The Social Value of Public Information with Costly Private Information*

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Abstract

In a beauty contest framework, we show that a more precise public information is welfare enhancing when increasing the precision of private information is costly. The accuracy of public information is chosen by the public authority taking into account that an increase in the precision of public information reduces the incentives for private acquisition of information, thereby inducing socially valuable savings of private resources.

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1 Introduction

In a highly debated paper, Morris and Shin (2002) have shown that public information may have a detrimental effect on welfare in a beauty contest framework. In their framework, private agents who wish to coordinate their actions overweight public signals, partly disregarding socially valuable private information. In the same context, Cornand and Heinemann (2004) argue that social welfare rises when more precise public information reaches a fraction of market participants only, while Svensson (2006) questions the empirical plausibility of Morris and Shin's result. Angeletos and Pavan (2004) find that public information is welfare improving in presence of positive externalities; in a monopolistic competition framework, Hellwig (2005) finds that it can be socially valuable to disregard some private information, as firms partly neglect their contribution to aggregate risk. Finally, Angeletos and Pavan (2006) develop a unifying framework where the role of public information for equilibrium welfare depends on the degree of complementarity or substitutability among agents' actions.

In this note we aim at further investigating the welfare implications of information provision by focusing on the cost of information. In a beauty contest framework, we

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show that more public information is always welfare enhancing when increasing the precision of private information is costly.

In our model private agents choose their signals precision by paying a cost, and the public authority sets the precision of the public signal knowing that its decision affects those of private agents. The latter, in choosing the precision of their signals, take the precision of public information as given. Hence, the public authority acts as a Stackelberg leader, who optimally exploits the fact that an increase in the precision of public information reduces the incentive for private acquisition of information, thereby inducing socially valuable savings of private resources. Such modeling of the information acquisition process is consistent with the widespread idea that private agents are typically faster than public authorities in adjusting the accuracy of their information.

2 The Setup

We model an economy populated by a continuum of agents indexed by the unit interval $[0, 1]$. Every agent observes noisy *private* and *public* signals about the fundamental, $\theta \in \mathbb{R}$, so that information is incomplete. As in Angeletos and Pavan (2006), we consider a two periods setting. In period -1, agents observe the state of the economy (θ_{-1}), then a benevolent social planner chooses the precision of next period public information by maximizing the sum of individual expected utilities, and finally private agents decide how much to invest in the precision of their private signals. In period 0, every agent receives her signals (public and private) and chooses an action affecting her utility, as well as that of the other individuals.

The state of the economy evolves according to the stochastic process:

$$\theta = \theta_{-1} + \chi,$$

The shock χ (normally distributed with mean zero, variance σ_θ^2, and hence precision $\alpha_\theta \equiv \sigma_\theta^{-2}$) takes place at the beginning of period 0. Note that all agents - private and public - have a common *ex ante* expectation θ_{-1} on the state variable θ, which allows us to compute ex ante expected utilities, and hence welfare, on a common stand-point.

At the beginning of period 0, every agent receives both a public signal, y, and a private signal, x_i:

$$y = \theta + \eta,$$

$$x_i = \theta + \varepsilon_i,$$

where η is normally distributed, independent of θ, with mean zero and precision α_y, and the noise terms ε_i are normally distributed, independent of θ, η and ε_j $(j \neq i)$, with mean zero and precision β_i. Note that y is common knowledge to all agents, whereas x_i is specific to each agent i and not observable by the others. The precision of the private signal may vary across agents.

The common posterior for θ, given public information, is normally distributed with mean $E(\theta|\, y) = \frac{\alpha_\theta \theta_{-1} + \alpha_y y}{\alpha_\theta + \alpha_y}$, and precision $P(\theta|\, y) = \alpha_\theta + \alpha_y$. We define, for convenience, $z \equiv E(\theta|\, y)$, and $\alpha \equiv \alpha_\theta + \alpha_y$. Private posteriors are normally distributed, with mean $E(\theta|\, y, x_i) = \frac{\alpha z + \beta_i x_i}{\alpha + \beta_i}$ and precision $P(\theta|\, y, x_i) = \alpha + \beta_i$.

