

The Wealth-Consumption Ratio: A Litmus Test for Consumption-based Asset Pricing Models*

Hanno Lustig
UCLA and NBER

Stijn Van Nieuwerburgh
NYU Stern and NBER

Adrien Verdelhan
Boston University

April 11, 2007

Abstract

The volatility of the price-dividend ratio on stocks, the predictability of stock returns, and the lack of predictability in dividend growth are commonly interpreted as evidence of substantial time-variation in risk premia. We construct the wealth-consumption ratio for the U.S., the price-dividend ratio on *total wealth*. We show that it is at least five times less volatile than the price-dividend ratio on stocks. The wealth-consumption ratio encodes information about conditional market prices of risk, and hence about asset prices. Matching its properties is a litmus test for consumption-based asset pricing models. Models that match the predictability of equity returns impute too much predictability to total wealth returns and hence too much volatility to the wealth-consumption ratio, because they rely on time variation in the risk premium on total wealth. The smoothness of the wealth-consumption ratio suggests that there may be less time-variation in market prices of risk than commonly inferred from equity prices alone.

*Lustig: Department of Economics, University of California at Los Angeles, Box 951477, Los Angeles, CA 90095; hlustig@econ.ucla.edu; Tel: (310) 825-8018; <http://www.econ.ucla.edu/people/faculty/Lustig.html>. Van Nieuwerburgh: Department of Finance, Stern School of Business, New York University, 44 W. 4th Street, New York, NY 10012; svnieuw@stern.nyu.edu; Tel: (212) 998-0673; <http://www.stern.nyu.edu/~svnieuw>. Verdelhan: Department of Economics, Boston University, 270 Bay State Road, Boston, MA 02215; av@bu.edu; Tel: (617) 353-6324; <http://people.bu.edu/av>. The authors would like to thank Dave Backus, Geert Bekaert, John Campbell, John Cochrane, Ricardo Colacito, Pierre Collin-Dufresne, Greg Duffee, Darrell Duffie, Lars Peter Hansen, Francis Longstaff, Sydney Ludvigson, Thomas Sargent, Kenneth Singleton, Stanley Zin, and participants of the NYU macro lunch, Stanford GSB finance seminar for comments. This material is based upon work supported by the National Science Foundation under Grant No 0550910.

The log wealth-consumption ratio is at least five and possibly as much as twenty times less volatile than the log price-dividend ratio on equity, depending on how it is measured. The same way that volatility in the price-dividend ratio on equity signals (mostly) changes in expected future equity returns, the volatility in the wealth consumption ratio signals (mostly) changes in expected future *total wealth returns*. However, the volatility gap between the wealth-consumption and the price-dividend ratio implies that there is much less total wealth return predictability than equity return predictability. An important challenge for our leading asset pricing models is then to generate predictability in equity returns without too much time variation in expected total wealth returns.

Why should economists care? At the end of 2005, equity ownership accounted for only 20% of household financial wealth. As we broaden the class of assets under consideration and include other financial assets (bonds, bank accounts, life insurance, etc), housing wealth, non-corporate business wealth, and durable wealth, total household wealth looks increasingly less risky. A lower risk premium on total wealth implies an average wealth-consumption ratio well above the price-dividend ratio on equity of around 25. Models that match equity return predictability paint a very different picture. Their average total wealth risk premium is closer to the equity premium and the average wealth-consumption ratio is well below 15. Using data on aggregate consumption and labor income, it is straightforward to show that they imply too little human wealth. Put differently, their implied risk premium on human wealth is too high. By constructing a measure of total wealth, we put ourselves in a position to confront these models with the restrictions imposed by total wealth returns.

