.??

A sharper test of our theory concerns the response of fiscal policy to political shocks. We introduce cohort-specific preference shocks affecting the agents’ taste for public goods. Our theory predicts that an increase in the appreciation for public consumption will strengthen

⁵Since the political outcome is always influenced by the forward-looking young voters, such fiscal discipline is a persistent force in the model.

⁶In standard formulations, the planner only attaches a positive weight to the welfare of the first generations, while future generations enter the planner’s preferences indirectly through the altruism of the first generation. For an exception, see Farhi and Werning (2005) where the planner attaches a positive weight on the welfare of all generations, resulting in an effective social discount factors exceeding the private one.

fiscal discipline, inducing an increase in taxation and a reduction in debt. In order to identify such preference shifts empirically, we follow Persson and Svensson (1989), and assume that governments differ in their weights on public good provision: left-wing (right-wing) governments care more (less) for public goods. In other words, young right-wing voters are less concerned with future public-good provision, so their fiscal discipline is weaker than that of left-wing voters. Changes in the political color of governments are then associated with changes in fiscal policy regime: right-wing governments should run larger deficits and accumulate more debt, in spite of no difference in intergenerational altruism between left-wing and right-wing voters.

We test these predictions using both US time series and OECD panel data. The results confirm the prediction of our theory. For instance, in the US we find that a shift from a democrat president to a republican one induces an increase in the debt-output ratio of between 1.7% and 2% per year. These results are statistically significant and robust to a number of control variables. The long-run effects are sizable. According to our estimates, an infinite sequence of republican presidents would imply a 2.7 times larger debt-output ratio than a sequence of democrat presidents (15% versus 41%). Similar results obtain in a panel of 21 OECD countries. We estimate a regression including country and time fixed effects, and a number of control variables. We use various alternative measures of the political orientation of governments, and find that right-wing governments accumulate significantly more debt than left-wing governments, although the quantitative effects are smaller than for the US.

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Our paper is related to a number of contributions on the determination of government debt. The papers closest to us are Barro (1979), Aiyagari, Marcet, Seppälä, and Sargent (2002), and Krusell, Martin, and Rios-Rull (2005). These papers have, as in our model, distortionary taxation and non-state-contingent government debt (as opposed to Lucas and Stokey, 1983). Different from us, however, these papers focus on representative-agent economies, so there is no scope for political conflict. Moreover, these papers have no public-good consumption, so our disciplining effect is not present.??

Interestingly, Aiyagari et al. (2002) find that government debt should be stationary, essentially for the same reason that individual wealth is stationary in an Aiyagari-Bewley-Huggett economy.⁷ Hence, even with commitment government debt can be (weakly) auto-regressive. We view our mechanism as a complementary explanation for why government debt indeed

⁷In these economies agents have a precautionary savings motive due to stochastic income. Wealth is bounded below by a borrowing constraint. Due to the equilibrium interest rate being smaller than the discount rate and that the absolute risk aversion is falling in consumption, the intertemporal incentive to reduce wealth will dominate when wealth becomes sufficiently large. Therefore, individual wealth will be stationary if the income process is stationary.

is auto-regressive in response to surprising expenditure shocks. Moreover, we show that our mechanism gives rise to a quantitatively large reduction in mean reversion, compared to the mechanism in Ayiagari et al. (2002).

While Barro (1979) and Ayiagari et al. (2002) assume commitment, Krusell et al. (2005) focus on time-consistent policies without risk. Their main point is that, due to an incentive to manipulate interest rates, there exists time-consistent policies replicating the commitment solution of Barro (1979).?

The paper is organized as follows. In section 2 we describe the model environment and derive the Generalized Euler Equation which is key to the characterization of the political equilibrium. Section 3 provides two examples that admit an analytical solution. Section 4 analyzes the general case. Section ?? discusses two applications (fiscal and political shocks). Section 8 concludes.

2 Model Economy

The model economy is populated by overlapping generations of two-period lived agents who work in the first period and live off their savings in the second period of their lives. The population size is assumed to be constant. Agents earn utility from the consumption of two goods: a private good (c) and a public good (g) which is provided by the government.

Private goods can be produced via two technologies – market and household production. Market production is subject to constant returns, and agents earn a hourly wage w . The household production technology is represented by the following production function;

$$y_H = F(h - h_M), \quad F'(\cdot) > 0, \quad F''(\cdot) \leq 0,$$

where h is the total individual time endowment, h_M is the market labor supply, and $h - h_M \geq 0$ is the household activity. Since the government cannot tax household production, taxation distorts the share of time that agents devote to market activity. Agents choose the allocation of their time so as to maximize total labor income, which is denoted $A(\tau)$;

$$A(\tau) = \max_{h_M} \{(1 - \tau)wh_M + F(h - h_M)\}. \quad (1)$$

This program defines the optimal market labor supply as a function of the tax rate, τ . In particular, we denote its solution as

$$h_M = h_M(\tau), \quad h'_M(\cdot) \leq 0. \quad (2)$$

Consider the preferences of the young agent in dynasty i , who was born in period t . These are represented by the following utility function;

$$U_Y^{i,t} = \log \left(c_{Y,t}^{i,t} \right) + \theta \log (g_t) + \beta \left(\log \left(c_{O,t+1}^{i,t} \right) + \theta \log (g_{t+1}) + \lambda U_Y^{i,t+1} \right), \quad (3)$$

where the subscript Y and O stand for "young" and "old", respectively. Time (t) superscripts denote the age at which an agent is born, while time (t) subscripts denote the time at which consumption takes place. Thus, $c_{Y,t}^{i,t}$ and $c_{O,t+1}^{i,t}$ represent, respectively, the private consumption at t and t+1 of the cohort born at t belonging to dynasty i . β is the discount rate, θ is a parameter describing the intensity of preferences for public good consumption, and λ is the altruistic weight on the utility of the agent's child (denoted by $U_Y^{i,t+1}$). In the rest of the paper, we omit time superscripts and subscripts when there is no source of confusion.

We maintain throughout that agents do not leave any monetary bequests to their children. Namely, although agents' preferences may exhibit some intra-family altruism, this is insufficient to induce a bequest motive. This implies that the Ricardian equivalence does not hold, and that there exists an inter-generational conflict about the timing of taxation and public debt policy. Note, though, that the extent of altruism affects agents' political choices. Absent bequests, agents choose labor supply, h_M , according to (2), and the consumption sequence so as to maximize utility (3) subject to their lifetime budget constraint given by

$$c_Y^i + c_O^i / R = A(\tau), \quad (4)$$

where R is the gross interest rate, and we recall that τ is the tax rate prevailing in the first period of the agent's life (i.e., when he works). The solution yields

$$c_Y^i = c_Y = \frac{A(\tau)}{1 + \beta}, \quad c_O^i = c_O = \frac{\beta R A(\tau)}{1 + \beta}. \quad (5)$$

The fiscal policy is determined period-by-period through repeated elections. We model electoral competition as a two-candidate political model of probabilistic voting à la Lindbeck and Weibull (1987),[?] which is extensively discussed in Persson and Tabellini (2000)[?]. In this model, agents cast their votes on one of two office-seeking candidates. Voters have different preferences not only over fiscal policy, but also over some policy dimension that is orthogonal to fiscal policy and about which the candidates cannot make binding commitments. In a probabilistic voting equilibrium, both candidates propose the same fiscal policy, which turns out to maximize a weighted sum of individual utilities, where the weights are the same for all agents of a given age but may differ between young and old agents. Thus, the equilibrium

policy maximizes a “political objective function” which is a weighted average utility of all voters.

The elected government chooses the tax rate ($\tau \in [0, 1]$), the public good provision ($g \geq 0$) and the debt (b') to be passed through to the following generation, subject to the following dynamic budget constraint⁸

$$b' = g + Rb - \tau w h_M(\tau). \quad (6)$$

Both private agents and governments have access to an international capital market providing borrowing and lending at the gross rate $R > 1$. The government is committed not to repudiate the debt. This implies that debt cannot exceed the present discounted value of the maximum tax revenue that can be collected;

$$b \leq \frac{\max_{\tau} \{\tau w h_M(\tau)\}}{R - 1} \equiv \bar{b}, \quad (7)$$

where $\bar{b}$ denotes the endogenous debt ceiling. This constraint rules out government Ponzi schemes.

Since agents vote twice in their life, first when they are young, and then when they are old, the first step to characterize the political equilibrium is to write the indirect utility of young and old agents. In the case of the young, substituting (1) and (5) into (8) yields:

$$U_Y(\mathbf{b}, \tau, \mathbf{g}) = (1 + \beta) \log \left(\frac{(1 + \beta R) A(\tau)}{1 + \beta} \right) + \theta \log(g) + \beta (\theta \log(g') + \lambda U_Y(\mathbf{b}', \tau', \mathbf{g}')), \quad (8)$$

where the primes denote next period's variables and boldface variables are vectors, defined as follows:

$$\mathbf{x} = \begin{bmatrix} x \\ x' \\ x'' \\ \dots \end{bmatrix} = \begin{bmatrix} x \\ \mathbf{x}' \end{bmatrix}$$

Similarly, the indirect utility of old voters is given by⁹

$$U_O(\mathbf{b}, \tau, \mathbf{g}) = \log \left(\frac{(1 + \beta R) A(1 - \tau_{-1})}{1 + \beta} \right) + \theta \log(g) + \lambda U_Y(\mathbf{b}, \tau, \mathbf{g}), \quad (9)$$

where τ_{-1} denotes the tax rate in the period when the current old were young. Note that the old care about their children who are alive contemporarily with them. Thus, the children's utility, U_Y , is not discounted.

The equilibrium of a probabilistic voting model can then be represented as the choice over time of τ, g and b' maximizing a weighted average indirect utility of young and households,

⁸Hereafter, we switch to a recursive notation with primes denoting next-period variables.

⁹With some abuse of notation, we write $U_O(\mathbf{b}, \tau, \mathbf{g})$ instead of $U_O(\mathbf{b}, \tau_{-1}, \tau, \mathbf{g})$ since τ_{-1} is not relevant for the political choice, due to the focus on Markov equilibrium and because preferences are separable.

given b . We denote the weights of the old and young as, respectively, ω and $1 - \omega$. Then, the “political objective function” which is maximized by both political candidates is

$$U(\mathbf{b}, \boldsymbol{\tau}, g) = (1 - \omega) U_Y(\mathbf{b}, \boldsymbol{\tau}, g) + \omega U_O(\mathbf{b}, \boldsymbol{\tau}, g), \quad (10)$$

subject to (6) and (7).

2.1 The commitment solution

A key feature of our model is that fiscal policy is not time consistent. The source of time inconsistency is different from those identified by other papers, and stems from the fact that agents vote repeatedly over their lifetime, and can condition the fiscal policy choice at different stages of their life.¹⁰ To establish a benchmark, it is useful to characterize the fiscal policy sequence that would be chosen by the first generation of voters if they could commit the entire future path of fiscal policy.

Consider, first, a special case in which there is no time inconsistency. Namely, suppose that the first generation of old agents can dictate its preferred policy ($\omega = 1$). In this case $U(\mathbf{b}, \boldsymbol{\tau}, g) = U_O(\mathbf{b}, \boldsymbol{\tau}, g)$. From (8) and (9), it follows immediately that the problem admits the following recursive formulation;

$$V_O^{comm}(b) = \max_{\{\tau, g, b'\}} v(\tau, g) + \beta \lambda V_O(b') \quad (11)$$

subject to (6) and (7), where

$$v(\tau, g) \equiv (1 + \lambda) \theta \log g + (1 + \beta) \lambda \log A(\tau) \quad (12)$$

is the flow utility accruing to the initially old agents from the current public and private consumption, either directly or through their altruism for their children.

Note that (11) is a standard recursive problem. The solution is time consistent, and is the same irrespective of whether the entire sequence is dictated by the first generation of old voters or chosen period-by-period by subsequent generations of old voters.

To solve the program, note that the intra-temporal first-order condition linking g and τ in

¹⁰For instance... (DISCUSS LUCAS-STOKEY AND THE LITERATURE ON CAPITAL TAXATION).

problem (12) is given by¹¹

$$\frac{1 + \beta}{\left(1 + \frac{1}{\lambda}\right) \theta} g = A(\tau) (1 - e(\tau)), \quad (13)$$

where $e(\tau) \equiv -(\partial h_M(\tau) / d\tau) (\tau / h_M(\tau))$ is the elasticity of labor supply. The intertemporal first order condition leads then to the standard Euler equation for public consumption:

$$\frac{g'}{g} = \beta \lambda R. \quad (14)$$

If $\beta = R^{-1}$ and $\lambda = 1$ (perfect altruism), the solution features a stationary policy, whereby debt, taxes, and consumption are kept constant at their initial levels. An unexpected once-and-for all fiscal shock (e.g., a war) should be financed by a permanent increase in the debt level, to be financed through a time-invariant higher tax level in future (see Barro, 1979).¹²

Next, we move to the general case in which the young affect the fiscal outcome ($\omega < 1$). In this case, a standard recursive formulation of the problem does not exist. However, the program admits a “two-stage-recursive” formulation. This is formalized in the following lemma;

Lemma 1 *The “commitment” problem admits a “two-stage recursive” formulation where;*

(i) *In the initial period, policies are set through*

$$\{\tau_0, g_0, b_1\} = \arg \max_{\{\tau_0, g_0, b_1\}} v(\tau, g) - (1 - \psi \lambda) \theta \log g + \beta \lambda V_O^{comm}(b_1),$$

subject to (6) and (7), where the function $V_O(\cdot)$ is given by (11), and the constant ψ is

$$\psi \equiv \frac{\omega}{1 - \omega(1 - \lambda)} \in \left(0, \frac{1}{\lambda}\right).$$

(ii) *After the first period, the problem is equivalent to (11).*

Proof in Appendix.

Lemma 1 implies that a set of rules applies to the first period, and another set of rules applies recursively to all future periods.¹³ Namely, when the young have some political influence ($\omega < 1$), the solution is time inconsistent; the fiscal policy sequence chosen under commitment

¹¹Two first order conditions with respect to τ and g are

$$\begin{aligned} \frac{(1 + \beta) \lambda}{A(\tau)} \frac{\frac{\partial A(\tau)}{\partial \tau}}{\left(wh_M(\tau) + \tau w \frac{\partial h_M(\tau)}{\partial \tau}\right)} &= -\beta \lambda \hat{V}'_O(b'), \\ -\frac{(1 + \lambda) \theta}{g} &= -\beta \lambda \hat{V}'_O(b'), \end{aligned}$$

The two FOCs, together with the fact that $A'(\tau) = -wh(\tau)$, lead to (13).

¹²We will return to the analysis of fiscal shocks in Section 5.

¹³Note that when $\omega = 1$, $\psi \lambda = 1$, and there is no difference between the first-period problem and the continuation.

differs from the one resulting from repeated decisions. This is an important point to which we return after characterizing the political equilibrium without commitment.

In spite of the differences in the first period, the long-run properties of this model are observationally equivalent to Barro's solution. In particular, equation (14) governs the dynamics of public debt after the first period, and whether debt grows, fall or remain constant over time only depends on the term $\beta\lambda R$. The following Proposition follows immediately from Lemma 1:

Proposition 1 *The “commitment” solution is such that (i) if $\beta\lambda R < 1$, then $\lim_{t \rightarrow \infty} b_t = \bar{b}$, (ii) If $\beta\lambda R > 1$, then $\lim_{t \rightarrow \infty} b_t = -\infty$, (iii) if $\beta\lambda R = 1$, $b_{t+1} = b_t$ for $t \geq 1$.*

2.2 The political equilibrium

We now move to the main contribution of the paper, that is the characterization of the political equilibrium when fiscal policy is set through repeated elections whereby voters cannot commit future policies. In general, a dynamic game between the current and future voters arises, and the set of equilibria is potentially large. We restrict attention to Markov perfect equilibria where agents condition their choice only on pay-off relevant state variables. Subsequent periods are in principle linked by two state variables: the government debt, b , and the private wealth of the old. However, since preferences are separable between private consumption and public goods, the wealth of the old does not affect the preference of the old for public goods. Therefore, b is the only pay-off relevant state variable. Our Markov equilibria thus feature policy rules as functions of b only.

Definition 2 *A (Markov perfect) political equilibrium is defined as a 3-tuple of functions $\langle B, G, T \rangle$, where $B : (-\infty, \bar{b}] \rightarrow [\underline{b}, \bar{b}]$ is a debt rule, $b' = B(b)$, $G : (-\infty, \bar{b}] \rightarrow R^+$ is a government expenditure rule, $g = G(b)$ and $T : (-\infty, \bar{b}] \rightarrow [0, 1]$ is a tax rule, such that the following functional equations hold:*

1. $\langle B(b), G(b), T(b) \rangle = \arg \max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0, 1]\}} U(\mathbf{b}, \boldsymbol{\tau}, \mathbf{g})$, subject to (6) and (7), where

$$\boldsymbol{\tau} = \begin{bmatrix} \tau \\ T(b') \\ T(B(b')) \\ T(B(B(b')))) \\ \dots \end{bmatrix}, \mathbf{g} = \begin{bmatrix} g \\ G(b') \\ G(B(b')) \\ G(B(B(b')))) \\ \dots \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} b \\ b' \\ B(b') \\ B(B(b')) \\ \dots \end{bmatrix}$$

and $U(\mathbf{b}, \boldsymbol{\tau}, \mathbf{g})$ is defined as in (10).

2. $B(b) = G(b) + Rb - T(b) h_M(T(b))$.

In words, the government chooses the current fiscal policy (taxation, expenditure and debt level left to the next generation) subject to the budget constraint, and under the expectation that future fiscal policies will be conducted according to the equilibrium policies rules, $\langle B(b), G(b), T(b) \rangle$. Furthermore, the vector of policy functions must be a fixed point of the system of functional equations in part 1 and 2 of the definition (where part 2 requires that the equilibrium policy functions are consistent over time with the resource constraint).

The following Lemma (proof in the appendix) is a useful step to characterizing the Markov equilibrium.

Lemma 2 *The first functional equation in Definition 2 admits the following two-stage recursive formulation:*

$$\langle B(b), G(b), T(b) \rangle = \arg \max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\}} \{v(\tau, g) - (1 - \psi\lambda)\theta \log g + \beta\lambda V_O(b')\}, \quad (15)$$

where $v(\cdot)$ is defined as in 12, subject to (6) and (7), and where V_O satisfies the following functional equation

$$V_O(b') = v(T(b'), G(b')) + \beta\lambda V_O(B(b')). \quad (16)$$

The key difference between the commitment solution and the political equilibrium can be seen by comparing the expressions of V_O^{comm} in (11) and that of V_O in (16). In the political equilibrium, the first generation of voters cannot choose the whole sequence of future policies, but must take the mapping from the state variable into the (future) policy choices as given. For this reason, there is no max operator in the definition of V_O . However, the two programs are identical when $\omega = 1$ (only the old vote), and in this case fiscal policy is time consistent.

What is the source of time inconsistency? When $\omega < 1$, the young, who care directly (i.e., not only through their altruism) about next-period public expenditure, want more public savings than the old. Hence, the young are fiscally more disciplined than their parents. In the commitment solution, the effect of the conflict between “rotten parents” and “disciplined children” is limited to the first-period fiscal policy (as from the second period onwards, their preferences are perfectly aligned). In contrast, such effect is persistent in the political equilibrium since subsequent generations of young voters enter the stage in each new election. The result is less debt accumulation.

We characterize the political equilibrium as follows. First, similar to (13) in the commitment solution, the intra-temporal first-order condition linking g and τ in problem (15) is given by

$$\frac{1 + \beta}{(1 + \psi)\theta} g = A(\tau)(1 - e(\tau)). \quad (17)$$

The only difference between (13) and (17) is in the denominator of the left hand-side term, where λ^{-1} is replaced by ψ .

Next, applying standard recursive methods to the First Order Conditions of (15)-(16), together with (17), leads to the following generalized Euler equation (GEE) describing the equilibrium dynamics of public good provision:¹⁴

$$\frac{G(B(b))}{G(b)} = \beta\lambda R - \underbrace{\beta\lambda G'(B(b)) \left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1 \right)}_{\text{the disciplining effect}}, \quad (18)$$

which is a key equation to characterize the political equilibrium. Note, first, that when the political power lies entirely in the old's hands ($\omega = 1$), then $\psi = \lambda^{-1}$, and the “disciplining effect” of the young disappears. In this case, the GEE coincides with the commitment solution in which $g'/g = \beta\lambda R$.

Suppose that a steady-state debt level exists, and denote such level by b^* . Since, in steady state, $G(B(b)) = G(b) = G(b^*)$, then (18) implies that

$$G'(b^*) = -\frac{(1 + \psi)(1 - \beta\lambda R)}{\beta(1 - \lambda\psi)} \equiv \zeta < 0, \quad (19)$$

which is constant and independent of the value of b^* . Thus, $G'(\cdot)$ is negative in the neighborhood of any steady state; higher debt is associated, as one might expect, with lower public spending. Plugging-in $G'(b^*)$ into (18) shows immediately that – in the neighborhood of a steady state – the growth rate of public spending without commitment is higher than with commitment, with the difference being proportional to ζ .

The disciplining effect introduces a discrepancy between the commitment solution and the political equilibrium that can lead to qualitatively different dynamics. Note that it continues to be possible that the GEE admits a linear equilibrium solution, i.e., one of the type $G(b) = \alpha_{0G} + \alpha_{1G}b$ and $B(b) = \alpha_{0B} + \alpha_{1B}b$ (in this case, $G'(\cdot)$ is a constant). Indeed, we will see a special case featuring a linear equilibrium. However, the presence of $G'(\cdot)$ on the right hand-side opens up the possibility that the equilibrium dynamics of public expenditure and debt be non-linear, and possibly feature multiple steady states.

