

Real Estate Prices in Production Economies

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April 24, 2007

Abstract

The aim of this study is to develop a general equilibrium framework linking real estate prices to the real economy. The model is evaluated in terms of its ability to explain: (i) the high volatility of residential real estate prices, (ii) the fact that commercial real estate prices are more volatile than residential real estate prices, (iii) the low volatility of residential rents. This study finds that a production economy model, allowing prices and quantities to be endogenously determined, is able to explain the puzzling volatility of real estate prices while capturing the main empirical regularities of the business cycle.

- House Prices, Production Economies, Asset Pricing, General Equilibrium, Real Estate
- JEL: G1, E3, R31

*Visiting postdoctoral fellow, Wharton School of Finance. Financial support by the National Center of Competence in Research "Financial Valuation and Risk Management" (NCCR Finrisk) is gratefully acknowledged. NCCR Finrisk is a research instrument of the Swiss National Foundation. This project has benefited from the comments and suggestions of Pascal St-Amour, Urban Jermann, Jean-Pierre Danthine, Jean Imbs and the participants of the reading group of the Wharton School. Any remaining errors are my own responsibility.

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1 Introduction

1.1 Motivation and Overview

The high volatility of commercial real estate prices, while documented in several studies [see Kwong and Leung (2000); Leung (2004)], constitutes a challenging empirical fact to explain. Commercial real estate prices are not only more volatile than output, they are also considerably more volatile than residential real estate prices. Yet, while the case of equity and bond prices has been extensively treated, few studies have attempted to confront the implications of standard macroeconomic models to this striking empirical regularity.

Following Kan, Kwong and Leung (2004), this study proposes to assess whether an extended version of the neoclassical model [see King and Rebelo (2000)] is able to explain the puzzling volatility of house prices. Compared to the standard asset pricing literature, in this paper, we propose to develop a methodology allowing prices and quantities to be endogenously determined within a general equilibrium model. The implications of the model that is derived are then confronted to the empirical facts. The model is evaluated in terms of its ability to explain: (i) the high volatility of residential real estate prices, (ii) the low volatility of residential rents, and (iii) the fact that commercial real estate prices are more volatile than residential real estate prices.

While market failures are often thought to play a key role in accounting for the high volatility of real estate prices, surprisingly, very few studies have attempted to explain this important empirical regularity using equilibrium frameworks. This study proposes to fill this gap. In terms of methodology, the main contribution of this paper is to show that a simple social planner problem can be reinterpreted as a competitive equilibrium involving a representative consumer, a real estate sector, including an investment and a final good producer, and a representative firm. The equivalence between the competitive equilibrium and the planner's problem can be exploited to derive two equivalent expressions for real estate prices.

Using the competitive equilibrium, we firstly show that asset pricing formulae connecting residential real estate prices to residential rents, and commercial real estate prices to the marginal productivity of commercial structures can be derived. The problem of the social planner can then be used to derive a reduced form for real estate prices. Exploiting the tractability of

these reduced form expressions, we then propose to identify and decompose two mechanisms that could potentially explain the determinants of the high volatility of commercial and residential real estate prices.

1.2 Methodology and Results

In section 2, the main stylized facts characterizing the link between real estate prices and the business cycle are presented. In terms of asset pricing facts, the main puzzles to be explained are the following: (i) residential real estate prices are more volatile than output, (ii) commercial real estate prices are more volatile than residential real estate prices and more than twice as volatile than output, (iii) residential rents are less volatile than output. The challenge consists of being able to explain these facts in a general equilibrium business cycle also able to capture the main empirical regularities of the business cycle.

The model is presented in section 3. The competitive equilibrium, which is composed of a representative consumer, a real estate investment good producer, a real estate final good producer, and a representative firm, is firstly characterized. Asset pricing formulae linking residential real estate prices to residential rents, and commercial real estate prices to the marginal productivity of commercial structures are then derived. In section 4, the equivalence between the competitive equilibrium and the centralized equilibrium is exploited to derive a second interpretation of commercial and residential real estate prices.

In section 5, the theoretical implications of the model are confronted to the stylized facts presented in section 2, and the empirical relevance of the framework that is developed is assessed. As for residential real estate prices, the key ingredient allowing to generate the observed volatility is to introduce a real estate investment good sector. In this economy, residential real estate prices can equivalently be represented as the ratio of the marginal productivity of investing in the representative firm and in the real estate investment firm. In this setting, a higher willingness to invest in residential real estate translates into an increase in the volatility of residential real estate prices. The dynamics of rents being determined by the consumption to housing ratio, the combination of time to build in housing and high risk aversion allows to explain the low volatility of residential rents observed in the data.

As far as the volatility of commercial real estate prices is concerned, following Chatterjee and Cooper (1993), overhead costs associated to the main-

tenance of commercial structures that affect the productive capacity of the firm are introduced. Compared to the literature, allowing managers to decide how to allocate their workforce between productive and maintenance activities, leads overhead costs to be time-varying. The amplification mechanism is obtained through capital gains that arise as a result of procyclical variations in the number of workers allocated to maintenance activities. Section 6 concludes.

2 Some Stylized Facts

While it is unusual to observe declines in nominal house prices, declines in real house prices are relatively more common. In addition, real house price changes appear to be persistent; that is, positive price changes tend to be followed by more positive price changes, and vice-versa for negative price changes. This observation has stimulated considerable interest among researchers, since asset prices are usually expected to adjust immediately to reflect new information about fundamental value, and not gradually as seems to be the case with house prices. As explained by Krainer (2002), persistence in house prices could indicate that housing markets are inefficient either in the sense that the market takes time to clear, or that prices and expectations about future price changes are set in a backward-looking manner.

Still according to Krainer (2002), an alternative explanation for the persistence in house prices is that prices depend directly on economic variables that are themselves persistent. A natural explanation of house price fluctuations that is compatible with this last statement is that house prices are determined by variables that themselves depend directly on technology shocks. As shown by many econometric studies, the Solow residuals exhibit a high degree of persistence. The high persistence of house prices could thus be due to the fact that autocorrelated technology shocks are the main driving force of the cycle and therefore that all macroeconomic variables, including house prices, inherit the statistical properties of the Solow residuals.

