

The Macroeconomic Dynamics of Labor and Capital Market Imperfections*

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Abstract

This paper investigates the propagation properties of credit market imperfections when they affect the cyclical reallocation of labor. A costly state verification problem in the capital lending relationship leads to endogenous job separations of two types: workers lose their jobs either because firms downsize to meet their interest payments or because the production unit is shut down following bankruptcy. When calibrated on firm-level gross capital and worker flows, this generates a powerful propagation mechanism, measured as amplification of exogenous shocks and high persistence in output growth, as credit frictions affect the large flows that are worker layoffs. In addition, while matching key labor market statistics, the model implies that the cyclicity of employment adjustments are driven by both job losses and hirings, as documented in recent empirical research for the U.S..

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1 Introduction

Of the many potential explanations for labor being separated from its current employment, this paper investigates the role and implications of separations due to credit market imperfections. Informational asymmetries between capital lenders and firms can lead to signals of financial distress, such as bankruptcy filings and default rates, that are counter-cyclical. And although the capital flows concerned in asset liquidation during bankruptcy are modest, the large flows involved in labor separation imply that the potential implications of countercyclical layoffs due to credit market imperfections for business cycles are significant.¹ In addition, credit market imperfections provide a rationalization for endogenous labor separations which, as recent evidence shows, play an equally important role as hirings in explaining the variability of employment.²

Introducing endogenous layoffs to meet interest payments on debt contracts is motivated by three observations from firms having filed for bankruptcy. Using data for all publicly listed firms in the U.S. having filed for bankruptcy since 1980, it is possible to follow the changes to sales of property, plant and equipment (PP&E) and employment in the years preceding and following a filing for bankruptcy protection. First, as documented in Section 2, it appears that sales of PP&E show no discernible trend in the years running up to bankruptcy, and spike during the first two years after filing. Firms do not, as might have been expected, run down their assets to avoid having to file for bankruptcy protection. Second, firms tend to diminish the size of their labor force in the years running up to a bankruptcy filing, while the change in employment stabilizes afterwards. Third, while the workforce is contracting, output per employee increases up to the date of filing for bankruptcy.³

Section 3 models imperfections in the allocation of labor and capital, in the form of a search and matching process, in the presence of a costly state verification type of credit market

¹A model of search in physical capital was first explored in Kurmann and Petrosky-Nadeau (2007). An extension introducing endogenous capital separation, or liquidations, found the quantitative implications to be small.

² Not only are both flows very volatile over the business cycle - the standard deviation of total job creation and job destruction are as much as three times larger than that of GDP - separation hazard rates are strongly countercyclical and job finding rates strongly procyclical. See Davis, Faberman and Haltiwanger (2006), Fujita and Ramey (2006), Yashiv (2006), Elsby, Michaels and Solon (2007), Fujita and Ramey (2007).

³There are two possible interpretations to this observation. First, it may be that, in order to meet a certain level of production, firms require remaining employees to supply additional effort. Second, it may be that these firms are terminating the least productive jobs, raising the average labor productivity in the process.

imperfection (Townsend, 1979). As in Kurmann and Petrosky-Nadeau (2007), this takes the form of an idiosyncratic productivity shock - which occurs after all optimal decisions are made and factor price equilibria are established - observable to the lender only at a cost. This assumption of asymmetry between capital lender and firm gives rise to an agency problem, and results in a debt contract that is characterized by endogenous capital liquidation from firms whose productivity falls below a state-dependent threshold.

Given the debt contract, once firms observe the realization of the idiosyncratic productivity shock they may lay off a fraction of the workforce, conditional paying a firing cost proportional to the negotiated wage and before production takes place, thereby saving on labor costs in order to avoid monitoring or having to default on their interest payments. However, firms require additional effort from the remaining employees to meet the beginning of period production plan. This addition of layoff uncertainty and work effort is incorporated into the worker's value of the current job, thereby affecting the negotiated wage. On the one hand greater job loss risk pushes up the wage, as does the expected additional work effort. On the other hand, the wage is decreasing in the size of the severance payment.

The motivation for the approach chosen to model frictions to the allocation of labor and capital rests first on the observation of similar patterns of congestion on both markets, detailed in Section 2. While the phenomenon is well known for labor flows, it is also the case that physical capital is often specific to a particular task and location, making its reallocation to another firm time consuming and costly. Second, just as worker flows involved in employment adjustments are large, often amounting to over 10% of employment on a quarterly basis, gross flows of capital additions and subtractions average 9.7% and 7.3% of undepreciated capital stocks, respectively.⁴

The quantitative implications of the model, explored in Section 4, establish that credit market imperfections can have significant aggregate consequences when they affect the cyclical reallocation of labor. When calibrated on firm-level gross capital and worker flows, this generates a powerful propagation mechanism, measured as amplification of exogenous shocks and high persistence in output growth, in which counter-cyclical changes in the rate of layoffs

⁴As reported in Ramey and Shapiro (1998), and largely confirmed by the work of Eisfeldt and Rampini (2006). While this is documented in Section 2, the reader is referred to Kurmann and Petrosky-Nadeau (2007) for more details regarding congestion and reallocation on physical capital markets.

due credit frictions are central. This is because the flows involved are large enough to affect aggregate outcomes. A model with endogenous capital liquidation, which concern a very small fraction of the aggregate capital stock, but only exogenous worker separations generates little to no amplification. At the heart of the high persistence in output growth lies the congestion on labor and physical capital markets. The added contribution of endogenous layoffs is to contribute to higher autocorrelation in output growth at two and three lags, as is the case in the quarterly time series for U.S. GDP.

The dynamics of labor adjustments, which are key to the model's propagation properties, also fit closely the empirical evidence. The variability of aggregate employment and unemployment are consistent with their empirical counterparts, while it is too low in models with exogenous separation or search frictions in labor markets alone. As in the data, the job finding hazard increases steeply in an expansion, and as recent evidence has re-established, the job separation rate is strongly counter-cyclical.⁵ By decomposing the variability of employment into job creations and destructions, the benchmark calibration implies that 20% of the variability in aggregate employment is attributable to job separations, which is close to the share reported in Fujita and Ramey (2007). In addition, as in the data, the model predicts countercyclical gross job creation flows. This contrasts with models of constant job separation rates which, by construct, imply that the entirety of employment variability is due to changes in the rate of job creation, and generate total job creations are procyclical.⁶

The model also sheds some light on the joint observation of countercyclical signals of financial distress and mildly procyclical capital liquidation at these firms. Indeed, capital liquidation attributable to financial distress, defined as the sum of sales of PP&E by firms in financial distress and the stock of PP&E of firms exiting due to bankruptcy, shows a moderately positive correlation with output over the business cycle.⁷ Along with a counter-cyclical default rate, the

⁵See in particular Fujita and Ramey (2006), Yashiv (2006), Elsby, Michaels and Solon (2007), and Fujita and Ramey (2007).

⁶Examples of such models, motivated by the conclusions of Shimer (2005a,b) on the acyclical nature of job separation rates, include Blanchard and Gali (2006), Gertler and Trigari (2006), Rotemberg (2006) and Rudanko (2006).

⁷These correlation relate to a sample covering 1980-2004. In Kurmann and Petrosky-Nadeau (2007) the sample ends with the last reported retirements of PP&E in 1993. The different samples reveal opposite correlations of the liquidation rate. The present model can be calibrated to match either correlation. However, given the very small size of the flows involved this does not affect the main quantitative results related to propagation and

model generates the procyclicality of capital liquidations due to financial distress observed in the data. The latter is linked to the strong procyclicality of the capital matching hazard as the optimal liquidation decision involves an arbitrage between the value of the capital match net of the current reinjection of funds and the value of recuperated capital. This is strongly affected by the congestion on physical capital markets by which it is much less costly to reallocate capital in an upturn. Thus, the value of liquidated capital is comparatively lower in a downturn leading to fewer liquidations of capital at firms under bankruptcy.

This paper is closest in spirit to that of Wasmer and Weil (2004) who consider the implications of credit and labor market frictions when hiring is conditional on entrepreneurs having first located a creditor.⁸ While they establish an accelerator mechanism of endogenous layoffs due to credit frictions, their investigation is not in a quantitative DSGE framework. It is also closely related to the work, initiated by Mertz (1995) and Andolfatto (1996), of integrating the Mortensen-Pissarides (1994) framework for labor markets into quantitative DSGE models.⁹ In particular, it relates to the work of Den Haan et al. (2000) who investigate the propagation induced by incorporating endogenous job separations, and the case of frictions in the allocation of physical capital explored in Kurmann and Petrosky-Nadeau (2007). The former paper's result of propagation is corroborated, although for different reasons. The mechanism involved in capital separation, key to their strong result, is founded on credit frictions and is not central to the results of this paper. Central here are labor separations, which are modeled differently. The latter paper, while contributing by introducing search in physical capital to a DSGE framework, found no significant quantitative implications, even with endogenous countercyclical capital separation due to financial distress. As this paper shows, credit market imperfections can have quantitatively important consequences when they affect the dynamics of employment adjustments, as they can operate through the important flows that are worker layoffs.

labor markets.

⁸Indeed, this paper even borrows from the title of their 2004 article.

⁹Other recent contributions, in a New Keynesian setting, include Krause and Lubik (2007).

2 Worker flows, capital flows and the business cycle

This section serves as a motivation for the chosen approach to modelling worker and capital reallocation. The empirical evidence on gross worker and capital flows is reviewed first, before examining the evidence of congestion phenomena on both factor markets, and how these relate to the business cycle.¹⁰ Finally, the relation between separations, of both workers and capital, and financial distress is documented, motivating the structure for layoffs and capital liquidations in the model.

2.1 Labor and capital flows

The labor literature has long documented the large flows of labor in and out of employment. In their seminal work, Davis, Haltiwanger and Schuh (1996) report that annual flows of job creation and job destruction in the manufacturing sector amount to 9.1% and 10.3% of employment, respectively. More recently, data from the Business Employment Dynamics (BED) data set, a virtual census of establishments discussed in Davis, Faberman and Haltiwanger (2006), show average quarterly job creation and job destruction rates over the period 1990-2005 of 7.9 and 7.6% of employment, respectively.

Similar observations are being uncovered for the allocation of physical capital. While the capital stock computed in the NIPA is the sum of current and past new investment, adjusted for depreciation, a bottom-up approach is needed to measure gross flows of physical capital. Based on a sample of publicly traded firms, Eisfeldt and Rampini (2006) estimate that annual gross investment, defined as the sum of capital expenditure and acquisition of used capital, averages 22% of the total capital stock. The reallocation of used capital, as reported in Eisfeldt and Rampini (2007), makes up to 24% of gross investment. Using the same data set, Ramey and Shapiro (1998) estimate that gross flows of capital additions and subtractions average 9.7% and 7.3% of undepreciated capital stocks, respectively. Thus flows of physical capital in and out of firms, even though capital, new and used, is often specific and tied to a particular location, are comparable to the job creation and destruction flows reported above.

¹⁰The discussion on the empirical evidence for capital flows and congestion on capital markets draws on Kurmann and Petrosky-Nadeau (2007), wherein more details and references may be found.

2.2 The dynamics of factor adjustments

Employment adjustments operate through flows of new hirings and job losses. The cyclical component of the gross job creation and job destruction flows, constructed by Fujita and Ramey (2006) using the gross flows data from the Current Population Survey (CPS), are both more volatile than the cyclical component of the industrial production index, their chosen measure for the business cycle. The variability of both flows is comparable, contrary to the suggestions made by Shimer (2005a, b) and Hall (2005) concerning the acyclical nature of job separations, lending support to the view that separations play a substantial role in employment adjustment.

Similar conclusions are reached by Elsby, Michaels and Solon (2007) in parallel research. Bringing a modification to the method of treating the 1994 CPS redesign appears to strongly affect the estimation of inflows into unemployment. As a result, the 2000/2001 recession is no longer characterized by weak inflows into unemployment (e.g. Shimer 2005a). The same authors explore the heterogeneity in flow rates across job leavers, job losers and labor force entrants. They find that job loser inflows are clearly countercyclical, spiking upward in all recessions.

Total flows of hirings and job losses are affected by the corresponding job finding and separation hazard rates, both highly volatile. As calculated by Fujita and Ramey (2006), the standard deviation of the job finding rate is three times that of the industrial production index at the business cycle frequency, the separation rate is about 2.3 times more volatile, and both hazard rates move strongly over the cycle. The job finding rate is strongly procyclical, with a contemporaneous correlation with output near 0.8. The job separation rate is strongly countercyclical, with a correlation around -0.7, with its highest correlation (-0.8) at a three month lag meaning there is a wave of job losses shortly before a downturn (Fujita and Ramey 2006, 2007).¹¹ By decomposing the variability of unemployment, Fujita and Ramey (2007) attribute between 38 and 55 % of total unemployment variability to separation rates, revising Shimer's conclusions on the acyclical nature of job separations (Shimer 2005a, b).¹²

New investment, as reported in the NIPA tables, is known to be highly procyclical. The al-

¹¹Correlations reported were computed for band pass (Baxter and King, 1999) filtered data.

¹²As Fujita and Ramey (2007) argue, the approximation of the unemployment rate used by Shimer 2005a does not actually decompose total unemployment variability. Applying their alternative decomposition to Shimer's own data, these authors find that the fluctuations in the separation rate are not small relative to fluctuations in unemployment.

location of used capital, as evidenced by two measures reported in Eisfeldt and Rampini (2006), the sales of existing property, plant and equipment, and capital acquired through mergers and acquisitions with existing firms,¹³ is equally procyclical. The Hodrick-Prescott (H.-P.) filtered contemporaneous correlation of sales of PP&E with output is 0.329, while the same measure for acquisitions is 0.675. Thus gross flows of capital, both in and out of firms, move with the cycle.

The allocation of capital also appears to display congestions effects akin to the time varying unemployment duration observed on the labor market. As Kurmann and Petrosky-Nadeau (2007) document, this is substantiated by the time varying amount of office and industrial office space left vacant (see Figure 6), which clearly declined during the strong late 90s expansion, and rose rapidly during the recession following the bursting of the stock market bubble in 2000. This suggests that vacancy duration on the capital market is countercyclical. Further evidence suggesting that the allocation of physical capital is probabilistic in nature comes from the well-documented wide distribution of investment rates across firms. At any given point in time, there is a substantial mass of establishments with zero investment that coexists with establishments that have investment rates above 20% of their capital stock (i.e. investment spikes).¹⁴

Finally, the specificity embodied in most physical capital can lead to an equilibrium effect that Shleifer and Vishny (1992) call *asset illiquidity*.¹⁵ These authors argue that when firms sell assets or liquidate to meet financial constraints, the specific nature of capital means that the buyers who value these assets most are likely to be firms in the same industry. But financial distress often affects industries as a whole, which means that these buyers are likely to be financially constrained as well. As a result, the assets are sold at a steep discount within the

¹³This latter quantity is included since mergers and acquisitions not only represent a change of ownership but often involve important modifications to the composition and use of existing capital. See the references in Eisfeldt and Rampini (2006) for more details.