Another interesting observation is that if an “interior” steady-state, $b^* < \bar{b}$ exists, and b converges monotonically to b^* in a neighborhood of b^* , then $G(b)$ must be concave around b^* .¹⁵

¹⁴The notion of GEE was first introduced in the literature by XXX (add some discussion)

¹⁵Consider a small perturbation of debt from the steady state; $\tilde{b} = b^* + \varepsilon$, $\varepsilon > 0$. The monotone convergence implies that $B(\tilde{b}) \in (b^*, \tilde{b})$. Due to the negative slope of $G(b)$ around b^* , $G(B(\tilde{b})) > G(\tilde{b})$, which implies that $G'(B(\tilde{b})) < \zeta$ according to (18). Since $B(\tilde{b}) > b^*$, this establishes that $G'(b) < \zeta$ for $b > b^*$. A similar argument establishes that $G'(b) > \zeta$ for $b < b^*$, by letting $\varepsilon < 0$. So, $G(b)$ must be concave around b^* .

Intuitively, when debt is above the steady state, there must exist a stronger disciplining effect to tighten the public consumption, for debt to fall and go back to the steady state. Conversely, when debt is lower than the steady state, there must exist a weaker disciplining effect leading to an increasing debt towards the steady state.

3 Two Analytical Examples

In the rest of the paper we parameterize the household production technology as follows:

$$F(h - h_M) = X(h - h_M)^\xi,$$

where h is the total individual time endowment, $h - h_M \geq 0$ denotes household activity, X is a parameter and $\xi \in ([0, 1])$. To rule out trivial solutions where $h_M = 0$, we assume that $X < w$. An analytical solution of the political equilibrium cannot be obtained in general. However, the model can be solved analytically in some special cases.

In the first case, we set $\xi = 0$, implying that agents cannot substitute market hours with household activity. Due to the log-utility function, labor taxation does not distort labor supply. We will see that in this case, a linear equilibrium exists, and the dynamics of debt resemble qualitatively the commitment solution.

In the second case, we set $\xi = 1$. This implies that market hours are supplied inelastically as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$. However, if taxation exceeds $\bar{\tau}$ market labor supply and tax revenues fall to zero. In this case, the equilibrium expenditure function, $G(\cdot)$, is concave, and a stable interior steady state with positive public good provision can be attained.

3.1 Example I: $\xi = 0$

As in this example the market labor supply is constant at h , we have that $A(\tau) = (1 - \tau)wh$ and $e(\tau) = 0$. Furthermore, as the top of the Laffer Curve is attained at $\tau = 1$, the maximum debt is $\bar{b} = wh/(R - 1)$. The FOC, (17), can be written as

$$1 - \tau = \frac{1 + \beta}{(1 + \psi)\theta wh} g. \quad (20)$$

Plugging-in this solution into the government budget constraint yields

$$b' = \left(1 + \frac{1 + \beta}{\theta(1 + \psi)}\right) g + Rb - wh. \quad (21)$$

Next, we guess G to be linear, $G(b) = \gamma(\bar{b} - b)$. Then, the GEE, (18), yields:

$$\frac{\gamma(\bar{b} - B(b))}{\gamma(\bar{b} - b)} = \beta\lambda R - \beta\lambda\gamma \left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1 \right). \quad (22)$$

Then, using (22), the budget constraint, (21), the equilibrium condition that $b' = B(b)$, and the expression of $\bar{b}$ given above leads to the following solution for γ ;

$$\gamma = \frac{(1 - \beta\lambda)\theta(1 + \psi)R}{(1 + \theta)(1 + \beta) + (1 - \beta\lambda)\theta\psi}.$$

Finally, substituting g by its equilibrium expression, $g = \gamma(\bar{b} - b)$, into (20) and (21), leads to a complete analytical characterization. This is summarized in the following Proposition (proof in the text).¹⁶

Proposition 3 *Assume that $\xi = 0$. Then, the time-consistent equilibrium is given by the following policy functions*

$$\tau = T(b) = 1 - \frac{1}{wh} \frac{(1 - \beta\lambda)(1 + \beta)R}{(1 + \theta)(1 + \beta) + (1 - \beta\lambda)\theta\psi} (\bar{b} - b), \quad (23)$$

$$g = G(b) = \frac{(1 - \beta\lambda)\theta(1 + \psi)R}{(1 + \theta)(1 + \beta) + (1 - \beta\lambda)\theta\psi} (\bar{b} - b), \quad (24)$$

$$b' = B(b) = \bar{b} - \frac{\theta + \lambda(1 + \beta + \theta)}{(1 + \theta)(1 + \beta) + (1 - \beta\lambda)\theta\psi} \beta R (\bar{b} - b), \quad (25)$$

where $\bar{b} \equiv wh / (R - 1)$.

It is interesting to note that $G'(\cdot) = -\gamma < 0$, implying that the disciplining effect in (18) increases the growth rate of public spending, as in the general discussion above. Due to the linearity of $G(\cdot)$, however, the disciplining effect is a constant for any debt level. For this reason, the dynamics do not lead to any stable interior steady state. If $(\theta + \lambda(1 + \beta + \theta)) \cdot ((1 + \theta)(1 + \beta) + (1 - \beta\lambda)\theta\psi)^{-1} \cdot \beta R < 1$, the economy converges asymptotically to the maximum debt level $\bar{b}$, while if this inequality is reverted the debt approaches minus infinity in the long-run. Although the long-run properties of the debt dynamics are qualitatively identical to those of the commitment solution, two differences should be noted. First, there exists a range of parameters such that, under commitments, the economy would accumulate debt till the maximum level ($b \rightarrow \bar{b}$), while the political equilibrium leads to an ever growing surplus ($b \rightarrow -\infty$). Namely, the political empowerment of future generations is beneficial to them. Second, if we take an economy converging to $\bar{b}$ under both regimes, the slope of the debt function, $B(b)$, is always steeper in the political equilibrium. In other words, public debt grows more slowly in the political equilibrium than under commitment.

¹⁶The results of Proposition 3 extend to economies with population growth and technical change. The analysis of this extension is presented in appendix ??.

Figure 1 provides a geometric representation of an equilibrium converging to $\bar{b}$.¹⁷ Panel *a* shows the equilibrium tax policy: the tax rates increases linearly with the debt level. Panel *b* shows the equilibrium expenditure: public good provision declines linearly with the debt level. Panel *c*, finally, shows the upward sloping equilibrium debt dynamics.

FIGURE 1 (THREE PANELS) HERE

In this example, the economy depletes its resources over time: generation after generation, agents find their private and public consumption progressively crowded out by debt repayment to foreign lenders. This occurs gradually, even in a model without any altruism (i.e., if we set $\lambda = 0$, which would give a standard OLG model). In this case, in the commitment solution the debt converges to $\bar{b}$ in only two periods. In contrast, the political equilibrium features

$$\bar{b} - b' = \bar{b} - B(b) = \frac{\theta}{(1 + \theta)(1 + \beta) + \theta\psi} \beta R(\bar{b} - b),$$

where $\psi = \omega / (1 - \omega)$. As the expression above shows, in spite of the lack of concern for future generations, voters do not support a “big party” which would consume the present value of the entire future income stream. In fact, the old would always support a big party, but young voters disagree because they care about what public expenditure will be when they become old. If debt were set to its maximum level right away, the young would suffer from their public consumption falling to zero in their old age. To see how crucial the concern for *public* consumption is, observe that, if $\theta = 0$, then the initial young and old voters would agree to set $b = \bar{b}$, and the young would secure private consumption in old age through savings. Thus, the key assumption is that private savings cannot buy public goods. The concern for public consumption in old age becomes then a partial substitute for lack of altruism towards future generations.

As it is young voters who discipline fiscal policy, increasing the political influence of the old (i.e., increasing ω) leads to higher debt, higher taxes, and an increase in current public good provision. Changing ω does not affect the steady state, but a larger ω implies a faster depletion of both private and public consumption. If the young have no influence on the political process ($\omega = 1$), the maximum debt is attained in the first period. Conversely, if the old have no political representation ($\omega = 0$) the debt dynamics converge to $\bar{b}$ at the slowest rate.

¹⁷Parameter values are given in Table X-1 below except for wh , which is adjusted to make $\bar{b}$ equal to its counterparts in the following cases.

Finally, it is worth noting that the political equilibrium and the commitment solution are identical in the first period (proof available upon request). This equivalence implies that the disciplining effect in the political solution is of the same size as in the first period of the commitment solution. This is due to the log-preference assumption over public goods, and that future public goods are linear in $(\bar{b} - b)$. These two features imply the cancellation of two opposing effects; if public funds were to be spent more lavishly in the future, then current decision makers might be expected to leave less for the future. On the other hand, if future governments were to spend more lavishly, they would be driven into public poverty earlier, which might be expected to induce the current policy decision makers to increase public savings.¹⁸

3.2 Example II: $\xi = 1$

Next, we turn to the second tractable case, where we assume constant returns to labor in the household production technology, i.e., $\xi = 1$. In this case, taxation does not distort labor supply as long as $\tau \leq \bar{\tau} \equiv 1 - X/w$, namely, agents only work in the market. If $\tau > \bar{\tau}$, however, agents switch all their time endowment into household production, and the tax revenue falls to zero. Thus, $\bar{\tau}$ is the top of the Laffer curve. Since rational voters would never choose a tax rate inducing no public good provision, the political equilibrium necessarily features $\tau \leq \bar{\tau}$. Thus, the model is observationally equivalent to one in which the government is committed not to tax income over the upper bound rate $\bar{\tau}$.

Three sub-cases can be distinguished. First, when the interest rate is sufficiently low, the economy behaves similarly to the linear equilibrium case: debt converges asymptotically to its maximum level, which is now $\bar{b} = \bar{\tau}wh/(R - 1)$, and the economy features public poverty in the long run, i.e. $\lim_{t \rightarrow \infty} g_t = 0$. However, since taxes are bounded from above by $\bar{\tau}$, private consumption does not fall to zero, but converges to $(1 - \bar{\tau})wh > 0$. Since the equilibrium dynamics resembles those of the benchmark model, we omit the analysis of this case (details available upon request). Second, when the interest rate is sufficiently high, the economy accumulate a perpetual surplus, and again there are no novel aspects.

The third case, which corresponds to an intermediate range of R , is more interesting. Here, the equilibrium is qualitatively different; an economy starting from low initial debt converges in finite time to a steady-state equilibrium such that $\tau = \bar{\tau}$, but debt is strictly lower than $\bar{b}$. In a neighborhood of the steady state, the equilibrium dynamics of the fiscal variables are

¹⁸To see this result technically, note that whenever the policy rule is on the following form $G(b) = \gamma(\bar{b} - b)$ for some γ , the cross derivative $\frac{\partial^2 V_Y(b)}{\partial b \partial \gamma}$ is always equal to zero. This means that the future lavishness, i.e. γ , will not impact on current political decisions.

given by steady-state debt level is given by¹⁹

$$b' = B(b) = b_0^* \equiv \bar{b} \left(1 - \frac{\theta(1+\psi)(1-\bar{\tau})}{\bar{\tau}(1+\beta)} \right) \quad (26)$$

$$\tau = T(b) = \bar{\tau} - \frac{R(1+\beta)}{wh(1+\beta+\theta(1+\psi))} (b_0^* - b) \quad (27)$$

$$g = G(b) = \frac{wh\theta(1+\psi)(1-\bar{\tau})}{1+\beta} + \frac{\theta(1+\psi)R}{1+\beta+\theta(1+\psi)} (b_0^* - b) \quad (28)$$

Figure 2 provides a geometric representation of the equilibrium. Panel *a* shows the equilibrium tax policy: taxes increase linearly with the debt level as long as $b \leq b_0^*$. Thereafter, T is flat at $\tau = \bar{\tau}$. Panel *b* shows the equilibrium expenditure: public good provision declines linearly with the debt level as $b \leq b_0^*$. To the right of b_0^* , the government loses the ability to adjust taxes, and thus the government expenditure function becomes steeper. Panel *c*, finally, shows that the policy is flat around b_0^* . Therefore, if the debt level starts sufficiently close to b_0^* , it converges to b_0^* in one period and remains at b_0^* thereafter. In other words, debt is strongly mean-reverting after a shock. The figure also shows that the debt and expenditure policy function feature discontinuous dynamics for high initial debt levels. Moreover, there are multiple steady states. However, these are fragile features of this particular example which disappear when one consider smooth labor supply distortion. Instead, as the next section will show, the existence of a locally stable steady-state debt level lower than $\bar{b}$ with an associated tax level lower than one and positive public good provision are robust features that carry on to the more general case.

What is the intuition for the dynamics around b_0^* ? Imagine, to make the case sharper, that voters are not altruistic ($\lambda = 0$). Yet, young voters care about public good provision one-period ahead. In the linear equilibrium of example I, this concern for the near future did not prevent the debt from increasing in every period, progressively impoverishing the future generations. Why? Because future generations could not threaten credibly current voters to cut public good provision drastically should they inherit a large debt. Voters would anticipate that the next generation would make part of the adjustment to a larger debt in the form of higher taxes and debt. Although government expenditure would also fall, these adjustments mitigate the expenditure-cutting effect. As a result, each generation of voters “passes the bill” to the next and only suffers a partial sacrifice of public consumption.

Passing the bill to future generations becomes harder, however, when taxation is increasingly distortionary. In example II, this is particularly stark; the tax rate cannot exceed $\bar{\tau}$.

¹⁹ A formal Proposition with a complete characterization of the equilibrium and its proof are provided in the appendix.

As the debt approaches b_0^* (and taxes approach $\bar{\tau}$), Voters anticipate that future generations will not be able to contain the reduction of public expenditure by increasing taxes over $\bar{\tau}$. Hence, the disciplining effect becomes very strong. Note that $G(\cdot)$ is (piece-wise-linear-) concave around the steady state b_0^* . To the right of b_0^* , the disciplining effect is so strong that debt falls and reverts to b_0^* in just one period. In contrast, to the left of b_0^* , $G(b)$ is less steep, implying a smaller disciplining effect. In fact, voters support an increasing debt, and b_0^* is a steady state.²⁰

FIGURE 2 (THREE PANELS) HERE

4 The General Case: $\xi \in (0, 1)$

The intuition behind the result of example II carries over to the general case with $\xi \in (0, 1)$, with smooth labor supply distortions. In this case, however, the equilibrium policy function are non-linear (nor piecewise linear), and the model does not admit an analytical solution. We must therefore resort to numerical analysis.²¹

We calibrate parameters as follows. First we think of one period as corresponding to thirty years. Thus, we set $\beta = 0.98^{30}$ and $R = 1.025^{30}$, implying a 2% annual discount rate and a 2.5% annual interest rate. This value of β is standard in the macroeconomics literature, and the value of R is consistent with the average real long-term U.S. government bond yields (2.5%) between 1960 and 1990. There are few quantitative clues for ω and λ . So we simply set $\omega = 0.5$ (equal political weights on the young and old) and $\lambda = 0.75$.²² We use the results from our example II to calibrate the two parameters, θ and $\bar{\tau}$. In particular, we choose parameters so as to match, in the example, the average debt-GDP ratio (0.30) and the government expenditure-GDP ratio (0.18) in the U.S. from 1960 to 1990. The calibration yields $\bar{\tau} = 0.51$ and $\theta = 0.37$.²³ Finally, we normalize wh to unity in this tractable case. Table X-1 summarizes the parameters.

²⁰ A related intuition explains why there is no internal steady state when the interest rate is low? The reason is that G' is bounded from below by ζ . Since the function G is continuous, the GEE (18) therefore implies an ever-decreasing sequence of public goods. Hence, with a low interest rate, the disciplining effect is not strong enough to generate falling debt for any $b \leq \bar{b}$, so $b \rightarrow \bar{b}$, irrespectively of the initial b .

²¹ We adopt a standard projection method with Chebyshev collocation (Judd, 1992) to approximate T and G , according to the First Order Conditions (17) and (18). The basic idea of the projection method is to approximate some unknown functions on a basis of functional space.

²² We must also assume $\lambda \in (\lambda_{\min}, \lambda_{\max})$, where $\lambda_{\min}$ and $\lambda_{\max}$ are implied by the conditions $R > 1 + (1 + \psi) / \zeta$ and $\beta \lambda R < 1$. Given the parameter values of β , R and ω , $\lambda_{\min}$ and $\lambda_{\max}$ are equal to 0.68 and 0.87, respectively.

²³ More precisely, we use the steady-state expressions of g and b in (28)-(26), each divided by wh , as proxies

Table X-1

$\beta = 0.98^{30}$	$R = 1.025^{30}$	$\omega = 0.50$	$\lambda = 0.75$	$\theta = 0.37$	$\bar{\tau} = 0.51$	$wh = 1$
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In addition, we must assign values to w , X and ξ . To this aim, we normalize $w = 1$ in the tractable case with $\xi = 1$ and we let $h = 1$ in all cases. Then, to make it easier to compare the simulated economy with the tractable case in which $\xi = 1$, we set w and X in a sequence of economies with different ξ according to the following two conditions. First, the top of the Laffer curve is constant across experiments at $\tau = \bar{\tau}$, and second, the tax revenue at the top of the Laffer curve is also constant and equal to the one in the tractable case with $\xi = 1$. The details are given in the appendix.

Figure 3 describes the equilibrium dynamics of two simulated economies, with respectively $\xi = 0.90$ and $\xi = 0.50$.²⁴ In both cases, the tax policy function is increasing in b (panel a) while the public expenditure function is a decreasing in b (panel b). The debt policy is an increasing convex function of b which crosses the 45-degree twice: first at an interior steady-state level, and then at the maximum debt. Interestingly, only the interior steady-state is stable. Namely, as long as the economy starts at $b < \bar{b}$, it converges to the internal steady state with no public poverty.²⁵

FIGURE 3 (THREE PANELS) HERE

To earn intuition, it is useful to go back to the analytical examples. In all cases, the tax function is non-decreasing and concave (strictly concave if $\xi > 0$), while the expenditure function is decreasing and concave (strictly concave if $\xi > 0$). In example II, the policy functions are piece-wise linear with a kink at the steady state. This is because taxation is

for the debt-GDP ratio. Thus, we set

$$\begin{aligned} \frac{\theta(1+\psi)(1-\bar{\tau})}{(1+\beta)} &= 0.30, \\ \frac{1}{R-1} \left(\bar{\tau} - \frac{\theta(1+\psi)(1-\bar{\tau})}{1+\beta} \right) &= 0.18. \end{aligned}$$

Given the other parameters, these two equations identify $\bar{\tau}$ and θ .

²⁴In the internal steady state of the two simulated economies with $\xi = 0.90$ and $\xi = 0.50$, the elasticities of market labor supply with respect to w , denoted by $\chi(h_M^*) = \frac{\partial h_M^*}{\partial w/w}$, are equal to 0.2309 and 0.4878, respectively. (ADD SOME DISCUSSION ON THE ESTIMATED VALUE OF THE ELASTICITY).

²⁵Clearly, simulations do not establish that these equilibria are unique. However, we run many simulations and never found any qualitatively different equilibrium from those display in the figure.

non-distortionary to the left of $\bar{\tau}$ and infinitely distortionary to the right of it. In the general case of $\xi \in (0, 1)$, the tax function flattens as b increases, since larger b implies requires higher taxes to be financed, and tax-collection becomes increasingly ineffective. At high debt levels, governments tend to react to further debt increases by cutting expenditure more than by increasing taxes. This shows up in the concave shape of the χ and T functions. In example II, the slope of the G function changes discontinuously, whereas in the numerical examples the derivative of G falls smoothly. In example I ($\xi = 0$), taxation is not distortionary. Thus, a larger debt is matched by a proportional increase in taxation and cut in expenditure.

Table X-2 reports steady state values of variables of interests under different values of ξ .

Table X-2						
	$\chi(h_M^*)$	τ^*	g^*	b^*	g^*/wh_M^*	b^*/wh_M^*
$\xi = 1.000$	0.0000	0.5100	0.1843	0.2967	0.1843	0.2967
$\xi = 0.975$	0.0488	0.4724	0.1935	0.2639	0.1892	0.2580
$\xi = 0.950$	0.1150	0.4562	0.1938	0.2566	0.1859	0.2463
$\xi = 0.900$	0.2309	0.4388	0.1901	0.2551	0.1775	0.2381
$\xi = 0.700$	0.4373	0.4032	0.2139	0.2235	0.1878	0.1963
$\xi = 0.500$	0.4878	0.3696	0.2998	0.1277	0.2519	0.1073

5 Financing a Surprise War

In this section, we introduce uncertainty and a fiscal shock. In particular, we assume that the government is forced to "fight a war", whose financing requires an exogenous spending of Z units per war period. The metaphor of wars is intended to capture more generally fiscal shocks increasing the marginal value of government spending. During the war, the government's budget constraint (6) changes to

$$b' = g + Rb - \tau wh_M(\tau) + Z, \quad (29)$$

while during peace it reverts to (6).

For simplicity, we focus on a "surprise" transitory shock that hits an economy which is in the steady state (Appendix A extends the analysis to fiscal shocks with a non-degenerate probability distribution).²⁶ More formally, the war is a zero-probability event lasting for one period only. This is identical to a temporary increase in the debt, from b to $b + Z/R$. The local dynamics around the steady state determine how the economy reacts to the shock. The main

²⁶In example I, an economy starting in the steady state would have attained the debt limit and would be unable to finance a surprise war. In order to analyze such particular case, one must either assume that the economy is not initially in the steady state (and study the effects on the transition), or consider a "poitive" fiscal shock ($Z < 0$) such as a windfall oil discovery.

result is that, contrary to Barro (1979), the government will only partially use debt to absorb the shock. Part of the cost of financing the war is financed by a short-term cut in current government and increase in taxation.