2.1 Standard Deviations and Correlations

As proposed by Prescott (1986), one way of evaluating the predictions of a theoretical framework is to compare moments that summarize the actual experience of an economy with similar moments from the model. In our context,

the key moments to replicate are presented in Table 1, where $\hat{y}_t, \hat{c}_t, \hat{i}_t^b, \hat{p}_t^h, \hat{z}_t$ and $\hat{p}_t^q$ denote respectively output, consumption, business investment, residential real estate prices, rents for the use of residential real estate, and commercial real estate prices.

To generate these empirical moments, the variables are, firstly, expressed in logarithm. The cyclical component is then computed by subtracting the HP filtered series from the actual series. We denote variables that have been subject to this transformation by a hat. All series are expressed in real terms. Rents, commercial real estate prices and residential real estate prices are deflated using the Consumer Price Index. The volatility of the cyclical component of the variables under study are reported in the first column. The relative standard deviation and the correlation with respect to output of each variable are reported in the second and the third columns.

Table 1: Empirical Moments USA (1960-2004)

	σ_x	σ_x/σ_y	$CORR(\hat{x}_t, \hat{y}_t)$
$\hat{y}_t$	1.54	1	1
$\hat{c}_t$	1.23	0.80	0.86
$\hat{i}_t^b$	5.28	3.43	0.87
$\hat{p}_t^h$	2.41	1.56	0.60
$\hat{z}_t$	1.07	0.69	0.44
$\hat{p}_t^q$	3.68	2.39	0.26

Source: BIS for commercial real estate prices (series collected from national sources); Davis, Lehnert and Martin (2005) for the series on residential real estate prices and rents; Federal Reserve Bank of St-Louis for output, consumption and investment. All variables have been expressed in real terms and in logs and have been detrended using a HP filter.

The data on residential rents and on house prices are taken from the study of Davis, Lehnert and Martin (2005). The sample that is made available by the authors includes quarterly data on residential rents and house prices from 1960 to 2004. As for data on commercial real estate prices, series collected from national sources by the BIS are used. The series for commercial prices is only available on a quarterly basis from 1977 to 2002.

As pointed out by the study of Kwong and Leung (2000), commercial real estate prices tend to be significantly more volatile than residential real estate prices¹. As shown in Table 1, this is also the case for the United States where

¹In their study, they find that commercial real estate prices are significantly more

residential real estate prices, while slightly more volatile than output, are less volatile than commercial real estate prices. The relative standard deviation of residential and commercial real estate prices with respect to output is 1.56 and 2.39.

In contrast, rents for the use of residential real estate are considerably less volatile than prices, and less volatile than output. The relative standard deviation of rents with respect to output is 0.69. As for real estate pricing facts, to be successful, the candidate model will thus have to be able to explain the following facts: (i) residential real estate prices are more volatile than output; (ii) commercial real estate prices are more than twice as volatile as output and more volatile than residential real estate prices; and (iii) residential rents are less volatile than output. As regards the volatility of the main macroeconomic variables, the key empirical moments to explain are the low volatility of consumption, which is less volatile than output, and the high volatility of investment, which is more than twice as volatile as output.

3 The Model

The economy is composed of a representative consumer, a numeraire sector and a real estate sector. The real estate sector is composed of one real estate investment good producer and of one final good producer which transforms residential capital into new houses. The firms in the numeraire sector use capital, labor and commercial structures to produce an output good, y_t^n . Technology shocks in the numeraire or representative sector are considered as the only source of business cycle fluctuations. The subscript n is used to denote variables describing the numeraire or non-residential sector. In the real estate sector, the real estate investment good is denoted by y_t^i .

The quantity of the investment good that is produced corresponds to the amount of residential investment in the economy. Finally, we denote the output of the final good producer, the real estate final good, by y_t^h . The final good is the total stock of housing produced in the economy. This stock of housing is produced by the final good firm which converts residential capital into residential real estate. The subscripts i and h are used to denote respectively variables describing the investment real estate good producer and the real estate final good producer.

volatile than residential real estate prices in several cities including Hong Kong, Tokyo, Singapore, Bangkok, Jakarta and Kuala Lumpur.

3.1 The Consumer

The consumer maximizes lifetime utility:

$$Max E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \frac{1}{1-\sigma} [c_t^{\kappa} h_t^{1-\kappa}]^{1-\sigma} \right\} \quad (1)$$

where c_t denotes the standard numeraire consumption good, h_t , is the final real estate good. The coefficient κ , where $0 \leq \kappa \leq 1$, denotes the weight in the utility function that households assign to the numeraire consumption good, c_t . The curvature parameter is denoted by σ .

3.1.1 Budget Constraint

Each period, the problem of the consumer consists of deciding how much of the numeraire consumption good, c_t , and of the the final real estate good, h_t , to consume. Following Christiano and Eichenbaum (1992), government spending, g_t , are assumed to take the form of an exogenous resource cost. The investment decision takes the form of an investment in business capital, i_t^b , and in residential capital, i_t^h . In this economy, to invest in business capital, households accumulate a business capital good, b_t , that can be allocated indifferently to the production of the numeraire good, y_t^n , or to the production of the investment good, y_t^i .

Investment in residential capital, i_t^h , takes the form of an investment in the final good firm. Finally, consumers decide how much of the numeraire consumption good, c_t , to purchase from the firm in the numeraire sector, and how much of the final real estate good, h_t , to purchase from the final real estate good supplier. The relative price of the real estate final consumption good is given by z_t and the relative price of the residential investment good is p_t^h .

Consumer revenues consist, firstly, of a labor income, $W_t N_t$, and of a capital income $r_t^n k_t^n + \pi_t^n$ paid by the firm in the numeraire or representative sector. The first component of the capital income, $r_t^n k_t^n$, is due to the share of business investment that is allocated to the firm in the numeraire sector, r_t^n , being the rate of return on numeraire capital. The households being owners of the firm, they receive a dividend income denoted by π_t^n .

The second component, $r_t^i k_t^i + \pi_t^i$, consists of the revenue that is paid by the real estate investment good producer. The investment in business capital that is allocated to the real estate investment good producer is $r_t^i k_t^i$.

