

Credit and Inflation under Borrowers' Lack of Commitment *

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Abstract

In this paper we investigate the existence of credit in a cash-in-advance economy where there are complete markets but for the fact that agents cannot commit to repay their debts. The key feature of our model is that, in the case of default, although agents are banned from the credit market, they cannot be seized their money balances. In an economy without uncertainty if the government follows the Friedman rule at the steady state agents save money to attain a completely smooth consumption path but there is no credit. If the inflation rate is positive there is credit and if it is sufficiently high agents are able to smooth completely their consumption path.

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1 Introduction

There is an extensive literature that studies the implications of limited commitment for risk sharing across consumers and across countries. See, for instance, Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jermann (2000), Kehoe and Levine (2001), Kehoe and Perri (2004), or Krueger and Perri (2006). This literature emphasizes that if agents cannot commit to repay their debts the equilibrium amount of credit available in an economy will be lower than the efficient amount and full risk sharing is not possible. This literature has proven to be useful to understand the fact that both individual consumers and countries bear more idiosyncratic risk than is consistent with complete and frictionless Arrow-Debreu markets.

Here we want to study the relationship between fiat money and credit. The real return of each type of asset and the volume of credit will be interconnected. As Shubik (1999) points out, the value of fiat money depends on the agents' ability to commit to repay their debts. Here we study the reverse connection. We want to explore how the return to fiat money affects the agents' ability to commit. In order to do so we assume that when an agent declares bankruptcy he cannot be seized its money balances. Thus, as in Kehoe and Levine (1993), if an agent defaults on a contract, he can be excluded from future contingent claims markets trading and has his assets seized but its money holdings. Thus, he cannot be excluded from spot market trading. Thus, the return to money affects the return of default and, therefore, the equilibrium amount of credit.

We build a simple economy along the lines of Kehoe and Levine (2001) and introduce a cash-in-advance constraint as in Svensson (1989). We assume complete information. Households need money for transaction purposes and can borrow from others. In this type of economy an external agency is needed to enforce contracts. Thus, we assume the existence of financial intermediaries that charges the same interest rate to borrowers and lenders. Nevertheless, since there is complete information, it lends to borrowers the amount they can commit to repay. The type of situations we have in mind are those in which individual consumers can successfully hide their money holdings from the bankruptcy authority and also countries. Specifically, our results apply to countries within a monetary union.

We find that, in the case of no uncertainty, there is no credit unless the the inflation rate is positive. That is, if the inflation rate is non positive agent who hold debts always would choose to default on them. As a result, credit cannot exist. Thus, households could only use money to

smooth their consumption path. Nevertheless, under the Friedman rule they can attain a completely smooth, efficient, consumption path. We show that there exists a level of positive inflation for which households can attain the efficient allocation with credit. For any positive inflation rate below this threshold the volume of credit increases with inflation, as well as the real return to bonds.

We next turn to analyze a stochastic version of our benchmark model economy. There the equilibrium is not well defined under the Friedman rule. The reason is already shown by Bewley, (1983, 1984) and Aiyagari (1994). In that case the return to money would be equal to the household's discount rate and households would like to hold an infinite amount of assets. Nevertheless, we can show that near the Friedman rule credit cannot exist for sufficiently impatient households.

Our paper is close in spirit to Aiyagari and Williamson (2000). They build a private information model in which households hold money because a limited participation constraint and cannot commit to repay their debts. Thus, households hold money because they are randomly denied access to credit markets. As in our setup, households keep their money balances if they declare bankruptcy. Here the gain of increasing inflation is lower than in our setup since inflation lowers the welfare of those households denied access to credit even they want to repay their debts.

2 The benchmark model economy

2.1 Population, preferences, endowments and production possibilities.

There are an infinite number of discrete time periods $t = 0, 1, \dots$. In each period there are two types of households $i = 1, 2$ and a continuum of each type of households. The measure of each type of households is one half. There is a single consumption good c ; the representative household of type i consumes c_t^i in period t . Both types of households derive utility from consumption and do not value leisure. We write lifetime utility as $\sum_{t=0}^{\infty} \beta^t u(c_t)$. The period utility function is twice continuously differentiable with $Du(c) > 0$, satisfies the boundary condition $Du(c) \rightarrow \infty$ as $c \rightarrow 0$ and has $D^2u(c) < 0$. The discount factor satisfies $0 < \beta < 1$.

Households are endowed with one unit of time each period. Each period households receive a shock to their efficiency units of labor $w \in \{w_l, w_h\}$. We assume that when households of type 1 receive the shock w_l households of type 2 receive the shock w_h . We start by assuming that

productivity alternates between good and bad, so if $w_t^i = w_h$ then $w_{t+1}^i = w_l$. To ensure that trade is welfare improving we assume that $(1 + \beta)u\left(\frac{w_h + w_l}{2}\right) > u(w_h) + \beta u(w_l)$. Furthermore, we assume that $w_j > 0$, $j = h, l$.

The production of the unique consumption good requires labor. The production function is linear, $Y = E$, where E is aggregate labor.

2.2 Market arrangements

We assume that households need money for transaction purposes and, as in Svensson (1989), that consumption expenditures are determined by the amount of money balances held at the end of the previous period. In addition to money households can hold bonds, which can be used for borrowing and lending purposes. The timing of the model is as follows: people work, consume, they are paid their return to their labor endowments and assets and decide their next period wealth as well as the composition of their portfolio.

Households cannot commit to repay their debts. In the event of default they are seized their bond holdings are banned from the credit market forever. However, they cannot be seized their real money balances. There are financial institutions that take bonds from savers and lend to borrowers. We assume that financial institutions are perfectly competitive and that they cannot price discriminate in the sense that they charge the same interest rate to any household. Moreover, we assume that there is perfect entry and exit in the financial sector. As Alvarez and Jermann (2000) show, in absence of private information, all these assumptions imply that financial institutions will set a borrowing limit (a solvency constraint) so that all households are not worse off being in trade than defaulting. That is, there will not be default in equilibrium.

2.3 The government

The government injects money in the economy as lump-sum transfers to agents. The stock of money supply evolves according to the law

$$M_{t+1} = (1 + \theta) M_t. \tag{1}$$

It will be useful to express the law of motion of the aggregate supply of real money balances in per capita terms

$$\mathbf{m}_{t+1} = \frac{1 + \theta}{1 + \varepsilon_{t+1}} \mathbf{m}_t, \quad (2)$$

where ε_{t+1} is the inflation rate at period $t + 1$.

2.4 The household's problem

The problem solved at period 0 by household of type i is

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t^i) \\ \text{s. t.} \quad & p_t c_t^i + q_t^n A_{t+1}^i + M_{t+1}^i \leq p_t w_t^i + A_t^i + M_t^i + T_{t+1}^i, \\ & p_t c_t^i \leq M_t^i, \\ & \frac{A_t^i}{p_t} \geq \underline{b}. \end{aligned} \quad (3)$$

Where A_t^i denotes nominal bond holdings at the beginning of period t , M_t^i stands for money balances at the beginning of period t and T_{t+1}^i is the money injected by the government as transfers.

2.5 The credit market

The borrowing limit $\underline{b}$ set every period satisfies

$$\sum_{t=s}^{\infty} \beta^{t-s} u(c_t^i) \geq \sum_{t=s}^{\infty} \beta^{t-s} u(c_t^{i,D}), \text{ for all } s, \quad (4)$$

where $\{c_t^{i,D}\}_{t=s}^{\infty}$, for all $s = 0, \dots$, is the allocation that solves the household problem in the event of default,

$$\begin{aligned} \sum_{t=0}^{\infty} \beta^t u(c_t^{i,D}) &= \max \sum_{t=0}^{\infty} \beta^t u(c_t^i) \\ \text{s. t.} \quad & p_t c_t^i + M_{t+1}^i \leq p_t w_t^i + M_t^i + T_{t+1}^i, \\ & p_t c_t^i \leq M_t^i. \end{aligned} \quad (5)$$

That is, the value of continuing to participate in the economy is no less than the value of dropping out. In this setting, the absence of private information implies that no household actually goes bankrupt in equilibrium: the credit agency will never lend so much to consumers that they will choose bankruptcy. We are going to call expression (4) the incentive compatibility constraint. That is, financial intermediaries choose the maximum amount of credit so that is incentive compatible: households are better off repaying their debts than defaulting on them.

