

Job Creation and Job Destruction over the Life Cycle

The Older Workers in the Spotlight

10th December 2006

Abstract

This paper extends the job creation - job destruction approach to the labor market to take into account the life-cycle of workers. Forward looking decisions about hiring and firing depend on the time over which to recoup adjustment costs. The equilibrium is typically featured by increasing (decreasing) firing (hiring) rates with age, and a hump-shaped age-dynamics of employment. The empirical plausibility of the model is assessed by incorporating existing age-specific labor market policies in France. Finally we show that the age-dynamics of employment is optimal when the Hosios condition holds and we design the optimal age-pattern for employment policies when this condition does not apply.

1 Introduction

It is now well known that the low employment rate of older workers accounts for half of the European employment gap (see OECD [2006]). The long-run unemployment incidence is 50% higher for older workers (see Farber [1997] and Machin and Manning [1999]). A first strand of the empirical literature emphasizes the negative role played by labor market institutions (specific insurance programs) on the job-search decisions of older unemployed workers (see for instance Blöndal and Scarpetta [1998]). A second strand gives greater importance to skill obsolescence, arguing that older workers suffer from a biased technological progress. Under wage stickiness, this gives firms incentives to send older workers into early retirement (see Crépon, Deniau and Perrez-Duarte [2002] and Aubert, Caroli and Roger [2006]).¹

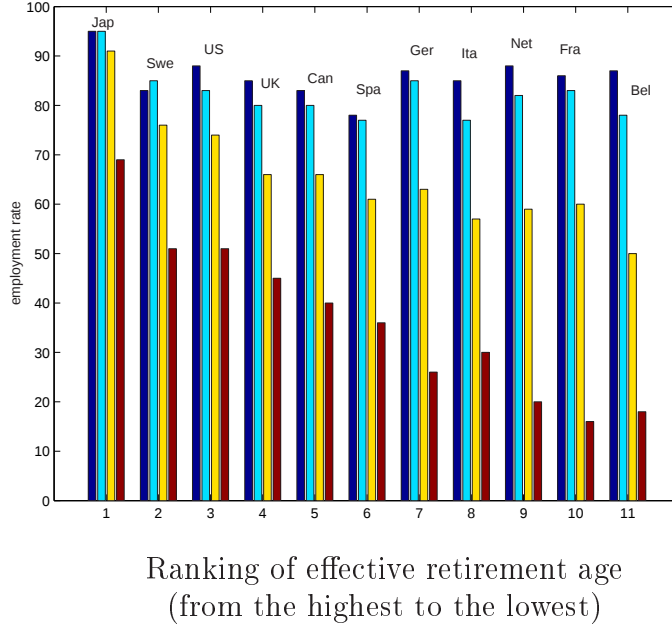
However, something is missing in this whole picture. Figure 1 shows that the fall in the employment rate of older workers is steeper when the retirement age gets closer, whatever the country considered. Two country groups emerged very clearly in the mid-nineties: those with high employment rates for workers aged 55-59 (Canada, Great Britain, Japan, the United States and Sweden) and those which experience a huge decrease in employment rates at these ages, around 25 points with respect to the 50-54 age group (Belgium, France, Italy and the Netherlands). As documented by Gruber and Wise [1999], the second group of countries is characterized by an effective retirement age of 60 (versus 65 in the first group). However, there is no reason to believe that these countries are more sensitive to ongoing technological progress.

In this paper, it is first argued that the proximity of the retirement age is the primary cause of the decrease in the employment rate of older workers. We study the direct influence of impending retirement on both job creation and job destruction, and abstract it from the labor productivity dimension.² Since Oi [1962], labor is indeed viewed as a quasi-fixed input

¹This point has already been put forward by Lazear [1979], from a theoretical standpoint.

²See Bartel and Sicherman [1993] and Friedberg [2003] for an investigation of the relation between the impending retirement and the labor productivity.

Figure 1: Employment rates from age 30 to 64 for OECD Countries



Source: OECD data for 1995 (authors' calculation). In each country, each bar refers to employment rates of the age groups : 30 - 49 (first bar on the left), 50 - 54, 55 - 59 and 60 - 64 (last bar on the right)

factor so that the hiring process is costly, leading firms to implement labor-hoarding strategies. An important contribution of Mortensen and Pissarides [1994] (MP hereafter) has been to provide theoretical foundations for these mechanisms in an overall theory of equilibrium unemployment. In that context, endogenous hirings and separations depend on the expected duration of jobs. But because workers live forever, workers' age plays no role on MP's labor market equilibrium. Surprisingly enough, the impact of the life cycle of workers in this extensively-used framework has not been yet addressed³. From this point of view, our paper fills a gap. Because the horizon of older workers is shorter, we show that firms and workers invest less in job-search

³The proximity of retirement has been scrutinized by Seater [1977] and Lungqvist and Sargent [2005], but only to explain the low search intensity of unemployed older workers. Bettendorf and Broer [2003] examine the age-dynamics of a labor market equilibrium with matching frictions, but firings are exogenous and there exists a perfect insurance assumption against unemployment and death risks.

and labor-hoarding activities at the end of the life cycle. This implies that the hiring (firing) rate decreases (increases) with the age of the worker. This approach could help to better understand the economic rationale underlying the “discrimination” against older workers in the process of hirings and firings. Furthermore, since all new entrants (youngest workers) are unemployed, we show that the overall equilibrium age-dynamics of employment is hump-shaped. Lastly, the first part of this paper also revisits the positive impact of conventional labor market policy tools in our life cycle setting. Importantly, we find the assumption of infinite-lived agents understates (overstates) the potential employment gains (costs) related to firing taxes (unemployment benefits).

Beyond its theoretical interest, we believe that this approach is able to deliver realistic empirical predictions. More particularly, we assess the ability of the model to mimic the older workers’ experience, for the French economy. This leads to taking into account the existing age-dependent policies that have been put in place in order to compensate for the fall in the older workers’ employment (specific employment protection and assistance programs). Simulation results show that our theoretical framework is able to replicate the main features of the French labor market. The distance from retirement appears as the key variable, much more than labor market institutions, to explain higher (lower) firing (hiring) rates for older workers. These empirical results confirm our basic intuition: the retirement age is the central factor to take into consideration in any analysis of older worker employment rates. On the other hand, the employment rate of younger workers is more conventionally governed by labor market institutions, because the influence of the retirement age vanishes as the distance from retirement increases. The last step of our quantitative investigation emphasizes that firms’ behaviors play a primary role in accounting for the decrease of employment at the end of the life cycle, that is, more than that of workers.

If the low employment rate of older workers can be explained by the short horizon created by impending retirement, the next issue is then to examine the social optimality of such outcomes. Before engineering any policy devices to prevent firms from discriminating against older workers, it is necessary to study the social optimality of such behaviors. By maximizing

the social welfare, we show that older workers come first (last) in the firing (hiring) process. Without any other distortions than the matching process, it is optimal to discriminate against older workers due to their impending retirement. The decentralized equilibrium even coincides with the first best outcome when the Hosios condition holds. However, what should be the age profile of firing costs and hiring subsidies when the Hosios condition no longer holds or unemployment benefits exist? As the effect of any distortion depends on the expected life-time of the match, we show that age constitutes the cornerstone of any optimal labor market policies, and in a way which is sometimes quite opposed to the sense of current OECD legislation.

The first section presents the benchmark model and the age-dynamics properties of the equilibrium. The second section addresses the issue of older workers' employment, by examining the impact of old-specific labor market policies and assessing the empirical performance of the model, based on the French experience. The last section examines the first best allocation and establishes the optimal age-profile for employment policies.

2 How Do Job Creation and Job Destruction Vary over the Life Cycle?

Let us consider an economy *à la* Mortensen - Pissarides [1994]. Labor market frictions imply that there is a costly delay in the process of filling vacancies, and endogenous job destructions closely interact with job creations. Wages are determined by a specific sharing rule of the rent generated by a job. The latter can be interpreted as the result of a bargaining between workers and employers. At this stage, no other frictions or inefficiencies are introduced.⁴

Contrary to the large literature following Mortensen - Pissarides [1994], we consider a life cycle setting characterized by a deterministic age at which workers exit the labor market. Firms are free to target their hirings by age: directed-search by age is technologically possible and legally authorized. This

⁴Unemployed workers job-search effort is discarded at this point for clarity of exposition. We present in Appendix A an extended version of the model with endogenous job-search effort for unemployed people. It allows us to assess the robustness of our results.

means that we are considering at this stage a “laissez-faire” economy.⁵

2.1 Benchmark Model Description

2.1.1 Worker Flows

We consider a discrete time model and assume that at each period the older worker generation retiring from the labor market is replaced by a younger worker generation of the same size (normalized to unity) so that there is no labor force growth in the economy. We denote i the worker’s age and T the exogenous age at which workers exit the labor market: they are both perfectly known by employers. There is no other heterogeneity across workers. The economy is at steady-state, and we do not allow for any aggregate uncertainty. We assume that each worker of the new generation enters the labor market as unemployed.

Job creation takes place when a firm and a worker meet. Firms are small and each has one job. The flows of newly created jobs result from a matching function the inputs of which are vacancies and unemployed workers. The destruction flows derive from idiosyncratic productivity shocks that hit the jobs at random. Once a shock arrives, the firm has no choice but either to continue production or to destroy the job. Then, for age $i \in (2, T - 1)$, employed workers are faced with layoffs when their job becomes unprofitable. At the beginning of each period, a job productivity ϵ is drawn in the general distribution $G(\epsilon)$ with $\epsilon \in [0, 1]$. The firms decide to close down any jobs whose productivity is below an (endogenous) productivity threshold (productivity reservation) denoted R_i .

Let u_i be the unemployment rate and v_i the vacancy rate of age i . For any age, we assume that there are matching functions that give the number of hirings as a function of the number of vacancies and the number of unemployed workers, $M(v_i, u_i)$, where M is increasing and concave in both its arguments, and with constant returns-to-scale. Let $\theta_i = v_i/u_i$ denote the

⁵If we consider *ex-ante* undirected search but *ex-post* match-specific heterogeneity, it adds some complexities but our main results should hold. Typically, the reservation productivity allowing the job to start will increase with a worker’s age, hence reducing the probability of being employed. In our equilibrium the latter result will hold due to the decrease in the number of vacancies targeted at older workers.

tightness of the labor market of age i . It is then straightforward to define the probability of filling a vacancy as $q(\theta_i) \equiv \frac{M(u_i, v_i)}{v_i}$ and the probability for unemployed workers to meet a vacancy as $p(\theta_i) \equiv \frac{M(u_i, v_i)}{u_i}$.

At the beginning of their age i , the realization of the productivity level on each job is revealed. Workers hired when they were $i - 1$ years old (at the end of the period) are now productive. Workers whose productivity is below the reservation productivity R_i are laid off. For any age i , the flow from employment to unemployment is then equal to $G(R_i)(1 - u_{i-1})$. The other workers who remain employed $(1 - G(R_i))(1 - u_{i-1})$ can renegotiate their wage. The dynamics by age of unemployment are then given by:

$$u_i = u_{i-1}(1 - p(\theta_{i-1})) + G(R_i)(1 - u_{i-1}) \quad \forall i \in (2, T - 1) \quad (1)$$

for a given initial condition $u_1 = 1$. The overall unemployment rate u is then defined by $u = \frac{\sum_{i=1}^{T-1} u_i}{T-1}$.

2.1.2 The Hiring Decision

Any firm is free to open a job vacancy and engage in hiring. c denotes the flow cost of recruiting a worker and $\beta \in [0, 1]$ the discount factor. Let V_i be the expected value of a vacant job directed to a worker of age i :

$$V_i = -c + \beta [q(\theta_i)J_{i+1}(1) + (1 - q(\theta_i))V_i]$$

where $J_i(\epsilon)$ is the expected value of a filled job by a worker of age i with idiosyncratic productivity ϵ . Following Mortensen and Pissarides, we assume that new jobs start at the highest productivity level, $\epsilon = 1$.

As $J_T(1) = 0$, no firms search for workers of age $T - 1$, that is $\theta_{T-1} = 0$. The zero-profit condition $V_i = 0 \quad \forall i \in (1, T - 2)$ allows us to determine the labor market tightness for each age θ_i from the following condition:

$$\beta J_{i+1}(1) = \frac{c}{q(\theta_i)} \quad (2)$$

As $1/q(\theta_i)$ is the expected duration of a vacancy directed to a worker of age i , the market tightness is such that the expected and discounted job value is equal to the expected cost of hiring a worker of age i .

2.1.3 The Firing Decision

For a bargained wage $w_i(\epsilon)$, the expected value $J_i(\epsilon)$ of a filled job by a worker of age i is defined by:

$$J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \max_i \{V_i\} \quad \forall i \in [1, T-1] \quad (3)$$

A first thing to note is that with probability $G(R_{i+1})$ the job is destroyed and the firm can freely choose to direct its vacant job to workers of any age. It is also worth emphasizing that the deterministic exit at age T leads to an exogenous job destruction, whatever the productivity realization: $J_T(\epsilon) = 0 \quad \forall \epsilon$.

The (endogenous) job destruction rule $J_i(\epsilon) < 0$ leads to a reservation productivity R_i defined by $J_i(R_i) = 0 \quad \forall i \in [2, T-1]$:

$$R_i = w_i(R_i) - \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) - \beta G(R_{i+1}) \max_i \{V_i\} \quad \forall i \in [2, T-1] \quad (4)$$

The higher the wage, the higher the reservation productivity, and hence the higher the job destruction flows. On the other hand, the higher the option value of filled jobs (expected gains in the future), the weaker the job destructions. Because the job value vanishes at the end of the working life, labor hoarding of older workers is less profitable. It is again worth determining the terminal age condition: $R_{T-1} = w_{T-1}(R_{T-1})$.

2.1.4 Wage Bargaining

The rent associated with a job is divided between the employer and the worker according to a wage rule. Following the most common specification, wages are determined by the Nash solution to a bargaining problem⁶.