The return on the share of business investment allocated to the real estate investment good producer is denoted by r_t^i . The dividend income paid by the real estate investment good producer to households is denoted by π_t^i . Finally, the last component of total income, $z_t k_t^h$, is the revenue from the investment in the final real estate good producer.

$$c_t + g_t + z_t h_t + i_t^b + p_t^h i_t^h = W_t N_t + r_t^n k_t^n + \pi_t^n + \pi_t^i + r_t^i k_t^i + z_t k_t^h \quad (2)$$

The investment in business capital allows the household to accumulate a capital good, b_t , that can be used indifferently in the numeraire sector or by the real estate investment good producer. As a result, we have that:

$$b_t = k_t^n + k_t^i \quad (3)$$

The accumulation of business and residential capital is governed by the following laws of motion:

$$\gamma k_{t+1}^h = i_t^h + (1 - \delta^h) k_t^h \quad (4)$$

$$\gamma b_{t+1} = i_t^b + (1 - \delta^b) b_t \quad (5)$$

where δ^h and δ^b denote respectively the depreciation rates of residential and business capital.

3.1.2 Optimality

The asset pricing formula describing the evolution of residential real estate prices can be derived by taking the first-order condition with respect to k_{t+1}^h :

$$p_t^h = \tilde{\beta}^* E_t \frac{\lambda_{t+1}}{\lambda_t} [z_{t+1} + (1 - \delta^h) p_{t+1}^h] \quad (6)$$

where² $\tilde{\beta}^* = \beta \gamma^{-\sigma}$. The payoff associated with investing in real estate, z_t , is given by the ratio of the relative marginal utility of the real estate good and of the standard consumption good:

$$z_t = \frac{(1 - \kappa)}{\kappa} \frac{c_t}{h_t} \quad (7)$$

²The fact that quantities of real estate vary, and are growing at a constant rate in the steady state, implies this modified discount factor.

This equation states that the consumer continues to invest in residential real estate until the marginal loss of investing equals the marginal gain, which represents the increase in utility obtained by the consumer from the extra payoff at $t + 1$.

The Euler condition describing the optimal accumulation of business capital, b_t , is given by:

$$\lambda_t = \tilde{\beta}^* E_t \lambda_{t+1} [(1 - \delta^b) + r_{t+1}^n] \quad (8)$$

The first-order condition with respect to k_t^i implies that:

$$r_t^i = r_t^n \quad (9)$$

This non-arbitrage condition states that the return to investing in business capital, b_t , in either the numeraire good producer, k_t^n , or the real estate investment good producer, k_t^i , must be equalized.

3.1.3 The Real Estate Investment Good Producer

The real estate investment good is produced using business capital, k_t^i , and land, l_t . We assume that in this economy there is an initial endowment of land that is owned by the real estate investment good producer. The problem of the investment good producer consists of choosing the optimal quantity of capital to rent from the consumer such as to maximize profits. Profits are given by:

$$\pi_t^i = p_t^h y_t^i - r_t^i k_t^i \quad (10)$$

where y_t^i is the quantity of output produced by the investment good producer. The production function is assumed to take the following form:

$$y_t^i = A_t^i k_t^i{}^\theta l_t^{1-\theta} \quad (11)$$

where A_t^i is a technology parameter and where $0 \leq \theta \leq 1$. Optimality implies that:

$$r_t^i = \frac{\theta p_t^h y_t^i}{k_t^i} \quad (12)$$

Total profits of the real estate investment good producer are thus given by:

$$\pi_t^i = (1 - \theta) p_t^h y_t^i \quad (13)$$

3.1.4 The Real Estate Final Good Producer

The final real estate good, y_t^h , is produced using residential capital, k_t^h . Profits are given by:

$$\pi_t^h = z_t y_t^h - z_t k_t^h \quad (14)$$

where z_t is the relative price of the real estate good and y_t^h denotes total output. As a result of perfect competition, firms in the real estate sector make zero profit, implying that:

$$y_t^h = k_t^h \quad (15)$$

In this economy, houses are thus produced by the final good firm. This production of residential real estate, by the final good firm, is done by converting residential capital, k_t^h , into residential real estate, y_t^h . Consumers can thus influence the stock of residential real estate by choosing how much to invest in residential capital.

3.1.5 The Numeraire Sector

The firm in the numeraire or representative sector produces an output good, y_t^n , using capital, k_t^n , commercial real estate, q_t , and labor, N_t . Moreover, there is an overhead cost associated to the investment in commercial structure, τ_t . The production function is Cobb-Douglas and is given by:

$$y_t^n = A_t^n k_t^\xi (N_t^T)^\alpha (\tau_t q_t)^{1-\alpha-\xi} \quad (16)$$

where A_t^n denotes the standard random technology shock, which is the main driving force of this economy.

We assume that the numeraire firm owns the initial endowment of commercial real estate. The variable or subscript q is used to characterize all the variables that are related to the modelling of commercial real estate: q_t denotes commercial structures, investment in commercial structure is denoted by i_t^q , and the price of commercial real estate is p_t^q . The two remaining production factors, capital and labor, are rented from the representative household.

Compared to business capital, in this economy, commercial real estate, q_t , is assumed to differ in that it requires to be maintained. Following Chatterjee and Cooper (1993), this maintenance cost takes the form of an overhead costs.

This assumption is meant to capture the fact that investing in commercial structures is associated with substantial costs that are linked to maintenance and that affect the productive capacity of the firm. Compared to the standard literature, in this economy, managers are confronted to another decision. They have to determine how to optimally allocate their total workforce, N_t^T , between productive activities, N_t^p and maintenance of commercial structures, N_t^m , so that:

$$N_t^T = N_t^m + N_t^p \quad (17)$$

If managers decide to allocate a larger fraction of their workforce to productive activities, N_t^p , they also have to integrate that this increase in production will come at the expense of having less workers in charge of maintaining commercial structures. When deciding how to allocate their workforce, managers also have to take into account the fact that having less workers in charge of maintenance leads to a faster rate of depreciation of commercial structures.