2.6 Cash-in-advance constraint and precautionary money balances

In order to write the problem in real terms we are going to denote as m_t^i real money balances at the beginning of period t . Thus, the budget constraint and the cash-in-advance constraint can be written as

$$\begin{aligned} c_t^i + q_t b_{t+1}^i + (1 + \varepsilon_{t+1}) m_{t+1}^i &\leq w_t^i + b_t^i + m_t^i + (1 + \varepsilon_{t+1}) \tau_{t+1}^i, \\ c_t^i &\leq m_t^i, \end{aligned} \tag{6}$$

where $b_t^i = A_t^i/p_t$, $1 + \varepsilon_{t+1} = p_{t+1}/p_t$, and $q_t = (1 + \varepsilon_{t+1}) q_t^n$. Notice that the difference $m_t^i - c_t^i$ could be thought of as the precautionary money balances held at the beginning of period t .¹ Thus, we are going to denote it as d_t^i . In real terms the budget constraint and the cash-in-advance constraint can be written as

$$\begin{aligned} (1 + \varepsilon_{t+1}) c_{t+1}^i + q_t b_{t+1}^i + (1 + \varepsilon_{t+1}) d_{t+1}^i &\leq w_t^i + b_t^i + d_t^i + (1 + \varepsilon_{t+1}) \tau_{t+1}^i, \\ d_{t+1}^i &\geq 0, \end{aligned} \tag{7}$$

This formulation shows clearly that choosing next period real money balances amounts to choosing next period consumption and next period precautionary real money balances. It also shows that consumption at period $t + 1$ depends on the current state. Thus we are going to proceed to define a recursive stationary equilibrium.

¹Using the term precautionary may seem incorrect since there is no uncertainty. Nevertheless, since we will study the case with uncertainty we prefer to abuse language a bit instead changing names.

2.7 Recursive stationary equilibrium

In a steady state the inflation rate is constant and equal to the money growth rate, θ . Since there is an equal measure of each type of households, the aggregate level of production stays constant and is equal to $\bar{w} = (w_h + w_l) / 2$. For simplicity, we are going to assume that the money transfers are non redistributive,

$$(1 + \theta) \tau_{t+1}^i = \frac{w_t^i}{\bar{w}} \theta \mathbf{m}. \quad (8)$$

Moreover, the per capita amount of money balances $\mathbf{m}_t$ is equal to the aggregate consumption, which equals the aggregate endowment plus the aggregate precautionary money balances, $\mathbf{m}_t = \bar{w} + d_t^1 / 2 + d_t^2 / 2$. Thus, aggregate money balances and prices will only depend on the monetary policy, that is completely summarized by θ , and the borrowing limit $\underline{b}$ set by the financial intermediaries. The measure of households over the state space is μ .

The household's problem

The individual state variable is the variable $x = \{j, b, d\}$, where j denotes the household's productivity level, $j = h, l$. The problem solved by a household of productivity level i stated recursively is

$$\begin{aligned} V(j, b, d; \underline{b}, \theta) = & \max_{\substack{c' \geq 0, \\ b' \geq \underline{b}, \\ d' \geq 0}} \{u(c') + \beta V(-j, b', d'; \underline{b}, \theta)\} \\ \text{s. t.} \quad & (1 + \theta) c' + q(\underline{b}, \theta) b' + (1 + \theta) d' \leq w_j + b + d + \frac{w_j}{\bar{w}} \theta \mathbf{m}(\underline{b}, \theta), \\ & \text{if } j = h, \text{ then } -j = l, \text{ and vice versa.} \end{aligned} \quad (9)$$

The maximum amount of credit

The limit $\underline{b}$ is set such that the incentive compatibility constraint is satisfied,

$$V(j, b, d; \underline{b}, \theta) \geq V_D(j, d; \theta), \quad \text{for all } b \geq \underline{b}, j = h, l, d \in \mathbb{R}_+, \text{ given } \theta, \quad (10)$$

where $V_D(j, d; \theta)$ satisfies

$$\begin{aligned}
V_D(j, d; \theta) &= \max_{\substack{c' \geq 0, \\ d' \geq 0}} \{u(c') + \beta V_D(-j, d'; \theta)\} \\
\text{s. t.} \quad &(1 + \theta)c' + (1 + \theta)d' \geq w_j + d + \frac{w_j}{w} \theta \mathbf{m}(\underline{b}, \theta), \\
&\text{if } j = h, \text{ then } -j = l, \text{ and vice versa.}
\end{aligned} \tag{11}$$

Steady state equilibrium

Definition 1. A limited borrowing steady state equilibrium for this economy is a monetary policy, θ , a borrowing limit $\underline{b}$, an aggregate amount of money balances, $\mathbf{m}(\underline{b}, \theta)$, a price for bonds, $q(\underline{b}, \theta)$, a set of functions $\left\{V(x; \underline{b}, \theta), g^c(x; \underline{b}, \theta), g^b(x; \underline{b}, \theta), g^d(x; \underline{b}, \theta)\right\}$, and a measure of households μ such that:

1. given μ, q , the monetary policy, θ , and $\mathbf{m}(\underline{b}, \theta)$, the functions $\left\{V(x; \underline{b}, \theta), g^c(x; \underline{b}, \theta), g^b(x; \underline{b}, \theta), g^d(x; \underline{b}, \theta)\right\}$ solve the household's problem,

2. markets clear:

$$\begin{aligned}
(a) \quad &\int_X g^c(x) d\mu = \int_X w d\mu, \\
(b) \quad &\int_X g^b(x) d\mu = 0, \\
(c) \quad &\mathbf{m}(\underline{b}, \theta) = \int_X (g^d(x) + g^c(s)) d\mu,
\end{aligned}$$

3. the measure of household is stationary, $x(B) = \int_X P(x, B) d\mu$, for all $B \subset \mathcal{B}$.

Definition 2. A limited borrowing steady state is incentive compatible if $V(j, \underline{b}, d; \underline{b}, \theta) \geq V_D(j, d; \theta)$, for all $j = h, l, d \in \mathbb{R}_+$, given θ , where the functions $\left\{V_D(j, d; \theta), f^d(j, d; \theta)\right\}$ solve the default problem given $\mu, q(\underline{b}, \theta), \mathbf{m}(\underline{b}, \theta)$, and the monetary policy θ .

We are going to examine symmetric incentive compatible steady states. In a symmetric steady state the level of consumption of each household only depends on its level of productivity and not on its type. That is, $c_t^i = c_h$ if $w_t^i = w_h$ and $c_t^i = c_l$ if $w_t^i = w_l$. Thus, the equilibrium bond and

precautionary money holdings satisfy

$$b_j = g^b(-j, b_{-j}, d_{-j}, j; \underline{b}, \theta), \quad j = h, l, \quad (12)$$

$$b_h + b_l = 0, \quad (13)$$

$$d_j = g^d(-j, b_{-j}, d_{-j}, j; \underline{b}, \theta), \quad j = h, l, \quad (14)$$

where $g^b(\cdot)$ and $g^d(\cdot)$ are the policy functions for bonds and precautionary money balances, respectively. This system of equations determines an equilibrium level of credit and precautionary money balances that we will denote as $b_j(\underline{b}, \theta)$ and $d_j(\underline{b}, \theta)$, $j = h, l$.

3 Efficiency and full commitment

In this section we investigate the existence of credit when borrowers can commit to repay their debts.

3.1 The efficient allocation

The efficient allocation is the one that solves the social planner's problem

$$\begin{aligned} \max_{c_t^1, c_t^2} \quad & \frac{1}{2} \sum_{t=0}^{\infty} \beta^t u(c_t^1) + \frac{1}{2} \sum_{t=0}^{\infty} \beta^t u(c_t^2) \\ \text{s. t.} \quad & \frac{c_t^1}{2} + \frac{c_t^2}{2} = \overline{w}. \end{aligned} \quad (15)$$

Denoting as c_h the consumption level when a household of type i has the high productivity shock and c_l when it is low we can simplify the social planner's problem to the static problem

$$\begin{aligned} \max_{c_h, c_l} \quad & \frac{1}{2} u(c_h) + \frac{1}{2} u(c_l) \\ \text{s. t.} \quad & \frac{c_h}{2} + \frac{c_l}{2} = \overline{w}, \end{aligned} \quad (16)$$

where it is easy to see that the efficient allocation entails full risk sharing, $c_h = c_l = \overline{w}$. Thus, efficiency implies that household's consumption is invariant with respect to household's income. Now we turn to the decentralization of the efficient allocation.

3.2 Equilibrium under full commitment

The symmetric efficient allocation can be decentralized under full commitment. Solving the problem of full commitment only requires to drop the borrowing constraint and the incentive compatibility constraint. A symmetric steady state equilibria is characterized by the following set of equations

$$q u'(c_j) = \beta u'(c_{-j}), \quad j = h, l, \quad (17)$$

$$(1 + \theta) u'(c_j) \geq \beta u'(c_{-j}), \quad j = h, l, \quad (18)$$

$$(1 + \theta) c_{-j} + q b_{-j} + (1 + \theta) d_{-j} = \left(1 + \theta \frac{\mathbf{m}}{\bar{w}}\right) w_j + b_j + d_j, \quad j = h, l, \quad (19)$$

$$\frac{1}{2} c_h + \frac{1}{2} c_l = \bar{w}, \quad (20)$$

$$\bar{w} + \frac{1}{2} d_h + \frac{1}{2} d_l = \mathbf{m}. \quad (21)$$

Equations (17) and (20) imply that the consumption equilibrium allocation is the efficient allocation, $c_h = c_l = \bar{w}$, and that the price of the bond satisfies $q = \beta$. That is, the nominal gross return of the bond is $(1 + \theta)/\beta$ whereas its real return is $1/\beta$ and the real return to money is $1/(1 + \theta)$. Thus, although the consumption allocation does not depend on inflation the household's portfolio does. There are two possible cases. Either $1 + \theta > \beta$, case in which households do not want to keep precautionary money balances and $1 + \theta = \beta$, case in which they are indifferent. We are going to review each case.

