Asset Pricing in a General Equilibrium Production Economy with Chew–Dekel Risk Preferences *

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Version 07.3

Abstract

In this paper we provide a thorough characterization of asset returns as implied by a simple general equilibrium production economy with convex investment adjustment costs. When households have Epstein–Zin preferences, the model is able to match the estimated volatility of consumption growth and unconditional mean risk–free rate, while generating a sizeable equity premium and a moderate volatility of stock returns. Consistently with the data, the model's implied price–dividend ratio is pro–cyclical and stock returns are predictable (and increasingly so as the time horizon increases), while dividend growth is not. We argue that the main shortcomings of the model are (i) the excessive volatility of the risk–free rate and (ii) the lack of predictability of the equity premium. The latter problem is expected to arise, given that both the volatility of the productivity shock and the price of risk are a–cyclical. We show that, similarly to the case of endowment economy, Gul (1991)'s disappointment aversion as generalized in Routledge and Zin (2004) has the potential of addressing the second of the two shortcomings.

Key words. Equity Premium, Business Cycle, Predictability, Disappointment Aversion.

JEL Codes: D81, E32, E43, E44, G12.

*We thank Dave Backus, Alberto Bisin, Satyajit Chatterjee, Hal Cole, Tom Cosimano, Jean–Sébastien Fontaine, René Garcia, Sydney Ludvigson, Dino Palazzo, B. Ravikumar, Bryan Routledge, Stan Zin, as well seminar attendants at NYU, Notre Dame, Philadelphia Fed, Purdue, Texas A&M, and Wisconsin for their comments and suggestions. All remaining errors are our own responsibility. Clementi thanks CIREQ and the Department of Economics at the Université de Montréal for their generous hospitality at various stages of this project.

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1 Introduction

In this paper we provide a thorough characterization of the asset returns implied by a simple variant of the neoclassical growth model, the workhorse of business cycle analysis since the seminal work of Kydland and Prescott (1982). First, we show the existence of reasonable parameter values such that the model matches the unconditional first moments of risk–free rate and equity premium, while generating volatility of output growth and consumption growth consistent with the empirical evidence. Then, we go on to study the dynamics of asset returns. We show that, again consistently with quarterly data for the US economy, (i) the price–dividend ratio is strongly pro–cyclical, (ii) the risk–free rate is essentially uncorrelated with output growth, and (iii) risk–free rate and equity return are moderately positively correlated. Furthermore, stock returns are forecastable using price–dividend ratios and the predictive power of the latter increases with the time horizon. Dividend growth, on the other hand, is not forecastable. These facts are consistent with what first found by Campbell and Shiller (1988) and Fama and French (1988a). The main shortcomings of the model appear to be (i) the excessive volatility of the risk–free rate and (ii) the lack of predictability of the equity premium.

Since the provocative paper by Mehra and Prescott (1985), scholars working on general equilibrium dynamic models of asset returns have made giant steps forward. However, most of this progress is the result of the analysis of endowment economies, along the lines of Lucas (1978). During the same period, following the lead of Kydland and Prescott (1982), business cycle analysis has also made great advances. For example, it was shown that the neoclassical stochastic growth model, when parameterized to match stylized low–frequency facts about the US economy, generates a joint high–frequency dynamics of aggregate output, consumption, and investment which is also consistent with NIPA data (see Cooley and Prescott (1995)). Here, by neoclassical stochastic growth model we mean a general equilibrium dynamic economy with capital accumulation, where households maximize expected discounted utility, markets are complete, and technology shocks to the production function are the only stochastic disturbance.

The asset pricing implications generated by the stochastic growth model should be of interest to both finance and business cycle scholars. On the one hand, as argued by Cochrane (2006) among others, its implications for macroeconomic quantities arise from the equality of marginal rates of substitution to marginal rates of transformation.

These, in turn, directly identify securities returns. Squaring the latter against the data provides a further opportunity for model's falsification. On the other hand, endowment economies are not that useful when the objective is to understand what macroeconomic factors lie behind the factor portfolios that have been shown to have predictive power for asset returns. Yet, the literature on the asset pricing implications of general production economies such as the stochastic growth model, is still in its infancy.

To our knowledge, the first comprehensive analysis of the asset pricing implications of the stochastic growth model is due to Rouwenhorst (1995). His main result is that, for such model, matching the unconditional mean equity premium is even harder than for Mehra and Prescott (1985)'s economy. This is the case for two main reasons. First, differently from what happens in an endowment economy, raising the coefficient of relative risk–aversion does not help to increase the volatility of the stochastic discount factor. In fact, higher risk aversion implies lower elasticity of substitution, and therefore lower volatility of consumption growth. Second, the price of capital being constant at 1, the volatility of stock returns is equal to that of the marginal product of capital, which is quite limited in this framework.

Tallarini (2000) shows that the first of the two issues outlined above can be addressed by allowing for the decoupling of risk aversion and intertemporal elasticity of substitution. By assuming Epstein–Zin preferences, he is able to raise risk aversion at arbitrarily high levels, while keeping the elasticity of substitution anchored at 1. Tallarini (2000) shows that there exist values of relative risk aversion that allow the stochastic growth model to generate a market price of risk consistent with the empirical evidence. However, the price of capital being constant at 1, his model cannot generate sizeable equity premia.

Jermann (1998) and Boldrin, Christiano, and Fisher (2001) deal with both of the issues raised by Rouwenhorst (1995), by assuming that agents form consumption habits and by impeding the quick adjustment of the capital stock to productivity shocks. In Jermann (1998), the latter feature is obtained by means of convex adjustment costs, while Boldrin, Christiano, and Fisher (2001) assume the existence of two production sectors, with limited inter–sectoral factor mobility. Everything else equal, these impediments to the smooth adjustment of capital imply that the price of capital is not anchored to 1 any longer. However, without limiting households' willingness to substitute consumption intertemporally, this will not have much of an impact on the volatility of capital gains and therefore on the volatility of stock returns. The main effect will be to lower the volatility of investment and increase that of consumption growth (via the economy's resource constraint). The low intertemporal elasticity of substitution implied by habit preferences takes care of this, leading to higher volatility of stock returns without counterfactual implications for the second moments of investment and consumption growth. The increase in the curvature of the utility function, by increasing risk–aversion, takes care of increasing the volatility of the stochastic discount factor.

Here we show that similar results can be obtained under the assumption that households' risk preferences over atemporal lotteries fall in the Chew–Dekel class. We consider two of such preference relationships. One is the well known structure leading to the expected utility representation. The other, due to Gul (1991), allows for disappointment aversion. Following Kreps and Porteus (1978) and Epstein and Zin (1989), we imbed these preference relations in a standard intertemporal choice problem, to obtain two utility criteria for evaluating stochastic sequences, known in the literature as Epstein–Zin and disappointment aversion, respectively.

We assign values to parameters, following the methodology typical of most modern macroeconomic studies. Whenever possible, we use direct empirical evidence. Alternatively, we choose them so that the model is consistent with certain low–frequency statistics for the US economy. This procedure leaves us with three parameters, governing relative risk–aversion, the elasticity of intertemporal substitution, and the elasticity of investment to Tobin's q, respectively. We pick their values so that our model matches the unconditional mean risk–free rate and equity premium, and the ratio of volatility of consumption growth to volatility of output growth. When households have Epstein–Zin preferences, the coefficient of relative risk aversion with respect to atemporal bets is about 21, the elasticity of intertemporal substitution is about 1/36, and the average elasticity of investment to Tobin's q is 0.77. Such low value for the elasticity of substitution falls within the confidence interval for this parameter, as estimated by several empirical studies (see Hall (1988), for example). Unfortunately, the parsimony of our model does not allow to directly compare the model's implied average elasticity of investment with respect to q to estimates obtained by the empirical literature on investment. However, the evidence discussed in Section 4 suggests that our value is by no means unreasonable. Admittedly, the RRA coefficient is larger than most economists have so far been comfortable with. In Section 4, we argue that the RRA coefficient is not the ideal measure of risk aversion. We find it more compelling to consider the level of relative risk aversion implied by the model along the equilibrium path. In the case of our model, such level seems to be fairly reasonable to us. Obviously, the same specification of risk preferences, when used to evaluate atemporal lotteries involving much greater risk than that generated by the model under consideration here, implies dramatically higher relative risk aversion. In Section 5, we document that this issue does not arise when agents are disappointment averse. To this end, the relevant property of this preference relation is first–order risk aversion. As far as we know, this insight is due to Epstein and Zin (1990a), who consider rank–dependent preferences in a Mehra–Prescott economy. Epstein and Zin (2001) and Bonomo and Garcia (1994) exploited it when considering disappointment aversion, still in the context of an endowment economy. Here we show that it works also in the case of a production economy. That is, we show that 1) there exists a value for the disappointment aversion parameter, such that the model has essentially the same implications for both quantities and prices as the model with Epstein–Zin preferences and that 2) when the disappointment averse agent is faced with an atemporal lottery that has a coefficient of variation 100 times as large as that implied by our model, he displays the same relative risk aversion as an expected utility agent with logarithmic utility.

The main shortcomings of the model are (i) the excessive volatility of the risk–free rate and (ii) the lack of predictability of the equity premium. For the expected equity premium to be time–varying, the volatility of stock returns or the price of risk, or both, must also be time–varying. Since the stochastic process for the only shock in the model is homoscedastic and risk aversion is a–cylical, the occurrence of the latter problem is not surprising. We show that Routledge and Zin (2004)'s generalization of disappointment aversion, by generating a time–varying price of risk, has the potential of addressing the second of the two shortcomings. This result mirrors that obtained by Routledge and Zin (2004) in the case of a Mehra–Prescott economy. However, the counterfactually high volatility of the risk free rate undermines the model's ability to generate predictability to the extent implied by the data.

A corollary of our results is that, when considering the asset pricing implications of general equilibrium, complete–market production economies, there is no apparent reason why one should abandon Chew–Dekel preferences in favor of habit–based preference specifications. This conclusion arises from the comparison of our results to those obtained by Jermann (1998) and Boldrin, Christiano, and Fisher (2001). Everything else equal, working with Chew–Dekel preferences is more appealing to us. To start with, the largest part the business cycle literature does not rely on habit formation. Secondly, Chew–Dekel preferences clearly allow for aggregation, in complete market economies. We have not found any proof that this is the case when some form of habit is assumed. Finally, a shortcoming of habit models is that the dependence of utility on the reference point (i.e. the habit) and the evolution of that reference point over time and states, depends on the particular application. As argued by Pesendorfer (2006), "This research seeks the *right* utility function for a particular application". This criticism does not apply to axiomatic–based preference relations such as those postulated in this paper. Once again in the words of Pesendorfer (2006), "Standard economic models relate behavior in different situations. For example, the Epstein–Zin axioms describe how the decision–maker behaves in simple situations and the formula derived in the representation theorem (applied to an economic decision problem) describes how decisions are made in more complicated economic problems."

