Learning and the Great Moderation

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28 February 2007

Abstract

We study a stylized theory of the volatility reduction in the U.S. after 1984—the Great Moderation—which attributes part of the stabilization to less volatile shocks and another part to more difficult inference on the part of Bayesian households attempting to learn the latent state of the economy. We use a standard equilibrium business cycle model with technology following an unobserved regime-switching process. After 1984, according to Kim and Nelson (1999), the variance of U.S. macroeconomic aggregates declined because boom and recession regimes moved closer together, keeping conditional variance unchanged. In our model this makes the signal extraction problem more difficult for Bayesian households, and in response they moderate their behavior, reinforcing the effect of the less volatile stochastic technology and contributing an extra measure of moderation to the economy. We construct example economies in which this learning effect accounts for about 30 percent of a volatility reduction.

Keywords: Bayesian learning, information, regime switching.
JEL codes: E3, D8.

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1 Introduction

1.1 Overview

The U.S. economy experienced a significant decline in volatility—on the order of fifty percent for many key macroeconomic variables—sometime during the mid-1980s. This phenomenon, sometimes called the *Great Moderation*, has been the subject of a large and expanding literature. The main question in the literature has been the nature and causes of the volatility reduction. In some of the research, better counter-cyclical monetary policy has been promoted as the main contributor to the low volatility outcomes. In other strands, the lower volatility is attributed primarily or entirely to good luck, in the sense that the shocks buffeting the economy have generally been less frequent and smaller than those from the high volatility 1970s era. In fact, the good luck story is probably the leading explanation in the literature to date. Yet, the pure good luck explanation strains credulity as the full amount of the volatility reduction is simply due to smaller shocks. Why should shocks suddenly be 50 percent less volatile?

In this paper, we study a version of the good luck story, but one which we think is more credible. In our version, the economy is indeed buffeted by smaller shocks after the mid-1980s, but this lessened volatility is coupled with changed equilibrium behavior of the private sector in response to the smaller shocks. The changed behavior comes from a learning effect which is central to the paper. The learning effect reduces overall volatility of the economy still further in response to the smaller shock volatility. Thus, in our version of the good luck story, the Great Moderation is due partly to less volatile shocks and partly to a learning effect, so that the shocks do not have to account for the entire volatility reduction. Quantifying the magnitude of this effect in an equilibrium setting is the primary purpose of the paper.

1.2 What we do

There have been many attempts to quantify the volatility reduction in the U.S. macroeconomic data. In this paper we follow the regime-switching approach to this question, as that will facilitate our learning analysis. The regimes can be thought of as expansions and recessions. According to Kim and Nelson (1999), expansion and recession states moved closer to one another after 1984, but in a way that kept conditional, within-regime variance unchanged. These results imply that recessions and expansions were relatively distinct phases and hence easily distinguishable in the pre-1984 era. In contrast, during the post-1984 era, the two phases were much less distinct.

The Kim and Nelson (1999) study is a purely empirical exercise. We want to take their core finding as a primitive for our quantitative-theoretic model: Regimes moved closer together, but conditional variance remained constant. The economies we study and compare will all be in the context of this idea.

We assume that the two phases are driven by an unobservable latent variable, and that economic agents must learn about this variable by observing other macroeconomic data, such as real output. Agents learn about the unobservable state via Bayesian updating.[1] When the two states are closer together, agents find it harder to infer whether the economy is in a recession or in an expansion based on observable data since the two phases of the business cycle are less distinct. Therefore, learning becomes more difficult and leads to an additional change in the behavior of households. In particular, volatility in macroeconomic aggregates will be moderated since the households are more uncertain which regime they are in at any point in time.

We wish to study this phenomenon in a model which can provide a well-known benchmark. Accordingly, we use a simple equilibrium busi-

[1] Learning as a signal extraction problem is a distinct issue from expectational stability as described by Evans and Honkapohja (2001). We do not study expectational stability in this paper.

ness cycle model in which the level of productivity depends in part on a first-order, two-state Markov process. The complete information version of this model is known to be very close to linear, so that a reduction in the variance of the driving shock process translates one-for-one into a reduction in the variance of endogenous variables in equilibrium. The incomplete information, Bayesian learning version of the model is nonlinear. Reductions in driving shock variance result in more than a one-for-one reduction in the variance of endogenous variables. The difference between what one observes in the complete information case and what one observes in the incomplete information, Bayesian learning case is the learning effect we wish to focus upon. We solve the model using perturbation methods.

1.3 Main findings

We begin by establishing that the baseline model with regime switching behaves nearly identically to standard models in this class under complete information when we use a suitable calibration that keeps driving shock variance and persistence at standard values. We then use the incomplete information, Bayesian learning version of this model as a laboratory to attempt to better understand the learning effect in which we are interested.

We report results for a comparison of economies in which both unconditional and conditional variances of the shock process are held constant, but where the distance between the two regimes is altered.[2] These economies have nearly identical driving shock processes when viewed from an $AR(1)$ perspective. The economies differ only to the degree that the two regimes are closer to each other or farther apart. We establish that this alone creates a learning effect—private sector behavior is substantially altered and endogenous variables are less volatile when the two regimes are closer. We attribute this result to the increased difficulty of the inference problem when the two regimes are closer together.

We then extend these first findings, allowing unconditional variance to

[2]There are clear limits to how well this task can be accomplished.

rise as regimes are moved farther apart, still keeping conditional variance constant. We then compare the resulting volatility of endogenous variables to a complete information benchmark. The complete information model is close to linear, and so the volatility of endogenous variables relative to the volatility of the shock is a constant. For the incomplete information, Bayesian learning economies, endogenous variable volatility rises with the volatility of the shock. This ratio begins to approach the complete information constant for sufficiently high shock variance. Thus the incomplete information economies begin to behave like complete information economies when the two regimes are sufficiently distinct. This is because the inference problem is simplified as the regimes move apart, and thus agent behavior is moderated less.

Finally, we compare incomplete information economies in which observed volatility in macroeconomic variables is substantially different, with one economy enjoying on the order of 50 percent lower output volatility than the other. This volatility difference is then decomposed into a portion due to lower shock variance—"good luck" as it is known in the literature—and another portion due to more difficult inference, the learning effect in which we are interested. We find that the learning effect accounts for about 30 percent of the volatility reduction, and the good luck portion accounts for about 70 percent. This suggests that learning effects may help account for a substantial fraction of observed volatility reduction in more elaborate economies which can confront more aspects of observed macroeconomic data.