In the model, this trade-off is captured by the introduction of a time-varying overhead cost, τ_t . This overhead cost is assumed to be an increasing function of the ratio of the total workforce, N_t^T , to the number of workers allocated to maintenance activities, N_t^m , so that:

$$\tau_t = f\left(\frac{N_t^T}{N_t^m}\right) \quad (18)$$

In other words, when managers decide to allocate a larger fraction of their workforce to productive activities, this translates into a rise in³ $\frac{N_t^T}{N_t^m}$, which in turn leads to an increase in τ_t . This allows to increase production but leads commercial structures to depreciate more quickly. This latter effect is captured by making the depreciation rate depend on τ , so that:

$$\delta^q(\tau) \text{ where } \delta^{q'}(\tau) > 0 \text{ and } \delta^{q''}(\tau) > 0$$

The overhead cost, τ_t , being a choice variable, the model dynamic is invariant to the choice of the functional form $f()$. As in the case of varying rate of utilization in capital [see King and Rebelo (2000)], the impact on

³or equivalently, by (17), into a rise in $1 + \frac{N_t^p}{N_t^m}$.

the model dynamics of time variation in overhead costs works through the elasticity coefficient ε_τ , where:

$$\varepsilon_\tau = \frac{\delta''^q(\tau)}{\delta'^q(\tau)} \tau$$

The problem of the firm consists in maximizing the infinite discounted sum of expected profits:

$$Max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \pi_t^n$$

where profits are given by:

$$\pi_t^n = y_t^n - r_t^n k_t^n - W_t N_t^T - p_t^q i_t^q \quad (19)$$

and where p_t^q denotes the relative price of commercial real estate.

The law of commercial structures accumulation is given by:

$$\gamma q_{t+1} = [1 - \delta^q(\tau_t)] q_t + i_t^q + \tilde{\varepsilon}_t \quad (20)$$

where $\tilde{\varepsilon}_t$ is a random shock affecting the supply of commercial structure⁴.

The demand for capital and labor is given by the standard conditions relating real wages and the rental rate of capital to their marginal productivity:

$$W_t = \alpha \frac{y_t^n}{N_t^T} \quad (21)$$

$$r_t^n = \xi \frac{y_t^n}{k_t^n} \quad (22)$$

⁴We assume that in the steady state, $\tilde{\varepsilon}_t$ is equal to a constant fraction of commercial structure, q_t :

$$\tilde{\varepsilon}^{ss} = \varrho q^{ss}$$

In the steady state, the value of the parameter ϱ is determined by the system of first-order conditions, given values for γ and δ^q .

3.1.6 Commercial Real Estate Prices

The dynamics of commercial real estate prices can be derived using the optimality condition associated with q_{t+1} :

$$p_t^q = \tilde{\beta}^* E_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \alpha - \xi) \frac{y_{t+1}^n}{q_{t+1}} + [1 - \delta^q(\tau_{t+1})] p_{t+1}^q \right] \quad (23)$$

The first component of the asset pricing formula:

$$(1 - \alpha - \xi) \frac{y_{t+1}^n}{q_{t+1}} \quad (24)$$

denotes the payoff associated with investing in commercial structures given by the marginal productivity of commercial real estate. Compared to the standard central asset pricing formula, the introduction of an overhead cost affecting the depreciation rate implies that a time-varying component, $\delta^q(\tau_{t+1})$, enters the capital gain component of the valuation.

3.2 Market Clearing

Finally the characterization of the competitive equilibrium requires that all markets clear. Firstly, equilibrium in the factor market implies that the quantity of labor and business capital supplied by the household equals the demand from the numeraire sector producer and the investment good producer.

$$k_t^n S = k_t^n D \quad (25)$$

$$N_t^S = N_t^D \quad (26)$$

$$k_t^i S = k_t^i D \quad (27)$$

Equilibrium on the final real estate market implies that the quantity of the real estate good demanded by the consumers equals the quantity produced by the final real estate good producer:

$$h_t = y_t^h \quad (28)$$

Second, equilibrium on real estate investment good market implies that the quantity demanded by the household, i_t^h , equals the quantity produced by the investment good producer, y_t^i :

$$i_t^h = y_t^i \quad (29)$$

In addition, land is assumed to be in fixed supply so that:

$$l_t = \bar{l} \quad (30)$$

and finally the quantity produced by the numeraire good producer equals the quantity of investment and consumption goods demanded by the household and the amount of resources used by the government:

$$y_t^n = c_t + i_t^b + g_t \quad (31)$$

4 The Centralized Equilibrium

In this economy without market imperfections, the competitive equilibrium and the centralized equilibrium are equivalent and the centralized problem can be used to derive an alternative expression of commercial and residential real estate prices.

Starting from the consumer budget constraint (2) and using the conditions (12), (21) and (22) characterizing optimality in production, the market clearing condition in the final real estate good market, (28), the optimality condition in the final real estate good sector (15), and the fact that profits in the numeraire sector and in the real estate investment good sector are given by:

$$\pi_t^i = (1 - \theta) p_t^h y_t^i \quad (32)$$

and:

$$\pi_t^n = (1 - \alpha - \xi) y_t^n - p_t^q i_t^q \quad (33)$$

the budget constraint can be rewritten as:

$$y_t^n + p_t^h (y_t^i - i_t^h) = c_t + g_t + i_t^b + p_t^q i_t^q \quad (34)$$

The Lagrangian for this problem is:

$$L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^{*t} \frac{1}{1 - \sigma} [c_t^\kappa h_t^{1-\kappa}]^{1-\sigma} + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t [y_t^n + p_t^h [y_t^i - i_t^h] - c_t - g_t - i_t^b - p_t^q i_t^q] \right\}$$

where λ_t is the Lagrange multiplier associated to the budget constraint (2). Substituting out, firstly, i_t^h, i_t^b and i_t^q using the accumulation equations (4),(5), and (20); and secondly y_t^n and y_t^i using the production functions (11) and (16) as well as condition (3), this Lagrangian can also be rewritten as:

$$\begin{aligned}
L = E_0 & \left\{ \sum_{t=0}^{\infty} \beta^{*t} \frac{1}{1-\sigma} [c_t^\kappa h_t^{1-\kappa}]^{1-\sigma} \right. \\
& + \sum_{t=0}^{\infty} \beta^{*t} \lambda_t [A_t^n (b_t - k_t^i)^\xi (N_t^T)^\alpha (\tau_t q_t)^{1-\alpha-\xi} - c_t - g_t + (1 - \delta^b) b_t - \gamma b_{t+1}] \\
& + \sum_{t=0}^{\infty} \beta^{*t} \psi_t [A_t^i k_t^{i\theta} l_t^{1-\theta} + (1 - \delta^h) h_t - \gamma h_{t+1}] \\
& \left. + \sum_{t=0}^{\infty} \beta^{*t} \varphi_t [(1 - \delta^q(\tau_t)) q_t + \tilde{\varepsilon}_t - \gamma q_{t+1}] \right\}
\end{aligned}$$

Defining $\psi_t = \lambda_t p_t^h$ and $\varphi_t = \lambda_t p_t^q$, the dynamic system can be derived. The control state variables are b_t, h_t and q_t and the 3 co-states or shadow prices associated to each control state variables are λ_t, ψ_t and φ_t . Prices can then be recovered using the ratios of the Lagrange multipliers.