¹⁴Most of the literature has interpreted this large distribution of investment rates across establishments as the result of plant-specific productivity and non-convex adjustment costs that lead to (S,s) type investment rules. See Kurmann and Petrosky-Nadeau (2007) for the discussion and references therein.

¹⁵As direct evidence of these costs, Ramey and Shapiro (2001) use equipment level data about closures of aeronautical plants, and find that other aerospace companies are overrepresented among buyers, and that even after taking into account age-related depreciation, the average resale value of equipment is only 28% of the replacement cost.

same industry or to less constrained industry outsiders who have a lower valuation.

2.3 Financial distress and separations

Separation of labor and physical capital from firms can occur, of course, for many different reasons. One of these reasons are credit constraints that require firms to sell assets, reorganize or even declare bankruptcy. While there is convincing evidence that financial distress is counter-cyclical, it is difficult to evaluate its effects on labor and capital separation.¹⁶

Information contained in the Bankruptcy Research Database enable us to get a step closer to quantifying the importance and behavior of separations at financially distressed firms. This database, compiled by Lynn PoLucky at UCLA, contains information on every large public firm in the United States having filed for bankruptcy since October 1979. The database contains 751 separate cases which can be matched with the Compustat database to obtain detailed information on firms having filed for bankruptcy over the last 25 years.¹⁷ With this data set it is possible to explore, in the years preceding and following filing for bankruptcy, actual firm behavior with regards to employment and capital decisions.

Figure 1 plots the change in the growth rate of employment at financially distressed firms over a 10 year window around the bankruptcy filing date. The growth rate of employment slows down during the two years preceding filing for bankruptcy. The year previous to filing, firms actually reduce the size of their workforce, while the strongest reduction occurs during the year of the filing, stabilizing thereafter. Unfortunately this information cannot distinguish changes in employment due to a hiring freeze or a rise in job separations, due to layoffs or quits, as only information on the number of employees is provided.¹⁸

Given the prospect and the consequences of defaulting on interest payments, one might

¹⁶As evidence to the first point, Covas and Den Haan (2006) document that in the U.S., default rates for corporate bonds peak at the end of recessions. The H.-P. filtered contemporaneous correlation of default rates with real GDP is -0.33 for the period 1971-2004. Second, as shown in Figure 7, bankruptcy filings by publicly traded firms spike during recession years, and the rate of filing has a contemporaneous correlation with GDP of -0.14.

¹⁷In some instances it was not possible to find the corresponding firm in Compustat. See appendix for details concerning the data and empirical evidence on employment changes and capital separations at financially distressed firms.

¹⁸Indeed, it may be that current employees, observing private information concerning the productivity of the firm before outsiders, begin to search for other jobs on the current job.

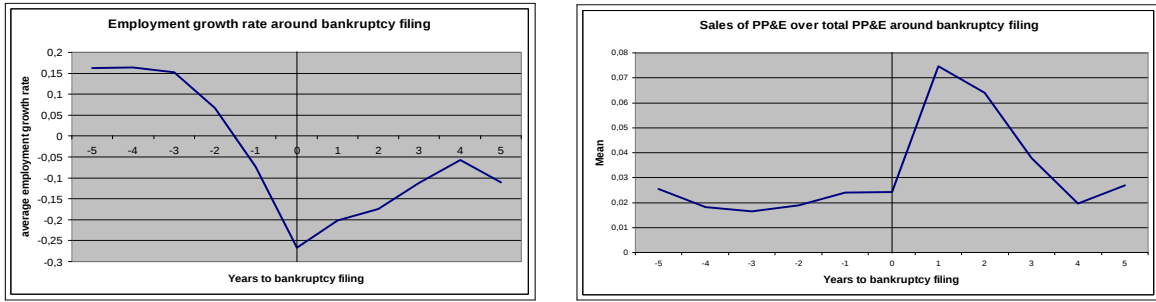


Figure 1: Employment and sales of PP&E during financial distress.

expect that firms would run down their assets in order to avoid having to file for bankruptcy. The ratio of sales of property, plant and equipment (PP&E) to the total stock of PP&E reported by the firm is computed to investigate this. Figure 1 reports the results for a ten year window around the year of filing for bankruptcy. It is striking that the ratio of sales of PP&E to the total stock remains relatively constant during the years leading up to a bankruptcy filing, giving no indication of the coming financial troubles. Sales of PP&E spike during the first two years after filing as creditors seek to recoup their debt by liquidating the firm's physical assets. This lends support to the notion that firms lay off workers to avoid bankruptcy procedures rather than sell physical capital.

There remains the question of whether, during episodes of layoffs, output at the firm declines with the workforce. As Figure 2 shows, it appears that output per employee at firms under financial distress increases steadily in the run-up to a bankruptcy filing. While there are many candidate explanations for this observation, one is that these firms are requiring more effort from each of the remaining employees leading to an actual increase in production per employee.

The last observation concerns the business cycle properties of capital liquidations due to financial distress. As in Kurmann and Petrosky-Nadeau (2007), this is defined as the PP&E of firms exiting following bankruptcy and the sales of PP&E of financially distressed firms, which includes the years (-2 -1 0 1 2) around a bankruptcy filing. While bankruptcy filings rise in a recession, the amount of property, plant and equipment reallocated in connection with financial distress does not. In fact, it moves moderately with the business cycle, with a contemporaneous correlation of 0.28 (see Table 1).¹⁹ To gain a clearer image, consider the

¹⁹The footnotes to the Compustat database provide information on the reasons for, and date of, deletion from

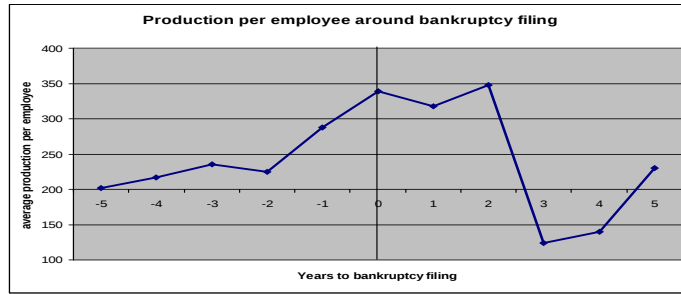


Figure 2: Production per employee during financial distress.

cyclicality of each component of capital liquidations. The share due to exits is acyclical, while sales are highly pro-cyclical. Debtors are more likely to liquidate the physical assets of the firm during an expansion. Given the specificity and congestion effects mentioned earlier this is not so surprising. The value of liquidated physical assets is much lower in a downturn, while finding buyers for capital is easier in an expansion. Thus debtors are able to recoup more of their loan in an expansion than a recession. The model developed in the next section will attempt to capture these stylized facts.

Table 1: Business cycle of capital liquidation under financial distress

1980-2004	Std. Deviation*	Contemp. Correlation ⁺
PP&E of exiting firms	2.14	0.07
Sales of PP&E	0.94	0.56
Capital liquidation rate	2.48	0.28

*: relative to output; ⁺ : with output.

Second moments are computed for Hodrick-Prescott filtered data.

the data set. Figure 11 plots the business cycle components of the capital liquidation rate and real GDP over the period 1980-2004. It is first notable that, contrary to financial distress itself, the rate of capital liquidation due to financial distress appears to be slightly pro-cyclical. It is also apparent that the correlation is much stronger in the second half of the sample. In fact, when the sample ends the same years as the last reported capital retirements in Compustat, or 1993 as in Kurmann and Petrosky-Nadeau (2007), the correlation is actually slightly counter cyclical. See appendix A for details.

3 Model

The model is populated by two types of agents: firms that produce using labor and capital; and households who decide on optimal consumption and investment in productive capital. The allocation of labor and capital from households to firms involves a similar costly and time consuming matching process. As in Kurmann and Petrosky-Nadeau (2007), the lending relationship between households and firms is subject to a credit market friction of the costly state verification type initiated by Townsend (1979), resulting in a debt contract characterized by endogenous liquidation of capital from firms whose productivity falls below a state-dependent threshold.

As households monitor when firms are unable to meet their interest payments and acquire the monitored firm's output, firms engage in layoffs, subject to paying a firing cost proportional to the wage bill, in order to save costs and pay the agreed interest on capital, thereby avoiding monitoring. Firms then require additional effort from the remaining workforce to compensate for the laid off workers when production occurs. Also, workers at liquidated production units lose their job.

Several simplifying assumptions made are worth mentioning here. First, there is an insurance scheme financed out of aggregate firm profits to pay the remaining wage bill owed to workers laid off in the period. Second, the idiosyncratic shock, observable only to firms, is i.i.d and occurs after the aggregate technology is observed and production decisions are made. These first two assumptions eliminate differences between ex ante and ex post wages. Third, there is no distinction between firms because the modeling assumptions, in particular the constant return to scale in production, imply that all firms are identical and that firm size is indeterminate.

3.1 The allocation of labor and capital

The set up of the labor market is quite standard in the labor search literature.²⁰ Firms post job vacancies v_t^n at a unit cost of γ . Jobs are filled via a constant returns to scale matching function taking vacancies and unemployed workers $M(u_t^n, v_t^n)$. Define $\theta_t = \frac{v_t^n}{u_t^n}$ as the labor

²⁰For a formal treatment of the set-up, see Mortensen and Pissarides (1994). The introduction of labor search to quantitative business cycle research is owed to the contributions of Merz (1995) and Andolfatto (1996). Labor force participation choices are not considered here, individuals are either employed or unemployed.

market tension, the matching probabilities are $\frac{M(u_t^n, v_t^n)}{v_t^n} = p(\theta_t)$ and $\frac{M(u_t^n, v_t^n)}{u_t^n} = q(\theta_t)$ for firms and workers respectively. Note that $q(\theta_t) = \theta_t p(\theta_t)$. Jobs are destroyed at the endogenous rate φ_t . Thus employment and unemployment evolve according to

$$n_{t+1} = (1 - \varphi_t)n_t + p(\theta_t)v_t^n \quad (1)$$

$$u_{t+1}^n = (1 - q(\theta_t))u_t^n + \varphi_t n_t \quad (2)$$

Physical capital is either in a productive state or in a liquid state. Define by k_t the capital stock that enters the production function of a representative firm in period t . Liquid capital l_t , in turn, is made up of two components: used capital that has been separated previously from other firms and new capital made available by households. To undertake investments, firms must post projects, v_t^k , and search for liquid capital at cost κ per project. Projects and liquid capital, set aside by households, are matched through a constant returns to scale function $G(l_t, v_t^k)$. Define $\phi_t = \frac{l_t}{v_t^k}$ as the capital market tension, the matching probabilities are $\frac{G(l_t, v_t^k)}{l_t} = p(\phi_t)$ and $\frac{G(l_t, v_t^k)}{v_t^k} = q(\phi_t)$ for firms and lenders respectively. Also, note that $p(\phi_t) = \phi_t q(\phi_t)$. Capital is separated (i.e. liquidated) at the endogenous rate s_t . Thus the law of motion for the stock of matched capital is

$$k_{t+1} = (1 - \delta)(1 - s_t)k_t + q(\phi_t)l_t. \quad (3)$$

Liquid capital not matched in the current period, u_t^k , is simply carried forward to the next period such that $u_{t+1}^k = (1 - q(\phi_t))l_t$.

3.2 Debt contract

To model endogenous capital liquidation, as in Kurmann and Petrosky-Nadeau (2007), while firms perfectly observe the realization of the idiosyncratic shock a_t , households can only do so after paying an auditing cost. This asymmetric information assumption creates an agency problem because, in the absence of auditing, the firm would always want to misreport a_t . The debt contract to deal with this problem is structured as follows. Households and firms negotiate the rental rate ρ_t per unit of matched capital *prior to* the realization of the idiosyncratic productivity shock a_t . Then, if $a_t \geq \bar{a}_t$ the firm pays $\rho_t k_t$, the household refrains from auditing and the capital match continues. If, on the other hand, $a_t < \bar{a}_t$ the firm is unable to pay the negotiated capital rental. In this situation, the household pays the auditing cost to verify the

firm's production and decides on the continuation of the capital match. If a_t is above some threshold $\underline{a}_t$ that is associated with the household's choice of optimal liquidation, the household takes all of the firm's production and covers for the totality of $w_t n_t$, γv_t^n and κv_t^k so as to continue the capital match into the next period. If instead a_t is below the optimal threshold $\underline{a}_t$, the household separates the match and takes back its capital stock without receiving nor paying anything. In this case, the firm is liquidated and the difference between production and the operating costs of $w_t n_t$, γv_t^n and κv_t^k is picked up by an insurance that is funded with the dividends from profit-making firms (see below).

Anticipating on the household's optimization problem, the optimal liquidation threshold, $\underline{a}_t$, is chosen such that the marginal utility from last unit of capital separated (net of depreciation and a loss of value due to specificity) equals the expected discounted value of the last unit of matched capital carried over into the next period net of the current period revenue from the monitored firm :

$$\lambda_t(1 - \delta)\varsigma_t k_t = \lambda_t \left[\underline{a}_t y_t - w_t n_t - \gamma v_t^n - \kappa v_t^k \right] + (1 - \delta)k_t \beta E_t V_k(n_{t+1}, k_{t+1}). \quad (4)$$

Note that if a_t is sufficiently low, this may entail injecting additional funds,²¹ and that $a_t > 0$ is independently distributed over time with probability density $h(a)$, cumulative density $H(a)$ and mean $E(a) = 1$. Given these assumption, the endogenous part of capital liquidation is defined as

$$s_t^e = H(\underline{a}_t). \quad (5)$$

In addition, capital matches are discontinued either for exogenous reasons or for reasons associated with credit frictions. Hence total capital separations are

$$s_t = s^x + s_t^e, \quad (6)$$

where s^x denotes (constant) exogenous separation and s_t^e denotes liquidation.

