A dynamic theory of the pecking-order based upon repeated signaling*

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Abstract

We examine the effect of Markovian hidden information about the marginal product of capital on the dynamics of financing and investment. The model features endogenous investment, debt, default, dividends, equity flotations and share repurchases. Since deadweight signaling costs are necessarily high when net worth is low, forward-looking risk-neutral shareholders behave as if risk-averse. Consequently, in each period's least-cost separating equilibrium, firms can signal positive information with high leverage and investment. Firms with negative information have no debt and raise external funds with equity. Pareto dominant pooling equilibria also exist, but only if net worth is sufficiently low. In the pooling equilbria, firms issue positive amounts of debt and investment is between respective first-best levels. The model is rich in testable predictions and consistent with a broad set of established stylized facts regarding leverage ratios and announcement effects, and can also explain observed violations of the pecking-order hypothesis.

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1 Introduction

Three decades have elapsed since Ross (1977) and Leland and Pyle (1977) first introduced the "incentive signaling" approach to corporate finance. However, we still lack a dynamic (infinite-horizon) quantitative theory of financing and investment in economies with hidden information.[1] This state of affairs was recently noted by Ross (2005) himself, who stated, "The introduction of the issues raised by the presence of asymmetric information in the determination of the capital structure and the integration of these issues into the intertemporal neoclassical model are a major challenge." Recently, a number of researchers have tried to analyze the effect of hidden information on investment by adopting the intuitively plausible assumption that it gives rise to linear-quadratic costs of external equity.[2] Recognizing the potential problems associated with this approach, Gomes (2001) states, "Ideally we would prefer to model financial intermediation endogenously."

The objective of this paper is to start from primitives and develop a dynamic quantitative model of corporate financing and investment when controlling insider-shareholders (e.g. institutional investors) have superior information ex ante about the marginal product of capital, which evolves as a Markov process. The dynamic pecking-order that emerges from the model can be viewed as an alternative to the static pecking-order hypothesis of Myers (1984). Of empirical interest, the model can explain observed departures from Myers' pecking-order, such as equity issuance by firms with ample debt capacity. More generally, the dynamic model developed here is readily mapped to real-world panel data sets since financing, payouts and investment all evolve endogenously.

The baseline model confines attention to separating equilibria in an infinite sequence of signaling games. In each signaling game, an insider proposes an *allocation* (a vector determining financing and investment) to an uninformed investor. We show that the procedure used to construct a dynamic least-cost separating equilibrium (LCSE) is logically the same as that used in single-period signaling models, e.g. Ambarish, John and Williams (1987). The first additional source of complexity is that the budget and incentive constraints in our model involve endogenously determined value functions, while single-period models involve terminal cash flows. The second additional source of complexity relative to one-shot signaling models is determining the endogenous default threshold accounting for the option value inherent in continuation. The two hurdles are readily cleared using recursive methods (e.g. Stokey and Lucas (1989)).

A thorny issue in all signaling models is multiple equilibria. Maskin and Tirole (1992) and Tirole (2006) show the equilibrium set can be refined by allowing insiders to propose an *option contract*, as opposed to a single allocation. In our setting with two possible types, the option contract allows the insider to choose between the two contractually specified allocations *after* the investor has committed to finance either option. Roughly speaking, the equilibrium set in this option contract game consists of all allocations that weakly Pareto dominate the LCSE. Thus, the LCSE is always in the equilibrium set. Conveniently, the Pareto

[1] In contrast, substantial progress has been made on dynamic models with hidden action. For a recent example, see DeMarzo and Fishman (2006).

[2] For examples, see Gomes (2001), Cooley and Quadrini (2001), and Hennessy and Whited (2006).

criterion allows us to cleanly characterize the set of points on the state (net worth) space at which it is possible for the firms to pool at the same allocation. Again, recursive techniques must be exploited since changes in equilibria lead to changes in the value functions, and vice versa.

The broad details of the model are as follows. We start with a neoclassical investment framework. The firm has a stochastic concave profit function of the form $\theta_t \varepsilon_t k_t^{\alpha}$. The shock ε_t is public and observed simultaneously by all agents at the end of period t. For example, ε can be interpreted as representing a macroeconomic shock. In the spirit of Lucas and McDonald (1990), the shock $\theta_t \in \{\theta_L, \theta_H\}$ is privately observed by a controlling insider-shareholder at the start of period t, with outsiders observing the private shock at the same time they observe ε_t. That is, insiders enjoy a one-step-ahead informational advantage over outsiders. In this way, the model captures the notion that insiders tend to enjoy temporary informational advantages over outsiders.[3] Each insider-shareholder has a one-year holding period and maximizes the cum dividend value of his equity stake. The insider is forward-looking and recognizes that his decisions affect end-of-period net worth and the value for which his equity stake will sell. The endogenous evolution of net worth is the underlying sources of dynamics.

We begin by discussing the predictions emerging from the baseline model, which confines attention to separating equilibria. A central insight is that anticipation of future signaling costs causes risk-neutral insiders to exhibit *pseudo-risk-averse* preferences.[4] In purely technical terms, this means the market value of total shareholder's equity is a concave transformation of internal resources. Stated differently, deadweight signaling costs are decreasing in net worth. To see this, suppose the insider has positive information about the marginal product of capital, but net worth is low. Here the firm would need to raise a large amount of funds in order to implement first-best investment. However, the issuance of such a large block of securities would create a strong temptation for firms with negative information to mimic, since they stand to gain a great deal from security mispricing. In order to separate, the insider with positive information must utilize costly signals such as overinvestment and issuance of defaultable debt. In this example, the marginal value of internal funds is high, since internal funds reduce the need for external funds and concomitant signaling costs. At the opposite extreme, a firm with high net worth does not require external funding. Such a firm will use a marginal dollar to pay dividends, implying that internal funds have a shadow value of one.

With this in mind, consider the nature of the LCSE. In order to minimize the temptation of insiders with negative information to mimic, they should be made as well off as possible. Since the insider is pseudo-risk-averse, it is optimal for firms reporting low expected profitability to be financed with external equity while still holding a cash buffer stock. In this way, outside investors provide the firm with insurance in the form of financial slack. In contrast, firms can credibly signal positive information by substituting debt for equity. Relative to symmetric information models, optimality conditions include additional terms related

[3] We would argue that permanent informational advantages are empirically implausible since numerous forces induce the revelation (acquisition) of information about public firms. See Sannikov (2006) for a model of ex ante adverse selection cum repeated moral hazard ex post.

[4] Alternatively, one can view the model as rationalizing corporate hedging as a response to repeated hidden information. Froot, Scharfstein and Stein (1993) derive a hedging motive in a single-period setting with costly verification of cash flows.

to the precautionary value of internal resources as well as the signal content of real and financial decisions. It is also worth noting that the effect of private information is not isomorphic to linear-quadratic costs of external equity. In equilibrium, the low type is free to issue as much equity as it would like with zero price impact. Conversely, the high type perceives a discontinuity in his payoff when he has issued sufficient equity to induce the low type to mimic.

The possibility of pooling equilibria is addressed using our extended model. Based upon the observations above, it is apparent that high types must incur significant deadweight losses in order separate when net worth is low. As intuition would suggest, a pooling equilibrium Pareto dominates the LCSE in low net worth states. Consequently, the equilibrium in the extended model entails pooling in low net worth states, followed by costly separation in intermediate net worth states, followed by a region where the incentive constraints are slack and both types implement second-best. On the pooling interval, the firms have positive amounts of debt and utilize a capital stock that is between the respective first-best levels. The switch from pooling to separating equilibria causes policies to become non-monotonic and/or discontinuous in net worth.

The model can be mapped directly to real-world data since investment, debt, equity flotations, dividends, share repurchases, and default are all endogenous. The behavior of the low type in the model is consistent with empirical observation. In particular, Leary and Roberts (2006) document that "more often than not, when firms issue equity, they do so before turning to the debt market or in lieu of issuing debt." Such behavior is inconsistent with the pecking-order of Myers (1984). In our model, investors reward the firm for revealing negative information by providing it with ample equity financing and a cash buffer stock that can be used when investment becomes profitable once more.

The baseline model also generates predictions consistent with observed patterns of abnormal equity returns surrounding corporate announcements. Such announcement effects do not emerge in trade-off theoretic models with common knowledge.[5] Simulated share prices exhibit positive (negative) abnormal returns in response to increases (decreases) in capital expenditures. This is consistent with the findings of McConnell and Muscarella (1985), who document raw equity returns of 1.21% following announced increases in capital budgets and -1.52% for decreases. The model also predicts that a high debt percentage in total external finance leads to positive abnormal returns, even when the investment rate is included as a conditioning variable. Consistent with this prediction, Masulis (1983) documents a 13.97% primary announcement return in debt for equity exchanges and a -9.91% return in equity for debt exchanges.

The model is also consistent with stylized facts regarding financial structure dynamics documented by Fama and French (2002). In simulated data, the leverage ratio is inversely related to lagged profitability and exhibits mean-reversion. Since the shock ε is public, it can be viewed as a proxy for macroeconomic shocks.[6] Following low realizations of ε, firms have low net worth. In such states, simulated leverage ratios are particularly high for firms with positive private information regarding the marginal product of capital

[5] In such environments, only changes in commonly observed state variables (e.g. earnings) induce changes in stock prices. Given these state variables, investors can predict firm actions, so policy announcements have no information content.

[6] In contrast, there is no reason to assume that the insiders are privately informed about the macroeconomy. Hence, θ cannot proxy for the macroeconomy.

since they must send strong signals in conjunction with their major fund-raising efforts. Thus, the model predicts that firms will have higher average leverage ratios at the tail-end of recessions. This prediction is consistent with evidence presented by Korajczyk and Levy (2004).

In the most closely related paper, Lucas and McDonald (1990) develop a dynamic model of investment under hidden information. However, they constrain the firm to finance with equity. Our model is also similar to that of Constantinides and Grundy (1990), but their model is static. Our paper is closely related to that of Myers and Majluf (1984), despite the fact that we predict departures from their asserted pecking-order. Myers and Majluf present a simple model illustrating that a firm constrained to finance with equity may rationally forego positive NPV projects if the lemons problem is sufficiently acute. They briefly (and less formally) consider the choice between debt and equity. They argue that firms should minimize the informational sensitivity of their securities, which generally implies a preference for debt over equity.

Nachman and Noe (1994) develop a formal theoretical foundation for the pecking-order hypothesis. They assume the scale of investment is fixed and attention is confined to securities with payoffs that increase monotonically with cash flow. Under these assumptions, there can be no separating equilibrium, as firms would always benefit from reporting the highest type, receiving the highest security value (due to monotonicity), and paying the additional funds as a dividend. Nachman and Noe derive sufficient conditions such that debt will be the unique source of financing in the pooling equilibrium. The essential idea is that debt minimizes cross-subsidies. The central difference between our setting and that considered by Nachman and Noe is that we allow the firm to signal using investment scale and share repurchases. The broader, and arguably more realistic, action space creates the possibility of separating equilibria that are impossible in their setting.

Brennan and Kraus (1987) also consider a setting with fixed investment scale. However, they allow for nonmonotone financings. In their model, a *financing* is defined by the net cash flow attributable to the issuance and/or repurchase of outstanding securities. They show nonmonotone financings are necessary to achieve a separating equilibrium if there is a continuum of types ranked according to first-degree stochastic dominance. Of particular interest, they show a firm with outstanding debt can costlessly overcome the problem of hidden information by simultaneously issuing equity and repurchasing debt. In this nonmonotone financing, the debt repurchase serves as a positive signal partially offsetting the negative signal content of the equity flotation. Clearly, the complex financings described by Brennan and Kraus are difficult to reconcile with the pecking-order hypothesis.

Two potential weaknesses of the Brennan and Kraus (1987) construction are that it relies upon preexisting debt and also ignores the free-rider problem in debt repurchases.[7] Constantinides and Grundy (1990) show the assumption of preexisting debt can be relaxed if (as in our model) the firm is run in the interest of insider-shareholders. In this setting, share repurchases serve as a positive signal, filling the role played by debt repurchases in the model of Brennan and Kraus. Constantinides and Grundy show that a firm with

[7]See Miranda (2006) for a discussion.

variable investment scale can costlessly implement first-best provided that there are zero costs of bankruptcy. Under this assumption, first-best is achieved with the firm issuing debt in excess of the amount needed to fund investment. Excess funds are then used for share repurchases. Effectively, the negative signal content of the debt issuance is compensated by the positive signal provided by the share repurchase.

Our model alters that of Constantinides and Grundy (1990) along the following dimensions: 1) default creates deadweight losses; 2) the hidden information problem repeats; and 3) net worth evolves endogenously. Each generalization provides new insights. The *combination* of hidden information with default costs induces departures from first-best. Intuitively, the firm would like to avoid costly default, but the lemons problem limits the ability to substitute equity for debt as a source of external funds. Conversely, default costs make it expensive to signal by substituting debt for equity. The repetition of this problem generates the pseudo-risk-aversion that encourages saving and adds to the signal content of debt. Finally, allowing for endogenous fluctuations in net worth allows the model to explain mean-reverting and countercyclical leverage ratios, as well as the negative relationship between leverage and lagged profitability.

In our model, the signal content of debt depends on firms' willingness to exchange equity for reductions in the face value of debt. For a limited class of distributions, we show analytically that low types are more willing to exchange equity for debt reductions, implying that debt is a positive signal. Part of the signal value of debt stems from the fact that low types know their equity is less valuable. This effect was first noted by Leland and Pyle (1977). In addition, concavity of the equity value function implies that low types attach a higher shadow cost to servicing debt, since their operating cash flows are lower. In the model of Leland and Pyle, a similar effect is present, but their model relies upon the assumption that the firm is controlled by a risk-averse manager. Similarly, Ross (1977) invokes a default penalty borne by a controlling manager. In our model, an otherwise risk-neutral shareholder has similar preferences, although the pseudo-risk-aversion arises endogenously.

The remainder of this paper is organized as follows. Section 2 presents assumptions on technology and describes the signaling game in which the informed party offers a single allocation to the investor. Section 3 characterizes the least-cost separating equilibrium of that game. Section 4 uses simulated data to evaluate empirical implications of the baseline model. Section 5 analyzes the option contract game and the possibility of pooling equilibria. Section 6 extends the baseline model to incorporate a tax advantage to debt. Section 7 concludes.

2 Economic Environment

2.1 Technology and Timing

Time is discrete and the horizon is infinite. There is a risk-free asset paying a constant rate of interest $r > 0$. All agents are risk-neutral and share the discount factor $\beta \equiv (1 + r)^{-1}$. Capital ($k$) decays exponentially at rate $\delta \in [0, 1]$. The following two assumptions describe the production technology and timing of information

revelation.