4.1 An Alternative Expression for Real Estate Prices

As shown in the previous section, the competitive equilibrium can be used to derive asset pricing formulae for both commercial and residential real estate prices. These equations linking prices to the payoffs of investing in each asset can easily be interpreted as arbitrage conditions describing the intertemporal trade-off that agents are facing in this economy. Exploiting the equivalence between the competitive and the centralized problem, we now present a different interpretation of residential and commercial real estate prices.

4.1.1 Residential Real Estate Prices

The centralized problem can be used to derive an alternative expression for residential real estate prices. Equivalence between the centralized problem and the competitive equilibrium implies that residential real estate prices can

equivalently be expressed as the ratio of the Lagrange multipliers associated to the budget and the residential investment constraints:

$$p_t^h = \frac{\psi_t}{\lambda_t} \quad (35)$$

From the first-order condition with respect to k_t^i , it can be shown that this ratio is determined by:

$$\frac{\psi_t}{\lambda_t} = \xi \frac{y_t^n}{b_t - k_t^i} \frac{1}{\theta(y_t^i/k_t^i)} \quad (36)$$

This formula illustrates that in equilibrium, house prices, as measured by the marginal benefit of increasing the stock of residential real estate, $\frac{\psi_t}{\lambda_t}$, has to be equal to its marginal cost. The right handside of the formula describes the trade-off that agents are facing when deciding where the investment in business capital, b_t , should be allocated⁵. The first component, $\xi \frac{y_t^n}{b_t - k_t^i}$, is what agents would have received if the investment in business capital had been allocated to the numeraire sector. This opportunity cost is thus measured by the marginal productivity of numeraire capital. The second component, $\frac{1}{\theta(y_t^i/k_t^i)}$, illustrates that this opportunity cost has to be adjusted to integrate that this investment has a positive impact on the capital stock of the investment good producer. In this model, in equilibrium, house prices will thus be determined by the ratio of marginal productivity of investing in each firm.

4.1.2 Commercial Real Estate Prices

Similarly, as shown above, commercial real estate prices can equivalently be expressed as the ratio of the Lagrange multipliers associated to the budget and the accumulation of commercial structures equation.

$$p_t^q = \frac{\varphi_t}{\lambda_t} \quad (37)$$

From the first-order condition with respect to τ_t , this ratio can be expressed as:

$$\frac{\varphi_t}{\lambda_t} = \frac{(1 - \alpha - \xi)y_t^n}{\tau_t} \frac{1}{\delta'(\tau_t)q_t} \quad (38)$$

⁵Business capital, b_t , can either be allocated to the numeraire firm or to the investment good firm:

$$b_t = k_t^n + k_t^i$$

This second expression illustrates the mechanism characterizing the determination of commercial real estate prices. In this economy, the rate at which structures depreciate determines the evolution of the stock of commercial real estate. On the one hand, if firms decide that increasing profits today is a priority, managers can decide to allocate less workers to the maintenance of commercial structures, N_t^m , in order to increase production immediately, by allocating more workers to productive activities, N_t^p . This increase in the productive capacity of the firm, captured by a rise in $\tau_t = f(N_t^T/N_t^m)$, however comes at the cost of implying a faster rate of depreciation for commercial structures $\delta^q(\tau)$.

On the other hand, managers also understand that if they decide to increase the number of workers in charge of maintenance, this decrease in the productive capacity of the firm, which translates into a decline in τ_t , while affecting the current productive capacity of the firm, also implies that commercial structures will depreciate more slowly. Allocating more workers to the maintenance of commercial structures is therefore a way of sacrificing productive capacity today to spare the stock of structures so as to reach higher levels of production in the future.

The right handside of equation (38) represents the opportunity cost of investing in commercial structure, which is determined by what managers could have earned by increasing the productive capacity of the firm, by decreasing the share of workers in charge of maintaining commercial structures. The first component, $\frac{(1-\alpha-\xi)y_t^p}{\tau_t}$, is the benefit of increasing the productive capacity, which is given by its marginal productivity. The second component, $\frac{1}{\delta'(\tau_t)q_t}$, illustrates that agents also integrate that increasing the productive capacity of the firm is costly, since the resulting decrease in maintenance leads to a higher rate of depreciation of commercial structures.

4.2 Solution Method

The solution is obtained by firstly solving for the macroeconomic variables using the method outlined by King, Plosser and Rebelo (2002). This method involves transforming the economy and log-linearizing the first-order conditions and then solving a linear dynamic system. The solution of the model economy can then be represented by a log linear state space system, with the vector of state variables, $\mathbf{Q}_t$, following a first-order autoregressive process with multivariate normal *i.i.d* impulses:

$$\mathbf{Q}_t = \mathbf{M}\mathbf{Q}_{t-1} + \boldsymbol{\varepsilon}_t$$

where the square matrix $\mathbf{M}$ governs the dynamics of the system. For instance, in the benchmark model, $\mathbf{Q}_t$ contains all control state variables defined above as well as the technology level. The dynamics of the other variables of interest can then be derived by exploiting the properties of the state space system.

4.3 Calibration

The structure of the model allows for several exogenous shocks: standard technology shocks, A_t^n , government spending shocks, g_t , technology shocks affecting the production of residential capital, A_t^i , and commercial real estate supply shocks, $\tilde{\varepsilon}_t$. As shown by the real business cycle literature [see Long and Plosser (1983); King and Rebelo (2000)], standard technology shocks are often found to constitute the main source of business cycle fluctuations. To assess whether technology shocks alone can account for the link between real estate prices and the business cycle, the model is simulated with standard technology shocks, A_t^n , as the only source of aggregate fluctuations⁶.