1.4 Recent related literature

Broadly speaking, there are two strands of literature concerning the Great Moderation. One focuses on dating the Great Moderation and the other looks into the causes that led to it. The dating literature, including Kim and Nelson (1999) and McConnell and Perez-Quiros (2000), typically assumes that the date when the structural break occurred is unknown and then identifies it using either classical or Bayesian methods. According to the other strand, there are three broad causes of the sudden reduction in volatility—better monetary policy, structural change, or good luck. Clarida, Gali and Gertler (2000) argued that better monetary policy in the Volker-Greenspan era led to lower volatility. The proponents of the structural change argument mainly emphasize one of two reasons for reduced volatility: a rising share of services in total production, which is typically less volatile than goods sector production (Burns (1960), Moore and Zarnowitz (1986)), and better inventory management (Kahn, McConnell, and Perez-Quiros (2002)). A number of authors, including Ahmed, Levin, and Wilson (2004) and Arias, Hansen, and Ohanian (2006), have compared competing hypotheses and concluded that in the recent years the U.S. economy has simply been hit by smaller shocks.

In the literature, learning has often been used to explain fluctuations in output. In Cagetti, Hansen, Sargent and Williams (2001) agents solve a filtering problem since they are uncertain about the drift of the technology. In Nieuwerburgh and Veldkamp (2006) agents solve a problem similar to the one posed in this paper. They use their model to help explain business cycle asymmetries. In their paper learning asymmetries arise due to endogenously varying rate of information flow. Another paper that incorporates learning in a dynamic stochastic general equilibrium model is Andolfatto and Gomme (2003). Here agents learn about the monetary policy regime, instead of technology, and learning is used to explain why real and nominal variables are highly persistent following a regime change.

Arias, Hansen and Ohanian (2006) employ a standard equilibrium business cycle model as we do, but with complete information. They conclude that the Great Moderation is most likely due to a reduction in the volatility of technology shocks. Our explaination does rely on a reduction in the volatility of technology shocks but that reduction accounts for only a fraction of the moderation according to the model in this paper.

1.5 Organization

In the next section we present our model. In the following section we calibrate and solve the model using perturbation methods. We then turn to results.

2 Environment

2.1 Overview

We study a version of an equilibrium business cycle model with exogenous growth. We think of this model as a laboratory to study the effects in which we are interested. Time is discrete and denoted by $t = 0, 1, ...\infty$. The economy consists of an infinitely-lived representative household that derives utility from consumption of goods and leisure. Aggregate output is produced by competitive firms that use labor and capital.

2.2 Households

The representative household is endowed with 1 unit of time each period which it must divide between labor, ℓ_t, and leisure, $(1 - \ell_t)$. In addition, the household owns an initial stock of capital k_0 which it rents to firms and may augment through investment, i_t. Household utility is defined over a stochastic sequence of consumption c_t and leisure $(1 - \ell_t)$ such that

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - \ell_t), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, E_0 is the conditional expectations operator and period utility function u is given by

$$u(c_t, \ell_t) = \frac{[c_t^{\theta}(1 - \ell_t)^{1-\theta}]^{1-\tau}}{1 - \tau}. \tag{2}$$

The parameter τ governs the elasticity of intertemporal substitution for bundles of consumption and leisure, and θ controls the intratemporal elasticity of substitution between consumption and leisure. At the end of each period t the household receives wage income and interest income which is used to buy consumption goods and to invest. Thus the household's end-of-period budget constraint is[3]

$$c_t + i_t = w_t \ell_t + r_t k_t, \tag{3}$$

where i_t is investment, w_t is the wage rate, and r_t is the interest rate. The law of motion for the capital stock is given by

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{4}$$

where δ is the net depreciation rate of capital.

2.3 Firms

Competitive firms produce output y_t according to the constant returns to scale technology

$$y_t = e^{z_t} f(k_t, \ell_t) = e^{z_t} k_t^{\alpha} \ell_t^{1-\alpha}, \tag{5}$$

where k_t is aggregate capital stock, ℓ_t is the aggregate labor input and z_t is a stochastic process representing the level of technology relative to a balanced growth trend.

2.4 Shock process

We assume that the level of technology is dependent on a latent, unobserved variable. Accordingly, we let z_t follow the stochastic process

$$z_t = (a_H + a_L)(s_t + \varsigma \eta_t) - a_L, \tag{6}$$

with

$$z_t = \begin{cases} a_H + (a_H + a_L)\varsigma\eta_t & \text{if } s_t = 1 \\ -a_L + (a_H + a_L)\varsigma\eta_t & \text{if } s_t = 0 \end{cases}, \tag{7}$$

where $a_H \geq 0$, $a_L \geq 0$, $\eta_t \sim i.i.d.\ N(0,1)$, and $\varsigma > 0$ is a weighting parameter. The variable s_t is the unobserved latent state of the economy where

[3] We stress "end of period" since in the begining of the period the agent has only expectations about the wage and the interest rate.

$s_t = 0$ denotes a "recession" state, and $s_t = 1$ denotes a an "expansion" state. We assume that s_t follows a first-order Markov process with transition probabilities given by

$$\Pi = \begin{pmatrix} q & 1-q \\ 1-p & p \end{pmatrix}, \tag{8}$$

where $q = \Pr(s_t = 0|s_{t-1} = 0)$ and $p = \Pr(s_t = 1|s_{t-1} = 1)$. Hamilton (1989) shows that the stochastic process for s_t is stationary and has an $AR(1)$ specification such that

$$s_t = \lambda_0 + \lambda_1 s_{t-1} + v_t, \tag{9}$$

where $\lambda_0 = (1-q)$, $\lambda_1 = (p+q-1)$, and v_t has the following conditional probability distribution: If $s_{t-1} = 1$, $v_t = (1-p)$ with probability p and $v_t = -p$ with probability $(1-p)$; and, $v_t = -(1-q)$ with probability q and $v_t = q$ with probability $(1-q)$ conditional on $s_{t-1} = 0$. Thus

$$z_t = \xi_0 + \xi_1 z_{t-1} + \sigma\epsilon_t, \tag{10}$$

where $\xi_0 = (a_H + a_L)\,\lambda_0 + \lambda_1 a_L - a_L$, $\xi_1 = \lambda_1$, $\sigma = (a_H + a_L)$, and

$$\epsilon_t = v_t + \varsigma\eta_t - \lambda_1\varsigma\eta_{t-1}. \tag{11}$$

The stochastic process for z_t, equation (10), has the same $AR\,(1)$ form as in a standard equilibrium business cycle model even though we have incorporated regime-switching. In our quantitative work, we use σ as the perturbation parameter in order to approximate a second-order solution to the equilibrium of the economy. The distribution of ϵ is nonstandard, being the sum of discrete and continuous random variables. Since η is *i.i.d.*, v and η are uncorrelated. The mean of ϵ_t is zero and the variance is given by

$$\sigma_\epsilon^2 = p\,(1-p)\,\frac{\lambda_0}{1-\lambda_1} + q\,(1-q)\left(1 - \frac{\lambda_0}{1-\lambda_1}\right) + \varsigma^2\left(1 + \lambda_1^2\right). \tag{12}$$

We draw from this distribution when solving the model. The variance is in part a function of ς, which will play a role in the analysis below.