Agent i is characterized by the utility function

$$u_i(a, \theta, \beta_i) \equiv -(1-r)(a_i - \theta)^2 - r\left(L_i - \bar{L}\right) - C(\beta_i), \tag{1}$$

where a is the action profile over all agents, $a_i \in (0,1)$ denotes agent i's action, θ represents the state of the economy, $r \in (0,1)$,

$$L_i \equiv \int_0^1 (a_j - a_i)^2 dj, \qquad \bar{L} \equiv \int_0^1 L_j dj,$$

and $C(\beta_i)$ is the cost (in consumption equivalent units) of improving precision for a private agent, which we assume to be linear, i. e.

$$C(\beta_i) \equiv C \cdot \beta_i.$$

The latter seems a natural assumption when there is competition among a large number of private information providers, a case in which the price of the precision of any private signal must be the same for every provider. Note that the first two terms in Equation (1) coincide with agent i's utility function in Morris and Shin (2002), while the third addendum accounts for the impact on utility of the precision cost of private information. Note also that the loss L_i increases in the distance between agent i's action and the action profile of the whole population. Hence, r captures the weight of a 'beauty contest' externality: the larger is r, the more important is the second guessing motive for each agent.

3 The Equilibrium

The model is solved by backward induction. In period 0, for given signal precisions, agent i chooses action a_i by maximizing $E(u_i(a, \theta, \beta_i) \mid y, x_i, \theta_{-1})$, which gives

$$a_i = (1-r) E_i(\theta) + r E_i\left(\int_0^1 a_j dj\right). \tag{2}$$

We follow an equilibrium reasoning analogous to that in Morris and Shin (2002) and Angeletos and Pavan (2004), guessing that agents' actions are a linear function of signals. However, departing from the literature, we consider the possibility of an agent deviating from the symmetric Nash equilibrium by choosing a different signal precision in period -1. Denoting with i the deviating agent, she follows a linear strategy of the type

$$a_i = \gamma_i z + (1-\gamma_i) x_i, \tag{3}$$

while all agents j other than i receive a private signal of common precision β, with $\beta \neq \beta_i$, thus following the linear strategy

$$a_j = \gamma z + (1-\gamma) x_j, \tag{4}$$

with $(\gamma_i, \gamma) \in (0,1]$ being parameters determined in the equilibrium.

The action chosen in equilibrium by agent i can now be determined as the best response to the actions of all other agents. By taking agent i's expected value of a_j (conditional on her information set), we obtain

$$\begin{aligned} E_i(a_j) &= \gamma z + (1-\gamma) E_i(x_j) \\ &= \gamma z + (1-\gamma) E_i(\theta), \end{aligned} \tag{5}$$

and by substituting this expression into Equation (2):

$$\begin{aligned} a_i &= (1-r) E_i(\theta) + r(\gamma z + (1-\gamma) E_i(\theta)) = \\ & (1-r\gamma) E_i(\theta) + r\gamma z. \end{aligned} \tag{6}$$

Based on both private and public information agent i's (conditional) expected value of θ is given by

$$E_i(\theta) = \delta_i z + (1-\delta_i) x_i, \tag{7}$$

where, by Bayes Theorem, $\delta_i = \frac{\alpha}{\alpha+\beta_i}$.

By substituting (3) and (7) into (6), and rearranging, we obtain

$$\gamma_i z + (1-\gamma_i) x_i = ((1-r\gamma)\delta_i + r\gamma) z + (1-r\gamma)(1-\delta_i) x_i,$$

from which it follows immediately that

$$\gamma_i = (1-r\gamma)\delta_i + r\gamma. \tag{8}$$

Since all agents j other than i receive a signal of common precision, the fact that agent i alone, having zero-measure, receives a different signal does not affect their behavior. The equilibrium value of γ can then be easily obtained by considering the case in which all agents in the economy observe private signals of the same precision: $\beta_i = \beta$, $\forall i$. Because in this case $\gamma_i = \gamma$ and $\delta_i = \delta$, one can immediately conclude that

$$\gamma = (1-r\gamma)\delta + r\gamma, \text{ i. e. } \gamma = \frac{\delta}{1-r(1-\delta)}, \tag{9}$$

with $\delta = \frac{\alpha}{\alpha+\beta}$.

Substituting for γ from (9), Equation (8) can now be rewritten as

$$\gamma_i = \frac{(1-r)\delta_i + r\delta}{1-r(1-\delta)}. \tag{10}$$

Note that $\gamma_i = \gamma$ if $\delta_i = \delta$, and that $\gamma_i < \gamma$ if $\delta_i < \delta$, implying that a higher precision of agent i's private signal (i. e. a lower δ_i) implies a smaller weight being assigned to the public signal.