4.3.1 Preferences and Long Run Behavior

As for κ , the weight attached to consumption in the utility function, it is calibrated such as to replicate the fact that, in the data, the average consumption share, c/y , is about 1/2 and it is set to 0.75. The curvature parameter, σ , is set to 10, which is the upper bound of the range suggested by Mehra and Prescott (1985). The modified subjective discount factor, $\tilde{\beta}^*$, is set to 0.9897. The calibration of the share of government spending parameter, s_g , is based on National Income Account data, and is set to 0.22.

4.3.2 Technology

The constant labor share in the Cobb-Douglas production function of the numeraire good producer, α , is 0.66, the capital share ξ is 0.27, which as suggested by the facts reported by Thalmann and Zarin-Nejadan (2003), implies a share of commercial structure of 0.06. We use a similar calibration

⁶While government spending shocks are assumed to be constant, government spending still affects the steady state of the economy.

for the capital share in the production function of the real estate investment good producer, and set θ to 0.27.

4.3.3 Depreciation Rates and Overhead Costs

The parameters described in the section offer some degree of freedom that is exploited so as to maximize the model ability to explain the empirical facts. Given that for some of these parameters, empirical studies are available, we report values that have been adopted and, when possible, contrast them with values chosen or estimated in previous studies.

The depreciation parameter on numeraire capital, δ^n , is set to 0.038. In King and Rebelo (2000), δ^n is set to 0.025. The rate of depreciation of residential capital, δ^h , is 0.00166. In Davis and Heathcote (2005), an estimated value for this parameter of 0.004 is found. The depreciation rate of commercial structures, δ^q , is set to 0.0044⁷. Compared to the literature, the high value for δ^n is chosen so as to make consumption smoothing more difficult in order to generate enough volatility in aggregate consumption⁸. Compared to the empirical estimates of Davis and Heathcote (2005), the value for δ^h used in this study is lower. As discussed in the next section, this is to maximize the model ability to explain the high volatility of residential real estate prices. The impact of setting δ^h to 0.004 on the results is discussed.

The introduction of time-varying overhead costs implies the calibration of an additional parameter, ε_τ . In steady state, this parameter, which denotes the elasticity of the depreciation rate relative to the overhead cost, τ_t is given by:

$$\varepsilon_\tau = \frac{\delta''^q(\tau)}{\delta'^q(\tau)} \tau \quad (39)$$

The calibration of this parameter is similar to the case of varying rate of capital utilization. The case $\varepsilon_\tau = \infty$ corresponds to the standard case without time-variation in τ_t . In the case of capital utilization, in King and Rebelo (2000), this parameter is set to 0.01 so as to maximize the ability of the model to amplify technology shocks. Given that previous studies do not offer precise guidance when it comes to calibrating this parameter, to generate time variation in the depreciation rate of commercial structures, ε_τ is set to 0.05.

⁷We are not aware of any empirical estimates for this parameter

⁸Another alternative would be to introduce adjustment costs.

4.3.4 Technology Shock and Deterministic Trend

The standard deviation of the technology shock is chosen such as to replicate U.S. postwar quarterly output growth volatility and is set to be 1.131 percent. To calibrate the persistence parameter, the value estimated in King and Rebelo (2000) is adopted and ρ_A is set to 0.979. The deterministic component of technology, γ , is assumed to grow at a quarterly rate of 1.004.

4.3.5 Summary

The calibration of the key parameters of the benchmark model is summarized in the two following Tables:

Table 4: Long Run Behavior

$\tilde{\beta}^*$	α	ξ	θ	γ	σ	κ	ρ^A	σ_ε^2	s_g
0.9897	0.66	0.27	0.27	1.004	10	0.75	0.979	0.0131	0.22

Table 5: Depreciation Rates and Overhead Costs

δ^h	δ^q	τ
0.00166	0.0044	0.05

5 Results

As shown in Table 6, which compares the theoretical implications of the model with the empirical facts presented earlier, the model that is developed seem to be able to characterize the relationship between real estate prices and the business cycle. It is possible to explain accurately the volatility of both residential and commercial real estate prices. The volatility of rents can also be explained. As regards, business cycle facts, while the volatility of investment is slightly underestimated, the model is able to account for the low volatility of consumption.

While significant progresses have been made when it comes to explain the volatility of real estate prices, explaining correlations, and in particular the low correlation between commercial real estate prices and output remains challenging for a model where technology shocks are the unique source of fluctuations. Allowing for additional shocks could allow to improve these correlations significantly⁹. It would however come at the cost of introducing

⁹The model could be simulated with shocks to A_t^i, g_t and $\tilde{\varepsilon}_t$.

additional degrees of freedom. The difficulty of a business cycle model driven solely by technology shocks to explain correlations could be interpreted as the presence of idiosyncratic shocks, specific to real estate markets, that do not have implications at the aggregate level.

Table 6: Theoretical vs Empirical Moments

Theoretical Moments	σ_x	σ_x/σ_y	$corr(\hat{x}_t, \hat{y}_t)$	Empirical Moments	σ_x	σ_x/σ_y	$corr(\hat{x}_t, \hat{y}_t)$
$\hat{y}_t$	1.54	1	1	$\hat{y}_t$	1.54	1	1
$\hat{c}_t$	1.14	0.74	0.99	$\hat{c}_t$	1.23	0.80	0.86
$\hat{i}_t^b$	3.72	2.41	0.99	$\hat{i}_t^b$	5.28	3.43	0.87
$\hat{p}_t^h$	2.41	1.56	0.99	$\hat{p}_t^h$	2.41	1.56	0.60
$\hat{z}_t$	1.14	0.74	0.99	$\hat{z}_t$	1.07	0.69	0.44
$\hat{p}_t^q$	3.68	2.39	0.99	$\hat{p}_t^q$	3.68	2.39	0.26

5.1 Residential Real Estate Prices

The mechanism generating the high volatility of residential house prices works as follows. In periods of booms, technology shocks in the numeraire sector lead to a rise in the first component, $\xi_{k_t^n}^{y_t^n}$, which is the marginal productivity of numeraire capital.

$$p_t^h = \xi_{k_t^n}^{y_t^n} \frac{1}{\theta(y_t^i/k_t^i)} \quad (40)$$

In a model where labor is fixed, on impact, the responses of marginal productivity and output to a one percent technology shock are similar, and are essentially driven by the technology shock, A_t^n . To generate a volatility for residential real estate prices that is higher than output, an additional effect is thus needed. This additional volatility is obtained via the second component, $\frac{1}{\theta(y_t^i/k_t^i)}$, which is the inverse of the marginal productivity of investment capital.