The empirical failure of the canonical consumption-based asset pricing model has spawned a large literature that addresses its shortcomings. Within the representative agent paradigm, two main avenues have been successfully explored. The first approach introduces time-varying risk-aversion in preferences. The external habit model of Campbell and Cochrane (1999), henceforth EH model, fits in this category.¹ The EH model was designed to show that equilibrium asset prices can be made to look like the data in a world without predictability in cash-flows, i.e. aggregate consumption and dividend growth are i.i.d. The second approach introduces predictability in aggregate consumption growth. The long-run risk model of Epstein and Zin (1991) and Bansal and Yaron (2004), henceforth LRR, falls under this heading.^{2,3} The LRR model embodies a different

¹Early contributions in the habit literature include Abel (1990), Constantinides (1990), Ferson and Constantinides (1991), Abel (1999). See Menzly, Santos, and Veronesi (2004) and Wachter (2006) for more recent contributions. Chen and Ludvigson (2007) estimate the habit process for a class of EH models.

²Hansen, Heaton, and Li (2006), Parker and Julliard (2005) and Malloy, Moskowitz, and Vissing-Jorgensen (2005) measure long-run risk based on leads and long-run impulse responses of consumption growth. Bansal, Kiku, and Yaron (2006) estimate the long-run risk model. Piazzesi and Schneider (2006) study its implications for the yield curve, Bansal, Dittmar, and Lundblad (2005) and Yang (2007) study the implications for the cross-section of equity portfolios, and Benzoni, Goldstein, and Collin-Dufresne (2005) for credit spreads.

³Bekaert, Engstrom, and Grenadier (2005) are the first to combine features of both models. Bansal, Gallant,

philosophy: it tries to make sense of asset prices in a world where persistent shocks to cash-flows are the driving force. Because these shocks are small, predictability in consumption and dividend growth is hard to detect. These two models are the workhorses of modern finance, because reasonably calibrated versions deliver a large equity premium, a low risk-free rate, and time-varying expected returns.

Despite this dichotomy, we show that both models have a log stochastic discount factor (SDF) that is linear in the same two asset pricing factors: the change in log consumption and the change in the log wealth-consumption ratio. This SDF representation highlights the importance of the wealth-consumption ratio for asset prices. By constructing the wealth-consumption ratio in the data we render the SDF observable. Thus, the properties of the wealth-consumption ratio are intimately linked to the conditional market prices of risk in both frameworks. The low volatility of the wealth-consumption ratio indicate that expected total wealth returns have only moderate time variation. The Euler equation for the total wealth return links the expected total wealth return to the conditional volatilities of the log SDF and the log return, and their conditional correlation. Because the log total wealth return and the log SDF are linear in the same two factors, they are (almost) perfectly negatively correlated, and the conditional variance of the log SDF is (almost) proportional to the expected log total wealth return. Hence, the moderate time variation in expected total wealth returns suggests that the conditional volatility of the stochastic discount factor only fluctuates moderately. The low volatility of the wealth-consumption ratio suggests that there is less variation in these market prices of risk than commonly inferred from equity markets.

Measuring the wealth-consumption ratio requires an estimate of total wealth. Total wealth is the sum of broadly-defined financial wealth and human wealth. Human wealth is unobservable: we observe its cash-flows, labor income, but not its discount rate. Lettau and Ludvigson (2001a, 2001b) measure the cointegration residual between log consumption, broadly-defined financial wealth, and labor income, “*cay*”. Their construction relies on cointegration between log human wealth and log labor income, i.e. the existence of a stationary price-dividend ratio on human wealth. That measure is not directly useful for our purposes because *cay* does not take into account the contribution of the volatility of price-dividend ratio on human wealth to the volatility of the the wealth-consumption ratio, the price-dividend ratio on total wealth. The literature contains (at least) three proposals to model expected returns on human wealth. Campbell (1996) equates them to expected returns on financial wealth, Shiller (1995) argues that they are constant, and Jagannathan and Wang (1996) implies that they equal expected labor income growth.⁴ In Section 1, we use these models to construct total wealth returns, and ultimately the wealth-consumption ratio *wc*.

and Tauchen (2007) estimate both long-run risk and external habit models.

⁴Lustig and Van Nieuwerburgh (2007) provide a fourth approach which is to back out returns on human wealth from observed aggregate consumption.