Assumption 1. Periodic operating profits are $\theta_t \varepsilon_t k_t^\alpha$ where $\alpha \in (0,1)$. The private shock θ takes values in $\Theta \equiv \{\theta_L, \theta_H\}$ with $0 \leq \theta_L < \theta_H$ and follows a first-order Markov process. The public shock ε is independently and identically distributed in the set of positive real numbers $\Re_+$. The density function $f : [\underline{\varepsilon}, \infty) \to [0,1]$ is continuous and differentiable with $f(\varepsilon) > 0$ for all $\varepsilon \geq \underline{\varepsilon}$.

Assumption 2. At the start of each period, the current θ is privately observed by a controlling insider-shareholder. Financing and investment are then contractually determined. At the end of the period, ε is revealed simultaneously to all agents. Realized output is observable, allowing investors to infer θ at the end of the period.

As a technical assumption, we shall assume that, despite being observable, the shock θ is nonverifiable in a court. This prevents punishing the insider directly based on false announcements of θ. We let π_{ij} denote the probability of θ_i conditional on lagged type θ_j. The shock θ is persistent with $1 > \pi_{HH} \geq \pi_{HL} > 0$.

The firm can raise funds by borrowing or issuing new shares. The borrowing technology is a one-period debt contract. Although we do not analyze optimal security design, we here note that if the formal bankruptcy process is inherently costly, it would be more efficient to transfer ownership in low profit states using equity warrants to dilute current shareholders. However, a realistic assessment of optimal security design must account for the tax advantages of debt. If the debt tax shield is sufficiently large relative to bankruptcy costs, debt will dominate equity warrants as a device for eliciting truthful revelation. Consistent with this argument, McDonald (2004) shows that written put options, which have signal value, entail large tax costs relative to synthetic equivalents with explicit borrowing. Based on this discussion, we incorporate a tax advantage to debt in Section 6. Presently, we abstract from taxes in order to isolate the effect of hidden information.

The face value of debt for the type-i is denoted b_i.[8] After observing the public shock ε, shareholders default only if they are unable raise a sufficient amount of internal and external funds to deliver the promised amount b_i. Consistent with the absolute priority rule, in the event of default the lender demands a cash payment that pins shareholders to their reservation value of zero. Cooley and Quadrini (2001) consider a similar debt contract, but they allow shareholders to inject cash directly into the firm. Anticipating, hidden information substantially alters the analysis of endogenous default.

The variable $\widetilde{w}$ denotes *provisional net worth,* which is the sum of capital net of depreciation plus operating profits less promised debt service

$$\widetilde{w}(b, k, \varepsilon, \theta) \equiv (1-\delta)k + \theta\varepsilon k^\alpha - b. \tag{1}$$

There are two state variables in the model: the lagged value of θ and *net worth* (w). In general, there are

[8] With some abuse of terminology, we call a firm that has realized the shock θ_i a type-i firm. However, it must be stressed that the type refers to a temporary shock.

two *end-of-period* equity value functions, with $v_i(w)$ representing that total market value of shareholders' equity given that the firm started the period with type θ_i and ended with net worth w. In general, the type at the beginning of the period is relevant to equity value since the lagged type determines the probability law governing the future type. However, in the special case where the shock θ is i.i.d. there is no need to keep track of the lagged type as a state variable and $v_H = v_L \equiv v$.

As mentioned above, default occurs when shareholders cannot possibly deliver the promised debt payment even after accounting for the external funds that can be raised in the next financing round. In particular, there exists $w^d < 0$ such that shareholders must default whenever $\widetilde{w} < w^d$. If $\widetilde{w} \geq 0$ it is obvious that default does not occur since the promised debt payment can be delivered using gross internal resources (operating profits plus installed capital). When $\widetilde{w} \in [w^d, 0)$, shareholders deliver the promised debt payment using a portion of the external funds raised in the next financing round. In fact, when $\widetilde{w} = w^d$, shareholders can only deliver the promised debt payment by selling off all the equity in the firm ($s = 1$). This implies that the model has an endogenous default condition of the form

$$v_L(w^d) = v_H(w^d) = 0. \tag{2}$$

In the event that $\widetilde{w} < w^d$, the lender demands the maximum cash payment consistent with the limited liability enjoyed by shareholders. Therefore, the lender will demand a cash payment that leaves current shareholders with net worth equal to w^d in the event of default. The resulting law of motion for net worth is

$$w(b, k, \varepsilon, \theta) = \max\{w^d, \widetilde{w}(b, k, \varepsilon, \theta)\}. \tag{3}$$

The domain for the two equity value functions is $W \equiv [w^d, \infty)$. Below, we will speak of $\left|w^d\right|$ as representing the firm's *going-concern value*. Heuristically, the going-concern value constitutes the future NPV that a penniless owner of the firm could expect to capture in this economy.

In a separating equilibrium, each period's true θ is revealed to the investor. However, to establish separation it will be necessary to consider the payoffs to an insider of type-i who receives the allocation of type-j where j does not necessarily equal i. Let ε_{ij}^d denote the critical value of ε such that type-i would default given that he has received the capital stock and debt obligation of type-j. Recalling that shareholders default when $\widetilde{w}(b, k, \varepsilon, \theta) < w^d$, ε_{ij}^d is determined by

$$\widetilde{w}(b, k, \varepsilon_{ij}^d, \theta) = w^d \Rightarrow \varepsilon_{ij}^d \equiv \frac{b_j - (1-\delta)k_j + w^d}{\theta_i k_j^\alpha}. \tag{4}$$

If $\varepsilon_{ij}^d \leq \underline{\varepsilon}$, there is zero probability of default for type-i that has received the allocation of type-j.

Recall that in the event of default the lender demands a cash payment from shareholders, call it $b_i^r \neq b_i$,

that leaves them with net worth equal to w^d. Therefore, we compute b_i^r using

$$(1-\delta)k_i + \theta_i \varepsilon k_i^\alpha - b_i^r = w^d \Rightarrow b_i^r = (1-\delta)k_i + \theta_i \varepsilon k_i^\alpha + \left|w^d\right|. \tag{5}$$

Equation (5) indicates that the lender seizes the firm's physical assets, all operating profits, and the going-concern value. We assume a fraction ($\phi > 0$) of going-concern value is spent on legal fees in the event of default. The lender's net recovery in the event of default is

$$b_i^r + \phi w^d = (1-\delta)k_i + \theta_i \varepsilon k_i^\alpha + (1-\phi)\left|w^d\right|. \tag{6}$$

The firm is also allowed to hold financial slack ($b < 0$) earning the interest rate r. As argued by Shyam-Sunder and Myers (1999) and Cooley and Quadrini (2001), there must be some cost to holding slack or firms facing financial market imperfections would never make cash distributions. Jensen (1986) argues that shareholder value may be dissipated if managers spend a portion of financial slack on perquisites. Others have argued that financial slack creates incentives for workers to demand higher wages (e.g. Bronars and Deere (1993)). Letting χ be an indicator for $b < 0$, we assume the firm incurs agency costs on holdings of financial slack equal to $\chi\gamma b^2/2$. Section 6 ignores agency costs of financial slack and instead considers tax costs of financial slack.

Assumption 3 summarizes the firm's borrowing technology.

Assumption 3. The debt contract is single-period. Default is endogenous and the lender enjoys strict seniority. There is a deadweight default cost equal to ϕ times the going-concern value $\left|w^d\right|$. The yield on corporate saving is r. The firm incurs agency costs equal to $\chi\gamma b^2/2$ when it holds cash.

In the interest of brevity, let Ω^{ij} denote the expected discounted end-of-period value of total shareholders' equity for a type-i that has taken the type-j allocation:

$$\Omega^{ij} \equiv \beta \int_{\varepsilon_{ij}^d}^{\infty} v_i[(1-\delta)k_j + \theta_i \varepsilon k_j^\alpha - b_j] f(\varepsilon) d\varepsilon. \tag{7}$$

Also, let ρ^i denote the market price of the debt issued by type-i in equilibrium.

$$\rho^i \equiv \beta \left[b_i \int_{\varepsilon_{ii}^d}^{\infty} f(\varepsilon) d\varepsilon + \int_{\underline{\varepsilon}}^{\varepsilon_{ii}^d} [(1-\delta)k_i + \theta_i \varepsilon k_i^\alpha - (1-\phi)w^d] f(\varepsilon) d\varepsilon \right]. \tag{8}$$

2.2 Insider Objectives

Insider objectives are identical to those assumed by Constantinides and Grundy (1990).

Assumption 4. The firm is controlled by a risk-neutral insider who will hold his shares for one period. The insider will not buy additional shares in the event of an equity flotation and will not tender shares in the event of a share repurchase.

Myers and Majluf (1984) also adopt the assumption that the firm is run in the interest of shareholders who do not alter their stakes even as the firm alters the number of shares outstanding.[9] The assumption that the insider holds his shares for one period plays an important technical role, allowing us to avoid the curse of dimensionality associated with keeping track of yet another state variable–the shares held by a multi-period insider. It is worth stressing, however, that the insider is forward-looking since the equity value functions capitalize the effects of future rounds of play.

The insider holds $m > 0$ shares of stock. The number of shares outstanding at the start of the period, inclusive of insider shares, is $c > m$. The number of new shares issued is denoted n, with $n < 0$ indicative of a share repurchase. Let $s \equiv n/(c+n)$ represent the percentage equity stake sold (repurchased).

Shares are issued and repurchased ex dividend. Total dividends are denoted d and are constrained to be nonnegative. Absent such a constraint, the firm could avoid signaling costs by having shareholders inject their own funds directly into the firm. Under the stated assumptions, the insider receives a fraction m/c of total dividends and holds an equity stake of $m/(c+n)$ at the end of the period. Therefore, the *objective function* of the informed insider of type-i is

$$\left(\frac{m}{c}\right)d + \left(\frac{m}{c+n}\right)\beta \int_{\varepsilon^d}^{\infty} v_i[(1-\delta)k + \theta_i \varepsilon k^{\alpha} - b]f(\varepsilon)d\varepsilon. \tag{9}$$

It will be convenient to think of the insider as proposing an allocation to the investor. An *allocation* is a vector $a \equiv (b, d, k, s)$. Suppose the insider observes θ_i at the start of the period. Using the definition of s, the objective function for the insider (9) simplifies to

$$\left(\frac{m}{c}\right)\left[d + (1-s)\beta \int_{\varepsilon^d}^{\infty} v_i[(1-\delta)k + \theta_i \varepsilon k^{\alpha} - b]f(\varepsilon)d\varepsilon\right]. \tag{10}$$

Conveniently, the multiplicative term m/c has no effect on the insider's program, since the vector a contains all relevant choice variables.

Assume the type-i insider reveals the firm's true type by choosing a separating allocation (b_i, d_i, k_i, s_i). If the firm issues new shares ($n > 0$), the equity flotation raises $s_i\Omega^{ii}$. In the event of a share repurchase, each outsider-shareholder must be indifferent between tendering or not at the margin. In this case, $s_i\Omega^{ii}$ also represents the cash outflow from a share repurchase. To demonstrate this claim, consider a simple example. Suppose $\Omega^{ii} = 100$, $c = 50$ and $n = -10$. After the share repurchase, the ex dividend value of each remaining share is $100/(50-10) = 2.5$. Therefore, in order to induce ten shareholders to sell, the firm must pay 25 $(= 10 \times 2.5)$. Now note that $s_i\Omega^{ii} = -(10/40) \times 100 = -25$.

[9] See page 189 of Myers and Majluf (1984) for a discussion.

2.3 The Simple Signaling Game

In the *simple signaling game*, as distinct from the option contract game presented in Section 5, the insider moves first and offers a technologically feasible allocation to the investor. The investor then updates his beliefs and either accepts or rejects the allocation. If the investor accepts, the allocation is implemented. If the offer is rejected, the firm is then constrained to finance with internal funds (i.e. $s = 0$ and $b \leq 0$).

Each insider is forward-looking by construction. In particular, the equity value functions (v_L, v_H) entering the insider's objective function (10) are constructed to correctly capitalize the outcome of all signaling games played by future insiders. The equilibrium concept in each signaling game is perfect Bayesian equilibrium (PBE). A PBE imposes the following requirements: 1) For any feasible offer, the investor must have some belief. 2) The investor's beliefs must be derived from Bayes' rule for any offer made in equilibrium. 3) The investor accepts an offer only if it yields weakly positive profits given his post-offer (interim) beliefs. 4) Each type's offer is optimal given the lender's actions.

We begin by defining the set of technologically feasible allocations $\mathcal{A}$, where

$$\mathcal{A} \equiv \{(b, d, k, s) : d \geq 0,\, k \geq 0,\, s \leq 1\}.$$

It is worth noting that this formulation of the signaling game assumes that investment is contractible, as in the model of Constantinides and Grundy (1990). If investment were not contractible, signaling would be more difficult since the firm would need to rely exclusively upon financial signals. In reality, the terms of financing are routinely (although not exclusively) linked to firms' stated use of funds. For example, a secured debt contract links financing to a specific capital investment.

To derive the LCSE we begin by solving Program L.

$$\text{PROGRAM L:} \quad \max_{a \in \mathcal{A}} \quad d_L + (1 - s_L)\Omega^{LL}$$

subject to the budget constraint

$$BC_L : d_L + k_L + \chi\gamma b_L^2/2 - w \leq \rho^L + s_L\Omega^{LL}.$$

The left side of BC_L measures the amount of funding provided by the investor and the right side represents the true value of the claims received by the investor. Thus, BC_L ensures the investor makes weakly positive profits in expectation. Program L is solved for each $w \in W$.

Letting the solution to Program L be denoted $a_L^* \equiv (b_L^*, d_L^*, k_L^*, s_L^*)$, we next solve Program H.

$$\text{PROGRAM H:} \quad \max_{a \in \mathcal{A}} \quad d_H + (1 - s_H)\Omega^{HH}$$

subject to the budget constraint

$$BC_H : d_H + k_H + \chi\gamma b_H^2/2 - w \leq \rho^H + s_H\Omega^{HH}$$

and a no-mimic constraint

$$NM_{LH} : d_L^* + (1 - s_L^*)\Omega^{LL} \geq d_H + (1 - s_H)\Omega^{LH}.$$

Program H is solved for each $w \in W$. The solution is denoted $a_H^* \equiv (b_H^*, d_H^*, k_H^*, s_H^*)$.

Notice that in solving Program L we did not impose a no-mimic constraint. Lemma 1 shows this is without loss of generality.[10]

Lemma 1. *Assume that a_H^* and a_L^* solve Program H and Program L respectively, and that $s_L^* \geq 0$. Then the high type prefers a_H^* to a_L^*.*

Proof. See Appendix A.