2.5 Information structure

2.5.1 Overview

In any period t, the agent enters the period with beliefs about the latent state s_t. The state of the economy, s_t, the level of technology, z_t, as well as the two shocks η_t and v_t, are all unobservable by the agent. Based on the expected wage and the expected interest rate, firms decide how much labor and capital to employ and the representative household makes labor supply and consumption decisions. After these decisions are made, shocks are realized, output y_t is produced. In our formulation, investment is the residual $y_t - c_t$. The level of technology z_t can be inferred based on the amount of inputs used and the realized output. This z_t is then used to form next period's beliefs about the state of the economy s_t. Given this timing, the information available to the agent at the time decisions are made is $F_t = \{y^{t-1}, c^t, z^{t-1}, i^{t-1}, k^t, \ell^t, w^{t-1}, r^{t-1}\}$. Here $a^t = \{a_0, a_1, ...a_t\}$ represents the history of any series a.

2.5.2 Beliefs

At date t, agents forecast s_{t-1} given information available at date t. Let $b_t = P(s_{t-1} = 0|F_t)$, then

$$\begin{aligned} b_t &= \sum_{s_{t-2}=0,1} P(s_{t-1} = 0, s_{t-2}|F_t) \\ &= P(s_{t-1} = 0, s_{t-2} = 0|F_t) + P(s_{t-1} = 0, s_{t-2} = 1|F_t), \end{aligned} \tag{13}$$

where the joint probability that the economy was in a recession in the last two periods is given by

$$\begin{aligned} P(s_{t-1} = 0, s_{t-2} = 0|F_t) &= P(s_{t-1} = 0, s_{t-2} = 0|z_{t-1}, F_{t-1}) \\ &= \frac{\phi(z_{t-1}, s_{t-1} = 0, s_{t-2} = 0|F_{t-1})}{\phi(z_{t-1}|F_{t-1})} \\ &= \frac{\phi_L(z_{t-1}|s_{t-1} = 0, s_{t-2} = 0, F_{t-1})}{\phi(z_{t-1}|F_{t-1})} \times \\ &\qquad P(s_{t-1} = 0|s_{t-2} = 0, F_{t-1}) P(s_{t-2} = 0|F_{t-1}). \end{aligned}$$

Similarly,

$$P(s_{t-1}=0,s_{t-2}=1|F_t)=$$
$$\frac{\phi_L(z_{t-1}|s_{t-1}=0,s_{t-2}=1,F_{t-1})P(s_{t-1}=0|s_{t-2}=1,F_{t-1})P(s_{t-2}=1|F_{t-1})}{\phi(z_{t-1}|F_{t-1})}.$$

Using the transition probabilities define g_L and g_H as

$$\begin{aligned}g_L &= \phi_L(z_{t-1}|s_{t-1}=0,s_{t-2}=0,F_{t-1})qb_{t-1}\\ &\quad+\phi_L(z_{t-1}|s_{t-1}=0,s_{t-2}=1,F_{t-1})(1-p)(1-b_{t-1}),\end{aligned}$$

and

$$\begin{aligned}g_H &= \phi_H(z_{t-1}|s_{t-1}=1,s_{t-2}=0,F_{t-1})(1-q)b_{t-1}\\ &\quad+\phi_H(z_{t-1}|s_{t-1}=1,s_{t-2}=1,F_{t-1})p(1-b_{t-1}),\end{aligned}$$

where ϕ_i denotes the density function under regime $i\in\{L,H\}$, and $\phi(z_{t-1} \mid F_{t-1})=\Sigma_{s_{t-1}}\Sigma_{s_{t-2}}\phi(z_{t-1},s_{t-1},s_{t-2}|F_{t-1})$.

Since $z_{t-1}=\xi_0+\xi_1 z_{t-2}+\sigma(v_{t-1}+\varsigma\eta_{t-1}-\lambda_1\varsigma\eta_{t-2})$, then conditional on v_t, z_t has a normal distribution. Let $s_{t-1}=0,s_{t-2}=0$, then $v_{t-1}=-(1-q)$ and $z_{t-2}=-a_L+\sigma\varsigma\eta_{t-2}$. Therefore, if in the last two periods the economy was in a recession, $z_{t-1}=\xi_0+\xi_1(-a_L)-\sigma(1-q)+\sigma\varsigma\eta_{t-1}$. Therefore, we can write the conditional density function as

$$\begin{aligned}\phi_{L00} &= \phi_L(z_{t-1}|s_{t-1}=0,s_{t-2}=0,F_{t-1})\\ &= \frac{1}{\sqrt{2\pi\sigma^2\triangle^2}}\exp\left(-\frac{(z_{t-1}-\xi_0-\xi_1(-a_L)+\sigma(1-q))^2}{2\sigma^2\varsigma^2}\right),\end{aligned}$$

and when $s_{t-1}=0,s_{t-2}=1$, then $v_{t-1}=-p$ and $z_{t-2}=a_H+\sigma\varsigma\eta_{t-2}$, the density function is

$$\begin{aligned}\phi_{L10} &= \phi_L(z_{t-1}|s_{t-1}=0,s_{t-2}=1,F_{t-1})\\ &= \frac{1}{\sqrt{2\pi\sigma^2\varsigma^2}}\exp\left(-\frac{(z_{t-1}-\xi_0-\xi_1(a_H)+\sigma p)^2}{2\sigma^2\varsigma^2}\right).\end{aligned}$$