Our benchmark wc series is the one arising from the Campbell (1996) assumption. It has the desirable theoretical feature that the human wealth share is stationary in the asset pricing models that are to follow. More importantly, it generates a log wealth-consumption ratio that is four times more volatile than the other two methods and than $-cay$. As such, it is a conservative gauge on the amount of variability in the wealth-consumption ratio. Its volatility is 8.4% per quarter, still five times lower than the volatility of the price-dividend ratio on the CRSP stock universe. A standard Campbell and Shiller (1988) equation links the wealth-consumption ratio to expected future total wealth returns (with a negative sign) and expected future consumption growth rates (with a positive sign). A similar relationship links the price-dividend ratio on equity (pd^m) to future equity returns and dividend growth rates. Therefore, a lower volatility for wc than for pd^m implies lower variation in expected future total wealth returns and/or consumption growth rates than in future equity returns and/or dividend growth rates. The *decomposition* facts for the wc ratio are similar to what many have found for the pd ratio before. First, the predictability of wc is concentrated in returns (96%) and not in cash-flow (consumption) growth (4%). Second, most of the variation in future expected returns is associated with variation in future risk premia rather than future risk-free rates.

With those volatility and predictability facts in hand, we turn to the LRR and the EH model. They constitute additional moments to confront the models with. Because the models' asset pricing implications are tied to the properties of the wealth consumption ratio through the SDF, matching these moments is of first order importance. In Section 2, we study the benchmark calibration of the long-run risk model. Consumption growth features a small, persistent component as well as heteroscedasticity. The calibration successfully generates a large equity risk premium and a low risk-free rate. Because the dividend claim is exposed to more long-run risk than the consumption claim, the model's equity risk premium is substantially higher than the total wealth risk premium. Correspondingly, the wealth-consumption ratio is about five times less volatile than the price-dividend ratio on equity, and the average wealth-consumption ratio is much higher than the average price-dividend ratio on stocks. The implied human wealth share is around 80% and the corresponding expected return on human wealth around 4%. Hence, the LRR model successfully replicates the relative volatility facts, and delivers a plausible wealth composition. The volatility level of the wealth consumption ratio is somewhat too low compared to the data. More importantly, the mechanism that generates these appealing properties has several drawbacks. First, when the inter-temporal elasticity of substitution is greater than 1, as in the benchmark calibration, the wealth consumption ratio predicts future consumption growth rates with the same sign but future total wealth returns with the opposite sign as in the data. Second, too much of the volatility in wc derives from consumption growth predictability rather than return predictability, compared to the data. Third, most of the predictability in expected returns comes from predictability in

risk-free rates rather than predictability in risk premia. Fourth, because the dividend claim has more long-run risk than the consumption claim, it suffers from these problems to an even larger extent.

In Section 3, we turn to the external habit model. The EH model provides a powerful mechanism to generate time-varying risk premia. Its predictability decomposition properties are very much in line with the data. After all, it predicts that 100% of variability in the wc ratio and pd ratio is due to discount rates, because cash-flows are assumed i.i.d. However, the volatility of the wc ratio of 29% in the benchmark EH calibration is about as high as that for the pd^m ratio, and more than three times higher than the 8.4% in the data. In order to generate the right amount of time-variation in equity risk premia, the EH model induces too much predictability in total wealth risk premia. Because the consumption claim is too risky an asset, the average wealth-consumption ratio is low. The EH model fails to drive enough of a wedge between total wealth and equity.

Given the centrality of the wealth-consumption ratio for understanding asset prices, Section 4 uses a simulated method of moments exercise to investigate whether there exist calibrations of both models that are consistent with the properties of the observed wc ratio. We find that such a calibration exists for both models, but also that imposing these restrictions comes at a high price. The best-fitting calibration of the LRR model chooses an intertemporal elasticity of substitution below one in order to better match the return predictability facts. Unfortunately, this reinstates the risk-free rate puzzle and makes the total wealth risk premium disappear. Matching the predictability facts for equity requires giving up on the equity risk premium as well. An equity risk premium only survives in the presence of counter-factually high dividend growth predictability. The best-fitting calibration on the EH model moderates the volatility of the surplus-consumption ratio substantially in order to match the low volatility of the wealth-consumption ratio. This resulting time-variation in consumption and equity risk premia is lower by a factor of four. Still, the best-fitting calibration of the EH model matches many of the properties of the wealth-consumption ratio while avoiding the risk-free rate and equity premium puzzles. The main issue with the EH model is of a different nature: it generates an average wealth-consumption ratio that is too low.