Let $\widehat{\theta}(a)$ denote the type inferred by the investor conditional upon receiving an arbitrary offer $a \in \mathcal{A}$. A PBE in which the firm receives the type-contingent allocations (a_L^*, a_H^*) can be supported by the following investor beliefs.

$$\begin{aligned}
\widehat{\theta}(a_H^*) &= \theta_H, \qquad (11)\\
\widehat{\theta}(a_L^*) &= \theta_L,\\
\widehat{\theta}(a) &\in \arg\min_{\theta\in\Theta} \quad s\int_{\varepsilon_{\theta\theta}^d}^{\infty} v_\theta[(1-\delta)k + \theta\varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon +\\
&\qquad b\int_{\varepsilon_{\theta\theta}^d}^{\infty} f(\varepsilon)d\varepsilon + \int_{\underline{\varepsilon}}^{\varepsilon_{\theta\theta}^d}[(1-\delta)k + \theta\varepsilon k^\alpha - (1-\phi)w^d]f(\varepsilon)d\varepsilon,\\
\forall a &\notin \{a_L^*, a_H^*\}.
\end{aligned}$$

On the equilibrium path, the beliefs in (11) are consistent with Bayes' rule. Off the equilibrium path, the investor imposes worst-case beliefs in the sense of Brennan and Kraus (1987). In particular, the investor attaches the lowest possible valuation to any package of securities not issued in equilibrium. For example, if the firm were to issue shares and/or debt, a worst-case belief imputes type θ_L. If the firm were to repurchase shares (and issue no debt), then a worst-case belief imputes type θ_H.

We now verify that the solutions to Programs L and H in conjunction with beliefs (11) constitute a PBE. First note that any offer $a_0 \notin \{a_L^*, a_H^*\}$ that would be acceptable to the investor must satisfy both BC_L and BC_H, because the beliefs minimize the value of the package of securities. We next verify that it is optimal for insiders observing θ_L and θ_H to offer to the investor a_L^* and a_H^*, respectively. Consider first the low type's incentive to offer some allocation $a_0 \notin \{a_L^*, a_H^*\}$ that would be acceptable to the investor. Since a_0 is acceptable, it must satisfy BC_L. Thus, a_0 was in the feasible set for Program L and optimality

[10] In solving Program L, we may confine attention to $s_L \geq 0$ without loss of generality. To see this, note that a share repurchase by the low type can be replaced with a dividend without affecting the value of the objective function.

implies the low type prefers a_L^* to a_0. This same argument shows the low type will not make an offer that is rejected, since such an offer induces zero outside funding, and zero outside funding is always acceptable to the investor. Therefore, the low type prefers a_L^* to all offers other than a_H^*. Finally, NM_{LH} ensures the low type prefers a_L^* to a_H^*.

Consider next the high type's incentive to offer some allocation $a_0 \neq a_H^*$ that would be accepted by the investor. Note that the allocation a_0 was in the feasible set for Program H. To see this, note that BC_H is necessarily satisfied, since the offer is acceptable even with worst-case beliefs. Since a_0 also satisfies BC_L, we know a_0 was in the feasible set for Program L. The optimality of a_L^* in Program L implies that NM_{LH} would be satisfied if the high type were to offer a_0. Thus, a_0 was in the feasible set for Program H, and the optimality of a_H^* for this program implies the high type prefers a_H^* to a_0. This same argument shows that the high type will not make an offer that is rejected, since such an allocation is equivalent to zero outside funding, and zero outside funding is always acceptable to the investor.

The next step in the construction is to define the equity value functions recursively. In order to derive the end-of-period value of total shareholders' equity, we begin by noting that next period's insider holds a fraction equal to m/c of all shares. Therefore, the total value of all shares is simply c/m times the payoff to the next insider as given in equation (10). Recalling that the next realized type is stochastic, we have:

$$\begin{aligned} v_j(w) \equiv\ & \pi_{Hj}\left[d_H^* + (1-s_H^*)\beta\int_{\varepsilon_{HH}^d}^{\infty} v_H[(1-\delta)k_H^* + \theta_H\varepsilon(k_H^*)^\alpha - b_H^*]f(\varepsilon)d\varepsilon\right] \\ & +(1-\pi_{Hj})\left[d_L^* + (1-s_L^*)\beta\int_{\varepsilon_{LL}^d}^{\infty} v_L[(1-\delta)k_L^* + \theta_L\varepsilon(k_L^*)^\alpha - b_L^*]f(\varepsilon)d\varepsilon\right]. \end{aligned} \tag{12}$$

Although above we omitted w as an argument in the optimal allocation vectors, it is worth stressing that the allocations $(a_L^*, a_H^*)(w)$ are contingent upon net worth at the start of the period.

The final step in the construction is the determination of the minimal level of net worth, denoted w^d. To this end, we begin by noting that BC_L is clearly binding in the optimal program. Substituting this constraint into the maximand, we can rewrite Program L as

$$\text{Program L}: \max_{b_L, k_L} \quad w + \rho^L(b_L, k_L) + \Omega^{LL}(b_L, k_L) - k_L - \chi\gamma b_L^2/2.$$

It follows that the objective function for Program L is linear in net worth. Therefore, there exists a minimal level of net worth, denoted $\widehat{w}$, at which the maximized objective in Program L is just equal to zero. For any $w < \widehat{w}$, the limited liability of shareholders would be violated. Next note that the maximized objective in Program H is necessarily equal to zero at this same level of net worth $\widehat{w}$. To see this, note that any allocation with $d_H > 0$ or $s_H < 1$ would violate NM_{LH}. It follows that regardless of the realized type, shareholders achieve a continuation value of zero if they start a financing round with net worth equal to $\widehat{w}$. Therefore, in

the baseline model, where attention is confined to LCSE, the default threshold is $w^d = \widehat{w}$, where

$$\widehat{w} \equiv -\max_{b_L, k_L} \quad \rho^L(b_L, k_L) + \Omega^{LL}(b_L, k_L) - k_L - \chi\gamma b_L^2/2. \tag{13}$$

Finally, we note that deadweight signaling costs shift down the equity value functions and limit the amount of external equity the firm can raise in order to extinguish prior debt obligations. To the best of our knowledge, such informational effects are absent from all existing structural models.

3 Equilibrium of Simple Signaling Game

The full-information first-best investment policy is

$$k_i^{FB} = \left[\frac{\alpha\theta_i E(\varepsilon)}{r+\delta}\right]^{1/(1-\alpha)}. \tag{14}$$

The following remark provides a description of the full-information economy.

Remark. If the profit shock θ_i is public information, default is costly ($\phi > 0$), and financial slack is costly ($\gamma > 0$), then the firm implements first-best investment (k_i^{FB}) using a combination of equity and default-free debt. The firm will never hold financial slack ($b_i \geq 0$).

The above remark shows that bankruptcy costs ($\phi > 0$) are not sufficient to induce distortions away from first-best or pseudo-risk-aversion. For if there is symmetric information, the firm can still achieve first-best by financing entirely with equity. However, if there is hidden information, financing entirely with equity becomes problematic since low types would have an incentive to mimic high types and capture gains from mispricing.

3.1 Low Type Policies

In order to express the optimality conditions compactly, we define some additional variables. The marginal effect of b on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned} \Omega_b^{HH} &\equiv -\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H'[(1-\delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H] f(\varepsilon) d\varepsilon \\ \Omega_b^{LH} &\equiv -\beta \int_{\varepsilon_{LH}^d}^{\infty} v_L'[(1-\delta)k_H + \theta_L \varepsilon k_H^\alpha - b_H] f(\varepsilon) d\varepsilon. \end{aligned} \tag{15}$$

The marginal effect of k on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned}\Omega_k^{HH} &\equiv \beta \int_{\varepsilon_{HH}^d}^{\infty} v_H'[(1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H][1-\delta+\alpha\theta_H \varepsilon k_H^{\alpha-1}]f(\varepsilon)d\varepsilon \\ \Omega_k^{LH} &\equiv \beta \int_{\varepsilon_{LH}^d}^{\infty} v_L'[(1-\delta)k_H + \theta_L \varepsilon k_H^{\alpha} - b_H][1-\delta+\alpha\theta_L \varepsilon k_H^{\alpha-1}]f(\varepsilon)d\varepsilon.\end{aligned} \tag{16}$$

Let $\mu(\cdot)$ denote the wealth-contingent multiplier on the NM_{LH} constraint in Program H.

In order to interpret the optimality conditions, it is useful to know the shadow value of internal resources. This is the subject of Lemma 2.

Lemma 2. *For all net worth levels $w > w^d$, the shadow value of internal resources is*

$$v_i'(w) = 1 + \pi_{Hi} \times \mu(w) \times [\Omega^{HH}(w) - \Omega^{LH}(w)]/\Omega^{HH}(w). \tag{17}$$

Proof. See Appendix A.

The intuition for Lemma 2 is as follows. If the realized type in the subsequent period is θ_L, a dollar of internal funds is just worth a dollar, as the firm essentially obtains financing on fair terms. However, if the realized type in the subsequent period is θ_H, a dollar of internal funds can be worth more than a dollar if it allows the firm to avoid signaling costs. By way of contrast, one-period models implicitly force $v' = 1$ since the firm never confronts informational asymmetries after the initial round of financing.

Define λ_i, η_i and ψ_i as the multipliers on the budget (BC_i), dividend non-negativity, and equity constraints ($s_i \leq 1$), respectively. The Lagrangian for Program L is

$$L = d_L + (1-s_L)\Omega^{LL} + \lambda_L[w - d_L - k_L - \chi\gamma b_L^2/2 + s_L\Omega^{LL} + \rho^L] + \eta_L d_L + \psi_L(1-s_L).$$

The first-order conditions for d_L and s_L are

$$1 - \lambda_L + \eta_L = 0 \tag{18}$$

$$(\lambda_L - 1)\Omega^{LL} - \psi_L = 0. \tag{19}$$

From equations (18) and (19) it follows that $\eta_L\Omega^{LL} = \psi_L$. Since ε has unbounded support, $\psi_L(w) > 0$ cannot occur for $w > w^d$. To see this, note that the proposed equilibrium would entail the low type getting zero. But then the low type would always opt for the high type allocation unless $s_H = 1$ and $d_H = 0$. Of course, this implies both types get a payoff of zero which contradicts being on shareholders' continuation region.

The first-order condition pinning down b_L^* is

$$\beta\left[\int_{\varepsilon_{LL}^d}^{\infty}[v_L'((1-\delta)k_L+\theta_L\varepsilon k_L^{\alpha}-b_L)-1]f(\varepsilon)d\varepsilon\right]=-\chi\gamma b_L+\beta\frac{\partial\varepsilon_{LL}^d}{\partial b_L}f(\varepsilon_{LL}^d)\phi w_L^d. \tag{20}$$

From Lemma 2 it follows that the left side of (20) is positive. It follows that b_L must be negative. The optimality condition for k_L takes a similar form, with

$$\beta\left[\int_{\underline{\varepsilon}}^{\infty}[v_L'((1-\delta)k_L+\theta_L\varepsilon k_L^{\alpha}-b_L)][1-\delta+\alpha\theta_L\varepsilon k_L^{\alpha-1}]f(\varepsilon)d\varepsilon\right]=1.$$

The next proposition follows directly from the first-order conditions for the debt and capital of the low type.

Proposition 1. *In the least-cost separating equilibrium, the low type chooses second-best policies. In particular,* $k_L^*(w)=k_L^{SB}>k_L^{FB}$ *and* $b_L^*(w)=b_L^{SB}<0$ *where*

$$b_L^{SB}=-\frac{\beta}{\gamma}\left[\int_{\underline{\varepsilon}}^{\infty}[v_L'((1-\delta)k_L^{SB}+\theta_L\varepsilon(k_L^{SB})^{\alpha}-b_L^{SB})-1]f(\varepsilon)d\varepsilon\right]. \tag{21}$$

Dividends and equity issuance for the low type are contingent upon net worth with

$$\begin{aligned} w &< k_L^*-\beta b_L^*+\gamma(b_L^*)^2/2\Rightarrow d_L^*=0 \;\; \textit{and} \;\; s_L^*>0\\ w &\geq k_L^*-\beta b_L^*+\gamma(b_L^*)^2/2\Rightarrow d_L^*=w-(k_L^*-\beta b_L^*+\gamma(b_L^*)^2/2) \;\; \textit{and} \;\; s_L^*=0. \end{aligned}$$

The intuition for the low type policies are as follows. In order to discourage imitation by the low type, the LCSE makes the low type as well off as possible. This is a standard feature of signaling models. However, in most signaling models, the optimal policy entails giving the low type the full-information first-best allocation. In the present model, the low type is given a "second-best" allocation which accounts for the fact that the shadow value of internal resources exceeds unity. Consequently, the low type overinvests relative to first-best and maintains financial slack. It is also worth emphasizing that both b_L^* and k_L^* are invariant to net worth. Therefore, the low type satisfies BC_L by varying dividends and equity issuance only. Note, this is the exact opposite of the pecking-order hypothesis that debt (and only debt) is used to achieve budget balance.

3.2 High Type Policies

The Lagrangian for Program H is

$$\begin{aligned} L &= d_H+(1-s_H)\Omega^{HH}+\lambda_H[w-d_H-k_H-\chi\gamma b_H^2/2+s_H\Omega^{HH}+\rho^H]\\ &\quad+\mu[d_L^*+(1-s_L^*)\Omega^{LL}-d_H-(1-s_H)\Omega^{LH}]+\eta_H d_H+\psi_H(1-s_H). \end{aligned}$$

The first-order conditions for d_H and s_H are

$$1 - \lambda_H - \mu + \eta_H = 0 \tag{22}$$

$$(\lambda_H - 1)\Omega^{HH} + \mu\Omega^{LH} - \psi_H = 0 \tag{23}$$

Substituting (22) into (23), we obtain

$$\eta_H \Omega^{HH} = \mu(\Omega^{HH} - \Omega^{LH}) + \psi_H. \tag{24}$$

It is straightforward to establish $\psi_H(w) = 0$ on the continuation region ($w > w^d$). To see this, suppose to the contrary that $\psi_H > 0$. It follows from (24) that $\eta_H > 0$ and $d_H = 0$. Thus, the high type gets a payoff of zero. However, the low type would then also receive an allocation with a payoff of zero since the NM_{HL} constraint demands

$$d_H + (1 - s_H)\Omega^{HH} \geq d_L + (1 - s_L)\Omega^{HL} \geq d_L + (1 - s_L)\Omega^{LL}. \tag{25}$$

But this contradicts being on shareholders' continuation region. Without loss of generality we shall treat $\psi_H = 0$ as we solve for optimal policies on the continuation region.

Rearranging (24) we obtain

$$\eta_H = \mu[(\Omega^{HH} - \Omega^{LH})/\Omega^{HH}]. \tag{26}$$

Condition (26) is of fundamental importance. It tells us that whenever NM_{LH} binds, the insider would be better off if the firm could implement a negative dividend. The lemons problem is manifest in condition (26), with the term in squared brackets representing the relative difference in true equity values for the two types. From this optimality condition it also follows that the high type will not pay a dividend if NM_{LH} binds.