3.3 Layoffs and labor relations

After markets have cleared, but before production takes place, firms observe the realization of the idiosyncratic shock. They are now presented with the option of laying off a fraction of the

²¹It can be shown that $\underline{a}_t y_t < w_t n_t + \gamma v_t^n + \kappa v_t^k$; i.e. the household is willing to inject additional funds into loss-making firms to cover $w_t n_t + \gamma v_t^n + \kappa v_t^k$ so as to continue the capital match. This is because separated capital yields zero return in the next period and comes with the risk that rematching takes time.

workforce, which entails paying a firing cost proportional to the wage bill, in order to save on labor costs and avoid defaulting on interest payments. However if they choose to do so, in order to meet beginning of period production decisions they are required to have the remaining workers provide additional effort. This effort is such that the labor effectively entering the production function is beginning of period employment. It follows that the threshold productivity that triggers monitoring/default, $\bar{a}_t$, is defined as $\bar{a}_t y_t = fcw_t n_t + \rho_t k_t + \gamma v_t^n + \kappa v_t^k$, where $0 < fc < 1$ is the firing cost. Defining, $\tilde{a}_t$ as the zero profit threshold $\tilde{a}_t y_t = w_t n_t + \rho_t k_t + \gamma v_t^n + \kappa v_t^k$, given these assumption, the remaining $(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a))n_t$ workers are required to engage in additional effort to meet the beginning of period production and labor input decisions. This additional effort required by each of the remaining workers, $e_t \equiv \frac{\int_{\bar{a}_t}^{\tilde{a}_t} dH(a)}{1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a)}$, is increasing in the spread between $\tilde{a}_t$ and $\bar{a}_t$, which itself depends on the firing cost.

Finally, it is assumed that workers at firms with idiosyncratic productivity $a < \underline{a}_t$, which triggers liquidation, also lose their job at the end of the period. Their wage is covered by an insurance scheme funded out of aggregate firm profits. It follows that the endogenous separation of labor is defined as

$$\varphi_t^e = \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) + \int_0^{\underline{a}_t} dH(a). \quad (7)$$

As for capital, matches are discontinued either for exogenous reasons or for reasons associated with credit frictions. Hence,

$$\varphi_t = \varphi^x + \varphi_t^e, \quad (8)$$

where φ^x denotes (constant) exogenous separation, and φ_t^e denotes layoffs.²²

3.4 Firms

At the beginning of each period, households and firms observe the exogenous aggregate technology, x_t . Firms maximize profits subject to the laws of motion for matched labor and capital, and a constant returns to scale production technology, $y_t = x_t f(n_t, k_t)$. Given the existing capital stock k_t , and employment n_t , firms post job vacancies and investment projects v_t^n and v_t^k , respectively. This yields the following Bellmann equation, where w and ρ are the remuneration

²²Figure 13 in the appendix illustrates the labor and capital separation decisions as a function of the realized idiosyncratic productivity.

of labor and capital:

$$J(n_t, k_t) = \max_{v_t^n, v_t^k} \int_{\bar{a}_t}^{\infty} \left[ay_t - w_t n_t - \rho_t k_t - \gamma v_t^n - \kappa v_t^k \right] dH(a) \\ + (1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} w_t n_t dH(a) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J(n_{t+1}, k_{t+1}).$$

Given the debt contract described above, firms optimize over realization of the idiosyncratic shock for which they retains their production. The term $(1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} w_t n_t dH(a)$ represent labor costs saved through layoffs. The firm's discount rate is $\beta E_t \frac{\lambda_{t+1}}{\lambda_t}$ because all profits are returned to the household at the end of the period. The exogeneity of the factor remunerations implies that firms do not internalize the effects of their employment and capital stocks on marginal productivities during repayment negotiations (see below).

The first order conditions to this problem,

$$(v_t^n) : \left[1 - \int_{\bar{a}_t}^{\infty} dH(a) \right] \frac{\gamma}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(n_{t+1}, k_{t+1}) \quad (9)$$

$$(v_t^k) : \left[1 - \int_{\bar{a}_t}^{\infty} dH(a) \right] \frac{\kappa}{p(\phi_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_k(n_{t+1}, k_{t+1}), \quad (10)$$

equate average vacancy costs to expected returns to each factor of production.

The marginal value of an employed worker to the firm, $J_n(n_t, k_t) = \int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} - w_t \right] dH(a) + (1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} w_t dH(a) + (1 - \varphi_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(n_{t+1}, k_{t+1})$, can be combined with (9) to form a forward looking equation for vacancy postings:

$$\frac{\gamma [1 - H(\bar{a}_t)]}{p(\theta_t)} = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \bar{\mu}_{t+1} (1 - \alpha) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} [1 - H(\bar{a}_{t+1})] \right. \\ \left. + (1 - fc) \int_{\bar{a}_{t+1}}^{\tilde{a}_{t+1}} w_{t+1} dH(a) + (1 - \varphi_{t+1}) \frac{\gamma [1 - H(\bar{a}_{t+1})]}{p(\theta_{t+1})} \right\}. \quad (11)$$

The marginal value of matched capital to the firm, $J_k(n_t, k_t) = \int_{\bar{a}_t}^{\infty} \left[a \alpha \frac{y_t}{k_t} - \rho_t \right] dH(a) + (1 - \delta)(1 - s_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_k(n_{t+1}, k_{t+1})$, combined with the optimality condition (10), yields a forward looking equation for project postings:

$$\frac{\kappa [1 - H(\bar{a}_t)]}{p(\phi_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \bar{\mu}_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} - \rho_{t+1} [1 - H(\bar{a}_{t+1})] + (1 - \delta)(1 - s_{t+1}) \frac{\kappa [1 - H(\bar{a}_{t+1})]}{p(\phi_{t+1})} \right\}. \quad (12)$$

3.5 Households

The representative household, given the existing capital stock k_t , and employment n_t , makes optimal consumption and investment decisions in order to maximize the value function:

$$V(n_t, k_t) = \max_{c_t, l_t, a_t} [\log(c_t) + \beta E_t V(n_{t+1}, k_{t+1})],$$

subject to the budget constraint $c_t + l_t = R_t^n + R_t^k + u_t^k + u_t^n b + \Pi_t - T_t$ and the laws of motion for matched labor (1) and capital (3). R_t^n and R_t^k are, respectively, the expected revenues from employment and matched capital. The government raises T_t in taxes to fund unemployment benefits $u_t^n b$. Π_t are firm profits defined below. $u_{t+1}^k = (1 - q(\phi_t))l_t$ is unmatched liquid capital in the period carried forward.

Given the presence of credit market imperfections, and the possibility of being laid off in the period, the expect return from employment is

$$R_t^n = \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a)\right) [w_t - e_t] n_t + \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) f c w_t n_t$$

The marginal values of employment and unemployment are obtained by differentiating the value function:

$$\begin{aligned} V_{u^n}(n_t, k_t) &= \lambda_t b + \beta E_t [q(\theta_t) V_n(n_{t+1}, k_{t+1}) + (1 - q(\theta_t)) V_{u^n}(n_{t+1}, k_{t+1})], \\ V_n(n_t, k_t) &= \lambda_t \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a)\right) [w_t - e_t] + \lambda_t \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) f c w_t \\ &\quad + \beta E_t [\varphi_t V_{u^n}(n_{t+1}, k_{t+1}) + (1 - \varphi_t) V_n(n_{t+1}, k_{t+1})]. \end{aligned}$$

From the employed worker's point of view at the beginning of the period (after the aggregate state has been observed), the probability of not being laid off is $\left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a)\right)$, in which case the negotiated wage w_t is paid but a loss of utility from the extra work, e_t , is incurred. With probability $\int_{\bar{a}_t}^{\tilde{a}_t} dH(a)$ the worker is laid off and receives a severance payment that is a fraction of the negotiated wage, $f c$. The value of being in an employment relationship at the beginning of the period is therefore an average of these two outcomes, and will enter the wage rule.

Given the debt contract described above, the household's expected revenue from matched capital is

$$\begin{aligned} R_t^k &= \int_{\bar{a}_t}^{\infty} \rho_t k_t dH(a) + \int_{\bar{a}_t}^{\tilde{a}_t} \left[(1 - \tau) a y_t - w_t n_t - \gamma v_t^L - \kappa v_t^k \right] dH(a) - \int_0^{\bar{a}_t} \tau a y_t dH(a) \\ &\quad + (1 - \delta) \varsigma_t s_t k_t, \end{aligned} \tag{13}$$

where the auditing cost to the household is a fixed proportion $\tau > 0$ of the firm's revenue, $\int_{0_t}^{\bar{a}_t} \tau a y_t dH(a)$. The last term, $(1 - \delta)\varsigma_t s_t k_t$, corresponds to separated capital returned to the household, net a depreciation and a separation cost. This is a convex cost to adjusting the liquidation rate introduced in order to capture the loss of value due to the specificity of capital mentioned above. Thus the value of capital actually returned to the household's budget constraint is diminished by a fraction $\varsigma_t \equiv g(s_t/s_{t-1})$, with $\varsigma'(\bullet) < 0$ and $\varsigma(\bullet) < 0$.

Household investment decisions are determined by the optimality condition for liquid capital:

$$\lambda_t = \beta E_t [q(\phi_t) V_k(n_{t+1}, k_{t+1}) + (1 - q(\phi_t)) V_u(n_{t+1}, k_{t+1})], \quad (14)$$

which equates the opportunity cost of forgone consumption to the expected value of additional liquid capital, composed of the marginal values for matched and unmatched capital. The corresponding marginal values for capital are:

$$\begin{aligned} V_{u^k}(n_t, k_t) &= \lambda_t, \\ V_k(n_t, k_t) &= \lambda_t \left[\frac{\partial R_t^k}{\partial k_t} \right] + (1 - \delta)(1 - s_t) \beta E_t V_k(n_{t+1}, k_{t+1}). \end{aligned}$$

where $\frac{\partial R_t^k}{\partial k_t} = \int_{\bar{a}_t}^{\infty} \rho_t dH(a) + \int_{\underline{a}_t}^{\bar{a}_t} (1 - \tau) a \alpha \frac{y_t}{k_t} dH(a) - \int_0^{\underline{a}_t} \tau a \alpha \frac{y_t}{k_t} dH(a)$.

3.6 Factor remunerations

In line with much of the labor search literature, the surplus of the worker-firm and lender-firm relationship is split according to Nash bargaining. As discussed above, this bargaining process takes place before the idiosyncratic shock a_t is realized. In each case, the surplus is the sum of marginal benefits to each party, and marginal factor products are taken as exogenous in each negotiation. For simplicity the household's relative bargaining power, η , is the same in each negotiation.

3.6.1 Labor

The surplus from the worker-firm relationship is defined as $S_t^n = J_n(n_t, k_t) + \frac{V_n(n_t, k_t) - V_u^n(n_t, k_t)}{\lambda_t}$. Nash bargaining implies that the worker receives $\frac{V_n(n_t, k_t) - V_u^n(n_t, k_t)}{\lambda_t} = \eta S_t^n$, and the firm receives $J_n(n_t, k_t) = (1 - \eta) S_t^n$. The negotiated wage is given by the expression

$$w_t = \eta \left[(1 - \alpha) \int_{\bar{a}_t}^{\infty} a dH(a) \frac{y_t}{n_t} + \gamma \theta_t \left[1 - \int_{\bar{a}_t}^{\infty} dH(a) \right] \right] + (1 - \eta) \left[b + \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) e_t \right] + \Omega_t. \quad (15)$$

where $\Omega_t \equiv w_t \left[\eta [1 - \int_{\bar{a}_t}^{\infty} dH(a)] + (1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right]$. Aside from being increasing in the marginal product, as will be the capital repayment rule, the negotiated wage contains two terms related to layoffs due to credit frictions. First, the negotiated wage is increasing in the additional effort required. Second, the term Ω_t captures two elements. The first term in brackets corresponds to a sharing between worker and firm of the fact that firms are optimizing only over the range $[\bar{a}_t, \infty)$. The second term in brackets corresponds to a sharing the benefits from being able to lay off workers, by which the lower the firing cost, the higher the negotiated wage. Finally, the labor tension term captures the effect of market conditions on the outside option during negotiations, while b captures the opportunity cost of the match to the worker.

3.6.2 Capital

The surplus from the lender-firm relationship is defined as $S_t^K = J_k(n_t, k_t) + \frac{V_k(n_t, k_t) - V_{uk}(n_t, k_t)}{\lambda_t}$. The rental rate resulting from Nash bargaining is given by the expression:

$$\rho_t = \eta \left[\bar{\mu}_t \alpha \frac{y_t}{k_t} + (1 - \delta)(1 - s_t) \frac{\kappa}{\phi_t} [1 - H(\bar{a}_t)] \right] + (1 - \eta) [\delta + (1 - \delta)(1 - \varsigma_t)s_t] + \Phi_t \quad (16)$$

where $\Phi_t = \rho_t H(\bar{a}_t) - (1 - \eta)(\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t)) \alpha \frac{y_t}{k_t}$.

Again, the market tension term captures the effect of market conditions on the outside option during negotiation, while δ captures the opportunity cost of the match. The term Φ_t in the repayment rule for capital captures the risk that follows from asymmetric information and the debt contract: it is expected that some firms will default and that a monitoring cost will be paid. The Details concerning the derivation of (15) and (16) are presented in the appendix.

3.7 Closing the model

A simplifying assumption was the existence of a state contingent insurance that covers for any shortfall in wage payments and costs of job vacancy and project postings left over by bankrupt firms. Aggregating over the different firms, the total amount of profits transferred to the household is defined as

$$\begin{aligned} \Pi_t &= \int_{\tilde{a}_t}^{\infty} \left[af(n_t, k_t) - \rho_t k_t - w_t n_t - \gamma v_t^n - \kappa v_t^k \right] dH(a) + \int_{\bar{a}_t}^{\tilde{a}_t} \left[af(n_t, k_t) - \rho_t k_t - fc w_t n_t - \gamma v_t^n - \kappa v_t^k \right] dH(a) \\ &\quad + \int_0^{\bar{a}_t} \left[af(n_t, k_t) - w_t n_t - \gamma v_t^n - \kappa v_t^k \right] dH(a) - \int_{\bar{a}_t}^{\tilde{a}_t} (1 - fc) w_t n_t dH(a). \\ \Pi_t &= \int_{\tilde{a}_t}^{\infty} \left[af(n_t, k_t) - \rho_t k_t - w_t n_t - \gamma v_t^n - \kappa v_t^k \right] dH(a) + \int_0^{\bar{a}_t} \left[af(n_t, k_t) - w_t n_t - \gamma v_t^n - \kappa v_t^k \right] dH(a) \end{aligned} \quad (17)$$

Combining this definition with that for the return to employment and productive capital, and the household's budget constraint, leads to the following familiar aggregate resource constraint:

$$y_t [1 - \tau(1 - \bar{\mu}_t)] = c_t + i_t^{new} + [\kappa v_t^k + \gamma v_t^L], \quad (18)$$

where $i_t^{new} = l_t - (1 - \delta)\varsigma_t s_t k_t - u_t^k$ corresponds to new investment. Resources consumed due to monitoring of firms is captured by the term $\tau(1 - \bar{\mu}_t)$. The bracketed term on the expenditure side corresponds to posting costs.