A useful illustration can be provided with the aid of the tractable example II of Section 3.2 ($\xi = 1$). Assuming the fiscal shock to be relatively small (so that the local analysis applies), it would shift the real debt from b_0^* to $b_0^* + Z/R$. Clearly, since the tax constraint ($\tau \leq \bar{\tau}$) was already binding before the war, an additional exogenous spending need will only make the constraint more binding and τ remains constant at $\bar{\tau}$. Moreover, the government sets $b' = b_0^*$. Consequently, the war is financed entirely by a reduction in spending:

$$G(b_0|z_W) = G(b_0|z_P) - X.$$

This example is extreme, insofar as the government does not engage in any (non-war) expenditure smoothing, and the debt level returns to the pre-war level in just one period after the war ends. The dotted lines of Figure 4 shows the post-war expenditure and debt dynamics of this simple case (clearly, taxes do not move as the constraint is binding).

In the general case with $\xi < 1$ (see the solid line in Figure 4), supposing that the economy was in a stable interior steady state before the war, the tax rate shoots up and public expenditure shoots down. Public debt first increases to finance the war, and then returns smoothly (as opposed to the case of $\xi = 1$, when this happened in just one period) to its steady-state level. Now, debt is used to finance the war, albeit only partially. Moreover, the impulse-response dynamics would feature a reversion to the mean of the debt level.

FIGURE 4 (THREE PANELS) HERE

Appendix A extends the analysis to the more general case in which the economy is hit by recurrent wars, and the state of the economy evolves following a first-order stationary Markov process. The results are similar to those emerging from the analysis of a surprise war. However, the anticipation of the possibility of future wars induces an additional precautionary motive for public savings in times of peace.

6 Political Shocks

In this section, we introduce time-varying preferences in the form of cohort-specific shocks affecting agents' appreciation for public-good (relative to private) consumption. For simplicity,

we assume that the realization of the shock is identical across all voters of a given age. In particular, we now let $\theta_Y \in \{\theta_r, \theta_l\}$ and $\theta_O \in \{\theta_r, \theta_l\}$ to denote the preference of the young and of the old, respectively, where $\theta_r < \theta_l$ (R stands for "right-wing" and L stands for "left-wing"). The late 1960's can be regarded as a leftist wave, where for no evident reason, agents' taste for government size increased. The neo-cons revolution of the 1980's is an example of a right-wing wave.

The realization of preference shocks is assumed to follow a first-order Markov process. More specifically, we denote by $p_{l,r}$ the probability that, conditional on the current young generation being rightist, a leftist young generation materializes in the next period. Equivalently, $p_{l,r}$ is the probability that, conditional on the current young voters being rightist, next period's voting population will consist of rightist old and leftist young agents. We define $p_{l,l}, p_{r,l}$ and $p_{r,r}$ in a similar fashion. By these definition,

$$p_{l,l} + p_{r,l} = p_{l,r} + p_{r,r} = 1.$$

We impose no restriction on the persistence of political shocks.

The equilibrium definition must be amended to allow for heterogenous preferences of young and old over public good provision (formally, we have additional state variables). Thus, the equilibrium policy functions will be denoted by $T(b|\theta_Y, \theta_O), G(b|\theta_Y, \theta_O)$ and $B(b|\theta_Y, \theta_O)$, where θ_Y, θ_O denotes the state of preferences of the current voters, young and old. We focus our main discussion on a version of the model with $\lambda = 0$, and then discuss separately the effect of altruism. Also, we start from the tractable case where $\xi = 0$ (the analogue of example I). In this case, a linear equilibrium exists, which the following Proposition characterizes (proof in appendix).

Proposition 4 *Assume that $\xi = 0$ and $\lambda = 0$. Then, the equilibrium with political uncertainty is given by the following policy functions.*

$$\begin{aligned} T(b|\theta_Y, \theta_O) &= 1 - \frac{(1-\omega)R(1+\beta)}{wh((1-\omega)(1+\theta_Y)(1+\beta) + \omega\theta_O)} (\bar{b} - b), \\ G(b|\theta_Y, \theta_O) &= \frac{((1-\omega)\theta_Y + \omega\theta_O)R}{\omega\theta_O + (1-\omega)(1+\theta_Y)(1+\beta)} (\bar{b} - b), \\ B(b|\theta_Y, \theta_O) &= \bar{b} - \frac{(1-\omega)\theta_Y\beta R}{\omega\theta_O + (1-\omega)(1+\theta_Y)(1+\beta)} (\bar{b} - b), \end{aligned}$$

where $\bar{b} \equiv wh/(R-1)$, and $\theta_Y \in \{\theta_r, \theta_l\}$ and $\theta_O \in \{\theta_r, \theta_l\}$ denotes the political preferences of the young and old voters, respectively.

A first interesting observation is that neither the variance nor the persistence of shocks have any effect on the equilibrium. A permanent change in political preferences, for instance, has the same effect as a temporary one, and more generally, the probabilities $p_{j,i}$ do not enter the equilibrium functions $T(\cdot)$, $G(\cdot)$ and $B(\cdot)$ – they only depend on the state of debt and on the current distribution of political preferences –. This surprising result depends on the cancellation of two opposite effects, an income and a substitution effect. To understand this point, suppose both the young and the old to be initially (say, at t) leftist, but they anticipate that the next generation (born at $t + 1$) will be rightist, i.e., has a low appreciation for government expenditure. Clearly, this has no influence on old voters at t . Consider the young at t . Since the next generation will spend a small share of $\bar{b} - b'$ into public good provision, a “responsible” fiscal policy has a lower return in terms of future public good consumption. The “substitution effect” calls for an increase in current debt. But the “income effect” goes in the opposite direction: precisely because the next generation will not deliver much public good, it is important that it inherits a low public debt. So, the expectation of a shift to the right strengthen (from the income effect standpoint) the fiscal policy of the leftist young. In the log-specification (and under no altruism).

This result is of independent interest. Persson and Svensson (1989)(?) argued in an influential article that strategic considerations affect the debt policy of governments with heterogeneous preferences for public consumption when there is a positive probability of non-reelection. For instance, a right-wing government with a low taste for public consumption may issue more debt when it knows that it will be replaced by a left-wing government with a stronger taste for expenditure. Their result is derived in a two-period model. Our generalization to an infinite horizon shows that in general strategic effects can go either way, due to the concomitant presence of income and substitution effects. This may explain why the empirical literature has found mixed support to this prediction [...].

Second, the policy functions in Proposition 4 provide some interesting comparative statics. First, as expected, both T and G are increasing in both θ_O and θ_Y (taxes and public good provision are increasing with the appreciation for public consumption). However, θ_O and θ_Y have opposite effects on the debt policy, B . An increase in θ_O increases B whereas an increase in θ_Y decreases B . As the old become more eager to consume public goods, they push for more debt. In contrast, more appreciation for public consumption make the young wary of debt. Since they care about next-period public consumption, the more they care for public consumption the more they are debt averse.

An interesting experiment is that of a two-period transition from a society where all agents

have a high preference for public consumption ($\theta_Y = \theta_O = \theta_L$) to one where all agents have a low preference for public consumption ($\theta_Y = \theta_O = \theta_R$). In the first period, θ_Y falls and θ_O does not change, whereas in the second period θ_O falls and θ_Y remains low. Figure 7 shows the effects of such sequence of policy shocks on the equilibrium policy function.²⁷ The tax policy, T , shifts down in the first period, and then further down. The policy function G shifts down in the first period, and then up in the second. Finally, the policy function B shifts up in the first period, and then down in the second. However, the net effects of the ideological shift on G and B are unambiguous. In particular, $G(b|\theta_r, \theta_r) < G(b|\theta_l, \theta_l)$, and $B(b|\theta_r, \theta_r) > B(b|\theta_l, \theta_l)$, namely a shift to the right leads unambiguously to more debt accumulation.

Similar results obtain in the case of an elastic labor supply. While in the linear case political shocks have only transitory effects (as in the long run the economy falls in all cases into immiserization), in the case of elastic labor supply political shocks have both short-term and long-term effects. We calibrate the model as in Table X-1, with two exceptions. First, since we have set $\lambda = 0$, we must reparameterize R that we set equal to 1.06 ³⁰ in order to have interior steady states under different political regimes. Second, we set $\omega = 0$, i.e., we assume that all political power is in the hands of the young. This simplification is introduced for purely expositional purposes, as it allows us to analyze changes in political preferences in the form of one-period shocks, since when $\omega = 0$, θ_O has no effect on the equilibrium (see the expressions in Proposition 4 for the linear case).²⁸ Finally, we assume that θ_L and θ_R are 10% above and below θ , respectively. We consider two alternative cases: in the first $p_{l,l} = p_{r,r} = 1$ (unanticipated shocks), and in the second $p_{l,l} = p_{r,r} = 0.5$ (i.i.d. shocks). The values of parameters are summarized in Table X-4.

Table X-4				
$\beta = 0.98$ ³⁰	$R = 1.06$ ³⁰	$\omega = 0$	$\lambda = 0$	$\bar{\tau} = 0.51$
$\theta_L = 0.40$	$\theta_R = 0.33$	$p_{l,l} = 0.5/1$	$p_{r,r} = 0.5/1$	$wh = 1$

Figure 7 plots the equilibrium policy rules under the two different political regimes for the case in which the political change is both permanent and totally unanticipated, $p_{l,l} = p_{r,r} = 1$. Dotted lines are for the left-wing regime ($\theta_Y = \theta_l$), while solid lines for the right-wing regime ($\theta_Y = \theta_r$). The arrows in the first panel also shows the dynamic adjustment of an economy

²⁷The parameter values are given in Table X-4. See the discussion below for the parameterization.

²⁸The reason why θ_O and θ_Y have different effects lies in the mechanics of the probabilistic voting model. Old voters only care about public good provision. Moreover, they support the maximum feasible budget deficit. Reducing θ_O is identical to reducing ω , since the model is sensitive to the intensity of voters preferences. Therefore, lowering θ_O will lead to lower taxes, lower spending and lower debt due to a loss of political influence of the old.

Details of simulations with $\omega = 0.5$ are available upon request.

starting as in the steady state of the left-wing regime and moving to the right-wing regime. As one can see, though the political regime switch leads to less public good provision and more public debt over time, the evolution of tax rates can be non-monotonic. This is because more public debt leaves heavier financial burden on the government budget, which forces the subsequent rightists to raise tax rates. Figure 8 plots the time-series dynamics of g , τ and b under the political regime shift. The solid lines show the case described in Figure 7 ($p_{l,l} = p_{r,r} = 1$). The dashed lines give the results with persistent political regimes ($p_{l,l} = p_{r,r} = 0.9$) and the dotted lines show the i.i.d. case ($p_{l,l} = p_{r,r} = 0.5$). Two remarks are in order. First, public policies in all cases feature the same dynamic features. Public spending decreases and public debt increases over time, while the tax rate goes down in the first period, and then increases for financing larger public debt. Second, there is a tendency of policy convergence between the left-wing and the right-wing as political regimes being less persistent. [...]

FIGURE 7 (Three Panels) HERE

FIGURE 8 (Three Panels each with two cases) HERE

Finally, we discuss altruism. Altruism raises the issue of whether parents value future public consumption according to their own taste (“paternalistic preferences”), or they do respect the preference of their children and grand-children. If they have paternalistic preferences, altruism makes no major difference to the analysis above. If they have paternalistic preferences, however, the persistence of preference shocks becomes important. To see why, consider a leftist parent expecting all future dynasties to be, without uncertainty, rightist. Clearly, such parent would choose a high debt since she must not be worried about the implication of her choice on the future public consumption of the members of her dynasty. More generally, there is now an intertemporal substitution effect inducing dynasties to consume more (and issue more debt) in times of strong preferences for public consumption. When political shocks are highly persistent, this effect is dominated by the mechanism discussed above. For instance, consider again the calibrated example with $\lambda = 0.75$. Suppose that the unconditional probability of θ_L and θ_R is the same, but the realization of the shocks are highly persistent, ($p_{r,r} = p_{l,l} = 0.9$). Then,

the dynamics are similar to those discussed earlier on, as shown in Figure 8.²⁹ However, in economies characterized by a combination of high (non-paternalistic) altruism, and a low persistence of preference shock the results can be reverted (i.e., leftist governments issue more debt). Leaving aside this curiosum, the prediction that left-wing governments issue less debt than right-wing governments is robust to the introduction of altruism.

FIGURE 9 (Three Panels) HERE

7 Empirical Analysis

In this section, we test the main prediction of the theory concerning the effect of political shocks on debt accumulation. A robust prediction of our theory is that right-wing governments are more prone to issue public debt than left-wing governments. Although our theory also has implications about taxation, these are less clear-cut. In particular, our theory predicts that left-wing governments tax less than right-wing governments in the long-run. However, our model ignores other rationales for taxation. For instance, countries spend a large share of their tax revenue on intragenerational transfers (unemployment benefits, welfare subsidies, sick and parental leave, etc.). Since left-wing governments have a stronger taste for transfers, the net effect on taxes may be ambiguous. Transfer motives would instead strengthen the predictions of our theory about debt policy: left-wing voters would dislike public debt insofar as it crowds out future transfers as well as public goods. Therefore, we restrict attention to debt policy.

We first consider the time-series evidence for the United States. The advantage of focusing on post-war US data is that this is a two-party presidential system where the definition of "left" and "right" is not controversial. In addition, the debt is measured consistently, and there are neither world wars nor special episodes such as the Great Depression in the sample period. The only disadvantage is the limited number of observations. We use annual data for the period 1948-2005 from the Economic Report of the President. We run the following regression

$$\Delta d_t = \alpha_0 + \alpha_1 DEM_t + \alpha_2 d_t + \alpha_3 (U_t - \bar{U}) + \varepsilon_t.$$

The dependent variable, Δd_t , is the annual change in the debt-GDP ratio. Coherently with the timing of our theory, we define $\Delta d_t \equiv D_{t+1}/Y_{t+1} - D_t/Y_t$, namely, the government in office at t sets (through its budget law) the surplus or deficit in the following year. The explanatory

²⁹ Similar results hold with this parameterization for $p_{l,l} = p_{r,r} < 0.75$.

variables include the debt-GDP ratio ($d_t = D_t/Y_t$), intended to capture the autoregressive component of debt (see Bohn, 1998); an indicator of the party affiliation of the president in office, and unemployment. The latter is intended to capture cyclical components of debt policy that are independent of politics.³⁰ We net unemployment of its sample average in order to ease the interpretation of the coefficients. The main variable of interest is DEM_t . This a dummy variable that takes on the value one when the president is a Democrat and the value zero when the president is a Republican. Our theory predicts α_1 to be negative, namely, debt growth should be lower under Democrat administrations. Note that α_0 measures the conditional mean of debt growth under Republicans, whereas $\alpha_0 + \alpha_1$ measures the conditional mean of debt growth under Democrats.

TABLE 1 HERE

Table 1 summarizes the results. The baseline regression (column 1) shows that Republican administrations, controlling for the autoregressive component only, induce an average increase in the debt-GDP ratio of 3.8 percentage points per year. Given the autocorrelation coefficient (-0.088), an infinite sequence of Republican governments would lead to a steady-state debt-GDP ratio of 42.7%. In contrast, Democrat administrations induce an average increase in the debt-GDP ratio of 1.7 percentage points, implying a steady-state debt-GDP ratio of 19.3%. The estimated difference is both large and statistically significant. The autoregressive coefficient is significant and has the expected negative sign. Controlling for unemployment (column 2) has no major effects on the results. The difference between Republicans and Democrats remains highly significant (well above 99%). The steady-state debt-GDP ratios become, respectively, 40.7% (Republicans) and 15.1% (Democrats).³¹ The autoregressive coefficient remains negative but drops to -0.067, becoming marginally insignificant. Interestingly, this estimate is very similar

³⁰One might argue that the ideology of governments may affect their response to business cycle fluctuations. However, an interaction between unemployment and the political measure has an insignificant effect in the regression.

³¹We have tested the stationarity of the unemployment series, and could reject the null hypothesis of a unit root. Adding a linear-quadratic time trend to the regression does not change the result of interest: the difference between Democrat and Republican administrations remain significant above 99%.

to that of Bohn (1998) who finds – after controlling for cyclical components in output and government expenditures – an autoregressive coefficient of -0.064 for the period 1948-95 (see Table II, p. 956). We also checked the sample stability by allowing the effect of Democrats to be different before and after 1980 (column 3). This test addresses the concern that the political effect may be driven by the policy of the Reagan and Bush administrations. We find no significant difference between the early and late part of the sample (the test that the two coefficients are identical is not rejected). In both subperiods Republicans have a higher propensity to accumulate debt.

We next extend the analysis to a sample of 21 OECD countries. A complete description of the data is provided in the appendix. Here we note that the major issue concerns measuring the political color of governments across countries and over time. Many countries have parliamentary systems and frequent coalition governments that are difficult to classify precisely. We will avoid problems of cross-country comparability between governments' political ideologies by including country-specific fixed effects in the regressions. In addition, we will filter out shocks common to all countries by including time effects. Nevertheless, defining the character of changes in government over time remains difficult. To address this problem, we use three different (though not independent) measures of the color of governments. Two measures are taken "off-the-shelf" from other studies. The first measure (POL_{FR}) was constructed by Franzese (2003), henceforth FR. It codes all parties in government from 1948 to 1997 from far left (value 0) to far right (value 10). The second measure (POL_{WKB}) is constructed by Woldendorp, Keman and Budge (1993, 1998), henceforth WKB. They assign scores for government and parliament from "right-wing dominance" (value 1) to "left-wing dominance" (value 5). The criterion for "dominance" is set by the share of seats in government and parliament. The third measure (POL), which is our most preferred one, is constructed by extending and simplifying the data of WKB (see the appendix for details). It assigns the value -1 for RIGHT, 0 for CENTRE and 1 for LEFT.

We run the following basic specification for the panel regressions

$$\Delta d_{ct} = f_c + f_t + \alpha_1 POL_{ct} + \alpha_2 d_{ct} + \alpha_3 U_{ct} + \varepsilon_{ct},$$

where f_c and f_t are country and time fixed effects, respectively. In some specifications, we run this regression with some additional control variables including GDP per capita, openness, Gini coefficient and two measures of the age structure of the population (proportion below 14 and above 65). Our preferred specification uses POL as the measure of political color (Table 2) over the sample period 1948-2005. The results with the alternative measures, POL_{FR} and POL_{WKB} , are reported in Table 3. In all regressions we exclude non-democratic governments.

TABLE 2 HERE

Table 2 summarizes the results. In the baseline specification (column 1) the coefficient of interest (POL) is negative and significant. Since this measure is increasing as governments move to the left, the regression confirms the theoretical implication that right-wing governments run larger debt as it was the case for the US. The quantitative effect is sizeable although smaller than for the US. A shift from a left-wing (+1) to a right-wing (-1) government increases the debt-GDP ratio by 0.6 percentage points per year. This is about a third of the effect that was estimated for the US. The autoregressive coefficient (d_t) is insignificant. This seems to suggest that debt is not mean reverting. However, the apparent lack of mean reversion is driven by an outlier, Japan, whose debt has risen sharply in recent years. If we introduce an interaction between d_t and a dummy variable for Japan (namely, we allow the autoregressive coefficient of Japan to be different), the process is significantly mean reverting. The Japanese dummy is positive and very highly significant. Controlling for unemployment does not change the results, and unemployment has the expected positive effect on debt accumulation. Adding further control variables strengthens the results slightly: the political effect becomes larger and debt becomes more mean reverting.³²

TABLE 3 HERE

³²We have also run separate regressions for each country in the same way as we did for the United States. There are several limitations to this approach. For many countries the number of observations is very small. Canada, Japan and Switzerland have almost no variation in political variables, and were excluded. The results are in accordance with our theory for a large majority of the countries, although in four cases (Australia, Austria, Germany and Denmark) the political effect goes in the wrong direction.

We also tested for possible non-linearities. We found no significant difference between CENTRE and LEFT, whereas there is a large and significant difference between these and governments labelled as RIGHT (note that more than half of the observations are coded as RIGHT). The quantitative effects are about the same as in the benchmark specification.

In Table 3 we check the robustness of the results to the alternative political measures. These have the disadvantage that the sample ends in 1996. The results confirm the previous analysis. The political variable POL_{FR} is positive (recall that POL_{FR} takes on higher values for right-wing governments) and significant, and the debt process is significantly mean reverting. In this case, Japan is less important, since the main increase in Japanese debt occurred after 1995. The second set of regressions uses POL_{WKB} as the political measure (recall that POL_{WKB} takes on lower values for right-wing governments). The results are again in line with our theory. The political effects are quantitatively comparable to those in Table 2 (e.g., going from one to five implies 7.2 percentage point yearly decrease in the debt-GDP ratio), but the coefficients are estimated less precisely and are only significant at the 10% level.

In conclusion, the empirical confirm the main theoretical predictions: debt-GDP ratio is mean reverting and right-wing governments tend to increase debt.

8 Conclusion

TO BE WRITTEN...

9 Appendix A: Anticipated Fiscal Shocks

In this appendix, we extend the analysis of Section 5 to a case in which the probability distribution of fiscal shocks is non-degenerate. Since there is a positive probability that the country experiences a perpetual war, and the government must be solvent in all states of nature, the maximum debt level now becomes

$$b \leq \frac{\max_{\tau} \{\tau w h_M(\tau)\} - Z}{R - 1} \equiv \bar{b}. \quad (30)$$

We denote by z^i the state of the economy, where $i \in \{P, W\}$ stands for peace and war, respectively. The state of the economy is assumed to evolve following a first-order Markov process, with transition probability matrix Π , whose elements we denote by p_{ij} (where $p_{i,i} + p_{j,i} = 1$, and $j \neq i$).