In periods of economic booms, to increase their stock of housing, h_t , agents start investing in the real estate investment firm and k_t^i rises. Since the technology shock does not affect the production function of the real estate investment good producer, the marginal productivity of investment capital, $\theta(y_t^i/k_t^i)$, decreases. From the structure of the model, k_t^i , being a "jump"

variable¹⁰, the rise in investment capital leads to a decline in the marginal productivity of investing in the investment good producer. On impact, this discrete jump inducing $\theta(y_t^i/k_t^i)$ to fall generates the rise in volatility needed to explain the fact that house prices are about 1.5 time more volatile than output. The response of residential real estate prices to a technology shock is presented in Figure 2 (see the appendix).

The depreciation rate, δ^h , has a significant impact on the volatility of house prices. A decline in the depreciation rate, δ^h , implies that a smaller share of residential investment is needed in the steady state to replace existing structures and therefore that larger variations in investment will be needed to achieve consumption smoothing. In response to a shock, housing smoothing leads agents to invest more in the real estate investment firm. The resulting larger increase in k_t^i generates a higher response of house prices via a larger decline in the marginal productivity of investment capital, $\theta(y_t^i/k_t^i)$. Setting δ^h to 0.004, which is the value suggested by the study of Davis and Heathcote (2005), would generate a decline in the volatility of house prices from 2.41 down to 2.08. A similar amplification mechanism could be obtained by introducing habit formation¹¹.

5.2 Rents

In this model, rents are determined by the standard relative scarcity principle:

$$z_t = \frac{(1 - \kappa)}{\kappa} \frac{c_t}{h_t}$$

The fact that housing, h_t , is a predetermined variable implies that, on impact, the dynamics of rents is mainly determined by the behavior of consumption, c_t . This explains why, as illustrated in Table 6, the volatility of residential rents, $\hat{z}_t$, tracks closely the volatility of aggregate consumption, $\hat{c}_t$. In this economy, the fact that building new houses takes time, and in contrast, that aggregate consumption can be increased immediately, implies that, in response to a positive shock, the relative quantity of housing declines. Consumers being willing to smooth consumption over time, this relative decline induced by the jump in consumption is moderate, which explains the low

¹⁰ k_t^i is a control variable.

¹¹ In the initial version of this paper, habit formation has been introduced. Given that habit formation is not crucial to generate this volatility, it has been removed to decrease the number of free parameters.

volatility of residential rents. The response of residential rents to a technology shock is presented in Figure 2.

5.3 The Volatility of Commercial Real Estate Prices

The key ingredient that allows the model to generate the high volatility of commercial real estate is the introduction of time-varying overhead costs. As can be seen from equation (41) below:

$$p_t^q = \tilde{\beta}^* E_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \alpha - \xi) \frac{y_{t+1}^n}{q_{t+1}} + [1 - \delta^q(\tau_{t+1})] p_{t+1}^q \right] \quad (41)$$

the valuation depends on the payoff associated with investing in commercial real estate, which is given by the marginal productivity of commercial structures and the capital gain component, $[1 - \delta^q(\tau_{t+1})] p_{t+1}^q$. The amplification of the volatility of prices is obtained through variations in the overhead cost, τ , that affect the capital gain component of the valuation. Whether an increase or a decrease in the volatility of commercial prices is obtained, therefore depends on how managers decide to allocate their workforce between productive and maintenance activities.

As shown in Table 6, in this model, the fact that agents are patient and risk averse generates an amplification of the volatility of commercial real estate prices which allows the model to match the observed volatility. This amplification is obtained through countercyclical variations in τ_t . In terms of the competitive equilibrium described earlier, in response to a positive shock, having patient and risk averse shareholders leads managers to choose to allocate more workers to the maintenance of commercial structures. By implying a slower rate of depreciation of commercial structures in period of booms, this mechanism allows to amplify the volatility of prices by generating procyclical capital gains.

Equation (42) allows to decompose the mechanism at work. After rearranging terms, commercial real estate prices can be expressed as:

$$p_t^q = \frac{(1 - \alpha - \xi) y_t^n}{q_t} \frac{1}{\delta'(\tau_t) \tau_t} \quad (42)$$

The stock of commercial structure, q_t , being predetermined, the response of the first term, $\frac{(1 - \alpha - \xi) y_t^n}{q_t}$, which is the marginal productivity of commercial real estate, is mainly driven by the technology shock. Compared to output,

the higher volatility of commercial real estate prices, in response to a positive shock, is obtained via the second component, $1/\delta'(\tau_t)\tau_t$, and an increase in the number of workers in charge of maintenance, or equivalently in a decline in τ_t .

Assuming that agents are very patient and highly reluctant to substitute consumption over time is key to generate this result. In terms of the calibration of the model, this translates into a value for the modified discount factor, $\tilde{\beta}^*$, close to 1, and a high value for the curvature coefficient, σ ¹². In such an economy, the fact that agents are highly reluctant to substitute consumption over time, induce them to use commercial structures as a saving device, is the key ingredient generating the high volatility of commercial real estate prices. The impulse responses of commercial real estate prices and τ_t to a technology shock are presented in Figure 2.

Decreasing the modified discount factor, from 0.9897 down to 0.92 would lead to a decline in volatility from 3.68 down to 1.38. This illustrates that when agents become more impatient, managers no longer allocate workers to maintenance in good time. When agents are more impatient to consume, commercial structures are no longer used as a saving device, but rather as a way of increasing present consumption. In terms of the decentralized equilibrium, having impatient shareholders leads managers to be willing to increase profits immediately, in response to a positive shock, by maximizing the current productive capacity of the firm.

The key mechanism leading to an amplification of prices being based on the fact that agents, in our economy, choose to use commercial structure as a saving device, the impact of varying the persistence of the technology shock, ρ_A is now discussed. Compared to a transitory shock, having more persistent shocks would lead agent to be willing to increase consumption on impact and reduce their incentive to save. Setting ρ_A to 0.99 would imply a rise in the volatility of consumption from 1.14 to 1.75, and a decline in the volatility of commercial real estate prices from 3.68 to 2.65. By implying countercyclical variations in τ_t , setting ρ_A to 0.99 would still generate some amplification in the volatility of commercial real estate prices.