Values of employed (on a job of productivity ϵ) and unemployed workers

⁶Recently, this wage setting rule has been somewhat disputed (See *e.g.* Shimer [2005] and Hall [2005]). We leave for future research the exploration of alternative wage rules.

of any age i , $\forall i < T$, are respectively given by:

$$\mathcal{W}_i(\epsilon) = w_i(\epsilon) + \beta \left[\int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x) dG(x) + G(R_{i+1}) \mathcal{U}_{i+1} \right] \quad (5)$$

$$\mathcal{U}_i = b + \beta [p(\theta_i) \mathcal{W}_{i+1}(1) + (1 - p(\theta_i)) \mathcal{U}_{i+1}] \quad (6)$$

with $b \geq 0$ denoting the opportunity cost of employment.⁷

For a given bargaining power of the workers, considered as constant across ages, the global surplus generated by a job, $S_i \equiv J_i(\epsilon) + W_i(\epsilon) - U_i$, is divided according to the following sharing rule:

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma [J_i(\epsilon) + \mathcal{W}_i(\epsilon) - \mathcal{U}_i] \quad (7)$$

As in MP, a crucial implication of this rule is that the job destruction is optimal not only from the firm's point of view but also from that of the worker. $J_i(R_i) = 0$ indeed entails $\mathcal{W}_i(R_i) = \mathcal{U}_i$.

According to this Nash bargaining solution, we then derive the following expression for the wage (see Appendix C for details):

$$w_i(\epsilon) = (1 - \gamma)b + \gamma(\epsilon + c\theta_i) \quad \forall i \in [1, T - 1] \quad (8)$$

This is a traditional wage equation, except that age matters through the market tightness. If this latter diminishes along the life cycle, the age profile of wages is decreasing. This could counteract the incentives for firms to fire older workers.⁸

2.2 The “*Laissez-Faire*” Equilibrium

The main objective of this section is to characterize the life cycle pattern of hirings and firings. For didactic reasons, we first rely exclusively on the firm behavior, without considering wage retroactions. Wages are assumed

⁷We assume that $\mathcal{W}_T = \mathcal{U}_T$ so that the social security provisions do not affect the wage bargaining and the labor market equilibrium.

⁸From an empirical standpoint, such a decrease in wages along the life cycle is a short-coming. A similar result holds also in MP, where wages are expected to decrease after hiring. As stated in Appendix A, it could be easily overcome by allowing for general human capital accumulation. If the worker's reservation wage and the productivity grow at the same rate over the life cycle, only the discount factor is changed at the equilibrium (see Appendix A).

to be fixed at the reservation wage level b . This “wage posting” case could be rationalized by a bargaining power for workers equal to 0 ($\gamma = 0$ in (8)). Then, we will turn to our benchmark labor market equilibrium when it allows for wages adjustments over the life cycle. The introduction of endogenous job-search effort and its implications on equilibrium job creations and job destructions will be also examined. Lastly, we will show that the age profile of employment rates is typically hump-shaped.

2.2.1 The Wage Posting Equilibrium

If wages are equal to b , the firing policy, defined by R_i , is independent of the hiring one.

Proposition 1. *If $\gamma = 0$, a labor market equilibrium with wage posting exists and it is characterized by a sequence $\{R_i, \theta_i\}$ solving:*

$$\begin{aligned} \frac{c}{q(\theta_i)} &= \beta(1 - R_{i+1}) & (JC_{PartialEq}) \\ R_i &= b - \beta \int_{R_{i+1}}^1 [1 - G(x)] dx & (JD_{PartialEq}) \end{aligned}$$

with terminal conditions $R_{T-1} = b$ and $\theta_{T-1} = 0$.

Proof. See Appendix D.1. □

It is then possible to derive the age profile of hirings and firings along the life cycle.

Property 1. $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \quad \forall i$.

Proof. See appendix D.2. □

Older workers are more vulnerable to idiosyncratic shocks. A shortened horizon relative to younger workers make them more exposed to firings. Otherwise stated, this reflects that labor-hoarding decreases with worker’s age. In turn, it creates a downward pressure on the hirings of older workers. It reinforces the decrease in hirings at the end of the working cycle due to a shortened horizon which makes vacancies unprofitable.

2.2.2 The Equilibrium with Wage Bargaining

If wages are bargained according to the equation (8), the firing policy depends now on the market tightness. The wage decrease over the life cycle is then likely to offset the direct effect of the shortening horizon on firings. This could put into question the decreasing age profile of firings, hence of hirings.

Proposition 2. *A labor market equilibrium with wage bargaining exists and it is characterized by a sequence $\{R_i, \theta_i\}$ solving:*

$$\frac{c}{q(\theta_i)} = \beta(1 - \gamma)(1 - R_{i+1}) \quad (JC)$$

$$R_i = b + \left(\frac{\gamma}{1-\gamma}\right) c\theta_i - \beta \int_{R_{i+1}}^1 [1 - G(x)] dx \quad (JD)$$

with terminal conditions $R_{T-1} = b$ and $\theta_{T-1} = 0$.

Proof. See Appendix D.3. □

Corollary 1. *Let be $M(v, u) = v^\psi u^{1-\psi}$ with $0 < \psi < 1$, and $G(\epsilon) = \epsilon$, $\forall \epsilon \in [0, 1]$, with $b \leq 1 \leq 2b/\beta$,⁹ the labor market equilibrium with wage bargaining can be summarized by a sequence $\{R_i\}_{i=2}^{T-1}$ solving:*

$$R_i = b + \left(\frac{\gamma c}{1 - \gamma}\right) \left[\frac{\beta(1 - \gamma)}{c} (1 - R_{i+1}) \right]^{\frac{1}{1-\psi}} - \frac{\beta}{2} (1 - R_{i+1})^2 \quad (9)$$

with terminal condition $R_{T-1} = b$.

Proof. Straightforward. □

The sequence of R_i is no longer necessarily monotonic. If the wage decreases sufficiently at the end of working life because of the weakness of the market tightness, then firms might be less inclined to fire these older workers.¹⁰ The following property and corollary state restrictions, implying that this indirect effect of age through wages does not dominate the direct impact of age on labor-hoarding and firing.

⁹From (9) it is straightforward to see that $b \leq 1 \leq 2b/\beta$ is a sufficient condition for an interior solution to exist ($R_i \geq 0 \forall i$).

¹⁰It is worth emphasizing that this incentive to keep older workers still remains when there is human capital accumulation that allows wages to grow over the life cycle at the same rate as productivity (see Appendix A).

Property 2.

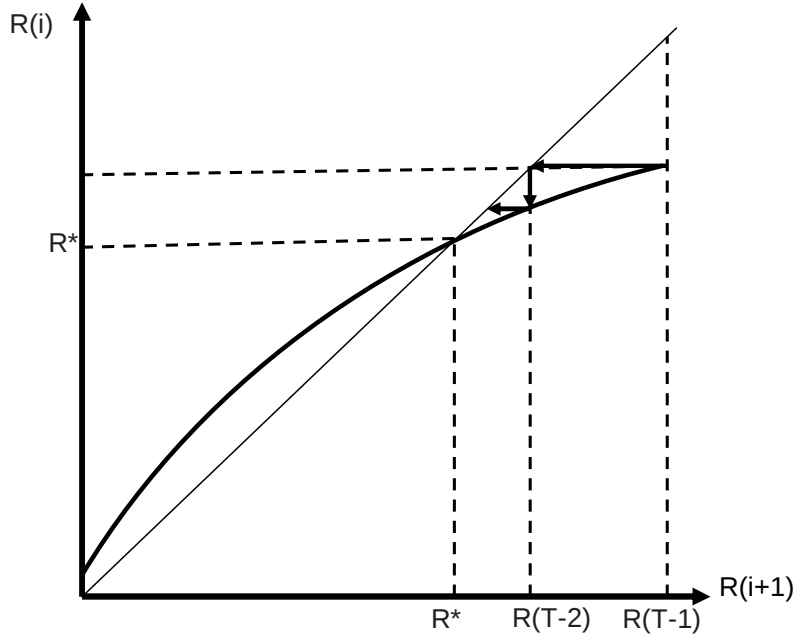
$$if \ 1 \geq \begin{cases} \frac{\gamma}{1-\psi} \left[\frac{\beta(1-\gamma)}{c} \right]^{\frac{\psi}{1-\psi}} & for \ \psi \geq 1/2 \\ 2\gamma \left[\frac{\beta(1-\gamma)}{c} \right]^{\frac{\psi}{1-\psi}} (1-b)^{\frac{2\psi-1}{1-\psi}} & for \ \psi \leq 1/2 \end{cases}$$

then the labor market equilibrium verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \ \forall i$.

Proof. See Appendix D.4. □

Figure 2 gives the age dynamics of the labor market. This figure shows that if $\frac{dR_i}{dR_{i+1}} \geq 0$ and $b \geq R_i, \forall i$, the model generates a monotonous increasing sequence of R_i . While this result is not ambiguous in the case of an exogenous wage, a parameter restriction is required when the wage derives from a Nash bargaining. The value of recruiting costs (c) is central for understanding this

Figure 2: Equilibrium dynamics of R_i



result. It determines how age influences the vacancy rate. The higher the recruiting cost, the less sensitive labor market tightness to age, the steeper the age profile of wages. If c is sufficiently high, the wage effect cannot

counteract the horizon effect on the reservation productivity: the age-profile of the firing rate is increasing.

Corollary 2. *If $\psi = 1/2$ the condition $c \geq \beta\gamma(1-\gamma)2$ ensures that the labor market equilibrium verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \quad \forall i$.*

Proof. Straightforward from Property 2 with $\psi = 1/2$. $\square$

Lastly, it should be emphasized that if Property 2 (hence Corollary 2) are not satisfied, the age dynamics of the labor market is oscillatory¹¹. In the light of empirical facts on employment, hiring and firing rates by age, we rule out such solutions of the labor market equilibrium. As it will be stated below, this is also consistent with realistic calibrations of the model.

2.2.3 The Role of the Endogenous Job-Search Effort

For didactic reasons, until now we have neglected the influence of the life cycle hypothesis on workers' job-search effort. Making the latter endogenous would actually reinforce the decrease in the employment rate at the end of working life. As the retirement age gets closer, the return on job-search investments decreases because the horizon (the expected job duration) over which they can recoup their investment is reduced. This point can easily be stated by considering the following unemployed problem to define job-search intensity:¹²

$$\mathcal{U}_i = \max_{e_i} \left\{ b - \frac{e_i^2}{2} + \beta [e_i p(\theta_i) \mathcal{W}_{i+1}(1) + (1 - e_i p(\theta_i)) \mathcal{U}_{i+1}] \right\}$$

where the labor market tightness is now defined by $\theta_i \equiv v_i / [\bar{e}_i u_i]$ with a matching function $M(v_i, \bar{e}_i u_i)$ where $\bar{e}_i$ is the average job-search effort of workers of age i .

The optimal decision rule shows that it is in the older unemployed workers' interest to reduce their job-search intensity, since the discounted sum of surplus related to employment is decreasing with age:

¹¹Proof available upon request.

¹²See the Appendix A for a detailed description of the model with a more general specification of preferences.

$$e_i = \beta p(\theta_i) [\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1}]$$

This provides an addition to existing forces that lead job creation to decrease with age. This point can be highlighted by examining the equilibrium properties with job-search effort.

Proposition 3. *A labor market equilibrium with wage bargaining and endogenous job-search effort exists and it is characterized by a sequence $\{R_i, \theta_i\}$ solving:*

$$\begin{aligned} \frac{c}{q(\theta_i)} &= \beta(1 - \gamma)(1 - R_{i+1}) & (JC_{eff}) \\ R_i &= b + \frac{1}{2} \left(\frac{\gamma}{1-\gamma} c \theta_i \right)^2 - \beta \int_{R_{i+1}}^1 [1 - G(x)] dx & (JD_{eff}) \end{aligned}$$

with terminal conditions $R_{T-1} = b$ and $\theta_{T-1} = 0$.

Proof. See Appendix D.5. □

Property 3. *Let be $M(v, eu) = v^\psi (eu)^{1-\psi}$ with $0 < \psi < 1$, and $G(\epsilon) = \epsilon$, $\forall \epsilon \in [0, 1]$, if $1 \geq \beta \frac{\gamma^2}{1-\psi} \left(\frac{\beta(1-\gamma)}{c} \right)^{\frac{2\psi}{1-\psi}}$, then the labor market equilibrium with wage bargaining and endogenous job-search effort verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \ \forall i$.*

Proof. See Appendix D.6. □

Corollary 3. *If $\psi = 1/2$ the condition $c \geq \beta\gamma(1 - \gamma)\sqrt{2\beta}$ ensures that the labor market equilibrium verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \ \forall i$.*

Proof. Straightforward from Property 3 with $\psi = 1/2$. □

This corollary shows that the restriction on c to ensure the existence of an equilibrium with $R_{i+1} \geq R_i$ is weaker than that imposed by corollary 2. This point demonstrates the positive role of job-search effort in explaining the increase (decrease) of firings (hirings) with age.

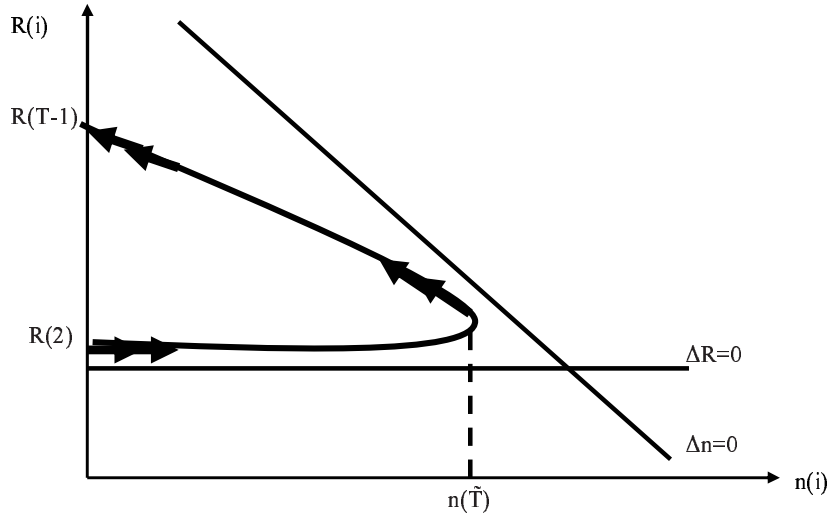
2.2.4 The Age Profile of the Employment Rate

The age profile of hirings and firings has been recursively determined from terminal conditions. On the other hand, the age profile of unemployment u_i

(or employment $n_i = 1 - u_i$) depends on the arbitrary initial condition u_1 . This explains why it is ambiguous:

$$u_i \geq \frac{G(R_{i+1})}{G(R_{i+1}) + p(\theta_i)} \Rightarrow n_{i+1} \geq n_i \quad \forall i$$

Figure 3: Equilibrium dynamics of $\{R_i, n_i\}$



Property 4. For $u_1 = 1$, there exists a threshold age $\tilde{T}$ so that $n_i \geq n_{i-1} \quad \forall i \leq \tilde{T}$ and $n_i \leq n_{i-1} \quad \forall i \geq \tilde{T}$.