The political equilibrium is characterized formally by the following fixed-point problem;

$$\left\langle \begin{array}{c} B(b|z^i), \\ G(b|z^i), \\ T(b|z^i) \end{array} \right\rangle = \arg \max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\}} \left\{ \begin{array}{c} (1 + \psi) \theta \log g + (1 + \beta) \log A(\tau) \\ + \beta (p_{i,i} V_O(b'|z^i) + p_{j,i} V_O(b'|z^j)) \end{array} \right\},$$

subject to either (6) or (29), and (30). $V_O(b|z^i)$, denoting the utility of the old $V_O(b|z^i)$, is given by the following functional equation

$$\begin{aligned} V_O(b|z^i) &= (1 + \lambda) \theta \log(G(b|z)) + (1 + \beta) \lambda \log A(T(b|z)) \\ &\quad + \beta \lambda (p_{z,z} V_O(b'|z) + p_{z',z} V_O(b'|z')). \end{aligned} \quad (31)$$

The analysis leads to the following generalization of the GEE to a stochastic environment (see appendix for the derivation);³³

$$E \left(\frac{G(B(b))}{G(b)} \middle| z = z^i \right) = \beta \lambda R - \beta \lambda E(G'(B(b)) | z = z^i) \left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1 \right). \quad (32)$$

Figure 5 show the dynamics of a simulated economy where we assume the following transition Markov matrix

$$\Pi = \begin{bmatrix} p_{PP} = 0.9 & p_{PW} = 0.1 \\ p_{WP} = 0.75 & p_{WW} = 0.25 \end{bmatrix}.$$

This implies that war is less likely than peace, and that the state is characterized by some persistence.

³³Note that the left hand-side of (32) is the conditional expectation of the marginal rate of substitution of public consumption between time t and $t+1$, given the state of nature (war or peace) at t .

FIGURE 5 (six panels) HERE

The first three panels represent, respectively, g , τ and b' as function of b and z .³⁴ Continuous (dotted) lines represent the level of the policy conditional on war (peace). The first panel shows that taxes are increasing in b and larger in war than in peace times. The second panel shows that expenditure (excluding war expenditure) is decreasing in b and larger in peace than in war times. Finally, the third panel show the dynamics of debt. In all panels, the continuous (dotted) line can be interpreted as the decisions rule associated with one particular history, namely when the economy experiences an infinite sequence of war (peace) times. The stationary distribution of debt is between the upper and lower steady states.

Panels 4-6 plot the evolution of policies. The results are qualitatively similar to those solid lines of Figure 4. The main differences are that in this case the anticipation of the possibility of future wars induces an additional precautionary motive for public savings in times of peace.

It is also useful to analyze the commitment solution in this stochastic environment. A simple generalization of Lemma 1 holds.³⁵ The analysis of the First Order Conditions leads to a stochastic version of the Euler equation under commitment, (14);

$$E \left(\frac{g'}{g} \middle| z = z^i \right) = \beta \lambda R, \quad i \in \{P, W\}. \quad (35)$$

FIGURE 6 (3 panels) HERE

Figure 6 is the analogue of Figure 5. In particular, the third panel shows that an economy experiencing perpetual war converges to the debt limit, while an economy experiencing perpetual peace (but perceiving a positive probability that a war starts) settles down below the maximum debt. Note that an even under commitment there is some scope for the government to reduce debt in times of peace. However, such scope is limited to a precautionary motive: agents anticipate that some future generation may suffer war, and wish to limit the extent to

³⁴We assume that a war costs 10% of maximum tax revenues.

³⁵In particular, after the first period, the problem can be expressed by the following recursive programme;

$$V_O(b|z^i) = \max_{\{\tau, g, b'\}} v(\tau, g) + \beta \lambda \left(p_{i,i} V_O(b'|z^i) + p_{j,i} V_O(b'|z^j) \right) \quad (33)$$

subject to (6) or (29) and (30). The functional equation (33) is the stochastic analogue of (11).

The analysis of the First Order Conditions leads to the following generalization, state-by-state, of equation (13);

$$\frac{1 + \beta}{(1 + \frac{1}{\lambda}) \theta} g^i = A(\tau^i) \left(1 - e(\tau^i) \right), \quad (34)$$

See the appendix for the details of the analysis.

which future government consumption must be cut. This effect is significantly smaller than in the politico-economic model. In Table X-3, we denote by b_P^* and b_R^* the steady-state debt levels with perpetual peace in the political equilibrium and the Ramsey allocation, respectively, for two different values of ξ . In all cases b_P^* is substantially lower than b_R^* , and is in fact rather close to $\bar{b}$, showing that the precautionary motive can only induce a limited amount of public saving.

Table X-3		
	b_P^*	b_R^*
$\xi = 0.90$	0.2295	0.4047
$\xi = 0.70$	0.1883	0.4047
$\xi = 0.50$	0.0932	0.4046

10 Appendix B: proofs of Lemmas and Propositions

10.1 Proof Lemma 1

We rewrite the political objective function (10) as follows:

$$\begin{aligned}
& \frac{U(\mathbf{b}, \tau, g)}{(1 - \omega + \omega\lambda)} \\
= & \frac{(1 - \omega) U_Y(\mathbf{b}, \tau, g) + \omega (\theta \log(g_0) + \lambda U_Y(\mathbf{b}, \tau, g))}{(1 - \omega + \omega\lambda)} \\
= & \frac{\omega \theta \log(g_0) + (1 - \omega + \lambda\omega) U_Y(\mathbf{b}, \tau, g)}{(1 - \omega + \omega\lambda)} \\
= & \frac{\omega}{(1 - \omega + \omega\lambda)} \theta \log(g_0) + \sum_{t=0}^{\infty} (\lambda\beta)^t ((1 + \beta) \log(A(\tau_t)) + \theta \log(g_t) + \beta \theta \log(g_{t+1})) \\
= & (1 + \beta) \log(1 - \tau_0) + (1 + \psi) \theta \log(g_0) + \sum_{t=1}^{\infty} (\lambda\beta)^t \left((1 + \beta) \log(A(\tau_t)) + \left(1 + \frac{1}{\lambda}\right) \theta \log(g_t) \right) \\
= & (1 + \beta) \log(1 - \tau_0) + (1 + \psi) \theta \log(g_0) + \sum_{t=1}^{\infty} (\lambda\beta)^t v(g_t, \tau_t).
\end{aligned}$$

It follows that

$$\begin{aligned}
& \max_{\{\tau_t, g_t, b_{t+1}\}_{t=0}^{\infty}} \left\{ \frac{U(\dots)}{(1 - \omega + \omega\lambda)} \right\} \Big|_{b_0} \\
= & \max_{\{\tau_0, g_0, b_1\}} \left\{ (1 + \beta) \log(A(\tau_0)) + (1 + \psi) \theta \log(g_0) + \max_{\{\tau_t, g_t, b_{t+1}\}_{t=1}^{\infty} | b_1} \left\{ \sum_{t=1}^{\infty} (\lambda\beta)^t v(g_t, \tau_t) \right\} \Big|_{b_1} \right\} \Big|_{b_0} \\
= & \max_{\{\tau_0, g_0, b_1\}_{t=0}^{\infty}} \{ (1 + \beta) \log(A(\tau_0)) + (1 + \psi) \theta \log(g_0) + \beta \lambda V_O(b_1) \} \Big|_{b_0},
\end{aligned}$$

where all maximizations are subject to (6), and the last step follows from equation (11).

10.2 Proof Lemma 2

Given policy rules $T(b)$, $G(b)$, and $B(b)$, the discounted utility of the young is defined by the function:

$$V_Y(b) \equiv \sum_{t=0}^{\infty} (\lambda\beta)^t \left((1+\beta) \log(A(T(B^t(b)))) + \theta \log(G(B^t(b))) + \beta\theta \log(G(B^{t+1}(b))) \right).$$

When ignoring the predetermined term with $\log(1 - \tau_{-1})$, the discounted utility of the old can be expressed as:

$$\begin{aligned} V_O(b) &\equiv \theta \log G(b) + \lambda V_Y(b) \\ &= \theta \log G(b) + \lambda \sum_{t=0}^{\infty} (\lambda\beta)^t \left((1+\beta) \log(A(T(B^t(b)))) + \theta \log(G(B^t(b))) + \beta\theta \log(G(B^{t+1}(b))) \right) \\ &= \lambda(1+\beta) \log(A(T(b))) + (1+\lambda)\theta \log G(b) \\ &\quad + \lambda \sum_{t=1}^{\infty} (\lambda\beta)^t \left((1+\beta) \log(A(T(B^t(b)))) + \theta(1+\lambda) \log(G(B^t(b))) \right). \end{aligned}$$

Rewrite $V_O(b)$ in a recursive fashion:

$$V_O(b) = (1+\lambda)\theta \log(G(b)) + (1+\beta)\lambda \log(A(T(b))) + \beta\lambda V_O(B(b)).$$

It is straightforward that, due to the Contraction Mapping Theorem, $V_O(b)$ is the unique solution to the above functional equation.

Moreover, the political objective function can be expressed as

$$\begin{aligned} &\frac{U(\mathbf{b}, \boldsymbol{\tau}, g)}{(1 - \omega + \omega\lambda)} \\ &= \psi\theta \log g + U_Y(\mathbf{b}, \boldsymbol{\tau}, g) \\ &= (1+\beta) \log(A(\tau)) + (1+\psi)\theta \log g + \beta\theta \log(G(b')) + \beta\lambda V_Y(b') \\ &= (1+\beta) \log(A(\tau)) + (1+\psi)\theta \log g + \beta V_O(b'). \end{aligned}$$

Now the political problem can be rewritten as

$$\begin{aligned} &\max_{\{g, \tau\}} \left\{ (1+\beta)\lambda \log(A(\tau)) + (\lambda + \lambda\psi)\theta \log g + \beta\lambda V_O(b') \right\} \\ &\text{s.t.} \\ &b' = Rb + g - \tau wh_M(\tau). \end{aligned}$$

10.3 Proof of the Generalized Euler Equation

The FOCs for problem (15) are

$$\begin{aligned}\frac{(1+\beta)\lambda A'(\tau)}{A(\tau)} - \beta\lambda V'_O(b') (wh_M(\tau) + \tau wh'_M(\tau)) &= 0, \\ \frac{\lambda\theta(1+\psi)}{g} + \beta\lambda \hat{V}'(b') &= 0.\end{aligned}$$

By the definition of $e(\tau)$ and the fact that $A'(\tau) = -wh_M(\tau)$, the FOCs can be rewritten as

$$-\frac{(1+\beta)\lambda}{A(\tau)} - \beta\lambda V'_O(b') (1 - e(\tau)) = 0, \quad (36)$$

$$\frac{\lambda\theta(1+\psi)}{g} + \beta\lambda V'_O(b') = 0. \quad (37)$$

Combining two FOCs delivers (17):

$$\frac{1+\beta}{(1+\psi)\theta}g = A(\tau)(1 - e(\tau)).$$

. Then we can rewrite (16) and the government budget constraint (6) as

$$\begin{aligned}V_O(b) &= ((1+\beta)\lambda + (1+\lambda)\theta) \log(G(b)) - (1+\beta)\lambda \log(1 - e(T(b))) + \beta\lambda V_O(B(b)), \\ B(b) &= G(b) + Rb - T(b)wh_M(T(b)).\end{aligned}$$

Differentiating $V_O(b)$ and $B(b)$ yields

$$\begin{aligned}V'_O(b) &= ((1+\beta)\lambda + (1+\lambda)\theta) \frac{G'(b)}{G(b)} - \frac{-(1+\beta)\lambda e'(T(b))T'(b)}{1 - e(T(b))} + \beta\lambda V'_O(B(b))B'(b), \\ B'(b) &= G'(b) + R - T'(b)wh_M(T(b))(1 - e(T(b))) \\ &= \left(1 + \frac{1+\beta}{\theta(1+\psi)}\right)G'(b) + R + e'(T(b))T'(b)A(T(b)).\end{aligned}$$

The last equality comes from the fact that

$$-T'(b)wh_M(T(b))(1 - e(T(b))) - e'(T(b))T'(b)A(T(b)) = \frac{1+\beta}{\theta(1+\psi)}G'(b),$$

as implied by (17) and $A'(\tau) = -wh_M(\tau)$. Plugging $V'_O(b)$ into the second FOC (37), we have

$$\frac{1}{G(b)} = \frac{\beta\lambda}{G(B(b))} \left(B'(B(b)) - \frac{1+\beta + (1+\frac{1}{\lambda})\theta}{\theta(1+\psi)}G'(B(b)) - \frac{(1+\beta)G(B(b))e'(T(B(b)))T'(B(b))}{\theta(1+\psi)(1 - e(T(B(b))))} \right).$$

Now substituting for $B'(B(b))$

$$\frac{1}{G(b)} = \frac{\beta\lambda}{G(B(b))} \left(R + \left(1 - \frac{1+\frac{1}{\lambda}}{1+\psi}\right)G'(B(b)) + e'(T(B(b)))T'(B(b))A(T(B(b))) - \frac{(1+\beta)G(B(b))e'(T(B(b)))T'(B(b))}{\theta(1+\psi)(1 - e(T(B(b))))} \right).$$

The FOC (17) implies

$$A(T(B(b)))(1 - e(T(B(b)))) = \frac{1 + \beta}{\theta(1 + \psi)} G(B(b)).$$

Therefore, we obtain the generalized Euler equation (18):

$$\frac{1}{G(b)} = \frac{\beta\lambda R}{G(B(b))} - \frac{\beta\lambda G'(B(b))}{G(B(b))} \left(\frac{1 + \frac{1}{\lambda}}{1 + \psi} - 1 \right).$$

10.4 Proof of Lemma ??

Consider a small perturbation of debt from the steady state; $\tilde{b} = b^* + \varepsilon$, $\varepsilon > 0$. The stability establishes that $b' < \tilde{b}$, which implies that $g' > g$ due to the negative slope of $G(b)$. From (18), we know that $G'(b') < \zeta$. Similar argument establishes that $G'(b') > \zeta$ for $\varepsilon < 0$, so $G(b)$ must be concave around b^* .

OLD We need to show that $B'(b^*) < 1$ at the steady state. From (6), (??) and Lemma ??, we know that $B'(b^*) = G'(b^*) + R - \frac{\partial(T(b^*)h_M(T(b^*)))}{\partial b} \leq G'(b^*) + R = \zeta + R$, where the inequality is due to the fact that the economy is located in the left of the top of the Laffer curve. Then the standard algebra shows that when (??) is satisfied, $\frac{\partial b'}{\partial b} < 1$ must hold.

Next consider a small perturbation of debt from the steady state; $\tilde{b} = b^* + \varepsilon$, $\varepsilon > 0$. The stability establishes that $b' < \tilde{b}$, which implies that $g' > g$ due to the negative $G'(b^*)$. From (18), we know that $G'(b') < \zeta$. Similar argument establishes that $G'(b') > \zeta$ for $\varepsilon < 0$, so $G(b)$ must be concave around b^* .

10.5 Step Function Equilibrium

Proposition 5 Suppose $R \in [1 + (1 + \psi)/\zeta, R_h]$, and let the initial debt level be $b = b^0 \in [\underline{b}, \bar{b}]$, where $\bar{b} \equiv \bar{\tau}wh/(R - 1)$ and $\zeta \equiv (1 + \lambda)\beta/(1 - \beta\lambda)$, and R_h and $\underline{b}$ are defined in the appendix. Then, the equilibrium is given by the following policy functions

$$\begin{aligned} \tau = T(b) &\equiv \begin{cases} \bar{\tau} - \frac{R(1+\beta)}{wh(1+\beta+\theta(1+\psi))} (b_0^* - b) & \text{if } b \in [\underline{b}, b_0^*) \\ \bar{\tau} & \text{otherwise} \end{cases}, \\ g = G(b) &\equiv \begin{cases} g_0^* + \frac{\theta(1+\psi)R}{1+\beta+\theta(1+\psi)} (b_0^* - b) & \text{if } b \in [\underline{b}, b_0^*) \\ b_n^* + \bar{\tau}wh - Rb & \text{if } b \in [b_n^*, b_{n+1}^*) \end{cases}, \\ b' = B(b) &\equiv \begin{cases} b_0^* \equiv \bar{b} \left(1 - \frac{\theta(1+\psi)(1-\bar{\tau})}{\bar{\tau}(1+\beta)} \right) & \text{if } b \in [\underline{b}, b_1^*) \\ b_n^* & \text{if } b \in [b_n^*, b_{n+1}^*) \end{cases}, \end{aligned}$$

where $g_0^* \equiv wh\theta(1 + \psi)(1 - \bar{\tau})/(1 + \beta) > 0$, and the sequence $\{b_n^*\}_{n=0,1,2,\dots,\infty}$ is the unique solution to the difference equation

$$(b_n^* - b_{n+1}^* + \bar{\tau}wh)^{1+\psi} (b_n^* - Rb_{n+1}^* + \bar{\tau}wh)^\zeta = (b_{n+1}^* - Rb_{n+1}^* + \bar{\tau}wh)^{1+\psi+\zeta},$$

given b_0^* . The sequence $\{b_n^*\}_{n=0,1,2,\dots,\infty}$ is monotonically increasing in n and $\lim_{n \rightarrow \infty} b_n^* = \bar{b}$.

10.6 Calibrating X and w

For $\xi \in (0, 1)$, total tax revenue Y and its derivative are given by

$$\begin{aligned} Y(\tau) &= w\tau \left(h - \left(\frac{(1-\tau)w}{\xi X} \right)^{\frac{1}{\xi-1}} \right), \\ \frac{\partial Y}{\partial \tau} &= w \left(h - \frac{1-\xi(1-\tau)}{(1-\xi)(1-\tau)} \left(\frac{(1-\tau)w}{\xi X} \right)^{\frac{1}{\xi-1}} \right). \end{aligned}$$

We set X and w as follows:

$$w = \frac{1-\xi(1-\bar{\tau})}{\bar{\tau}}, \quad (38)$$

$$X = \frac{(1-\bar{\tau})^{2-\xi}}{\bar{\tau}} \frac{(1-\xi)^{1-\xi}}{\xi} (1-\xi(1-\bar{\tau}))^\xi h^{1-\xi}. \quad (39)$$

Note first that as $\xi \rightarrow 1$, then X and w converge to their respective values in the analytical example $\lim_{\xi \rightarrow 1} X = (1-\bar{\tau})$ and $\lim_{\xi \rightarrow 1} w = 1$ (we set w equal to 1 in the analytical case). Moreover, these choices for X and w yield that the top of the Laffer curve is at $\tau = \bar{\tau}$ and that maximal tax revenue is equal to $\bar{\tau}h$. The details are given by:

$$\begin{aligned} \frac{\partial Y(\bar{\tau})}{\partial \tau} &= w \left(h - \frac{1-\xi(1-\tau)}{(1-\xi)(1-\tau)} \left(\frac{(1-\tau)w}{\xi X} \right)^{\frac{1}{\xi-1}} \right) \\ &= w \left(h - \frac{1-\xi(1-\tau)}{(1-\xi)(1-\tau)} \left(\frac{(1-\tau)}{\xi \frac{(1-\bar{\tau})^{2-\xi}}{\bar{\tau}} \frac{(1-\xi)^{1-\xi}}{\xi} (1-\xi(1-\bar{\tau}))^\xi h^{1-\xi}} \frac{1-\xi(1-\bar{\tau})}{\bar{\tau}} \right)^{\frac{1}{\xi-1}} \right) \\ &= 0. \end{aligned}$$

$$\begin{aligned} Y(\bar{\tau}) &= \bar{\tau}w \left(h - \left(\frac{(1-\bar{\tau})w}{\xi X} \right)^{\frac{1}{\xi-1}} \right) \\ &= \bar{\tau} \frac{1-\xi(1-\bar{\tau})}{\bar{\tau}} \left(h - \left(\frac{(1-\bar{\tau})}{\xi \frac{(1-\bar{\tau})^{2-\xi}}{\bar{\tau}} \frac{(1-\xi)^{1-\xi}}{\xi} (1-\xi(1-\bar{\tau}))^\xi} \frac{1-\xi(1-\bar{\tau})}{\bar{\tau}} \right)^{\frac{1}{\xi-1}} \right) \\ &= \bar{\tau}h. \end{aligned}$$

10.7 Political Uncertainty

The value function (conditional on the political state θ_Y and θ_O) can be written as

$$\left\langle \begin{array}{c} B(b|\theta_Y, \theta_O), \\ G(b|\theta_Y, \theta_O), \\ T(b|\theta_Y, \theta_O) \end{array} \right\rangle = \arg \max_{\{b' \leq \bar{b}, g \geq 0, \tau \in [0,1]\}} \left\{ \begin{array}{c} (\lambda\theta_Y + \psi\lambda\theta_O) \log g + (1+\beta)\lambda \log(1-\tau) \\ + \beta\lambda L(\theta_Y) (p_{l,l}V_O(b'|\theta_l, \theta_l) + p_{r,l}V_O(b'|\theta_r, \theta_l)) \\ + \beta\lambda (1-L(\theta_Y)) (p_{l,r}V_O(b'|\theta_l, \theta_r) + p_{r,r}V_O(b'|\theta_r, \theta_r)) \end{array} \right\}, \quad (40)$$

subject to (6) and (7), the utility of the old $V_O(b|\theta_Y, \theta_O)$ is given by the functional equation

$$\begin{aligned} V_O(b|\theta_Y, \theta_O) &= (\lambda\theta_Y + \theta_O) \log(G(b|\theta_Y, \theta_O)) + (1 + \beta) \lambda \log(1 - T(b|\theta_Y, \theta_O)) \\ &\quad + \beta \lambda L(\theta_Y) (p_{l,l} V_O(b'|\theta_l, \theta_l) + p_{r,l} V_O(b'|\theta_r, \theta_l)) \\ &\quad + \beta \lambda (1 - L(\theta_Y)) (p_{l,r} V_O(b'|\theta_l, \theta_r) + p_{r,r} V_O(b'|\theta_r, \theta_r)) \end{aligned} \quad (41)$$

and

$$\begin{aligned} L(\theta_Y) &= 1 \text{ if } \theta_Y = \theta_l, \\ L(\theta_Y) &= 0 \text{ if } \theta_Y = \theta_r, \\ p_{l,l} + p_{r,l} &= p_{l,r} + p_{r,r} = 1, \end{aligned}$$