Finally, we notice that our work is also quite close to other recent analyses of asset returns in general equilibrium production economies. Danthine and Donaldson (2002) and Guvenen (2005) focus on household heterogeneity, constructing environments in which stock market participants's consumption growth is more volatile because these agents provide insurance against aggregate shocks to non–participants. In Danthine and Donaldson (2002), the fraction of agents excluded from the stock market coincide with workers. While being denied the possibility of using capital accumulation for consumption smoothing, these agents obtain insurance from the firms' owners by means of labor contracts. In Guvenen (2005), non–participants are assumed to have a lower elasticity of intertemporal substitution. This stronger consumption smoothing motive can be satisfied only by trading in bonds with the owners of capital. In turn, this implies a more volatile consumption growth for the latter. Finally, Croce (2006)'s main objective is to the evaluate the welfare costs of business cycles implied by an economy with Epstein–Zin preferences and long–run risk à la Bansal and Yaron (2004), parameterized to match some moments of asset returns.

The remainder of the paper is organized as follows. The model is introduced in Section 2. Section 3 characterizes the properties of asset returns in the case of Epstein–Zin preferences. Section 4 assesses the parameter values implied by our calibration procedure. The asset prices implications of disappointment aversion are characterized in Section 5. The concluding remarks are in Section 6.

2 The Model

Ours is a simple version of the standard neoclassical growth model. Time is discrete and runs from $t = 0$ to infinity. The economy consists of a large number of identical and infinitely lived households that derive utility uniquely from consumption. Production takes place in firms, that finance the purchase of capital by selling claims to their cash flows to households. Two are the differences from the standard framework used for business cycle analysis: labor supply is inelastic (it is assumed that each agent provides one unit of labor every period) and the adjustment of the capital stock is costly.

We denote by $S_t = \{s_v\}_{v=0}^{t}$ the history of the economy between dates 0 and t. In other words, S_t contains all payoff–relevant information as of time t. Once specified the other assumptions, we will be able to define its elements.

2.1 Preferences

We follow Kreps and Porteus (1978) and Epstein and Zin (1989) in constructing preference orderings over stochastic sequences of consumption by adopting a Koopman's time aggregator and risk–preferences in the Chew–Dekel class.[1]

In particular, we assume that at any state S_t, agents value stochastic sequences of consumption $\{c_v\}_{v=t}^{\infty}$ by means of the time aggregator

$$v(S_t) \equiv [c_t^{\gamma} + \beta\mu^{\gamma}(S_t)]^{1/\gamma}, \quad \gamma \leq 1,\ \beta > 0,$$

where $\mu(S_t)$ represents the certainty equivalent of the lottery over the streams of utility associated to all histories $S_t \cup \{s_v\}_{v=t+1}^{\infty}$, and β defines the relative importance of future versus current utility. Obviously, the nature of the certainty equivalent $\mu(\cdot)$ will depend on the preference relation over atemporal lotteries that is assumed. In this paper we will consider two popular formulations, known in the literature as *expected utility* and *disappointment aversion*.[2]

[1] By time–aggregator we simply mean a criterion for evaluating deterministic sequences of consumption. Koopmans (1960) characterizes the set of aggregators that satisfy the conditions of *history independence*, *future independence*, and *stationarity.* Chew (1989) and Dekel (1986) derive a class of risk preferences that include expected utility as a special case and lead to first–order conditions that are linear in probabilities. See Backus, Routledge, and Zin (2005) for a careful yet readable summary of this literature.

[2] As is well known, expected utility is the representation of a preference relation satisfying the axioms of monotonicity, completeness, transitivity, continuity, and independence. Disappointment Aversion and its generalization relax the latter, replacing it with weaker requirements, known as the Gul's *weak independence* axiom and the *δ–weak independence* axiom, respectively. See Routledge and Zin (2004) for a careful but readable exposition of these issues.

It turns out that the utility representations admitted by both relations can be expressed as special cases of the representation of *generalized disappointment aversion*, due to Routledge and Zin (2004). Under the generalized disappointment aversion criterion, an agent evaluating a lottery attaches a penalty in utility terms to events that she considers disappointing. These are all events that fall below her disappointment threshold, given by the certainty equivalent itself, scaled by the parameter $\xi > 0$. Therefore, the certainty equivalent $\mu(S_t)$ satisfies the following condition:

$$\mu^\eta(S_t) = \sum_{S_{t+1}} \pi(S_{t+1}|S_t) v^\eta(S_{t+1}) - \theta \sum_{S_{t+1}\in\Delta_{t+1}} \pi(S_{t+1}|S_t) \left\{ [\xi\mu(S_t)]^\eta - v^\eta(S_{t+1}) \right\},$$
$$\Delta_{t+1} = \{S_{t+1} : v(S_{t+1}) < \xi\mu(S_t)\}, \;\; \eta \leq 1,\; \theta \geq 0,\; \xi \geq 0.$$

The set Δ_{t+1} is the set of disappointing payoffs. Disappointment aversion is obtained by setting $\xi = 1$. Finally, expected utility obtains for $\theta = 0$.

2.2 Production and Capital Accumulation

We assume that population grows at the constant rate $\varphi \geq 0$. For simplicity, we are going to express all variables in per–capita terms. Aggregate output y_t is produced according to

$$y_t = k_t^\alpha \left(z_t l_t\right)^{1-\alpha},$$

where k_t and l_t are the capital and labor inputs, respectively, and $0 < \alpha < 1$. Labor augmenting technological progress at time t is given by

$$z_t = e^{\lambda t + \varepsilon_t},$$

where $\lambda > 0$ and $\varepsilon_t = \rho\varepsilon_{t-1} + \zeta_t,\;\; \zeta_t \sim N(0, \sigma^2)$. As commonly assumed in theoretical studies of the business cycle, deviations from the linear time–trend follow a first–order autoregressive process. This formulation implies that productivity growth follows a stationary $MA(\infty)$ process:

$$\log(z_{t+1}/z_t) = \lambda + (\rho - 1)\sum_{s=0}^{\infty} \rho^s \zeta_{t-s} + \zeta_{t+1}.$$

Investment at time t is subject to adjustment costs according to the function $g(k_t, k_{t+1})$, where $g(\cdot,\cdot)$ is twice differentiable, strictly increasing and strictly convex in k_{t+1}, and linearly homogenous in k_t. In the analysis that follows, we will assume that

$$g(k_t, k_{t+1}) \equiv \left| \frac{k_{t+1}}{k_t} - \psi \right|^\iota k_t, \quad \iota > 1, \psi > 0.$$

Finally, letting i denote gross investment, the per–capita capital stock evolves according to

$$k_{t+1} = i_t + (1-\delta)k_t$$

and the aggregate resource constraint is

$$y_t = c_t + i_t + g(k_t, k_{t+1}).$$

The timing and the resolution of uncertainty are as usual. It follows that the history S_t can be summarized by the pair (k_t, z_t).

2.3 Planner's problem

Following much of the literature, we characterize the equilibrium allocation by solving the planner's problem. Expressed in terms of trend-stationary variables, the latter can be written in recursive form as

$$v(k,\varepsilon) = \max_{k'} \left\{ c^{\gamma} + \beta e^{[\varphi+\gamma\lambda]} \mu \left[v(k',\varepsilon') \right]^{\gamma} \right\}^{1/\gamma} \tag{1}$$

subject to

$$\begin{aligned} c + e^{(\lambda+\varphi)} k' &= k^{\alpha} (e^{\varepsilon} l)^{1-\alpha} + (1-\delta)\, k - \left| e^{(\lambda+\varphi)} \frac{k'}{k} - \psi \right|^{\iota} k \\ \varepsilon' &= \rho\varepsilon + \zeta', \quad \zeta' \sim N(0,\sigma^2), \end{aligned}$$

where $\mu \equiv \mu \left[v(k',\varepsilon') \right]$ solves

$$\mu^{\eta} = \mathrm{E}_{\varepsilon'} \left\{ \frac{[1+\theta I(k',\varepsilon',\mu)] v^{\eta} (\varepsilon', z')}{1+\theta \mathrm{E}_{\varepsilon'} \left[I(k',\varepsilon',\mu) \right]} \right\}, \quad I(k',\varepsilon',\mu) = \begin{cases} 1 & \text{if } v(k',\varepsilon') < \xi\mu, \\ 0 & \text{otherwise.} \end{cases}$$

This framework nests some of the most popular models in the asset pricing literature. By setting $\xi = 1$, one obtains the case of Disappointment Aversion. Further, by posing $\theta = 0$, one recovers what is known as the *Epstein-Zin* model. Finally, for $\gamma = \eta$, we obtain the classical case of expected discounted utility. Throughout this paper, we will assume $\psi = e^{\lambda+\varphi}$. That is, we will assume that adjustment costs are strictly positive if and only if capital grows at a rate different from its balanced growth level. Under this assumption, as pointed out by Abel (2002), the balanced growth rate of the economy will be invariant to the parameter ι.

2.4 Asset Returns

As in most of the asset pricing literature, our analysis will focus on two assets, which we call risk–free asset and equity, respectively. In the data, we will identify the former with the 3–month Treasury Bill and the latter with a portfolio of stocks traded on the New York Stock Exchange. Details on data sources and elaboration are confined to Appendix A. The theoretical counterpart of the 3–month T-Bill will be a 1–period lived asset that delivers one unit of consumption in all states of nature. Its conditional gross return, denoted as $R^f(k,z)$, satisfies[3]

$$\sum_i \pi(\varepsilon_i|\varepsilon)m(\varepsilon_i|k,\varepsilon)R^f(k,\varepsilon) = 1.$$

The term $m(\varepsilon_i|k,\varepsilon)$ is known in the literature as the stochastic discount factor. Its specification depends on the assumption on risk–preferences.