The optimality condition pinning down b_H^* is

$$\begin{aligned} & \beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [v_H'((1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H) - 1] f(\varepsilon) d\varepsilon - \frac{\partial \varepsilon_{HH}^d}{\partial b_H} f(\varepsilon_{HH}^d) \phi w_H^d \right] + \chi\gamma b_H \qquad (27) \\ = & \left(\frac{\mu\Omega^{LH}}{\lambda_H} \right) [1 - s_H] \left[\frac{\Omega_b^{HH}}{\Omega^{HH}} - \frac{\Omega_b^{LH}}{\Omega^{LH}} \right]. \end{aligned}$$

The term $v_H' - 1$ on the left side of (27) captures a deadweight cost of debt service since shareholders value a dollar of internal funds (weakly) more than the lender receiving that dollar. The next term measures the deadweight costs of default. The third term measures the marginal cost of saving. The right side measures the signal value of debt, discussed in greater detail below. From (27) and Lemma 2 it follows immediately that $\mu = 0 \Rightarrow b_H \leq 0$. Condition (27) therefore leads to a key implication of the model, that signaling must

be a concern if the firm issues a positive amount of debt.[11]

The optimality condition pinning down k_H^* is

$$
\begin{aligned}
1 = \beta & \left[\int_{\varepsilon_{HH}^d}^{\infty} [v_H'((1-\delta)k_H + \theta_H \varepsilon k_H^{\alpha} - b_H)][1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon \right] \\
+\beta & \left[\int_{\underline{\varepsilon}}^{\varepsilon_{HH}^d} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha-1}] f(\varepsilon) d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial k_H} f(\varepsilon_{HH}^d) \phi w_H^d \right] \\
+ & \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) [1 - s_H] \left[\frac{\Omega_k^{HH}}{\Omega^{HH}} - \frac{\Omega_k^{LH}}{\Omega^{LH}} \right]
\end{aligned}
\tag{28}
$$

Equation (28) states that the optimal investment policy equates marginal benefits with the unit price of capital. The first term on the right side of the equation measures the benefit to shareholders from an additional unit of installed capital. The second term measures the benefit of investment accruing to bondholders. The last term measures the signaling benefit provided by investment, which is discussed in greater detail below.

From (27) and (28) it also follows that

$$
\begin{aligned}
\mu(w) &= 0 \Rightarrow b_H^*(w) = b_H^{SB} \text{ and } k_H^*(w) = k_H^{SB} \\
&\text{where} \\
b_H^{SB} &= -\frac{\beta}{\gamma} \left[\int_{\underline{\varepsilon}}^{\infty} [v_H'((1-\delta)k_H^{SB} + \theta_H \varepsilon (k_H^{SB})^{\alpha} - b_H^{SB}) - 1] f(\varepsilon) d\varepsilon \right] \\
&\text{and} \\
1 &= \beta \left[\int_{\underline{\varepsilon}}^{\infty} [v_H'((1-\delta)k_H^{SB} + \theta_H \varepsilon (k_H^{SB})^{\alpha} - b_H^{SB})][1 - \delta + \alpha \theta_H \varepsilon (k_H^{SB})^{\alpha-1}] f(\varepsilon) d\varepsilon \right].
\end{aligned}
$$

Proposition 2 spells out some important implications of the above optimality conditions.

Proposition 2. *In the least-cost separating equilibrium, there exists a net worth level w_{slack} at which the low type's no-mimic constraint becomes slack. For all $w > w_{slack}$, the high type implements second-best policies with $k_H^* = k_H^{SB} > k_H^{FB}$ and $b_H^* = b_H^{SB} < 0$. For $w > w_{slack}$, dividends and equity issuance are contingent upon net worth with*

$$
\begin{aligned}
w &\in [w_{slack}, k_H^{SB} - \beta b_H^{SB} + \gamma (b_H^{SB})^2/2) \Rightarrow d_H^* = 0 \text{ and } s_H^* > 0 \\
w &\in [k_H^{SB} - \beta b_H^{SB} + \gamma (b_H^{SB})^2/2, \infty) \Rightarrow d_H^* = w - [k_H^{SB} - \beta b_H^{SB} + \gamma (b_H^{SB})^2/2] \text{ and } s_H^* = 0.
\end{aligned}
$$

Proof. See Appendix A.

Discussion of the economic content of conditions (27) and (28) for $\mu > 0$ is delayed until the next subsection which links the optimality conditions to single-crossing conditions common in the signaling literature.

[11] This result no longer holds once we introduce tax advantages to debt finance.

3.3 Single-Crossing Conditions

In this subsection we consider low net worth states such that the no-mimic constraint is binding. In the previous subsection it was established that $d_H = 0$ when NM_{LH} binds. Let us now consider the normalized payoff $u(b, k, s; \theta_i)$ to a type-i insider that takes some arbitrary allocation (b, k, s) such that $d = 0$

$$u(b, k, s; \theta_i) \equiv (1 - s)\beta \int_{\varepsilon^d}^{\infty} v_i[(1 - \delta)k + \theta_i \varepsilon k^\alpha - b]f(\varepsilon)d\varepsilon. \tag{29}$$

Next, we compute the total differential of u, evaluated at the high type allocation. We have

$$du(b_H, k_H, s_H; \theta_i) = (1 - s_H)\Omega_k^{iH} * dk + (1 - s_H)\Omega_b^{iH} * db - \Omega^{iH} * ds. \tag{30}$$

Setting the derivative to zero, one obtains the slope of indifference curves evaluated at the high type allocation. The insider's willingness to exchange equity ownership for additional capital is determined by

$$\frac{ds}{dk}(b_H, k_H, s_H; \theta_i) = \frac{(1 - s_H)\Omega_k^{iH}}{\Omega^{iH}}. \tag{31}$$

This indifference curve notation allows us to rewrite the optimality condition for the high-type capital stock (28) as

$$\begin{aligned} 1 = {} & \beta \left[\int_{\varepsilon_{HH}^d}^{\infty} [v_H'((1 - \delta)k_H + \theta_H \varepsilon k_H^\alpha - b_H)][1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha - 1}]f(\varepsilon)d\varepsilon \right] \\ & + \beta \left[\int_{\underline{\varepsilon}}^{\varepsilon_{HH}^d} [1 - \delta + \alpha \theta_H \varepsilon k_H^{\alpha - 1}]f(\varepsilon)d\varepsilon + \frac{\partial \varepsilon_{HH}^d}{\partial k_H} f(\varepsilon_{HH}^d)\phi w_H^d \right] \\ & + \left(\frac{\mu \Omega^{LH}}{\lambda_H} \right) \left[\frac{ds}{dk}(b_H, k_H, s_H; \theta_H) - \frac{ds}{dk}(b_H, k_H, s_H; \theta_L) \right]. \end{aligned} \tag{32}$$

The optimality condition above tells us that the high-type's capital stock increases with the signal content of capital, as measured by the difference in the slope of the two type's indifference curves in k-s space. For example, in figure 1A the indifference curves are drawn under the assumption that the high type has a greater willingness to exchange equity for capital. In this case, higher capital investment would provide a positive signal which encourages overinvestment relative to first-best.

Intuition suggests that four factors determine the relative slopes of the types' indifference curves in k-s space. First, the high type generates more future cash than a low type for a given level of capital. Second, persistence in θ implies that a high type has a high probability of being a high type in the subsequent period. For such a firm, the future internal cash generated by installed capital may be more valuable since internal funds reduce exposure to signaling costs. These first two effects serve to increase the slope of the high-type indifference curve. However, the low type knows his equity is less valuable than that of the high type, which increases his willingness to exchange equity for capital. In addition, the low type *may* place a higher shadow

value on a marginal dollar at the end of the period, since it necessarily realizes lower net worth when it receives the high-type allocation.

The insider's willingness to exchange equity for debt reductions is determined by

$$\frac{ds}{db}(b_H, k_H, s_H; \theta_i) = \frac{(1-s_H)\Omega_b^{iH}}{\Omega^{iH}}. \tag{33}$$

Using this indifference curve relationship allows us to rewrite the debt optimality condition for the high type (27) as

$$\begin{aligned} & \beta\left[\int_{\varepsilon^d_{HH}}^{\infty}[v'_H((1-\delta)k_H+\theta_H\varepsilon k_H^{\alpha}-b_H)-1]f(\varepsilon)d\varepsilon-\frac{\partial\varepsilon^d_{HH}}{\partial b_H}f(\varepsilon^d_{HH})\phi w_H^d\right]+\chi\gamma b_H \qquad (34)\\ = & \left(\frac{\mu\Omega^{LH}}{\lambda_H}\right)\left[\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_L)\right|-\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_H)\right|\right]. \end{aligned}$$

Equation (34) tells us that the high-type's borrowing depends upon the signal content of debt, as measured by the difference in the slope of the indifference curves in b-s space. Recall that when the constraint NM_{LH} is slack, the high type will choose to save an amount $b_H^{SB} < 0$ such that the left side of (34) is equal to zero. Starting at this point, if the firm were to decrease its saving, the left side of (34) would increase. With this in mind, consider the indifference curves in figure 1B. In figure 1B, the low type is assumed to be more willing to exchange equity for a reduction in debt. Therefore, debt provides a positive signal and the high type borrows more/saves less in order to separate from the low type.

An efficient mix of real and financial signals equates the ratio of distortion to signal content at the margin. In particular, equations (32) and (34) imply that

$$\begin{aligned} & \frac{1-\beta\left[\int_{\varepsilon^d_{HH}}^{\infty}(v'_H)*[1-\delta+\alpha\theta_H\varepsilon k_H^{\alpha-1}]f(d\varepsilon)+\int_{\underline{\varepsilon}}^{\varepsilon^d_{HH}}[1-\delta+\alpha\theta_H\varepsilon k_H^{\alpha-1}]f(d\varepsilon)+\frac{\partial\varepsilon^d_{HH}}{\partial k}f(\varepsilon^d_{HH})\phi w_H^d\right]}{\frac{ds}{dk}(b_H,k_H,s_H;\theta_H)-\frac{ds}{dk}(b_H,k_H,s_H;\theta_L)}\\ = & \frac{\beta\left[\int_{\varepsilon^d_{HH}}^{\infty}[(v'_H)-1]f(d\varepsilon)-\frac{\partial\varepsilon^d_{HH}}{\partial b_H}f(\varepsilon^d_{HH})\phi w_H^d\right]+\chi\gamma b_H}{\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_L)\right|-\left|\frac{ds}{db}(b_H,k_H,s_H;\theta_H)\right|}. \end{aligned}$$

The numerator of the left side of the above equation measures the deviation of capital from second-best (where second-best accounts for precautionary value and default costs). The numerator of the right side measures the deviation of debt from second-best. The denominators measure the marginal information content of real and financial signals, respectively.

It is difficult to derive general results for the signaling terms that will hold for all possible distributions (f) of the public shock. However, we obtain strong results on the signal content of safe debt provided the private shocks are i.i.d. Since the high type's equity is more valuable ($\Omega^{HH} > \Omega^{LH}$), a sufficient condition for debt to be a positive signal, in the sense of the low type being more willing to exchange equity for debt reductions, is that the low type attach a higher marginal cost to debt service. Using integration by parts it

is possible to show that

$$\left|\Omega_b^{HH}\right| \leq \left|\Omega_b^{LH}\right| \Leftrightarrow [F(\varepsilon_{LH}^d) - F(\varepsilon_{HH}^d)]v'(w^d) \leq -\int_{w^d}^{\infty} v''(w)\left[F\left(\frac{w-(1-\delta)k+b}{\theta_L k^\alpha}\right) - F\left(\frac{w-(1-\delta)k+b}{\theta_H k^\alpha}\right)\right]dw. \tag{35}$$

It is readily observed that the bracketed term on the right side of equation (35) is strictly positive. Lemma 3 follows directly.

Lemma 3. *If the private shocks are i.i.d. and the equity value function is weakly concave, then a marginal increase in safe (across types) debt is a positive signal.*

The intuition for Lemma 3 is simple. When debt is safe, both types are servicing debt with probability one. If the value function is concave, the low type necessarily attaches a higher shadow cost to servicing debt since his realized net worth will be lower for any realization of ε.

3.4 Example: Exponentially Distributed Public Shocks

This subsection assumes ε is exponentially distributed, with $f(\varepsilon) \equiv \xi e^{-\varepsilon\xi}$. Under this distributional assumption, we obtain unambiguous results regarding the signal content of debt even allowing for correlation in the private shocks. We confine attention to the signal content of defaultable debt, although similar results are obtained when we consider the issuance of safe debt.

We begin by rewriting the expression for the marginal cost of debt (15) as follows

$$\Omega_b^{ij} = \frac{-\beta}{\theta_i k_j^\alpha}\int_{\varepsilon_{ij}^d}^{\infty} f(\varepsilon)\left(\frac{\partial}{\partial\varepsilon}v_i((1-\delta)k_j + \theta_i\varepsilon k_j^\alpha - b_j)\right)d\varepsilon. \tag{36}$$

Using integration by parts it follows that

$$\Omega_b^{ij} = \frac{-\xi\Omega^{ij}}{\theta_i k_j^\alpha}. \tag{37}$$

It follows directly that debt issuance is a positive signal, with

$$\frac{\Omega_b^{HH}}{\Omega^{HH}} - \frac{\Omega_b^{LH}}{\Omega^{LH}} = \frac{\xi(\theta_L^{-1} - \theta_H^{-1})}{k_H^\alpha} > 0. \tag{38}$$

We have thus established Proposition 3, our strongest analytical result on the signal content of debt.

Proposition 3. *If the public shocks have the exponential distribution, then a marginal increase in debt is a positive signal.*

A similar calculation allows us to write

$$\begin{aligned}\frac{\Omega_k^{HH}}{\Omega^{HH}} - \frac{\Omega_k^{LH}}{\Omega^{LH}} &= \frac{\alpha\xi\beta}{k_H}\left[\frac{Cov[v_H(w(b_H,k_H,\varepsilon,\theta_H)),\varepsilon]}{\Omega^{HH}} - \frac{Cov[v_L(w(b_H,k_H,\varepsilon,\theta_L)),\varepsilon]}{\Omega^{LH}}\right] \\ &\quad - \frac{(1-\delta)\xi(\theta_L^{-1}-\theta_H^{-1})}{k_H^\alpha}.\end{aligned} \tag{39}$$

The sign of the signal content of investment, given above, is ambiguous, reflecting the competing factors discussed above. On one hand, the high type can put his capital to more productive use and may have a stronger precautionary motive, since he has a higher probability of realizing a high type in the following period. However, it is also possible for the low type to have a stronger precautionary motive, since it will have lower net worth for any realization of ε. These effects are captured by the covariance terms. In general, one expects firm value to have a higher covariance with ε when the type is high since $dv/d\varepsilon = \theta k^{\alpha} v'$. However, the fact that the high type's equity is more valuable ($\Omega^{HH} > \Omega^{LH}$) makes it impossible to determine whether a high type is more willing to exchange equity for capital. The term on the second line is negative, and equal to -$(1-\delta)$ times the debt signaling term. To understand this term, note that part of the return to installed capital is nonstochastic, since a unit of capital will be worth $(1-\delta)$ at the end of the period. The signaling expression for debt measures the insider's willingness to exchange equity for a reduction in a fixed payment obligation. Similarly, the signaling expression for capital measures, in part, the insider's willingness to exchange equity for an increased fixed payment.