Having characterized the equilibrium at time 0, we can now move backward studying agent i's utility maximization problem at -1. Agent i affects the precision of her private signal, β_i, at a cost. In doing so she takes into account the level of precision of public information. From Equation (1), her expected utility at time -1 can be written as

$$\begin{aligned} E[u_i(a,\theta,\beta_i) \mid \theta_{-1}] = -(1-r) E\left[(a_i-\theta)^2 \mid \theta_{-1}\right] - \\ +rE\left[\int_0^1 (a_j-a_i)^2 dj \mid \theta_{-1}\right] + rE\left[\int_0^1 (a_h-a_i)^2 dh \mid \theta_{-1}\right] - C\cdot\beta_i, \end{aligned} \tag{11}$$

which after some algebra (see the Appendix) becomes:

$$E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right]=-\frac{\left(1-r\right)\left(\alpha+\beta\right)^2}{\left(\alpha+\left(1-r\right)\beta\right)^2}\left(\frac{1-r}{\alpha+\beta_i}+\frac{r\left(\alpha+\left(r-1\right)\beta\right)}{\left(\alpha+\beta\right)^2}\right)-C\cdot\beta_i. \quad (12)$$

Therefore, agent i's individual expected utility maximization problem,

$$\max_{\beta_i\geq 0}\quad E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right],$$

gives the first order condition

$$\frac{\left(1-r\right)^2\left(\alpha+\beta\right)^2}{\left(\alpha+\left(1-r\right)\beta\right)^2\left(\alpha+\beta_i\right)^2}-C+\mu=0,$$

where μ is the Lagrange multiplier associated to the constraint $\beta_i\geq 0$. Second order conditions are obviously satisfied.

It is worth noting that, if we ignored the cost of precision of the private signal, the expected utility (11) would always be increasing in β_i, as agents could target more precisely the fundamental (see Equation (2)).

Consider first the case in which the optimal unconstrained β_i would be negative, implying $\mu>0$ and $\beta_i=0$. Focusing on the unique symmetric equilibrium,[1] the expected utility (12) reduces to

$$E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right]=-\frac{1-r}{\alpha}, \quad (13)$$

and hence, recalling that $\alpha\equiv\alpha_\theta+\alpha_y$,

$$\frac{\partial E\left[u_i\left(a,\theta,\beta_i\right)\mid\theta_{-1}\right]}{\partial\alpha_y}>0. \quad (14)$$

Consider now the case where $\beta_i\geq 0$ and hence $\mu=0$. In the symmetric equilibrium, we have $\beta_i=\beta$, and therefore

$$\left(1-r\right)^2=C\left(\alpha+\left(1-r\right)\beta\right)^2. \quad (15)$$

By totally differentiating (15), we obtain

$$\frac{d\beta}{d\alpha_y}=-\frac{1}{1-r}<0. \quad (16)$$

From (16), it is apparent that an increase in the precision of public information renders less appealing the investment in the precision of the private signal. Note that the ratio $1/\left(1-r\right)$ is greater than one, implying that an increase in the precision of public information causes a larger reduction in the precision of private information. Moreover, the ratio increases with the externality parameter r. In fact, when the beauty contest effect is large (i. e. r is large), an increase in public information has a strong coordinating power for agents, hence reducing their willingness to buy private information.

[1] Uniqueness follows by the same argument in Morris and Shin (2002).

4 Aggregate welfare

At the symmetric equilibrium, the benevolent social planner maximizes the sum of individual expected utilities at time -1, that are given by

$$E\left[u_i\left(a,\theta,\beta\right)\mid\theta_{-1}\right]=-\frac{\left(1-r\right)\left(\alpha+\left(1-r\right)^2\beta\right)}{\left(\alpha+\left(1-r\right)\beta\right)^2}-C\cdot\beta. \tag{17}$$

Therefore, aggregate expected welfare net of private precision costs ($\tilde{W}\left(\alpha\right)$) is

$$\begin{aligned}\tilde{W}\left(\alpha\right) &\equiv \int_0^1 E\left[u_i\left(a\left(\beta\right),\theta\right)\mid\theta_{-1}\right]di= \\ &= -\frac{\left(1-r\right)\left(\alpha+\left(1-r\right)^2\beta\right)}{\left(\alpha+\left(1-r\right)\beta\right)^2}-C\cdot\beta.\end{aligned} \tag{18}$$

As we want to characterize the conditions under which welfare is increasing in the precision of public information, we do not need to explicitly consider its cost. In fact, whenever the marginal impact of α_y on welfare is positive and decreasing, there will be an interior optimal solution to the social planner problem under the natural assumption that the marginal cost of public information provision is non-decreasing.