Equilibrium without credit

For the efficient allocation to be decentralized without credit the nominal return to bonds have to be zero, $\beta = 1 + \theta$, that is the Friedman rule must hold. In this case, the full risk sharing allocation can be decentralized with households only keeping precautionary money balances. The portfolio composition is

$$d_l = \frac{\beta \bar{w} (w_h - w_l)}{w_h + (1 + \beta) w_l}, \quad (22)$$

$$d_h = 0, \quad (23)$$

$$b_h = b_l = 0, \quad (24)$$

$$\mathbf{m} = \bar{w} + \frac{1}{2} d_l. \quad (25)$$

Thus, it is possible to sustain the full risk sharing allocation only with money as long as the Friedman rule holds. In any other case, $\beta < 1 + \theta$, credit is needed to decentralize the full risk sharing allocation. Let us turn to analyze the decentralization with credit.

Equilibrium with credit and the borrowing limit

In the case in which $1 + \theta > \beta$ the nominal return to bonds is positive and households do not want to hold precautionary money balances. This also implies that the total amount of money has to coincide with total consumption, $\mathbf{m} = \bar{w}$. In the case in which $1 + \theta = \beta$ the household is indifferent about the composition of its portfolio. In this case, the maximum amount of debt occurs when agents hold no precautionary money balances. We call this equilibrium the *credit equilibrium*. The household's portfolio is

$$b_h^E = -\frac{(1 + \theta)}{1 + \beta} (\bar{w} - w_l), \quad (26)$$

$$b_l^E = -b_h^E, \quad (27)$$

$$d_h^E = d_l^E = 0. \quad (28)$$

Notice that b_h^E is negative. This implies that the household with high productivity has a beginning of period negative position in bonds. That is, if households cannot commit to repay their debts households with high productivity may have incentive to default on their debts. We will come back to this issue in the following section.

4 The incentive compatible steady state

The section is organized in the following way. We first characterize symmetric steady states for a given level of inflation and a borrowing limit. Then we investigate which of those equilibria are incentive compatible in the sense specified in section 2.7.

4.1 Characterization of the symmetric steady state

The steady state allocation is characterized by

$$q u'(c_j) \geq \beta u'(c_{-j}), \quad j = h, l, \quad (29)$$

and equations (18) to (21). Notice that if the borrowing constraint is not binding the equilibrium allocation must be the full commitment allocation. Thus, the amount of borrowing in the equilibrium with full commitment b_h^E is the natural debt limit below which no borrowing constraint is binding. The following propositions characterize the equilibrium allocation for any borrowing constraint tighter than the natural debt limit.

Proposition 1. *Given a monetary policy θ such that $1 + \theta = \beta$, and a borrowing limit that satisfies $\underline{b} > b_h^E$, then there exists a continuum of equilibria for which the equilibrium price of bonds satisfies $q(\theta, \underline{b}) = \beta$, the consumption allocation is $c_h = c_l = \bar{w}$, its amount of borrowing satisfies $b_h \in (\underline{b}, 0]$, $b_l = -b_h$, and its money holdings satisfy*

$$\beta \bar{w} + \beta (b_h + d_h) = \left(1 + (\beta - 1) \frac{\mathbf{m}}{\bar{w}}\right) w_l - b_h + d_l, \quad d_h \geq 0, d_l \geq 0, \quad (30)$$

$$\mathbf{m} = \bar{w} + \frac{1}{2} d_h + \frac{1}{2} d_l. \quad (31)$$

Proof: see Appendix A. Notice that in this case the Friedman rule holds: the real return of money equals the household's discount rate. In equilibrium, the real return to bonds has to be the same. The borrowing constraint is not binding in this case because there is a continuum of portfolio allocations that support the full commitment allocation. That is, households can attain the efficient allocation, even there is a debt restriction, by accumulating money.

Proposition 2. *Given a monetary policy θ such that $1 + \theta > \beta$, and a borrowing limit that satisfies $\underline{b} \geq b_h^E$, the equilibrium bond price satisfies $\beta \leq q(\theta, \underline{b}) \leq 1 + \theta$, being increasing with $\underline{b}$, $b_h = \underline{b}$ and the equilibrium satisfies*

$$q(\theta, \underline{b}) u'(c_l) = \beta u'(c_h), \quad (32)$$

$$q(\theta, \underline{b}) u'(c_h) > \beta u'(c_l), \quad (33)$$

$$(1 + \theta) u' (c_l) \geq \beta u' (c_h), \quad (34)$$

$$(1 + \theta) u' (c_h) > \beta u' (c_l), \quad (35)$$

$$d_h = d_l = 0, \quad b_h = -b_l = \underline{b}, \quad (36)$$

$$(1 + \theta) c_h + q(\theta, \underline{b}) \underline{b} = (1 + \theta) w_l - \underline{b}, \quad (37)$$

$$\frac{1}{2} c_h + \frac{1}{2} c_l = \overline{w} = \mathbf{m}, \quad (38)$$

and as $\underline{b} \rightarrow b_h^E$ the equilibrium consumption allocation converges to the full commitment allocation.

This proposition tells us that for any borrowing limit tighter than the natural debt limit the price of the bond is higher (its real return is lower) and households do not attain full risk sharing, $c_l > c_h$. Notice that consumption is higher when the household has low productivity. This is so because consumption depends on last period level of productivity. Now the question is for which debt limit the incentive compatibility constraint is satisfied in the limited borrowing case. Thus, we need to turn to analyze the default allocation.

4.2 The default allocation

Now we turn to investigate the household's decision in the default case. In this case, the problem solved by the household is shown in expression (11). Notice that, in the event of default, households cannot borrow and they are restricted to use money whose gross return is $1/(1 + \theta)$. The first order conditions of the problem are

$$(1 + \theta) u' (z_l) \geq \beta u' (z_h), \quad d_l \geq 0, \quad (39)$$

$$(1 + \theta) u' (z_h) \geq \beta u' (z_l), \quad d_h \geq 0, \quad (40)$$

$$(1 + \theta) z_l + (1 + \theta) d_l = \left(1 + \theta \frac{\mathbf{m}}{\overline{w}}\right) w_h + d_h, \quad (41)$$

$$(1 + \theta) z_h + (1 + \theta) d_h = \left(1 + \theta \frac{\mathbf{m}}{\overline{w}}\right) w_l + d_l, \quad (42)$$

Notice that since $w_h > w_l$ and $\beta \leq 1 + \theta$ the amount of savings when having the good shock is positive, $d_l > 0$ whereas $d_h = 0$. Thus, the equations that characterize the default allocation are

$$(1 + \theta) u'(z_l) = \beta u'(z_h), \quad (43)$$

$$z_l + (1 + \theta)z_h = \left(1 + \theta \frac{\mathbf{m}}{w}\right) w_l + \frac{1 + \theta \frac{\mathbf{m}}{w}}{1 + \theta} w_h, \quad (44)$$

$$z_h \geq w_l. \quad (45)$$

4.3 Credit in deflationary economies

The value function for any holdings of bonds and assets is $V(i, b, d; \underline{b}, \theta)$ which is increasing in $\underline{b}$. Thus, if $V(i, b, d; \underline{b}, \theta) \leq V_D(i, d; \underline{b}, \theta)$ for some $d \in \mathbb{R}_+$, we can show that there cannot exist an equilibrium with credit.

Proposition 3. *Let the borrowing limit satisfy $\underline{b} \in [b_h^E, 0)$. For any non positive rate of inflation the limited borrowing steady state associated to $\underline{b}$ is not incentive compatible. In particular,*

$$V(h, \underline{b}, 0; \underline{b}, \theta) < V_D(h, 0; \theta).$$

Proof: see Appendix A. That is, the default allocation gives higher utility than the limited borrowing equilibrium allocation to the household with high productivity. Thus, credit collapses and only money will be used in equilibrium.

Corollary 1. *Under non positive inflation the incentive compatible equilibrium allocation is characterized by*

$$(1 + \theta) u'(c_l) = \beta u'(c_h), \quad d_l \geq 0, \quad (46)$$

$$d_h = 0, \quad (47)$$

$$(1 + \theta) c_l + (1 + \theta) d_l = \left(1 + \theta \frac{\mathbf{m}}{w}\right) w_h, \quad (48)$$

$$(1 + \theta) c_h = \left(1 + \theta \frac{\mathbf{m}}{w}\right) w_l + d_l, \quad (49)$$

$$\frac{1}{2} c_h + \frac{1}{2} c_l = \bar{w}. \quad (50)$$

Whenever $\beta < 1 + \theta$ there cannot be full risk sharing since the real return to money is low. Nevertheless, if the Friedman rule holds, $\beta = 1 + \theta$, the economy can attain full risk sharing with only money. Moreover, notice that the non existence of credit is not equivalent to the existence of trade. There is trade in the spot market of goods, but the credit market is shut down.