Proof. See Appendix D.7. □

In the case where all the new entrants are unemployed, high vacancy rates and low firing rates at the beginning of the working life cycle make the employment rate increasing with age until the age $\tilde{T}$. Until this threshold age, this increase in employment rate is simply the result of a queue phenomenon. From $\tilde{T}$ on, the employment rate evolution by age mimics the age profile of firings and hirings. The age heterogeneity across workers then leads to a low employment rate for older workers. The overall age-dynamics of employment is thus hump-shaped, as found in OECD data.

2.2.5 The Age Profile of Wages

Since the seminal empirical work of Mincer [1962], it is well-known that the wage increases with age and declines at the end of the life cycle. This stylized fact can be explained in our model simply by including general human capital accumulation as shown in Appendix A. The wage increases with the accumulation of human capital, but the decrease of the labor market tightness slows down this growth. At the end of the life cycle, the large decrease of the tightness can dominate the dynamic of human capital accumulation.

Property 5. *Let $\psi = 1/2$, if $c > 2\beta(1 - \gamma)^2$, then $w_{i+1}(R_{i+1}) > w_i(R_i)$, $\forall i \leq \tilde{T}$.*

Proof. See Appendix D.8. □

The discrimination against the older workers leads also to another phenomenon: at the end of the life cycle, firms hoard their workers less (the reservation productivity increases with the age of the worker). Only the more productive remain at work. Property 5 illustrates this result. Accordingly, the average wage can increase with age due to a composition effect.

2.2.6 Distance from Retirement instead of Age

An important parameter of the model is the retirement age. Only the distance between the current age and the retirement age matters according to a horizon effect. On the contrary, the biological age does not matter in itself.

Property 6. *For two retirement ages, T and $T + N$, we have $R_{T-1-i} = R_{T+N-1-i}$ and $\theta_{T-1-i} = \theta_{T+N-1-i}$, $\forall i$.*

Proof. Proposition 2 and Corollary 1 clearly show that for all T , we have the same terminal condition: for two retirement ages, T and $T + N$, we have $R_{T-1} = R_{T+N-1} = b$ and $\theta_{T-1} = \theta_{T+N-1} = 0$. Then, using backward induction, the equations (JC) and (JD) show that the $R_{T-1-i} = R_{T+N-1-i}$ and $\theta_{T-1-i} = \theta_{T+N-1-i}$, $\forall i$. □

This explains why some countries experience a drop in their employment rate as soon as the age of 55 (see figure 1 in the introduction): the 55-59 years-old workers are close to their retirement age. The model is thus

able to provide some foundations to the observed OECD employment rate differences, by relying on a pure horizon effect.

2.3 Labor Market Policy Revisited in a Life Cycle Setting

In a “laissez-faire” economy, the labor hoarding strategy is less valuable for older workers. This section revisits the impact of firing taxes and unemployment benefits in a finite horizon context. It analyzes to what extent labor market policy tools either reinforce or counteract this primary mechanism at work.

We first emphasize that a constant firing tax implies a higher decrease in firings in our economy than in an infinite-lived agents’ model *à la* MP. The short horizon of older workers implies that firms can escape from the tax by waiting for the pending retirement age. Secondly, the unemployment benefit system has a lower negative impact on the employment of older workers. Indeed, the horizon during which the firms must bear higher labor costs is shorter for older workers. Overall, these results highlight that conventional labor market instruments introduce a bias in favor of the older worker employment due to a “horizon effect”. We will also show that the global impact on employment of these policies is more favorable in our life-cycle setting.

2.3.1 The Equilibrium with Unemployment Benefits and Firing Costs

Let us denote z the unemployment benefit financed by a non-distortionary tax. F denotes a firing cost which refers to the implicit costs in mandated employment protection legislation and in experience-rated unemployment insurance taxes. Furthermore, we consider a two-tier wage structure in line with Mortensen and Pissarides [1999] and Pissarides [2000].

Proposition 4. *A labor market equilibrium with wage bargaining exists and it is characterized by a sequence $\{R_i, \theta_i\}$ solving:*

$$\begin{aligned} \frac{c}{q(\theta_i)} &= \beta(1 - \gamma)(1 - R_{i+1} - F) \\ R_i &= b + z - (1 - \beta)F + \frac{\gamma c}{1 - \gamma} \theta_i - \beta \int_{R_{i+1}}^1 [1 - G(x)] dx \end{aligned}$$

with terminal conditions $R_{T-1} = b + z - F$ and $\theta_{T-1} = 0$.

Proof. See Appendix B and consider $F_i = F_{i+1} = F$, $z_i = z$ as well as $H_i = 0$. $\square$

In a MP type of economy, $T \rightarrow \infty$, and the productivity threshold and labor market tightness jump on stationary values that we denote, R and θ .¹³

Corollary 4. *If $T \rightarrow \infty$, the labor market equilibrium is characterized by $\{R, \theta\}$ solving:*

$$\begin{aligned} \frac{c}{q(\theta)} &= \beta(1 - \gamma)(1 - R - F) \\ R &= b + z - (1 - \beta)F + \frac{\gamma c}{1 - \gamma}\theta - \beta \int_R^1 [1 - G(x)] dx \end{aligned}$$

Proof. Straightforward from Proposition 4. $\square$

Corollary 5. *Let be $M(v, u) = v^\psi u^{1-\psi}$ and $G(\epsilon) = \epsilon$, when $\psi = 1/2$ the condition $c \geq \beta\gamma(1 - \gamma)2$ ensures that the labor market equilibrium with policy verifies $R_{i+1} \geq R_i$ and $\theta_{i+1} \leq \theta_i \ \forall i$.*

Proof. Similar to proof of Corollary 2. $\square$

2.3.2 The Impact of Firing Costs Revisited

We first re-examine the impact of the firing tax in our life cycle setting.

Property 7. *If Corollary 5 is satisfied, the labor market equilibrium is characterized by, $\forall i$:*

$$\begin{aligned} 0 &\geq \frac{dR}{dF} > \frac{dR_i}{dF} > \frac{dR_{i+1}}{dF} \\ 0 &\geq \frac{d\theta_{i+1}}{dF} > \frac{d\theta_i}{dF} > \frac{d\theta}{dF} \end{aligned}$$

Proof. See Appendix D.9 $\square$

¹³When $F = 0$, R is equivalent to R^* on figure 2.

Property 7 emphasizes that the older workers benefit more from the employment protection, and, by consequence, that the firing tax reduces more job destructions than in an infinite horizon economy.¹⁴

At the end of the working cycle, introducing a firing tax increases the present firing cost without any future consequences on the job value as the worker will be retired in the next period. On the other hand, in an infinite horizon, the present firing cost increases in the same proportion as in our life-cycle model, but the job value decreases, as the firm rationally expect the future cost of the firing tax. In some sense, retirement allows firms to escape from the firing tax, leading them to more labor hoarding for older workers.

By backward induction, firings of younger workers are also reduced since expected durations of jobs increase. This explains why the global effect of a firing tax on job destructions is higher in our life-cycle economy. Similarly, these longer expected durations for jobs also translate into an increase in labor market tightness. This counteracts the direct negative impact of the firing cost on job creation and this explains why $0 \geq \frac{\partial \theta_i}{\partial F} \quad \forall i > \frac{\partial \theta}{\partial F}$.

This suggests that evaluating employment protection in an infinite-lived agents context underestimates the potential positive impact on employment. Overall, however the effect of a firing tax in our life cycle setting remains ambiguous, since job creation is still negatively affected by the level of the firing costs.

2.3.3 The Impact of Unemployment Benefits Revisited

We now analyze the impact of unemployment benefits.

Property 8. *If Corollary 5 is satisfied, the labor market equilibrium is characterized by, $\forall i$:*

$$\begin{aligned} \frac{dR}{dz} &\geq \frac{dR_i}{dz} > \frac{dR_{i+1}}{dz} > 0 \\ \frac{d\theta}{dz} &\leq \frac{d\theta_i}{dz} < \frac{d\theta_{i+1}}{dz} < 0 \end{aligned}$$

¹⁴Ultimately, when $\gamma = 0$ and $\beta \rightarrow 1$, $\frac{dR}{dF} = 0$ whereas $\frac{dR_i}{dF} < 0$ (for instance $\frac{dR_{T-1}}{dF} = -1$).

Proof. See Appendix D.9 □

In our life cycle setting, unemployment benefits imply higher distortions for younger workers than for older workers. Labor hoarding strategies are indeed directly related to the sum of labor costs until retirement. The expected cost of unemployment benefits is thus higher for younger workers. This also explains why the unemployment benefit level has a larger global impact on job destruction in a MP type of economy than in our life cycle setting. In turn, the labor market tightness is less affected in our setup.

3 The Older Workers in the Spotlight

It is in the interest of firms to differentiate their hiring and firing decisions by age in a life cycle setting. It potentially delivers an original explanation for the observed decrease in the employment rate at the end of the working cycle. This section is precisely focused on older workers' employment. Beyond their proximity to retirement, the labor market of older workers is also characterized by specific policies which have been designed to compensate for age discrimination. Our life-cycle model allows us to study the theoretical impact of the age-dependent policies. They could explain a large part of the employment apart from the retirement age. In a second step, we propose a quantitative analysis to measure to what extent the distance from retirement is indeed the main feature of older workers' employment, based on the French experience.

3.1 Older Worker Policies and Labor Market Equilibrium

Faced with the low employment rate of older workers, governments have put in place different policies to counteract this trend or to compensate for it by specific assistance programs for older workers. These policies are certainly crucial to understanding the different forces at work when considering the end of the working life cycle. Our approach provides a theoretical framework to analyze their consequences by age on hirings and firings.

It is possible to distinguish two types of labor market policy oriented toward older workers.

- The first type is designed to protect them. Some policies for older workers are indeed characterized by (i) higher firing taxes (ii) higher "unemployment" benefits through specific assistance programs.

In some countries, (*e.g.* Belgium, Finland, France, Japan, Korea, Norway), it is indeed more costly for firms to lay off older workers because of longer notice periods or higher severance pay. Another important feature concerning older workers is that there exist specific inactivity and disability programs in most European countries. They provide generous substitution incomes for people eligible for these programs until retirement: this leads to an increase in the non-employment incomes. Moreover, these workers must be considered as inactive (the job-search costs are higher than for the younger) as the benefits are not conditional on a job-search activity.

- The second type of policy aims at increasing the likelihood for older workers to find a job. In the UK, firms receive a subsidy if they hire an older worker. In the US, the objective of the anti-age discrimination law is to give the same employment opportunities to all individuals, whatever their age.

3.1.1 The Equilibrium with Age-dependent Labor Market Policies

We now let z_i be the age-dependent unemployment benefit financed by a non-distortionary tax.¹⁵ F_i is the tax that the firm must pay when it fires a worker of age i , and we introduce H_i as the hiring subsidy that the firm gets when it hires a worker of age i . The equilibrium allocation with wage bargaining and endogenous search effort is now featured by (see Appendix B for derivation details):

Proposition 5. *For given sequences of policy instruments $\{H_i, F_i, z_i\}$, a labor market equilibrium with wage bargaining and endogenous search effort*

¹⁵We assume that unemployed and employed workers face an age-specific lump sum tax, X_i , so that $X_i = z_i u_i \quad \forall i$.

exists and it is characterized by a sequence $\{R_i, \theta_i\}$ solving:

$$\begin{aligned} \frac{c}{q(\theta_i)} &= \beta(1 - \gamma)(1 - R_{i+1} + H_{i+1} - F_{i+1}) & (JC_{pol}) \\ R_i &= b + z_i + \frac{1}{2} \left(\frac{\gamma c}{1 - \gamma} \theta_i \right)^2 - \beta \left[\int_{R_{i+1}}^1 [1 - G(x)] dx - F_{i+1} \right] - F_i & (JD_{pol}) \end{aligned}$$

with terminal conditions $R_{T-1} = b + z_{T-1} - F_{T-1}$ and $\theta_{T-1} = 0$.

Proof. See Appendix B. □

3.1.2 The impact of age-dependent labor market policies

From the system (JC_{pol}) -(JD_{pol}), it can easily be shown that age-specific policies have an impact on the targeted population, but also on younger agents due to the forward-looking behaviors of firms.

The impact of age-increasing unemployment benefits. The unemployment benefit (z_i) is found to exert a conventional upward pressure on wages. This leads to an increase in the productivity threshold R_i : the number of job destructions rises and the hiring rate decreases.

To get more insights into the role of the age profile of unemployment benefits, let us examine the impact of an additional unemployment income for a senior. For simplicity, let us now consider that $z_{T-1} = z$ and $z_i = 0 \ \forall i \in [2, T-2]$. Assume also $H_i = F_i = 0 \ \forall i$ and $\gamma = 0$. From proposition 5, it is straightforward to see that:

$$\begin{aligned} R_{T-1} &= b + z_{T-1} \quad ; \quad \frac{c}{q(\theta_{T-2})} = \beta(1 - b - z_{T-1}) \\ R_{T-2} &= b - \beta \int_{R_{T-1}}^1 [1 - G(x)] dx \quad ; \quad \frac{c}{q(\theta_{T-3})} = \beta(1 - R_{T-2}) \end{aligned}$$

As could be expected, this first shows that the greater generosity of assistance programs at the end of the working life increases firings and decreases hirings of older workers ($\frac{\partial R_{T-1}}{\partial z_{T-1}} > 0$ and $\frac{\partial \theta_{T-2}}{\partial z_{T-1}} > 0$). But it is also important to notice that the value of labor-hoarding for younger workers is also reduced ($\frac{\partial \beta \int_{R_{T-1}}^1 [1 - G(x)] dx}{\partial z_{T-1}} = \frac{\partial \beta \int_{R_{T-1}}^1 [1 - G(x)] dx}{\partial R_{T-1}} \times \frac{\partial R_{T-1}}{\partial z_{T-1}} < 0$). This leads to higher (lower) firing (hiring) rate for workers who are not yet eligible for the generous unemployment benefit system ($\frac{\partial R_{T-2}}{\partial z_{T-1}} > 0$ and $\frac{\partial \theta_{T-3}}{\partial z_{T-1}} < 0$).

Unambiguously, the age-increasing non-employment income implies a rise in the non-employment rate of all workers (see equation (1)). Nevertheless, the firing probability of older workers is higher than that of the younger workers because the expected effects of this policy are smaller than its instantaneous impact.