The following proposition illustrates the impact of the precision of public information on welfare once its effect on the precision of private signals is taken into account.

Proposition 4.1
At the symmetric equilibrium, it is

(i) $\frac{d\tilde{W}(\alpha)}{d\alpha_y}>0$,

(ii) $\frac{d^2\tilde{W}(\alpha)}{d\alpha_y^2}<0$,

for all $(\alpha,\beta)\geq 0$.

Proof.
(i) Consider first the case when $\beta_i=\beta=0$. From Equation (18), aggregate expected welfare can be written as $\tilde{W}\left(\alpha\right)=-\frac{1-r}{\alpha}$, from which it follows immediately that $\frac{d\tilde{W}(\alpha)}{d\alpha_y}>0$.

Second, focus on the case when $\beta_i=\beta\geq 0$. From Equation (18), we have

$$\begin{aligned}\frac{d\tilde{W}\left(\alpha\right)}{d\alpha_y}&=\frac{\left(1-r\right)}{\left[\alpha+\beta(1-r)\right]^3}\cdot \\ \cdot\left\{\alpha+\beta(1-r)(1-2r)+\frac{d\beta}{d\alpha_y}(1-r)\left[(1+r)\alpha+(1-r)^2\beta\right]\right\}&-C\frac{d\beta}{d\alpha_y}.\end{aligned}$$

Exploiting Equation (15) to substitute for C in the last addendum and Equation (16) to substitute for $\frac{d\beta}{d\alpha_y}$, the expression above becomes:

$$\frac{d\tilde{W}\left(\alpha\right)}{d\alpha_y}=\frac{\left(1-r\right)^2}{\left[\alpha+\beta(1-r)\right]^2}>0$$

for all $(\alpha, \beta) \geq 0$.
(ii) In both cases, the second statement of the proposition follows immediately by differentiating $\frac{d\tilde{W}(\alpha)}{d\alpha_y}$. ■

When the precision of the private signal is costly, an increase in the precision of public information is always welfare enhancing. This occurs because an increase in the precision of the public signal reduces the individual incentives to invest in the precision of the private signal (see Equation (16)). The latter is already sub-optimally weighted in the individual decision problem due to the beauty contest externality. Hence, an increase in α negatively affects the expected social welfare $\tilde{W}(\alpha)$. Nevertheless, such a decrease is dominated by the reduction in the cost of precision, that becomes larger as the beauty contest externality r increases. In fact, in a beauty contest framework, agents end up underweighting the precision of the public signal, so that the resulting reduction of the expenditure in private information is strong enough for the overall impact of an increase of the precision of the public signal to be welfare improving.

5 References

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6 Appendix

To obtain the single-individual utility function (12), we concentrate on the first three terms of Eq. (11), i. e.

$$-(1-r)\,E\left[(a_i-\theta)^2\right]-rE\left[\int_0^1(a_j-a_i)^2\,dj\right]+rE\left[\int_0^1(a_h-a_j)^2\,dh\right]. \qquad \text{(A1)}$$

In the following, we take as understood that the expectation is computed from the standpoint of the period -1 information set. First, we focus on the first addendum in Eq. (A1). Exploiting Eq. (3) we obtain:

$$-(1-r)\,E\left[(a_i-\theta)^2\right]=-(1-r)\,E\left[(\gamma_i z+(1-\gamma_i)x_i-\theta)^2\right].$$

Recalling that $z\equiv E(\theta|\,y)=\frac{\alpha_\theta\theta_{-1}+\alpha_y y}{\alpha_\theta+\alpha_y}$, and substituting it in the expression above yields:

$$-(1-r)\,E\left[\left(\gamma_i\frac{\alpha_\theta\theta_{-1}+\alpha_y y}{\alpha_\theta+\alpha_y}+(1-\gamma_i)x_i-\theta\right)^2\right].$$