The equilibrium of this economy is defined by the system of equations (1)-(17), aggregate production $x_t f(n_t, k_t)$ and the aggregate resource constraint.

4 Model dynamics

The quantitative implications are evaluated by comparing the business cycle performance of the model to a model without endogenous layoffs, one with credit market imperfections but only exogenous labour separation, and one with search frictions in labor the market alone, in terms of impulse response functions and unconditional second moments. Labor market implications are also evaluated by computing contributions to employment fluctuations due to job creations and job destructions.

4.1 Shocks and functional forms

Following much of the real business cycle literature, aggregate technology is assumed stationary and to evolve according to

$$\log x_t = \rho_x \log x_{t-1} + \varepsilon_t^x,$$

with $\varepsilon_t^x \sim (0, \sigma_x^2)$ and $0 < \rho_x < 1$.

For household preferences, period utility is defined as $u(c) = \log c$. The production technology is a Cobb-Douglas function with constant returns to scale of the form $x f(n, k) = x n^{1-\alpha} k^\alpha$ with $0 < \alpha < 1$. The idiosyncratic shock a is assumed to follow a log-normal distribution with mean $E(a) = 1$; i.e. $\log(a) \sim N(-\frac{\sigma_{\log(a)}^2}{2}, \sigma_{\log(a)}^2)$. Finally, following much of the labor literature, the matching technologies are Cobb-Douglas, $M(u^L, v^L) = \xi (u^L)^\chi (v^L)^{1-\chi}$ and $G(v^k, l) = \varkappa (l)^{1-\epsilon} (v^k)^\epsilon$, with $0 < \epsilon < 1$ and $0 < \chi < 1$.

4.2 Calibration

The model is calibrated to quarterly data. For the parameters that are common with the RBC literature the calibration is standard. The discount factor $\beta = 0.99$ is set so as to match an average annual real yield on a risk less 3-month treasury bill of 4.1%. The share of capital in the production function is set to $\alpha = 1/3$, and the rate of depreciation of capital to $\delta = 0.025$. This corresponds to an annual decline of productive use of capital of 10%. Finally, the parameters of the exogenous process are $\rho_x = 0.979$ and $\sigma_x = 0.0072$.²³ A list of parameters and their values is reported in Table 2.

Table 2 : Parameter values

Discount factor	β	0.99
Capital depreciation rate	δ	0.025
Production function parameter	α	0.33
Cost of employment vacancy	γ	0.05
Elasticity in labor matching function	χ	0.6
Elasticity in capital matching function	ε	0.75
Household bargaining power	η	0.5
Capital matching rate	$q(\phi)$	0.5

The calibration reported in Kurmann and Petrosky-Nadeau (2007) is adopted for the parameters proper to the capital market. The quarterly steady state separation rate of $s = 0.01$ is such as to match the long-run averages from the firm-level data discussed in Section 2. Together with $\delta = 0.025$, this calibration implies that 71% of all separations are due to depreciation (i.e. retirement) and 29% are due to sales and firm exits / acquisitions, which is consistent with the proportions reported by Ramey and Shapiro (1998). Furthermore, the values of s and δ result in the following steady state *gross* investment rate (using the capital accumulation equation)

$$\frac{G(v^k, l)}{k} = [1 - (1 - \delta)(1 - s)] = 0.0348,$$

which translates into a yearly investment rate of 14% – a rate that corresponds to the range of values computed by Ramey and Shapiro (1998) and Eisfeldt and Rampini (2006). The loss of

²³These are the values used in Kurmann and Petrosky-Nadeau (2007), and were obtained by extracting a Solow residual from the data, removing a linear trend and estimating the AR(1) process.

capital value following liquidation is set to $\varsigma = 0.95$ such that investment in used capital as a fraction of gross investment, $\varsigma(1 - \delta)s$, coincides with the 24% reported by Eisfeldt and Rampini (2007). The share of capital separations attributable to financial distress the sample period 1980-2004 is approximately 5%, and the elasticity of the loss of value due to capital specificity is set such that the relative volatility of s^e in the model coincides with the one in the data.

Much recent work has been done in the business cycle literature to incorporate matching on the labor market, and the calibration reported in Gertler and Trigari (2006) is followed closely here. The cost of job vacancies is set to $\gamma = 0.05$. The elasticity in the labor matching function, χ , is set to 0.6. Steady state unemployment is set to 7%. Finally, the firing cost is calibrated such that the steady state endogenous job destruction rate is 0.03, or 1/2 of total separations, which is in-line with evidence on the layoffs rate in Davis, Faberman and Haltiwanger (2006). This calibration implies a standard deviation of the idiosyncratic productivity, σ_a , of 0.23.

This calibration implies a steady state labor share of approximately two thirds, as in the data. Consumption is approximately three quarters of output, while total unemployment benefits constitute around 4% of GDP. The remaining steady state values are presented in Table 3.

Table 3: Implied steady state values

Job finding rate	$q(\theta)$	0.79
Total Job destruction rate	φ	0.06
Share due to endogenous separations	φ^e/φ	1/2
Labor share		0.67
New investment-capital ratio	i/y	0.025
Total investment-capital ratio	$\frac{m(v^k, L)}{K}$	0.0348
consumption-output ratio	c/y	74.5 %
Vacancy cost-output ratio	$\gamma v^L/y$	0.25 %
Project posting-output ratio	$\kappa v^k/y$	0.41 %
Total unemployment benefits - ouput ratio	$(1 - n)b/y$	4.75 %

4.3 Simulation results

The model's performance is analyzed in two stages. First, the propagation poperties and the dynamics of employment adjustments are examined, evaluating the contributions of job

creations and job separations. The dynamics of physical capital are evaluated second. In both instances the analysis is based on impulse response functions to an expansionary innovation and a variety of unconditional 2nd moments.

4.3.1 Propagation and the dynamics of employment adjustments

To begin investigating the propagation properties of credit market imperfections when they affect the cyclical reallocation of labor, Figure 3 plots the impulse response function to an expansionary technological innovation for output for the model and the two comparison cases. The solid line plots the responses for the benchmark model with endogenous labor separation and capital liquidation. The first alternative model, characterized by exogenous separation of labor and endogenous capital liquidation, is represented by the starred line. Finally, a model with matching only on the labor market is represented by the circled lines.

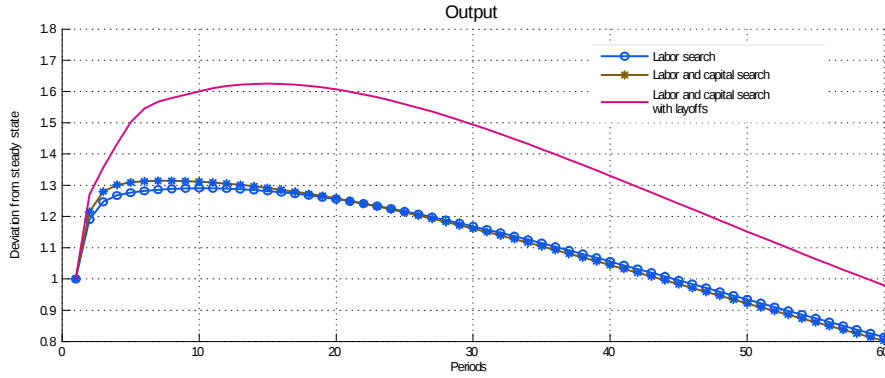


Figure 3: Impulse responses to an expansionary technological innovation.

Comparing impulse responses, the striking feature of the benchmark model is the amplification of the response of output to the technological innovation. This is both in terms of the impulse responses and unconditional second moments presented in Table 4. The standard deviation of output for both comparison models is approximately 1.2, while in the proposed model it is of 1.33. The source of the difference is the state dependent rate of layoffs affecting directly employment, as the response for output in both comparison models are almost identical. The first quadrant in Figure 4, which plots the response of the aggregate employment, makes this clear: a labor matching model with perfect capital markets, and a model of labor and capital matching with exogenous labor separation have the same dynamics for employment, while a

model with endogenous separation due to credit frictions amplifies the response of aggregate employment to a small exogenous shock. Despite the different adjustment mechanisms at play in dynamics of employment adjustments, the three responses are similar in shape. This is reflected in the similar correlations with output reported in Table 4. We will return to the sources of the difference later when considering the dynamics of employment adjustment.

In order to evaluate the model's performance in terms of 2nd moments for key macroeconomic variables, Table 5 presents the Hodrick-Prescott filtered standard deviations relative to, and contemporaneous correlations with, output observed in the data and generated by the models. As Cogley and Nason (1995) document, a major weakness of quantitative business cycle models is their inability to generate the sizable positive autocorrelation for output growth over several quarters that is observed in the data. While a model of labor search alone, as Andolfatto (1997) demonstrated, can generate some persistence in output growth, when it is appropriately calibrated on gross worker flows it falls short of the number in the data. As the last three rows of Table 4 report, the combination of search frictions in labor and physical capital, along with endogenous separations due to financial distress, ameliorates on this dimension.

Table 4: 2nd moments

1952:1-2005:4	U.S. data		Labor and Capital search				Labor matching	
			endogenous		exogenous			
Variable:	a	b	a	b	a	b	a	b
c	0.58	0.69	0.52	0.96	0.48	0.98	0.50	0.98
n	0.48	0.82	0.43	0.84	0.30	0.84	0.30	0.83
i^{new}	3.08	0.79	4.06	0.83	2.66	0.99	2.62	0.99
k			0.42	0.19	0.27	0.32	0.25	0.06
$\sigma(y)$	1.59		1.33		1.21		1.18	
$corr(\Delta y, \Delta y_{-1})$	0.264		0.30		0.23		0.28	
$corr(\Delta y, \Delta y_{-2})$	0.227		0.13		0.08		0.07	
$corr(\Delta y, \Delta y_{-3})$	0.057		0.11		0.03		0.03	

a: Standard deviation relative to output; b: contemporaneous correlation with output.

All moments, but output growth, are Hodrick-Prescott filtered;

Data sources: BLS, BEA.

Statistics for variables central to the labor matching process - job vacancies, the unem-

ployment rate and the labor market tension (the ratio of vacancies to unemployment) - are presented in Table 5.²⁴ As noted by Shimer (2005a), the labor market tension is very volatile and strongly procyclical in the data: the H.-P. filtered standard deviation is 15 time that of real GDP with a contemporaneous correlation of 0.89. As is also well know, job vacancies are strongly procyclical and unemployment is strongly countercyclical, and both series are also highly volatile. As seen in Table 5, the proposed model with endogenous labor and capital separations outperforms both comparison models in terms of matching both the relative volatilities and contemporaneous correlation. In particular, the correlation of job vacancies is lower in the proposed model and closer to the data than in the two comparison models. This is because the countercyclical layoff rate, by increasing employment at the firm rapidly, decreases the marginal value of an additional job vacancy, thus reducing the correlation between vacancies and output. By the same token it reduces the relative volatility of vacancies, seen in Figure 4 as the smaller response to an innovation. Finally, just as for employment, the volatility of unemployment generated is much closer to what is found in the data than in the comparison models, with a H.-P. filtered standard deviation of 5.68 compared to 6.82 in the data.

In all three models the job finding rate is strongly procyclical and very volatile (see Table 5), although the volatility in the proposed model almost perfectly matches the data while for the comparison model it is significantly greater. As the matching depends solely on the labor market tension, the different stems principally from the lower volatility of vacancies.

As seen in the second quadrant of Figure 4, the job finding rate jumps upward following a positive innovation to the exogenous process. While there is broad consensus concerning the strong procyclicality of the job finding hazard, Fujita and Ramey (2006) report job separation rates that are also volatile (see Table 5) and strongly negatively correlated with the business cycle.²⁵ It is remarkable that the proposed model performs quite well on both counts, with a relative standard deviation of labor separation nearly twice that of output, and a contemporaneous correlation of -0.95 with the business cycle.

²⁴Job vacancies are proxied for by the Conference Board's help wanted index, as in Shimer (2005a). The unemployment series was obtained from the BLS (series LNS14000000). Quaterly series were computed by averaging over monthly observations.

²⁵I thank Shigeru Fujita and Garey Ramey for kindly sharing their data. The raw monthly series were first adjusted by a 12 month backward-looking moving average, as in Fujita and Ramey 2006. Quaterly series were then computed by averaging over monthly observations.

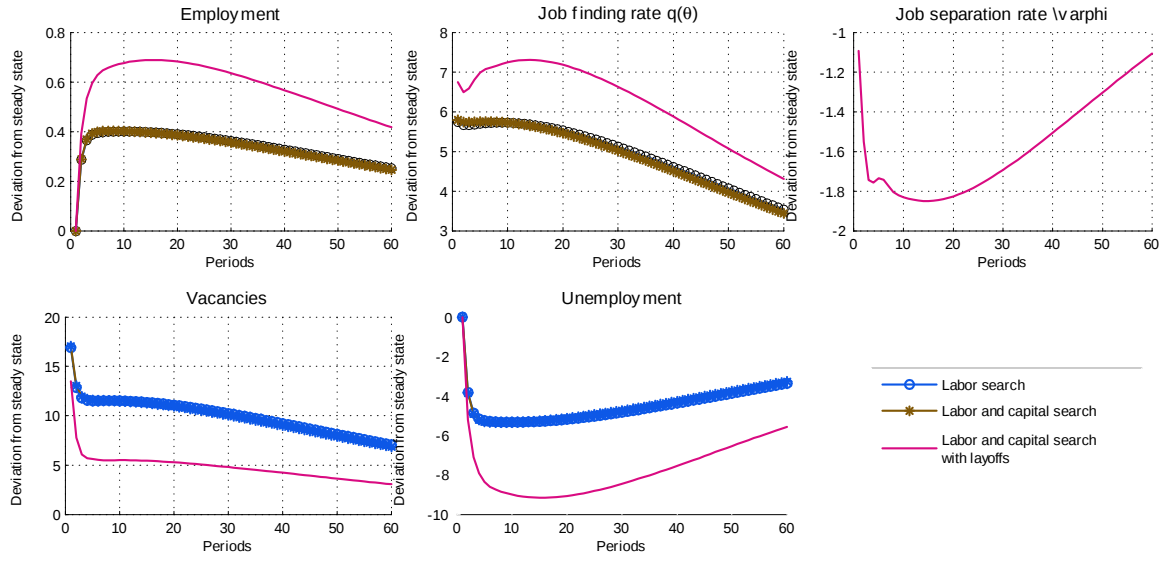


Figure 4: The dynamics of employment adjustments following an expansionary technological innovation.