Identifying the theoretical counterpart of the return on equity is not as straightforward. The literature identifies the return on equity with the marginal gross return of investment. It is rather easy to see why this is the case. The necessary conditions of problem (1) imply that the marginal return on investment in state i is

$$R^e(k,\varepsilon;\varepsilon_i) = \frac{\alpha k'^{\alpha-1}(\varepsilon_i l')^{1-\alpha} + (1-\delta) - \frac{\partial}{\partial k'}g[k',k''(k',\varepsilon_i)]}{1+e^{-(\lambda+\varphi)}\frac{\partial}{\partial k'}g(k,k')}.$$

The expected conditional return is thus

$$E[R^e(k,\varepsilon)] = \sum_{\varepsilon'} \pi\left(\varepsilon_i|\varepsilon\right) R^e\left(k,\varepsilon;\varepsilon_i\right).$$

Notice that the term $[1+e^{-(\lambda+\varphi)}\frac{\partial}{\partial k'}g(k,k')]$ is nothing but the relative price of capital in the current period, which we will denote as $P_{k'}$. With a little algebra, we can rewrite

$$R^e(k,\varepsilon;\varepsilon_i) = \frac{c(k',\varepsilon_i) - (1-\alpha)y(k',\varepsilon_i) + e^{\lambda+\varphi}P_{k''}k''}{P_{k'}k'} = \frac{d(k',\varepsilon_i) + e^{\lambda+\varphi}P_s'}{P_s}.$$

The marginal return of investment turns out to be equal to the return from owning the capital stock. This implies that the literature identifies $P_s \equiv P_{k'}k'$ with the market value of capital and $d \equiv c-(1-\alpha)y$ with aggregate dividend. Without adjustment costs, the price of capital is always equal to 1 and the share price equals the capital stock. In order to allow direct comparisons with the literature, we are also going to define the marginal return on investment as the theoretical counterpart of the return on equity. It is obvious however, that there are many reasons why this definition is short of ideal.

The variable d defined above identifies the net resource flow from the corporate sector to the household sector. When consumption is greater than the labor share

[3]In the remainder of the paper, our notation will reflect the fact that our numerical procedure involves approximating the stochastic process for the productivity shock ε by means of a 6–state Markov chain.

of income, the net payout of the corporate sector is positive. This is the case in which the capital share of output is greater than gross investment. However, when consumption is lower than the labor share of income, the net payout of the corporate sector is negative. This is because the capital share of income is not sufficient to finance gross investment. Households invest part of their labor income in the corporate sector. Therefore, $R^e(k, \varepsilon; \varepsilon_i)$ is the return to the owners of the capital stock, once eventual new investments are taken into account. The empirical counterpart of the net payments to shareholders consists of the gross flow of resources from the corporate sector to the household sector (dividends, share repurchase, interest payments on bonds, bonds repurchase, interest payments on other loans, extinguishment of other loans) minus the gross flow of resources from the household sector to the corporate sector (gross issue of equity and bonds plus new loans). This implies that in general the marginal return to the capital stock will be a rather poor proxy for the return on equity, which is defined as the return to one share owned at the beginning of the period.[4]

2.5 Numerical Approximation and Simulation

As it should be clear by now, we are interested in comparing our model's implications for a set of moments of prices and quantities, with their empirical counterparts. The moments' point estimates reported in the remainder of the paper are the output of an algorithm that involves i) obtaining numerical approximations to the optimal policy for consumption generated by program (1) and ii) applying standard Montercarlo simulation methods in order to approximate the stationary distribution of capital implied by the same program.

In the business cycle literature, the policy functions implied by models as simple as ours are often computed by means of low–order perturbation methods. Most commonly, by means of log–linear approximation. For standard preference specification and parameter values, the validity of this approach has been established by several authors. For example, Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) carry out a very detailed comparison of several approximation algorithms, to conclude that perturbation methods are fairly accurate when risk-aversion and the

[4]This observation is also the premise of a recent paper by Gomme, Ravikumar, and Rupert (2006). However, they follow a different, very interesting route. Their benchmark for the return on equity generated by the model is not the return on an equity portfolio, but rather the return to a measure of aggregate business capital, which they construct using data from the National Income and Product Account.

variance of the shock are small. When these conditions are not met, finite element methods and Chebyshev polynomials always perform better. Unfortunately for us, Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) restrict their tests to quantities, i.e. they do not compare the asset pricing implications obtained by applying different methods. Recently, John Cochrane expressed some skepticism about the accuracy of asset pricing moments' approximations obtained by low–order perturbation. In Cochrane (2006), he writes: "*I remain a bit worried about the accuracy of approximations in general equilibrium model solutions. Most papers solve their models by making a linear–quadratic approximation about a non–stochastic steady–state. But the central fact of life that makes financial economics interesting is that risk premia are not at all second order.*"

Our algorithm uses finite element methods. We define grids for the two state variables and we approximate the value function along the capital dimension by means of a low–degree spline. The stochastic process is approximated by a 6×6 markov chain, along the lines of Rouwenhorst (1995). The fixed point of the Bellman equation (1) is computed by repeatedly iterating on the operator. When we compared the unconditional asset returns generated by our algorithm with those implied by log–linearization around the deterministic steady–state, we found sizeable differences even for low levels of risk aversion. Another sign that accuracy may be a greater issue when dealing with prices than when dealing with quantities, is that when iterating on the Bellman operator, quantities settle much earlier than prices. It should be clear, however, that these considerations do not necessarily generalize to other frameworks.

3 The Case of Epstein–Zin Preferences

Here we consider the case of $\theta = 0$. The resulting specification of the utility function is known in the literature as Epstein–Zin. The stochastic discount factor is

$$m(\varepsilon_i|k,\varepsilon) = \beta e^{\lambda(\gamma-1)} \left[\frac{c(k',\varepsilon_i)}{c(k,\varepsilon)}\right]^{\gamma-1} \left[\frac{v(k',\varepsilon_i)}{\mu(k',\varepsilon)}\right]^{\eta-\gamma},$$

where

$$\mu(k',\varepsilon) = \left[\sum_i \pi(\varepsilon_i|\varepsilon) v^{\eta}(k',\varepsilon_i)\right]^{1/\eta}.$$

3.1 Calibration

We borrow most of the parameter values from the real business cycle literature. The model period is one quarter. The income share of capital (α) is equal to 0.36. Following Cooley and Prescott (1995), we set λ and φ so that the yearly growth rates of population and per–capita output are 1.2% and 1.56%, respectively. Once again following Cooley and Prescott (1995), the autocorrelation coefficient ρ is set to 0.95. The standard deviation σ is chosen so that the model generates a standard deviation of output growth of about 1%.

The parameters γ, η, and ι are chosen to match the relative standard deviation of quarterly consumption growth, the quarterly expected risk–free rate, and the quarterly expected equity premium. Our targets are 0.5, 0.322%, and 1.836%, respectively.

Simple computations show that the optimality conditions imply restrictions for the parameters β and δ on the balanced growth path, in the deterministic version of our model. These conditions are

$$\begin{aligned} \delta &= \frac{y-c}{k} + 1 - e^{\lambda+\varphi}, \\ \alpha\frac{y}{k} &= \frac{e^{\lambda+\varphi}}{\beta^*} + \delta - 1, \end{aligned}$$

where $\beta^* \equiv \beta e^{\varphi+\lambda\gamma}$ is the discount factor in the Bellman Equation. Following other contributions in the the literature on asset pricing in production economies, such as Jermann (1998) and Danthine and Donaldson (2002), we decide to set $\delta = .025$, implying an annual depreciation rate of 10%, and $\beta^* = 0.99$. These values imply an annual capital–output ratio of 2.14, once is lower than most available estimates (Cooley and Prescott (1995) report a value of 3.32, when housing is accounted for), and a 12.7% investment–capital ratio at the annual frequency, which is higher than the 7.6% value reported by Cooley and Prescott (1995).

The calibration is summarized in Table 1. The plausibility of the parameters γ, η, and ι is assessed in Section 4. Notice that, as it is the case in Jermann (1998) and Boldrin, Christiano, and Fisher (2001), our values for $\beta^*, \varphi, \lambda$, and γ imply that $\beta = 1.136$. For $\beta > 1$, a version of our model with no growth would not allow equilibria with positive interest rates. However, as shown by Kocherlakota (1990b) in the case of endowment economies, equilibria with positive interest rates may exist in growing economies, in spite of the fact that $\beta > 1$.[5]

[5]In the past, Many scholars have manifested uneasiness with respect to discount rates greater than 1. A commonly used argument is that such an assumption implies that households prefer future consumption to current consumption. This is always true in the case of constant streams of consumption. It does not have to be true, however, in the case, like ours, of growing consumption. Finally, it is worth recalling that several econometric studies, among which Hansen and Singleton (1982) estimated β to be significantly greater than 1.

α	δ	λ	φ	β	γ
0.36	0.025	0.00387	0.00298	1.136	-36.4
η	θ	ξ	ρ	σ	ι
-20	0	1	0.95	0.0164	1.222

Table 1: Calibration with Epstein–Zin Preferences

	σ_C/σ_Y	σ_I/σ_Y	$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$
Model	0.5	2.457	0.322%	2.81%	1.708%	11.86%
Data	0.499	2.647	0.322%	0.902%	1.836%	7.702%

Table 2: Unconditional Moments with Epstein–Zin Preferences

3.2 Results: Unconditional Moments

Recall that our parameterization is designed to match the ratio of the standard deviation of consumption growth to the standard deviation of output growth σ_C/σ_Y, the unconditional expected risk–free rate ($E(r^f)$), and the expected unconditional equity premium ($E(r^e - r^f)$). The purpose of this sub–section is to compare the model's predictions for other unconditional moments of asset returns, to their empirical counterparts. The model generates an annual equity premium of about 6.8%. While short of our target, this rate is higher than 6.2%, the benchmark value used by Mehra and Prescott (1985) and a host of other papers after them, among which Jermann (1998). Our target for the equity premium is higher because, differently from most other studies, we include the 1990's in our sample. The volatility of the term $\frac{v(k',\varepsilon_i)}{\mu(k',\varepsilon)}$ is greater, the more persistent the productivity shock. If we assumed a higher value for the parameter ρ, the value of η implied by our calibration procedure would be lower, with the best result (along this dimension) obtained for i.i.d. TFP growth. Nevertheless, we decide to follow the macroeconomics literature and stick to $\rho = 0.95$.