Similarly

$$
\begin{aligned}
\phi_{H01} &= \phi_H(z_{t-1}|s_{t-1}=1, s_{t-2}=0, F_{t-1}) \\
&= \frac{1}{\sqrt{2\pi\sigma^2\varsigma^2}} \exp\left(-\frac{(z_{t-1}-\xi_0-\xi_1(-a_L)-\sigma q)^2}{2\sigma^2\varsigma^2}\right),
\end{aligned}
$$

and

$$
\begin{aligned}
\phi_{H11} &= \phi_H(z_{t-1}|s_{t-1}=1, s_{t-2}=1, F_{t-1}) \\
&= \frac{1}{\sqrt{2\pi\sigma^2\varsigma^2}} \exp\left(-\frac{(z_{t-1}-\xi_0-\xi_1(a_H)-\sigma(1-p))^2}{2\sigma^2\varsigma^2}\right).
\end{aligned}
$$

Thus we can write b_t as

$$
b_t = \frac{g_L}{g_L+g_H}. \tag{14}
$$

2.5.3 Expectations

Since b_t is the probability that the economy was in a recession and $(1-b_t)$ is the probability that the economy was in an expansion in the last period, we determine the probability distribution of the current state by allowing for the possibility of state change. In particular,

$$
\begin{aligned}
&[P(s_t=0|F_t), P(s_t=1|F_t)] \\
&\quad = [P(s_{t-1}=0|F_t), P(s_{t-1}=1|F_t)] \begin{bmatrix} q & 1-q \\ 1-p & p \end{bmatrix}
\end{aligned} \tag{15}
$$

which can be rewritten as

$$
[P(s_t=0|F_t), P(s_t=1|F_t)] = [b_t, (1-b_t)] \begin{bmatrix} q & 1-q \\ 1-p & p \end{bmatrix}. \tag{16}
$$

Given that $P(s_t=0|F_t) = b_t q + (1-b_t)(1-p)$ and $P(s_t=1|F_t) = b_t(1-q) + (1-b_t)(p)$, the current expected state is given by

$$
s_t^e = b_t(1-q) + (1-b_t)(p), \tag{17}
$$

and the expected technology at date t is given by

$$
z_t^e = (a_H + a_L)s_t^e + (-a_L).
$$

Equivalently

$$z_t^e = [b_t(1-q) + (1-b_t)(p)]\, a_H - [b_t q + (1-b_t)(1-p)]\, a_L. \tag{18}$$

2.6 Household's problem

The household's decision problem is to choose a sequence of $\{c_t, (1-\ell_t) | t \geq 0\}$ that maximizes (1) subject to (3) and (4) given a stochastic process for $\{w_t, r_t | t \geq 0\}$, interiority constraints $c_t \geq 0,\ 0 \leq \ell_t \leq 1$, and given k_0. Expectations are formed rationally given the assumed information structure.

Assuming an interior solution, the optimality conditions imply that

$$\frac{u_\ell(c_t, 1-\ell_t)}{u_c(c_t, 1-\ell_t)} = E_t[w_t] \tag{19}$$

Before the shocks are realized, households make their consumption and labor decisions based on expected wage and interest rates. In this model investment is a residual and absorbs unexpected shocks to income. The Euler equation is

$$u_c(c_t, 1-\ell_t) = \beta E_t \left[u_c(c_{t+1}, 1-\ell_{t+1})(r_{t+1} + (1-\delta)) \right]. \tag{20}$$

2.7 Firm's problem

Firms produce a final good by choosing capital k_t and labor ℓ_t such that they maximize their expected profits. The firms period t problem is then

$$\max_{k_t, \ell_t} E_t[e^{z_t} k_t^\alpha \ell_t^{1-\alpha} - w_t \ell_t - r_t k_t]\ \forall t. \tag{21}$$

The first order conditions for the firm are

$$r_t = E_t[e^{z_t} f_k(k_t, \ell_t)] \tag{22}$$

$$w_t = E_t[e^{z_t} f_\ell(k_t, \ell_t)] \tag{23}$$

These conditions differ from the standard condition because technology level z_t is not observable.

2.8 Competitive equilibrium

Given initial capital k_0 and initial state s_0, an equilibrium is a sequence of quantities $\{c_t, i_t, \ell_t, k_{t+1}, y_t, \}_{t=0}^{\infty}$ and prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that

- Given prices, the household solve its maximization problem,
- Given prices, firms solve the profit maximization problem,
- Markets for goods, labor, and capital clear,

$$y_t = c_t + i_t \tag{24}$$

$$k_{t+1} = (1-\delta)k_t + i_t. \tag{25}$$

- Beliefs are updated according to equations (13), (15), and (18).

2.9 Social planner's problem

The planner's problem is to maximize the households utility (1) subject to the production technology (5), law of motion of capital (4), the resource constraint, and the evolution of beliefs about technology. In this model, the timing and informational constraints faced by the planner are the same as in the decentralized economy. Due to the lack of active learning as in Nieuwerburgh and Veldkamp (2006), the competitive equilibrium and the Pareto optimum problem are equivalent. The planner does not take into account the effect of consumption and labor choices on the evolution of beliefs.

In the recursive formulation of the planner's passive learning problem, the state variables are the capital stock and the expected value of the technology at date t. Thus the value function $V\left(k_t, z_t^e\right)$ solves the following equation

$$V\left(k_t, z_t^e\right) = \max_{c_t, \ell_t} u(c_t, 1-\ell_t) + \beta E[V(k_{t+1}, z_{t+1}^e)|F_t] \tag{26}$$

subject to

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{27}$$

$$y_t = c_t + i_t, \tag{28}$$

$$y_t = e^{z_t} k_t^{\alpha} \ell_t^{1-\alpha}, \tag{29}$$

interiority constraints $c_t \geq 0,\ 0 \leq \ell_t \leq 1$, $k_t \geq 0$, given k_0, and the rules for expectation formation given by equation (13), (15), and (18). We solve our model using second order perturbation methods.

2.10 Second order approximation

The perturbation methods we use are standard and are described in Arouba, Fernandez-Villaverde, and Rubio-Ramirez (2006). Our parameter $\sigma = a_H + a_L$ plays the role of σ in Arouba, et al. (2006). This is possible because the regime switching process can be written as an $AR\,(1)$ process. We use and modify the Mathematica code provided by Arouba, et al., (2006). That code is written for a standard normal random variable ϵ_t in (10), whereas ϵ_t follows the mean zero process (11) in this paper. We construct this distribution empirically and draw from it in our implementation of the Arouba, et al., (2006) code.