The impact of the age-increasing firing tax. Whereas F_i tends, as expected, to push down R_i by increasing the current cost of firing, F_{i+1} increases R_i by reducing the value of labor-hoarding, *i.e.* the expected future gain associated with the job (see the term in brackets, equation JD_{pol}). This suggests that job destruction is crucially related to the age profile of the employment protection, $F_{i+1} - F_i$.

More severe employment legislations protect workers who already have a job, but at the expense of those without a job. This point has already been stated theoretically by Mortensen and Pissarides [2000].

To get further insights into the role of the age profile of firing taxes, let us, for instance, examine the impact of a policy introducing an additional tax when laying off older workers. For simplicity, consider $F_{T-1} > 0$ and $F_i = 0 \quad \forall i \in [2, T-2]$. Without loss of generality, we assume again that the bargaining power of the workers is equal to zero ($\gamma = 0$). Assume also $H_i = 0$ and $z_i = z, \forall i$. From proposition 5, it is straightforward to see that:

$$\begin{aligned} R_{T-1} &= b + z - F_{T-1} \quad ; \quad \frac{c}{q(\theta_{T-2})} = \beta(1 - b - z) \\ R_{T-2} &= b + z + \beta F_{T-1} - \beta \int_{R_{T-1}}^1 [1 - G(x)] dx \quad ; \quad \frac{c}{q(\theta_{T-3})} = \beta(1 - R_{T-2}) \end{aligned}$$

This clearly shows that for workers of age $T-1$, the introduction of the tax reduces firings ($\frac{\partial R_{T-1}}{\partial F_{T-1}} < 0$) while the number of newly-hired workers who enter to employment at age $T-1$ is unchanged ($\frac{\partial \theta_{T-2}}{\partial F_{T-1}} = 0$): unambiguously, the firing tax implies an increase in the employment rate of the oldest workers (see equation (1)). On the contrary, for workers of age $T-2$, it appears that the firing probability increases and simultaneously the hiring one decreases ($\frac{\partial R_{T-2}}{\partial F_{T-1}} > 0$ and $\frac{\partial \theta_{T-3}}{\partial F_{T-1}} \equiv \frac{\partial \theta_{T-3}}{\partial R_{T-2}} \times \frac{\partial R_{T-2}}{\partial F_{T-1}} < 0$). The expected firing tax in $T-1$ indeed reduces the value of job continuation when the worker is $T-2$ years old on the labor market (less labor-hoarding). The associated

increase in productivity thresholds translates into lower expected duration of jobs, hence lower hirings: the employment rate of workers of age $T - 2$ falls unambiguously with F_{T-1} .

Overall, our framework illustrates that the aggregate employment impact of age-dependent employment protection is not determined *a priori*.

The impact of the age-increasing hiring subsidy. In the equations (JC_{pol}) and (JD_{pol}) , the hiring subsidies have a direct impact only on the job creation rule, unlike the firing taxes which affect both (JC_{pol}) and (JD_{pol}) .

Let us, for instance, examine the impact of a policy introducing an additional subsidy when hiring older workers. For simplicity, consider $H_{T-1} > 0$ and $H_i = 0 \ \forall i \in [2, T - 2]$. We assume that the bargaining power of the workers is equal to zero ($\gamma = 0$), and $F_i = 0$ and $z_i = z, \forall i$. From Proposition 5, it is straightforward to see that:

$$\begin{aligned} R_{T-1} &= b + z \quad ; \quad \frac{c}{q(\theta_{T-2})} = \beta(1 - b - z + H_{T-1}) \\ R_{T-2} &= b + z - \beta \int_{R_{T-1}}^1 [1 - G(x)] dx \quad ; \quad \frac{c}{q(\theta_{T-3})} = \beta(1 - R_{T-2}) \end{aligned}$$

It clearly appears that when the bargaining power of the workers is equal to zero, the hiring subsidy targeted for the age $T - 1$ workers has an impact only on this age group, through an increase in their hiring rate. This policy unambiguously increases the employment rate of the older workers, without introducing any distortions for the younger workers.

Nevertheless, this result is not general. Indeed, when the bargaining power of the workers is strictly positive, the higher exit rate from unemployment tomorrow pushes up the wage today and then the productivity threshold. This leads to an increase (decrease) of the firing (hiring) rate for the younger workers. This adjustment of the wage could be neglected in the analysis of the unemployment benefit and the firing tax because it amplifies the direct effects of the institutional changes. This is no longer the case for the hiring subsidies.

3.1.3 The Influence of Legislation Prohibiting Age-Discrimination

The second issue of this section is about legislation prohibiting age discrimination. This legislation in the US dates back to the 60's (Age Discrimination in Employment Act in 1967, and subsequent amendments). In 2000, the European Union Council Directive also required all 15 EU countries to introduce legislation prohibiting direct and indirect discrimination at work on the grounds of age.

To extend our benchmark economy we should notice that when age discrimination is efficiently prohibited, the job-search is undirected and there is only one labor market tightness, θ . However, there still exists distinct equilibrium wages since productivity shocks are job-specific. Lastly, under the assumption one job - one firm, it not possible to examine the question of the age-discrimination legislation related to firings.

Let consider that search effort is constant, and denote by V the value of a vacant position, value functions now solve:

$$V = -c + \beta \left[q(\theta) \sum_{i=1}^{T-2} \frac{u_i}{u} J_{i+1}(1) + (1 - q(\theta))V \right] \quad (10)$$

$$J_i(\epsilon) = \epsilon - w_i(\epsilon) + \beta \left[G(R_{i+1})V + \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) \right] \quad (11)$$

$$\mathcal{W}_i(\epsilon) = w_i(\epsilon) + \beta \left[G(R_{i+1})\mathcal{U}_{i+1} + \int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x) dG(x) \right] \quad (12)$$

$$\mathcal{U}_i = b + z + \beta [p(\theta)\mathcal{W}_{i+1}(1) + (1 - p(\theta))\mathcal{U}_{i+1}] \quad (13)$$

The free entry condition, $V = 0$, now implies:

$$\frac{c}{q(\theta)} = \beta \sum_{i=1}^{T-2} \frac{u_i}{u} J_{i+1}(1)$$

which means that the average recruiting cost equals the firm's expected gain related to the match according to the age of the worker she meets. With probability $q(\theta)$ the vacant job is filled, and with probability u_i/u it is filled with a worker of age i .

Proposition 6. *Let $G(\epsilon) = \epsilon$, $\forall \epsilon \in [0, 1]$, a labor market equilibrium with wage bargaining and age discrimination legislation exists and it is character-*

ized by a sequence $\{R_i, \theta, u_i\}$ solving:

$$\begin{aligned}\frac{c}{q(\theta)} &= \beta(1 - \gamma) \sum_{i=1}^{T-2} \frac{u_i}{u} (1 - R_{i+1}) \\ R_i &= b + z + \gamma p(\theta) \beta (1 - R_{i+1}) - \frac{\beta}{2} (1 - R_{i+1})^2 \\ u_i &= G(R_i) (1 - u_{i-1}) + (1 - p(\theta)) u_{i-1}\end{aligned}$$

with boundary conditions $R_{T-1} = b + z$ and $u_1 = 1$.

Proof. See Appendix C.3 for the equilibrium derivation. $\square$

If we assume for simplicity that the bargaining power of the workers is equal to zero ($\gamma = 0$), Proposition 6 implies that the dynamic of the productivity threshold is the same as if there were no anti-discrimination law. Then, this law has no impact on firing rates by age in this case. Turning to the hiring process, if we assume that $\theta = \theta_{T-2}$, with θ_{T-2} obtained in the model with discrimination, then $\theta > \theta_{T-1}$ since $\theta_{T-2} > \theta_{T-1}$. Hence, unambiguously, the employment rate of the older workers increases with the anti-discrimination law.

More generally, such a law imposes the same job creation rate whatever the worker's age, on the basis of the average expected gain. As a consequence, it is favorable to older workers' recruitment as regards the laissez-faire economy. Labor market tightness for older workers is indeed increased by the age discrimination legislation. But, in turn, since wages are positively related with labor market tightness, equilibrium wages and destruction rates of older workers are also increased. The overall impact on employment is thus theoretically undetermined, but one might expect that the direct impact on hirings is less than offset by the indirect impact on firings, that is, this legislation increases the employment rate of older workers.

3.2 A Quantitative Investigation: the French Older Worker Experience

The objective of this section is twofold. On the one hand, we aim at showing that the model is able to account for the observed age-dynamics of job creation and job destructions flows. On the other hand, we verify that the

feedback effect of the retirement age is crucial in the explanation of the decrease in the employment rate of older workers, relative to existing labor market institutions that are traditionally favored in any model aiming at explaining labor market flows. We completely discard the entry-stage process of the labor market and so the younger worker flows, typically focusing on employment age heterogeneity after the age of 30 in our empirical investigations.

We choose to evaluate the model on French data (males). Firstly, the retirement age is known without any uncertainty. All men exit the labor force when they reach their full pension age and they are not allowed to continue working after drawing their pension. Until recently, more than 90% of men retired at 60. All these features make France very close to our framework, which leaves aside any questions related to heterogeneity and uncertainty about the exit date from the labor force.

France is also characterized by the existence of specific and generous assistance programs for older workers. Workers are eligible for these programs in the case of layoffs, conditional on being aged more than 57. They then receive a generous income until retirement. Another important feature of the French labor market is the specific firing tax that firms incur in the case of laying off older workers (“Delalande Tax”).

3.2.1 Calibration and Quantitative Assessment

The empirical performance of the model is now evaluated on French data (male workers) by simulating the equilibrium with age policies (JC_{pol})-(JD_{pol}). The calibration is based on the period prior to 1993, that is before the Social Security reform which has introduced more heterogeneity in the retirement age. Similarly, we discard modifications to the layoff tax scheme that have been implemented from 1993 on.

We consider flows between employment and non-employment as older workers are mostly entitled to specific assistance programs which are slightly more generous than unemployment benefits.

Calibration. Specifications of functional forms for the distribution of idiosyncratic shocks and the matching function are first required. As is usually assumed, we consider an uniform distribution and a Cobb-Douglas matching function. More precisely, we set:

$$\begin{aligned} G(\epsilon) &= \epsilon \quad \forall \epsilon \in [0, 1] \\ \ln(p(\theta_i)) &= \psi \theta_i \quad 0 < \psi < 1 \end{aligned}$$

A first set of parameters $\Phi_1 = \{\beta, \psi, \gamma, F, z, F(old), z(old)\}$ is then based on external information, where F (resp. z) is the firing tax (resp. unemployment benefits) for workers less than 55 (resp. 57) years old, and $F(old)$ (resp. $z(old)$) refers to older worker-specific labor market policies for workers more than 55 (resp. 57) years old.

The discount factor β equals 0.96, which yields an annual interest rate of 4%. The elasticity of the matching function is set to the extensively-used value $\psi = 0.5$ (see, among others, Mortensen and Pissarides [2000]). The bargaining power of the workers is fixed at $\gamma = 0.3$, which is the mean of admissible values estimated on French microdata (see Abowd and Allain [1996]). Overall firing costs represent in France 15 months of average wage earnings (see Abowd and Kramarz [2003]). No evidence exists for France on the decomposition of these costs into severance transfers to workers and administrative costs. Garibaldi and Violante [2002] provide such a breakdown for Italy, whose employment protection OECD indicator is reported to be very close to France: a third of firing costs are related to administrative costs. Applying this value for the overall firing cost in France leads the layoff tax to represent in our economy 5 months of the average annual wage earnings, which is obtained by setting $F = 0.2$. For workers between 55 and 60, we also include the additional administrative cost targeted at older workers (the “Delalande tax”). This additional component represents 25% of the annual wage earnings (French law until July 1992). This leads to $F(old) = 0.32$, which implies that the overall firing tax for older workers is 8 months of the average annual wage earnings. Unemployment benefits are set to reproduce an average net replacement ratio of 55% (Martin [1996]) for workers less than 57 years old: this implies $z = 0.275$. For workers over 57, we add a premium

of 10% which reflects the fact that they are exempt from the decrease in the replacement ratio throughout the unemployment spell.¹⁶

Lastly, and due to a lack of information on the value of leisure and the flow of recruiting costs, we choose to calibrate a second set of parameters $\Phi_2 = \{b, c\}$ in order to reproduce stylized facts on the employment rate in France in 1993. Accordingly, the model will be assessed on its ability to account for job flows in and out of non-employment rather than on the replication of the age dynamics of the employment stock variable. The values of b and c are set such that the model matches the observed average employment rate for 30-44 and 55-59 year old male workers, denoted respectively $n_{(30-44)}$ and $n_{(55-59)}$. This implies $b = 0.18$ and $c = 0.46$.