Table 5: 2nd moments - labor market variables

1977:1-2005:4	U.S. data		Labor and Capital matching				Labor matching	
			endogenous		exogenous			
Variable:	a	b	a	b	a	b	a	b
v^n	8.82	0.89	9.07	0.78	11.72	0.86	11.85	0.87
u^n	6.82	-0.84	5.68	-0.84	4.32	-0.84	4.37	-0.83
θ	15.41	0.89	12.13	0.98	14.91	0.98	15.05	0.98
Hazard rates:								
$q(\theta)$	4.83	0.6	4.85	0.98	5.96	0.98	6.02	0.98
φ	3.34	-0.86	1.56	-0.98	-	-	-	-
Job flows:								
$Job\ creation$	2.02	-0.53	4.11	-0.03	4.31	0.31	4.37	0.32
$Job\ destruction$	2.98	-0.86	1.18	-0.99	-	-	-	-

a: Standard deviation relative to output

b: contemporaneous correlation with output. All moments are H-P filtered.

Data sources: BLS, BEA, Fujita and Ramey (2006)

A useful decomposition of employment change, from the law of motion for aggregate employment, separates all changes into flows of job creation and job destruction:

$$\begin{aligned} n_{t+1} - n_t &= q(\theta_t)u_t^n - \varphi_t n_t \\ \Delta n_{t+1} &= JC_t - JD_t \end{aligned}$$

where job creation is given by $JC_t = q(\theta_t)u_t^n$ and job destruction by $JD_t = \varphi_t n_t$. Thus as all employment changes can be attributed to either job creation, or job destruction, a log-linear approximation of employment change is given by

$$\widehat{\Delta n}_{t+1} = \frac{jc}{n} \widehat{JC}_t - \frac{jd}{n} \widehat{JD}_t$$

where hatted variables denote percentage deviations from steady state values.

The three models generate gross job creation flows four time more volatile than output, twice what is observed in the data. However, the proposed model also generates gross job creation flows that are moderately negatively correlated with the cycle. While the work of Blanchard and Diamond (1990) and Fujita and Ramey (2006) mentioned earlier uncovers stronger correlation coefficients (-0.53 vs -0.03), this results for the proposed model is a stark improvement on models of constant job separation rates. These models actually imply clearly pro-cyclical total job creations.

Using a variance decomposition as in Fujita and Ramey (2007), the variance of the change in employment may be written :

$$var(\widehat{\Delta n}_{t+1}) = \left(\frac{jc}{n}\right)^2 var(\widehat{JC}_t) + \left(\frac{jd}{n}\right)^2 var(\widehat{JD}_t) - 2\frac{jcd}{n} cov(\widehat{JC}_t, \widehat{JD}_t).$$

Using the same concepts as these authors, denote the proportion of variation in employment changes attributable to job creation and job destruction by β^{jc} and β^{jd} respectively.²⁶ Postulating constant job separations has the immediate implication that $\beta^{jc} = 1$ while $\beta^{jd} = 0$, that is, the entirety of employment adjustments are explained by changes in job creation. In the proposed model however the decomposition is closer to the empirical evidence related above. For the current calibration, 79% of employment change is attributable to job creation, while job destructions contribute 20% to the total variance. The remaining discrepancy is due to the covariance term. This result is below the share attributable to job destructions computed by

²⁶Where $\beta^{jc} = \frac{(\frac{jc}{n})^2 var(\widehat{JC}_t)}{var(\widehat{\Delta n}_{t+1})}$ and $\beta^{jd} = \frac{(\frac{jd}{n})^2 var(\widehat{JD}_t)}{var(\widehat{\Delta n}_{t+1})}$.

Fujita and Ramey (2007), but it nonetheless a clear improvement on models of constant labor separation.

4.3.2 Capital dynamics

The rapid rise of the capital stock in the benchmark model is attributable at first to the drop, on impact, of the capital liquidation rate (see Figure 5). The ensuing rise in the capital stock is due to the strong response in employment which, by increasing the marginal product of capital, raises the level of the desired stock.

Table 6: 2nd moments - capital allocation

1980-2004	U.S. data		Labor and Capital matching			
			endogenous		exogenous	
	a	b	a	b	a	b
s^e	2.48	0.28	2.16	0.32	-	-
$q(\phi)$	-	-	4.07	0.58	2.47	0.93

a: Standard deviation relative to output.

b: contemporaneous correlation with output

Moments are H.-P. filtered; data: Compustat and BRD.

Table 6 presents the second moments for capital liquidation and matching rates. As discussed earlier, while the default rate is countercyclical (with a contemporaneous correlation with output of -0.08 compared to -0.14 in the data), capital liquidation due to financial distress is moderately procyclical. The model's standard deviation for the liquidation rate relative to output is remarkably close to its empirical counterpart, while the contemporaneous correlation with the business cycle is nearly identical to what was found in the data. The rational can be found in the strongly procyclical capital matching rate, $q(\phi)$. After the initial drop of the capital separation rate (see Figure 5), capital liquidation increases just as the capital matching hazard increases sharply. As a result, debtors are more likely to liquidate the physical assets of the firm during an expansion. This is because they are able to recoup more of their loan in an expansion than a recession.

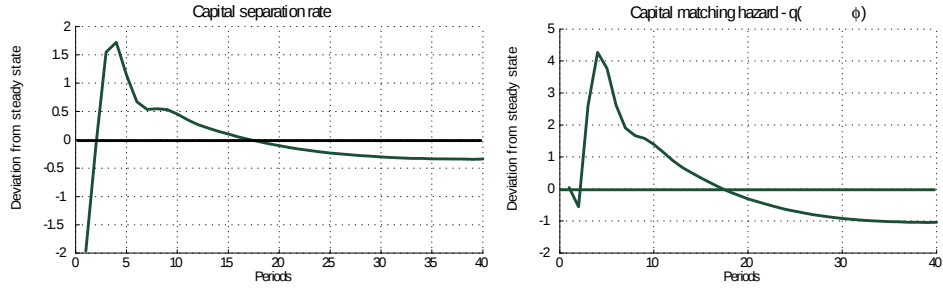


Figure 5: Impulse responses to an expansionary innovation - capital liquidation and matching hazards.

5 Conclusion

This paper investigated, in a Dynamic General Equilibrium model, the role of credit market imperfections of the costly state verification type for the cyclical reallocation of labor and physical capital. Introducing endogenous job separations related to credit frictions - workers lose their jobs either because firms downsize to meet their interest payments or because the production unit is shut down following bankruptcy - has quantitatively important consequences as credit frictions can operate through the large flows that are flows of laid-off workers. This contrasts with earlier quantitative investigations into the implications of credit market imperfections for business cycles, such as Christensen and Dib (2004), and Meier and Muller (2005).

In addition, the model implies that the cyclicity of employment adjustments are driven by both job losses and hirings, as documented in recent empirical research for the U.S.. An interesting future investigation would consider the role of monetary policy and interest rate shocks which, by affecting the cost of physical capital, may have strong effects on the cyclical reallocation of labor.

References

- [1] Andolfatto, David (1996), "Business Cycles and Labor-Market Search", *American Economic Review* vol. 86(1), pp. 112-132.
- [2] Becker., Randy, John Haltiwanger, Ron Jarmin, Shawn Klimek and Dan Wilson (2005). "Micro and Macro Data Integration: The Case of Capital." U.S. Census Bureau working paper.
- [3] Blanchard, Olivier and Peter Diamond(1990), "The Cyclical Behavior of the Gross Flows of U.S. Workers", *Brookings Papers on Economic Activity* 2, pp. 85-155
- [4] Blanchard, Olivier and Jordi Gali (2006)
- [5] Covas, Francisco and Wouter den Haan (2006), "The Role of Debt and Equity Financing over the Business Cycle," Working paper.
- [6] Davis, Steven, John C. Haltiwanger and Scott Schuh (1996), *Job Creation and Destruction*, Cambridge: MIT Press.
- [7] den Haan, Wouter J., Garey Ramey and Joel Watson (2003), "Liquidity Flows and Fragility of Business Enterprises", *Journal of Monetary Economics* vol. 50(6), pp. 1215-1241.
- [8] Eisefeldt, A. and A. Rampini (2006), "Capital Reallocation and Liquidity." *Journal of Monetary Economics* vol. 53(3), pp. 369-399.
- [9] Eisefeldt, A. and A. Rampini (2007), "New or Used: Investment with Credit Constraints." *Journal of Monetary Economics*, forthcoming.
- [10] Elsby, Micheal W., Ryan Michaels and Gary Solon (2007), "The Ins and Outs of Cyclical Unemployment", NBER working paper No. 12853.
- [11] Fallick, Bruce and Charles A. Fleischmann (2004), "Employer-to-Employer Flows in the U.S. Labor Market: The Complete Picture of Gross Worker Flows, "Finance and Economics Discussion Series 2004-34. Washington: Board of Governors of the Federal Reserve System.

- [12] Fujita, Shigeru and Gary Ramey (2006), "The Cyclicalities of Job Loss and Hiring", mimeo, Federal Reserve Bank of Philadelphia and University of California at San Diego.
- [13] Fujita, Shigeru and Ramey, Gary (2007), "Reassessing the Shimer Facts", mimeo, Federal Reserve Bank of Philadelphia and University of California at San Diego.
- [14] Gertler, Mark and Antonella Trigari (2006), "Unemployment fluctuations with staggered Nash wage bargaining."Draft, New York University and Bocconi University.
- [15] Hall, Robert E. (2005), "Job Loss, Job finding, and Unemployment in the U.S. Economy over the Past Fifty Years," in Gertler and Rogoff (eds.), NBER Macroeconomics Annual, 101-137, MIT Press, Cambridge.
- [16] King, Robert G. and Sergio T. Rebelo (1999), "Resuscitating Real Business Cycles", in John B. Taylor and Michael Woodford (eds), Handbook of Macroeconomics, volume 1.
- [17] Krause, Michael and Thomas Lubik (2007), "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions", *Journal of Monetary Economics*, forthcoming.
- [18] Kurmann, André and Nicolas Petrosky-Nadeau (2007), "Search in Physical Capital Markets as a Propagation Mechanism", UQAM working paper.
- [19] Nagypal, Eva (2004), "Worker Reallocation over the Business Cycle: The Importance of Job-to-Job Transitions", working paper, Northwestern University.
- [20] Merz, Monika (1995), "Search in the Labor Market and the Real Business Cycle", *Journal of Monetary Economics* vol. 36(2), pp. 269-300.
- [21] Mortensen, Dale T. and Christopher A. Pissarides (1994), "Job Creation and Job Destruction in the Theory of Unemployment", *The Review of Economic Studies* vol. 61(3), pp. 397-415.
- [22] Ramey, Valerie and Matthew D. Shapiro (1998). "Capital Churning." Working paper.
- [23] Ramey, Valerie and Matthew D. Shapiro (2001). "Displaced Capital: A Study of Aerospace Plant Closings" *Journal of Political Economy* vol. 109(5), pp. 958-992.

- [24] Rotemberg, Julio (2006), "Cyclical Wages in a Search-and-bargaining model with large firms," Draft, Harvard University.
- [25] Rudanko, Leena (2006), "Idiosyncratic and Aggregate Risk in a Frictional Labor Market", draft, University of Chicago.
- [26] Townsend, Robert M. (1979), "Optimal Contracts and Competitive Markets with Costly State Verification", *Journal of Economic Theory* vol. 21(2), pp. 265-293.
- [27] Shimer, Robert (2005a), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", *American Economic Review* vol. 95(1), pp. 25-49.
- [28] Shimer, Robert (2005b), "Reassessing the Ins and Outs of Unemployment." Draft, University of Chicago.
- [29] Shleifer, Andrei and Robert W. Vishny (1992), " Liquidation Values and Debt Capacity: A Market Equilibrium Approach," *Journal of Finance*, vol. 47(4), pp. 1343-1366.
- [30] Wasmer, Étienne and Philippe Weil, (2004), "The Macroeconomics of Labor and Credit Market Imperfections", *American Economic Review* vol. 94(4), pp. 944-963
- [31] Yashiv, Eran (2006), "U.S. Labor Market Dynamics Revisited", IZA Discussion Paper no. 2455.

A Empirical evidence

A.1 Capital flows and financial distress

This first figure, reproduced from Kurmann and Petrosky-Nadeau (2007), represent the vacancy rate of office and industrial space over approximately two decades.

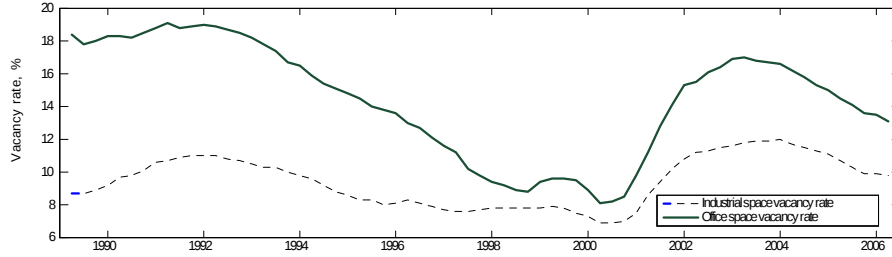


Figure 6: Evidence of congestion on physical capital markets - vancancy rates for office and industrial space. Source: Torto Wheaton Richard Ellis.

The evidence on labor and capital at financially distressed firms reported in this paper uses Standard and Poors' Compustat Data base on all publicly traded firms. The data set is matched with the Bankruptcy Research Database (BRD), compiled by Prof. Lynn M. LoPucki at the UCLA Law department. Of the 751 reported cases of bankruptcy filings by large publicly traded firms since October 1979, it was possible to match 623 firms with the unique firm identifiers used by Compustat (mnemonic: gvkey).

Figure 7 plots the number of bankruptcy filings recorded in years from 1980 to 2006, with NBER recession years represented as colored bands. It is immediately apparent that the number of filings spikes during the recession years.

The correlation of the bankruptcy filing rate, defined as the ratio of filings to the number of firms active in Compustat in a given year, with the business cycle component of GDP, is reported in Table 7.