3 Learning effects

3.1 Calibration

Following the equilibrium business cycle literature, we calibrate the model at a quarterly frequency. The discount factor is set to $\beta = 0.9896$, the elasticity of intertemporal substitution is set to $\tau = 2$, and $\theta = 0.357$ is chosen such that labor supply is 31 percent of discretionary time in the steady state. We set $\alpha = 0.4$ to match the capital share of national income and the net depreciation rate $\delta = 0.0196$. This is also the benchmark calibration used by Auroba, et al., (2006). Apart from the parameters commonly seen in the business cycle literature, there are some additional parameters that capture the regime switching process. In particular, the parameters $-a_L$ and a_H reflect the level of technology z_t relative to trend in recessions and expansions respectively. We also have the transition probabilities p and q as well as a weighting parameter ς.

To obtain a baseline economy that can be compared to the benchmark calibration of Auroba, et al., (2006), (AFR), we use equation (10), reproduced here for convenience

$$z_t = \xi_0 + \xi_1 z_{t-1} + \sigma \epsilon_t. \tag{30}$$

We wish to choose the values of p, q, a_H, a_L and ς such that $\xi_0 = 0$, $\xi_1 = 0.95$, $\sigma = 0.007$, and $\sigma_\epsilon^2 = 1$, the standard equilibrium business cycle values and the ones used in the benchmark calibration of AFR. To remain comparable to AFR, we would like ϵ_t to be close to a standard normal random variable, with $\sigma = a_H + a_L = 0.007$. To meet this latter requirement, we choose symmetric regimes by setting $a_H = 0.0035 = a_L$. Since

$$\xi_1 = \lambda_1 = (p + q - 1),$$

we set $p = q = 0.975$. These values imply $\xi_0 = (a_H + a_L)\,\lambda_0 + \lambda_1 a_L - a_L = 0$. This leaves the mean and variance of ϵ_t. The mean is zero, but to get the variance

$$\sigma_\epsilon^2 = p\,(1-p)\,\frac{\lambda_0}{1-\lambda_1} + q\,(1-q)\left(1 - \frac{\lambda_0}{1-\lambda_1}\right) + \varsigma^2\left(1 + \lambda_1^2\right)$$

equal to one, we set the remaining parameter $\varsigma = 0.719$.[4] Thus the unconditional standard deviation of the shock process, $\sigma\sigma_\epsilon$, is 0.007, and the conditional standard deviation, $\sigma\varsigma\sigma_\eta = \sigma\varsigma$ is 0.005 (since $\sigma_\eta = 1$ by assumption). For this calibration, the nonstochastic steady state values are given by: $z_{ss} = 0$, $k_{ss} = 23.141$, $c_{ss} = 1.288$, $\ell_{ss} = 0.311$, and $y_{ss} = 1.742$.

3.2 A complete information benchmark comparison

The benchmark calibration involves choosing parameters for the regime switching economy such that the economy is as close as possible to a standard equilibrium business cycle model. We now investigate whether the

[4] We chose the positive value for Δ that met this requirement.

TABLE 1.
Benchmark comparison.

Variable	AFR	Model
Output	1.2418	1.2398
Consumption	0.4335	0.4323
Hours	0.5714	0.5706
Investment	3.6005	3.5953
Capital	0.2490	0.2466

Table 1: A comparison of standard deviations of key endogenous variables for a standard equilibrium business cycle model (AFR) and the complete information version of the present model with regime switching, calibrated to mimic the standard case. The addition of regime switching does not change the standard deviations appreciably. In both cases, each simulation has 200 observations. For each simulation we compute the standard deviations for percentage deviations from Hodrick-Prescott filter with $\lambda = 1600$ and then average over 250 simulations.

benchmark equilibrium is comparable to the equilibrium of a standard model. For this purpose we endow the agents with complete information. Table 1 shows that the regime switching economy with complete information and the baseline calibration delivers results almost identical to a standard equilibrium business cycle model, that is, the same results as AFR. Since we use perturbation methods to solve our model, it seems natural then to compare our model with AFR.

This shows that despite the addition of regime switching, the complete information economy calibrated to look like the standard case delivers results very similar to the standard case. We now turn to incomplete information economies with Bayesian learning.

3.3 Allowing only regime distance to change

We ideally would like to keep economies as similar as possible while changing the distance between expansionary and recessionary regimes. Our intuition is that, keeping conditional (within regime) variance constant, regimes which are closer together pose a more difficult inference problem for agents.

The agents then base behavior on an expectation of the latent state, and take actions which are not as extreme as they would be under complete information. The result is a moderating force in the economy.

To investigate this hypothesis in similar economies we proceed as follows. Except for a_H, a_L, ς, and σ_ϵ, all parameters are the same as in the baseline economy. Our goal is to move the values of a_H and a_L farther apart but keep the standard deviation of the shock, $\sigma\sigma_\epsilon$, equal to 0.007, and simultaneously keep the conditional standard deviation, (*a.k.a*, within-regime standard deviation) $\sigma\varsigma = 0.005$. If we achieve this, then the shock processes for each of these economies will look the same from an $AR(1)$ perspective.

The conditional standard deviation requires

$$\sigma\varsigma = (a_H + a_L)\,\varsigma = 0.005. \tag{31}$$

The unconditional standard deviation requires

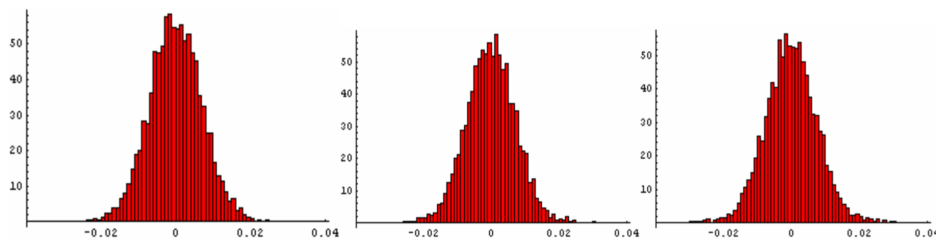
$$\sigma\sigma_\epsilon = (a_H + a_L)\left[\kappa + \varsigma^2\left(1 + \lambda_1^2\right)\right]^{\frac{1}{2}} = 0.007 \tag{32}$$

where $\kappa = p\,(1-p)\,\frac{\lambda_0}{1-\lambda_1} + q\,(1-q)\left(1 - \frac{\lambda_0}{1-\lambda_1}\right)$. With $p = q$, then $\kappa = p\,(1-p) = 0.024375$. Also, $1 + \lambda_1^2 = 1.9025$. These considerations indicate that there is only one available parameter, ς, that can be calibrated to match both targets. We can write

$$\frac{0.005033}{\varsigma}\left[0.024375 + \varsigma^2\,(1.9025)\right]^{\frac{1}{2}} = 0.007. \tag{33}$$

Taking the positive value of ς that solves this equation yields $\varsigma = 0.87435$, so that $a_H + a_L = 0.005756$.