4.4 Credit in inflationary economies

Proposition 4. *There exists a level of positive inflation θ^* for which the full commitment allocation is the incentive compatible equilibrium allocation,*

$$V(i, b_i^E, 0; b_h^E, \theta) \geq V_D(i, 0; \theta).$$

Proof: see appendix A.

Proposition 5. *For any level of inflation $0 < \theta \leq \theta^*$ there exists a borrowing limit $b_h^E \leq \underline{b}(\theta) < 0$ for which its associated limited borrowing steady state is incentive compatible. This borrowing limit is decreasing with θ .*

The intuition is as follows. $V(h, b_h(\underline{b}, \theta), 0; \underline{b}, \theta)$ increases with $\underline{b}$ and θ . $V_D(h, 0; \theta)$ decreases with θ . For any $\theta \in (0, \theta^*]$ there exists a level of $\underline{b}$ for which

$$V(h, b_h(\underline{b}, \theta), 0; \underline{b}, \theta) = V_D(h, 0; \theta), \quad \text{for all } \underline{b} \geq b_h^E.$$

For any $\theta \in (0, \theta^*)$ $\underline{b} > b_h^E$. Proposition 2 ensures that $q(\underline{b}(\theta), \theta)$ decreases with θ . Thus, for any level of inflation $0 < \theta \leq \theta^*$ the incentive compatibility constraint is binding for the household with high productivity. The amount of credit and risk sharing increases with inflation. Inflation acts as a discipline device for households to repay their debts.

Proposition 6. *For any $\theta \in [0, \theta^*]$ the level of consumption with high productivity c_h is a function $c_h : [0, \theta^*] \rightarrow (w_l, \bar{w})$, the price of the bond $q : [0, \theta^*] \rightarrow [1, \beta]$ are continuous differentiable strictly increasing functions.*

5 A stochastic environment

We modify our physical environment so that the type with high productivity is chosen randomly. As before, household 1 has high productivity when household 2 has low productivity and we denote as π the probability of switching from one state to another one. We assume that there are complete markets but for the fact that households cannot commit to repay their debts. That is, households trade Arrow securities in bonds. Therefore, each household issues every period as many bonds as possible states there will be next period. Notice, however, that money balances are not contingent. The following assumption ensures that trade is always efficient.

Assumption 1.

$$\left(\frac{u(\bar{w})}{1-\beta}, \frac{u(\bar{w})}{1-\beta} \right) > (u(w_h), u(w_l)) (I - \beta P)^{-1},$$

where P is the Markov transition matrix that governs the type with high productivity.

5.1 Equilibrium definition

The household's problem

The individual state variable is the variable $x = \{j, b, d\}$, where j denotes the household's productivity level, $j = h, l$. The problem solved by a household of productivity level j stated recursively is

$$\begin{aligned} V(j, b_j, d; \underline{b}, \theta) = & \max_{c' \geq 0} \left\{ u(c') + \beta \pi V(j, b'_j, d'; \underline{b}, \theta) + \beta (1 - \pi) V(-j, b'_{-j}, d'; \underline{b}, \theta) \right\} \\ \text{s. t.} \quad & (1 + \theta) c' + \sum_{s=j}^{-j} q(j, s, \theta, \underline{b}) b'_s + (1 + \theta) d' \leq w_j \left(1 + \theta \frac{\mathbf{m}(\underline{b}, \theta)}{\bar{w}} \right) + b_j + d, \\ & b'_s \geq \underline{b}, \quad s = j, -j, \\ & d' \geq 0, \\ & \text{if } j = h, \text{ then } -j = l, \text{ and vice versa.} \end{aligned} \tag{51}$$

Notice that c' denotes consumption next period.

The maximum amount of credit

The limit $\underline{b}$ is set such that the incentive compatibility constraint is satisfied,

$$V(j, \underline{b}, d; \underline{b}, \theta) \geq V_D(j, d; \theta), \quad \text{for all } j = h, l, d \in \mathbb{R}_+, \text{ given } \theta, \quad (52)$$

where $V_D(j, d; \theta)$ satisfies

$$\begin{aligned} V_D(j, d; \theta) &= \max_{c' \geq 0} \{u(c') + \beta \pi V_D(j, d'; \theta) + \beta (1 - \pi) V(-j, d'; \theta)\} \\ \text{s. t.} \quad &(1 + \theta) c' + (1 + \theta) d' \geq w_j \left(1 + \theta \frac{\mathbf{m}(\underline{b}, \theta)}{\underline{w}}\right) + d, \\ &d' \geq 0, \\ &\text{if } j = h, \text{ then } -j = l, \text{ and vice versa.} \end{aligned} \quad (53)$$

Notice that the problem of default amounts to a situation with incomplete markets.

Equilibrium

Definition 3. A limited borrowing steady state equilibrium for this economy is a monetary policy, θ , a borrowing limit $\underline{b}$, an aggregate amount of money balances $\mathbf{m}(\underline{b}, \theta)$, a set of bond prices, $q(j, s, \underline{b}, \theta)$, a set of functions $\left\{V(x; \underline{b}, \theta), g^c(x; \underline{b}, \theta), g^b(x; \underline{b}, \theta), g^d(x; \underline{b}, \theta)\right\}$, and a measure of households μ such that:

1. given μ , the monetary policy θ , $q(j, s, \underline{b}, \theta)$, $\mathbf{m}(\underline{b}, \theta)$, and the functions $\left\{V(x; \underline{b}, \theta), g^c(x; \underline{b}, \theta), g^b(x; \underline{b}, \theta), g^d(x; \underline{b}, \theta)\right\}$ solve the household's problem,

2. markets clear:

$$\begin{aligned} (a) \quad &\int_X g^c(x) d\mu = \int_X w d\mu, \\ (b) \quad &\int_X g^b(x) d\mu = 0, \\ (c) \quad &\mathbf{m}(\underline{b}, \theta) = \int_X (g^d(x) + g^c(s)) d\mu, \end{aligned}$$

3. the measure of household is stationary, $x(B) = \int_X P(x, B) d\mu$, for all $B \subset \mathcal{B}$.

Definition 4. A steady state is incentive compatible if $V(j, \underline{b}, d; \underline{b}, \theta) \geq V_D(j, d; \theta)$, for all $j = h, l$, $d \in \mathbb{R}_+$, given θ , where the functions $\left\{V_D(j, d; \theta), f^d(j, d; \theta)\right\}$ solve the default problem given μ , $q(j, s, \underline{b}, \theta)$, and the monetary policy θ .

We concentrate on symmetric steady states.

5.2 The full commitment equilibrium allocation

Proposition 7. *If $1 + \theta = \beta$ a full commitment symmetric steady state in which households hold precautionary money balances cannot exist.*

Intuition: See Bewley (1983), Bewley (1984), Aiyagari (1994), Kehoe and Levine (2001). The idea is the following: Suppose that households did not borrow to the limit. Thus, in order to support the full commitment allocation they need to save in the form of money. Since the return to money is not contingent they will save more than if there were complete markets in the states with high productivity. As a result, the precautionary money balances should go to infinity. Thus, an equilibrium may not exist and a symmetric steady state does not exist. Thus for a symmetric steady state to exist, precautionary money balances should be zero.

Corollary 2. *For any $1 + \theta \geq \beta$ a full commitment symmetric steady state is characterized by the following set of equations*

$$q(j, j) u'(c^j) = \beta \pi u'(c^j), \quad j = h, l, \quad (54)$$

$$q(j, -j) u'(c^j) = \beta (1 - \pi) u'(c^{-j}), \quad j = h, l, \quad (55)$$

$$(1 + \theta) u'(c^j) \geq \pi \beta u'(c^j) + (1 - \pi) \beta u'(c^{-j}), \quad j = h, l, \quad (56)$$

$$(1 + \theta) c^j + q(j, j) b_j + q(j, -j) b_{-j} = (1 + \theta) w_j + b_j, \quad j = h, l, \quad (57)$$

$$\frac{1}{2} c_h + \frac{1}{2} c_l = \bar{w}, \quad (58)$$

$$\bar{w} = \mathbf{m}. \quad (59)$$

The expression c^j denotes the level of consumption next period when today's level of productivity is j . Notice that we have changed the notation for consumption since households do not know their future productivity. This also points out that consumption depends on the state at the previous period. Notice that the economy achieves full risk sharing, $c^j = \bar{w}$, and that the price of a bond

that gives one unit of the good next period is β . The amount of borrowing is

$$b_h = -\frac{(1+\theta)(\bar{w} - w_l)}{1 + \beta(1 - 2\pi)}. \quad (60)$$

As in the case without uncertainty, we can think of this amount borrowed as the natural borrowing limit. We denote it as b_h^E .