4 Simulation of Baseline Model

The following parameter values are used in the simulations: $\beta = 1/1.065$; $\alpha = 0.60$; $\delta = 0.10$; $\gamma = 0.05$; $\phi = 0.20$; $\theta_H = 0.8$; $\theta_L = 0.6$; and $\pi_{HL} = \pi_{HH} = 1/2$. The public shocks are exponentially distributed with $f(\varepsilon) = 2\exp(-2\varepsilon)$. The procedure used to solve the model numerically is described in Appendix B. Once the model is solved, the wealth-contingent policy functions $(a_L^*, a_H^*) : W \to \mathcal{A} \times \mathcal{A}$ are used to generate a simulated panel data set. We draw 3000 sample paths of public and private shocks consisting of 31 draws for each. All firms start with zero net worth and the initial period is dropped from the sample. We then use the policy functions generated by the model to determine shock-contingent policy and wealth paths. The simulated panel data set is similar in size to those commonly used in empirical testing.

To illustrate concavity of the equity value function, or pseudo-risk-aversion, we begin by plotting the firm's enterprise value. The enterprise value of a firm is traditionally defined as the difference between firm value and its cash balance. Since physical capital is perfectly reversible in the model, a dollar in cash and a dollar in capital have equivalent implications for equity's continuation value. In this setting, it is appropriate to define the *enterprise value function* (e) as the difference between ex ante equity value and net worth[12]

$$e(w) \equiv v(w) - w.$$

The slope of the enterprise value function is a direct measure of the precautionary motive for avoiding debt, since $e' = v' - 1$. The slope of the enterprise value function measures the marginal net gain current shareholders would capture if they could inject cash directly into the firm. We can express the enterprise

[12]This definition assumes the θ draws are i.i.d., as is the case in the numerical analysis.

value as

$$e(\overline{w}) = \left|w^d\right| + \int_{w^d}^{\overline{w}} [v'(w) - 1]dw \qquad \forall \quad \overline{w} > w^d. \tag{40}$$

Thus, the enterprise value, at any point in the wealth space is equal to the going-concern value plus the cumulative precautionary value of internal resources. Figure 2 plots the enterprise value function. Consistent with concavity of the equity value function (v), the slope of the enterprise value function is positive but decreasing in net worth. The enterprise value has a slope of zero for high levels of net worth where the no-mimic constraint is slack. Examining figure 2, we see that starting at the default threshold, the value gained from a one unit cash infusion is roughly equal to 1.05. This magnitude is sensitive to the parameterization of the model. For example, Lemma 2 indicates that the gain from cash infusions increases in the probability of the high type.

Figure 3 plots the capital allocations of each type relative to first-best. In the LCSE, the low type invests slightly above first-best regardless of realized net worth. This overinvestment reflects the fact that informational asymmetries create a precautionary motive for capital accumulation. The high type also invests more than first-best, with the extent of the distortion decreasing in net worth. When net worth is low, the no-mimic constraint binds and the high type overinvests in order to signal positive information. In addition, this installed capital reduces bankruptcy costs. If net worth is sufficiently high, the no-mimic constraint is slack, but the high type still overinvests due to precautionary motives.

Figures 4 plots the wealth-contingent financing policies for each type. Consistent with Proposition 1, the low type uses dividends and equity issuance as the sole means of achieving budget-balance, while retaining a wealth-invariant level of savings. When net worth is low, the low type sets the dividend to zero and issues a large amount of equity. Equity issuance for the low type then declines monotonically in net worth.

The debt of the high type declines monotonically in net worth. Intuitively, increases in net worth reduce the need for external financing. This allows the high type to cut back on both costly signals (overinvestment and risky debt). Therefore, after a high realization of the public (ε) shock, the model predicts that firms will choose low leverage ratios. Thus, the model is consistent with the empirical observation that leverage ratios are countercyclical (e.g. Korajczyk and Levy (2004)). This is also consistent with the observation that leverage ratios are decreasing in lagged profitability (e.g. Fama and French (2002)).

It is worth noting that when the high type issues equity, it almost always conducts a *joint offering* which combines equity with debt. In contrast, the low type issues equity without any debt. This is consistent with existing studies. Asquith and Mullins (1986) document negative abnormal returns in a sample of pure common stock offerings. Masulis and Korwar (1986) find that seasoned equity offerings are associated with negative price changes on average. However, the announcement return is positively related to leverage changes.

Turning next to dividend policy, we see that both types pay dividends only if net worth is sufficiently high. This prediction is consistent with the empirically observed positive relation between dividends and firm size. It is also apparent that the low type initiates dividends at a lower net worth threshold than the

high type. This difference reflects the fact that the low type has inferior investment opportunities.

Table 1 reports reduced-form leverage regressions similar to those reported in the empirical literature. In the first three regressions, the dependent variable is the book leverage ratio. In the last regression, the dependent variable is the change in the book leverage ratio. The first row utilizes a specification from Shyam-Sunder and Myers (1999). Shyam-Sunder and Myers use this regression to test the pecking-order hypothesis that debt is used to fill all financing gaps. In our model, the financing gap is $k + d - w$. According to the pecking-order as traditionally specified, the predicted coefficient on the (capital-normalized) financing gap is one. Inspecting Table 1, we see that the simulated firms fill only 24% of the financing gap with debt. Leary and Roberts (2006) show that the test constructed by Shyam-Sunder and Myers is prone to Type II errors. Based on this simulated regression, we argue that the statistical test proposed by Shyam-Sunder and Myers is also prone to a form of Type I error. In our simulated data, the null hypothesis that "hidden information influences financing decisions" would be incorrectly rejected if the econometrician were to assume that the coefficient on the financing gap should be one.

The next two regressions are similar to those reported by Fama and French (2002). The objective here is to show that the model is broadly consistent with the stylized facts. Fama and French report a negative relationship between leverage and lagged profits. In the simulated data, we also find that leverage is declining in lagged measures of profitability. In the model, high realizations of the public shock ε result in high lagged profitability and high net worth. Firms with positive information use the increase in net worth to reduce their reliance on debt. Also consistent with the estimates of Fama and French, the coefficient on the market-to-book ratio is positive in the simulated data.

Shyam-Sunder and Myers (1999) and Fama and French (2002) both estimate variants of the following equation to test for mean-reversion in leverage

$$LEV_t - LEV_{t-1} = B_0 + B_{TA} * [LEV^* - LEV_{t-1}] + u_t. \tag{41}$$

We follow Shyam-Sunder and Myers in using the firm-specific sample average leverage ratio as an estimate of the "target" leverage ratio. Shyam-Sunder and Myers report a significant estimate of $B_{TA} = 0.41$. Fama and French (2002) estimate smaller, yet significant, coefficients with B_{TA} in the range of 0.07 to 0.10. These regressions are of interest since the empirical literature often treats mean-reverting leverage as prima facie evidence in favor of trade-off theory cum transactions costs. For example, Fama and French (2002) state that, "the simple pecking order predicts that... the speed of adjustment is indistinguishable from zero, whereas the trade-off model says it is reliably positive." In the fourth regression presented in Table 2, the estimated value of B_{TA} is 0.18. Thus, the simulated firm exhibits mean-reversion speeds consistent with those actually observed–absent taxes or direct transactions costs.

The model provides a laboratory for analyzing the cross-sectional determinants of abnormal returns associated with investment and financing announcements. We begin by noting that this analysis is not

always directly comparable to existing empirical studies. In the simulated data, we can measure the stock price after the market has fully incorporated information about lagged earnings. This *ex ante stock price* is equal to $v(w)/c$. We can then measure the stock price just after the firm announces its financing plans. This *ex post stock price* is equal to $[d + (1 - s)\Omega]/c$. The pure abnormal return (AR) associated with the announced financing is the ex post price less the ex ante price normalized by the ex ante price

$$AR_t \equiv \frac{d_{it} + (1 - s_{it})\Omega_t^{ii} - v(w_t)}{v(w_t)}. \tag{42}$$

In contrast, empirically observed abnormal returns may reflect market inferences about lagged earnings in addition to inferences about future prospects.

Table 2 analyzes the cross-sectional determinants of abnormal returns by treating the simulated AR_t as a dependent variable. Such regressions are common to the event study literature. A common theme running through the regressions is that high investment rates are statistically and economically significant signals. This is consistent with the empirical evidence presented by McConnell and Muscarella (1985) that increases in capital expenditures are associated with positive abnormal returns. In the first reported regression, the abnormal return is positively related to the investment rate. In the second and third regressions we see a high percentage of debt in total external financing is also a positive signal. This is consistent with Masulis' (1983) finding that debt for equity exchanges are associated with positive abnormal returns. In the fourth regression we see that, unconditionally, the abnormal return is increasing in leverage. However, the fifth regression reveals that the significance of leverage is very sensitive to the regression specification. In particular, the leverage ratio becomes insignificant once we include the investment rate as a regressor. This is because the high type always invests much more than the low type in our model. However, the high type does not necessarily issue more debt. For example, when net worth is sufficiently high both firms actually save. Therefore, capital expenditures are the most informative signal.

5 The Option Contract Game

The objective of this section is to determine whether it is possible for the two firm types to pool at the same allocation, say $a^P(w)$. To this end, we rely on results derived by Maskin and Tirole (1992) and Tirole (2006), who consider an option contract game.[13] The utility of the option contract game is that it allows one to narrow the set of possible equilibria in an analytically tractable manner. In reality, one can view a shelf-registration as resembling an option contract since a shelf-registration narrows the menu of securities from which a firm may (costlessly) select after it acquires new information.

The timing and equilibrium concept in the option contract game are the same as that for the simple signaling game. However, the initial offer is a menu rather than an allocation. In particular, the game begins with the informed insider offering an *option contract* consisting of a pair $(a_1, a_2) \in \mathcal{A} \times \mathcal{A}$. Technically,

[13]The exposition closely follows Tirole's Section 6.4.

the contract represents an agreement by the two parties to enter into a direct revelation mechanism. If he accepts, the investor has entered into a binding agreement to fund the firm regardless of which option on the menu the insider subsequently chooses.

We look for PBE at each point in the net worth space. Importantly, each point in the net worth space has measure zero. Therefore, at each point in the state space the agents can treat the value functions as *given*. Similarly, we the modelers can treat the value functions as fixed even as we vary the equilibria at *particular* points. Having said this, all discussions regarding equilibrium at particular points in the net worth space should be understood as being conditioned on internally consistent value functions. Recursive methods will be used to ensure internal consistency.

We begin by noting that for all $w \geq \widehat{w}$, the LCSE entailed $s_L^*(w) \geq 0$. Therefore, for all $w \geq \widehat{w}$, Tirole's weak monotonic profit condition is satisfied and the pair $(a_L^*, a_H^*)(w)$ represents the Rothschild-Stiglitz-Wilson allocation (RSW). For $w < \widehat{w}$, the constraint BC_L cannot be satisfied by any $a \in \mathcal{A}$ and at these points the RSW allocation necessarily corresponds to the reservation allocation with $s = 0$ and $b \leq 0$. Of course, with $w < \widehat{w} < 0$, the lack of outside funding would cause both types to default, resulting in a payoff of zero. With some abuse of jargon, we shall refer to the RSW payoffs as the "separating payoffs."

The following is a restatement of Tirole's Proposition 6.1.

Lemma 4. *The separating payoffs are always in the equilibrium set. The payoffs from pooling at an option contract (a^P, a^P) are in the equilibrium set only if the investor earns a weakly positive profit in expectation (based upon his prior beliefs) and the payoffs Pareto dominate the separating payoffs from the perspective of both insider types.*

Recall that when attention was confined to separating equilibria, the default threshold was given by $w^d = \widehat{w}$ as defined in equation (13). We conjecture, and verify, that the default threshold in the present setting, which allows for pooling, is $w^d = \widehat{w}_P < \widehat{w}$. That is, the possibility of pooling increases the continuation region relative to what is possible if only separating equilibria are considered.

Proposition 4 states that it is possible to support an equilibrium in which pooling occurs for low net worth states while separation occurs for higher net worth states.

Proposition 4. *In the option contract game it is possible to support an equilibrium such that $w^d < \widehat{w}$. In this equilibrium, there exists $\overline{w}^p > \widehat{w}$ such that for $w \in [w^d, \overline{w}^p]$ the payoffs resulting from pooling at an allocation $a^P(w)$ are in the equilibrium set. For $w > \overline{w}^p$ there is no pooling equilibrium.*

Proof. We begin by establishing the existence of a pooling region. Choose ϵ arbitrarily small and let $w \equiv \widehat{w} - \epsilon$. The RSW payoffs at this point are zero. However, if the types were to pool using $d = 0$, $b_L^*(\widehat{w})$ and $k_L^*(\widehat{w})$, they could achieve a strictly positive payoff at some $s < 1$ while the investor earns zero expected profit based upon priors. The claimed results then follows from Lemma 4. Next recall that there exists a $w \equiv w_{slack}$ at which the no-mimic constraint in Program H is slack. For $w \geq w_{slack}$, the solution to Program

H yield a higher payoff for the high type than any pooling allocation that gives the investor weakly positive expected profits based upon prior beliefs. From Lemma 4 it follows that no pooling equilibrium can exist on this interval.■

Although pooling extends the continuation region, the firm will still default if net worth is sufficiently low. In particular, the attempt to pool would be unsuccessful if the investor could not break even at $s = 1$. Therefore, we redefine the default threshold as $w^d = \widehat{w}_P$, where

$$\widehat{w}_P \equiv -\max_{b,k} \quad \pi[\rho^H(b,k) + \Omega^H(b,k)] + (1-\pi)[\rho^L(b,k) + \Omega^L(b,k)] - k - \chi\gamma b^2/2. \tag{43}$$

Based upon Lemma 4, we may use the following Program P to determine whether pooling is possible at a particular point. Program P makes the high type as well off as possible subject to: 1) The pooling allocation is improving for the low type relative to his separating allocation (LTI). 2) Provision of outside funding is positive in expectation (PIE) based upon the investor's prior beliefs.

$$\text{PROGRAM P:} \quad \max_{a \in \mathcal{A}} \quad d + (1-s)\Omega^H$$

subject to

$$\begin{aligned} LTI \quad &: \quad d + (1-s)\Omega^L \geq d_L^* + (1-s_L^*)\Omega^{LL} \\ PIE \quad &: \quad \pi[\rho^H(b,k) + \Omega^H(b,k)] + (1-\pi)[\rho^L(b,k) + \Omega^L(b,k)] \geq k + \chi\gamma b^2/2 - w. \end{aligned}$$

We denote the solution to Program P as $a^P \equiv (b^P, d^P, k^P, s^P)$. From Lemma 4 we know that pooling is only possible at point w if $d^P + (1-s^P)\Omega^H \geq d_H^* + (1-s_H^*)\Omega^{HH}$.