Attempts to move the regimes farther apart or closer together than $a_H + a_L = 0.005756$ will cause us to miss on either the unconditional or the conditional variance target. However, the deterioration is not too dramatic for values of $\sigma = a_H + a_L$ as low as 0.0035 or as high as 0.0105. These are convenient values as the former is one half of the benchmark calibration value, and the latter is three times the former. Thus we set $a_H = a_L = 0.0035$



A B C

Figure 1: Shock distributions for economies A, B, and C are very similar, but the regimes for economy B are twice as far apart as those for economy A. For economy C, the regimes are three times as far apart. Figures are drawn for $6,500$ draws from each distribution.

for economy A, $a_H = a_L = 0.007$ for economy B, and $a_H = a_L = 0.0105$ for economy C. In each case we set ς to keep the conditional standard deviation equal to 0.005.

In terms of the equation for the shock process

$$z_t = \xi_0 + \xi_1 z_{t-1} + \sigma\epsilon_t,$$

these economies are all very similar. In each case, $\xi_0 = 0$, $\xi_1 = 0.95$, and the standard deviation of the shock is not too far from 0.007. From the perspective of the $AR(1)$ process describing the stochastic technology, the economies are nearly the same. The distribution of $\sigma\epsilon$ for the three economies, A, B, and C is depicted in Figure 1.

Even though these economies have similar $AR(1)$ shock processes, they differ in $\sigma = a_H + a_L$, the distance between the recession and expansion states. In economy A the two states are not very distinct relative to economies B and C, and in economy C the states are the most distinguishable. Because conditional variance is constant, we expect the inference problem of the agents to be relatively difficult in Economy A, leading to moderated behavior, and relatively easy in Economy C. Table 2 illustrates how this dif-

TABLE 2. INCREASING σ

	Economy		
	A	B	C
	Parameter values		
a_H	0.0035	0.0070	0.0105
a_L	0.0035	0.0070	0.0105
σ	0.0070	0.0140	0.0210
$\sigma\sigma_\epsilon$	0.0070	0.0073	0.0077
$\sigma\varsigma$	0.0050	0.0050	0.0050
	Volatility, in percent s.d.		
Output	0.9300	1.0100	1.1060
Consumption	0.1450	0.1730	0.2050
Hours	0.1240	0.2850	0.4450
Investment	3.5580	3.8040	4.1060
Capital	0.2430	0.2640	0.2870
$\frac{1}{T}\sum(s^e - s)^2$	0.3900	0.3680	0.2970

Table 2: Economies with different conditional expectation of technology z_t but calibrated in a way such that the AR(1) shock process is almost identical, having the same persistence as well as the same conditional and nearly the same unconditional standard deviation.

ference in the distance between the regimes is enough to induce differences in aggregate volatility.

Table 2 illustrates a learning effect across economies that appear to be nearly identical when looking at the nature of the productivity shock process driving the economy. The inference problem is more severe in some cases, leading agents to make different decisions and changing the nature of the equilibrium. In economy A,the states are twice as close as in economy B and three times as close relative to economy C. This makes the inference problem harder in economy A and agents experience greater uncertainty about the true state in this economy. The result is that standard deviation of output falls by almost 19 percent in economy A when the states are three times closer relative to economy C. The volatility of other endogenous variables also falls precipitously.

Our hypothesis is that the inference problem is more severe in economy

A. For each economy A, B, and C we compute the average sum of squared deviations of the expected latent state from the true latent state. This statistic measures the confusion faced by agents in their respective economies about the underlying state of the economy. From the last row of Table 2 we can see that as the states become more and more distinguishable, that is, as we move from economy A to economy C, the confusion as measured by this statistic decreases.

Table 2 also indicates that $\sigma\sigma_\epsilon$ increases somewhat as we move from Economy A to Economy C, so that these economies are not exactly the same in terms of unconditional volatility. As discussed above, we can only keep either unconditional volatility or conditional volatility completely constant while still meeting other calibration goals, but not both. Still, according to the Table, output volatility increases by 19 percent moving from economy A to economy C, while the unconditional standard deviation increases by only 10 percent (that is, 0.007 versus 0.0077) for the same economies. This suggests a fairly substantial learning effect for economies that, on the face of it, would appear to be very similar. We now turn to another set of experiments to explore this finding more systematically.

3.4 Incomplete approaches complete information

In this subsection we abandon the attempt to keep all aspects of the economies the same as we move regimes farther apart or closer together. Instead, in this subsection we allow the standard deviation of $\sigma\epsilon$ to vary when we change $\sigma = a_H + a_L$, but we hold conditional (within regime) variance constant using the parameter ς. This means that the economies we compare in this subsection will have substantially different unconditional variances, different levels of volatility coming directly from the driving shock process in the economy. We then need a benchmark against which we can compare these economies. The benchmark we choose is the counterpart, complete information version of these economies.

Standard equilibrium business cycle models are known to be very close to linear when there is complete information. In these economies, the standard deviation of key endogenous variables increases one-for-one with increases in $\sigma\sigma_\epsilon$. We expect that our regime switching model with complete information is also very close to linear with respect to the unconditional standard deviation, $\sigma\sigma_\epsilon$. The only difference between this case and the economies we study is the addition of incomplete information and Bayesian learning. The latter economies are nonlinear, so that the standard deviation of key endogenous variables no longer increases one-for-one with increases in $\sigma\sigma_\epsilon$. By comparing the complete and incomplete information versions of the same economies, we can infer the size of the learning effect in which we are interested. In addition, we expect the inference problem to become less severe as regimes move farther apart. The economies with learning should begin to look more like complete information economies as regimes become more distinct.

In Figure 2 we plot the unconditional standard deviation on the horizontal axis. On the vertical axis, we plot the standard deviation of output relative to unconditional standard deviation. In our version of the standard "RBC" equilibrium business cycle model (no regime switching, complete information), the standard deviation of output is 1.2418 percent and the standard deviation of the shock is 0.7 percent. Thus the ratio of standard deviation of output relative to the standard deviation of the shock process is 1.77. Because of linearity, this ratio does not change as the unconditional variance of the shock increases. This is depicted by the horizontal dotted line in Figure 2. We also know that our complete information model with regime switching delivers results close to the standard equilibrium business cycle model for certain parameter values (see Table 1). Thus we expect the ratio of standard deviations of output and shocks to be constant for this case as well. This turns out to be verified in the Figure, as the solid line indicates only minor deviations from the standard business cycle model.[5]

The linear relationship between $\sigma\sigma_\epsilon$ and the standard deviation of endogenous variables breaks down in a model with incomplete information.