We assume that all policy functions are continuous and differentiable. Consider the case in which in the current period both the young and the old are left. Then, the solution must satisfy the following First Order Conditions.

$$-\frac{(1 + \beta) \lambda}{1 - \tau} = \beta \lambda L(\theta_Y) (p_{l,l} V'_O(b'|\theta_l, \theta_l) + p_{r,l} V'_O(b'|\theta_r, \theta_l)) \text{ wh} \quad (42)$$

$$\begin{aligned} &+ \beta \lambda (1 - L(\theta_Y)) (p_{l,r} V'_O(b'|\theta_l, \theta_r) + p_{r,r} V'_O(b'|\theta_r, \theta_r)) \text{ wh} \\ -\frac{\lambda\theta_Y + \psi\lambda\theta_O}{g} &= \beta \lambda L(\theta_Y) (p_{l,l} V'_O(b'|\theta_l, \theta_l) + p_{r,l} V'_O(b'|\theta_r, \theta_l)) \\ &+ \beta \lambda (1 - L(\theta_Y)) (p_{l,r} V'_O(b'|\theta_l, \theta_r) + p_{r,r} V'_O(b'|\theta_r, \theta_r)) \end{aligned} \quad (43)$$

The two equations, (42)-(43), together imply that

$$T(b|\theta_Y, \theta_O) = 1 - \frac{(1 + \beta) G(b|\theta_Y, \theta_O)}{(\theta_Y + \psi\theta_O) \text{ wh}}.$$

This allows us to obtain:

$$\begin{aligned} V_O(b|\theta_Y, \theta_O) &= ((1 + \beta) \lambda + \lambda\theta_Y + \theta_O) \log(G(b|\theta_Y, \theta_O)) \\ &\quad + \beta \lambda L(\theta_Y) (p_{l,l} V_O(b'|\theta_l, \theta_l) + p_{r,l} V_O(b'|\theta_r, \theta_l)) \\ &\quad + \beta \lambda (1 - L(\theta_Y)) (p_{l,r} V_O(b'|\theta_l, \theta_r) + p_{r,r} V_O(b'|\theta_r, \theta_r)), \end{aligned} \quad (44)$$

$$B(b|\theta_Y, \theta_O) = \left(1 + \frac{(1 + \beta)}{(\theta_Y + \psi\theta_O)}\right) G(b|\theta_Y, \theta_O) + Rb - wh. \quad (45)$$

Differentiating $V^O(b|\theta_Y, \theta_O)$ and $B(b|\theta_Y, \theta_O)$ yields, then,

$$V'_O(b|\theta_Y, \theta_O) = ((1 + \beta) \lambda + \theta_Y + \lambda\theta_O) \frac{G'(b|\theta_Y, \theta_O)}{G(b|\theta_Y, \theta_O)} \quad (46)$$

$$\begin{aligned} &+ \beta \lambda L(\theta_Y) (p_{l,l} V_O(b'|\theta_l, \theta_l) + p_{r,l} V_O(b'|\theta_r, \theta_l)) B'(b|\theta_Y, \theta_O) \\ &+ \beta \lambda (1 - L(\theta_Y)) (p_{l,r} V_O(b'|\theta_l, \theta_r) + p_{r,r} V_O(b'|\theta_r, \theta_r)) B'(b|\theta_Y, \theta_O) \\ B'(b|\theta_Y, \theta_O) &= \left(1 + \frac{(1 + \beta)}{(\theta_Y + \psi\theta_O)}\right) G'(b|\theta_Y, \theta_O) + R. \end{aligned} \quad (47)$$

Combining these with (43), we obtain

$$\begin{aligned}
V'_O(b|\theta_Y, \theta_O) &= ((1+\beta)\lambda + \lambda\theta_Y + \theta_O) \frac{G'(b|\theta_Y, \theta_O)}{G(b|\theta_Y, \theta_O)} - \frac{\lambda\theta_Y + \psi\lambda\theta_O}{G(b|\theta_Y, \theta_O)} B'(b|\theta_Y, \theta_O) \\
&= ((1+\beta)\lambda + \lambda\theta_Y + \theta_O) \frac{G'(b|\theta_Y, \theta_O)}{G(b|\theta_Y, \theta_O)} \\
&\quad - \frac{\lambda\theta_Y + \psi\lambda\theta_O}{G(b|\theta_Y, \theta_O)} \left(\left(1 + \frac{(1+\beta)}{(\theta_Y + \psi\theta_O)}\right) G'(b|\theta_Y, \theta_O) + R \right) \\
&= (1 - \psi\lambda)\theta_O \frac{G'(b|\theta_Y, \theta_O)}{G(b|\theta_Y, \theta_O)} - \frac{(\lambda\theta_Y + \psi\lambda\theta_O)R}{G(b|\theta_Y, \theta_O)},
\end{aligned}$$

Then, (43) leads to the GEE under political uncertainty

$$\begin{aligned}
\frac{1}{G(b|\theta_Y, \theta_O)} &= \beta\lambda R \left(\begin{aligned} &p(\theta'_Y = \theta_Y, \theta'_O|\theta_Y, \theta_O) \frac{\frac{\theta'_Y + \psi\theta_Y}{\theta_Y + \psi\theta_O}}{G(B(b|\theta_Y, \theta_O)|\theta'_Y = \theta_Y, \theta_Y)} \\ &+ p(\theta'_Y \neq \theta_Y, \theta'_O|\theta_Y, \theta_O) \frac{\frac{\theta'_Y + \psi\theta_Y}{\theta_Y + \psi\theta_O}}{G(B(b|\theta_Y, \theta_O)|\theta'_Y \neq \theta_Y, \theta_Y)} \end{aligned} \right) \\
&\quad - \beta\lambda \left(\frac{1 + \lambda^{-1}}{1 + \psi} - 1 \right) \left(\begin{aligned} &p(\theta'_Y = \theta_Y, \theta'_O|\theta_Y, \theta_O) \frac{G'(B(b|\theta_Y, \theta_O)|\theta'_Y = \theta_Y, \theta_Y)}{G(B(b|\theta_Y, \theta_O)|\theta'_Y = \theta_Y, \theta_Y)} \\ &+ p(\theta'_Y \neq \theta_Y, \theta'_O|\theta_Y, \theta_O) \frac{G'(B(b|\theta_Y, \theta_O)|\theta'_Y \neq \theta_Y, \theta_Y)}{G(B(b|\theta_Y, \theta_O)|\theta'_Y \neq \theta_Y, \theta_Y)} \end{aligned} \right),
\end{aligned} \tag{48}$$

If $\lambda = 0$, then the GEE becomes

$$\frac{1}{G(b|\theta_Y, \theta_O)} = -\frac{\beta}{1 + \psi} \cdot E \left[\frac{G'(B(b|\theta_Y, \theta_O)|\theta'_Y, \theta'_O)}{G(B(b|\theta_Y, \theta_O)|\theta'_Y, \theta'_O)} \right],$$

Next, we guess that

$$G(b|\theta_Y, \theta_O) = \gamma(\theta_Y, \theta_O) (\bar{b} - b). \tag{49}$$

Then (45) implies that

$$\bar{b} - B(b|\theta_Y, \theta_O) = \left(R - \left(1 + \frac{(1+\beta)}{(\theta_Y + \psi\theta_O)} \right) \gamma(\theta_Y, \theta_O) \right) (\bar{b} - b).$$

Then, (48) establishes

$$R \frac{(\theta_Y + \psi\theta_O)}{\gamma(\theta_Y, \theta_O)} = \psi\theta_O + (1+\beta)(1+\theta_Y),$$

which gives

$$\gamma(\theta_Y, \theta_O) = \frac{(\theta_Y + \psi\theta_O)R}{\psi\theta_O + (1+\beta)(1+\theta_Y)}.$$

It follows that

$$\begin{aligned}
b' &= \left(1 + \frac{(1+\beta)}{(\theta_Y + \psi\theta_O)} \right) \gamma(\theta_Y, \theta_O) (\bar{b} - b) + Rb - (R-1)\bar{b} \\
&= \bar{b} - \frac{\beta R \theta_Y}{\psi\theta_O + (1+\beta)(1+\theta_Y)} (\bar{b} - b), \\
\tau &= 1 - \frac{1}{wh} \frac{(1+\beta)R}{\psi\theta_O + (1+\beta)(1+\theta_Y)} (\bar{b} - b)
\end{aligned}$$

Since $\psi = \frac{\omega}{1-\omega}$ for $\lambda = 0$, this proves Proposition 4.

Finally, we need to show that (??) holds in the equilibrium. Note that in equilibrium, the policy rule $G(.)$ be of the form (49). Then 48 simplifies to

$$\frac{1}{g} = \frac{\beta}{1 + \psi} \frac{1}{(\bar{b} - b')}. \quad (50)$$

Using (50), we can rewrite (48) as (??).

Table 1: Regression for U.S. Data

Dep. Variable	change in the debt-GDP ratio Δd_t		
	(1)	(2)	(3)
constant	0.0378** (2.51)	0.0274 (1.61)	0.0272 (1.62)
d_t	-0.0885** (-2.34)	-0.0675 (-1.60)	-0.0670 (-1.61)
DEM	-0.0207*** (-4.02)	-0.0172*** (-3.27)	-
UNEMPL	-	0.0064*** (2.92)	0.0064*** (2.84)
DEM_PRE1980	-	-	-0.0182** (-2.61)
DEM_POST1980	-	-	-0.0156*** (-2.78)
Obs.	57	57	57
R ²	0.3974	0.4934	0.4942

Notes: DEM is a dummy variable which equals one or zero when the president is a Democrat or Republican, respectively. UNEMPL stands for the unemployment rate subtracted by the mean of the unemployment rate. DEM_PRE1980 is set equal to DEM before 1980 and zero afterwards, while DEM_POST1980 equals DEM after 1980 and zero otherwise. Robust t statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.

Table 2: Panel Regression

Dep. Variable	change in the debt-GDP ratio Δd_t				
	(1)	(2)	(3)	(4)	(5)
d_t	0.0051 (0.49)	0.0079 (0.69)	-0.0205*** (-2.65)	-0.0287*** (-3.16)	-0.0305*** (-3.42)
d_t *JPN	-	-	0.1137*** (6.82)	0.1264*** (7.26)	0.1120*** (6.11)
POL	-0.0031** (-2.19)	-0.0036** (-2.56)	-0.0031** (-2.25)	-0.0038*** (-2.73)	-0.0040*** (-2.82)
UNEMPL	-	0.0015** (2.32)	-	0.0027*** (4.30)	0.0026*** (4.29)
Control Variables	No	No	No	No	Yes
obs.	1005	948	1005	948	933
Ad. R ²	0.2995	0.2764	0.3483	0.3362	0.3590

Notes: Country dummies and year dummies are included to control for the fixed effects and time effects. JPN is a dummy variable which equals one for Japan and zero otherwise. POL codes left-right positions of government through a three-point scale: -1 for the right-wing government, 0 for the coalition government and 1 for the left-wing government. UNEMPL stands for the unemployment rate. Control variables are the log of real GDP per capita and openness. Robust t statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.

Table 3: Robustness

Dep. Variable	change in the debt-GDP ratio Δd_t					
	(1)	(2)	(3)	(4)	(5)	(6)
d_t	-0.0354*** (-3.60)	-0.0215** (-1.98)	-0.0206* (-1.79)	-0.0273*** (-2.78)	-0.01689 (-1.49)	-0.0186 (-1.55)
POL_FR	0.0023* (1.88)	0.0027*** (2.59)	0.0027*** (2.67)	-	-	-
POL_EJPR	-	-	-	-0.0018* (-1.78)	-0.0018* (-1.74)	-0.0017* (-1.66)
UNEMPL	-	0.0018** (2.43)	0.0034*** (4.62)	-	0.0015* (1.75)	0.0035*** (4.27)
Control Variables	No	No	Yes	No	No	Yes
obs.	835	769	752	749	692	672
Ad. R ²	0.2977	0.2850	0.3209	0.3410	0.2923	0.3325

Notes: Country dummies and year dummies are included to control for the fixed effects and time effects. POL_FR codes left-right positions of government at far left to 10 at far right. POL_EJPR assigns scores for government and parliament from 1, as "right-wing dominance", to 5, as "left-wing dominance". UNEMPL stands for the unemployment rate. Control variables are the log of real GDP per capita, openness, the sizes of population over 65 and below 14. Robust t statistics is in brackets. ***, ** and * is significant at 1%, 5% and 10%, respectively.