As mentioned in the Introduction, we are not the first to consider the asset pricing implications of the stochastic growth model with power risk preferences and convex adjustment costs. The conclusion reached by Jermann (1998) is that with $\gamma = \eta = -9$, i.e. with a coefficient of relative risk aversion of 10 and an elasticity of intertemporal substitution of 0.1, the model generates a counter–factually low annual risk–premium of just 0.26% and a counter–factually high risk–free rate of 3.36%. These findings motivated him to consider internal habit instead. On the one hand, internal habit increases the curvature of the utility function for given consumption. This implies higher risk–aversion. On the other hand, it also implies a very low intertemporal elasticity of substitution. The latter fact, together with convex adjustment costs, allows to raise the volatility of the stock return while keeping the standard deviation of consumption growth low. As Jermann very effectively puts it: "*They (consumers) have to both care (about consumption smoothing), and be prevented from doing anything about it.*" Here we show that essentially the same result can be achieved with Epstein–Zin preferences, simply by raising the RRA coefficient beyond 10 and lowering the elasticity beyond 0.1.[6] Higher risk–aversion will have the result of raising the equity premium and also the precautionary motive, thereby lowering the risk-free rate. At the same time, the elasticity of substitution will drop to a value that allows us match to the volatility of consumption growth while raising the standard deviation of stock returns at the same time. As the reader may expect, similar results can also be obtained under the restriction $\gamma = \eta$, i.e. in the case in which agents maximize expected discounted utility. We verify this conjecture in Section 3.5.

The standard deviation of equity return is also not far from its data counterpart, and is extremely close to the value obtained by Jermann (1998) in the case of his benchmark calibration. Where the model seems to miss completely the target is the standard deviation of the risk-free rate, which turns out to be about 3 times as large as in the data. This shortcoming, which is shared by the models with habit formation of Jermann (1998) and Boldrin, Christiano, and Fisher (2001), is due to the combination of a low elasticity of substitution (low γ) and rapidly increasing marginal adjustment costs (low ι). A low elasticity of substitution means that households are very eager to smooth consumption. In turn, this implies that the demand for bonds is very inelastic. A low value of ι means that adjustment costs are particularly effective in preventing consumption smoothing. This means that the productivity shocks imply wider shift of the bond demand schedule. Finally, since the supply of bonds is perfectly rigid, it follows that the risk–free rate must vary a lot.

3.3 Results: Business–Cycle Properties of Asset Returns

Figures 1 and 2 report impulse response functions for quantities and prices, respectively. The plots were obtained by averaging out the values generated by simulating the model for 200,000 runs of 250 periods each. In period 1 of each run, de–trended capital is set equal to the mean of its unconditional equilibrium distribution, while the

[6]The plausibility of such assumption in the context of our model, is assessed in Section 4.

shock is set to the highest value in the grid. For every statistic, we plot its percentage deviation from the unconditional mean.

The response to the innovation in the shock has a half life which is much greater than that implied by more standard versions of the stochastic growth model. This difference, which is clearly due to the investment adjustment cost, also shows up in terms of greater persistence of output. While the autocorrelation of the process for ε_t is only 0.95, that of detrended output turns out to be 0.989.

The bottom–left panel of Figure 1 reproduces the path of the price of capital, i.e. the cost of diverting one more unit from current consumption to capital accumulation. In the absence of investment adjustment costs, it would be constant at 1. In our case it is decreasing over time, as the progressive decline in the marginal product of capital calls for a decreasing pattern of investment. For the same reason, dividends drop on impact, and then grow back to their unconditional mean. The share price, which is the product of the price of capital and the capital stock, is also decreasing. The two latter observations directly imply that the return on equity must drop on impact and then go back to its unconditional mean. Finally, the risk–free rate follows closely the pattern of consumption growth.

The business cycle properties of the model are summarized in Table 3, which reports the unconditional correlations between output growth, consumption growth, and asset returns. The model does quite a good job in matching the correlation pattern of risk–free rate, stock return, and equity premium. On the other hand, it generates correlation coefficients between output growth and stock return and between consumption growth and stock return that are much larger than in the data. This feature is not that surprising, given that ours is a one–shock model.

	c'/c		r^f		r^e		$r^e - r^f$	
	Model	Data	Model	Data	Model	Data	Model	Data
y'/y	0.941	0.446	0.052	0.05	0.785	0.039	0.793	0.055
c'/c			0.132	0.023	0.926	0.161	0.917	0.161
r^f					0.223	0.193	-0.014	0.0758
r^e							0.971	0.993

Table 3: Unconditional Correlations

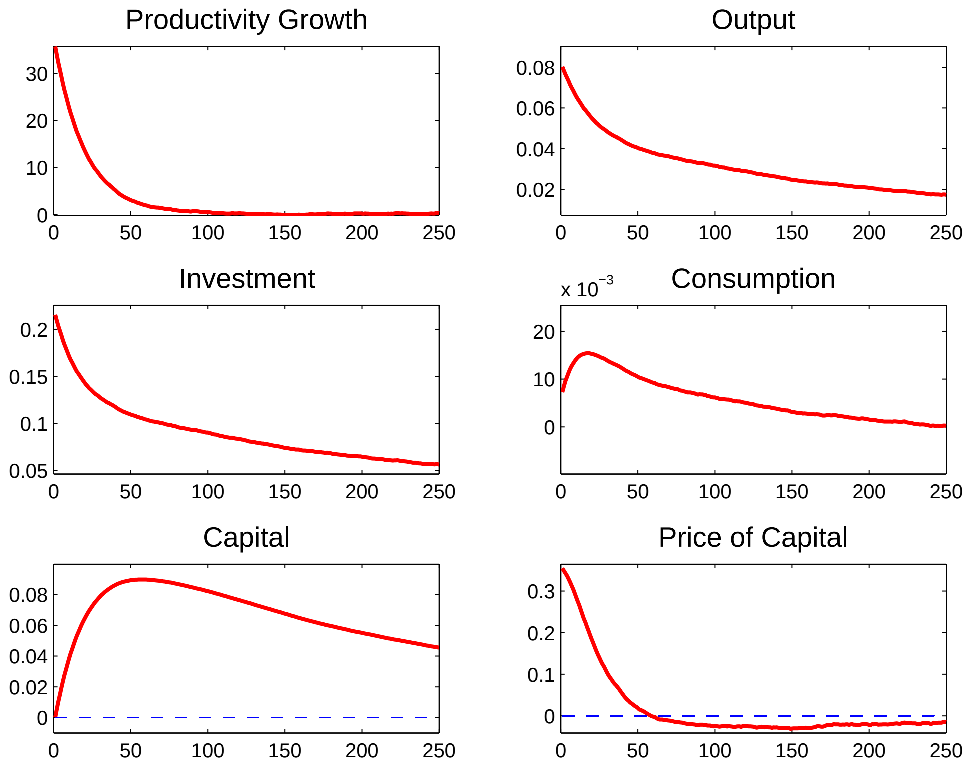

Figure 1: Impulse Responses: Quantities.

It's perhaps worth noticing that quantity dynamics is essentially invariant with respect to η, the parameter governing risk aversion. Hansen, Sargent, and Tallarini (1999) show that in the case of quadratic utility and linear constraints, this result holds exactly.[7]

3.4 Results: Predictability of Asset Returns

Every student of finance soon learns that stock prices, dividends, and returns are linked by an accounting identity. For any stock i,

$$R_{t+1}^i \equiv (P_{t+1}^i + D_{t+1}^i)/P_t^i.$$

It follows that if the price of an asset is high today, agents must be expecting that either next period's price will be high, or the dividends will be high, or the rate of return will be low. Or a combination of these events. In turn, this means that, if one

[7]This property has also the effect of dramatically reducing the computational burden. We can proceed in two steps. First, we compute the locus of parameters (γ, ι) such that the model matches exactly the relative standard deviation of consumption, for given η. Call this locus $\iota(\gamma)$. Then we look for the triple $[\eta, \gamma, \iota(\gamma)]$ such that the model matches exactly the remaining two moments.

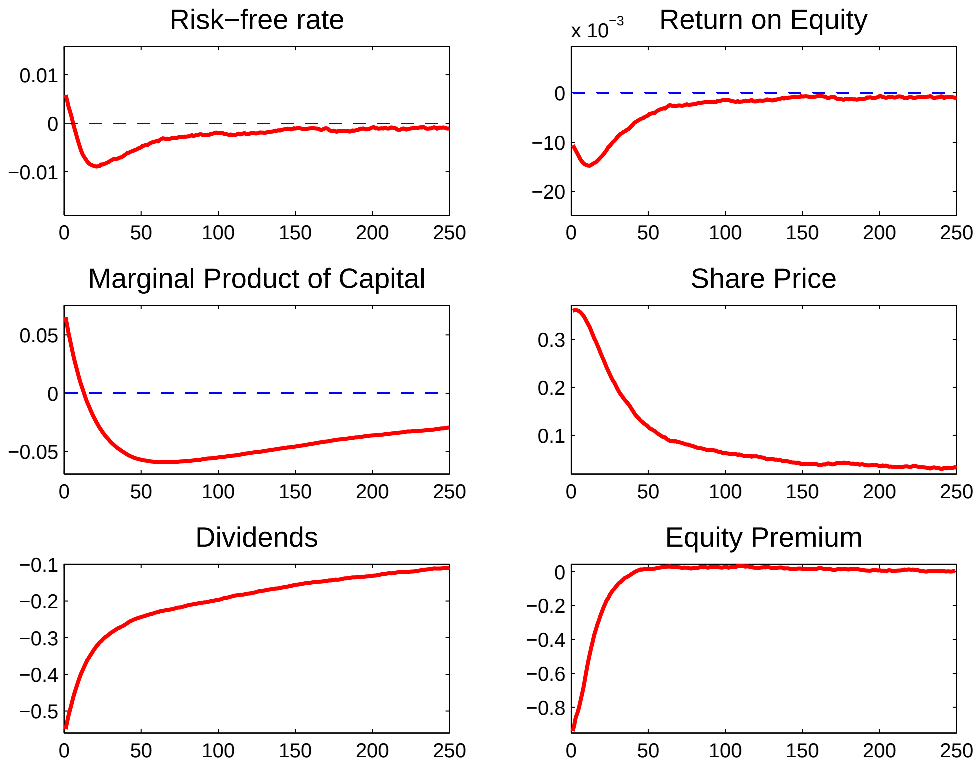

Figure 2: Impulse Responses: Prices.

imposes that stock prices are not explosive, a high price today must be associated with either low returns or high dividends in the future, or both.

Campbell and Shiller (1988) formalize the above argument, by log–linearizing the above identity and iterating forward, to obtain

$$\log(P_t^i/D_t^i) = \frac{b}{1-\nu} + E_t \sum_{s=0}^{\infty} \nu^s [\log(D_{t+1+s}^i/D_t^i) - \log(r_{t+1+s}^i)],$$

where b and $\nu < 1$ are linearization constants. Indeed, several authors, among which Campbell and Shiller (1988) and Fama and French (1988a,b), have shown that real stock returns, in particular at long horizons, are forecasted by the price–dividend ratio. On the other hand, price–dividend ratios do not forecast future dividend growth. These phenomena are summarized in the columns labeled "Data" in Table 4, where we report the results of regressing cumulative stock returns and dividend growth at different horizons, on the current price–dividend ratio. As expected, the regression coefficients associated with cumulative returns are negative. Furthermore, their absolute values and their R^2 are increasing with the horizon. Dividend growth, instead, is essentially not forecastable by means of the price–dividend ratio. The figures reported in the columns labeled "Model" are computed using data obtained simulating the model. The pattern is very similar to the one just discussed. Stock returns are forecastable, and increasingly so as the horizon increases. Dividend growth, instead, is not.