We shall denote the pointwise equilibrium allocations as (a_L^{**}, a_H^{**}), where

$$\begin{aligned} d^P(w) + [1-s^P(w)]\Omega^H \quad &\geq \quad d_H^*(w) + [1-s_H^*(w)]\Omega^{HH} \Rightarrow (a_L^{**}, a_H^{**})(w) \equiv (a^P(w), a^P(w)) \\ d^P(w) + [1-s^P(w)]\Omega^H \quad &< \quad d_H^*(w) + [1-s_H^*(w)]\Omega^{HH} \Rightarrow (a_L^{**}, a_H^{**})(w) \equiv (a_L^*, a_H^*)(w). \end{aligned} \tag{44}$$

The final step in the construction is to use a recursive equation to pin down the equity value functions

$$\begin{aligned} v_j(w) \quad \equiv \quad & \pi_{Hj}\left[d_H^{**} + (1-s_H^{**})\beta \int_{\varepsilon_{HH}^d}^{\infty} v_H[(1-\delta)k_H^{**} + \theta_H \varepsilon (k_H^{**})^\alpha - b_H^{**}] f(\varepsilon) d\varepsilon\right] \\ & + (1-\pi_{Hj})\left[d_L^{**} + (1-s_L^{**})\beta \int_{\varepsilon_{LL}^d}^{\infty} v_L[(1-\delta)k_L^{**} + \theta_L \varepsilon (k_L^{**})^\alpha - b_L^{**}] f(\varepsilon) d\varepsilon\right]. \end{aligned} \tag{45}$$

Before turning to the results of the numerical simulation, we present the optimality conditions from Program P. Letting λ denote the multiplier on the PIE constraint, the optimal financing policy in the

pooling equilibrium satisfies

$$\pi(\Omega_b^H + \rho_b^H) + (1-\pi)(\Omega_b^L + \rho_b^L) + \chi\gamma b = \frac{(1-\lambda\pi)\Omega^H(1-s)}{\lambda}\left[\left|\frac{\Omega_b^H}{\Omega^H}\right| - \left|\frac{\Omega_b^L}{\Omega^L}\right|\right] \tag{46}$$

and the optimal investment rule satisfies

$$\pi(\Omega_K^H + \rho_K^H) + (1-\pi)(\Omega_K^L + \rho_K^L) - 1 = \frac{(1-\lambda\pi)\Omega^H(1-s)}{\lambda}\left[\frac{\Omega_K^H}{\Omega_K^H} - \frac{\Omega_K^L}{\Omega_K^L}\right]. \tag{47}$$

From the optimality conditions on d and s it is also possible to show that $1 > \lambda\pi$. The left sides of the two optimality conditions represent the average deadweight loss associated with increases in debt and capital, respectively. Since the high type is pooling, he is concerned about the average level of efficiency. The right side of the optimality conditions represents the information content of increases in the relevant control variable. This term captures the costly cross-subsidy provided by the high type to the low type in any pooling equilibrium.

Figure 5 presents the results from simulations under the same parameterization as in Section 4, but allowing for the possibility of pooling. Consistent with Proposition 4, the firms pool when net worth is low and switch to the LCSE when net worth is sufficiently high. On the pooling interval, the high type underinvests and the low type overinvests relative to first-best. Once there is a switch to the LCSE, the high type sharply increases investment and the low type sharply cuts investment. Figure 6 depicts equilibrium financing policies. On the pooling region, investment is financed primarily with debt, since this reduces cross-subsidies from the high type to the low type. Once again, we see that the switch from pooling to separating equilibrium leads to interesting non-monotonicities in policies. For example, the debt of the high type spikes upward once there is a switch from pooling to separating equilibrium. The debt of the high type then begins falling as increases in net worth reduce the need for costly signaling in the separating equilibrium.

A unique feature of our structural model is that it differentiates between dividends and share repurchases as modes of distributing cash to shareholders. From this perspective, it is interesting to note the share repurchases occur in equilibrium once we allow for pooling. This did not occur when we only considered LCSE. The intuition is simple. When we allow for pooling, the continuation region is larger and default costs are lower. In this case, the logic of the model more closely approximates that of Constantinides and Grundy (1990), where firms issue debt in excess of the amount needed for investment and use the residual funds for share repurchases. In other words, when bankruptcy becomes less costly, the firm places greater reliance on financial signals (as distinct from real investment signals).

6 Integration with Trade-off Theory

The model can be easily extended to include a corporate income tax and the associated tax shield benefit of debt. There are a number of compelling rationales for extending the model to include taxes. First, there

is no reason why theories of financing based upon hidden information and the trade-off theory should be mutually exclusive. It is noteworthy that this point was stressed by Myers (1984) himself. Second, including tax advantages of debt will help the model to explain an even broader set of stylized facts. In particular, Graham (1996) presents empirical evidence in support of the hypothesis that increases in the value of the debt tax shield result in marginal increases in the propensity of firms to use debt finance.

Following the convention in the literature, we assume the corporate income tax rate is a constant $\tau \in (0,1)$. The base of the corporate income tax is operating income less economic depreciation less interest expense plus interest income. Let y denote the promised yield to maturity on debt. From the identity $\rho \equiv b/(1+y)$, it follows that interest expense is $y\rho = b - \rho$. The same expression represents taxable interest income if $b < 0$. In the U.S. payments to lenders are treated as principal first. To capture this rule, we assume interest deductions are disallowed in the event of a default on the debt obligation. Finally, we note that the inclusion of a corporate income tax creates a tax penalty to financial slack since shareholders can earn the rate of return r by saving on their own account while the corporation earns $r(1-\tau)$ after-tax. To isolate this tax effect, below we set agency costs of financial slack equal to zero ($\gamma = 0$).

To summarize, this subsection replaces Assumption 3 with Assumption 3′.

Assumption 3′. Corporate income is taxed at rate $\tau \in (0,1)$. In the event of default, interest deductions are disallowed. Default is endogenous. In the event of default, the lender has strict seniority. The deadweight default cost for a firm of type-i is a fraction (ϕ) of going-concern value $\left|w^d\right|$. The pre-tax yield on corporate saving is r.

For simplicity, we return now to the baseline model that considers only separating equilibria. Accounting for the corporate income tax, the provisional net worth of the firm is

$$\widetilde{w} \equiv (1-\delta)k + \theta\varepsilon k^\alpha - b - \tau[\theta\varepsilon k^\alpha - \delta k - (b-\rho)] = (1-\delta(1-\tau))k + (1-\tau)\theta\varepsilon k^\alpha - b + \tau(b-\rho). \quad (48)$$

From (48) it follows that sum of net worth and the debt payment is just equal to the internal resources of an unlevered firm plus the value of the debt tax shield. The default-inducing shock must be modified to account for the effect of taxes, with

$$[1-\delta(1-\tau)]k_j + (1-\tau)\theta_i\varepsilon_{ij}^d k_j^\alpha - b_j + \tau(b_j - \rho_j) \equiv w^d \Rightarrow \varepsilon_{ij}^d \equiv \frac{b_j - [1-\delta(1-\tau)]k_j + w^d - \tau(b_j-\rho_j)}{(1-\tau)\theta_i k_j^\alpha}. \quad (49)$$

Accounting for the fact that interest deductions are disallowed in default, lender recoveries in the event of default are equal to

$$(1-\delta(1-\tau))k_i + (1-\tau)\theta_i\varepsilon k_i^\alpha + (1-\phi)\left|w^d\right|. \quad (50)$$

That is, in the event of a default, the lender receives the value of the physical capital, earnings, and going-concern value net of the corporate income tax bill.

The expected end-of-period equity value must also be adjusted to account for the new definition of net worth

$$\Omega^{ij} \equiv \beta \int_{\varepsilon_{ij}^d}^{\infty} v_i[(1-\delta(1-\tau))k_j + (1-\tau)\theta_i \varepsilon k_j^{\alpha} - b_j + \tau(b_j - \rho_j)]f(\varepsilon)d\varepsilon, \tag{51}$$

and the bond value function must account for the new definition of lender recoveries in default

$$\rho^i \equiv \beta \left[b_i \int_{\varepsilon_{ii}^d}^{\infty} f(\varepsilon)d\varepsilon + \int_{\underline{\varepsilon}}^{\varepsilon_{ii}^d} [(1-\delta(1-\tau))k_i + (1-\tau)\theta_i \varepsilon k_i^{\alpha} - (1-\phi)w_i^d]f(\varepsilon)d\varepsilon \right]. \tag{52}$$

In the interest of brevity, we shall focus on optimal financial policies. Once again, the low type allocation is derived by solving Program L. Program L is unaffected save the new definitions for the equity value and debt value which account for taxes. The optimality condition for b_L^* is

$$\frac{\partial}{\partial b}(y_L \rho_L) \times \tau \int_{\varepsilon_{LL}^d}^{\infty} f(\varepsilon)d\varepsilon = f(\varepsilon_{LL}^d)\frac{\partial \varepsilon_{LL}^d}{\partial b}\phi \left| w_L^d \right| + \left[1 - \tau \frac{\partial}{\partial b}(y_L \rho_L)\right] \left[\int_{\varepsilon_{LL}^d}^{\infty} [v_L' - 1]f(\varepsilon)d\varepsilon \right]. \tag{53}$$

The left side of equation (53) is the marginal tax shield benefit provided by debt. The first term on the right side of the equation is the marginal bankruptcy cost. The second term on the right side is a measure of the precautionary cost of debt service. Since asymmetric information induces $v_L' > 1$ for low net worth states, the low type is predicted to have low debt relative to traditional trade-off theory. Thus, the precautionary effect offers a potential explanation for what would appear to be conservative debt policies.

The high type allocation again solves Program H. In the interest of brevity, we redefine the marginal cost of debt service to account for taxes. The marginal effect of b on the expected discounted end-of-period value of shareholders' equity for a high-type (low-type) that has taken the high-type allocation is

$$\begin{aligned} \Omega_b^{HH} &\equiv -\left[1 - \tau \frac{\partial y\rho}{\partial b}\right] \beta \int_{\varepsilon_{HH}^d}^{\infty} v_H'[(1-\delta(1-\tau))k_H + (1-\tau)\theta_H \varepsilon k_H^{\alpha} - b_H + \tau(b_H - \rho_H)]f(\varepsilon)d\varepsilon \quad (54) \\ \Omega_b^{LH} &\equiv -\left[1 - \tau \frac{\partial y\rho}{\partial b}\right] \beta \int_{\varepsilon_{LH}^d}^{\infty} v_L'[(1-\delta(1-\tau))k_H + (1-\tau)\theta_L \varepsilon k_H^{\alpha} - b_H + \tau(b_H - \rho_H)]f(\varepsilon)d\varepsilon. \end{aligned}$$

The optimality condition for b_H^* is

$$\begin{aligned} &\frac{\partial}{\partial b}(y_H \rho_H) \times \tau \int_{\varepsilon_{HH}^d}^{\infty} f(\varepsilon)d\varepsilon + \left(\frac{\mu \Omega^{LH}}{\beta \lambda_H}\right)[1 - s_H]\left[\frac{\Omega_b^{HH}}{\Omega^{HH}} - \frac{\Omega_b^{LH}}{\Omega^{LH}}\right] \\ = \; & f(\varepsilon_{HH}^d)\frac{\partial \varepsilon_{HH}^d}{\partial b}\phi \left| w_H^d \right| + \left[1 - \tau \frac{\partial}{\partial b}(y_H \rho_H)\right] \left[\int_{\varepsilon_{HH}^d}^{\infty} [v_H' - 1]f(\varepsilon)d\varepsilon \right]. \end{aligned} \tag{55}$$

This condition tells us that the optimal financing of the high type equates marginal tax and signaling benefits of debt with bankruptcy and precautionary costs. If net worth is sufficiently high, the no-mimic constraint will be slack and the precautionary effect will induce the high type to have low leverage relative to that predicted by traditional trade-off theory. This offers another explanation for Graham's (2000) finding that

large-liquid firms have conservative debt policies.[14] By way of contrast, if net worth is low, the signaling motive will be operative. This will induce "excessive" leverage relative to the trade-off theoretic benchmark.

7 Conclusions

The objective of this paper was to take a first step in constructing dynamic structural models of corporate financing when insiders enjoy superior information each and every period. We argue that the theory proposed here does a better job of explaining observed financing behavior than the pecking-order hypothesis. For example, the model can account for equity issuance by firms with ample debt capacity. In addition, the model can account for the positive abnormal returns that generally occur after increases in capital expenditures. The framework of Myers and Majluf (1984) predicts that higher investment is a negative signal.

Two important theoretical insights are delivered. First, anticipation of future signaling costs converts a risk-neutral insider into a pseudo-risk-averse insider. This risk-aversion weakens the attractiveness of debt relative to what one obtains in a single-period model where (by construction) the firm only faces the lemons problem once. This same risk-aversion adds to the signal content of debt, however. The optimal mix of debt and equity can be viewed as balancing efficient risk-sharing against information revelation. Thus, the results of our dynamic model have clear linkages with static contracting theory. Second, the nature of equilibria in such markets is contingent on the firm's net worth. In particular, when net worth is sufficiently low, the costs of separation are extremely high and it seems natural for firms to find their way to a pooling equilibrium.

[14] This explanation differs fundamentally from that offered by Hennessy and Whited (2005), which is based upon taxes on distributions and direct flotation costs. In the present model, we have shut off those two channels.

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Appendix A: Proofs

Proof of Lemma 1.

The allocation a_L^* was in the feasible set for Program H. To verify, the NM_{LH} constraint would be trivial as the allocations would be type-independent. Since BC_L is satisfied at a_L^* and $s_L^* \geq 0$ we know BC_H would be slack. The fact that a_H^* solves Program H tells us the high type's maximand must be at least as high at a_H^* as at a_L^*.

Proof of Lemma 2.

Applying the Envelope Theorem, the value of internal funds if the low type is realized is $\lambda_L = 1$. The value of a dollar of internal funds to the high type is given by $\lambda_H + \mu$. To see this, it must be noted that the low type's equilibrium payoff (which enters the NM_{LH} constraint) can be expressed as $w + \kappa^*$. This explains why the shadow value of internal funds to the high type is not simply λ_H. Taking a probability weighted average of these values yields (17).

Proof of Proposition 2.