Section 5 studies the implications of such a low wealth-consumption ratio. Given observed consumption, and an average wealth-consumption ratio of 9.3, the value implied by the best-fitting version of the EH model, US households had total per capita nominal wealth of \$300,000 in the last quarter of 2005. Their broadly defined financial wealth, which also includes real estate, net worth of non-corporate business, and consumer durable wealth, was \$168,000 in the same quarter. That only leaves \$130,000 or 43.6% of the total for human wealth. Put differently, the resulting price-dividend ratio on human wealth is 5.6, which corresponds to an average expected return on human wealth of almost 20%. This view of the world implies that an investment in human capital carries twice as much aggregate risk as an investment in equity. The best-fitting LRR model

provides very different predictions for human wealth: Its average wealth-consumption ratio of 30.7 translates into an average human wealth share of 84%, an expected return on human wealth of 5.2% per year, or a risk premium of 4.1% per year.

Many have argued that human wealth represents a much larger share than 45% of total wealth, as much as 90% (e.g., Jorgenson and Fraumeni (1989)). At the end of the paper, we provide independent evidence from the cross-section of stock and bond returns that corroborates the view that the human wealth share is large. We exploit the fact that the total wealth return has factor loadings of 1 and 1 on the two asset pricing factors that make up the SDF. We identify the total wealth risk premium as the sum of the estimated market prices of risk of consumption growth and the change in the wealth-consumption ratio. Across various specifications, we find a total wealth risk premium between 2.5% and 4.2%, substantially below the estimated equity risk premium. The implied wealth-consumption ratio varies between 33 and 76, and the corresponding human wealth share is between 85 and 94%.

Appendix A contains data sources and Appendix B provides details on how to deal with time-varying human wealth shares in the construction of the wealth-consumption ratio. Appendix C contains technical results for the LRR model, Appendix D does the same for the EH model. Appendix E contains supplementary material.

1 The Wealth-Consumption Ratio in the Data

In this section, we measure the wealth-consumption ratio in the data. This requires taking a stance on human wealth returns. Since we only observe the cash-flows on human wealth, it requires taking a stance on how to discount future labor income. We set up a VAR model in Section 1.1 to compute expected present values, evaluate several alternatives for human wealth returns in Section 1.2, and show what they imply for the properties of the wealth-consumption ratio in Section 1.3. The volatility of the wealth-consumption ratio is directly related to the amount of predictability in total wealth returns. We decompose the variability in wc into predictability of returns and predictability of consumption growth in Section 1.4. Finally, we compare this to the volatility and the predictability decomposition for stock returns in Section 1.5.

For consistency with the timing of the budget constraint in the models that are to follow, all returns in this paper are ex-dividend instead of cum-dividend. The return on a claim to aggregate consumption, the *total wealth return*, is defined as

$$R_{t+1} = \frac{W_{t+1}}{W_t - C_t} = \frac{C_{t+1}}{C_t} \frac{WC_{t+1}}{WC_t - 1}.$$

In what follows, we use lower-case letters to denote natural logarithms. The notation wc_t denotes

the log wealth-consumption ratio

$$wc_t = w_t - c_t = \log \left(\frac{W_t}{C_t} \right),$$

where wealth is measured at the beginning of the period and the consumption flow is over the ensuing period. Likewise $cw_t = -wc_t$ is the log consumption-wealth ratio. We start by using the Campbell (1991) approximation of the log total wealth return $r_t = \log(R_t)$ around the long-run average log wealth-consumption ratio A_0 . Because the return is ex-dividend, this log-linearization delivers:

$$r_{t+1} \approx \Delta c_{t+1} + wc_{t+1} + \kappa_0 - \kappa_1 wc_t, \quad (1)$$

with linearization coefficients that are non-linear functions of the long-run log wealth-consumption ratio A_0

$$\kappa_1 = \frac{e^{A_0}}{e^{A_0} - 1} > 1 \quad \text{and} \quad \kappa_0 = -\log(e^{A_0} - 1) + \frac{e^{A_0}}{e^{A_0} - 1} A_0. \quad (2)$$