Table 4: Long–Horizon Regressions of Equity Returns and Dividend Growth

Dependent Variables	$r^e_{t,t+s}$				D_{t+s}/D_t			
	Model		*Data*		*Model*		*Data*	
Horizon (s)	*Slope*	R^2	*Slope*	R^2	*Slope*	R^2	*Slope*	R^2
1	−.061	0.05	−.115	0.08	.0574	.00	−.031	.05
	(−51.22)		(−2.18)		(2.73)		(−1.64)	
2	−.1118	0.1	−.22	0.14	.6522	.01	−.059	.05
	(−72.60)		(−3.04)		(18.42)		(−1.76)	
3	−.155	0.14	−.284	0.18	.6681	.01	−.084	.05
	(−88.74)		(−3.46)		(17.06)		(−1.74)	
5	−.2208	0.20	−.509	0.29	1.4357	.00	−.131	.07
	(−110.59)		(−4.65)		(12.35)		(−1.96)	
7	−.2686	0.24	−.778	0.36	2.2512	.01	−.169	.05
	(−125.23)		(−5.35)		(16.14)		(−1.68)	

Note: t statistics in parenthesis. Specification: $x_{t,t+s} = a + b\log(P/D)_t + \epsilon_t$
Data sources: See Appendix A.

The relation between current price dividend–ratio and future stock returns may be due to the predictability of the risk–free rate, the equity premium, or both. The evidence, summarized in Table 5, shows that essentially the predictability of stock returns is entirely due to time–variation in the expected equity premium. This confirms the finding of many others before us, among which Campbell (1999). Table 5 also shows that, in the case of our model, the opposite occurs. Differently from what happens in the data, the expected equity premium is essentially acyclical, and the predictability of stock returns is due to the time-variation in the risk–free rate. These findings are not surprising. One the one hand, the model generates a much higher volatility of the risk–free rate than in the data. On the other hand, given that risk–aversion and the volatility of stock–returns is acylical by construction, there is no reason to expect expected equity premia to to be counter–cyclical.

Routledge and Zin (2004) have shown that, in the case of a Mehra–Prescott-type economy, generalized disappointment aversion generates time–varying risk premia. In Section 5 we will ask whether this is also a feature of our framework, and, in the affirmative case, whether it implies predictability of the equity premium to the extent present in the data.

Table 5: Long–Horizon Regressions of Equity Premia and Risk–Free Rate

Dependent Variables	$r^e_{t,t+s} - r^f_{t,t+s}$				$r^f_{t,t+s}$			
	Model		*Data*		*Model*		*Data*	
Horizon (s)	*Slope*	R^2	*Slope*	R^2	*Slope*	R^2	*Slope*	R^2
1	–.0054	.00	–.117	0.08	–.0556	0.2	.002	.00
	(–4.25)		(–2.27)		(-109.81)		(.18)	
2	–0.0088	.00	–.219	0.15	–.1030	0.2	–.001	.00
	(–4.86)		(–3.13)		(–110.01)		(–.03)	
3	–.0111	.00	–.28	0.19	–.144	0.19	–.004	.00
	(–4.96)		(–3.60)		(–109.02)		(–.17)	
5	–.0097	.00	–.484	0.31	–21.11	0.18	.04	.01
	(–3.37)		(–4.78)		(–105.52)		(–.69)	
7	–.0047	.00	–.712	0.36	–.2639	0.17	–.066	.03
	(–1.39)		(–5.29)		(–101.23)		(–1.33)	

Note: t statistics in parenthesis. Specification: $x_{t,t+s} = a + b\log(P/D)_t + \epsilon_t$
Data sources: See Appendix A.

3.5 The Case of Expected Discounted Utility

Here we characterize the equilibrium allocation under the restriction $\eta = \gamma$. This is the popular scenario in which agents maximize expected discounted utility. The stochastic discount factor is

$$m(\varepsilon_i|k,\varepsilon) = \beta e^{\lambda(\gamma-1)} \left[\frac{c(k',\varepsilon_i)}{c(k,\varepsilon)}\right]^{\gamma-1}.$$

The only innovation in the calibration procedure is that we now have only two free parameters, γ and ι, which we set in order to match σ_C/σ_Y and $E(r^f)$. The resulting values are $\iota = 1.257$ and $\gamma = -30.79$.

Table 6 shows the results of the numerical experiment. Although the model does not match the post–war mean equity premium, it is able to generate a value of about 5.78%, which is close to that targeted by Mehra and Prescott (1985) and several other studies after them. Interestingly, the volatility of the risk–free rate and stock return are lower than in the general EZ case, and they are closer to their empirical counterparts. Since the volatility of consumption growth is kept constant across parameterizations, the volatility of stochastic discount factor in the EU must be lower. The precautionary saving motive decreases. In order to keep the expected risk–free rate at the target level, we end up increasing relative risk aversion and the elasticity of substitution. Both changes have the effect of lowering the risk-free rate. Everything else equal, higher elasticity of substitution also means lower volatility of the risk–free rate. Finally, to keep the consumption growth rate at its target level in spite of a higher elasticity, we need an increase in ι. That is, we need slower–rising adjustment costs. Higher values for γ and ι imply a lower volatility of stock prices and therefore stock returns.

	σ_C/σ_Y	σ_I/σ_Y	$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$
Model	0.5	2.387	0.311%	2.39%	1.445%	9.362%
Data	0.499	2.647	0.322%	0.902%	1.836%	7.702%

Table 6: Unconditional Moments with Expected Discounted Utility

We do not show the impulse response functions and the results of the predictability regressions, as they are very close to those obtained in the case of general Epstein–Zin preferences.

4 Assessing Parameter Values

We now assess the plausibility of the values for the parameters γ, η, and ι, which govern elasticity of substitution, risk aversion, and adjustment costs, respectively.

4.1 Adjustment Costs

As argued above, investment adjustment costs are needed in our model in order to generate volatility in the price of capital, which in turn translates into volatility of the share price. As a matter of fact, essentially all studies in the literature on asset pricing in production economy assume the existence of such costs or other rigidities that impede the smooth adjustment of capital to shocks.[8] The question is whether the model produces overidentifying restrictions that can be used to assess how reasonable is the value for the parameter ι implied by our calibration procedure. Under our assumptions, marginal Tobin's q equals average q. Therefore, it is natural to think of validating the model by comparing the elasticity of the investment rate with respect to q to its estimated counterpart. In our model, q equals the price of capital and the elasticity is simply:

$$\frac{d\log(i/k)}{d\log(q)} = \frac{d(i/k)}{dP_{k'}}\frac{P'_k}{i/k} = \frac{1}{i/k}\frac{1+\chi\left(\frac{i}{k}\right)\iota\left|\frac{i}{k}+(1-\delta)-\psi\right|^{\iota-1}}{\iota(\iota-1)\left|\frac{i}{k}+(1-\delta)-\psi\right|^{\iota-2}},$$

where $\chi(i/k) = 1$ if $(i/k) > \psi - (1-\delta)$, and $\chi(i/k) = -1$ otherwise. In the stationary distribution, the point elasticity ranges between 0 and 3, with a mean of 0.77.

[8] This is the case for Jermann (1998), Boldrin, Christiano, and Fisher (2001), Danthine and Donaldson (2002), Guvenen (2005), and Croce (2006).

It is well–known that attempts at estimating the elasticity using aggregate data gave disappointing outcomes. Typically, the estimates are not significantly different from zero, and the variation in aggregate q accounts for a risible fraction of the variation in investment rates.[9] In the last 10 years or so, several authors have studied the relation between investment and Tobin's q using firm–level data. In particular, we refer to the work of Eberly (1997), who employed the Global Vantage dataset, and Barnett and Sakellaris (1998) and Abel and Eberly (2002), who worked with Compustat data. One of the main lessons learned from them is that the relationship between investment and q is highly non–linear. A corollary is that information on the cross–sectional distribution of q can be used to improve the predictive power of investment equations. For example, when they estimate aggregate elasticity by computing the increase in total investment implied by a 1% increase in q for all firms, Barnett and Sakellaris (1998) obtain a point estimate of 0.84. When controlling for higher moments of the distribution of q, Eberly (1997) finds that regressing $\log(\sum_i I_i/K_i)$ on $\sum_i q_i$ yields a point estimate of 0.62, with a R^2 of 0.08. Regressing $\sum_i \omega_i \log(I_i/K_i)$ over $\sum_i \omega_i q_i$ (with $\omega_i = K_i/\sum_i K_i$) yields a coefficient estimate of 0.72, with R^2 of 0.38. Interestingly, Abel and Eberly (2002) also estimate that adjustment costs amount to 1.1% of the cost of investment in manufacturing, and 9.7% in non–manufacturing. In the case of our model, the mean value is about 1.5%.

While the findings we have just summarized cannot be used for direct falsification of our model, they suggest that its implications for the magnitude of adjustment costs and the elasticity of investment to Tobin's q are hardly out of line.

4.2 Elasticity of Intertemporal Substitution

Our calibration also implies a value for the elasticity of intertemporal substitution that is unusually low for the macroeconomics literature.

Although very far from the unitary value that we are used to see assumed in macroeconomic models, an intertemporal elasticity of substitution of about 1/37 falls in the confidence interval of several econometric studies, among which Hall (1988).

Campbell and Cochrane (1999) pointed out a further restriction on the value of the intertemporal elasticity of substitution, by considering its implication for the cross–country variation of interest rates. A low elasticity may imply a counterfactually high variation. This can be easily seen in the case of an endowment economy with $\gamma = \eta$

[9] We refer to studies, such as von Furtstenburg (1977), where the investment to capital ratio is gross aggregate investment divided by an estimate of the aggregate capital stock, and q is the ratio of market capitalization to the capital stock.

and i.i.d. lognormal consumption growth with mean λ and standard deviation σ. In that case, one can write

$$\log(r_t^f) = \log(\beta) + (1-\gamma)\lambda - (1-\gamma)^2 \frac{\sigma^2}{2}.$$

Campbell and Cochrane (1999) argue that if preferences were the same across countries, in absence of international capital flows, a very low γ may translate minimal differences in growth rates λ into large differences in risk–free rates. Whether it will or not, depends on the cross–country relation between first and second moment of consumption growth. Since countries with high consumption growth rates also tend to have lower standard deviation of consumption growth, the precautionary motive will work towards reducing the dispersion in interest rates. Without considering actual data, it is not possible to draw a conclusion on the net effect. Kandel and Stambaugh (1991) report that during the period 1957–1987, average real consumption growth was 8.2% in South Korea and 3.2% in the United States. They go on to argue that, under the lognormality assumption and with an elasticity of 1/29, a 4% difference in the volatility of consumption growth would make the simple model consistent with the data. They argue that the actual measured difference was not that far, at 2.1%.