Taking into account the structure of the signals and the evolution of the fundamental we obtain

$$-(1-r)\,E\left[\left(\gamma_i\frac{\alpha_\theta\theta_{-1}+\alpha_y(\theta_{-1}+\chi+\eta)}{\alpha_\theta+\alpha_y}+(1-\gamma_i)(\theta_{-1}+\chi+\varepsilon_i)-(\theta_{-1}+\chi)\right)^2\right],$$

which gives:

$$-(1-r)\,E\left[\left(\gamma_i\frac{\alpha_y(\chi+\eta)}{\alpha_\theta+\alpha_y}+(1-\gamma_i)\varepsilon_i-\gamma_i\chi\right)^2\right],$$

and hence:

$$-(1-r)\left[\frac{\gamma_i^2}{\alpha}+\frac{(1-\gamma_i)^2}{\beta_i}\right].$$

Substituting for γ_i (Eq. (10)), we obtain

$$-(1-r)\left\{\frac{\left[(1-r)\delta_i+r\delta\right]^2/\alpha+\left[(1-r)(1-\delta_i)\right]^2/\beta_i}{\left[1-r(1-\delta)\right]^2}\right\},$$

and, after developing the terms in square brackets:

$$-\frac{(1-r)}{\left[1-r(1-\delta)\right]^2}\left\{(1-r)^2\left[\frac{\delta_i^2}{\alpha}+\frac{(1-\delta_i)^2}{\beta_i}\right]+\frac{2r(1-r)\delta\delta_i}{\alpha}+\frac{r^2\delta^2}{\alpha}\right\}.$$

Because $\delta_i=\frac{\alpha}{\alpha+\beta_i}$, and $(1-\delta_i)=\frac{\beta_i}{\alpha+\beta_i}$, we have that $\frac{\delta_i^2}{\alpha}+\frac{(1-\delta_i)^2}{\beta_i}=\frac{1}{\alpha+\beta_i}=\frac{\delta_i}{\alpha}$. Hence, we obtain:

$$-\frac{(1-r)}{[1-r(1-\delta)]^2}\left\{\frac{(1-r)^2\delta_i}{\alpha}+\frac{2r(1-r)\delta\delta_i}{\alpha}+\frac{r^2\delta^2}{\alpha}\right\} \tag{A2}$$

We now consider the second addendum in Eq. (A1). Exploiting Eqs. (3) and (4), we get:

$$-rE\left[\int_0^1 (a_j-a_i)^2\,dj\right]=-rE\left[\int_0^1 ((\gamma-\gamma_i)z+(1-\gamma)x_j-(1-\gamma_i)x_i)^2\,dj\right].$$

Substitution of z, x_i and x_j yields:

$$-rE\left[\left((\gamma-\gamma_i)\frac{\alpha_y(\chi+\eta)}{\alpha_\theta+\alpha_y}+(1-\gamma)(\chi+\varepsilon_j)-(1-\gamma_i)(\chi+\varepsilon_i)\right)^2\right],$$

Because $\alpha\equiv\alpha_\theta+\alpha_y$, $\alpha_\theta\equiv\sigma_\theta^{-2}$, and $\alpha_\eta\equiv\sigma_\eta^{-2}$ the above equation gives:

$$-r\left[\frac{(\gamma-\gamma_i)^2}{\alpha}+\frac{(1-\gamma)^2}{\beta}+\frac{(1-\gamma_i)}{\beta_i}\right]. \tag{A3}$$

Finally, as for the third addendum in Eq. (A1), we simply observe that, from the standpoint of agent i, all signals perceived by the other agents are identical, so that

$$rE\left[\int_0^1 (a_h-a_j)^2\,dh\right]=\frac{2r(1-\gamma)^2}{\beta}. \tag{A4}$$

Summing Eqs. (A3) and (A4), we readily obtain

$$-r\left[\frac{(\gamma-\gamma_i)^2}{\alpha}+\frac{(1-\gamma_i)}{\beta_i}-\frac{(1-\gamma)^2}{\beta}\right],$$

which – by substituting for γ and γ_i from Eqs. (9) and (10), respectively – becomes:

$$-\frac{r}{[1-r(1-\delta)]^2}\left[\frac{(1-r)^2(\delta-\delta_i)^2}{\alpha}+\frac{(1-r)^2(1-\delta_i)^2}{\beta_i}-\frac{(1-r)^2(1-\delta)^2}{\beta}\right].$$

Collecting the common $(1-r)^2$ term, developing the square terms and rearranging, we get:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta^2\left(\frac{\beta-\alpha}{\alpha\beta}\right)+\delta_i^2\left(\frac{\beta_i+\alpha}{\alpha\beta_i}\right)-\frac{2\delta\delta_i}{\alpha}+\frac{(1-2\delta_i)}{\beta_i}-\frac{(1-2\delta)}{\beta}\right].$$

Notice that, from the definition of δ_i, we have: $\delta_i^2(\alpha+\beta_i)/(\alpha\beta_i)=\delta_i/\beta_i$. Hence:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta^2\left(\frac{\beta-\alpha}{\alpha\beta}\right)-\frac{2\delta\delta_i}{\alpha}+\frac{(1-\delta_i)}{\beta_i}-\frac{(1-2\delta)}{\beta}\right].$$