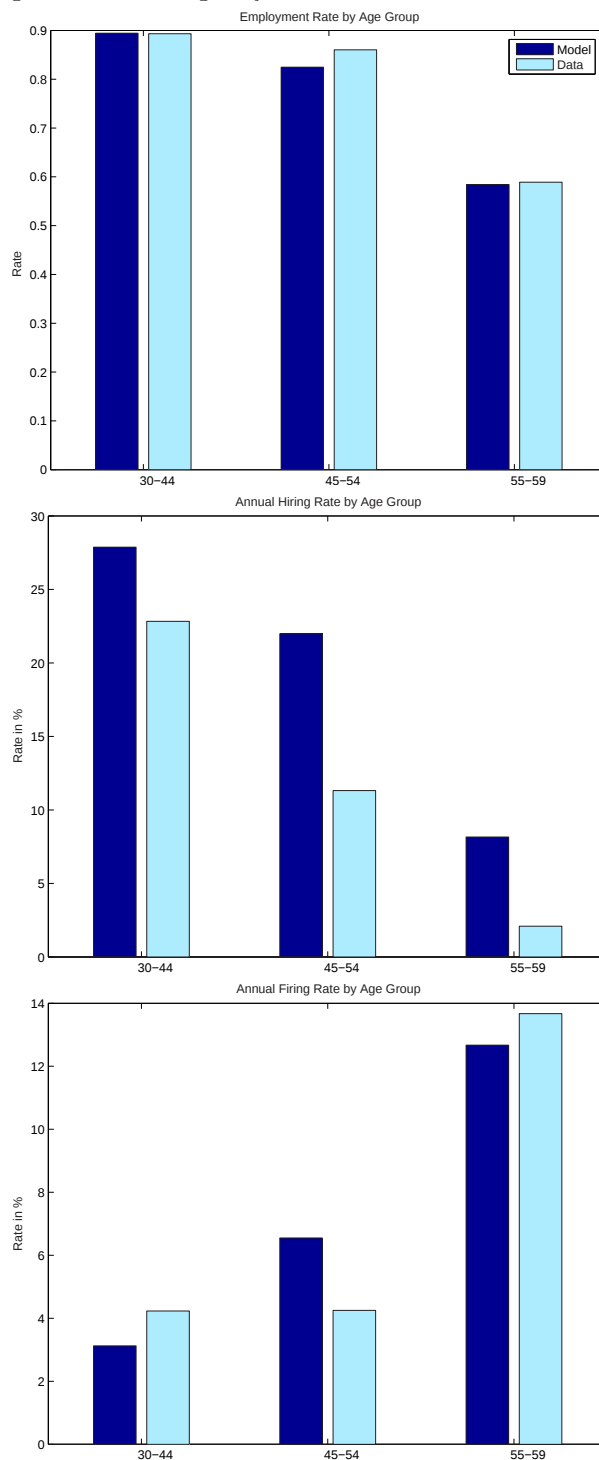
Model Assessment. Our calibration strategy hence implies that by definition the model matches the employment rate for the two age groups, 30-44 and 55-59. The employment rate of workers aged between 45 and 55 is also well replicated. The key issue about the empirical relevance of our model then relies on its ability to account for job creation and job destruction flows. The model could replicate the age-dynamics of the stock variable (employment), but understates or overstates by ten both hirings and firings. Figure 4 shows entry and exit rates for three age groups, 30-44, 45-54 and 55-59, so that our assessment is based on six moments. Interestingly, this figure shows that our model matches quite well the age-pattern of firing and hiring rates over the life cycle. However, it must be acknowledged that the levels are not perfectly reproduced. More particularly, the firing and hiring rates of the age group 45-55 are overestimated. Yet, we consider that this simple model works surprisingly well to generate the age profile of hirings and firings over the life cycle.

3.2.2 Identifying the mechanisms at work

What are the main driving forces behind the age-dynamics generated by our model? First, it is of primary interest to identify the relative importance of the age policies and the horizon effect in this result. It is often argued that

¹⁶The first year of the replacement rate is approximated around 0.65 (Martin [1996]). This implies $z(old) = 0.3025$.

Figure 4: The age-dynamics of the labor market



Data = French Labor Force Survey (males, averages over 1990-1993)

Hiring rate = outflows from non-employment / non-employment

Firing rate = inflows into non-employment / employment

when generosity of unemployment benefits is the primary cause of low employment (see Lunqvist and Sargent [2005]). Accordingly, one could expect that higher UB for workers aged 57 or more in France is the key feature at the origin of the decline in the older worker employment rate. We will show that the intrinsic horizon effect actually is dominant. Secondly, it is crucial to identify the respective contributions of the supply and demand for older workers. Which side of the labor market is the most sensitive to the horizon effect? It will be revealed that the demand side seems dominant in explaining the decrease in the employment rate when the retirement age comes closer.

Disentangling the horizon effect and the age policies. The generosity of assistance programs after 55 could explain the decline in the employment rate for these workers, but also that of the 45-54 age-groups by a feedback effect. Even if this last effect is inherent to our framework, this explanation would be a simple transposition to our life cycle model of a well-identified and classic mechanism.

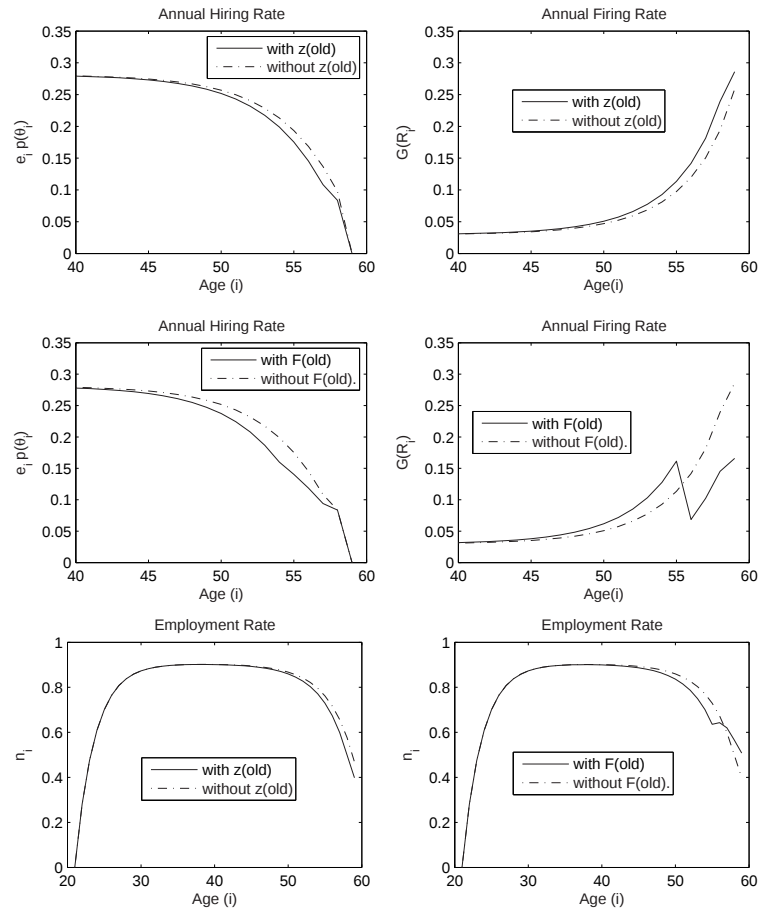
In order to identify the relative contribution of the horizon effect, we consider along our benchmark calibration the case where the unemployment benefit and/or the firing tax are constant throughout the working life cycle. Table 1 ("without age policies") gives a quantitative measure of the decrease of employment when both policies are the same for all ages. It appears that there still exists a large decrease in the employment rate after 55. Consistently, the hiring and firing rates are deeply affected by the proximity of the retirement age. Without any age policies, the decrease in the 55-59 employment rate (with resp. to 45-54) is equal to 25.8 points. These results clearly show that the "horizon effect" largely dominates the dynamics of the employment rate at the end of the life cycle.

By comparing the first and the second lines, it appears that the increasing

Table 1: Policies versus Horizon Effect

Age	Employment			Hiring			Firing		
	30-44	45-54	55-59	30-44	45-54	55-59	30-44	45-54	55-59
Without Age Policies	89.6	86.2	60.4	27.9	24.9	11.0	3.1	4.9	17.3
Without Age Firing Cost	89.6	85.2	54.2	27.6	22.0	7.3	3.3	7.3	20.3
Benchmark	89.4	82.5	58.4	27.9	22.0	8.2	3.1	6.5	12.7

Figure 5: Age-dynamics of the labor market with and without age-specific policies



profile of UB after 57 is responsible for a decrease of 6 points for the 55-59 employment rate, and also of 1 point for the 45-54 age-groups. Higher firing costs after 55 lead to better employment protection for the 55-59 years-old, who benefit from an increase in the employment rate of 4.2 points due to this legislation. However, this latter negatively affects the employment rate for workers aged 45-54 which decreases by 2.7 points. The net effect on employment rates is thus close to zero (see also figure 5). These results are consistent with a recent econometrical evaluation of the Delalande Tax (Behaghel, Crépon and Sédillot [2005]).

The respective role of the firms and the workers. If the horizon effect seems to play an important role at the end of the working life, both firms and workers are potentially responsible for this outcome. It is possible to identify their respective roles by considering economies with a constant job-search effort for workers. More precisely, we compare two economies, with and without an endogenous job-search effort at each age. In this latter case, the constant job-search effort is set to its average endogenous value over the 30-44 age range.

Table 2: Workers versus Firms

	Employment			Hiring			Firing		
	30-44	45-54	55-59	30-44	45-54	55-59	30-44	45-54	55-59
Benchmark	89.4	82.5	58.4	27.9	22.0	8.2	3.1	6.5	12.7
Benchmark $e_i = \bar{e}$	89.4	84.9	72.6	27.9	26.2	15.9	3.1	5.8	8.9
Without Age Policies	89.6	86.2	60.4	27.9	24.9	11.0	3.1	4.9	17.3
Without Age policies, $e_i = \bar{e}$	89.6	87.4	68.9	27.9	27.2	18.0	3.1	4.5	15.1

We first consider the benchmark economy (with age policies) in these two cases (the first two lines of Table 2). By comparing these two economies, we see that the employment rate of 55-59 years old workers is reduced by 12.8 points (with resp. to 45-54), instead of 24.1 points when search effort is endogenous. This suggests that more than 50% of the decrease in the employment rate is related to the firms' behavior (demand side). Moreover, the intrinsic role of the workers' behavior (supply side) is certainly over-emphasized, as the unemployment benefits are particularly generous at the end of the working life. The relative contribution of the labor demand could

be even greater when restricting our attention to the horizon effect. We now consider the two economies without age policies (the last two lines) with and without endogenous search efforts. The contribution of the firms' behavior goes up to more than 70%.

4 Efficient Job Creation and Job Destruction over the Life Cycle

Our model precisely highlights economic rationales sustaining age discrimination: there is less time over which to recoup creation costs and temporary negative productivity shocks for older workers. A *laissez-faire* equilibrium is then typically featured by job creation (destruction) rates decreasing (increasing) with age. Hence, there is age “discrimination” against older workers at the equilibrium. In some countries, policies have been implemented to sustain the demand for older workers. If the previous section has questioned their efficiency, their optimality is still an open question. This section is devoted to the analysis of this issue.

4.1 On the Optimality of Age Discrimination

We first wonder to what extent the *laissez-faire* equilibrium is optimal. We precisely show that the Hosios condition leads to equilibrium efficiency. In this particular case, the discrimination against the older workers is optimal. The intuition of the result is simple: a central planner maximising social welfare will allocate fewer resources to the hiring and the labor-hoarding of older workers, as the same phenomenon as in the decentralized equilibrium is at work: a short horizon makes less efficient any investments in older worker employment.

In line with the analysis of Pissarides [2000] with infinite-lived agents, we derive the optimal steady-state allocation by maximizing the sum of discounted output flows net of recruiting costs for a new entrant generation. This is done over the life cycle of workers¹⁷. In addition, since generations

¹⁷It is possible to show that it is equivalent to maximizing the expected gain of unemployed workers.

are independent of each other, doing this maximization for each one would lead to the same result.

The planner's problem is stated as:

$$\max_{\{R_i\}_{i=2}^{T-1}, \{\theta_i, e_i\}_{i=1}^{T-1}} \sum_{i=1}^{T-1} \beta^i \left(y_i + \left[b - \frac{1}{2} e_i^2 \right] u_i - c \theta_i u_i e_i \right) \quad (14)$$

under the constraints:

$$u_{i+1} = G(R_{i+1})(1 - u_i) + u_i (1 - e_i p(\theta_i)) \quad (15)$$

$$y_{i+1} = u_i e_i p(\theta_i) 1 + (1 - u_i) \int_{R_{i+1}}^1 \epsilon dG(\epsilon) \quad (16)$$

where y_i is the average output.

Proposition 7. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i)$ and $G(\epsilon) = \epsilon$, the efficient allocation exists and it is characterized by a sequence $\{R_i^*, \theta_i^*\}$ solving:*

$$\begin{aligned} \frac{c}{q(\theta_i^*)} &= \beta (1 - \eta(\theta_i^*)) (1 - R_{i+1}^*) & (JC^*) \\ R_i^* &= b + \frac{1}{2} \left(\frac{\eta(\theta_i^*)}{1 - \eta(\theta_i^*)} c \theta_i^* \right)^2 - \beta \int_{R_{i+1}^*}^1 [1 - G(x)] dx & (JD^*) \end{aligned}$$

with terminal conditions $R_{T-1}^* = b$ and $\theta_{T-1}^* = 0$.

Proof. See Appendix D.10. □

Property 9. *Let $\eta(\theta_i^*) = 1 - \psi \ \forall i$ and $G(\epsilon) = \epsilon$, if $1 \geq \beta(1 - \psi) \left(\frac{\beta\psi}{c} \right)^{\frac{2\psi}{1-\psi}}$, then the efficient allocation verifies $R_{i+1}^* \geq R_i^*$ and $\theta_{i+1}^* \leq \theta_i^* \ \forall i$.*

Proof. Let substitute $1 - \gamma$ by ψ in proof of property 3, and the proof is straightforward. □

These results therefore suggest that it is socially efficient to discriminate against older workers by providing them with a lower probability of hiring and a higher probability of firing.

Property 10. *Let $\eta(\theta_i^*) = 1 - \psi$, if $\gamma = 1 - \psi$ then $R_i = R_i^*$ et $\theta_i = \theta_i^* \ \forall i$.*

Proof. The proof is straightforward by substituting ψ by $1 - \gamma$ in Proposition 7 and by comparing with Proposition 2. □

As in Mortensen and Pissarides [1994], the equilibrium is in general not optimal. This is only in the case of the Hosios condition (Property 10). It is easy to show that a competitive job-search equilibrium *à la* Moen [1997] is also able to generate the socially optimal match surplus sharing rule characterized by the Hosios condition (proof available upon request). Our life cycle economy indeed does not introduce any additional source of externalities.

4.2 Optimal Age-Dependent Firing Costs and Hiring Subsidies

It is well-known that the existence of unemployment benefits distorts the *laissez-faire* equilibrium and legitimates the implementation of firing cost and hiring subsidy policies (Mortensen and Pissarides (1999)). We show in this section that the first best allocation can also be recovered in our life cycle setting by such policies, but only on condition that they are age-dependent.

Overall, we show that the age profile of hiring subsidies and firing taxes is crucially related to the value of the worker's bargaining power and the level of unemployment benefits. The intuition of these results is the following:

- Firstly, high unemployment benefits increase the labor cost and then reduce the incentives for firms to hire workers. Nevertheless the reduction of the labor demand is not the same at each age. Indeed, for an older worker, the firm only pays the tax introduced by the UB during a short period. On the other hand, for a younger worker, the firm anticipates paying this tax for a longer period. In this case the comparative advantage of the younger worker becomes a handicap. In order to restore the efficient allocation, firing taxes and hiring subsidies typically must decrease with age.
- Secondly, if the bargaining power of the workers is too low relatively to its optimal value, firms over-invest in the process of hiring the younger workers. Because of congestion effects, the rotation costs are higher than their optimal value. In order to restore the first-best allocation, firing taxes and hiring subsidies typically must increase with age.