We begin by demonstrating

$$\mu(\widehat{w}) = 0 \Rightarrow \mu(w) = 0 \; \forall \; w > \widehat{w}.$$

From the Envelope Theorem

$$\frac{\partial}{\partial w}\left[d_L^* + (1 - s_L^*)\beta \int_{\underline{\varepsilon}}^{\infty} v_L[(1-\delta)k_L^* + \theta_L \varepsilon (k_L^*)^{\alpha} - b_L^*] f(\varepsilon) d\varepsilon\right] = 1.$$

Next, consider that NM_{LH} demands

$$d_L + (1 - s_L)\beta \int_{\varepsilon_{LL}^d}^{\infty} v_L[(1-\delta)k_L + \theta_L \varepsilon k_L^{\alpha} - b_L] f(\varepsilon) d\varepsilon \geq d_H + (1 - s_H)\Omega^{LH}. \tag{56}$$

The left-side has a slope of one in wealth. Suppose now that $\mu(\widehat{w}) = 0$ which implies that the high type implements (b_H^P, k_H^P) at $\widehat{w}$. Now consider the slope of the right-side of NM_{LH} under the conjecture that the constraint remains nonbinding as w increases. Under the hypothesis that NM_{LH} remains nonbinding, the high type continues to implement (b_H^P, k_H^P) which are invariant to w. The slope of the right-side of (56) may then be computed as

$$\frac{d}{dw}[d_H + (1 - s_H)\Omega^{LH}] = \frac{\partial d_H}{\partial w} - \Omega^{LH}\frac{d}{dw}s_H. \tag{57}$$

If $\partial d_H / \partial w > 0$, it follows that $\mu = 0$. Suppose instead that $\partial d_H / \partial w \leq 0$. From BC_H it follows that

$$\frac{d}{\partial w}s_H = \frac{-1 + \partial d_H / \partial w}{\Omega^{HH}}. \tag{58}$$

Substituting (58) into (57) one obtains

$$\frac{\partial d_H}{\partial w} - \frac{ds_H}{dw} * \Omega^{LH} = \frac{\Omega^{LH}}{\Omega^{HH}} + \frac{\partial d_H}{\partial w}\left[1 - \frac{\Omega^{LH}}{\Omega^{HH}}\right] < 1.$$

Thus, the left-side of (56) has a steeper slope than the right and the NM_{LH} constraint remains nonbinding as conjectured. The high-type allocation for $\mu = 0$ follows directly from (27) and (28).■

Appendix B: Details of Computational Algorithm

The computational procedure is based on value function iteration. The individual steps are as follows. The idiosyncratic shock ε is implemented by discretizing its domain using N possible values. Each maximization is implemented by discretizing the domain of the decision variables.

1. Guess "going-concern" value w^d.

2. Guess the end-of-period equity value functions (v_j).

3. Solve for the low type allocation a_L that maximizes the objective function of the low type subject to the budget constraint. Since the dividend is not unique, we pick the allocation in the optimal set that minimizes the dividend.

4. Solve for the high type allocation a_H that maximizes the high type's objective subject to the budget and no-mimic constraints.

5. Compute new value functions $v_j^{'}$ using

$$v_j^{'} = \pi_{Hj}\left[d_H + \beta(1-s_H)\sum_{n=1}^{N} f(\varepsilon_n) v_H[(1-\delta)k_H + \theta_H \varepsilon_n k_H^{\alpha} - b_H]\right]$$
$$+ \pi_{Lj}\left[d_L + \beta(1-s_L)\sum_{n=1}^{N} f(\varepsilon_n) v_L[(1-\delta)k_L + \theta_L \varepsilon_n k_L^{\alpha} - b_L]\right].$$

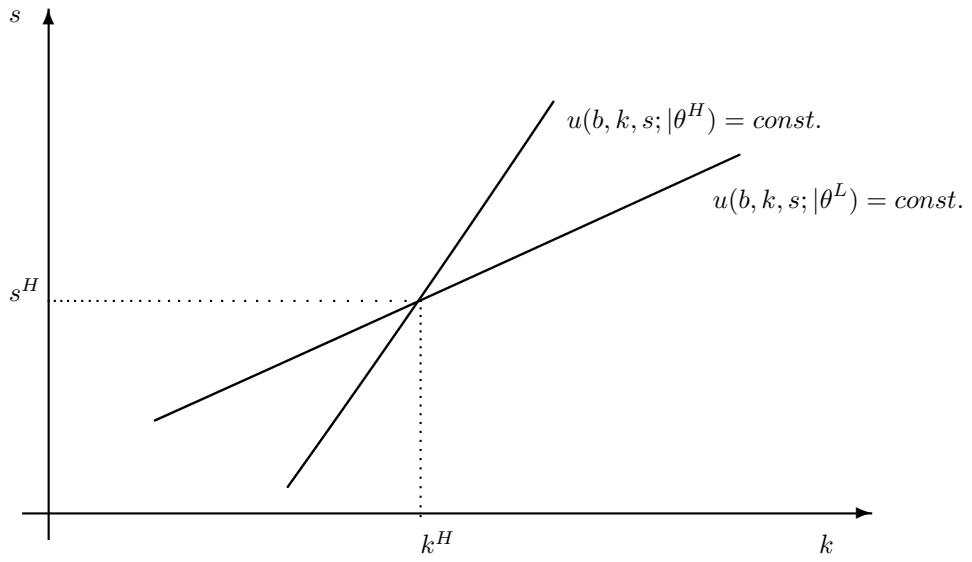
6. The functions $v_j^{'}$ from the previous step are the new guesses for v_j. The procedure is then restarted from step 2 until convergence.

7. Check the option value inherent in the firm by verifying $v_j(w^d) = 0$. If these conditions are not satisfied, update the initial guess w^d and restart the procedure from step 1 until convergence.

Figure 1: **Single-crossing conditions**

This figure presents single-crossing conditions. Panel A shows the indifference curves drawn under the assumption that the high type has a greater willingness to exchange equity for capital. In this case, higher capital investment provides a positive signal which encourages overinvestment relative to first-best. Panel B shows the indifference curves drawn under the assumption that the low type is more willing to exchange equity for a reduction in debt. Therefore, debt provides a positive signal and the high type borrows more/saves less in order to separate from the low type.

Panel A: Higher capital investment as positive signal



Panel B: Higher debt as positive signal

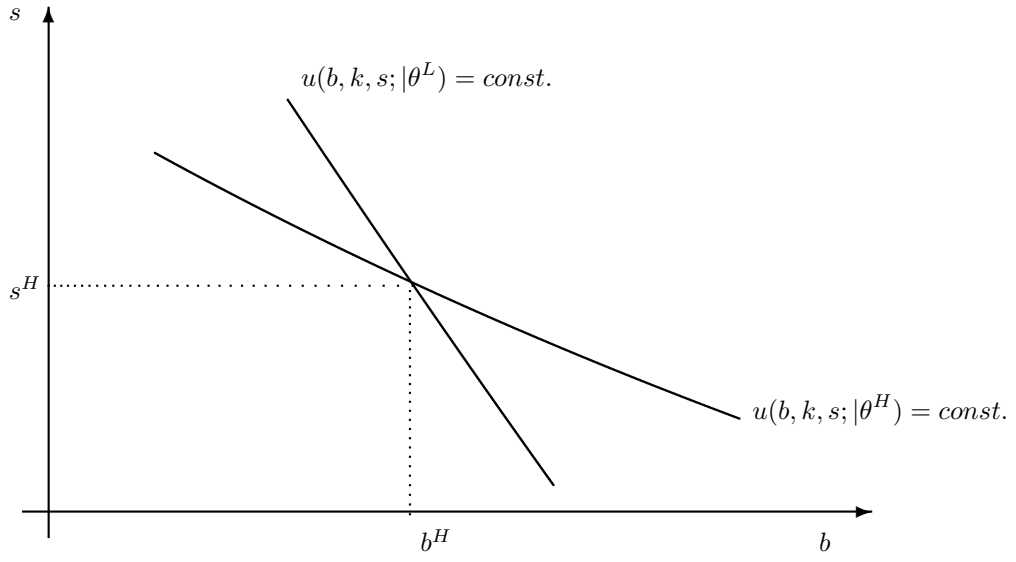


Figure 2: **Enterprise value of the firm**

The enterprise value of the firm $v - w$ is plotted as a function of the realized net worth, w. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

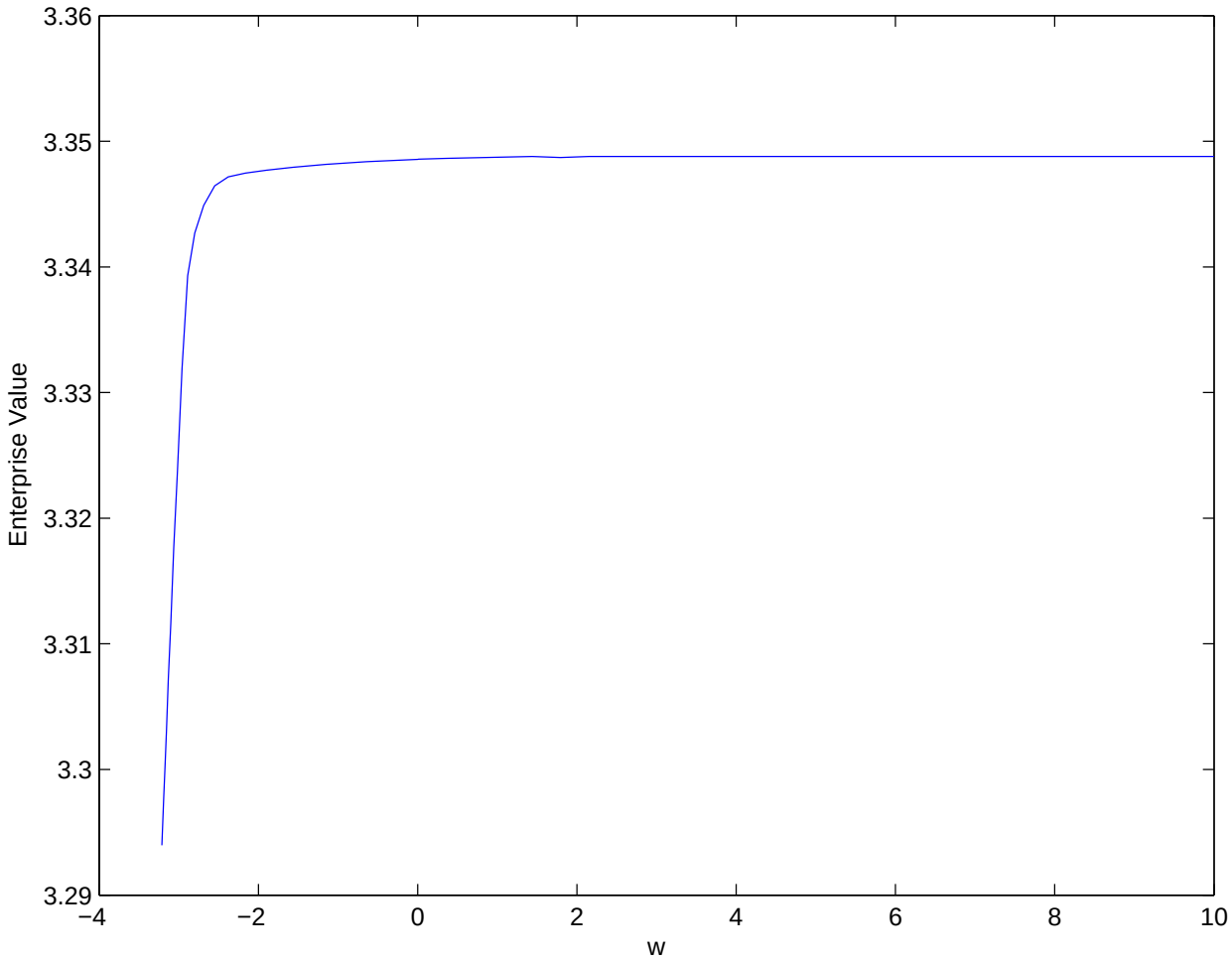


Figure 3: **Equilibrium capital allocations**

Equilibrium capital allocations, k_i^*, scaled by the first-best allocations, k_i^{FB}, are plotted as a function of the realized net worth, w, for both high and low values of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

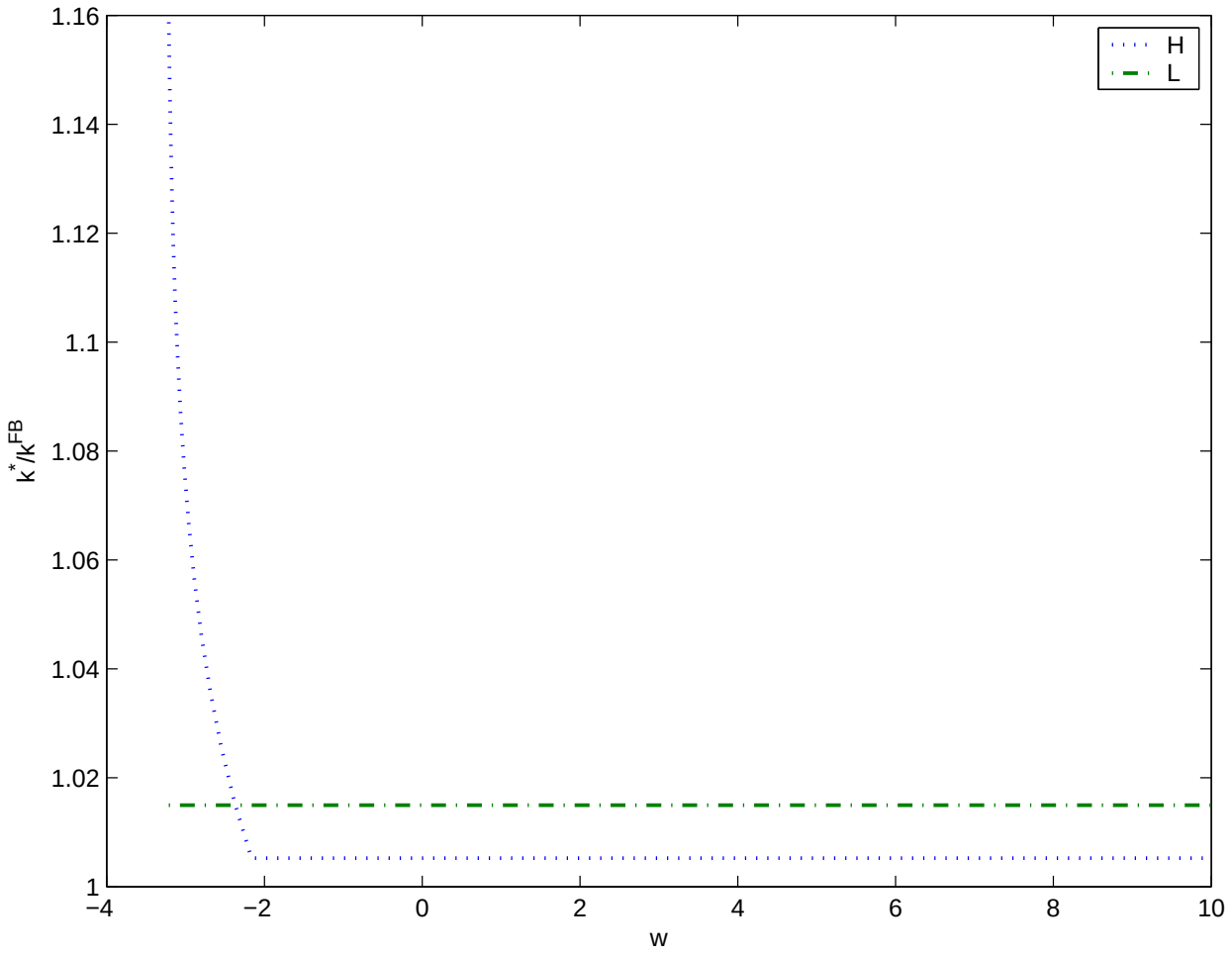
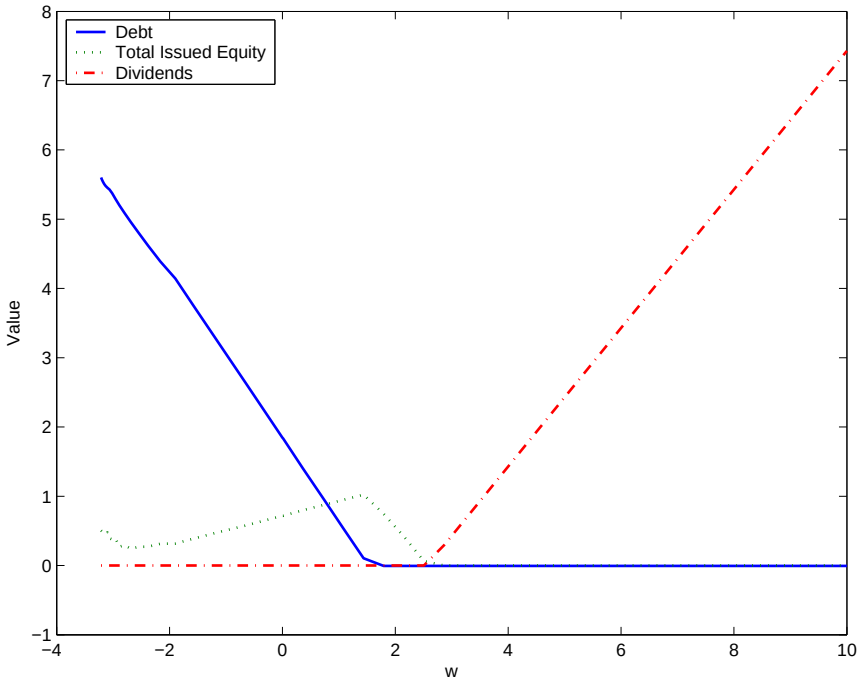