Figure 1: equilibrium policy rules when $\xi=0$

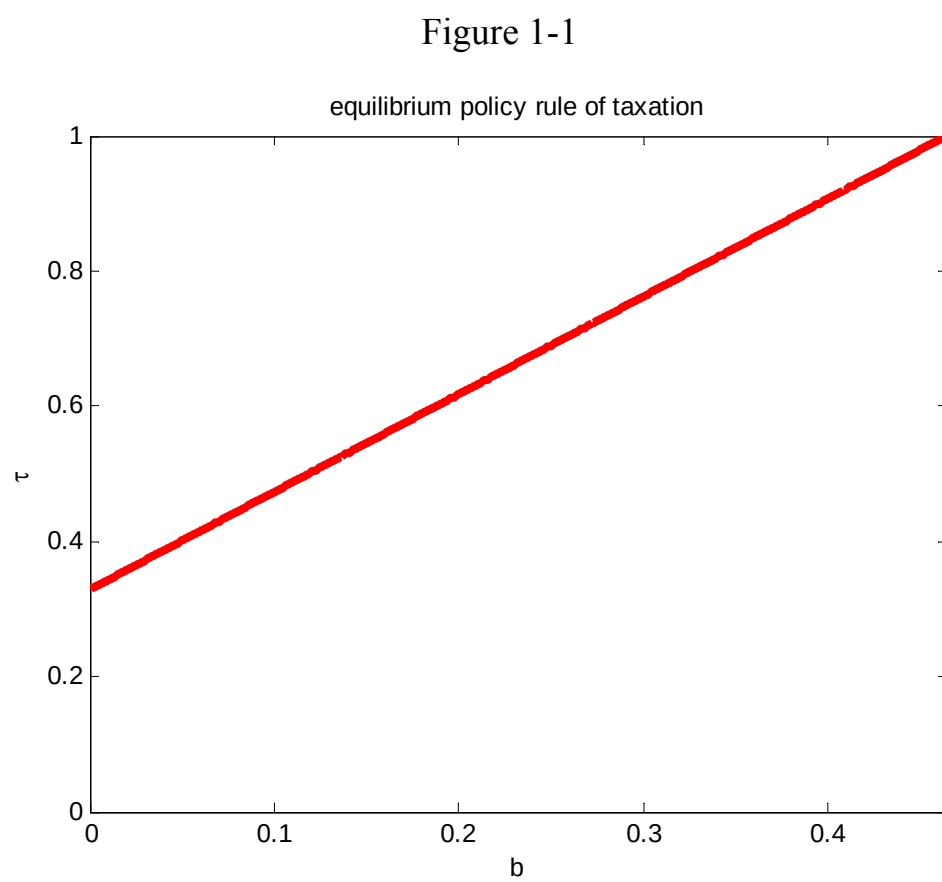


Figure 1-2

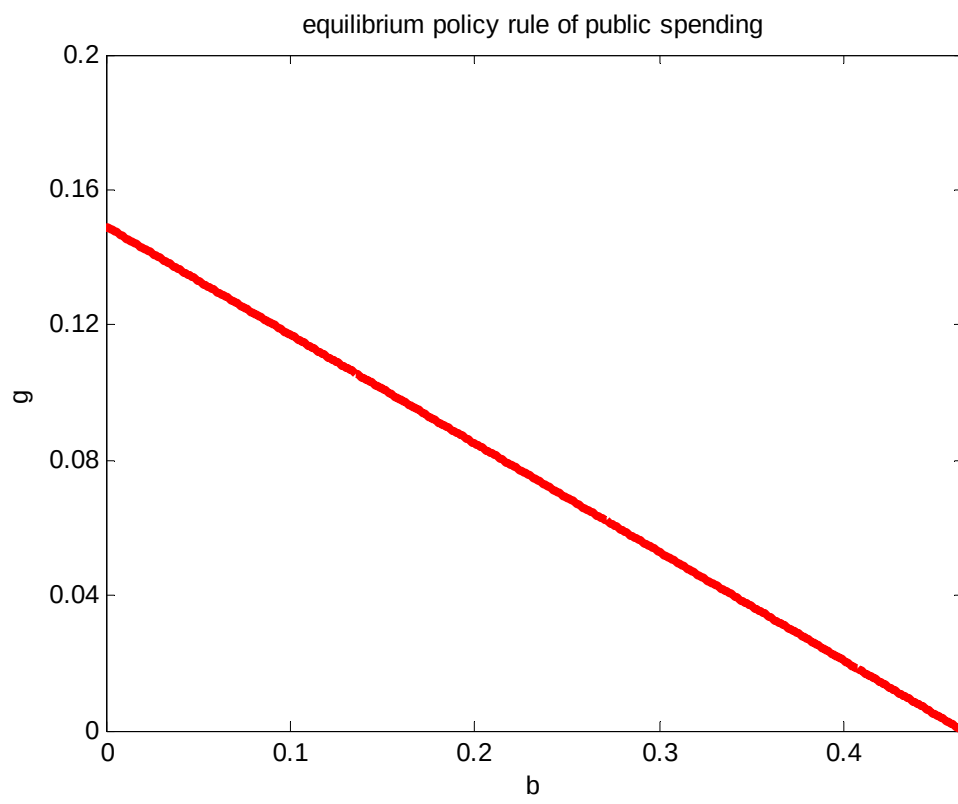


Figure 1-3

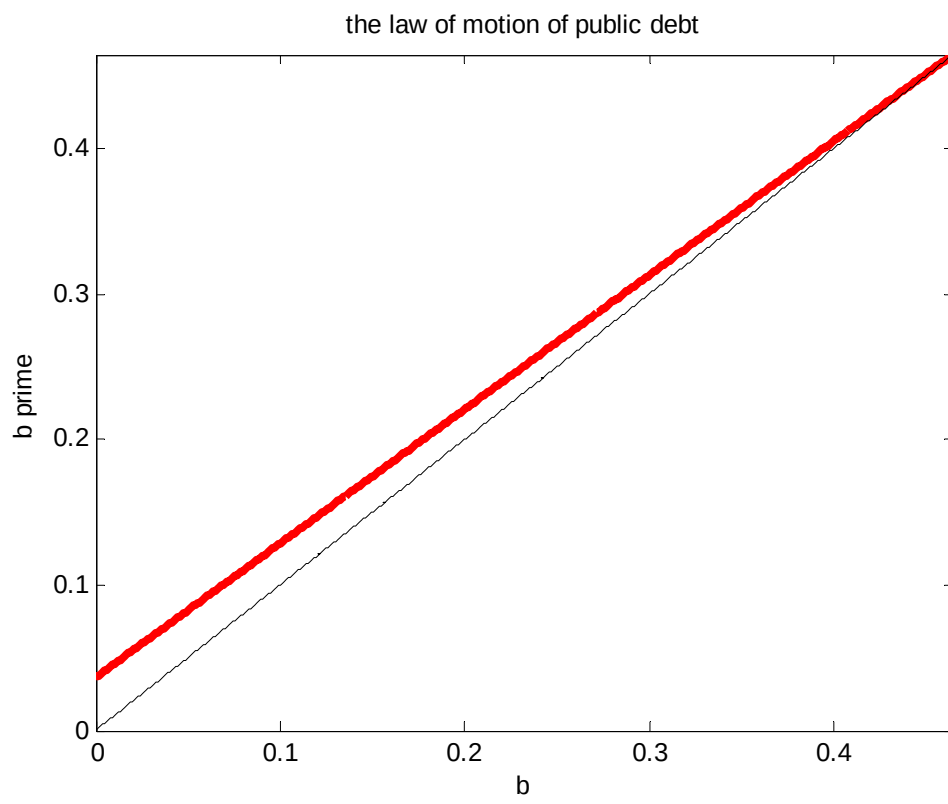


Figure 2: equilibrium policy rules when $\xi=1$

Figure 2-1

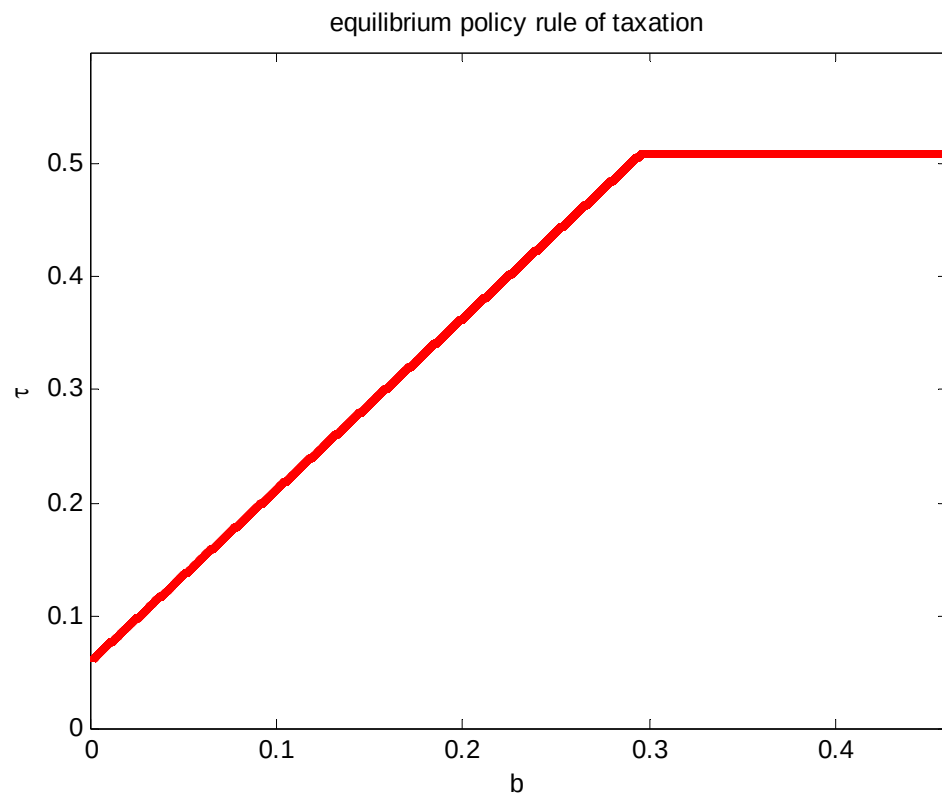


Figure 2-2

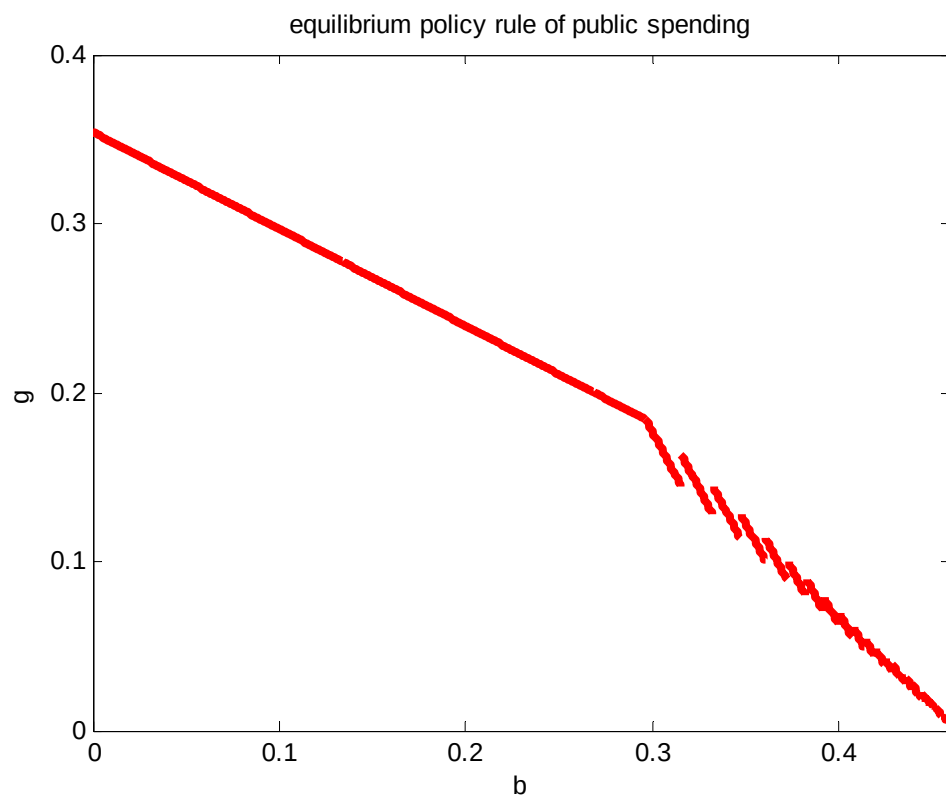


Figure 2-3

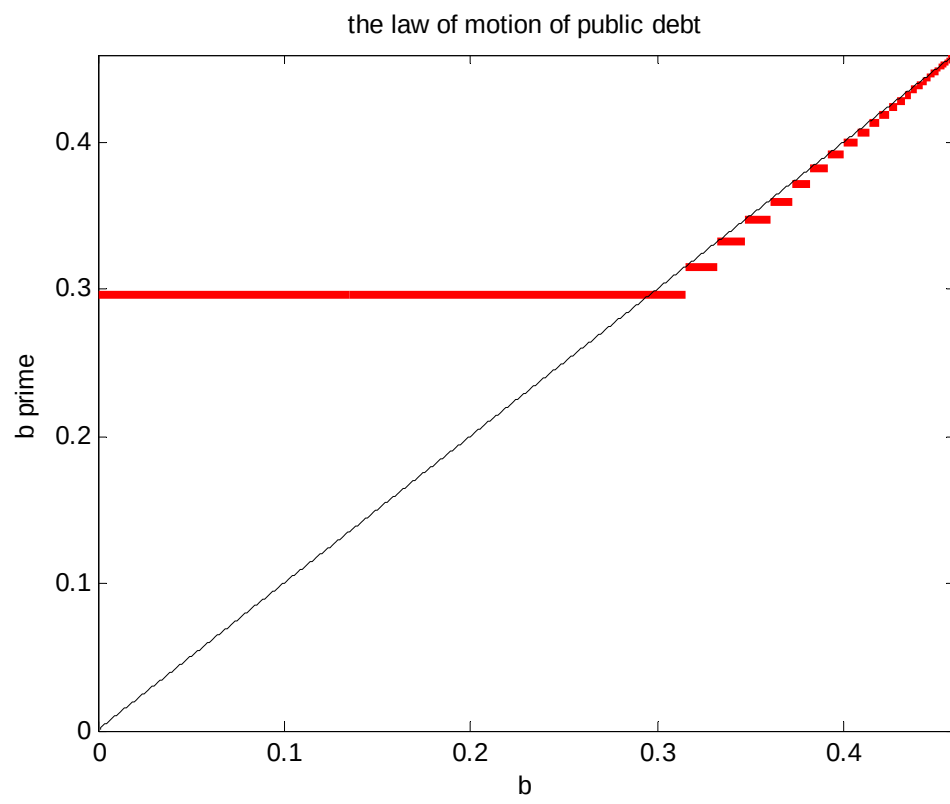


Figure 3: equilibrium policy rules when $\xi \in (0,1)$
(solid line and dotted line stand for $\xi=0.90$ and $\xi=0.50$, respectively)

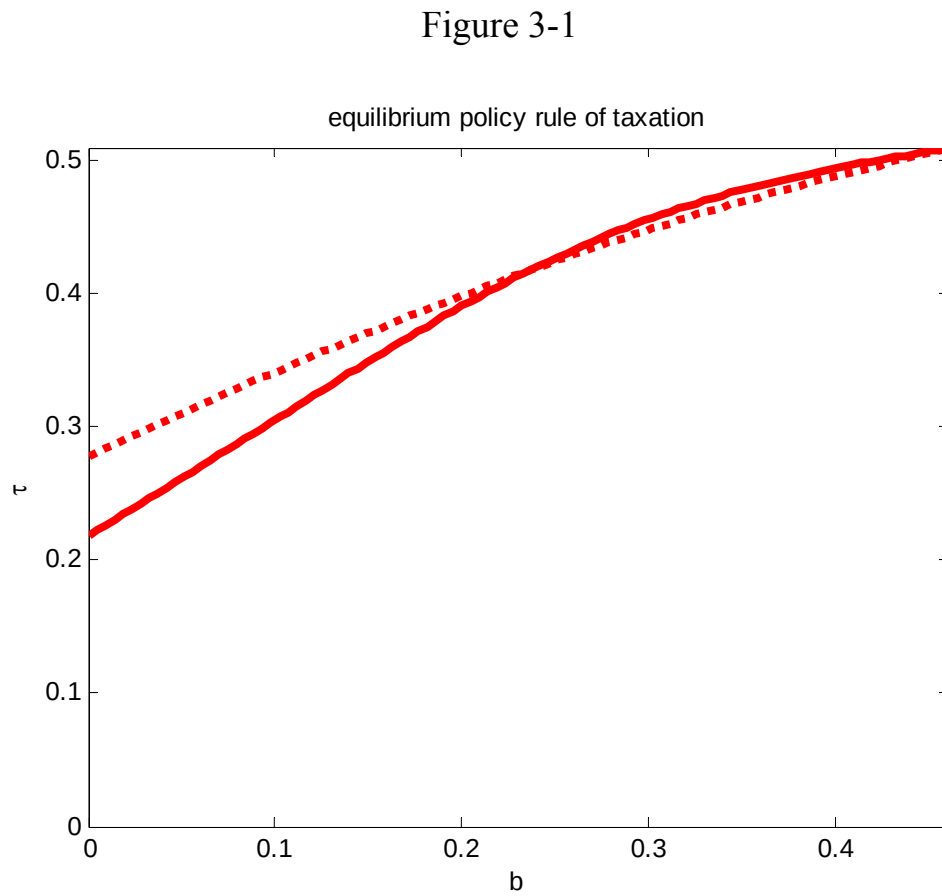


Figure 3-2

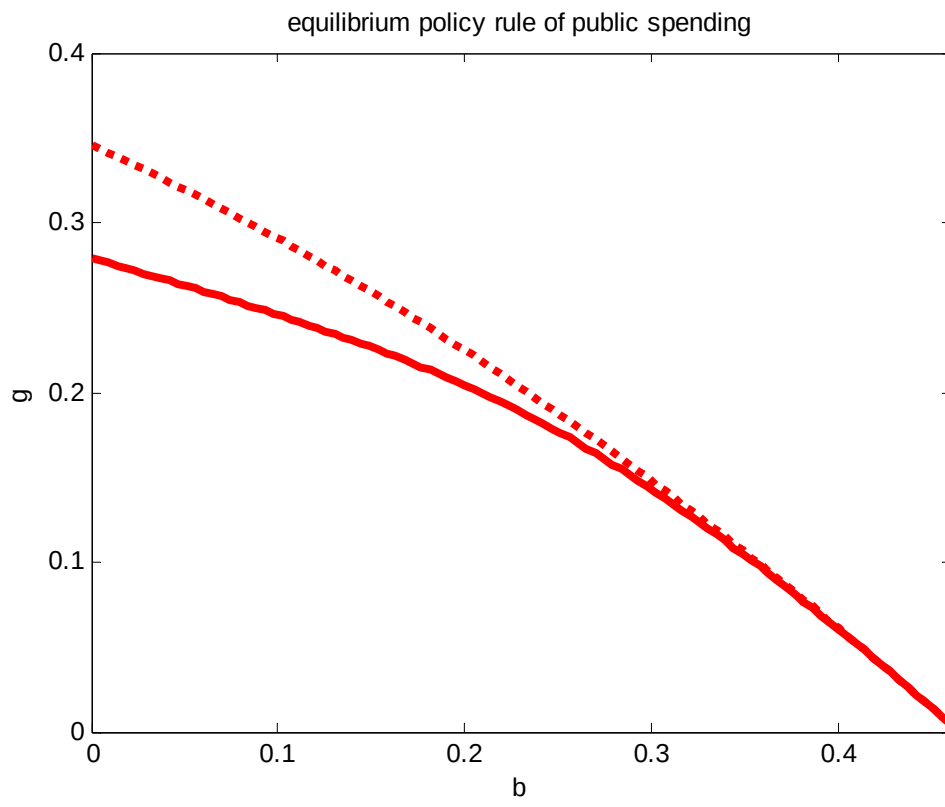


Figure 3-3

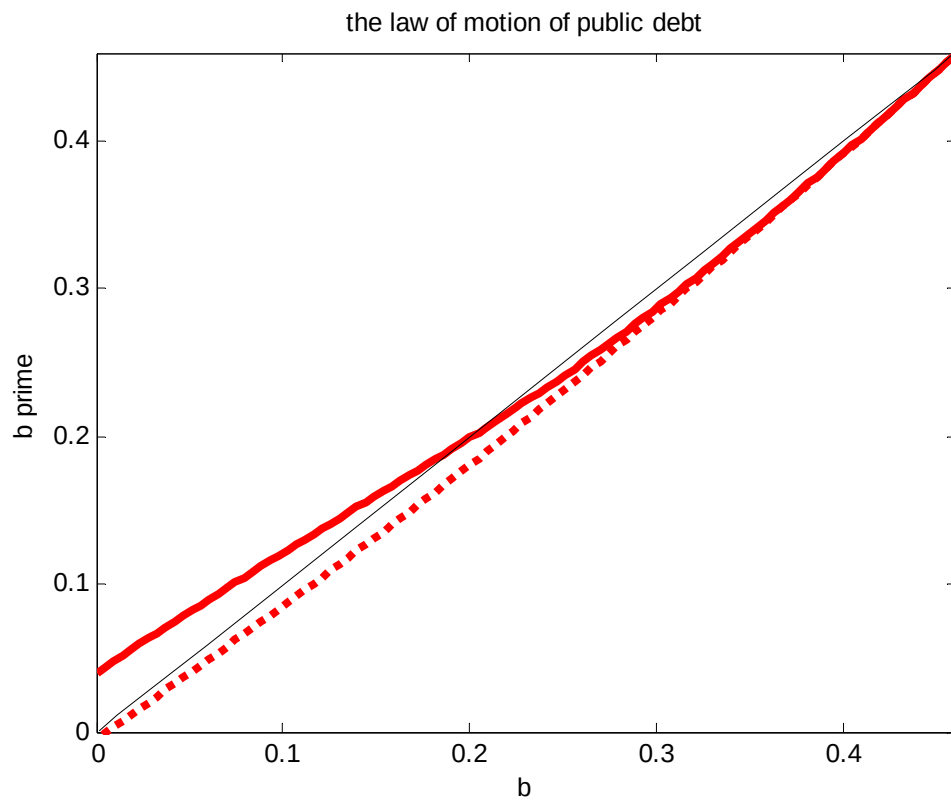


Figure 4: impulse response function for a unanticipated war
(solid line and dotted line stand for $\xi=0.90$ and $\xi=1$, respectively)

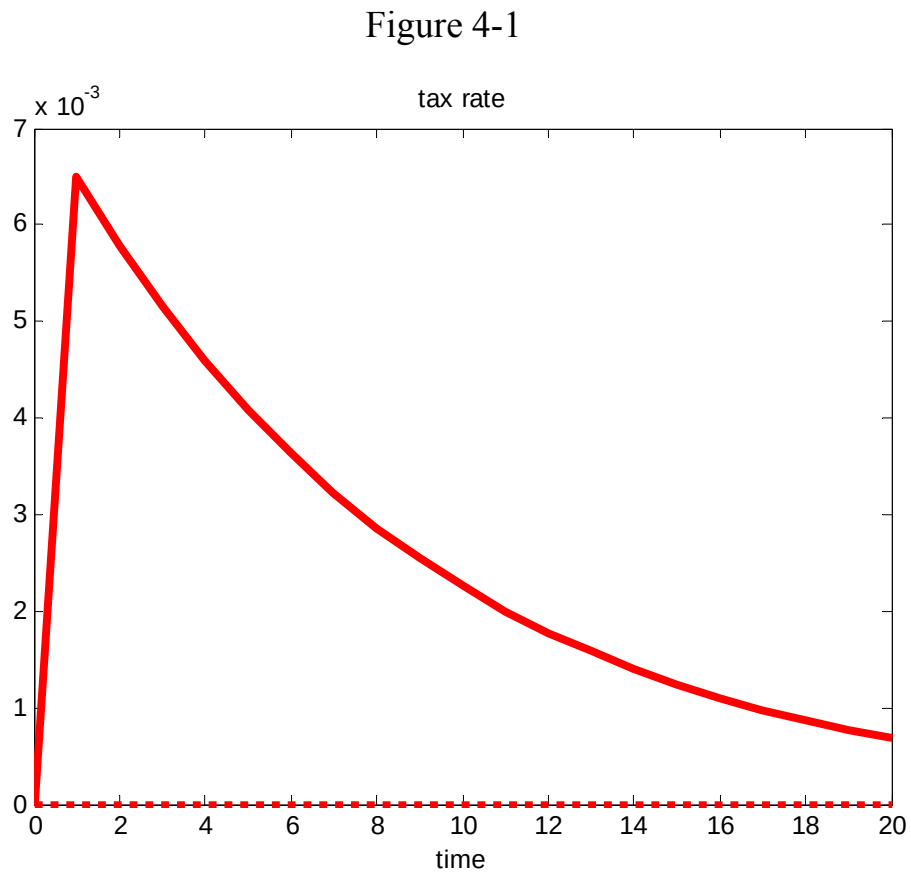


Figure 4-2

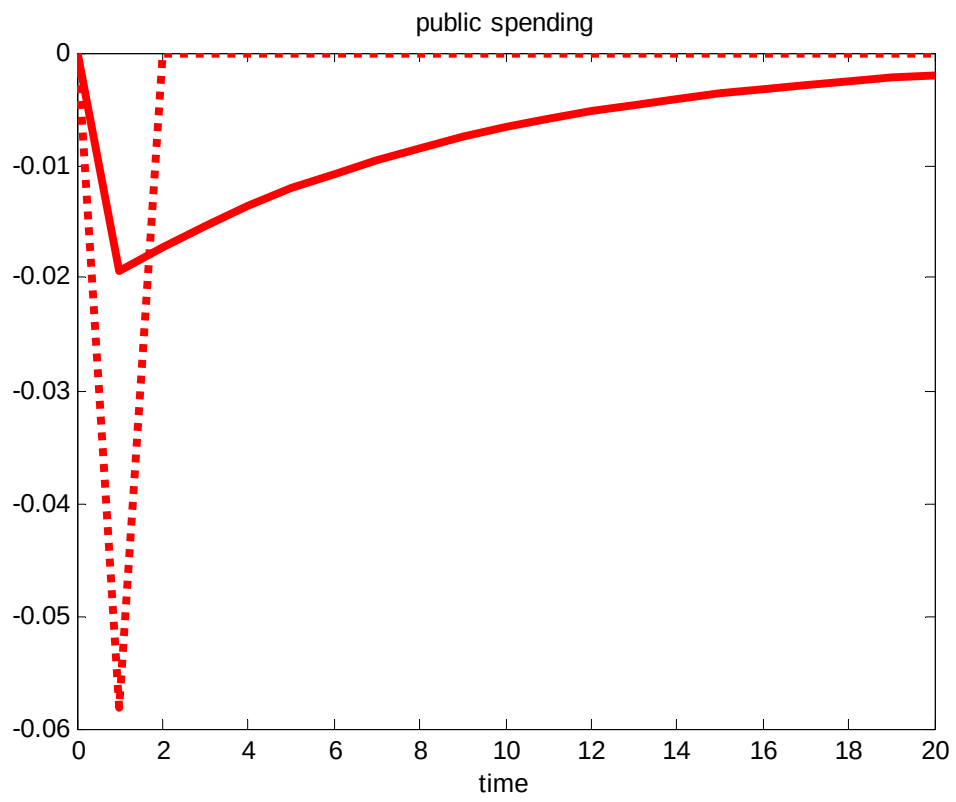


Figure 4-3

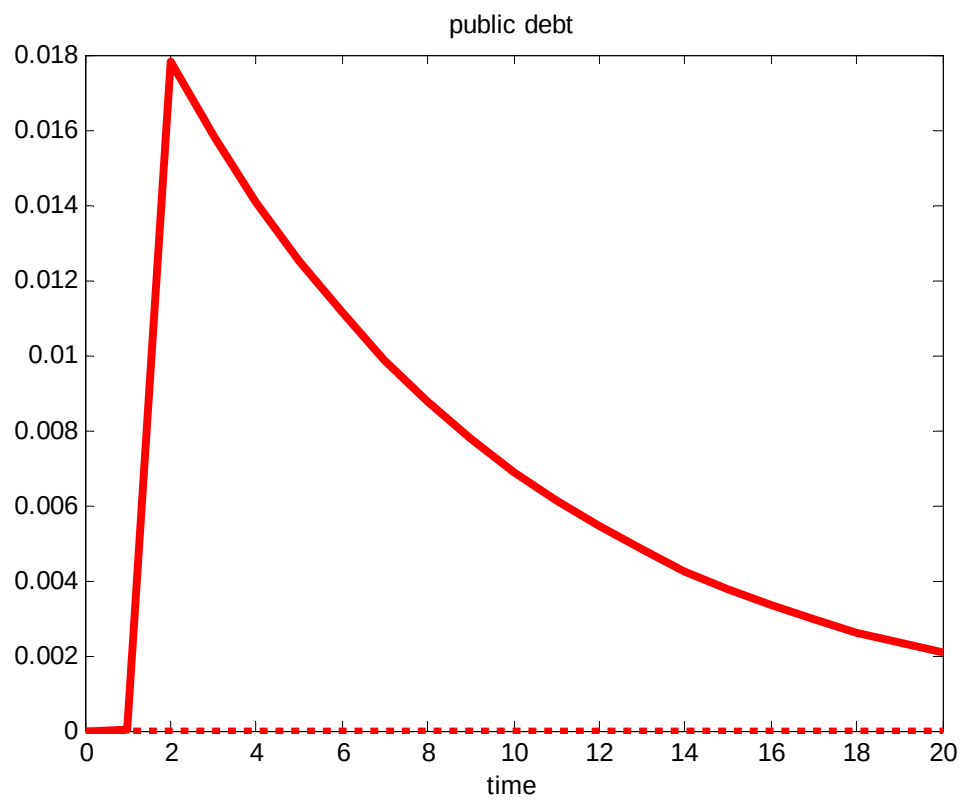


Figure 5: equilibrium policy rules with war
(dotted line stands for peace and solid line stands for war)