4.3 Risk Aversion

Since the path–breaking contribution by Mehra and Prescott (1985), the main objective of most contributions to the asset pricing literature has been to lower the level of risk–aversion needed to attain an unconditional expected equity premium consistent with the data. As pointed out by Kocherlakota (1996), this was – and still is – due to the fact that most economists believe a coefficient of relative risk aversion higher than 10 to be highly implausible.

Indeed, during the last thirty years or so there have been quite a few studies that showed how popular models of economic behavior imply a rather low coefficient of relative risk aversion. These studies have contributed to generate the belief that individuals are not nearly as risk–averse as it is needed by most asset pricing models to generate sizeable risk premia. However, as also acknowledged by Mehra and Prescott (1985) and Kocherlakota (1996), the conclusions of all of these studies have been successfully challenged. For example, among the arguments in support of an upper bound of 10, Mehra and Prescott (1985) cite the work of Kydland and Prescott (1990) and Friend and Blume (1975). Kydland and Prescott (1990) find that only RRA coefficients between 1 and 2 allow their model to replicate the observed relative variabilities of detrended investment and consumption. It is clear though, that such result is model–specific. Our work, for example, shows that it is possible to achieve the same result with higher risk–aversion. Friend and Blume (1975) argue that a low RRA coefficient is needed for models of portfolio allocation to generate allocations of wealth across risky and riskless assets that is not grossly counterfactual. There are at least two important issues that detract from the relevance of this result. Kandel and Stambaugh (1991) convincingly argue that it obtains in endowment economies under time–additive expected utility and i.i.d. consumption growth, but does not generalize to other settings. Kocherlakota (1990a) shows that it vanishes if one uses proxies for the market portfolio that are broader than the stock market. For some time, the most compelling argument against higher risk–aversion has been what is known as the risk–free rate puzzle. Weil (1989) showed that in an endowment economy where agents maximize expected discounted utility, the low elasticity of intertemporal substitution implied by high risk aversion also induces a conterfactually high risk–free rate. This argument, however, essentially lost all of his bite when Epstein and Zin (1990b) showed how to easily disentangle attitude towards risk from attitude towards growth. Furthermore, the results illustrated in the previous sections of this paper show that for our model, as well as for Jermann (1998)'s and Boldrin, Christiano, and Fisher (2001)'s, a low elasticity of substitution is actually a necessary condition for success.

Our opinion is that the debate on the plausibility of different levels of the RRA coefficient is misleading. The main reason is that it somewhat disregards the simple fact that commonly used measures of the attitude towards risk, among which the RRA coefficient, are local. Or that, in other words, agents' attitude towards risk depends, possibly in a dramatic way, on the variance of the bets they face.[10] Regardless of her risk preferences, an agent's relative risk aversion is defined as the amount she would pay in order to avoid a multiplicative atemporal bet. This means that, in general, relative risk aversion measures must be indexed by the lottery's probability distribution. The simple calculations shown in Section 5 remind us that the commonly used RRA coefficient is nothing else but the relative risk aversion that obtains for simple bets of infinitely small variance, in the case of expected utility.

We see two ways of assessing the relative risk aversion implied by a given model. First, we can ask whether the relative risk aversion computed at the "equilibrium bets" is at odds with experimental or direct empirical evidence. That is, we can investigate

[10]Notable exceptions are Kandel and Stambaugh (1991) and Epstein and Zin (1990a), who explicitly discuss this point.

the plausibility of the implied relative risk aversion, given the risks implied by the model. We have computed this measure in the case of our framework. For given pair (k, ε), and regardless of the shape of risk preferences, our agent faces a bet over the lottery $\{v[k'(k,\varepsilon),\varepsilon_i],\pi(\varepsilon_i|\varepsilon)\}_{i=1}^n$, where n is the number of values for the productivity shock. Our measure of risk–aversion is the value $P(k, \varepsilon)$ such that:

$$[1 - P(k,\varepsilon)] \sum_i \pi(\varepsilon_i|\varepsilon) v[k'(k,\varepsilon),\varepsilon_i] = \mu[v(k',\varepsilon')].$$

That is, the agent would be indifferent between the lottery and a sure amount equal to the fraction $1 - P(k,\varepsilon)$ of its expected value. In Table 7 we report the values $P(k,\varepsilon)$ (expressed as a percentage of the lottery's expected value) that obtain for a given value of the shock ε (the third) and for three levels of the detrended per–capita capital stock: the lowest in the ergodic set, the mean of the distribution, and the highest in the ergodic set, respectively. The columns labeled *C.V.* report our measure of risk: the coefficient of variation of the lotteries, expressed in percentage of their expected values. The first column refers to the case of Epstein–Zin preference under consideration here. The other two to the case of disappointment aversion to be discussed in Section 5.

Table 7: Implied Equilibrium Risk Aversion (in percentage points)

	EZ		*DA*		*GDA*	
k	*R.A.*	*C.V.*	*R.A.*	*C.V.*	*R.A.*	*C.V.*
Low	.019	0.34	.008	0.33	.002	.37
Medium	.005	0.22	.005	0.21	.00071	.23
High	.002	0.14	.003	0.13	.0001	.14

It appears that the risks faced by the agent are rather small. In the EZ case, at the mean of the distribution of capital the standard deviation is equal to only 0.22% of the lottery's expected value. In order to avoid the lottery and replace it with a sure amount, the agent would be willing to give up 0.005% of its expected value. Notice that any value $v(k,\varepsilon)$ could be implemented by a constant sequence of detrended consumption $c_t = v(k,\varepsilon)[1 - \beta e^{\varphi+\lambda\gamma}]^{1/\gamma}$. Therefore, the coefficient of variations and the risk–aversion measures can be recast as percentages of such constant consumption equivalent. For the parameter values adopted in Section 3, we have that $[1 - \beta e^{\varphi+\lambda\gamma}]^{1/\gamma} = 1.134867$. For the sake of illustration, consider an agent facing a lottery over future utilities, whose expected value is 25,000. For the agent, this value is equivalent to a constant stream of consumption equal to 28,372. If the payoffs of the lottery $v[k'(k,\varepsilon),\varepsilon_i]$ were to be implemented by constant consumption sequences, the coefficient of variation in terms of per–period consumption equivalent would be about 62. In order to avoid this risk, the agent would be willing to give up 1.4 units of consumption. Is it too much? Too little? Based on personal introspection, Cochrane (1997) does not find it unreasonable for a family earning $\$50,000$ per year to be willing to pay 25 cents in order to avoid a bet involving the win or loss of \$10 with even probability. This leads us to conclude that, most likely, he would not find unreasonable the behavior of our agent either. Unfortunately, the literature on the direct estimation of risk aversion is still fairly limited. Some of the studies suffer from the lack of background information about the subjects of the study. Others, from the fact that their results depend on the answers to hypothetical questions. Dohmen, Falk, Huffman, Sunde, Schupp, and Wagner (2005) gathered data about attitude towards risk by facing 22,000 individuals with a series of choices among hypothetical lotteries.[11] Unfortunately for us, they do not report their estimates by variance classes. They assume that agents are expected utility maximizers, and report that most agents have a relative risk aversion coefficient between 5 and 10. However, a non–negligible fraction of subjects displays a coefficient greater than 20.

A second way of assessing relative risk aversion is implied by the assumption of universality of risk preferences. That is, by the assumption that we require an agent's attitude towards risk to be described by the same preference relation, regardless of the risk she faces. It has been pointed out that this criterion constitutes a problem for expected utility.[12] Since under expected utility the bet's variance has only a second–order effect on risk–aversion, individuals are essentially risk–neutral when faced with low–risk, for a very large set of RRA coefficients. The implication is that RRA coefficients that generate realistic levels of risk aversion for small risk, also generate incredibly large risk–aversion with respect to larger risks. This point is very effectively demonstrated in Rabin (2000). This is also a shortcoming of our framework. Even though the relative risk aversion of an EZ agent with with $\eta = -20$ is plausible at the risk levels implied by our model's equilibrium, the same agent will display unreasonable risk aversion when facing considerably greater risks. The literature on decision theory under uncertainty pointed out that the issue we have just illustrated

[11] A strength of their work is that they have rather accurate information about gender, age, and parental background of the respondents. A further strength is that they validate the survey's results by means of an experiment implying actual payments. The problem is that while the hypothetical questions allowed for arbitrary variance, funding constraints implied that the experiments were limited to low–variance lotteries.

[12] See for example Kandel and Stambaugh (1991), Epstein and Zin (1990a), and Rabin (2000).

is much less of a problem when risk preferences feature first–order risk aversion. This is yet another reason why it is of interest to analyze a version of our model in which agents are disappointment averse. We now turn to that. Our digression on risk aversion will continue in Section 5.2.

5 Disappointment Aversion

In this section we consider the case of disappointment aversion preferences due to Gul (1991). Before us, Epstein and Zin (2001), Routledge and Zin (2004) and Bonomo and Garcia (1994) have studied the asset pricing implications of this preference specification in endowment economies. This is the first attempt at modeling it in a production economy. The reader that is familiar with the properties of this preference relation is advised to jump to the beginning of section 5.1.

In most applied work in economics, agents choose among risky outcomes (lotteries over simple events) by evaluating each of them according to the expected utility criterion. According to this paradigm, the certainty equivalent μ of a lottery over a finite set of payoffs $\{x_1, x_2, ..., x_N\}$ satisfies

$$u(\mu) = \sum_{i=1}^{N} \pi_i u(x_i),$$

where $\pi_i \geq 0$, $\sum_{i=1}^{N} \pi_i = 1$, is the probability of event i and u is an increasing and continuous function. The preference relation over lotteries represented by this utility specification is known to satisfy the axioms of monotonicity, completeness, transitivity, continuity, and independence. The latter has repeatedly come under attack, as experimental studies have found an increasing number of instances in which individuals' decision making seems to violate it. Perhaps the most famous of these violations is that known as the Allais Paradox.