Once the two data sets are matched, it is possible to investigate some dimensions of firm behavior during the years preceding and following a bankruptcy filing, looking first into changes in the filing firm's workforce. The firm level growth rate of employment is computed as

$$\gamma_{it} = \frac{n_{it} - n_{it-1}}{(n_{it} + n_{it-1})/2},$$

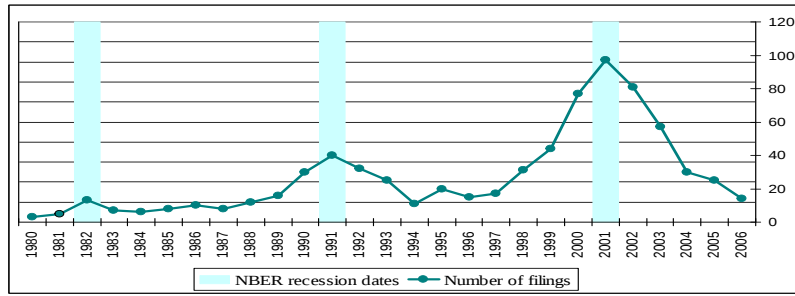


Figure 7: Bankruptcy filings and recessions.

where employment is the number of employees reported in Compustat (data item 29). The results for a 10 year window around the filing year are presented in Figure 8.

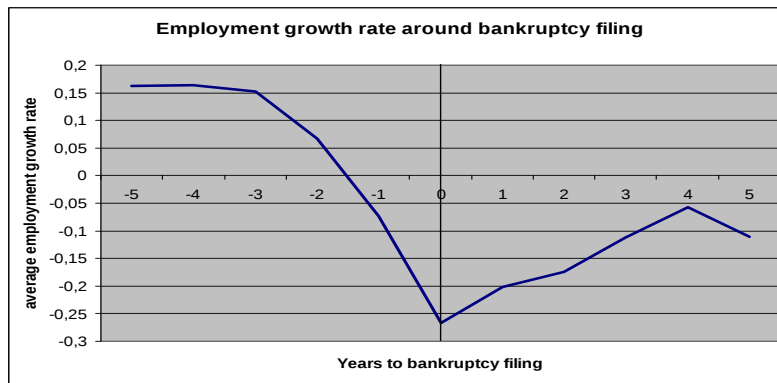


Figure 8: Bankruptcy filings and employment changes.

One might expect that firm run down their asset in order to avoid having to file for bankruptcy. To investigate this the ratio of sales of property, plant and equipment (Compustat data item 107) to the total stock of PP&E reported by the firm (Compustat data item 7) is computed. Figure 9 reports the results for a ten year window around the year of filing for bankruptcy.

The next figure plots the evolution of output (Compustat data item 12) per employee (Compustat data item 29) at firms under financial distress.

The cyclicalty of financial distress and the associated capital liquations, defined as the sum of sales of PP&E at financially distressed firms and the PP&E of firm exiting Compustat due to bankruptcy, is plotted in Figure 11 and moments are reported in Table 7. Reasons for deletion

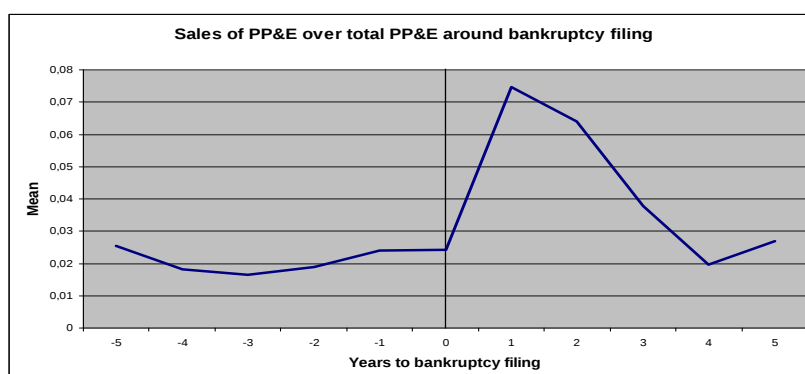


Figure 9: Bankruptcy filings and sales of property, plant and equipment.

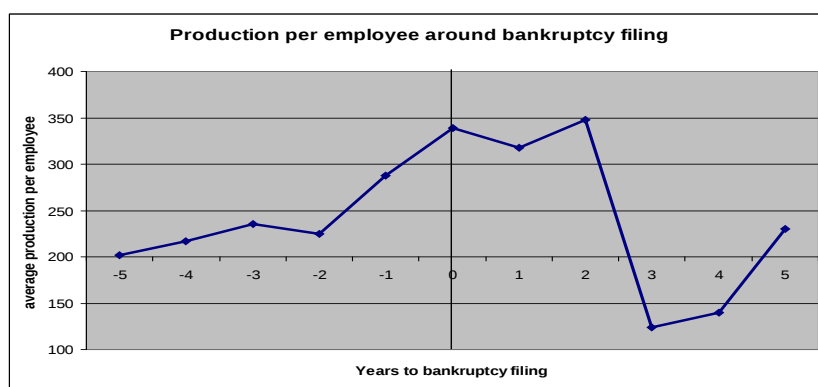


Figure 10: Production per employee at firms under financial distress.

from Compustat are detailed in footnote 35 (mnemonic `aftnt35`), while the month and year of deletion are given, respectively, in footnotes 33 and 34 (mnemonics `aftnt33` and `aftnt34`). Figure 12 plots the sample correlation between the liquidation rate and GDP.

The following table gives the standard deviation of the Hodrick-Prescott filtered time series relative to that of real GDP. The last column give the contemporaneous correlation with the business cycle component of GDP.

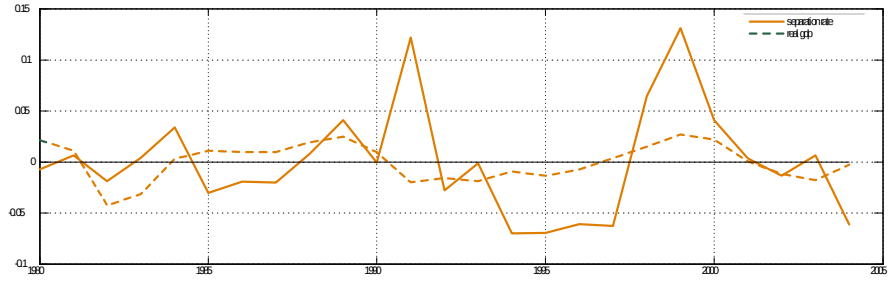


Figure 11: The business cycle of capital separations.

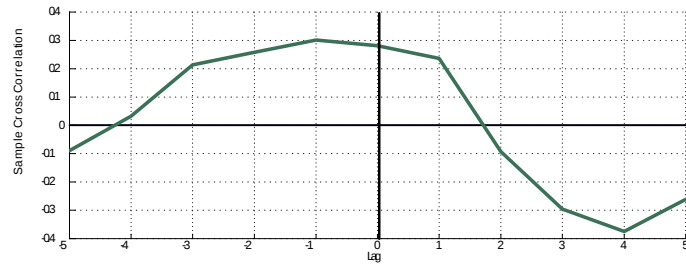


Figure 12: Sample cross correlation function between the rate of capital liquidation due to financial distress and real gdp.

Table 7: Business cycle of bankruptcies and capital liquidation

1980-2005	Rel. Std Dev	Contp. Corr
Rate of bankruptcy filings	0.11	-0.14
PP&E exit rate	2.14	0.07
Rate of sales of PP&E	0.94	0.56
Liquidation rate	2.48	0.28

All moments are H.-P. filtered

B Full model

TO BE COMPLETED

- Household

The representative household, given the existing capital stock k_t , and employment n_t , makes optimal consumption and investment decisions in order to maximize the value function:

$$V(n_t, k_t) = \max_{c_t, l_t, a_t} [\log(c_t) + \beta E_t V(n_{t+1}, k_{t+1})],$$

subject to the budget constraint $c_t + l_t = R_t^n + R_t^k + u_t^k + u_t^n b + \Pi_t - T_t$ and the laws of motion for matched labor and capital. The government raises T_t in taxes to fund unemployment benefits $u_t^L b$. Π_t are firm profits defined below. $u_{t+1}^k = (1 - q(\phi_t))l_t$ is unmatched liquid capital in the period carried forward.

The marginal values of employment and unemployment are obtained by differentiating the value function:

$$\begin{aligned} V_{u^n}(n_t, k_t) &= \lambda_t b + \beta E_t [q(\theta_t) V_n(n_{t+1}, k_{t+1}) + (1 - q(\theta_t)) V_{u^n}(n_{t+1}, k_{t+1})], \\ V_n(n_t, k_t) &= \lambda_t \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) [w_t - e_t] + \lambda_t \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) f c w_t \\ &\quad + \beta E_t [\varphi_t V_{u^n}(n_{t+1}, k_{t+1}) + (1 - \varphi_t) V_n(n_{t+1}, k_{t+1})]. \end{aligned}$$

From the employed worker's point of view at the beginning of the period (after the aggregate state has been observed), the probability of not being laid off is $\left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a)\right)$. If the individual is not laid off the negotiated wage w_t is paid but at loss of utility from the extra work is incurred. With probability $\int_{\bar{a}_t}^{\tilde{a}_t} dH(a)$ the worker is laid off and received a severance payment that is a fraction of the wage, $f c$. The value of being in an employment relationship is therefore an average of these two outcomes.

Given the debt contract described above, the household's expected revenue from matched capital, R_t^k ,

$$\begin{aligned} R_t^k &= \int_{\bar{a}_t}^{\infty} \rho_t k_t dH(a) + \int_{\bar{a}_t}^{\tilde{a}_t} \left[(1 - \tau) a y_t - w_t n_t - \gamma v_t^L - \kappa v_t^k \right] dH(a) - \int_0^{\bar{a}_t} \tau a y_t dH(a) \\ &\quad + (1 - \delta) \varsigma_t s_t k_t, \end{aligned}$$

where the auditing cost to the household is a fixed proportion $\tau > 0$ of the firm's revenue, $\int_0^{\bar{a}_t} \tau a y_t dH(a)$. The last term, $(1 - \delta) \varsigma_t s_t k_t$, corresponds to separated capital returned to the household, net a depreciation and a separation cost. This is a convex cost to adjusting the liquidation rate introduced in order to capture the loss of value due to the specificity of capital mentioned above. Thus the value of capital actually returned to the household's budget constraint is diminished by a fraction $\varsigma_t \equiv g(s_t/s_{t-1})$, with $\varsigma'(\bullet) < 0$ and $\varsigma(\bullet) < 0$.

Investment decisions are determined by the optimality condition for liquid capital:

$$\lambda_t = \beta E_t [q(\phi_t)V_k(n_{t+1}, k_{t+1}) + (1 - q(\phi_t))V_u(n_{t+1}, k_{t+1})],$$

which equates the opportunity cost of forgone consumption to the expected value of additional liquid capital, composed of the marginal values for matched and unmatched capital. The corresponding marginal values for capital are:

$$\begin{aligned} V_{u^k}(n_t, k_t) &= \lambda_t, \\ V_k(n_t, k_t) &= \lambda_t \left[\frac{\partial R_t^k}{\partial k_t} \right] + (1 - \delta)(1 - s_t)\beta E_t V_k(n_{t+1}, k_{t+1}). \end{aligned}$$

where $\frac{\partial R_t^k}{\partial k_t} = \int_{\bar{a}_t}^{\infty} \rho_t dH(a) + \int_{\underline{a}_t}^{\bar{a}_t} (1 - \tau)a\alpha_{\bar{k}_t}^{\underline{y}_t} dH(a) - \int_0^{\underline{a}_t} \tau a\alpha_{\bar{k}_t}^{\underline{y}_t} dH(a)$. The remaining first order conditions are

$$\begin{aligned} (c_t) &: 1/c_t = \lambda_t \\ (\underline{a}_t) &: \lambda_t(1 - \delta)k_t = \lambda_t \left[\underline{a}_t y_t - w_t n_t - \gamma v_t^L - \kappa v_t^k \right] + (1 - \delta)k_t \beta E_t V_k(n_{t+1}, k_{t+1}). \end{aligned}$$

- Firms

At the beginning of each period, households and firms observe the exogenous aggregate technology, x_t . Firms maximize profits subject to the laws of motion for matched labor and capital, and a constant returns to scale production technology, $y_t = x_t f(n_t, k_t)$. Given the existing capital stock k_t , and employment n_t , firms post job vacancies and investment projects v_t^n and v_t^k , respectively. This yields the following Bellmann equation, where w and ρ are the remuneration of labor and capital:

$$\begin{aligned} J(n_t, k_t) &= \max_{v_t^n, v_t^k} \int_{\bar{a}_t}^{\infty} \left[a y_t - w_t n_t - \rho_t k_t - \gamma v_t^n - \kappa v_t^k \right] dH(a) \\ &\quad + (1 - f c) \int_{\bar{a}_t}^{\tilde{a}_t} w_t n_t dH(a) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J(n_{t+1}, k_{t+1}). \end{aligned}$$

The exogeneity of the factor remunerations implies that firms do not internalize the effects of their employment and capital stocks on marginal productivities during repayment negotiations (see below). The first order conditions to this problem,

$$\begin{aligned} (v_t^n) &: \left[1 - \int_{\bar{a}_t}^{\infty} dH(a) \right] \frac{\gamma}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(n_{t+1}, k_{t+1}) \\ (v_t^k) &: \left[1 - \int_{\bar{a}_t}^{\infty} dH(a) \right] \frac{\kappa}{p(\phi_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_k(n_{t+1}, k_{t+1}), \end{aligned}$$

equate average vacancy costs to expected returns to each factor of production.