5.3 The default allocation

Notice that when if a household cannot borrow the problem it faces is a typical Bewley problem under incomplete markets (see Aiyagari 1994) and liquidity constraints. Thus, we cannot characterize the solution to its problem but we can describe some properties of the value function.

Proposition 8. *The utility under default, $V_D(j, d; \theta, \mathbf{m})$, is strictly decreasing with the inflation rate.*

Intuition. We have already argued that there cannot be precautionary money balances in a symmetric steady state. Thus, in equilibrium $\mathbf{m} = \bar{w}$. The problem solved by a household in the case of default can be written as

$$\begin{aligned} V_D(j, d; \theta) = & \max_{z \geq 0} \{u(z) + \beta \pi V_D(j, d'; \theta) + \beta(1 - \pi) V(-j, d'; \theta)\} \\ \text{s. t.} \quad & (1 + \theta)z + (1 + \theta)d' \geq w_j(1 + \theta) + d, \\ & d' \geq 0, \\ & \text{if } j = h, \text{ then } -j = l, \text{ and vice versa.} \end{aligned} \quad (61)$$

Notice that the first order condition is

$$(1 + \theta)u'(z) \geq \beta E_j u'(z'). \quad (62)$$

For θ sufficiently large, since consumption is bounded above, the above first order condition will hold with strict inequality. Thus, in such a case $z_t = w_t$, for all t . For any positive inflation rate, the budget constraint can be written as

$$z + d' \geq w_j + \frac{d}{1 + \theta}. \quad (63)$$

Clearly, the higher the inflation rate the smaller the budget set. Hence, utility decreases with inflation.

5.4 Existence of credit and inflation

Proposition 9. *For any inflation rate θ there exists a discount factor $\beta^*(\theta)$ such that for all $\beta < \beta^*(\theta)$ the full commitment equilibrium is not incentive compatible. In particular,*

$$V(h, b_h^E, 0; b_h^E, \theta) < V_D(h, 0; \theta).$$

The full commitment allocation is incentive compatible if the discount factor is sufficiently large. In order to assess the quantitative importance of this result we conduct several exercises

5.5 Some numerical examples

Here we conduct several quantitative exercises to assess the importance of our results. Table 1 shows the values chosen for the parameters of the stochastic version of our benchmark economy. Figure 1 shows that for a discount factor $\beta = 0.4$ and a deflation rate of almost 6 percent the full commitment allocation gives lower utility than default to the household with high productivity. Of course this result depends on the parameter values used. Our result applies to a larger class of economies. Let us assume that the idiosyncratic uncertainty is that faced by households in Aiyagari (1994). Table 2 shows a discretization of the process used by Aiyagari. In Figure 2 using a discount factor of $\beta = 0.9$ we can see that near the Friedman rule, a deflation rate of 9.9 percent, again, the household with high productivity prefers to default. That is, the full commitment allocation is not incentive compatible. Next we turn to use a process with higher variability in household's productivity. We have borrowed it from Díaz, Pijoan-Mas, and Ríos-Rull (2003). Table 3 shows the productivity values and the transition matrix associated to this process. Figure 3 shows that, for a discount factor of $\beta = 0.96$ the full commitment allocation is not incentive compatible near the Friedman rule, for a deflation rate close to 4 percent. As a matter of fact, the inflation rate for which the full commitment allocation is incentive compatible turns out to be 2.45 percent. Thus, we think that our theory does not predict too high optimal inflation rates.

6 Discussion of the results

Here we have presented a framework where, for credit to exist, the deflation rate must be away from the Friedman rule. In the case without uncertainty a Pareto optimal allocation can be achieved using only money. In the presence of uncertainty this is no longer possible since the return to money is not contingent. Thus, complete risk sharing needs of the existence of credit. At low levels of inflation, the real return to bonds increases with inflation.

We have assumed that there is complete information and, at the same time, money holdings cannot be seized upon default. We could think of this environment as complete information when households have access to the credit market and private information upon default. Nevertheless, we should remember that households do not carry precautionary money balances when they declare default, the key point is that they can save in the form of money after default. Thus, we think that our assumption is not inconsistent.

We also have assumed that financial intermediaries set the same borrowing limit for all agents. In a framework with complete markets this assumption is not restrictive. If we assumed that the return to bonds were not contingent this assumption would need to be changed.

We also have assumed that, under full commitment, money is super neutral. Inflation acts as a consumption tax and the proceeds of the tax are returned as lump sum transfers. Thus, the allocation is efficient. If we assume, instead, that households value leisure then inflation distorts the labor-leisure decision. In this case, the welfare loss due to the labor distortion would have to be compared to the welfare gain due to the existence of more credit. Thus, the welfare gains of inflation would be lower. This is argued by Aiyagari and Williamson (2000). In their framework households hold money because of a limited participation constraint even if they do not want to default on their debts. Thus, similarly to our case with leisure-labor decision, the welfare gain of inflation is smaller than in our benchmark model with uncertainty. The question, however, is why such a limited participation constraint is in place. In our framework the distortion imposed on the labor decision could be eliminated by giving appropriate subsidies to households that do not default on their debts. Thus, our main message would still hold.

We also have assumed that there are no other assets but credit and money. In a production economy households could also hold capital. The key issue here is which goods are credit goods

and which goods are capital goods. If capital were a credit good regardless of having defaulted or not and households could accumulate capital in the event of default our model falls apart. Credit cannot exist in this framework. Nevertheless, if capital is cash good under default, as in Stockman (1981), our main message goes through.

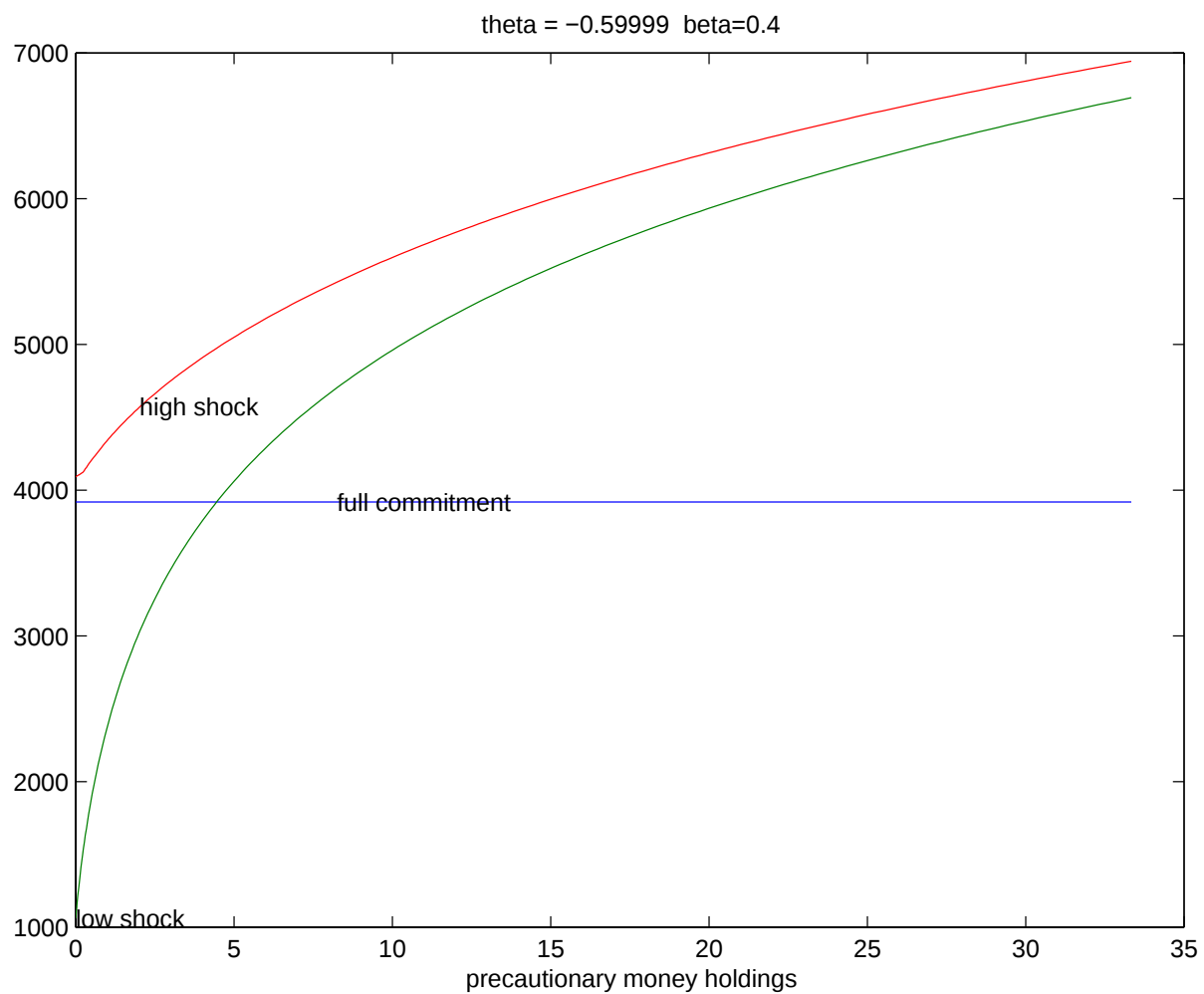


Figure 1: “full commitment” refers to the utility function under full commitment. “high shock” refers to the utility under default obtained by the household with high productivity.