[5]This figure is computed by simulating 200 quarters for each economy, and averaging results over 250 such economies.

When the states become less distinct, moving from the right to the left along the horizontal axis in Figure 2, the agents have to learn about the state of the economy and the learning effect moderates the behavior of all endogenous variables. But when states are distinct, toward the right in the Figure, the standard deviation of output rises more than one-for-one. Agents are more able to discern the true state when the states are more distinct.

Figure 2 shows that learning has a pronounced effect on private sector equilibrium behavior. Moreover, it shows that the learning effect becomes larger as regimes move closer together, keeping conditional (within regime) variance unchanged. This makes sense as the inference problem becomes more difficult for agents. Instead of knowing which regime they are in, the agents base behavior on an expected regime, which often lies midway between the two states. This leads the agents to take actions midway between the ones they would take if they were sure they were in one regime or the other. This provides a clear moderating force in the economy above and beyond the reduction in unconditional variance. We now turn to an assessment of how large this moderating force may be.

3.5 Comparing economies with observed moderation

In this subsection we compare two economies, both of which have incomplete information and Bayesian learning. The first economy has higher volatility than the second. We choose parameters so that the volatility of output in the second economy is on the order of 50 percent of the corresponding volatility in the first economy. Thus the second economy has greatly moderated endogenous variable volatility relative to the first. Some of this reduced volatility is due to reduced volatility of the shocks, but some is due to the learning effect. The main idea in this subsection is to understand whether this learning effect could be large enough to be a significant contributor to an output moderation of this magnitude in an equilibrium setting, or whether it would be trivially small and hence unlikely to provide much of an impact.

The empirical literature on the Great Moderation, including Kim and

Figure 2

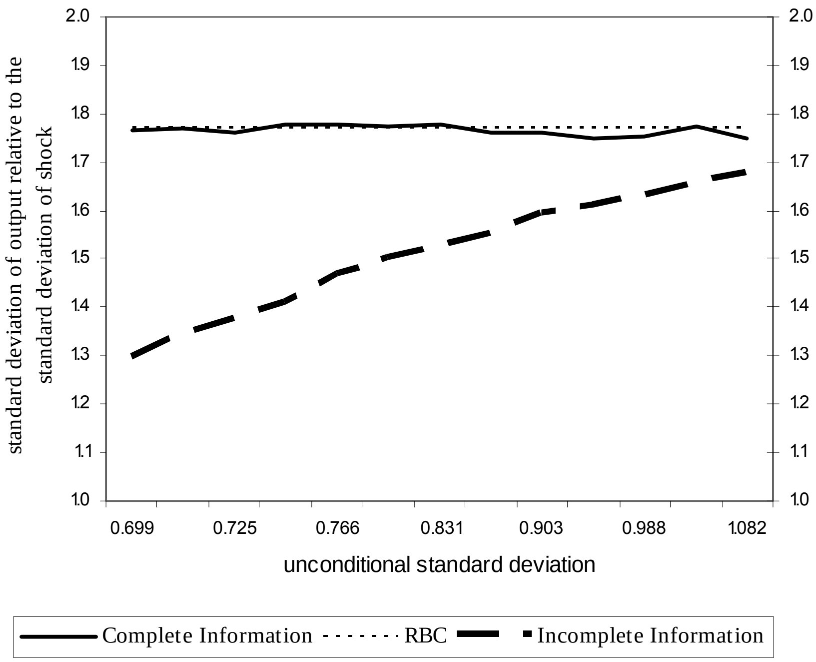


Figure 2: The complete information economy, like the RBC model, has volatility which is proportional to the volatility of the shock. This is indicated by the horizontal line in the Figure. In the incomplete information economy, this is no longer true because of the inference problem. This problem becomes less severe moving to the right in the Figure, and the incomplete information case approaches the complete information case.

TABLE 3. MODERATION.

	Economy	
	High volatility	Low volatility
	Parameter Values	
a_H	0.0265	0.0025
a_L	0.0265	0.0025
σ	0.053	0.005
$\sigma\sigma_\epsilon$	0.0108	0.007
$\sigma\varsigma$	0.005	0.005
	Volatility, in percent s.d.	
Output	1.816	0.908
Consumption	0.390	0.138
Hours	1.141	0.082
Investment	6.504	3.487
Capital	0.451	0.238
$\frac{1}{T}\sum(s^e - s)^2$	0.125	0.415

Table 3: Comparison high and low volatility incomplete information economies. The volatility reduction in output is about 50 percent, but the volatility reduction in the unconditional variance is only 35 percent. Learning accounts for on the order of 30 percent of the volatility reduction in output.

Nelson (1999), McConnell and Perez-Quiros (2000), and Stock and Watson (2003), focuses on documenting the large decline in the volatility of macroeconomic variables of many varieties after 1984. For this reason we want to compare economies in which the endogenous variables have substantially different volatility. From there, we want to decompose the sources of the volatility reduction.

For this purpose, we set $a_H = a_L = 0.0225$ in the high volatility economy and $a_H = a_L = 0.0025$ in the low volatility economy. This implies $\sigma = 0.045$ in the former case and $\sigma = 0.005$ in the latter case. We again choose ς to keep the conditional (within regime) standard deviation $\sigma\varsigma$ constant at 0.005. These parameter choices imply that $\sigma\sigma_\epsilon$, the unconditional standard deviation of the productivity shock, is 1.08 percent in the high volatility economy and 0.7 percent in the low volatility economy. We view these as plausible values. These parameter choices are described in the top panel of Table 3. With these parameter values, the endogenous output standard deviation in the high volatility economy is 1.816, whereas the corresponding standard deviation in the low volatility economy is 0.908, a reduction of 50 percent. Moreover, all endogenous variables are considerably less volatile. This is documented in the lower panel of Table 3.

If these were complete information economies, the volatility reduction would be proportional to the decline in the unconditional standard deviation of the productivity shock ϵ_t. If that was the case, the endogenous variables in the low volatility economy would be about 65 percent as volatile $\left(\frac{0.007}{0.0108}\right)$ as those in the high volatility economy—this would be a volatility reduction of 35 percent. The actual volatility reduction is 50 percent, and the extra 15 percentage points of volatility reduction can be attributed to the learning effect described in the previous subsection. Thus we conclude that for these two economies, the good luck part of the volatility reduction accounts for 35/50 or 70 percent of the total volatility reduction, and the learning effect accounts for 15/50 or 30 percent of the volatility reduction.