Figure 4: **Equilibrium financing policies**

Equilibrium financing policies: debt, ρ_i, and equity, $s_i^* \Omega^{ii}$, as well as equilibrium dividend policy, d_i^*, are plotted as functions of the realized net worth, w. Panel A presents the case of high value of θ, while Panel B presents the case of low value of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

Panel A: High value of θ



Panel B: Low value of θ

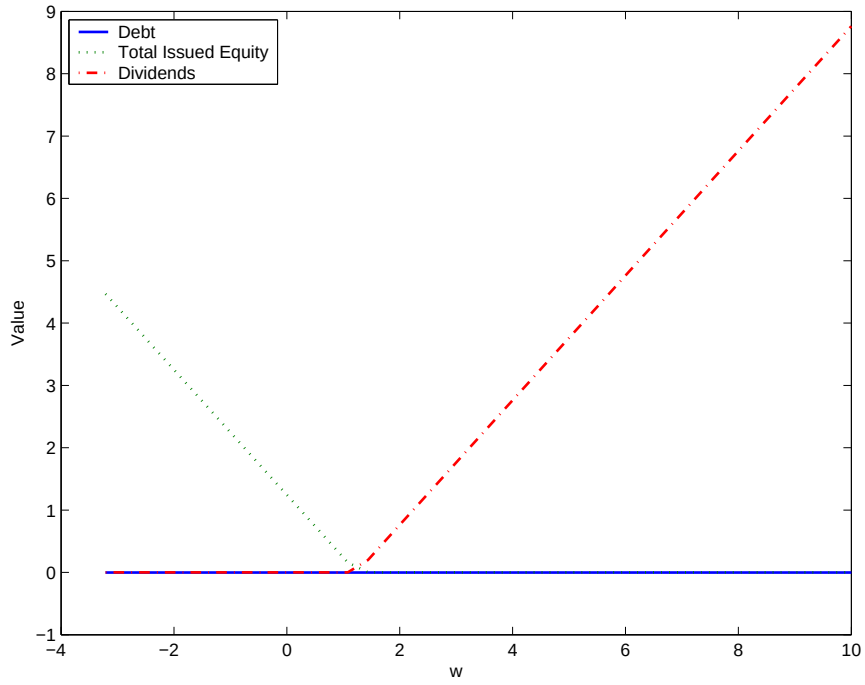


Figure 5: **Equilibrium capital allocations in option contract game**

Equilibrium capital allocations, k_i^*, scaled by the first-best allocations, k_i^{FB}, are plotted as a function of the realized net worth, w, for both high and low values of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

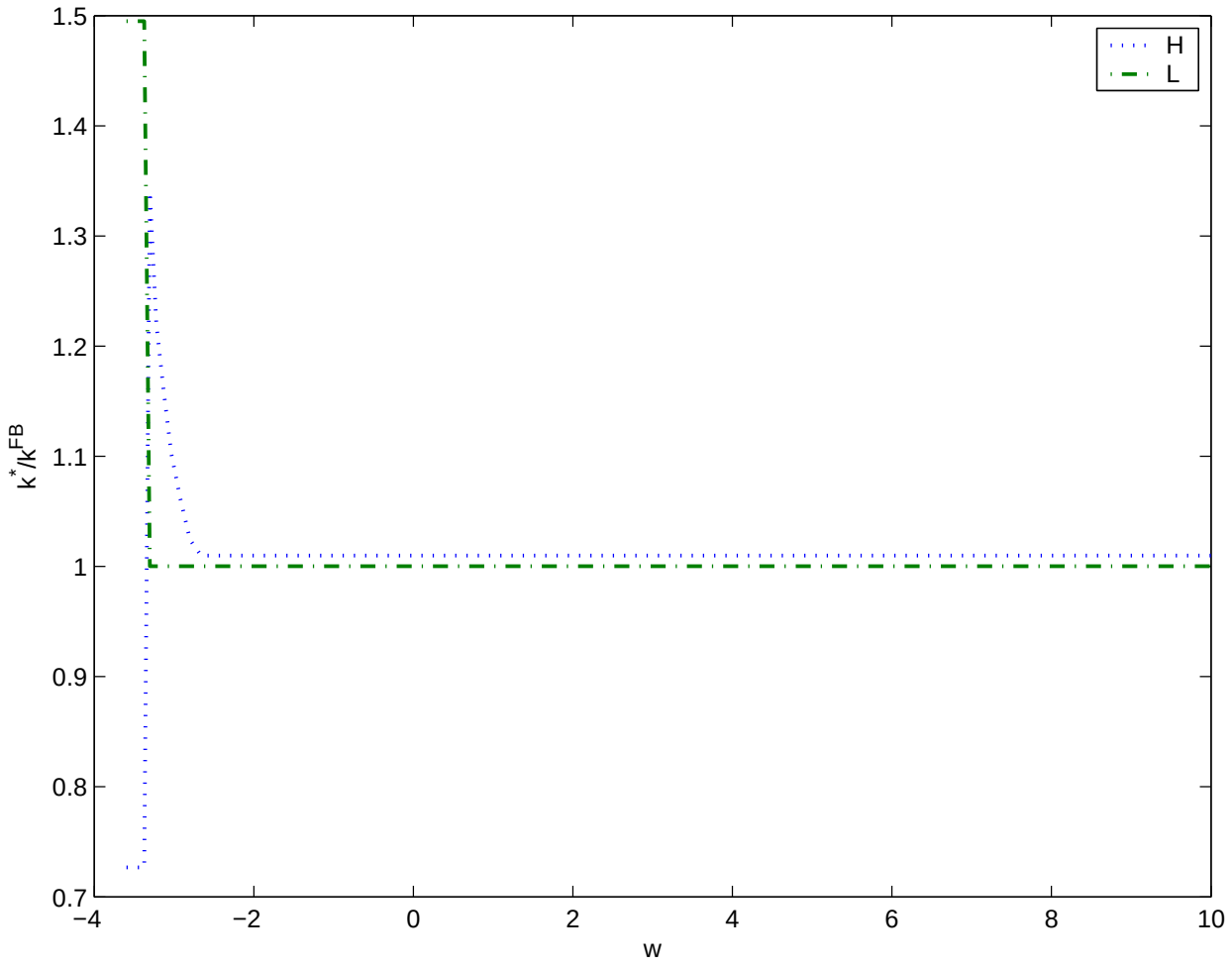
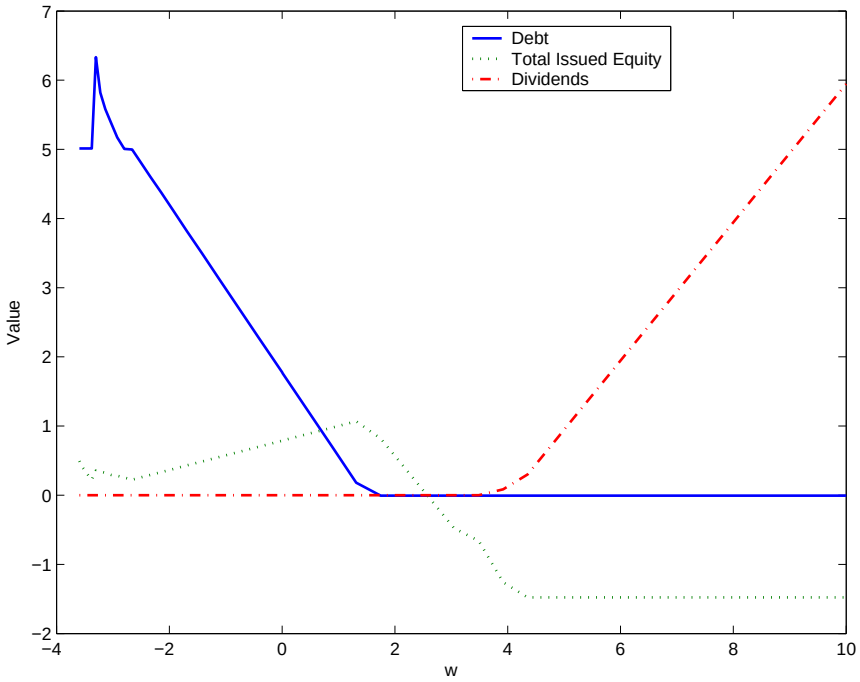


Figure 6: **Equilibrium financing policies in option contract game**

Equilibrium financing policies: debt, ρ_i, and equity, $s_i^* \Omega^{ii}$, as well as equilibrium dividend policy, d_i^*, are plotted as functions of the realized net worth, w. Panel A presents the case of high value of θ, while Panel B presents the case of low value of θ. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

Panel A: High value of θ



Panel B: Low value of θ

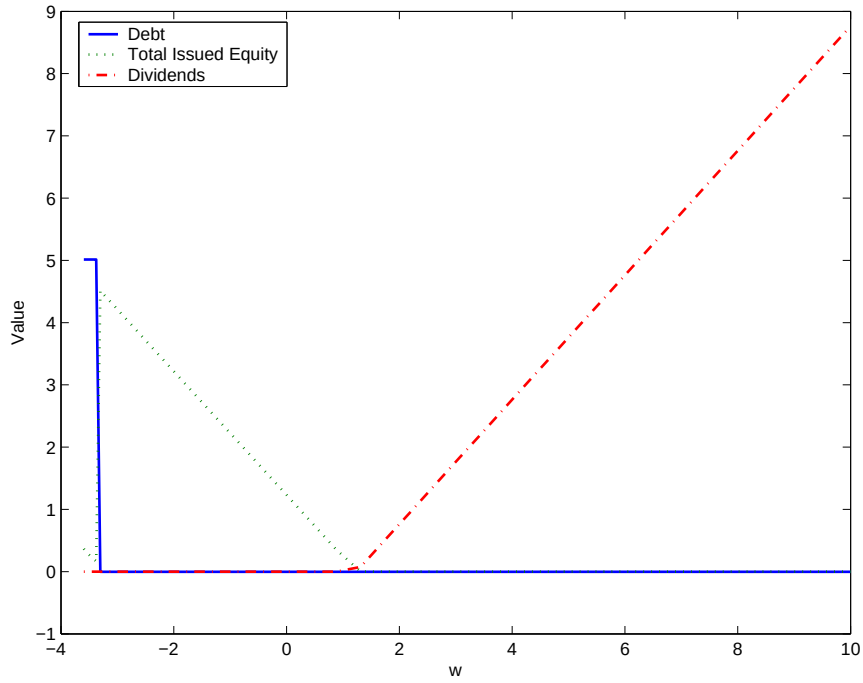


Table 1: **Leverage Regressions**

This table reports results of several regressions on the simulated data. The first three regressions have book leverage, $\frac{\rho_t}{k_t}$, as the dependent variable. The last regression has $\frac{\rho_t}{k_t} - \frac{\rho_{t-1}}{k_{t-1}}$ as the dependent variable. The financing gap is defined as $\frac{d_t + k_t - w_t}{k(t)}$ and operating profits are defined as $\theta_t \varepsilon_t k_t^{\alpha-1}$. The simulated panel of firms contains 3,000 firms over 31 time periods, where the initial period has been dropped for each firm. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

Financing Gap	Lagged Operating Profits	Book-to-Market	$\mathrm{E}[\frac{\rho}{k}] - \frac{\rho_{t-1}}{k_{t-1}}$
0.2405	-	-	-
(0.0912)	-	-	-
-	-0.0114	-	-
-	(0.0044)	-	-
-	-0.0075	0.0103	-
-	(0.0067)	(0.0090)	-
-	-	-	0.1813
-	-	-	(0.0194)

Table 2: **Announcement Effect Regressions**

This table reports results of several regressions on the simulated data with the abnormal return on the announcement day, $AR_t = \frac{d_{it} + (1 - s_{it})\Omega_t^{ii} - v(w_t)}{v(w_t)}$, as the dependent variable. Notation $a^{\pm}$ means conditioning on the positive(negative) values of a. The simulated panel of firms contains 3,000 firms over 31 time periods, where the initial period has been dropped for each firm. The productivity shock ε is exponentially distributed and both productivity shocks are *i.i.d.*

Investment Rate	$\frac{\rho_t^+}{\rho_t^+ + s_t^+ \Omega_t}$	$\frac{\rho_t}{k_t}$	$\frac{s_t \Omega_t}{k_t}$
0.0163	-	-	-
(0.0012)	-	-	-
-	0.0752	-	-
-	(0.0251)	-	-
0.0155	0.0103	-	-
(0.0017)	(0.0094)	-	-
-	-	0.1307	-
-	-	(0.0595)	-
0.0162	-	0.0222	-0.0042
(0.0052)	-	(0.0512)	(0.0277)