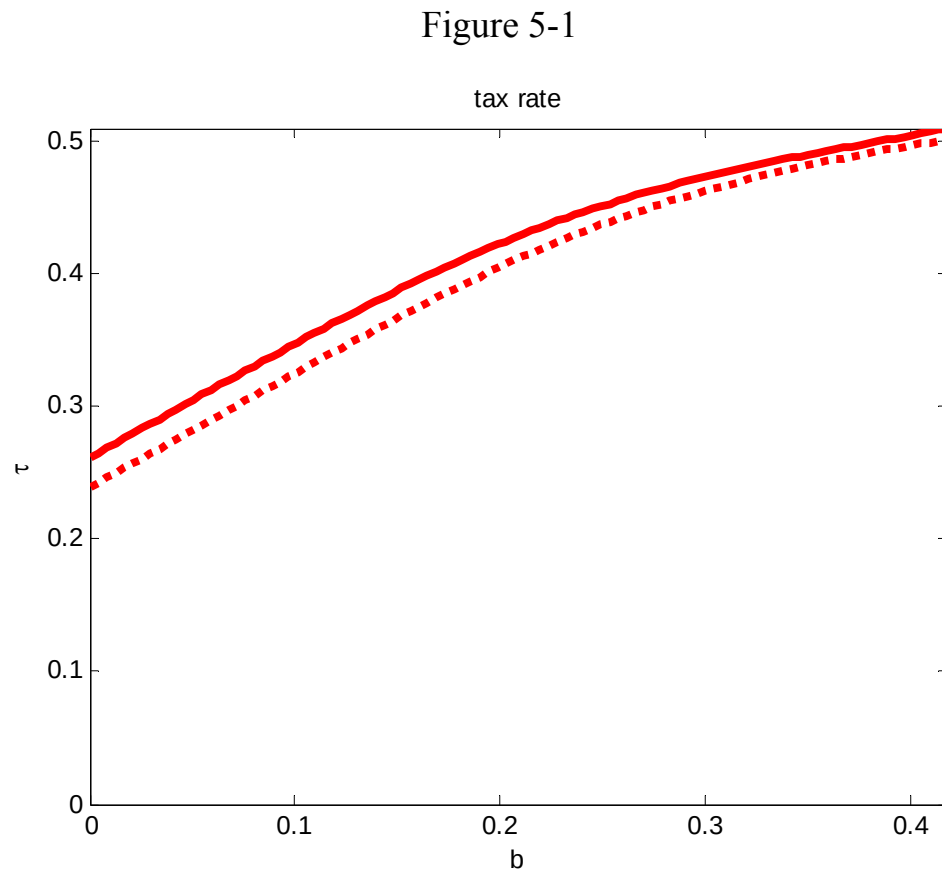


Figure 5-2

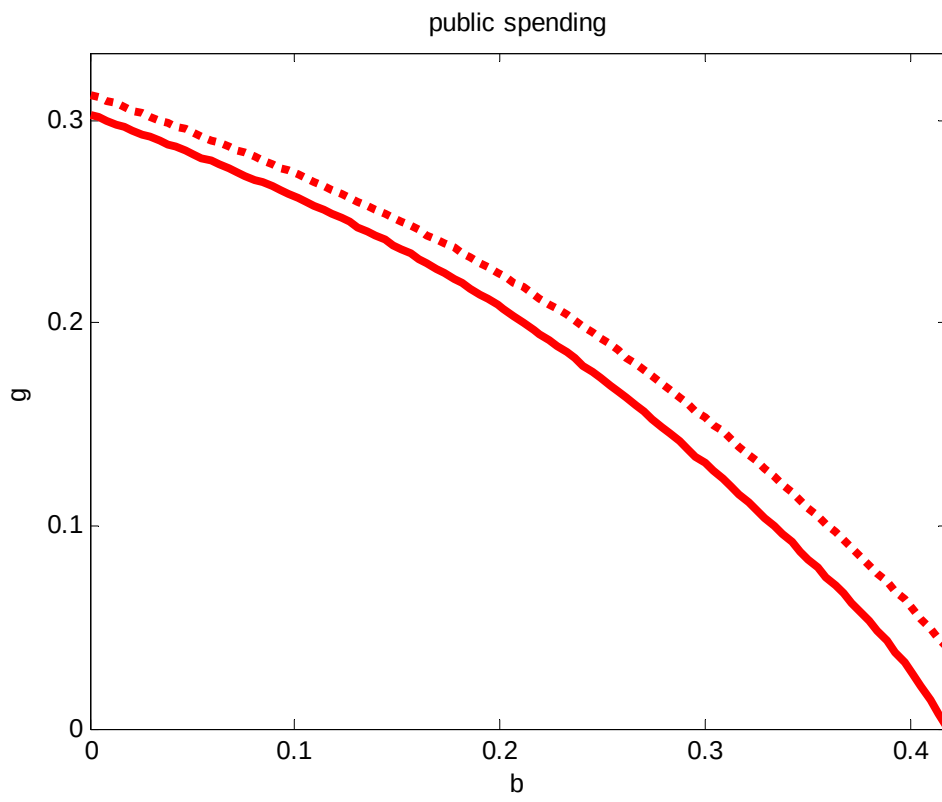
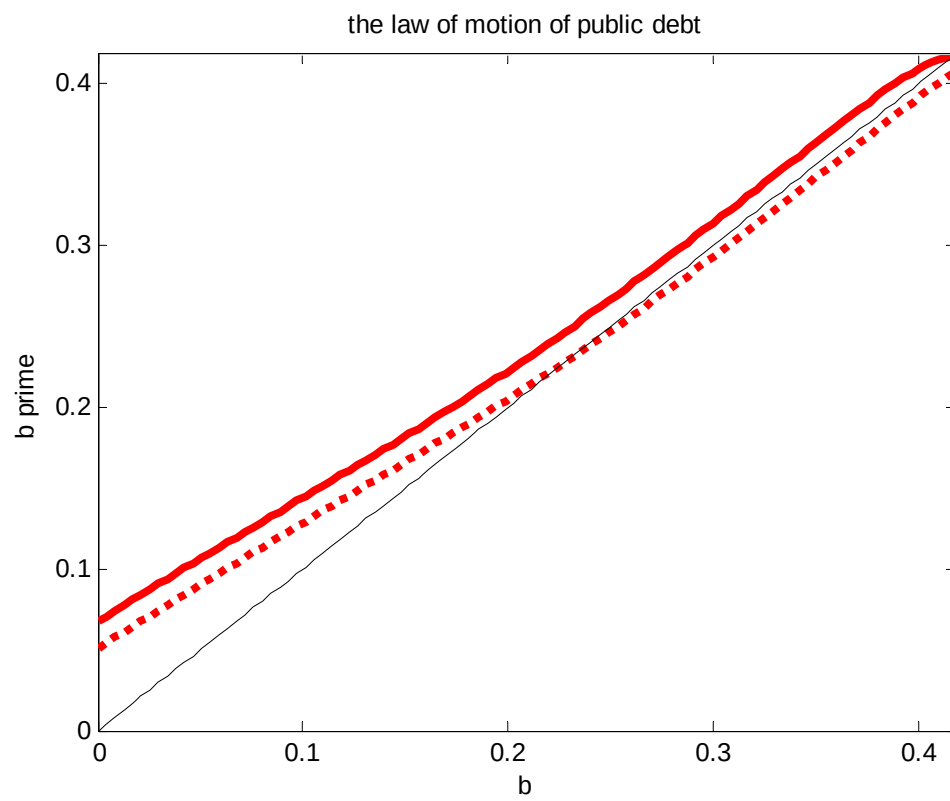
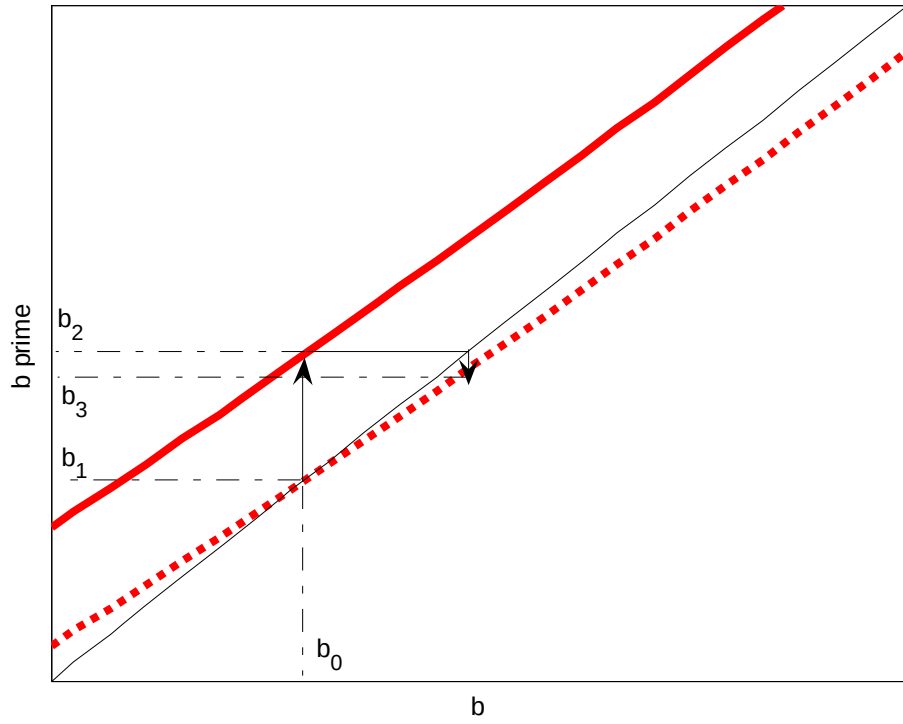


Figure 5-3

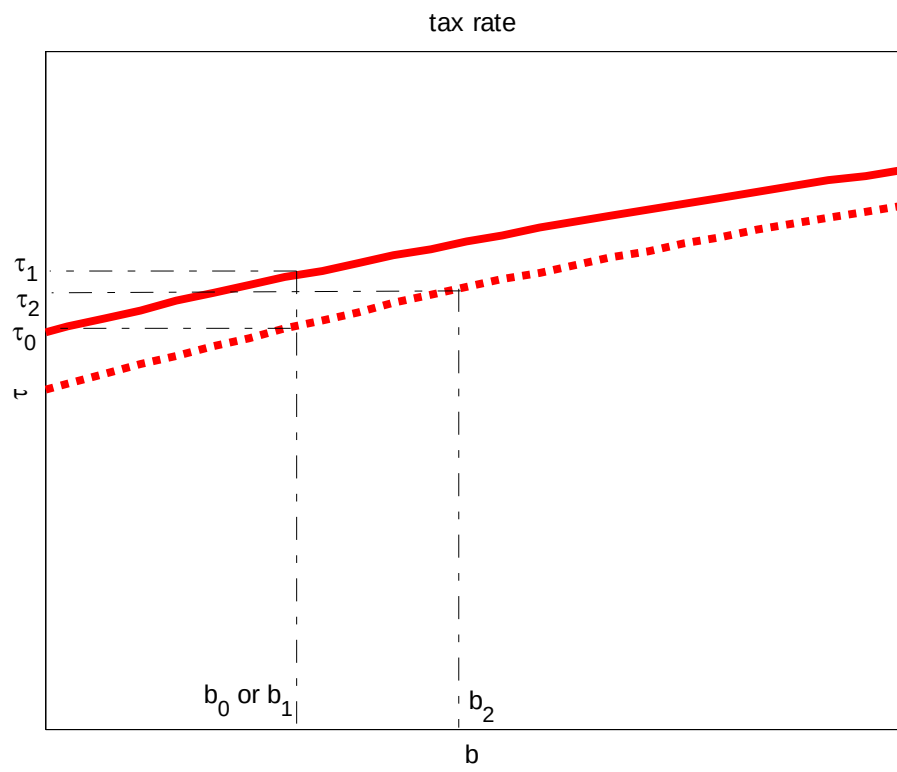


Panel 5-4

the law of motion of public debt



Panel 5-5



Panel 5-6

public spending

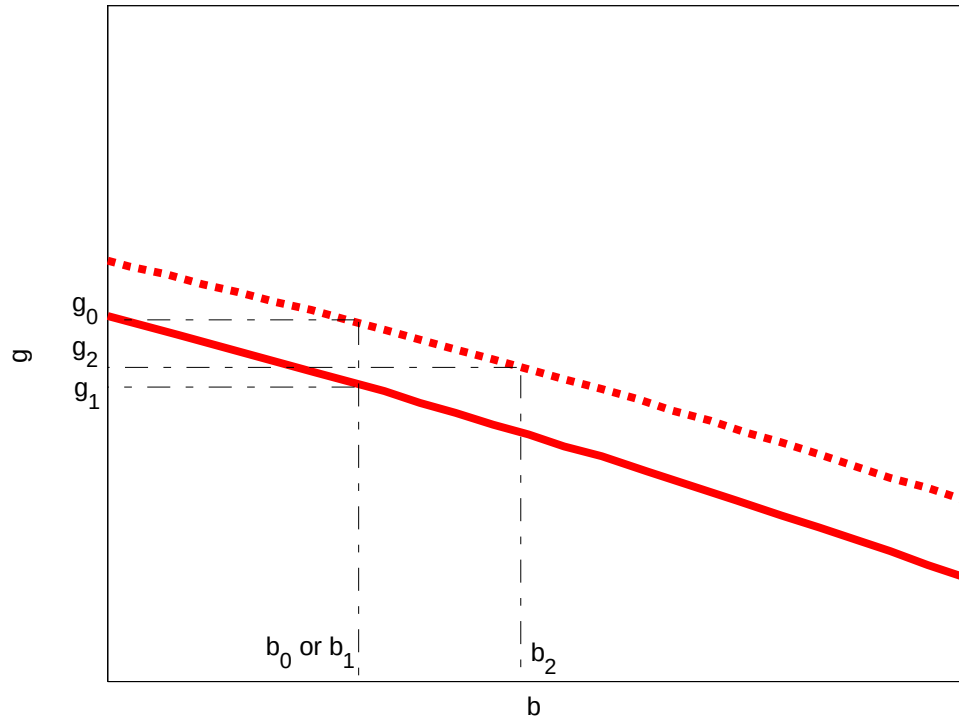


Figure 6: Ramsey policy rules with war after the initial period
(dotted line stands for peace and solid line stands for war)