Decision theorists have therefore sought to identify preference relations that satisfy a weaker version of the independence axiom. Here we consider the work of Faruk Gul. Gul (1991) defines as *disappointing* those outcomes that lie below a lottery's certainty equivalent. Its *weak independence axiom* requires independence only for lotteries that are disappointment–comparable. Gul shows that the preference relation satisfying this axiom along with those named above, can be represented by the certainty equivalent μ that solves

$$u(\mu) = \sum_{i=1}^{N} \pi_i u(x_i) - \theta \sum_{x_i \leq \mu} \pi_i \left[u(\mu) - u(x_i) \right].$$

Outcomes below the certainty equivalent receive a greater weight in the computation of overall utility. Such weight depends positively on the parameter θ and on the distance from the certainty equivalent itself. Notice that μ appears on both sides of the above condition, and therefore must be determined together with the set of disappointing states. It is sometimes handy to express μ as the following weighted average:

$$u(\mu) = \sum_{i=1}^{N} \tilde{\pi}_i u(x_i), \quad \tilde{\pi}_i = \pi_i \frac{1 + \theta \times I(x_i, \mu)}{1 + \theta \sum_{(x_i \leq \mu)} \pi_i},$$

where $\sum_i^N \tilde{\pi}_i = 1$ and $I(x_i, \mu)$ is an indicator function that takes value 1 if $x_i \leq \mu$ and 0 otherwise. This formulation makes it clearer that under DA the attitude towards risk not only depends on the curvature of u, but also on the value of θ. For the sake of illustration, consider the case of $N = 2$. In Figure 3 we have pictured the qualitative behavior of indifference curves, in the cases of expected utility ($\theta = 0$) and disappointment aversion ($\theta > 0$). It is important to notice that, contrary to the expected utility case, under disappointment aversion the indifference curve is not differentiable at the certainty equivalent. The kink reflects what Segal and Spigal (1990) called first–order risk aversion, as opposed to second–order risk aversion, which characterizes expected utility. The difference between first– and second–order risk aversion can be appreciated by computing the effect of θ on relative risk aversion, i.e. on the amount P (the risk premium) that an agent is willing to pay in order to avoid a given bet.[13] Consider the following example. For some $\kappa \geq 0$, let an agent endowed with wealth ω_0 consider an investment opportunity that pays $w_0(1+\kappa)$ and $w_0(1-\kappa)$ with equal probabilities. With disappointment aversion, the risk–premium associated with κ, $P(\kappa)$, solves

$$\frac{1+\theta}{2+\theta} u[w_0(1-\kappa)] + \frac{1}{2+\theta} u[w_0(1+\kappa)] = u[w_0(1 - P(\kappa))], \quad \kappa > 0$$

and satisfies $P(0) = 0$. In a neighborhood of $\kappa = 0$, we have that

$$P(\kappa) = P(0) + \frac{dP}{d\kappa} d\kappa + \frac{1}{2} \frac{d^2 P}{d\kappa^2} d\kappa^2 = \frac{\theta}{2+\theta} d\kappa - \frac{1}{2} \frac{u''(w_0) w_0}{u'(w_0)} \left[\frac{4(1+\theta)}{(2+\theta)^2} \right] dk^2.$$

If the utility function is isoelastic with $\frac{u''(w_0)w_0}{u'(w_0)} = \eta - 1$, we can write

$$P(\kappa) = \frac{\theta}{2+\theta} d\kappa + \frac{1}{2}(1-\eta) \left[\frac{4(1+\theta)}{(2+\theta)^2} \right] dk^2,$$

[13] Here we focus on multiplicative bets, but similar arguments can be made for additive bets. For an accessible review of the definition and measurement of risk aversion, see Chapter 2 of Gollier (2001).

Risk aversion depends negatively on both the disappointment parameter θ and on $1-\eta$. For $\theta > 0$, an increase in κ, i.e. an increase in risk, has a first–order effect on the risk premium. This is why in this case we talk of first–order risk aversion. In the case of expected utility ($\theta = 0$) instead, risk has only a second–order effect, as we obtain the familiar expression $P(\kappa) = [(1-\eta)/2]dk^2$. This is why $|\frac{u''(w_0)w_0}{u'(w_0)}| = 1-\eta$ is known as the de Finetti–Arrow–Pratt coefficient of relative risk–aversion.

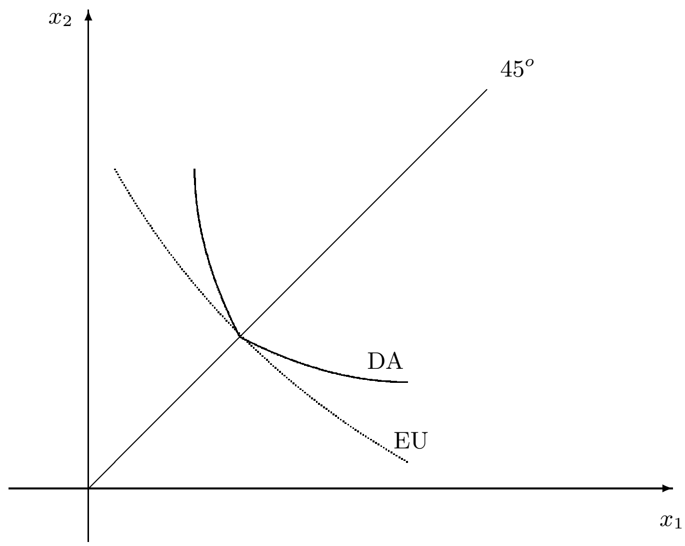

Figure 3: Indifference curves.

5.1 Asset Returns

In the case of disappointment aversion, the stochastic discount factor takes the following form:

$$m(\varepsilon_i|k,z) = \beta e^{\lambda(\gamma-1)} \frac{1+\theta I(k',\varepsilon_i,\mu)}{1+\theta\sum_i \pi(\varepsilon_i|\varepsilon)I(k',\varepsilon_i,\mu)} \left[\frac{c(k',\varepsilon_i)}{c(k,\varepsilon)}\right]^{\gamma-1} \left[\frac{v(k',\varepsilon_i)}{\mu(k',\varepsilon)}\right]^{\eta-\gamma},$$

where $I(k',\varepsilon_i,\mu) = 1$ if $v(k',\varepsilon_i) \leq \mu$, and $I(k',\varepsilon_i,\mu) = 0$ otherwise.

We decide to set $\eta = 0$ and pick the values of ι, γ, and θ in order to match the same moments as in Section 3: the mean risk–free rate, the mean equity premium, and the ratio of standard deviation of consumption growth with respect to output growth. Table 8 summarizes the parameters values that are different with respect to Section 3.

Table 9 reports the implied values for the second moment of risk–free rate and return on equity, and for the mean equity premium. The results are very close to

η	θ	ξ	β	γ	ι
0	0.15	1		-36.04	1.218

Table 8: Calibration under Disappointment Aversion

those obtained under Epstein–Zin preferences. This suggests that, for the purpose of computing these unconditional moments, first–order risk–aversion can be used as an alternative to second–order risk–aversion. As we will see, however, the two parameterizations have radically different implications for the attitude towards risk, for levels of uncertainty different from those implied by the equilibrium of our model.

	σ_C/σ_Y	σ_I/σ_Y	$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$
Model	0.5	2.48	0.322%	2.956%	1.701%	11.87%
Data	0.499	2.647	0.322%	0.902%	1.836%	7.702%

Table 9: Unconditional Moments – Disappointment Aversion

As it was the case for the preference specifications considered in previous sections, even under disappointment aversion the price–dividend ratio does not predict future levels of the equity premium. This is not surprising, given that even under DA, risk–aversion is essentially acyclical by construction.

5.2 Reconsidering Risk Aversion

In Section 3 we acknowledged that most economists would find a value of $\eta = -20$, necessary for the EZ model to match the average historical equity premium, to be implausible. We have argued, however, that the emphasis on a local measure such as the coefficient of relative risk aversion is misleading. In fact, in the case of our model, the relative risk aversion measured at the risk implied by the model does not seem to be excessive. We also noted that a value of $\eta = -20$ could still be problematic, as he would imply a much higher relative risk aversion for higher risks. Here we document that this criticism does not apply to the DA model.

Table 7 shows that the models with expected utility and disappointment aversion, calibrated to match the same asset pricing moments, obviously generate the same risk–aversion for equilibrium bets. However, they imply very different levels of risk aversion for greater risks. This phenomenon is illustrated in Figure 4, which plots the loci of $1-\eta$ and θ such that the EU and DA models generate the same level of relative risk aversion, for simple bets with different risks. To be precise, we have considered a simple atemporal bet that pays $1-\kappa$ or $1+\kappa$, with equal probability, similarly to what we have done in Section 4. We then identified, for given $0<\kappa<1$, the set of pairs (θ,η) such that

$$\frac{1+\theta}{2+\theta}(1-\kappa)+\frac{1}{2+\theta}(1+\kappa)=\left[\frac{1}{2}(1-\kappa)^{\eta}+\frac{1}{2}(1+\kappa)^{\eta}\right]^{1/\eta}.$$

The left–hand side is the certainty equivalent of the bet for a GDA agent with disappointment aversion parameter θ, $\eta=1$ (no curvature in the Bernoulli utility function), and $\xi=1$. The right–hand side is the certainty equivalent for an EU agent with relative risk–aversion coefficient $1-\eta$. For a few values of κ, which also equal the coefficient of variation of the bet, Figure 4 plots the loci of pairs $(\theta,1-\eta)$ that satisfy the above condition. The relative risk–aversion coefficient that is equivalent to any given value of θ is higher, the lower the risk. The second column of Table 7 indicates that at the mean of the distribution of capital, our calibration of the model with Epstein–Zin preferences generates a coefficient of variation of 0.22%. Figure 4 shows that in the case of the atemporal lotteries considered here, a disappointment averse agent displaying the same local risk aversion must have $\theta\approx 0.04$. For a coefficient of variation 100 times higher, i.e. $\kappa=0.22$, the relative risk aversion of the same EU agent goes through the roof. However, the DA agent exhibits an attitude towards risk similar to that of an EU agent who is almost risk–neutral![14]**??**

The bottom line is that with first–order risk aversion, it is possible to generate non–negligible levels of risk–aversion to small–risk bets, without implying enormous risk–aversion with respect to larger–risk bets.