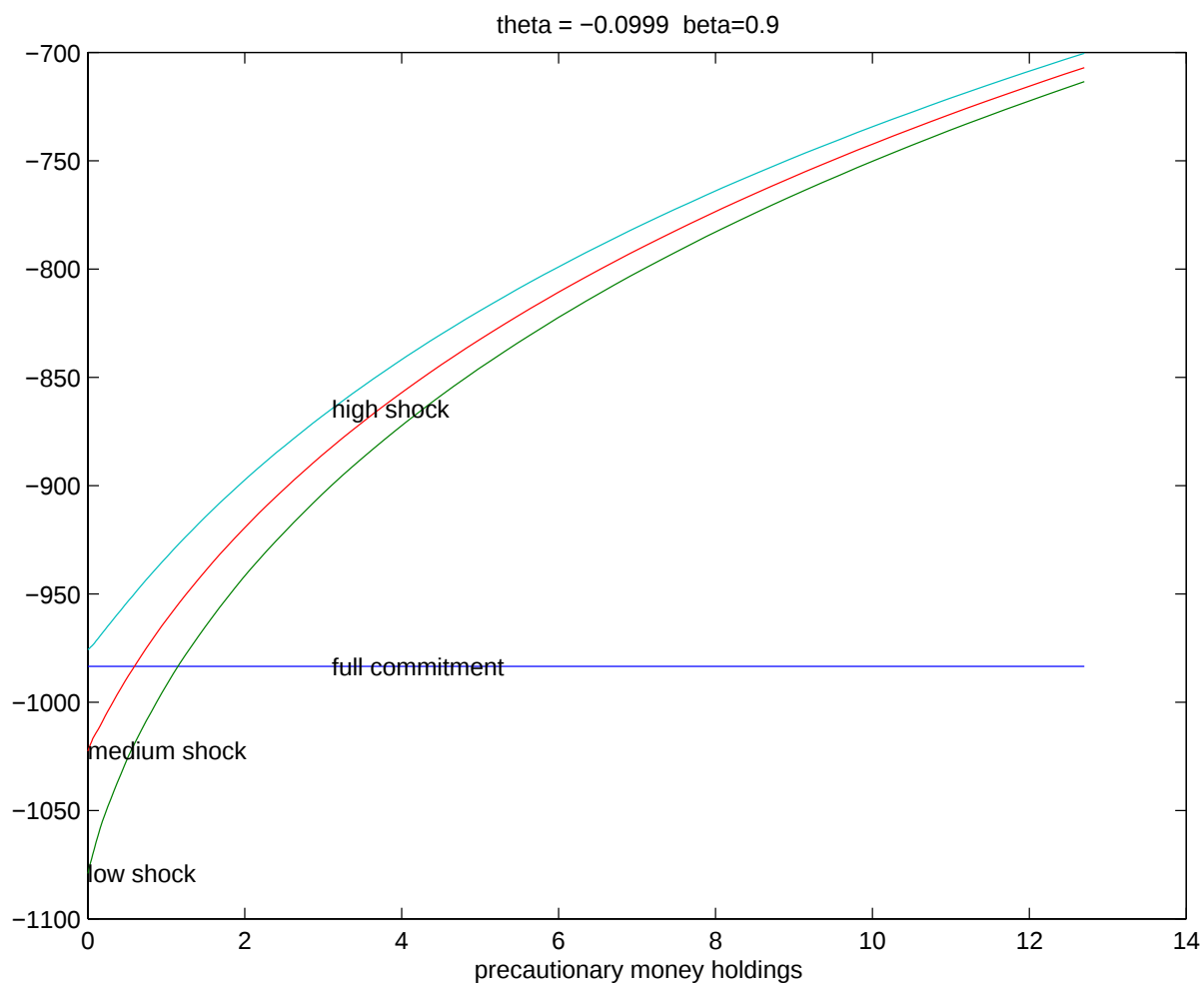


Figure 2: “full commitment” refers to the utility function under full commitment. “high shock” refers to the utility under default obtained by the household with high productivity. Likewise “medium shock” and “low shock”.

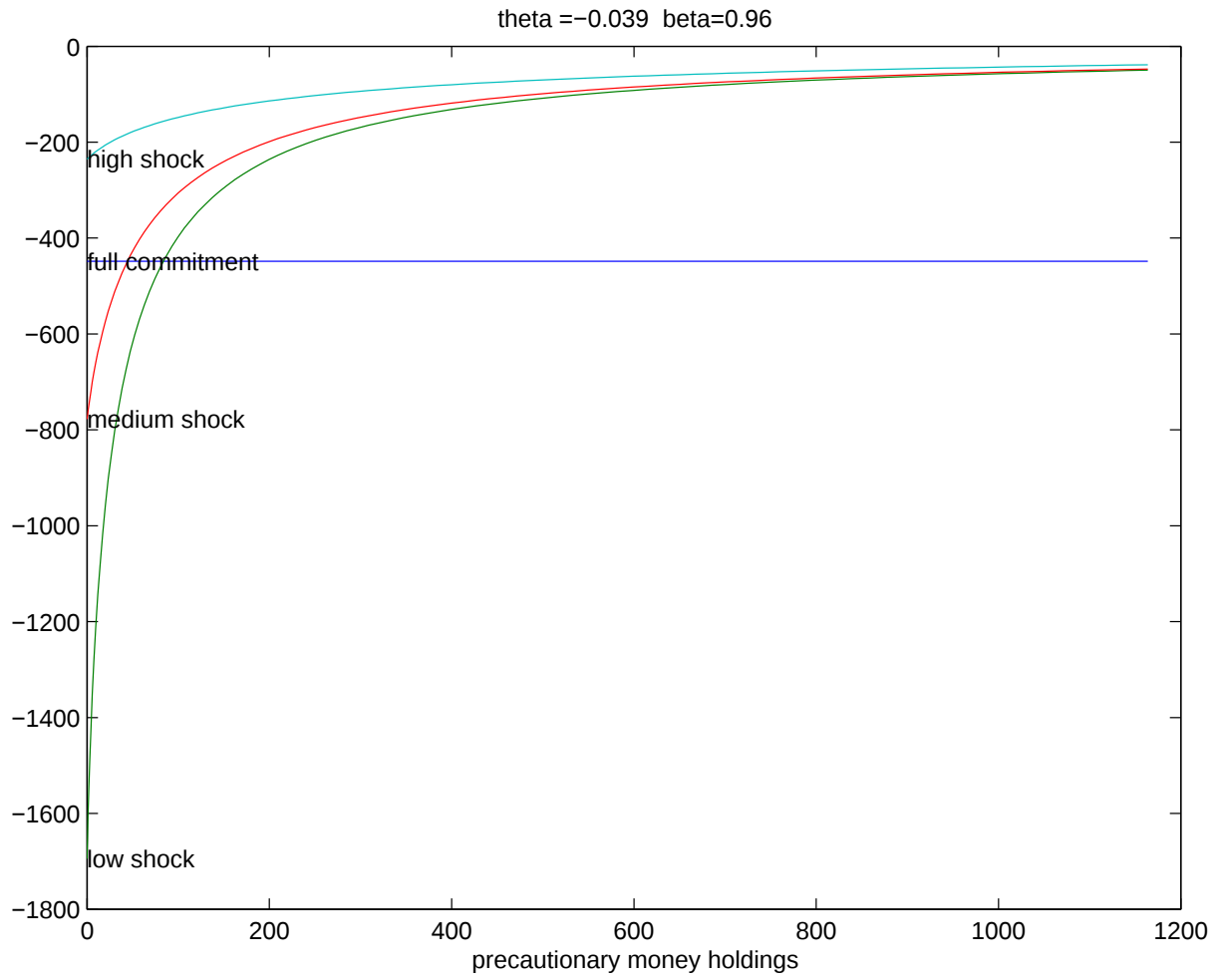


Figure 3: “full commitment” refers to the utility function under full commitment. “high shock” refers to the utility under default obtained by the household with high productivity. Likewise “medium shock” and “low shock”.