By comparing the equilibrium allocation with the first best one, it is possible to feature the first best policies.

Proposition 8. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) = 1 - \psi$, assume a given sequence for unemployment benefits $\{z_i\}$, the optimal labor market policy is a sequence $\{H_i^*, F_i^*\}$ solving:*

$$H_{i+1}^* = F_{i+1}^* + \left[\frac{\gamma - (1 - \psi)}{(1 - \gamma)\psi} \right] \frac{c}{\beta q(\theta_i^*)} \quad (17)$$

$$F_i^* = z_i + \beta F_{i+1}^* + \left[\left(\frac{\gamma}{1 - \gamma} \right)^2 - \left(\frac{1 - \psi}{\psi} \right)^2 \right] \frac{1}{2} (c\theta_i^*)^2 \quad (18)$$

with boundary conditions $H_{T-1}^* = F_{T-1}^* = z_{T-1}$, and where θ_i^* is given by the solution of the dynamical system (JC^*) - (JD^*) .

Proof. The proof is straightforward by allowing H_i and F_i in (JC_{pol}) and (JD_{pol}) to be consistent with (JC^*) and (JD^*) . $\square$

To understand the policy implications of this proposition, we disentangle the role played by each distortion, either related to unemployment benefits or search externalities. The two following corollaries deal successively with these two sources of distortions and their respective implications on policy.

Corollary 6. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) = 1 - \psi$, assume $\gamma = 1 - \psi$ and take $z_i = z \ \forall i$ as given, the age dynamics of hiring subsidies and firing taxes is characterized by $F_i \geq F_{i+1} \geq z$ and $H_i \geq H_{i+1} \geq z$.*

Proof. The proof is straightforward by considering $\gamma = 1 - \psi$ in Proposition 8. $\square$

Assuming $\gamma = 1 - \psi$ (job-search externalities are internalized), we are focusing on the policy implications of the distortions related to unemployment compensations.

Why does the firing tax decrease with worker's age? Let us first consider a job with a worker of age $T - 1$. Correcting for an excessive wage implies $F_{T-1} = z$. With a worker of age $T - 2$, not only z but also F_{T-1} must be internalized: both z , by increasing the wage, and F_{T-1} by reducing the

value of labor-hoarding are found to increase R_i . Accordingly, $F_{T-2} > F_{T-1}$. By backward induction, it thus appears that $F_i^* = z \sum_{j=0}^{T-1-i} \beta^j$: the firing tax internalizes the sum of discounted unemployment benefits until the exit from the labor market.¹⁸ Hiring subsidies are then introduced to avoid the distortion induced by termination costs, $H_i = F_i \ \forall i$.

If we allow for some exogenous heterogeneity in unemployment benefits, that is $z_i \neq z_{i+1}$, we have $F_i^* = z_i + \beta F_{i+1}^*$. Then, if $z_i \geq z_{i+1}$, it is straightforward to see that the optimal age profile of firing taxes is still decreasing with age, $F_i^* \geq F_{i+1}^*$. On the contrary, if $z_i \leq z_{i+1}$ it can be the case that the optimal employment protection for older workers who benefit from high unemployment compensations is higher than for the younger.

Corollary 7. *Assume $z_i = 0 \ \forall i$ and Proposition 7 is satisfied, the age dependence of hiring subsidies and firing taxes is characterized by:*

$$\text{if } \gamma > 1 - \psi, \text{ then } H_i^* > H_{i+1}^* \geq 0 \text{ and } F_i^* > F_{i+1}^* \geq 0.$$

$$\text{if } \gamma < 1 - \psi, \text{ then } H_i^* < H_{i+1}^* \leq 0 \text{ and } F_i^* < F_{i+1}^* \leq 0.$$

Proof. Imposing $\theta_{i+1}^* \leq \theta_i^*$ from Proposition 7 into proposition 8, the proof is straightforward. $\square$

If $\gamma > 1 - \psi$, the worker's bargaining power is higher than its efficient value. This implies that equilibrium wages are higher than required by the optimum. Consequently, there are not enough vacancies at the equilibrium. Hiring subsidies have to be introduced in order to be consistent with $\theta_i = \theta_i^*$. But at the same time, the large value of γ together with hiring subsidies are responsible for an excessive rate of job destruction: $\frac{\gamma}{1-\gamma} c \theta_i^*$ (from (JD_{pol})) $> \frac{1-\psi}{\psi} c \theta_i^*$ (from (JD^*)). This requires a positive tax on firings. Until now, the same results would have been obtained in a Mortensen-Pissarides economy with infinite life horizon.

Our additional point is that the size of the distortions related to $\gamma \neq 1 - \psi$ is decreasing with a worker's age. This is due to $\theta_i \geq \theta_{i+1}$, which indicates

¹⁸If it was assumed that agents have an infinite life horizon on the labor market, as in Mortensen-Pissarides ($T \rightarrow \infty$), it would be straightforward to see that $F_i = z/(1-\beta) \ \forall i$.

that the wage incidence of γ is as smaller the older the worker. Ultimately, even if $\gamma > 1 - \psi$, we have $F_{T-1} = H_{T-1} = 0$ (for $z = 0$). Consistently, when $\gamma > 1 - \psi$, we find it optimal to reduce the size of employment protection and the amount of hiring subsidies as a worker's age increases.

In turn, when $\gamma < 1 - \psi$, equilibrium wages are not high enough so that it is optimal to tax hirings and simultaneously encourage firings. For the same reason as before, distortions being lower for older workers, hiring taxes and firing subsidies are optimally increasing with a worker's age.¹⁹

Overall, the age dynamics of firing taxes and hiring subsidies depend both on the value of unemployment benefits and on worker bargaining power. In particular, even though $\gamma < 1 - \psi$, it can be the case that $F_i \geq F_{i+1}$ if the value of z is high enough to have equilibrium wages higher than their efficient value. In other words, higher unemployment benefits make a decreasing profile of hiring subsidies and firing taxes by age more likely. On the contrary, if γ and z are low enough, the dynamics are reversed. Interestingly, this can easily be formally stated by considering the case $\beta \rightarrow 1$.

Corollary 8. *Let $\eta(\theta_i) = -\theta_i q'(\theta_i)/q(\theta_i) = 1 - \psi$, assume $\beta \rightarrow 1$, $\gamma < 1 - \psi$, $z_i = z \ \forall i$ and property 9 is satisfied, then*

$z \geq \tilde{z}$ is a sufficient condition for $F_i > F_{i+1} \geq z$ and $H_i > H_{i+1} \geq z$,

$z \leq \hat{z}$ is a sufficient condition for $F_i < F_{i+1} \leq z$ and $H_i < H_{i+1} \leq z$.

where $\hat{z} = \left[\left(\frac{1-\psi}{\psi} \right)^2 - \left(\frac{\gamma}{1-\gamma} \right)^2 \right] \frac{c^2}{2} \left[\frac{\psi(1-b)}{c} \right]^{\frac{2}{1-\psi}}$ and $\tilde{z} = \left[\left(\frac{1-\psi}{\psi} \right)^2 - \left(\frac{\gamma}{1-\gamma} \right)^2 \right] \frac{c^2}{2} \left[\frac{\psi}{c} \right]^{\frac{2}{1-\psi}}$.

Proof. See Appendix D.11.²⁰ □

These results should provide important insights on the optimal age-pattern of employment protection and hiring subsidies in OECD countries. In Anglosaxon countries with low unemployment benefits, the shape of the firing

¹⁹Lastly, it is worth emphasizing that if we had assigned an efficiency motive to unemployment benefits by removing redistributive considerations, it is straightforward to see from Proposition 8 that for $F_i = 0 \ \forall i$ and $\gamma > 1 - \psi$, $z_i^* \leq z_{i+1}^* \leq 0$, whereas if $\gamma < 1 - \psi$, $z_i^* \geq z_{i+1}^* \geq 0$.

²⁰If $\gamma < 1 - \psi$ and $z \in [\hat{z}, \tilde{z}]$, the age dynamics of H_i and F_i are typically non-monotonous (first increasing and then decreasing).

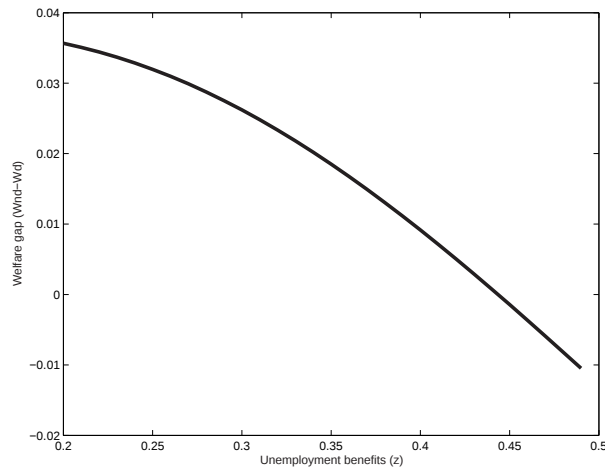
tax should be increasing, whereas the revers should hold in European countries such as in France.

4.3 The Role of Legislation Prohibiting Age Discrimination

As emphasized above, some OECD countries have more or less recently adopted a law to prohibit age discrimination. Despite its effectiveness being still controversial, it is of primary interest to examine whether this labor market policy is welfare-improving or not. Interestingly, our answer is again that it depends on the level of unemployment benefits.

Of course, this law cannot be a first best policy²¹. However, in a second best world, where job-search externalities are not internalized or unemployment benefits are paid to unemployed workers, prohibiting age discrimination can be optimal.

Figure 6: Welfare gain from age discrimination legislation



It is quite intuitive that the optimality of the law prohibiting age discrimination depends on the level of unemployment benefits. Indeed, when

²¹This can be easily demonstrated. The program of the social planner when age discrimination is prohibited incorporates a supplementary constraint, $\theta_i = \theta$.

unemployment benefits are high, wages are higher than required by the efficiency, so that employment rates are too low. The legislation is then counter-productive, since it gives more weight to less profitable workers (the older ones). On the other hand, when employment is too high (too much rotation), the legislation which reduces the expected gain from hirings is now welfare improving. Since no analytical results can be derived to state this point, we run some simulations of the dynamic system defined by Proposition 6 (the same parameters as detailed in the calibration of the French economy). The results are illustrated in Figure 6.²²

This suggests that when unemployment benefits are high, adopting a legislation prohibiting age-discrimination is welfare-degrading. This result calls into question the adoption of this legislation in some European economies such as France.

5 Conclusion

This paper puts the emphasis on a life cycle view of the labor market, both for understanding supply and demand characteristics, and for implementing welfare-improving policies. We first incorporate life-cycle features into the job creation - job destruction approach to the labor market. The equilibrium is typically featured by increasing (decreasing) firing (hiring) rates with age, and an hump-shaped age-dynamics of employment. In that context, we find that firing taxes and unemployment benefits introduce a bias in favor of older workers.

²²To compare equilibrium welfare with and without directed job-search we use the following definitions, respectively:

$$\begin{aligned}\mathcal{W}^d &= \sum_{i=1}^{T-1} \beta^i (y_i^d + bu_i^d - c\theta_i^d u_i^d) \\ \mathcal{W}^{nd} &= \sum_{i=1}^{T-1} \beta^i (y_i^{nd} + bu_i^{nd} - c\theta^{nd} u_i^d)\end{aligned}$$

where subscripts d and nd stand for the equilibrium with age discrimination (benchmark) and without age discrimination, respectively. In order to capture only the impact of a constant hiring rate implied by the age discrimination law on the labor market dynamics, we assume that the job-search effort is constant over the life-cycle.

The empirical plausibility of the model is assessed on French data by incorporating existing age-specific labor market policies. We show the primary role played by the retirement date in accounting for the observed fall in the older worker employment rate. This result relies neither on retirement programs nor on declines in productivity. It simply refers to the expected distance from retirement, that determines the expected duration of jobs and, in turn, both firms and workers' forward-looking strategies at the end of the working life.

Lastly, we emphasize that the optimal age profile of older worker policies should sharply differ among countries, according to differences in unemployment benefit institutions. While in a US-type economy hiring subsidies and firing taxes should be more favorable to older workers, the reverse holds true in European countries with high unemployment compensation. This clearly calls into question the optimality of higher employment protection for older workers and age discrimination legislation adopted in some European countries.

Several interesting aspects of a life cycle approach to the labor market have not been addressed in this paper. Our benchmark could also provide interesting insights to explain younger workers' employment from a quantitative standpoint. But it would imply modeling the specificities of the labor market at the first stages of the life cycle, for instance the imperfect information on the productive characteristics of younger workers. This line of research has, for instance, been examined by Pries and Rogerson [2005] in an infinite-lived agents context. Finally, the long run impact of the increasing weight of older workers in the labor force could be also addressed in our framework. All these topics are left for further research.

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A Extended Model with Endogenous Search Effort and Human Capital Accumulation

This section describes an extended version of our benchmark model that allows both for endogenous job-search effort and general human capital accumulation. Our objective is twofold:

- It is first to examine the robustness of our results with respect to the introduction of job-search effort.
- It is then to show that a basic incorporation of human capital accumulation reconciles the model with the hump-shaped profile of wages over the life cycle.

The matching function that gives the number of hirings, $M(v_i, \bar{e}_i u_i)$, now includes the average job-search effort $\bar{e}_i$. Accordingly, labor market tightness is defined as $\theta_i = v_i/[e_i u_i]$, so that the probability for unemployed workers to meet a vacancy is $e_i p(\theta_i) = e_i \frac{M(v_i, \bar{e}_i u_i)}{\bar{e}_i u_i}$ and the probability for a firm to fill a vacancy is $q(\theta_i) = \frac{M(v_i, \bar{e}_i u_i)}{v_i}$.

Let us consider some general preferences for the opportunity cost of employment and the search effort, $\phi(b_i, e_i)$,²³ the value of the unemployed position writes as:

$$\mathcal{U}_i = \max_{e_i \geq 0} \{ \phi(b_i, e_i) + \beta [e_i p(\theta_i) \mathcal{W}_{i+1}(1) + (1 - e_i p(\theta_i)) \mathcal{U}_{i+1}] \}$$

with $\phi'_1 > 0$, $\phi'_2 < 0$ and $\phi''_1 < 0$, $\phi''_2 > 0$. The optimal job-search effort decision rule then solves:

$$-\phi'_2(b_i, e_i) = \beta p(\theta_i) [\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1}] \quad (19)$$

General human capital accumulation over the life cycle is for further simplicity assumed to continuously increase with age, whatever the worker's

²³In the main text we consider a simpler specification, $\phi(b_i, e_i) = b_i - \frac{e_i^2}{2}$.

status on the labor market (either employed or unemployed).²⁴ Firms' value functions are now defined by:

$$\begin{aligned} V_i &= -c_i + \beta [q(\theta_i)J_{i+1}(1) + (1 - q(\theta_i))V_i] \\ J_i(\epsilon) &= h_i\epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^1 J_{i+1}(x)dG(x) + \beta G(R_{i+1}) \max_i \{V_i\} \quad \forall i \in [1, T-1] \end{aligned}$$

where h_i reflects upon the productivity impact of human capital for a worker of age i . c_i is the recruiting cost which eventually depends on the worker's age due to different levels in human capital levels (see hereafter).