Substituting δ_i in the third addendum in the big square brackets, and δ in the fourth, we obtain

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta^2\left(\frac{\beta-\alpha}{\alpha\beta}\right)-\frac{2\delta\delta_i}{\alpha}+\frac{\delta_i}{\alpha}-\delta\left(\frac{\beta-\alpha}{\alpha\beta}\right)\right],$$

and rearranging:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left[\delta(\delta-1)\left(\frac{\beta-\alpha}{\alpha\beta}\right)+\delta_i\frac{(1-2\delta)}{\alpha}\right].$$

Because $(1-2\delta)=\frac{\beta-\alpha}{\alpha+\beta}$, then $\frac{(1-2\delta)}{\alpha}=\frac{\beta-\alpha}{\alpha\beta}\frac{\beta}{\alpha+\beta}=\frac{\beta-\alpha}{\alpha\beta}(1-\delta)$. Therefore, we can write:

$$-\frac{r(1-r)^2}{[1-r(1-\delta)]^2}\left(\frac{\beta-\alpha}{\alpha\beta}\right)(1-\delta)(\delta_i-\delta) \tag{A5}$$

We now consider the sum of (A2) and (A5) obtaining:

$$-\frac{(1-r)}{(1-r(1-\delta))^2}\left[\frac{(1-r)^2\delta_i}{\alpha}+\frac{2(1-r)r\delta\delta_i}{\alpha}+\right.$$
$$\left.+\frac{r^2\delta^2}{\alpha}+r(1-r)\left(\frac{\beta-\alpha}{\alpha\beta}\right)(1-\delta)(\delta_i-\delta)\right].$$

Focus on the term inside the square brackets in the equation above, and collect the $\delta_i\alpha^{-1}$ terms that can be found in the first, in the second, and in the last addendum. This yields:

$$-\frac{(1-r)}{(1-r(1-\delta))^2}\left[\left((1-r)^2+2(1-r)r\delta+r(1-r)(1-\delta)\right)\frac{\delta_i}{\alpha}+\right. \tag{A6}$$
$$\left.+\frac{r^2\delta^2}{\alpha}-\frac{r(1-r)(1-\delta)\delta_i}{\beta}-r(1-r)(1-\delta)\delta\left(\frac{\beta-\alpha}{\alpha\beta}\right)\right].$$

Collecting $r\delta$, the second and the fourth addenda in the square brackets of the last equation become $r\delta\left[r\delta\alpha^{-1}-(1-r)(1-\delta)\left(\alpha^{-1}-\beta^{-1}\right)\right]$, that is: $r\delta[(-1+r+\delta)\alpha^{-1}$ $+(1-r)(1-\delta)\beta^{-1}]$. Because $(1-\delta)\beta^{-1}=\delta\alpha^{-1}$, the latter expression can be written as $r\delta\alpha^{-1}(\delta-(1-r)(1-\delta))$. Using $(1-\delta)\beta^{-1}=\delta\alpha^{-1}$ in the third addendum gives $-r(1-r)\delta\delta_i\alpha^{-1}$; finally, adding this expression to the first addendum and simplifying we obtain $(1-r)\delta_i\alpha^{-1}$. Accordingly, Eq. (A6) can be rewritten as:

$$-\frac{(1-r)}{\alpha(1-r(1-\delta))^2}\left[(1-r)\delta_i+r\delta(\delta-(1-r)(1-\delta))\right].$$

Substituting for δ and δ_i in the last expression, we obtain

$$-\frac{(1-r)}{\alpha\left[1-r\left(1-\frac{\alpha}{\alpha+\beta}\right)\right]^2}\left[(1-r)\frac{\alpha}{\alpha+\beta_i}+r\frac{\alpha}{\alpha+\beta}\left(\frac{\alpha}{\alpha+\beta}-(1-r)\left(\frac{\beta}{\alpha+\beta}\right)\right)\right],$$

and simplifying:

$$-\frac{(1-r)(\alpha+\beta)^2}{[\alpha+(1-r)\beta]^2}\left[\frac{1-r}{\alpha+\beta_i}+\frac{r}{(\alpha+\beta)^2}(\alpha-(1-r)\beta)\right],$$

from which Eq. (12) in the main text follows immediately. Notice that, when $\beta_i = \beta$, we obtain an expression that is equivalent to social welfare as defined by Morris and Shin (2002).