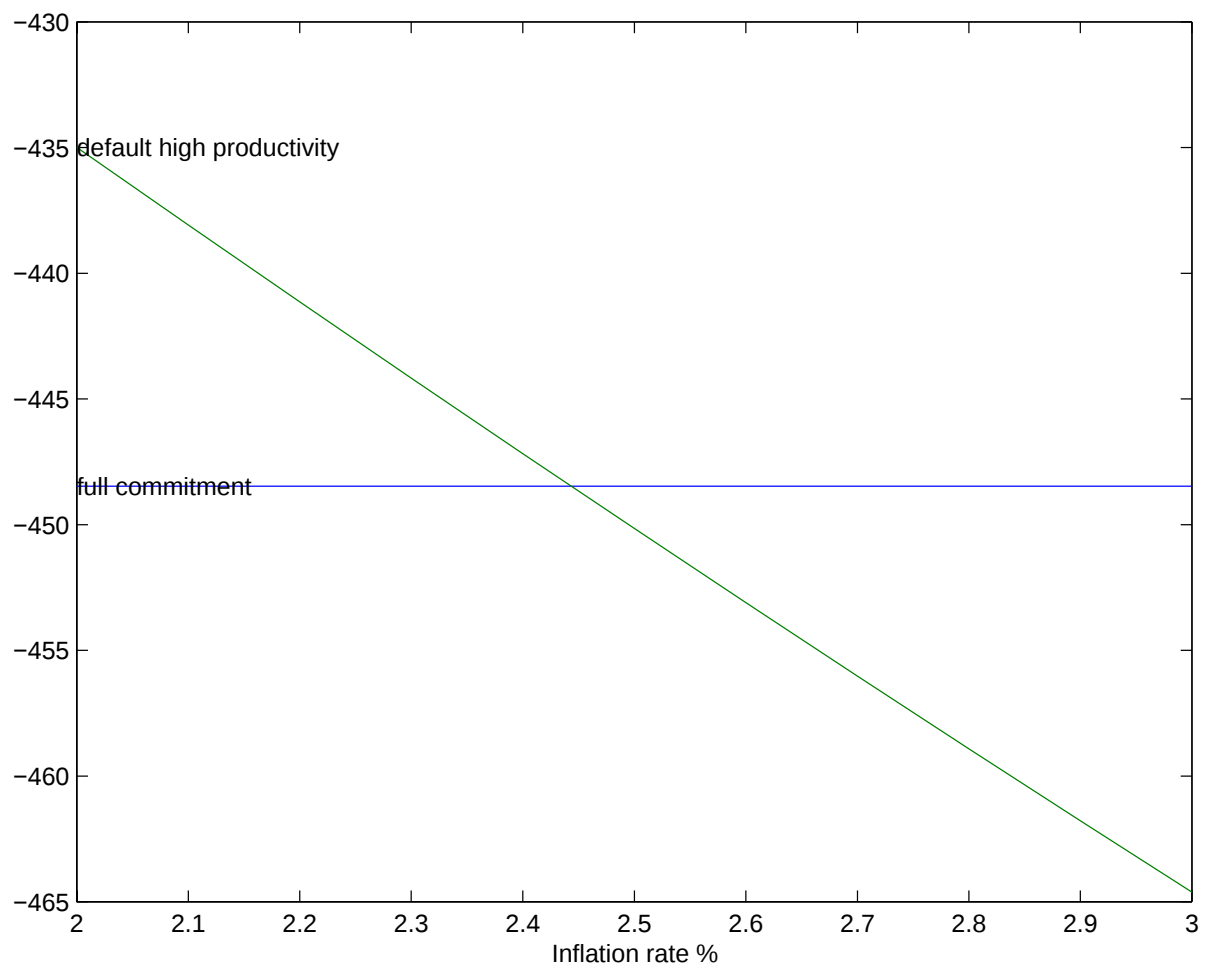


Figure 4: “full commitment” refers to the utility function under full commitment. “high shock” refers to the utility under the fault obtained by the household with high productivity.

Table 1: Example in the random switching economy

$w \in \{w_1, w_2\} =$			$\{1.00, 20.00\}$
$\pi_{e,e'} =$			$\begin{bmatrix} 0.60 & 0.40 \\ 0.40 & 0.60 \end{bmatrix}$
Stationary distribution			
$\pi^* =$			0.50 0.50

Table 2: Example of Bewley type of economy (I)

$w \in \{w_1, w_2, w_3\} =$				$\{0.78, 1.00, 1.27\}$
$\pi_{e,e'} =$				$\begin{bmatrix} 0.66 & 0.27 & 0.07 \\ 0.28 & 0.44 & 0.28 \\ 0.07 & 0.27 & 0.66 \end{bmatrix}$
Stationary distribution				
$\pi^* =$				0.3373 0.3253 0.3373

Table 3: Example of Bewley type of economy (II)

$w \in \{w_1, w_2, w_3\} =$					$\{1.00, 5.29, 46.55\}$
$\pi_{e,e'} =$					$\begin{bmatrix} 0.96500 & 0.00347 & 0.000333 \\ 0.03937 & 0.95000 & 0.010625 \\ 0.00000 & 0.08300 & 0.917000 \end{bmatrix}$
Stationary distribution					
$\pi^* =$					0.4983 0.4429 0.05870

Appendix

A The incentive compatible steady state

Proposition 1. *Given a monetary policy θ such that $1 + \theta = \beta$, and a borrowing limit that satisfies $\underline{b} > b_h^E$, then there exists a continuum of equilibria for which the equilibrium price of bonds satisfies $q(\theta, \underline{b}) = \beta$, the consumption allocation is $c_h = c_l = \bar{w}$, its amount of borrowing satisfies $b_h \in (\underline{b}, 0]$, $b_l = -b_h$, and its money holdings satisfy*

$$\beta \bar{w} + \beta (b_h + d_h) = \left(1 + (\beta - 1) \frac{\mathbf{m}}{\bar{w}}\right) w_l - b_h + d_l, \quad d_h \geq 0, d_l \geq 0, \quad (64)$$

$$\mathbf{m} = \bar{w} + \frac{1}{2} d_h + \frac{1}{2} d_l. \quad (65)$$

Proof. Using the first order condition (18) for $i = h, l$ we find that $c_h = c_l$. Feasibility implies that $c_h = c_l = \bar{w}$. The price of the bond has to satisfy $q = \beta$, otherwise either everybody would like to borrow or save. Therefore, the borrowing constraint on bonds cannot be binding, hence, $b_h \in (\underline{b}, 0]$. Market clearing conditions imply $b_l = -b_h$. The amount of precautionary money balances is obtained using the budget constraint shown in (19) and the total amount of money is given by (21). $\square$

Proposition 2. *Given a monetary policy θ such that $1 + \theta > \beta$, and a borrowing limit that satisfies $\underline{b} \geq b_h^E$, the equilibrium bond price satisfies $\beta \leq q(\theta, \underline{b}) \leq 1 + \theta$, being increasing with $\underline{b}$, $b_h = \underline{b}$ and the equilibrium satisfies*

$$q(\theta, \underline{b}) u'(c_l) = \beta u'(c_h), \quad (66)$$

$$q(\theta, \underline{b}) u'(c_h) > \beta u'(c_l), \quad (67)$$

$$(1 + \theta) u'(c_l) \geq \beta u'(c_h), \quad (68)$$

$$(1 + \theta) u'(c_h) > \beta u'(c_l), \quad (69)$$

$$d_h = d_l = 0, b_h = -b_l = \underline{b}, \quad (70)$$

$$(1 + \theta) c_h + q(\theta, \underline{b}) \underline{b} = (1 + \theta) w_l - \underline{b}, \quad (71)$$

$$\frac{1}{2} c_h + \frac{1}{2} c_l = \bar{w} = \mathbf{m}, \quad (72)$$

and as $\underline{b} \rightarrow b_h^E$ the equilibrium consumption allocation converges to the full commitment allocation.

Proof. We proceed by steps.

1. The price of bonds satisfies $\beta \leq q(\theta, \underline{b})$. Suppose, on the contrary, that $\beta > q(\theta, \underline{b})$. Then, in (29) it would imply simultaneously $u'(c_h) > u'(c_l)$ and $u'(c_l) > u'(c_h)$, which cannot be possible, hence $\beta \leq q(\theta, \underline{b})$.

2. *The price of bonds satisfies $\beta < q(\theta, \underline{b})$.* Suppose that it is not true, that is, $\beta \geq q(\theta, \underline{b})$. This assertion and result 1 implies that the only possibility is $\beta = q(\theta, \underline{b})$. In this case (29) and feasibility imply that $c_h = c_l = \bar{w}$. Since $\beta < 1 + \theta$, using the first order condition with respect to precautionary money balances (18) we have that $d_h = d_l = 0$. But in this case, using the household's budget constraint shown in (19) we have that $b_h = b_h^E$ which violates the borrowing constraint, unless d_l is positive. But this implies $\beta = 1 + \theta$. Thus, we arrive to a contradiction. Hence $\beta < q(\theta, \underline{b})$. It is easy to check that this implies, since $w_h > w_l$,

$$q(\theta, \underline{b}) u'(c_l) = \beta u'(c_h), \quad (73)$$

$$q(\theta, \underline{b}) u'(c_h) > \beta u'(c_l). \quad (74)$$

For any $\underline{b} > b_h^E$ the equilibrium price satisfies $\beta < q(\theta, \underline{b})$ thus, $c_l > \bar{w} > c_h$.

3. *The price of bonds satisfy $q(\theta, \underline{b}) \leq 1 + \theta$.* The price cannot be greater than $1 + \theta$, otherwise by (29) all households would like to borrow, which would not satisfy market clearing conditions.