In that context, the wage bargaining (see Appendix C for more details) solves:

$$w_i(\epsilon) = (1 - \gamma)\phi(b_i, e_i) + \gamma(h_i\epsilon + c_i\theta_i e_i) \quad (20)$$

Before turning to the equilibrium definition, one must provide some assumptions on laws of motion for human capital, recruiting costs and opportunity costs of employment. In a way consistent with an infinite-lived agents matching model with growth (see Pissarides [2000]), we consider a deterministic trend on human capital, and that recruiting costs and workers' reservation wage are indexed to this trend:

$$\begin{aligned} h_{i+1} &= (1 + \mu)h_i \\ c_i &= ch_i \\ \phi(b_i, e_i) &= h_i\phi(b, e_i) \end{aligned}$$

with $\mu \geq 0$, a given initial condition $h_1 = 1$ and some positive parameters $c, b \geq 0$.

²⁴A useful extension of this model would be to allow for some depreciation in human capital skill with periods of unemployment. Currently, our goal is primarily to show that human capital accumulation provides an opposite force to the age-horizon on the age dynamics of wages.

Proposition 9. *A labor market equilibrium with wage bargaining, general human capital accumulation and endogenous search effort exists and it is characterized by a sequence $\{R_i, \theta_i\}$ solving:*

$$\begin{aligned} \frac{c}{q(\theta_i)} &= \beta(1 + \mu)(1 - \gamma)(1 - R_{i+1}) & (JC_{eff}) \\ R_i &= \phi(b, \Phi(\theta_i)) + \left(\frac{\gamma}{1-\gamma}\right) c\theta_i e_i - \beta(1 + \mu) \int_{R_{i+1}}^1 [1 - G(x)] dx & (JD_{eff}) \end{aligned}$$

with terminal conditions $\theta_{T-1} = 0$ and $R_{T-1} = \phi(b, 0)$, and where $\Phi(\theta_i) \equiv -\phi_2'^{-1}\left(\frac{\gamma}{1-\gamma}c\theta_i\right)$.

Proof. Similar to the proof of Proposition 2, but by making in addition use of the fact that $-\phi_2'(b_i, e_i) = \beta p(\theta_i) [\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1}] = \frac{\gamma c_i}{1-\gamma} \theta_i$ which implies $e_i = -\phi_2'^{-1}\left(b, \frac{\gamma}{1-\gamma}c\theta_i\right) \equiv \Phi(\theta_i)$. $\square$

If $\mu = 0$, it is then possible to derive the Corollary 3 in the main text (see appendix D.6). This corollary demonstrates the positive role of the job-search effort in explaining the increase (decrease) of firings (hirings) with age.

Turning to the influence of human capital accumulation on equilibrium wages, let us remove the endogenous search effort. The equilibrium wage bargaining is:

$$w_i(\epsilon) = h_i [(1 - \gamma)b + \gamma(\epsilon + c\theta_i)]$$

Since h_i is increasing with age whereas $\theta_i \geq \theta_{i+1}$, according to the value of μ , the age dynamics profile of wages is typically hump-shaped.

B Benchmark Model with Labor Market Policy

Let us denote H_i a lump sum paid to the employer when a new worker of age i is hired, F_i the firing cost and z_i the unemployment benefits. We follow MP by considering that the wage structure that arises as a Nash bargaining solution has two tiers. The first tier wage reflects the fact that hiring subsidy is directly relevant to the decision to accept a match and that the possibility of incurring firing costs in the future affects the value the employer places

on the match. In turn, the second tier wage applies when firing costs are directly relevant to a continuation decision.

Let the subscript $i = 0$ index the initial wage and the value of a job under the terms of the two-tier contract, firms' value functions solve:

$$\begin{aligned} V_i &= -c + \beta q(\theta_i) [J_{i+1}^0 + H_{i+1}] + \beta(1 - q(\theta_i))V_i \\ J_i^0 &= 1 - w_i^0 + \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \left[\max_i \{V_i\} - F_{i+1} \right] \\ J_i(\epsilon) &= \epsilon - w_i(\epsilon) + \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) + \beta G(R_{i+1}) \left[\max_i \{V_i\} - F_{i+1} \right] \end{aligned}$$

The optimal firing decision rule now solves:

$$J_i(R_i) = -F_i$$

Adding the free entry condition, $V_i = 0$, it emerges that labor market tightness and productivity threshold are derived from the following two equations:

$$\frac{c}{q(\theta_i)} = \beta [J_{i+1}^0 + H_{i+1}] \quad (21)$$

$$R_i = w(R_i) - F_i - \beta \left[\int_{R_{i+1}}^1 J_{i+1}(x) dG(x) - G(R_{i+1}) F_{i+1} \right] \quad (22)$$

Let us now examine the derivation of the two-tier wage structure. The latter is characterized by the following two sharing rules (as a result of Nash bargaining):

$$\mathcal{W}_i(1) - \mathcal{U}_i = \gamma [J_i^0 + H_i + \mathcal{W}_i(1) - \mathcal{U}_i] \Rightarrow w_i^0 \quad (23)$$

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma [J_i(\epsilon) - (\max_i \{V_i\} - F_i) + \mathcal{W}_i(\epsilon) - \mathcal{U}_i] \Rightarrow w_i(\epsilon) \quad (24)$$

so that the equations for the initial and subsequent wage bargaining are (see Appendix C.2 for details):

$$w_i^0 = (1 - \gamma)(b + z_i) + \gamma(1 + c\theta_i + H_i - \beta F_{i+1}) \quad (25)$$

$$w_i(\epsilon) = (1 - \gamma)(b + z_i) + \gamma(\epsilon + c\theta_i + F_i - \beta F_{i+1}) \quad (26)$$

The derivation of (JC_{pol}) and (JD_{pol}) is then straightforward by noticing that $J'_{i+1}(\epsilon) = 1 - \gamma$ and $J_i(R_i) = -F_i$ implies that $J_i(\epsilon) = (1 - \gamma)(1 - R_i) - F_i$, and similarly making use of the fact that $J_i^0 = J_i^0 - J_i(R_i) - F_i$.

C Wage Equations Derivations

C.1 Wage Bargaining in a “*Laissez-Faire*” Equilibrium

Let first consider the derivation of the wage equation in a “*Laissez-Faire*” equilibrium with both endogenous job-search effort and human capital accumulation.

The sharing rule (7) can take the form:

$$-(1 - \gamma)\mathcal{U}_i = \gamma[J_i(\epsilon) + \mathcal{W}_i(\epsilon)] - \mathcal{W}_i(\epsilon) \quad (27)$$

>From value functions (3),(5) and (6), it turns out to be that:

$$\begin{aligned} \gamma[J_i(\epsilon) + \mathcal{W}_i(\epsilon)] - \mathcal{W}_i(\epsilon) &= \gamma h_i \epsilon - w_i(\epsilon) + \gamma \beta \int_{R_{i+1}}^1 [J_{i+1}(x) + \mathcal{W}_{i+1}(x)] dG(x) \\ &\quad - \beta \int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x) dG(x) \\ &\quad - (1 - \gamma)\beta G(R_{i+1})\mathcal{U}_{i+1} \end{aligned} \quad (28)$$

Similarly,

$$\begin{aligned} \gamma \beta \int_{R_{i+1}}^1 [J_{i+1}(x) + \mathcal{W}_{i+1}(x)] dG(x) &= \gamma \beta \int_{R_{i+1}}^1 [J_{i+1}(x) + \mathcal{W}_{i+1}(x) - \mathcal{U}_{i+1}] dG(x) \\ &\quad + \gamma \beta [1 - G(R_{i+1})]\mathcal{U}_{i+1} \\ \beta \int_{R_{i+1}}^1 \mathcal{W}_{i+1}(x) dG(x) &= \beta \int_{R_{i+1}}^1 [\mathcal{W}_{i+1}(x) - \mathcal{U}_{i+1}] dG(x) \\ &\quad + \beta [1 - G(R_{i+1})]\mathcal{U}_{i+1} \end{aligned}$$

Since (7) holds for each age:

$$\gamma \beta \int_{R_{i+1}}^1 [J_{i+1}(x) + \mathcal{W}_{i+1}(x) - \mathcal{U}_{i+1}] dG(x) = \beta \int_{R_{i+1}}^1 [\mathcal{W}_{i+1}(x) - \mathcal{U}_{i+1}] dG(x)$$

so that (28) can be written as:

$$\gamma [J_i(\epsilon) + \mathcal{W}_i(\epsilon)] - \mathcal{W}_i(\epsilon) = \gamma h_i \epsilon - w_i(\epsilon) - (1 - \gamma) \beta \mathcal{U}_{i+1} \quad (29)$$

In turn, the unemployed value (6) is:

$$\begin{aligned} \mathcal{U}_i &= \phi(b_i, e_i) + \beta [e_i p(\theta_i) (\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1}) + \mathcal{U}_{i+1}] \\ &= \phi(b, e_i) + \beta \left[e_i p(\theta_i) \frac{\gamma}{1 - \gamma} J_{i+1}(1) + \mathcal{U}_{i+1} \right] \end{aligned}$$

From the free entry, it derives:

$$J_{i+1}(1) = \frac{c_i v_i}{\beta M(\bar{e}_i u_i, v_i)} \equiv \frac{c_i}{q(\theta_i)}$$

so that since $p(\theta_i) = M(\bar{e}_i u_i, v_i) / [\bar{e}_i u_i]$ and $e_i = \bar{e}_i$ in equilibrium, we have:

$$(1 - \gamma) \mathcal{U}_i = (1 - \gamma) \phi(b_i, e_i) + \gamma c_i e_i \theta_i + (1 - \gamma) \beta \mathcal{U}_{i+1} \quad (30)$$

Let substitute for (29) and (30) in (27), we find:

$$-(1 - \gamma) \phi(b_i, e_i) - \gamma c_i e_i \theta_i - (1 - \gamma) \beta \mathcal{U}_{i+1} = \gamma h_i \epsilon - w_i(\epsilon) - \beta (1 - \gamma) \mathcal{U}_{i+1}$$

This lastly leads to the following wage equation:

$$w_i(\epsilon) = h_i [(1 - \gamma) \phi(b, e_i) + \gamma (\epsilon + c e_i \theta_i)]$$

Abstracting from the endogenous job-search effort and human capital accumulation, it is straightforward to get the equation (8).

C.2 Wage Bargaining with Labor Market Policy

For further simplicity, let us assume $h_{i+1} = h_i = 1$. Under this assumption, we derive the two-tier wage structure.

The sharing rule (23) first can be written as:

$$-\gamma H_i - (1 - \gamma) \mathcal{U}_i = \gamma [J_i^0 + \mathcal{W}_i(1)] - \mathcal{W}_i(1) \quad (31)$$

Following the same derivation strategy as for the case without policy, we find that:

$$\gamma [J_i^0 + \mathcal{W}_i(1)] - \mathcal{W}_i(1) = \gamma 1 - w_i^0 - (1 - \gamma)\beta \mathcal{U}_{i+1} - \gamma \beta F_{i+1} \quad (32)$$

$$(1 - \gamma)\mathcal{U}_i = (1 - \gamma)\phi(b, e_i) + \gamma c e_i \theta_i + (1 - \gamma)\beta \mathcal{U}_{i+1} \quad (33)$$

Substituting out for (32) and (33) in (31), it emerges that the initial wage solves:

$$w_i^0 = (1 - \gamma)\phi(b, e_i) + \gamma(1 + c e_i \theta_i + H_i - \beta F_{i+1})$$

Similarly, the sharing rule (24) can be written as:

$$-\gamma F_i - (1 - \gamma)\mathcal{U}_i = \gamma [J_i(\epsilon) + \mathcal{W}_i(\epsilon)] - \mathcal{W}_i(\epsilon) \quad (34)$$

This in turn leads to the following wage equation:

$$w_i(\epsilon) = (1 - \gamma)\phi(b, e_i) + \gamma(\epsilon + c e_i \theta_i + F_i - \beta F_{i+1})$$

C.3 Wage Bargaining with Age-Discrimination Legislation

From the equation (10)-(11)-(12)-(13) and the free-entry condition, it is possible to show that the total surplus related to the job match, $S_i(\epsilon) \equiv J_i(\epsilon) - V + W_i(\epsilon) - U_i$ solves

$$S_i(\epsilon) = \epsilon - b - z - \beta p(\theta)(\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1}) + \beta \int_{R_{i+1}}^1 S_{i+1}(x) dG(x)$$

In addition, since the sharing rule can be stated both as $\gamma S_i(\epsilon) = \mathcal{W}_i(\epsilon) - \mathcal{U}_i$ and $(1 - \gamma)S_i(\epsilon) = J_i(\epsilon)$, it emerges that:

$$J_i(\epsilon) = (1 - \gamma)[\epsilon - b - z - \beta p(\theta)(\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1})] + \beta \int_{R_{i+1}}^1 J_{i+1}(x) dG(x) \quad (35)$$

This entails $J_i'(\epsilon) = 1 - \gamma$, hence $J_i(\epsilon) = (1 - \gamma)(\epsilon - R_i)$ and $\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma(\epsilon - R_i)$. It is then straightforward to derive from the sharing rule the following wage equation:

$$w_i(\epsilon) = \gamma\epsilon + (1 - \gamma)(b + z) + \gamma(1 - \gamma)\beta p(\theta)(1 - R_{i+1})$$

In turn, the job destruction rule solves $S_i(R_i) = 0$,

$$R_i = b + z + \beta\gamma p(\theta)(1 - R_{i+1}) - \beta \int_{R_{i+1}}^1 [x - R_{i+1}] dG(x)$$

D Proofs of propositions, properties and corollaries

D.1 Proof of proposition 1

For $\gamma = 0$, the differentiation of (3) with respect to ϵ , implies that $J'_i(\epsilon) = 1 \ \forall i$. Since $J_i(R_i) = 0$, the value of a filled job verifies $J_i(\epsilon) = \epsilon - R_i$. The equation (2) can be written as:

$$\frac{c}{q(\theta_i)} = \beta(1 - R_{i+1})$$

This gives immediately ($JC_{PartialEq}$). Since by integrating by parts $\int_{R_{i+1}}^1 J_{i+1}(x) dG(x) = \int_{R_{i+1}}^1 J'_{i+1}(x)[1 - G(x)] dx = \int_{R_{i+1}}^1 [1 - G(x)] dx$ and $V_i = 0 \ \forall i$ (free entry condition), it is straightforward to see that the equation (4) verifies ($JD_{PartialEq}$).