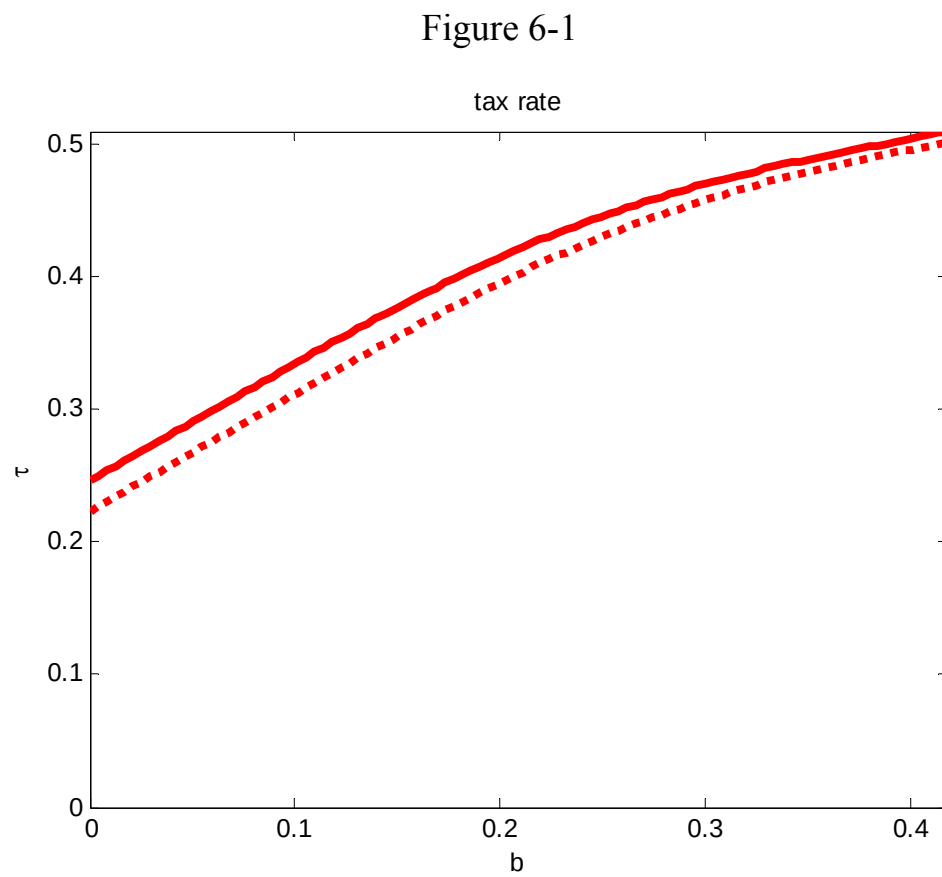


Figure 6-2

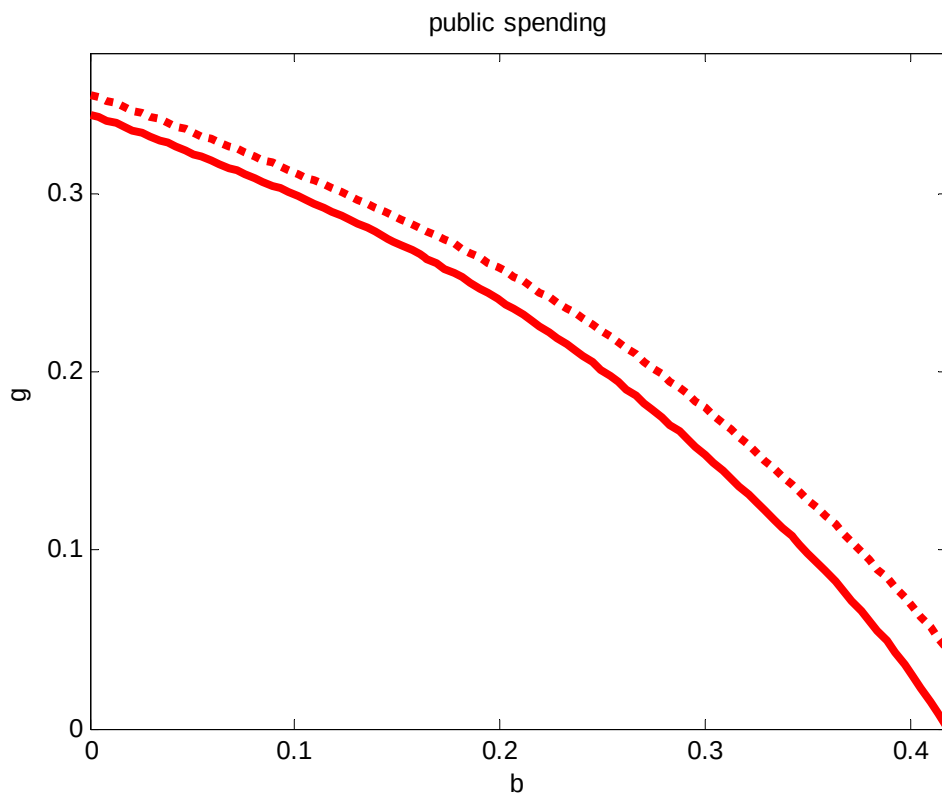


Figure 6-3

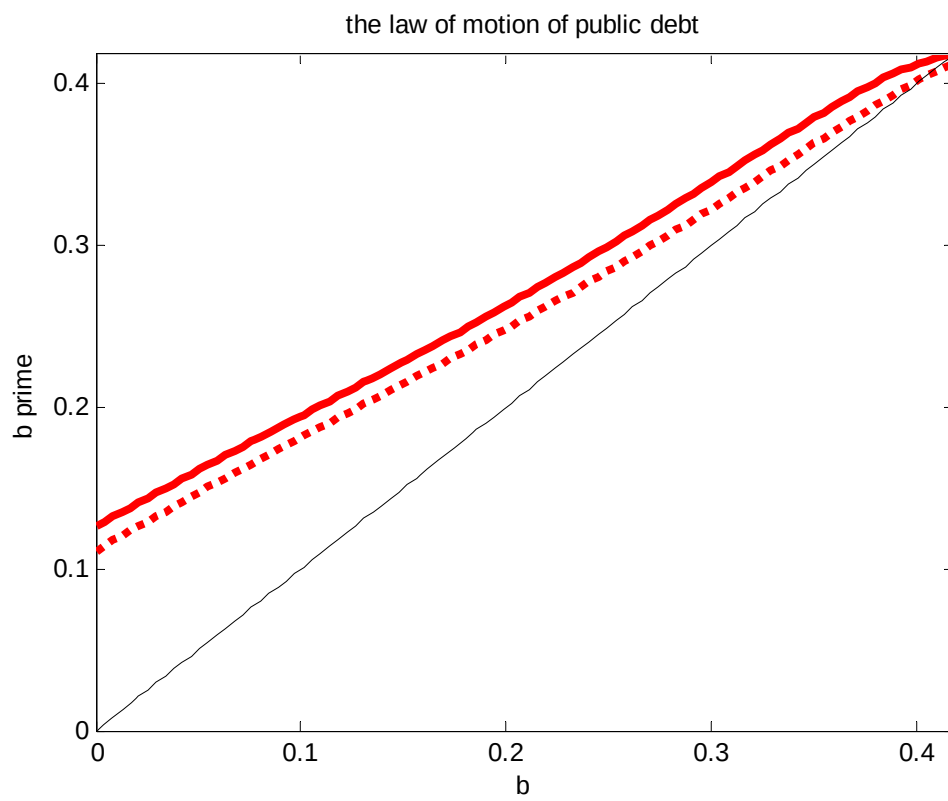
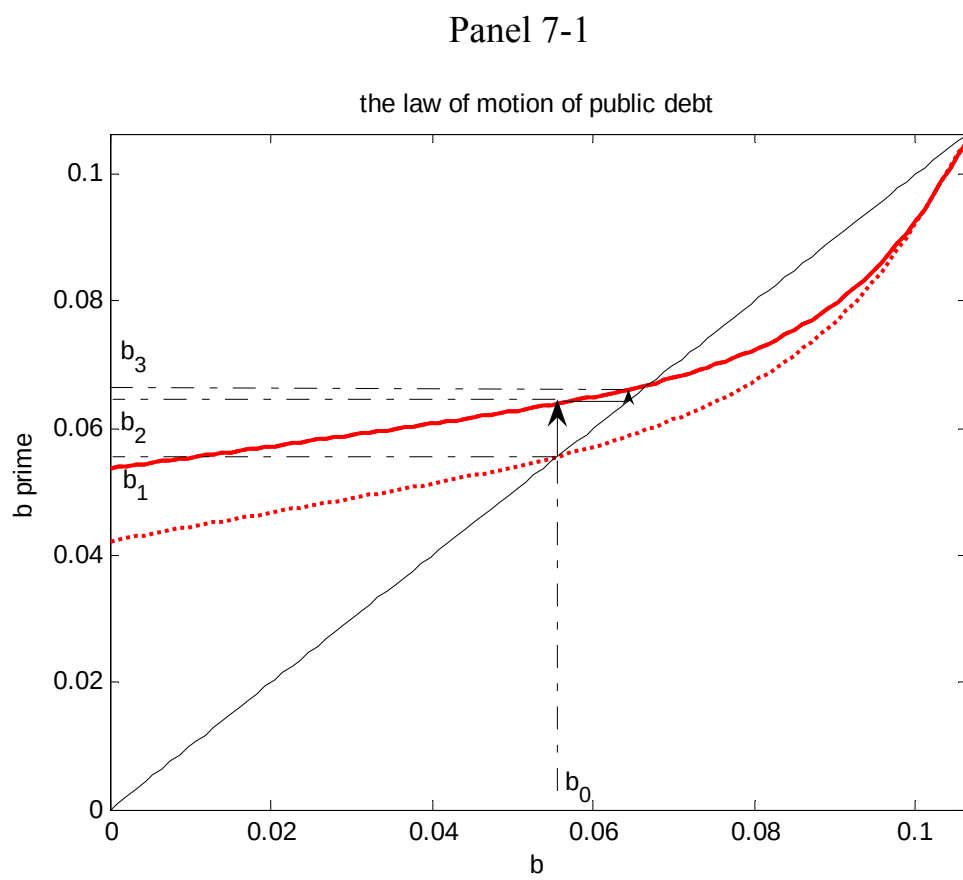
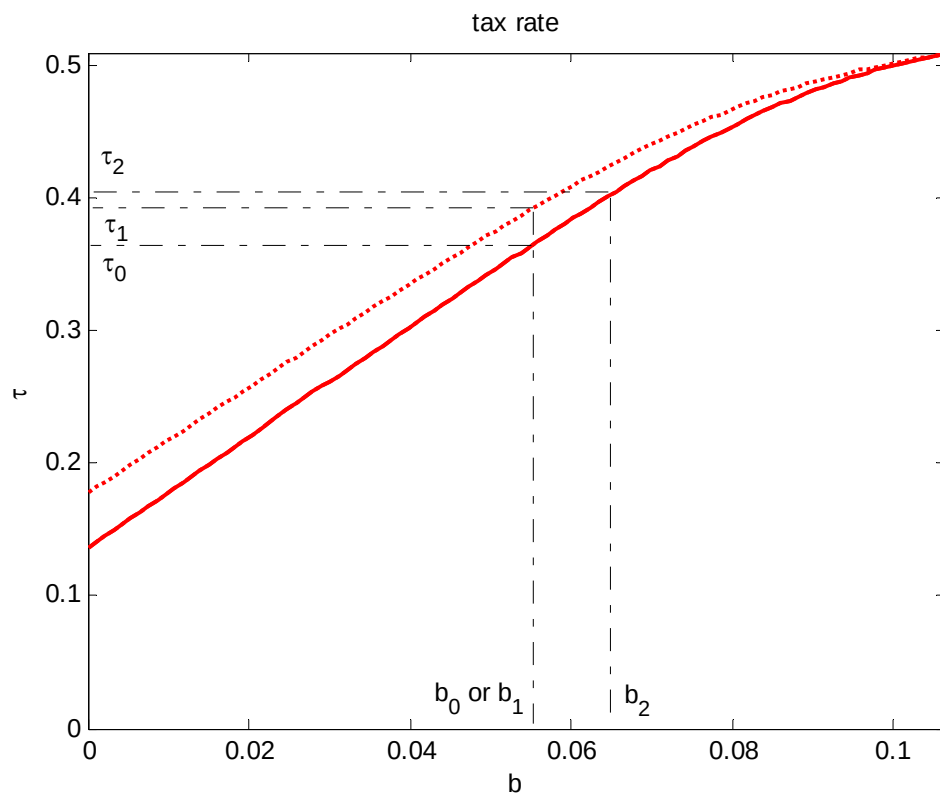


Figure 7: Political Regime Shifts

(dotted lines for the left-wing regime, and solid lines for the right-wing)



Panel 7-2



Panel 7-3

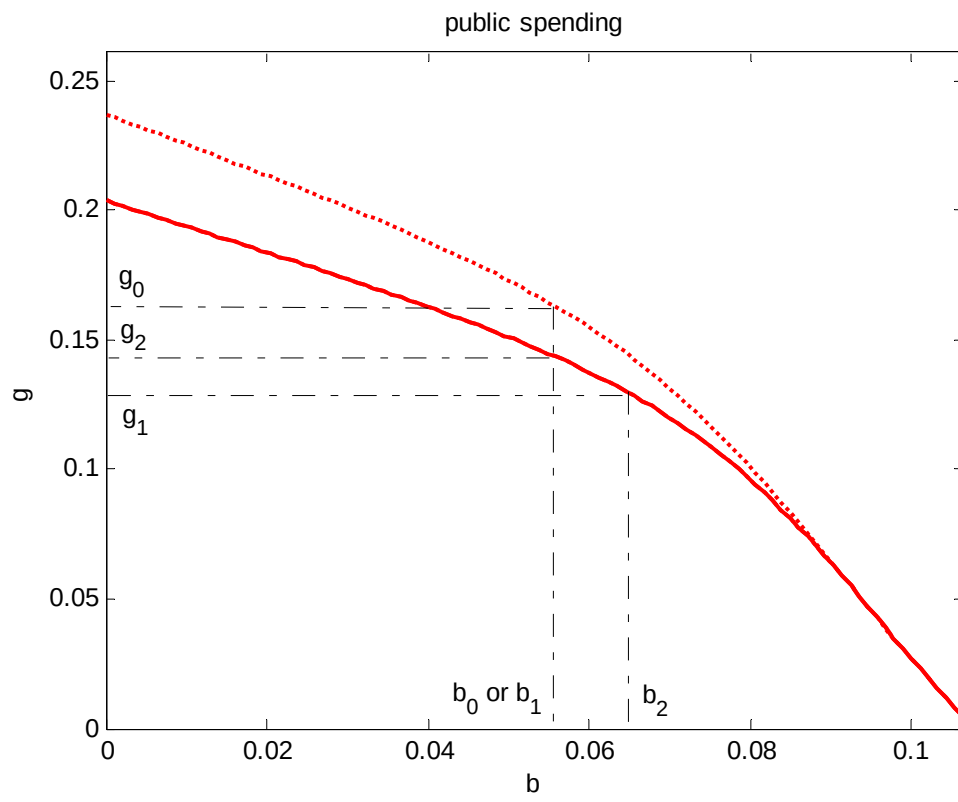
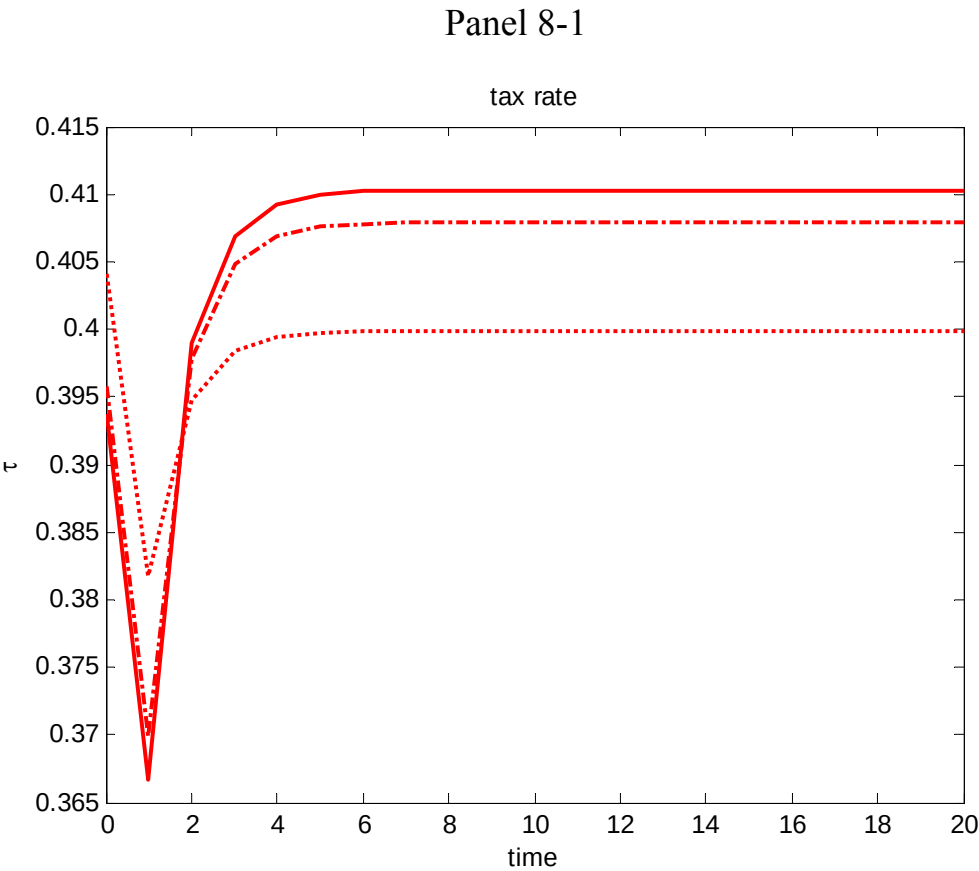
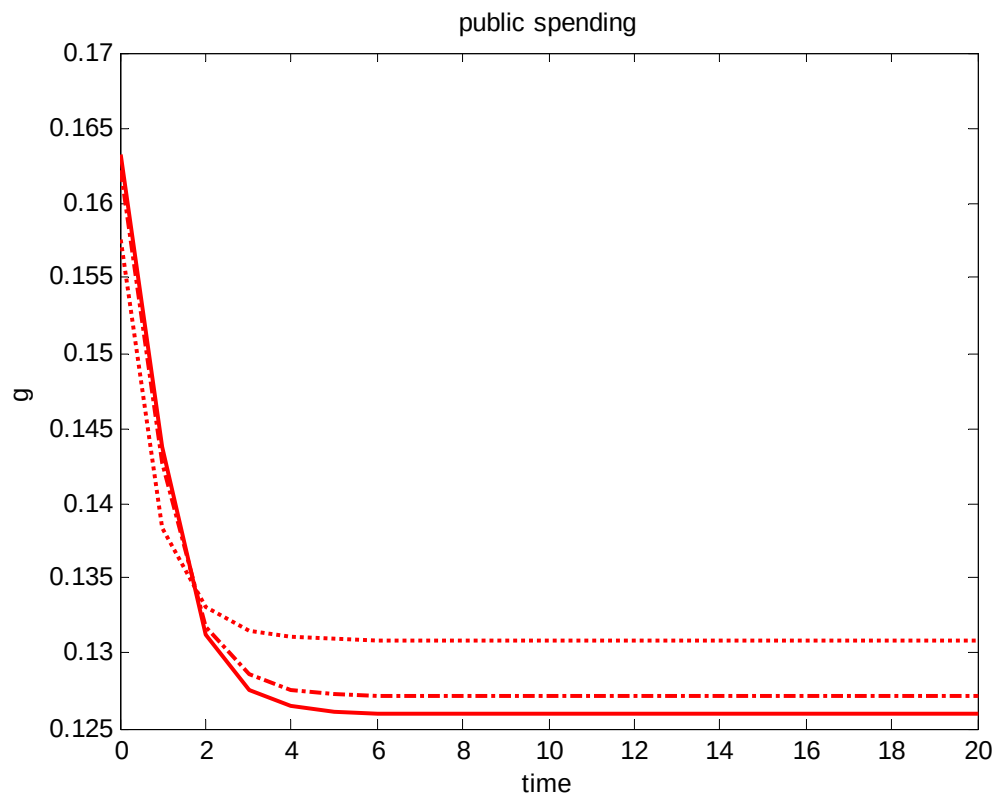


Figure 8: Time-Series for Political Regime Switches

(solid lines for $p=1$, dashed lines for $p=0.9$ and dotted lines for $p=0.5$)



Panel 8-2



Panel 8-3

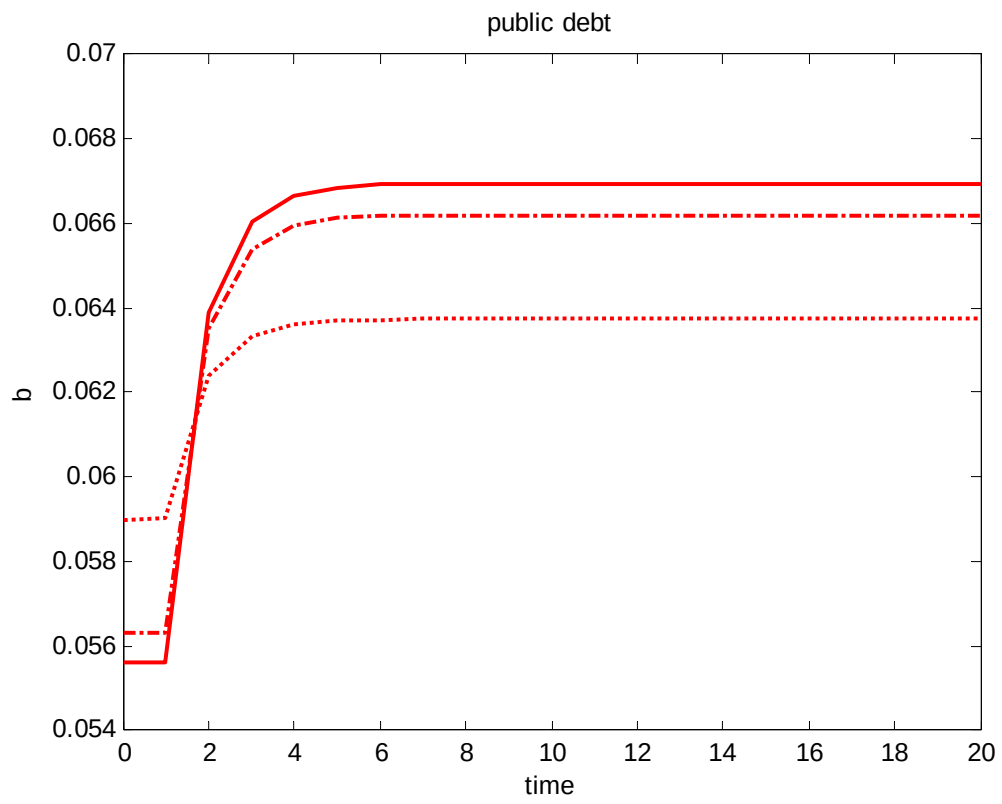
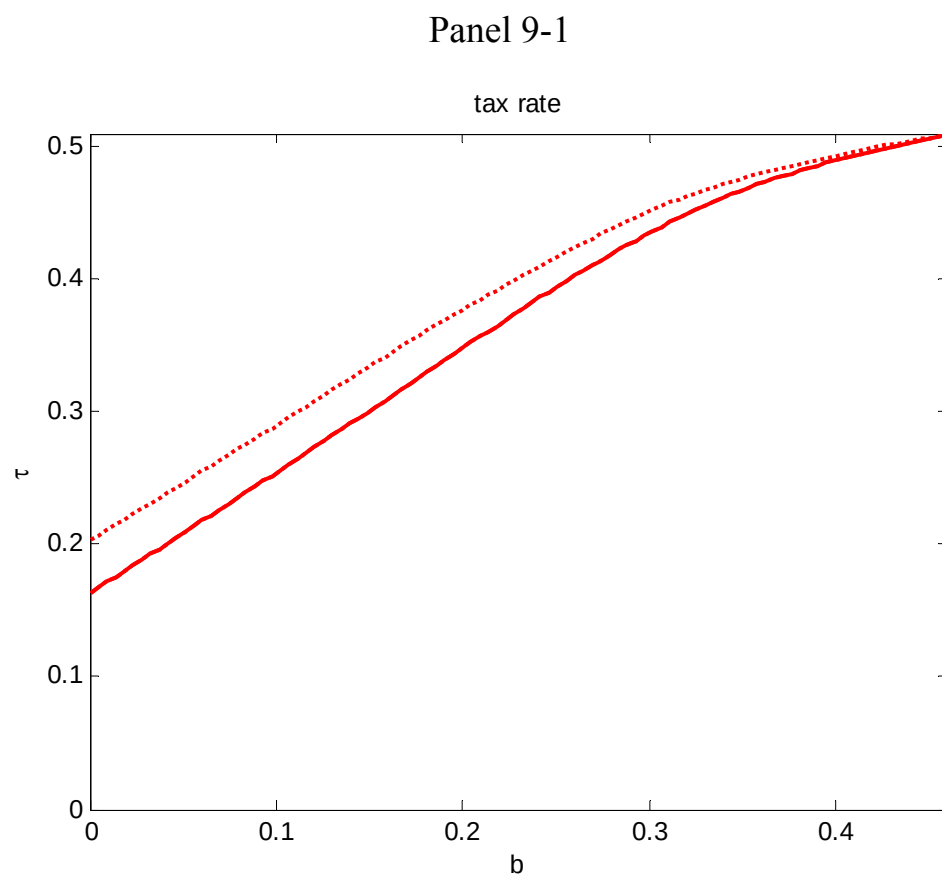
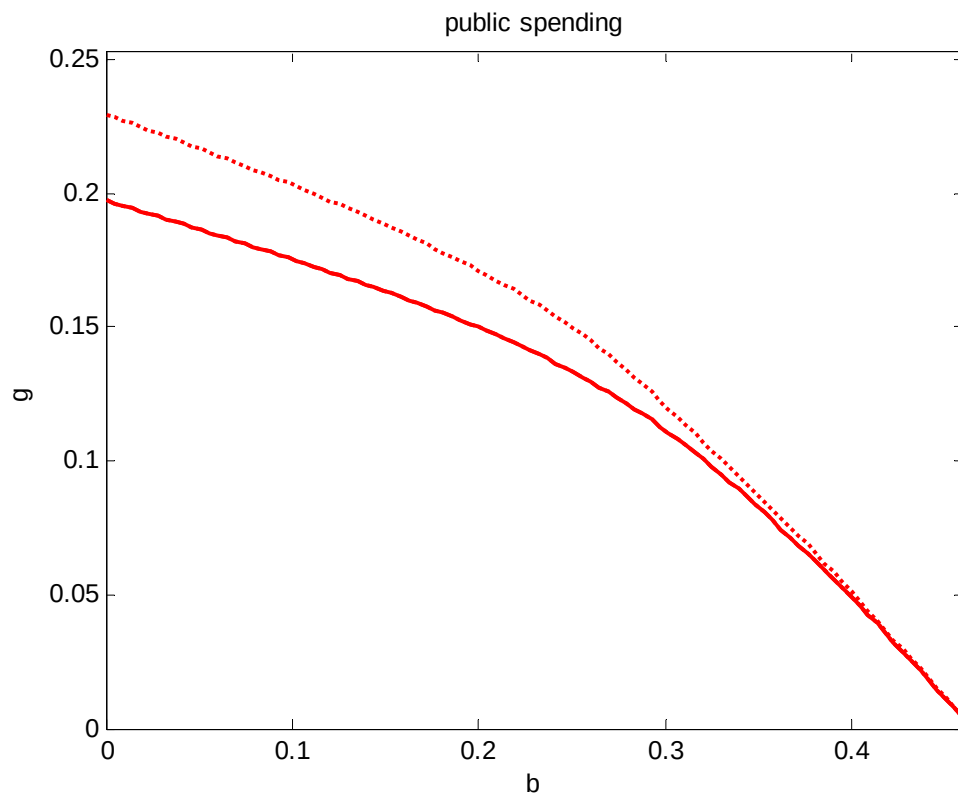


Figure 9: Political Uncertainty with Altruism

(dotted lines for the left-wing regime and solid lines for the right-wing)



Panel 9-2



Panel 9-3