In the case where ρ_A is set to 0.999, having shocks that are almost permanent would imply a reduction in the volatility of commercial real estate prices. In this case, the fact that the increase in income is almost permanent gives agents very little incentive to use commercial structures as a saving device

¹² $\sigma = 10$ is the upper bound of the range suggested by Mehra and Prescott (1985).

in response to a favorable shock. As a result, managers would find optimal to decrease the number of workers in charge of maintenance. By implying procyclical variations in τ_t and therefore capital losses due to higher depreciation in good times, having permanent shocks would lead the introduction of overhead costs to generate a reduction in the volatility of commercial real estate prices.

6 Conclusion

Following the literature on business cycles, the main contribution of this paper is to develop a tractable theoretical framework allowing to link residential and commercial real estate prices to the macro economy. In terms of methodology, the fact that, in our framework, both real estate and commercial real estate prices can be expressed as a ratio of Lagrange multipliers simplifies considerably the analysis. Exploiting the equivalence between the competitive and the centralized problems, formulae allowing to study the determinants of the volatility of residential and commercial real estate prices are provided. In both cases, it has been possible to decompose the final effect into two components. In the case of residential real estate prices, introducing a residential investment sector allows to generate the additional volatility needed to match the empirical facts. As for commercial real estate prices, introducing a time-varying overhead costs capturing the impact of maintenance costs on the productive capacity of the firm is the key ingredient.

When it comes to the asset pricing implications of macroeconomic models, the main contribution of this study is to have developed a business cycle model allowing to generate quantitative asset pricing implications that are in line with the empirical facts. When it comes to future work, while the focus is often on the demand side and the specification of preferences, the findings of this study seem to indicate that exploring the production side is likely to constitute a promising direction of research.

7 Appendix: Impulse Response Analysis

Figure 1: Technology Shock, Output, Consumption, Investment

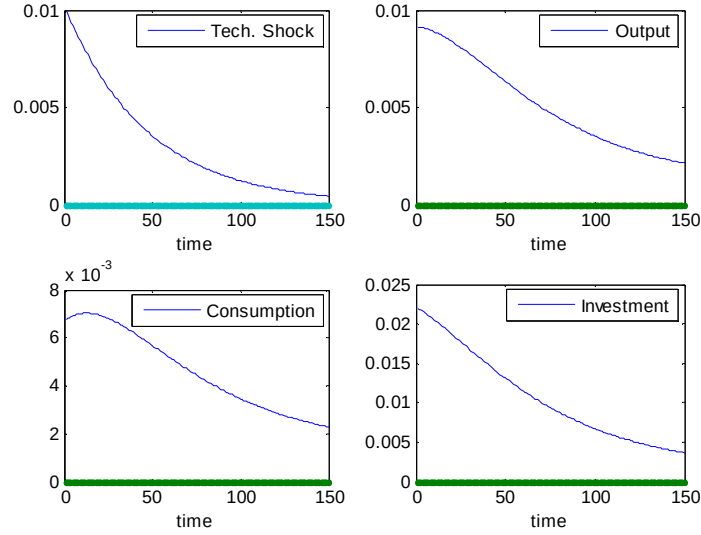


Figure 2: Residential Real Estate Prices, Residential Rents, Commercial Real Estate Prices, Utilization Rate Commercial Structures

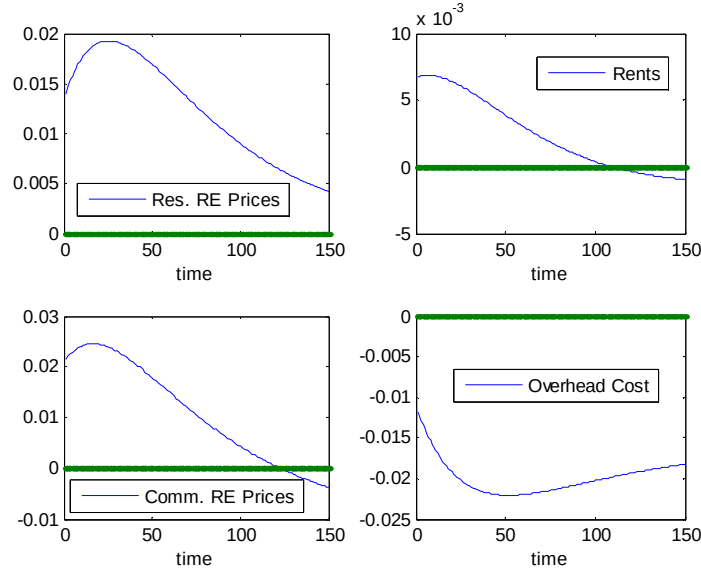
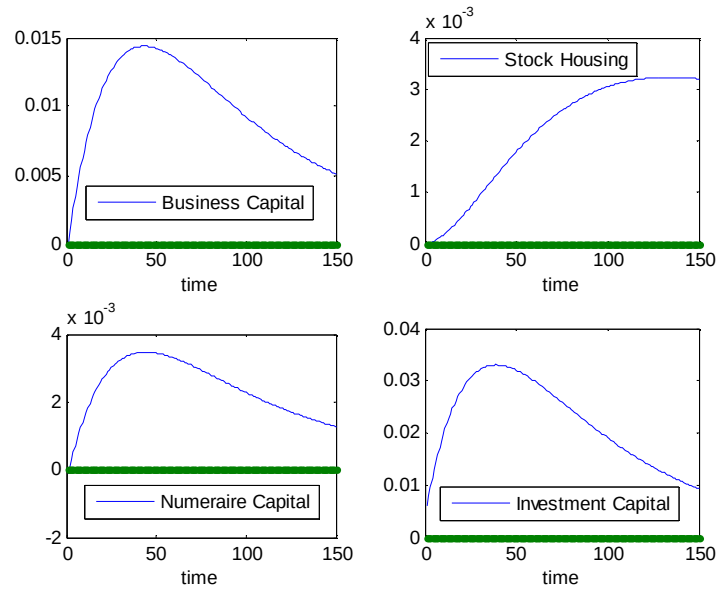


Figure 3: Business Capital, Housing Stock, Numeraire Capital, Investment Capital



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