D.2 Proof of property 1

The proof is straightforward. Making use of ($JD_{PartialEq}$), we indeed obtain:

$$\begin{aligned} R_{T-1} &= b \\ R_{T-2} &= b - \beta \int_{R_{T-1}}^1 [1 - G(x)] dx \leq R_{T-1} \\ R_{T-3} &= b - \beta \int_{R_{T-2}}^1 [1 - G(x)] dx \leq R_{T-2} \\ &\dots \end{aligned}$$

As can be seen in ($JC_{PartialEq}$), θ_i depends negatively on R_{i+1} , and it turns out that $\theta_{i+1} \leq \theta_i \ \forall i$.

D.3 Proof of proposition 2

By differentiating (3) with respect to ϵ , it emerges that $J'_i(\epsilon) = 1 - \gamma \ \forall i$. Since $J_i(R_i) = 0$, the value of a filled job verifies $J_i(\epsilon) = (1 - \gamma)(\epsilon - R_i)$.

The equation (2) can then be written in order to determine the sequence of θ_i as an expression of R_i (equation (JC)). Moreover, by combining the wage equation (8) and the equation (4), and integrating by parts as in proof of proposition 1, one gets the equation (JD) describing the age profile of R_i .

D.4 Proof of property 2

If $\frac{dR_i}{dR_{i+1}} \geq 0$ [condition (C1)] and $R_i \leq b$ [condition (C2)] the solution to the dynamical equation (9) necessarily verifies $R_{i+1} \geq R_i$. Given the definition of (JC) and $q'(\theta_i) \leq 0$, it then comes that $\theta_{i+1} \leq \theta_i$.

Making use of (9), we have:

$$\frac{dR_i}{dR_{i+1}} \geq 0 \iff 1 \geq \left(\frac{\gamma}{1-\psi} \right) \left[\frac{\beta(1-\gamma)}{c} \right]^{\frac{\psi}{1-\psi}} (1 - R_{i+1})^{\frac{2\psi-1}{1-\psi}} \quad (36)$$

$$R_i - b \leq 0 \iff 1 \geq 2\gamma \left[\frac{\beta(1-\gamma)}{c} \right]^{\frac{\psi}{1-\psi}} (1 - R_{i+1})^{\frac{2\psi-1}{1-\psi}} \quad (37)$$

If $\psi \geq 1/2$, evaluating (36) for $R_{i+1} = 0$ is sufficient to insure that both conditions (C1) and (C2) hold simultaneously. On the contrary, if $\psi \leq 1/2$, evaluating (37) for $\max\{R_i\} = b$ implies that (C1) and (C2) hold.

D.5 Proof of proposition 3

Similar to proof of proposition 2, but by making in addition use of the fact that:

$$\begin{aligned} e_i &= \beta p(\theta_i) [\mathcal{W}_{i+1}(1) - \mathcal{U}_{i+1}] \\ &= \frac{\gamma}{1-\gamma} c \theta_i \end{aligned} \quad (38)$$

Since the wage solves in that case the equation (20) for $\phi(b_i, e_i) = b - \frac{e_i^2}{2}$, combine this wage equation with (38) and plug into (4) we get (JD_{eff}). Proof of (JC_{eff}) is straightforward.

D.6 Proof of property 3

Following the same procedure as for the proof of Property 2, we now find that conditions (C1) and (C2) are (from Proposition 3):

$$\frac{dR_i}{dR_{i+1}} \geq 0 \iff 1 \geq \beta \frac{\gamma^2}{1-\psi} \left(\frac{\beta(1-\gamma)}{c} \right)^{\frac{2\psi}{1-\psi}} (1-R_{i+1})^{\frac{2\psi}{1-\psi}} \quad (39)$$

$$R_i - b \leq 0 \iff 1 \geq \beta \gamma^2 \left(\frac{\beta(1-\gamma)}{c} \right)^{\frac{2\psi}{1-\psi}} (1-R_{i+1})^{\frac{2\psi}{1-\psi}} \quad (40)$$

It is thus obvious that the condition (39) is more stringent than condition (40) whatever the value of ψ . Evaluating the former condition with $R_{i+1} = 0$, one gets the sufficient condition reported in the main text.

D.7 Proof of property 4

Let us denote $\Psi(R_{i+1}, \theta_i) = \frac{G(R_{i+1})}{G(R_{i+1})+p(\theta_i)}$. By definition, $\frac{\partial \Psi(R_{i+1}, \theta_i)}{\partial R_{i+1}} > 0$ and $\frac{\partial \Psi(R_{i+1}, \theta_i)}{\partial \theta_i} < 0$. For $\theta_{i+1} \leq \theta_i$ and $R_{i+1} \geq R_i$ (property 2), it thus appears that $\Psi(R_{i+1}, \theta_i) \leq \Psi(R_{i+2}, \theta_{i+1}) \forall i$.

Let us first reason by contradiction by assuming $u_1 < \Psi(R_2, \theta_1)$. Since $\Psi(R_{i+1}, \theta_i) \leq \Psi(R_{i+2}, \theta_{i+1})$ this necessarily implies $u_2 < u_1$. In turn since $\Psi(R_2, \theta_1) < \Psi(R_3, \theta_2)$, we have $u_3 < u_2$, and $\Psi(R_3, \theta_2) < \Psi(R_4, \theta_3)$... This shows that in that case we would have $n_{i+1} \leq n_i \forall i$.

On the contrary, since by definition $u_1 = 1 > \Psi(R_2, \theta_1)$, we have that $n_2 > n_1$. $\Psi(R_{i+1}, \theta_i) < \Psi(R_{i+2}, \theta_{i+1}) \forall i$ then insures that there exists an age $\tilde{T}$ which verifies $u_{\tilde{T}} = \frac{G(R_{\tilde{T}+1})}{G(R_{\tilde{T}+1})+p(\theta_{\tilde{T}})}$, and so that $n_{\tilde{T}+1} \leq n_{\tilde{T}}$.

D.8 Proof of property 5

The equation (8) gives the wage at age i for a productivity level ϵ . Then the wage gap by age, at the level of the productivity reservation by age is given by the difference of the equation (8) for age $i+1$ and age i , for the productivity R_{i+1}, R_i :

$$w_{i+1}(R_{i+1}) - w_i(R_i) = \gamma[(R_{i+1} - R_i) + c(\theta_{i+1} - \theta_i)]$$

Let substitute out from R_{i+1} (R_i) as a function of θ_{i+1} (θ_i) according to proposition 2 under the parameter restriction $\psi = 1/2$ and $G(\epsilon) = \epsilon$, we find:

$$w_{i+1}(R_{i+1}) - w_i(R_i) = \frac{c}{1-\gamma} \left[1 - \frac{c}{2\beta(1-\gamma)^2} \right] (\theta_{i+1} - \theta_i)$$

Then $w_{i+1}(R_{i+1}) - w_i(R_i) > 0$ if $1 - \frac{c}{2\beta(1-\gamma)^2} < 0$ because $\theta_{i+1} - \theta_i < 0$.

D.9 Proofs of properties 7 and 8

From proposition 4 and corollary 4, let consider $M(v, u) = v^\psi u^{1-\psi}$ with $\psi = 1/2$ and $G(\epsilon) = \epsilon$, it comes:

$$\begin{aligned} dR_i &= dz - \left[1 - \beta + \frac{2\gamma(1-\gamma)\beta^2}{c}(1 - R_{i+1} - F) \right] dF \\ &\quad + \beta dR_{i+1} \left[1 - R_{i+1} - \frac{2\beta\gamma(1-\gamma)}{c}(1 - R_{i+1} - F) \right] \\ dR_{T-1} &= dz - dF \\ dR &= dz - \left[1 - \beta + \frac{2\gamma(1-\gamma)\beta^2}{c}(1 - R - F) \right] dF \\ &\quad + \beta dR \left[1 - R - \frac{2\beta\gamma(1-\gamma)}{c}(1 - R - F) \right] \end{aligned}$$

Let examine the impact of firing cost, we have:

$$\begin{aligned} \frac{dR_{T-1}}{dF} &= -1 \\ \frac{dR}{dF} &= -\frac{1 - \beta + \frac{2\gamma(1-\gamma)\beta^2}{c}(1 - R - F)}{1 - \beta + \frac{2\gamma(1-\gamma)\beta^2}{c}(1 - R - F) + \beta R} > -1 \end{aligned}$$

so that we have $\frac{dR_{T-1}}{dF} < \frac{dR_i}{dF} < \frac{dR_2}{dF} \rightarrow \frac{dR}{dF} \leq 0$. It can be noticed that when $\beta = 1$ and $\gamma = 0$, $\frac{dR}{dF} = 0$

Similarly, the incidence of unemployment benefits is derived from:

$$\begin{aligned} \frac{dR_{T-1}}{dz} &= 1 \\ \frac{dR}{dz} &= \frac{1}{1 - \beta(1 - R) + \frac{2\gamma(1-\gamma)\beta^2}{c}(1 - R - F)} > 1 \text{ from corollary 5.} \end{aligned}$$

so that we have $0 \leq \frac{dR_{T-1}}{dz} < \frac{dR_i}{dz} < \frac{dR_2}{dz} \rightarrow \frac{dR}{dz}$.

Lastly, from corollary 5, it is straightforward to see that:

$$\begin{aligned}\frac{d\theta_i}{dF} &= -\frac{\beta(1-\gamma)}{c(1-\psi)}\theta_i^\psi \left[1 + \frac{dR_{i+1}}{dF}\right] \\ \frac{d\theta}{dF} &= -\frac{\beta(1-\gamma)}{c(1-\psi)}\theta^\psi \left[1 + \frac{dR}{dF}\right] \\ \frac{d\theta_i}{dz} &= -\frac{\beta(1-\gamma)}{c(1-\psi)}\theta_i^\psi \frac{dR_{i+1}}{dz} \\ \frac{d\theta}{dz} &= -\frac{\beta(1-\gamma)}{c(1-\psi)}\theta^\psi \frac{dR}{dz}\end{aligned}$$

Taking into consideration $\frac{dR_i}{dF} \leq \frac{dR}{dF} \leq 0$ and $0 \leq \frac{dR_i}{dz} \leq \frac{dR}{dz}$, as well as $\theta \geq \theta_i$, it comes that $\frac{d\theta_i}{dF} \geq \frac{d\theta}{dF}$ and $\frac{d\theta_i}{dz} \geq \frac{d\theta}{dz}$.

D.10 Proof of proposition 7

Let us denote λ_i and μ_i the Lagrange multiplier associated with constraints (15) and (16), optimal decision rules with respect to R_{i+1}, θ_i and u_i, y_i are respectively given by:

$$\begin{aligned}\lambda_i &= \mu_i R_{i+1} \\ \beta^i c &= (\mu_i - \lambda_i) p'(\theta_i) \\ \beta^i (e_i + c\theta_i) &= p(\theta_i) (\mu_i - \lambda_i) \\ \lambda_{i-1} &= \beta^i \left(b - \frac{1}{2}e_i^2 - c\theta_i e_i \right) + \lambda_i [1 - e_i p(\theta_i) - G(R_{i+1})] \\ &\quad + \mu_i \left[e_i p(\theta_i) - \int_{R_{i+1}}^1 \epsilon dG(\epsilon) \right] \\ \mu_i &= \beta^{i+1}\end{aligned}$$

It is then possible to derive the following expression of the productivity threshold:

$$R_i = b - \frac{1}{2}e_i^2 - c\theta_i e_i - \beta \int_{R_{i+1}}^1 \epsilon dG(\epsilon) + c \frac{e_i p(\theta_i)}{p'(\theta_i)} + \beta R_{i+1} (1 - G(R_{i+1}))$$

Let us remark that $c\theta_i + e_i = cp(\theta_i)/p'(\theta_i)$ implies $e_i = c \left[\frac{p(\theta_i)}{p'(\theta_i)} - \theta_i \right] = c\theta_i \frac{\eta(\theta_i)}{1-\eta(\theta_i)}$ (since $p(\theta_i)/p'(\theta_i) = \frac{\theta_i}{1-\eta(\theta_i)}$) and $\beta R_{i+1}(1-G(R_{i+1})) - \beta \int_{R_{i+1}}^1 \epsilon dG(\epsilon) = -\beta \int_{R_{i+1}}^1 (\epsilon - R_{i+1}) dG(\epsilon) = \int_{R_{i+1}}^1 [1-G(x)] dx$, one gets (JD^*) . The derivation of (JC^*) is straightforward from the optimality conditions of the planner's problem.

D.11 Proof of corollary 8

From proposition 8, we have that the optimal age profile of firings solve for $\beta \rightarrow 1$:

$$F_i - F_{i+1} = \left[\left(\frac{\gamma}{1-\gamma} \right)^2 - \left(\frac{1-\psi}{\psi} \right)^2 \right] \frac{1}{2} (c\theta_i)^2 - z$$

Accordingly, from (JC^*) since $\theta_i \leq \left[\frac{\psi}{c} \right]^{\frac{1}{1-\psi}} \quad \forall i, z \geq \left[\left(\frac{1-\psi}{\psi} \right)^2 - \left(\frac{\gamma}{1-\gamma} \right)^2 \right] \frac{c^2}{2} \left[\frac{\psi}{c} \right]^{\frac{2}{1-\psi}}$ insures that $F_i > F_{i+1}$. In turn, since for $i \in [1, T-2]$, $\theta_i \geq \left[\frac{\psi(1-b)}{c} \right]^{\frac{1}{1-\psi}}$, it emerges that $z \leq \left[\left(\frac{1-\psi}{\psi} \right)^2 - \left(\frac{\gamma}{1-\gamma} \right)^2 \right] \frac{c^2}{2} \left[\frac{\psi(1-b)}{c} \right]^{\frac{2}{1-\psi}}$ insures that $F_i < F_{i+1}$.