The marginal value of an employed worker to the firm, $J_n(n_t, k_t) = \int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} - w_t \right] dH(a) + (1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} w_t dH(a) + (1 - \varphi_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(n_{t+1}, k_{t+1})$, can be combined with (9) to form a forward looking equation for vacancy postings:

$$\begin{aligned} \frac{\gamma [1 - H(\bar{a}_t)]}{p(\theta_t)} &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \bar{\mu}_{t+1} (1 - \alpha) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} [1 - H(\bar{a}_{t+1})] \right. \\ &\quad \left. + (1 - fc) \int_{\bar{a}_{t+1}}^{\tilde{a}_{t+1}} w_{t+1} dH(a) + (1 - \varphi_{t+1}) \frac{\gamma [1 - H(\bar{a}_{t+1})]}{p(\theta_{t+1})} \right\}. \end{aligned}$$

The marginal value of matched capital to the firm, $J_k(n_t, k_t) = \int_{\bar{a}_t}^{\infty} \left[a \alpha \frac{y_t}{k_t} - \rho_t \right] dH(a) + (1 - \delta)(1 - s_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_k(n_{t+1}, k_{t+1})$, combined with the optimality condition (10), yields a forward looking equation for project postings:

$$\frac{\kappa [1 - H(\bar{a}_t)]}{p(\phi_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \bar{\mu}_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} - \rho_{t+1} [1 - H(\bar{a}_{t+1})] + (1 - \delta)(1 - s_{t+1}) \frac{\kappa [1 - H(\bar{a}_{t+1})]}{p(\phi_{t+1})} \right\}.$$

- Remunerations

- Labor

The surplus from the worker-firm relationship is defined as $S_t^n = J_n(n_t, k_t) + \frac{V_n(n_t, k_t) - V_u^n(n_t, k_t)}{\lambda_t}$. Nash bargaining implies that the worker receives $\frac{V_n(n_t, k_t) - V_u^n(n_t, k_t)}{\lambda_t} = \eta S_t^n$, and the firm receives $J_n(n_t, k_t) = (1 - \eta) S_t^n$. The negotiated wage is given by the expression

$$w_t = \eta \left[(1 - \alpha) \int_{\bar{a}_t}^{\infty} a dH(a) \frac{y_t}{n_t} + \gamma \theta_t \left[1 - \int_{\bar{a}_t}^{\infty} dH(a) \right] \right] + (1 - \eta) \left[b + \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) e_t \right] + \Omega_t.$$

where $\Omega_t \equiv w_t \left[\eta \left[1 - \int_{\bar{a}_t}^{\infty} dH(a) \right] + (1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right]$. The negotiated wage contains two terms related to layoffs due to credit frictions. First, the negotiated wage is increasing in the additional effort required. Second, the term Ω_t captures two elements. The first term in brackets corresponds to a sharing between worker and firm of the fact that firms are optimizing only over the range $[\bar{a}_t, \infty)$. The second term in brackets corresponds to a sharing the benefits from being able to lay off workers. The lower the firing cost, the higher the negotiated wage.

$$\begin{aligned}
S_t^n &= J_n(n_t) + \frac{V_n(t) - V_{u^n}(t)}{\lambda_t} \\
S_t^n &= \int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} - w_t \right] dH(a) + (1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} w_t dH(a) + (1 - \varphi_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(n_{t+1}, k_{t+1}) \\
&\quad + \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) [w_t - e_t] + \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) fc w_t + \varphi_t \beta E_t \frac{V_{u^n}(n_{t+1}, k_{t+1})}{\lambda_t} \\
&\quad + (1 - \varphi_t) \beta E_t \frac{V_n(n_{t+1}, k_{t+1})}{\lambda_t} - b - q(\theta_t) \beta E_t \frac{V_n(t+1)}{\lambda_t} - (1 - q(\theta_t)) \beta E_t \frac{V_{u^n}(t+1)}{\lambda_t} \\
S_t^n &= \int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} \right] dH(a) + \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right) w_t - b - \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) e_t \\
&\quad + (1 - \varphi_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[J_n(n_{t+1}) + \frac{V_n(t+1) - V_{u^n}(t+1)}{\lambda_{t+1}} \right] \\
&\quad - q(\theta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{V_n(t+1) - V_{u^n}(t+1)}{\lambda_{t+1}} \right] \\
S_t^n &= \int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} \right] dH(a) + \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right) w_t - b - \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) e_t \\
&\quad + (1 - \varphi_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^L - \eta q(\theta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^L \\
S_t^n &= \int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} \right] dH(a) + \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right) w_t - b - \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) e_t \\
&\quad + (1 - \varphi_t) \frac{\gamma \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right)}{p(\theta_t)(1 - \eta)} - \frac{q(\theta_t) \eta \gamma \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right)}{p(\theta_t)(1 - \eta)} \\
(1 - \eta) S_t^n &= (1 - \eta) \left[\int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} \right] dH(a) + \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right) w_t - b - \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) e_t \right] \\
&\quad + (1 - \varphi_t) \frac{\gamma \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right)}{p(\theta_t)} - \frac{\eta \gamma q(\theta_t) \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right)}{p(\theta_t)}
\end{aligned}$$

Combine this with $(1 - \eta) S_t^n = \int_{\bar{a}_t}^{\infty} \left[a(1 - \alpha) \frac{y_t}{n_t} - w_t \right] dH(a) + (1 - fc) \int_{\bar{a}_t}^{\tilde{a}_t} w_t dH(a) + (1 - \varphi_t) \frac{\gamma \left(1 - \int_{\bar{a}_t}^{\infty} dH(a) \right)}{p(\theta_t)}$ to obtain the wage equation.

- – Capital

The surplus from the lender-firm relationship is defined as $S_t^K = J_k(n_t, k_t) + \frac{V_k(n_t, k_t) - V_{u^k}(n_t, k_t)}{\lambda_t}$.

The rental rate resulting for Nash bargaining is given by the expression:

$$\rho_t = \eta \left[\bar{\mu}_t \alpha \frac{y_t}{k_t} + (1 - \delta)(1 - s_t) \frac{\kappa}{\phi_t} [1 - H(\bar{a}_t)] \right] + (1 - \eta) [\delta + (1 - \delta)(1 - \xi_t) s_t] + \Phi_t$$

where $\Phi_t = \rho_t H(\bar{a}_t) - (1 - \eta)(\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t))\alpha \frac{y_t}{k_t}$.

Both rules have remuneration increasing in the factor's marginal product. The market tension term in each remuneration rule captures the effect of market conditions on the outside option during negotiation, while b and δ capture the opportunity cost of the match. The term Φ_t in the repayment rule for capital capture the risk the follows from asymmetric information and the debt contract: it is expected that some firms will default and that a monitoring cost will be paid. . The Details concerning the derivation of (15) and (16) are presented in the appendix.

By definition $S_t^k = J_k(n_t, k_t) + \frac{V_k(n_t, k_t) - V_{u^k}(n_t, k_t)}{\lambda_t}$, thus

$$\begin{aligned} S_t^k &= \bar{\mu}_t \alpha \frac{y_t}{k_t} - \rho_t [1 - H(\bar{a}_t)] + (1 - \delta)(1 - s_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_k(n_t, k_t) \\ &\quad + \rho_t [1 - H(\bar{a}_t)] + (\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t)) \alpha \frac{y_t}{k_t} + (1 - \delta) \xi_t s_t \\ &\quad + (1 - \delta)(1 - s_t) \beta E_t \frac{V_k(n_{t+1}, k_{t+1})}{\lambda_t} - \frac{V_{u^k}(n_{t+1}, k_{t+1})}{\lambda_t} \\ S_t^k &= \alpha \frac{y_t}{k_t} \left[\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t)) \right] + (1 - \delta) \xi_t s_t - 1 \\ &\quad + (1 - \delta)(1 - s_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[J_k(n_{t+1}, k_{t+1}) + \frac{V_k(n_{t+1}, k_{t+1}) - V_{u^k}(n_{t+1}, k_{t+1})}{\lambda_{t+1}} \right] \\ &\quad + (1 - \delta)(1 - s_t) \beta E_t \frac{V_{u^k}(n_{t+1}, k_{t+1})}{\lambda_t} \end{aligned}$$

From the first order condition for liquid capital and the household's share of the total surplus, $\beta E_t \frac{V_{u^k}(n_{t+1}, k_{t+1})}{\lambda_t}$ can be written as $\left[1 - q(\theta_t) \eta \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^k \right]$. Thus,

$$\begin{aligned} S_t^k &= \alpha \frac{y_t}{k_t} \left[\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t)) \right] + (1 - \delta) \xi_t s_t - 1 + (1 - \delta)(1 - s_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^k \\ &\quad + (1 - \delta)(1 - s_t) \left[1 - \eta q(\theta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^k \right] \\ S_t^k &= \alpha \frac{y_t}{k_t} \left[\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t)) \right] + (1 - \delta) \xi_t s_t - 1 + (1 - \delta)(1 - s_t) \frac{\kappa [1 - H(\bar{a}_t)]}{p(\theta_t)(1 - \eta)} \\ &\quad + (1 - \delta)(1 - s_t) \left[1 - \eta q(\theta_t) \frac{\kappa [1 - H(\bar{a}_t)]}{p(\theta_t)(1 - \eta)} \right] \\ (1 - \eta) S_t^k &= (1 - \eta) \left\{ \alpha \frac{y_t}{k_t} \left[\bar{\mu}_t + (\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t)) \right] + (1 - \delta) \xi_t s_t - 1 + (1 - \delta)(1 - s_t) \right\} \\ &\quad + (1 - \delta)(1 - s_t) \frac{\kappa [1 - H(\bar{a}_t)]}{p(\theta_t)} - \eta q(\theta_t) \frac{\kappa [1 - H(\bar{a}_t)]}{p(\theta_t)} \end{aligned}$$

Combine this with $(1 - \eta) S_t^k = \bar{\mu}_t \alpha \frac{y_t}{k_t} - \rho_t [1 - H(\bar{a}_t)] + (1 - \delta)(1 - s_t) \frac{\kappa}{p(\theta_t)} [1 - H(\bar{a}_t)]$ to obtain the repayment equation.

- Profit and aggregate resource constraint

A simplifying assumption was the existence of a state contingent insurance that covers for any shortfall in wage payments and costs of job vacancy and project postings left over by bankrupt firms. Aggregating over the different firms, the total amount of profits transferred to the household is defined as

$$\begin{aligned}\Pi_t = & \int_{\bar{a}_t}^{\infty} \left[af(n_t, k_t) - \rho_t k_t - w_t n_t - \gamma v_t^L - \kappa v_t^k \right] dH(a) \\ & + \int_0^{\underline{a}_t} \left[af(n_t, k_t) - w_t n_t - \gamma v_t^L - \kappa v_t^k \right] dH(a).\end{aligned}$$

Combining this definition with that for the return to productive capital and the household budget constraint leads to the following aggregate resource constraint:

$$y_t [1 - \tau(1 - \bar{\mu}_t)] = c_t + i_t^{new} + \left[\kappa v_t^k + \gamma v_t^L \right],$$

where $i_t^{new} = l_t - (1 - \delta)\varsigma_t s_t k_t - u_t^k$ corresponds to new investment. Resources consumed due to monitoring of firms is captured by the term $\tau(1 - \bar{\mu}_t)$. The bracketed term on the expenditure side corresponds to posting costs.

The following figure plots the return to capital as a function of the realized idiosyncratic productivity, a . For all realizations above the monitoring threshold $\bar{a}$, households receive the negotiated repayment ρ . This threshold is located below the zero profit productivity, $\tilde{a}$, as firms layoff workers, subject to a firing cost, in order to cut on expenses and avoid monitoring. For realizations below the monitoring threshold, the return decreases linearly with the value of the idiosyncratic productivity such that a region exists where the lender actually reinjects funds to maintain the relationship. For all realizations below the separation threshold $\underline{a}$ the lender receives no payment and recuperates the unit of capital.

C A model with exogenous labor separation

D A simple labor and capital search business cycle model

- Household's dynamic program:

$$\begin{aligned}V(u_t^n, k_t) &= \max_{c_t, k_{t+1}} \left[\log(c_t) + \beta E_t V(u_{t+1}^n, k_{t+1}) \right] \\ \text{subject to } c_t + k_{t+1} &= w_t n_t + \rho_t k_t + (1 - \delta)k_t + (1 - n_t)b + \Pi_t - T_t\end{aligned}$$

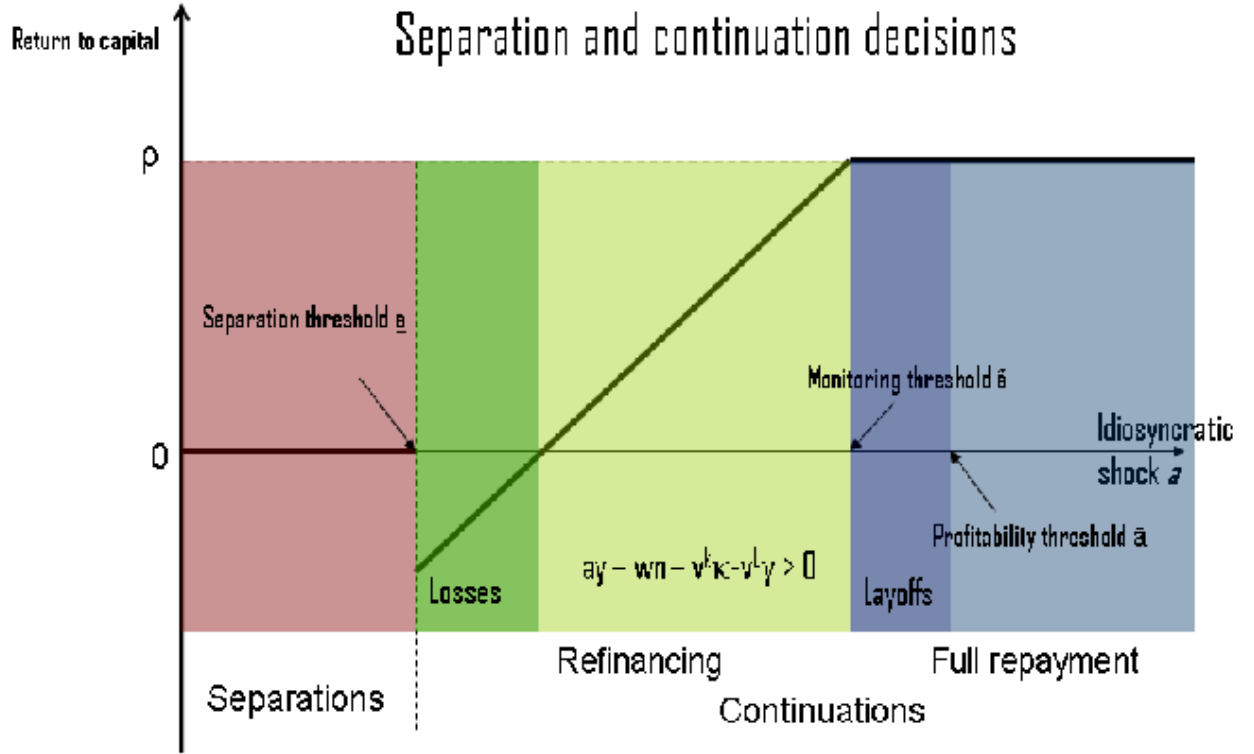


Figure 13: Idiosyncratic productivity, separation and continuation decisions.