5.3 Generalized Disappointment Aversion

Recently, Routledge and Zin (2004) have proposed a generalization of Gul's notion of disappointment aversion. Their work allows the disappointment threshold to differ

[14]Notice that the outcome of this simple exercise depends on the type of bet faced by the agent. The calibration results reported earlier in the paper show that, in the context of our model, a DA agents needs $\theta\approx 0.14$ – rather than $\theta\approx 0.04$ – in order to generate a relative risk aversion close to that of an EZ agent with $\eta=-20$. This caveat, however, does not affect the spirit of the exercise. Figure 4 shows that, in the case of the simple bets considered here, it takes $\kappa=0.006$ for an EZ agent with $\eta=-20$ and a DA agent with $\theta=.14$ to display the same relative risk aversion. Given the slope of the locus for $\kappa=0.2$, this does not change our conclusion in any appreciable way.

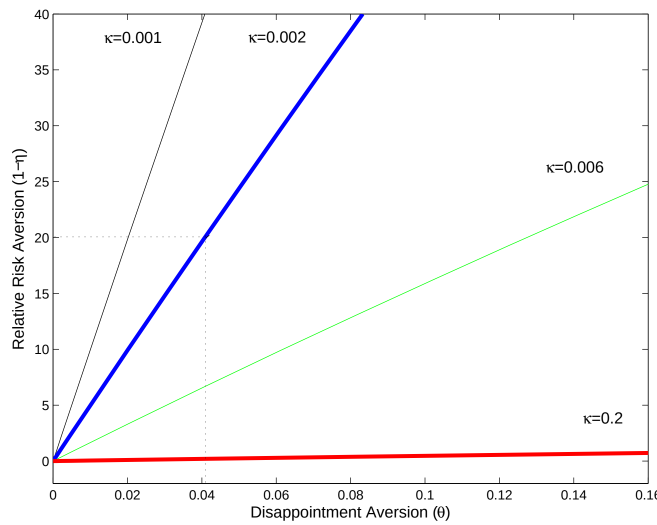

Figure 4: Comparison between risk–preferences.

from the certainty equivalent.[15] This implies a different utility representation:[16]

$$u(\mu) = \sum_{i=1}^{N} \pi_i u(x_i) - \theta \sum_{x_i \le \xi\mu} \pi_i \left[u(\xi\mu) - u(x_i)\right]$$

and a different stochastic discount factor:

$$m(\varepsilon_i|k,z) = \beta e^{\lambda(\gamma-1)} \frac{1+\theta I(k',\varepsilon_i,\mu)}{1+\theta\xi^{\eta}\sum_i \pi(\varepsilon_i|\varepsilon) I(k',\varepsilon_i,\mu)} \left[\frac{c(k',\varepsilon_i)}{c(k,\varepsilon)}\right]^{\gamma-1} \left[\frac{v(k',\varepsilon_i)}{\mu(k',\varepsilon)}\right]^{\eta-\gamma},$$

where $I(k',\varepsilon_i,\mu) = 1$ if $v(k',\varepsilon_i) \le \xi\mu$, and $I(k',\varepsilon_i,\mu) = 0$ otherwise.

Table 10 shows that by setting $\theta = 1.5$, $\xi = 1.007$, and leaving all other parameters at the same values as above, this specification implies similar values for the unconditional moments of asset returns.

	σ_C/σ_Y	σ_I/σ_Y	$E(r^f)$	$Std(r^f)$	$E(r^e - r^f)$	$Std(r^e)$
Model	0.5	2.38	0.322%	2.569%	1.781%	11.79%
Data	0.499	2.647	0.322%	0.902%	1.836%	7.702%

Table 10: Unconditional Moments – Generalized Disappointment Aversion

The important question, however, is whether this parameterization generates a counter–cyclical market price of risk, similarly to what shown in Routledge and Zin

[15]Routledge and Zin (2004)'s main theoretical innovation is to allow for a more general definition of disappointment comparability.

[16]In the case of $\xi > 1$, one needs to multiply the right–hand side by the constant $[1-\theta(\xi^{\eta}-1)]^{-1}$ and impose $\theta(\xi^{\eta}-1) < 1$ in order to maintain $\mu(x) = x$ and monotonicity. See Section 2.3 of Routledge and Zin (2004).

(2004) in the case of a Mehra–Prescott economy. The answer is positive. Table 11 reports the average values of the market price of risk, conditional on all levels of the productivity shock, in the cases of DA and GDA preferences, respectively. The shocks are listed in increasing order of magnitude.

Shock	DA	GDA
1	0.096	0.234
2	0.122	0.215
3	0.151	0.175
4	0.238	0.192
5	0.124	0.086
6	0.090	0.046

Table 11: Conditional Market Price of Risk

Unfortunately, the extent of the predictability of the equity premium generated by our model is not even close to that implied by the data. The results of regressing future cumulative equity premia on current price–dividend ratios, which we report in Table 12, make it clear. With respect to the DA case, the magnitudes of the slope parameters are greater in absolute value, and the regressions' R^2 are slightly larger. However, the results are still very far from what implied by the data. Once again, the main issue appears to be the large and counterfactual volatility of the risk–free rate.

Table 12: Predictability of the equity premium: DA Vs. GDA.

	EU		*Data*		*GDA*	
Horizon (s)	*Slope*	R^2	*Slope*	R^2	*Slope*	R^2
1	−.00777	.00	−.117	0.08	−0.019	.003
	(−7.20)		(−2.27)		(−13.84)	
2	−0.0142	.00	−.219	0.15	−0.035	.006
	−9.36		(−3.13)		(−18.15)	
3	−.0191	.00	−.28	0.19	−0.048	.008
	(−10.26)		(−3.60)		(−20.27)	
5	−.0259	.00	−.484	0.31	−0.067	.01
	(−10.79)		(−4.78)		(-22.22	
7	−.0287	.00	−.712	0.36	−0.08	.01
	(−10.2)		(−5.29)		(-22.73)	

Note: t statistics in parenthesis.

6 Conclusion

In this paper we have shown that when households have Epstein–Zin preferences, a general equilibrium production economy with convex investment adjustment costs produces asset pricing implications that are fairly close to those generated by similar economies with internal habit formation. In particular, the model generates a sizeable equity premium with a fairly low volatility of stock returns, pro–cyclical price–dividend ratio, and predictability of stock returns. Consistently with the empirical evidence, the predictability improves with the time horizon, and it is not matched by the predictability of dividend growth.

The levels of risk aversion implied by the model along the equilibrium path do not seem unreasonable. What appears to be unreasonable is the attitude towards risk that these preferences imply for bets with greater risk, under the universality assumption. Assuming risk preferences that allow for first–order risk aversion addresses this issue. In fact, there exists a value for the disappointment aversion parameter, such that the model has essentially the same implications for both quantities and prices as the model with Epstein–Zin preferences. Furthermore, when the disappointment averse agent is faced with an atemporal lottery that has a coefficient of variation 100 times as large as that implied by our model, he displays the same relative risk aversion as an expected utility agent with logarithmic utility.

No matter the preference specification, the main shortcomings of the model appear to be (i) the excessive volatility of the risk–free rate and (ii) the lack of predictability of the equity premium. For the expected equity premium to be time–varying, the volatility of stock returns or the price of risk, or both must be time–varying. Since the stochastic process for the only shock in the model is homoscedastic and risk aversion is a–cylical, the occurrence of the latter problem is not surprising. Gul (1991)'s disappointment aversion as generalized in Routledge and Zin (2004), by generating a time–varying price of risk, has the potential of addressing the second of the two shortcomings. This result mirrors that obtained by Routledge and Zin (2004) in the case of a Mehra–Prescott economy. However, the counterfactually high volatility of the risk free rate undermines the model's ability to generate predictability to the extent implied by the data.

We believe that understanding how to amend the model in order to reduce the volatility of the risk–free rate should be at the top of the research agenda. In the present framework, the excess volatility is the result of the perfect rigidity of bond supply, the high rigidity of bond demand, implied by the low intertemporal elasticity of substitution, the fast–rising investment adjustment costs, and the relatively low persistence of productivity shocks. A drastic change in the parameters governing the elasticity of substitution and the adjustment costs would impair the model's ability to generate a sizeable equity premium. On the one hand, increasing the persistence of the productivity shock would make our framework consistent with most consumption–based asset pricing studies, that assume i.i.d. consumption growth. On the other hand, doing so would represent somewhat of an infringement of the methodological approach that calls for the use of direct evidence for the parameterization, whenever possible. Allowing for a bond supply schedule with non–zero elasticity seems to us the most promising direction. This could be accomplished by assuming some form of heterogeneity across households, as in Guvenen (2005), for example, or by introducing a government entity that finances a deficit by issuing securities to the public.

A Data

Our data on quantities is drawn from FRED (Federal Reserve Economic Data) at `http://research.stlouisfed.org/fred2/`. Output is variable GDPC96 (Real Gross Domestic Output). Consumption is the sum of variables PCNDGC96 and PCESVC96 (Real Personal Consumption Expenditures on Nondurable Goods, and on Services, respectively). Investment is variable FPIC96 (Real Fixed Private Investment). All series are quarterly from 1947:1 to 2005:4, expressed in billions of chained 2000 dollars, and seasonally adjusted. Employment is variable AWHI (Total Weekly Hours in All Private Industries), which is monthly - we averaged to obtain quarterly observations.

Our data on asset returns is drawn from CRSP (Center for Research in Security Prices). Nominal Equity Returns correspond to NYSE variable VWRETD (Value-Weighted Return on the NYSE Index, including Dividends). Nominal Risk-Free Returns correspond to the yield to maturity on 90-day T-Bills (based upon the average between bid and ask prices, and drawn from the Fama T-Bill Term Structure Supplemental Files). All series are quarterly from 1947:1 to 2005:4. (Gross) real returns were computed as gross nominal returns divided by the gross inflation rate. The inflation rate is based upon the CPI from FRED, variable CPIAUCNS (CPI for all urban consumers, all items). This variable is not seasonally adjusted, and it is monthly - quarterly observations correspond to the value of the index in the last month of each quarter.

We computed price-dividend ratios as follows. The variable VWRETD described previously provides $(P_t + D_t)/P_{t-1} - 1$, where P_t is the value-weighted index and D_t are dividends at time t. From the CRSP dataset we also obtained NYSE variable VWINDX, which provides P_t (the value of the index relative to a base year). Price-Dividend ratios are thus the inverse of $(1+\text{VWRETD}_t)\times\ \text{VWINDX}_{t-1}/\text{VWINDX}_t - 1$.

We ran the return predictability regressions on an annual basis. We computed annual dividends by summing up quarterly dividends. Annual price-dividend ratios are the NYSE value-weighted index for the last quarter divided by annual dividends. The real returns between years t and $t+k$ were computed by summing up all real quarterly returns between the two dates.

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