We think this calculation, while far from definitive, clearly demonstrates that learning could play a substantial role in the observed volatility reduction in the U.S. economy, with a contribution that may have been on the order of 30 percent of the total. This is fairly substantial, and it suggests that it may be fruitful to analyze the hypothesis of this paper in more elaborate models which can confront the data on more dimensions.

The shock processes driving these two economies are displayed in Figure 3. The two economies are perhaps not too different from this perspective, but the higher volatility economy clearly has more tail events and is less like a normally distributed random variable than the one driving the low volatility economy.

The inference problem clearly becomes more severe in the low volatility economy. Our measure of confusion is given in the last line of Table 3. This measure increases substantially as the economy becomes less volatile. Figures 4 and 5 show more detail on the nature of this confusion. In these

Figure 3

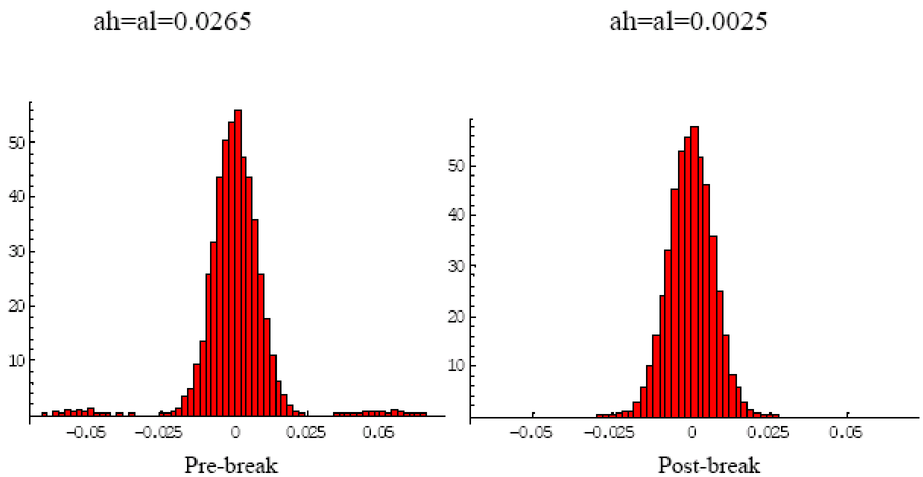


Figure 3: Shock distributions for the high volatility economy, on the left, and the low volatility economy, on the right.

Figures, we plot time series on the unobserved latent state s_t and the agent's expectations of that state at each date. The state s_t is either 0 or 1 and is indicated by solid bars at 0 and 1 in the Figure. The expectation is never exactly zero or one but is often close. For the high volatility economy, shown in Figure 4, the agent is only rarely confused about the state. This is characterized by relatively few dates at which the expectation is not close to zero or one. For the high volatility economy, shown in Figure 5, the agent is confused about the state much more often, as indicated by many more dates at which the state is far from zero or one. In addition, the agent sometimes believes in recession or expansion states when in fact the opposite is true; in other words the agent can be confidently wrong about the state. This can occur because of the amount of noise relative to the distance between the two regimes.

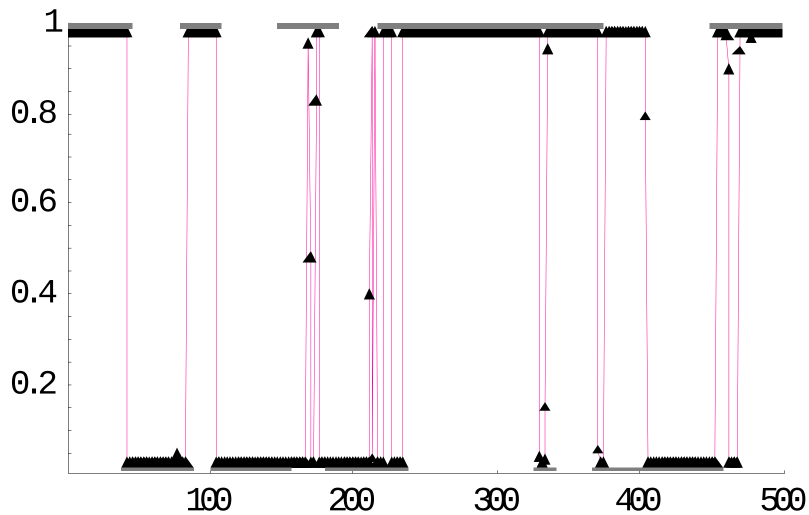


Figure 4: The true state s_t versus the expected state in the high volatility economy. The true state is indicated by bars at zero or one. The agent is relatively sure of the state in this economy.

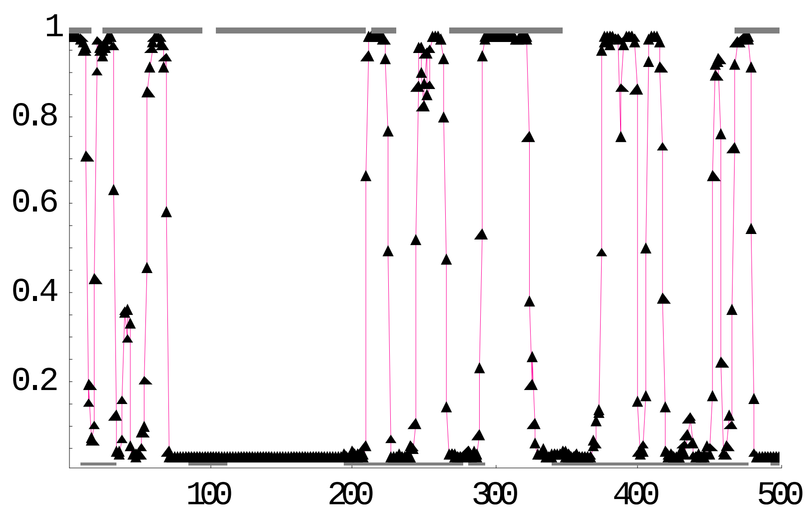


Figure 5: The true state versus the expected state in the low volatility economy. The agent is relatively confused about the true state, causing moderated behavior.

4 Conclusions

We have investigated the idea that learning may have contributed to the great moderation in a stylized regime-switching economy. The main point is that direct econometric estimates may overstate the degree of "good luck" or moderation in the shock processes driving the economy. This is because the changes in the nature of the shock process with incomplete information can also change private sector behavior and hence the nature of the equilibrium.

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