Define n as employment and $u^n = 1 - n$ as unemployment labor. Jobs are destroyed at the exogenous rate φ . Government raises T_t in taxes to fund unemployment benefits $(1 - n_t)b$. Π_t are firm profits defined below. Optimization leads to the familiar condition on the marginal utility of consumption and the Euler equation. The value a marginal unemployed and the marginal employed individual can be derived from the representative households value function:

$$\begin{aligned} V_{u^n}(t) &= \lambda_t b + \beta E_t [q(\theta_t) V_n(t+1) + (1 - q(\theta_t)) V_{u^n}(t+1)] \\ V_n(t) &= \lambda_t w_t + \beta E_t [\varphi V_{u^n}(t+1) + (1 - \varphi) V_n(t+1)] \end{aligned}$$

- Firm's dynamic program:

$$J(n_t, k_t) = \max_{v_t^L, v_t^k} y_t - w_t n_t - \rho_t k_t - \gamma v_t^n - \kappa v_t^k + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J(n_{t+1}, k_{t+1}).$$

Demand for capital equates its marginal product to its marginal cost. Vacancies are

posted such that the average cost of a vacancy is equal to its expected value:

$$\begin{aligned}\frac{\gamma}{p(\theta_t)} &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1-\alpha) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} + (1-\varphi) \frac{\gamma}{p(\theta_{t+1})} \right\} \\ \frac{\kappa}{p(\phi_t)} &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \alpha \frac{y_{t+1}}{k_{t+1}} - \rho_{t+1} + (1-\delta)(1-s) \frac{\kappa}{p(\phi_{t+1})} \right\}\end{aligned}$$

The value of a marginal worker to the firm is simply

$$J_n(n_t) = (1-\alpha) \frac{y_t}{n_t} - w_t + (1-\varphi) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(n_{t+1}).$$

Combining I obtain a forward looking equation for vacancy postings:

$$\frac{\gamma}{p(\theta_t)} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ (1-\alpha) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} + (1-\varphi) \frac{\gamma}{p(\theta_{t+1})} \right\}$$

The surplus to the labor relationship is $S_t^n = J_n(n_t) + \frac{V_n(t) - V_{un}(t)}{\lambda_t}$. A result of Nash bargaining is that $(1-\eta)S_t^n = J_n(n_t)$ and $\eta S_t^n = \frac{V_n(t) - V_{un}(t)}{\lambda_t}$ from

$w_t = \arg \max [J_n(n_t, k_t)]^{1-\eta} \left[\frac{V_n(n_t, k_t) - V_{un}(n_t, k_t)}{\lambda_t} \right]^\eta$ A little algebra yields the wage equation

$$w_t = \eta \left[(1-\alpha) \frac{y_t}{n_t} + \gamma \theta_t \right] + (1-\eta)b$$

- Wage resolution:

$$\begin{aligned}S_t^n &= J_n(n_t) + \frac{V_n(t) - V_{u^L}(t)}{\lambda_t} \\ S_t^n &= (1-\alpha) \frac{y_t}{n_t} - w_t + (1-\varphi) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_n(n_{t+1}) + w_t + (1-\varphi) \beta E_t \frac{V_n(t+1)}{\lambda_t} + \\ &\quad \varphi \beta E_t \frac{V_{un}(t+1)}{\lambda_t} - b - q(\theta_t) \beta E_t \frac{V_n(t+1)}{\lambda_t} - (1-q(\theta_t)) \beta E_t \frac{V_{un}(t+1)}{\lambda_t} \\ S_t^n &= (1-\alpha) \frac{y_t}{n_t} - b + (1-\varphi) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[J_n(n_{t+1}) + \frac{V_n(t+1) - V_{un}(t+1)}{\lambda_{t+1}} \right] \\ &\quad - q(\theta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{V_n(t+1) - V_{un}(t+1)}{\lambda_{t+1}} \right] \\ S_t^n &= (1-\alpha) \frac{y_t}{n_t} - b + (1-\varphi) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^n - \eta q(\theta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^n \\ S_t^n &= (1-\alpha) \frac{y_t}{n_t} - b + (1-\varphi) \frac{\gamma}{p(\theta_t)(1-\eta)} - \frac{q(\theta_t) \eta \gamma}{p(\theta_t)(1-\eta)} \\ (1-\eta) S_t^n &= (1-\eta) \left[(1-\alpha) \frac{y_t}{n_t} - b \right] + (1-\varphi) \frac{\gamma}{p(\theta_t)} - \eta \gamma \theta_t\end{aligned}\tag{19}$$

Combine this with $(1-\eta)S_t^n = (1-\alpha) \frac{y_t}{n_t} - w_t + (1-\varphi) \frac{\gamma}{p(\theta_t)}$ to obtain the wage equation.

- Solving the repayment rule:

$$\begin{aligned}
S_t^K &= J_k(k_t) + \frac{V_k(t) - V_{u^k}(t)}{\lambda_t} \\
S_t^K &= \alpha \frac{y_t}{k_t} - \rho_t + (1 - \delta)(1 - s)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_k(k_{t+1}) + \rho_t + (1 - \delta)s + (1 - \delta)(1 - s)\beta E_t \frac{V_k(k_{t+1})}{\lambda_t} - 1 \\
S_t^K &= \alpha \frac{y_t}{k_t} + (1 - \delta)s - 1 + (1 - \delta)(1 - s)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[J_k(k_{t+1}) + \frac{V_k(k_{t+1}) - V_{u^k}(k_{t+1})}{\lambda_t} \right] \\
&\quad + (1 - \delta)(1 - s)\beta E_t \frac{V_{u^k}(k_{t+1})}{\lambda_t}
\end{aligned}$$

Use the fact that $\beta E_t \frac{V_{u^k}(k_{t+1})}{\lambda_t} = 1 - \eta q(\phi_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^k$:

$$\begin{aligned}
S_t^K &= \alpha \frac{y_t}{k_t} + (1 - \delta)s - 1 + (1 - \delta)(1 - s)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^k \\
&\quad + (1 - \delta)(1 - s) \left[1 - \eta q(\phi_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}^k \right] \\
(1 - \eta)S_t^K &= (1 - \eta) \left[\alpha \frac{y_t}{k_t} + (1 - \delta)s - 1 \right] + (1 - \delta)(1 - s) \frac{\kappa}{p(\phi_t)} + (1 - \delta)(1 - s) \left[(1 - \eta) - \frac{\eta q(\phi_t) \kappa}{p(\phi_t)} \right]
\end{aligned}$$

Combine this with $S_t^K = \alpha \frac{y_t}{k_t} - \rho_t + (1 - \delta)(1 - s) \frac{\kappa}{p(\phi_t)}$ to obtain the repayment equation.

E Equilibrium systems of equations

E.1 Labor and capital matching with endogenous separations

The equilibrium system is defined by 29 equations identifying 29 variables:

$$y_t = x_t n_t^{1-\alpha} k_t^\alpha \quad (1)$$

$$\rho_t = \eta \left[\bar{\mu}_t \alpha \frac{y_t}{k_t} + (1-\delta)(1-s_t) \frac{\kappa}{\phi_t} [1-H(\bar{a}_t)] \right] + (1-\eta) [\delta + (1-\delta)(1-\xi_t)s_t] + \Phi \quad (2)$$

$$w_t = \eta \left[(1-\alpha) \int_{\bar{a}_t}^{\infty} a dH(a) \frac{y_t}{n_t} + \gamma \theta_t [1 - \int_{\bar{a}_t}^{\infty} dH(a)] \right] + (1-\eta) \left[b + \left(1 - \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) \right) e_t \right] + \Omega_t \quad (3)$$

$$\begin{aligned} \frac{\kappa [1-H(\bar{a}_t)]}{p(\phi_t)} &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left\{ \bar{\mu}_{t+1} \alpha \frac{y_{t+1}}{k_{t+1}} - \rho_{t+1} [1-H(\bar{a}_{t+1})] + (1-\delta)(1-s_{t+1}) \frac{\kappa [1-H(\bar{a}_{t+1})]}{p(\phi_{t+1})} \right\} \\ \frac{\gamma [1-H(\bar{a}_t)]}{p(\theta_t)} &= \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \bar{\mu}_{t+1} (1-\alpha) \frac{y_{t+1}}{n_{t+1}} - w_{t+1} [1-H(\bar{a}_{t+1})] \right. \\ &\quad \left. + (1-fc) \int_{\bar{a}_{t+1}}^{\tilde{a}_{t+1}} w_{t+1} dH(a) + (1-\varphi_{t+1}) \frac{\gamma [1-H(\bar{a}_{t+1})]}{p(\theta_{t+1})} \right\} \end{aligned} \quad (5)$$

$$\frac{1}{c_t} = \lambda_t \quad (6)$$

$$\lambda_t = \beta E_t \lambda_{t+1} + \frac{\eta}{1-\eta} \frac{\kappa [1-H(\bar{a}_t)]}{\phi_t} \lambda_t \quad (7)$$

$$y_t [1 - \tau(1 - \bar{\mu}_t)] = c_t + i_t^{new} + \kappa v_t^k + \gamma v_t^n \quad (8)$$

$$k_{t+1} = (1-\delta)(1-s_t)k_t + q(\phi_t)l_t \quad (9)$$

$$n_{t+1} = (1-\varphi_t)n_t + q(\theta_t)(1-n_t) \quad (10)$$

$$u_{t+1}^k = [1 - q(\phi_t)] l_t \quad (11)$$

$$u_{t+1}^n = (1 - q(\theta_t)) u_t^L + \varphi_t n_t \quad (12)$$

$$\phi_t = \frac{l_t}{v_t^k} \quad (13)$$

$$q(\phi_t) = \phi_t^{-\varepsilon} \quad (14)$$

$$p(\phi_t) = \phi_t^{1-\varepsilon} \quad (15)$$

$$\theta_t = \frac{v_t^n}{1-n_t} \quad (16)$$

$$q(\theta_t) = \iota \theta_t^{1-\chi} \quad (17)$$

$$p(\theta_t) = \iota \theta_t^{-\chi} \quad (18)$$

$$i_t^{new} = l_t - u_t^k - (1-\delta)s_t k_t \quad (19)$$

$$y_t \tilde{a}_t = w_t n_t + \rho_t k_t + \kappa v_t^k + \gamma v_t^n \quad (20)$$

$$y_t \bar{a}_t = fc w_t n_t + \rho_t k_t + \kappa v_t^k + \gamma v_t^n \quad (21)$$

$$y_t \underline{a}_t = w_t n_t + \kappa v_t^k + \gamma v_t^n + k_t \Psi_t \quad (22)$$

$$\varphi_t^e = \int_{\bar{a}_t}^{\tilde{a}_t} dH(a) + \int_0^{\underline{a}_t} dH(a) \quad (23)$$

$$\varphi_t = \varphi^x + \varphi_t^e \quad (24)$$

$$s_t = s^x + H(\underline{a}_t) \quad (23)$$

$$\varphi_t = H(\underline{a}_t) + H(\tilde{a}_t) - H(\bar{a}_t) \quad (24)$$

$$\bar{\mu}_t = \int_{\bar{a}_t}^{\infty} a dH(a) \quad (25)$$

$$\underline{\mu}_t = \int_{\underline{a}_t}^{\infty} a dH(a) \quad (26)$$

$$\Phi_t = \rho_t H(\bar{a}_t) - (1 - \eta)(\underline{\mu}_t - \bar{\mu}_t - \tau(1 - \bar{\mu}_t))\alpha \frac{y_t}{k_t} \quad (27)$$

$$\Psi_t = (1 - \delta) \left[\varsigma_t - \frac{1}{q(\phi_t)} \left[1 - (1 - q(\phi_t))\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \right] \right] \quad (29)$$

F Steady state computations

F.1 A simple labor matching business cycle model

The investment Euler equations pins down the steady state return to capital as $\rho = 1/\beta - 1 + \delta$. The firm's capital demand then determines the steady state capital to output ratio, $k/y = \alpha/\rho$. Given a steady state unemployment rate, the steady state capital stock can be identified using the production function, $k = (x \frac{k}{y})^{\frac{1}{1-\alpha}} n$, which also allows us to determine the steady state output.

The steady state equality of flows in and out of unemployment, $\varphi n = q(\theta)(1 - n)$, is used to compute the steady state job finding rate: $q(\theta) = \frac{\varphi n}{1-n}$. The level parameter in the matching function is then used match the chosen value for the vacancy matching probability. The wage rate and unemployment benefits are then computed with their respective definitions.

Finally, consumption is computed by using the definitions of firm profits, $\Pi = y - wn - \rho k - \gamma v^n$, and the government budget constraint to rewrite the households budget constraint as $y = c + \delta k + \gamma v^n$.

F.2 Labor and capital matching with exogenous separations

Just as in the preceding model, the budget constraint can be rewritten as $y = c + \delta k + \gamma v^n + \kappa v^k$. Then, combining the steady state optimality condition for liquid capital with the results from

Nash bargaining, concerning the splitting of the surplus, and the intertemporal condition for project postings, the steady state repayment equation can be expressed as

$$\rho = \delta + \left(\frac{1}{\beta} - 1\right) + \left(\frac{1 - q(\phi)}{q(\phi)}\right) \left[\left(\frac{1}{\beta} - 1\right) - (1 - \beta)(1 - \delta)(1 - s) \right].$$

The steady state capital-output ratio is then computed as $k/y = \alpha \left[\rho + \frac{\kappa}{\phi q(\phi)} \left(\frac{1}{\beta} - (1 - \delta)(1 - s) \right) \right]^{-1}$, where $\frac{\kappa}{\phi} = (1 - \beta)^{\frac{1-\eta}{\eta}}$. It is then straightforward to compute the steady state values of capital, output and the wage rate. Liquid capital is compute as $l = k [1 - (1 - \delta)(1 - s)] / q(\phi)$. To compute the steady state, first solve for the capital market, then for the labor market. Computing the remaining steady state values is straightforward.

F.3 Labor and capital matching with endogenous separations

With endogenous separations, the steady state repayment is given by

$$\rho = \delta + \left(\frac{1}{\beta} - 1\right) + \left(\frac{1 - q(\phi)}{q(\phi)}\right) \left[\left(\frac{1}{\beta} - 1\right) - (1 - \beta)(1 - \delta)(1 - s) \right] + \Phi / (1 - \eta).$$

Using the definitions for the thresholds $\tilde{a}$ and $\underline{a}$, the steady state capital-output ratio can be expressed as

$$\frac{k}{y} = \frac{\tilde{a} - \underline{a}}{\rho - \Psi}.$$

This yields an expression for the repayment as a function of the parameters and the productivity thresholds. The numerical strategy is then to solve for the variance of the idiosyncratic shock, σ_a , such that the definitions for the three thresholds are respected. Once this is the case, the remaining steady state values are straight foward to compute using the resulting capital-output ratio.

G Log-linear systems of equations

TO BE COMPLETED