4. *The precautionary money balances are zero, $d_h = d_l = 0$.* Since $q(\theta, \underline{b}) u'(c_h) > \beta u'(c_l)$ this implies that $(1 + \theta) u'(c_h) > \beta u'(c_l)$, thus $d_h = 0$. The first order condition with respect to d_l satisfies $(1 + \theta) u'(c_l) \geq \beta u'(c_h)$. Thus, in principle, the household with high productivity is indifferent between saving in bonds or money but he saves in form of money then $b_l < -\underline{b}$ which, by clearing market conditions, would imply that $b_h > \underline{b}$ which, in its turn implies full risk sharing, $c_l = c_h = \bar{w}$, which implies, using the first order condition with respect to precautionary money balances, $\beta = 1 + \theta$. We arrive to a contradiction and, hence, $d_l = 0$.

5. *The price of bonds increases with $\underline{b}$.* The steady state satisfies

$$c_h = w_l - \frac{1+q}{1+\theta} \underline{b}, \quad (75)$$

$$c_l = 2\bar{w} - c_h, \quad (76)$$

$$q = \frac{\beta u'(c_h)}{u'(c_l)}, \quad (77)$$

$$1 + \theta > \frac{\beta u'(c_h)}{u'(c_l)}. \quad (78)$$

Consider two borrowing limits that satisfy $b_h^E < \underline{b}^2 < \underline{b}^1$. Denote as $c_l^i, c_h^i, q^i, b_h^i, \dots$ the steady state associated at each limit. Concavity in the utility function implies that

$$q^1 > \frac{\beta u'\left(w_l - \frac{1+q^1}{1+\theta} \underline{b}_2\right)}{u'\left(w_h + \frac{1+q^1}{1+\theta} \underline{b}_2\right)}, \quad (79)$$

which implies that $q^2 < q^1$. This also implies that $c_l^1 > c_l^2$ and $c_h^1 < c_h^2$. In the limit the allocation converges to the full commitment allocation. $\square$

Proposition 3. *Let the borrowing limit satisfy $\underline{b} \in [b_h^E, 0)$. For any non positive rate of inflation the limited borrowing steady state associated to $\underline{b}$ is not incentive compatible. In particular,*

$$V(h, \underline{b}, 0; \underline{b}, \theta) < V_D(h, 0; \theta).$$

Proof. Notice that by Propositions 1 and 2 there are two possible cases: one in which $1 + \theta = \beta$ and another in which $1 + \theta > \beta$. We analyze each in turn.

1. $1 + \theta = \beta$. In this case, by Proposition 1 the equilibrium allocation is the full commitment allocation and the precautionary money balances are positive. Thus, $\mathbf{m} > \bar{w}$. In this case, the default allocation solves the problem

$$\begin{aligned} \max \quad & u(c_l) + \beta u(c_h) \\ \text{s. t.} \quad & z_l + (1 + \theta)z_h = \left(1 + \theta \frac{\mathbf{m}}{\bar{w}}\right) w_l + \frac{1 + \theta \frac{\mathbf{m}}{\bar{w}}}{1 + \theta} w_h, \\ & z_h \geq w_l. \end{aligned} \tag{80}$$

It is easy to check that the full commitment allocation is inside the default budget set,

$$(2 + \theta)\bar{w} \leq \left(1 + \theta \frac{\mathbf{m}}{\bar{w}}\right) w_l + \frac{1 + \theta \frac{\mathbf{m}}{\bar{w}}}{1 + \theta} w_h. \tag{81}$$

Thus, for any level of deflation, the full commitment allocation gives less utility than the default allocation. In the case of zero inflation the full commitment allocation is exactly in the boundary of the default budget set but it gives lower utility than the default allocation (due to concavity of the instantaneous utility function).

2. $1 + \theta > \beta$. In this case, by Proposition 2 the equilibrium allocation is the full commitment allocation and the precautionary money balances are zero. Thus, $\mathbf{m} = \bar{w}$. In this case, the default allocation solves the problem

$$\begin{aligned} \max \quad & u(c_l) + \beta u(c_h) \\ \text{s. t.} \quad & z_l + (1 + \theta)z_h = (1 + \theta) w_l + w_h, \\ & z_h \geq w_l. \end{aligned} \tag{82}$$

Here it is easy to check that the equilibrium allocation is inside the default budget set,

$$c_l + (1 + \theta)c_h = 2\bar{w} + \theta c_h < (1 + \theta) w_l + w_h$$

Since $\theta < 0$ and $c_h > w_l$. □

Proposition 4. *There exists a level of positive inflation θ^* for which the full commitment allocation is the incentive compatible equilibrium allocation,*

$$V(i, b_i^E, 0; b_h^E, \theta) \geq V_D(i, 0; \theta).$$

Proof. For a level of inflation $\theta \leq 0$ the default allocation gives higher utility than the full commitment allocation,

$$V(i, b_i^E, 0; b_h^E, \theta) < V_D(i, 0; \theta),$$

whereas for θ sufficiently large the default allocation converges to the autarkic allocation, $c_h = w_h$ and $c_l = w_l$. By assumption 1, the full commitment allocation gives higher utility than the autarkic allocation. By continuity, there exists $\theta^* > 0$ for which

$$V(i, b_i^E, 0; b_h^E, \theta) \geq V_D(i, 0; \theta).$$

Since the value functions are non decreasing in d , the result holds for any precautionary money balances. $\square$

Proposition 5. *For any level of inflation $0 < \theta \leq \theta^*$ there exists a borrowing limit $b_h^E \leq \underline{b}(\theta) < 0$ for which its associated limited borrowing steady state is incentive compatible. This borrowing limit is decreasing with θ .*

Proof. Coming soon. $\square$

Proposition 6. *For any $\theta \in [0, \theta^*]$ the level of consumption with high productivity c_h is a function $c_h : [0, \theta^*] \rightarrow (w_l, \bar{w})$, the price of the bond $q : [0, \theta^*] \rightarrow [1, \beta]$ are continuous differentiable strictly increasing functions.*

Proof. Coming soon. $\square$

B A stochastic environment

Let us write the household's budget constraint in a sequential form. z_t denotes the household that has the high productivity. This random variable is Markov and summarizes the state of the world. The vector z^t denotes a possible history of events up to time t . The monetary policy is not contingent to the state of the world and follows the law

$$M_{t+1} = (1 + \theta)M_t. \quad (83)$$

This also implies that, in steady state, the price of the consumption good does not depend on the state of the world. The problem solved by a household is

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t \sum_{i=0}^{\infty} \prod (z^t) u(c_t^i(z^t)) \\ \text{s. t.} \quad & p_t c_t^i(z^t) + \sum_l q_t^n(z^t, z_l) A_{t+1}^i(z^t, z_l) + M_{t+1}^i(z^t) \leq p_t w_t^i(z^t) + A_t^i(z^t) + M_t^i(z^{t-1}) + T_{t+1}^i(z^t), \\ & p_t c_t^i(z^t) \leq M_t^i(z^t), \\ & \frac{A_t^i(z^t)}{p_t} \geq \underline{b}. \end{aligned} \quad (84)$$

Denote $b_{t+1}^i(z^t, z_l) = A_{t+1}^i(z^t, z_l) / p_{t+1}$ and $q_t(z^t, z_l) = (1 + \theta)q_t^n(z^t, z_l)$. Further, $m_{t+1}^i(z^t) = M_{t+1}^i(z^t) / p_{t+1}$. Notice that consumption at any period $t + 1$ is completely determined by money holdings at the beginning of that period, which are contingent on information up to time t . Thus, without loss of generality we can write $c_{t+1}^i(z^t)$. Therefore, we can write the problem in real terms as

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^{t+1} \sum_{i=0}^{\infty} \prod (z^t) u(c_{t+1}^i(z^t)) \\ \text{s. t.} \quad & (1 + \theta)c_{t+1}^i(z^t) + \sum_l q_t(z^t, z_l) b_{t+1}^i(z^t, z_l) + (1 + \theta)d_{t+1}^i(z^t) \leq \\ & w_t^i(z^t) + b_t^i(z^t) + d_t^i(z^{t-1}) + (1 + \theta)\tau_{t+1}^i(z^t), \\ & d_{t+1}^i(z^t) \geq 0, \\ & b_{t+1}^i(z^t, z_l) \geq \underline{b}. \end{aligned} \quad (85)$$

Proposition 7. *If $1 + \theta = \beta$ a full commitment symmetric steady state in which households hold precautionary money balances cannot exist.*

Proof. Coming soon. □

Proposition 8. *The utility under default, $V_D(j, d; \theta, \mathbf{m})$, is strictly decreasing with the inflation rate.*

Proof. Coming soon. □

Proposition 9. *For any inflation rate θ there exists a discount factor $\beta^*(\theta)$ such that for all $\beta < \beta^*(\theta)$ the full commitment equilibrium is not incentive compatible. In particular,*

$$V(h, b_h^E, 0; b_h^E, \theta) < V_D(h, 0; \theta).$$

Proof. Coming soon. □

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