By iterating forward on equation (1), we arrive at an expression that links the log consumption-wealth ratio at time t to expected future total wealth returns and consumption growth rates:

$$wc_t = \frac{\kappa_0}{\kappa_1 - 1} + \sum_{j=1}^H \kappa_1^{-j} \Delta c_{t+j} - \sum_{j=1}^H \kappa_1^{-j} r_{t+j} + \kappa_1^{-H} wc_{t+H}. \quad (3)$$

Because this expression holds both ex-ante and ex-post, one is allowed to add the expectation sign on the right-hand side. Imposing the transversality condition as $H \rightarrow \infty$ drops the last term, and delivers the familiar Campbell-Shiller decomposition of the “price-dividend” ratio for the consumption claim, the wealth-consumption ratio:

$$wc_t = \frac{\kappa_0}{\kappa_1 - 1} + E_t \left[\sum_{j=1}^{\infty} \kappa_1^{-j} \Delta c_{t+j} \right] - E_t \left[\sum_{j=1}^{\infty} \kappa_1^{-j} r_{t+j} \right] \equiv \frac{\kappa_0}{\kappa_1 - 1} + \Delta c_t^H - r_t^H. \quad (4)$$

We denote the cash-flow component by Δc_t^H and the discount rate component by r_t^H .

The total wealth return is a weighted sum of the return on broad financial wealth r_{t+1}^a and the return on human wealth r_{t+1}^y :

$$r_{t+1} = (1 - \nu_t) r_{t+1}^a + \nu_t r_{t+1}^y, \quad (5)$$

where ν_t is the share of human wealth in total wealth at time t . Two issues stand in the way of implementing (4): we need a model for calculate expectations and discounted sums, and we need a model for expected returns (discount rates) on human wealth.

1.1 Vector Error Correction Model (VECM)

We introduce a VECM that takes a stance of what is in the information set, and allows us to form expectations of the discounted sums in (4). The state vector z contains, in order of appearance, the log change in real per capita broad financial wealth (Δp^a), the log change in real per capita labor income (Δy), the log dividend-price ratio on broad financial wealth (dp^a), the log real three-month T-bill return (r_t^f), the spread between the ten-year T-bond and the three-month T-bill return (y_{sp}), the labor income share (lis), the log change in real per capita consumption (Δc), the spread between the AAA bond and the BBB bond return (def), and the spread between the returns on small value stocks and small growth stocks (val):

$$z_t = [\Delta p_t^a, \Delta y_t, dp_t^a, r_t^f, y_{sp_t}, lis_t, \Delta c_t, def_t, val_t]$$

The inclusion of the yield spread, the default spread and the value spread as return predictors is common in the literature (e.g., Campbell, Polk, and Vuolteenaho (2007)). All other variables are strictly necessary to construct the objects of interest. The data series are quarterly and run from 1952.I-2005.IV (216 quarters). Appendix A contains detailed data definitions for the variables in the state vector z_t .

We posit an autoregressive law of motion for the state vector: $z_{t+1} = \Psi z_t + \varepsilon_{t+1}$. All variables in the VAR are demeaned. This law of motion is augmented with the law of motion for cay , the demeaned cointegration vector between log consumption c , log financial wealth p^a , and log labor income y . Appendix A provides evidence for the existence of this cointegration relationship. Lettau and Ludvigson (2001a, 2001b) argue that cay contains information about future returns.

$$\mathbf{z}_{t+1} \equiv \begin{bmatrix} z_{t+1} \\ cay_{t+1} \end{bmatrix} = \begin{bmatrix} \Psi & \Gamma \\ \tilde{\Psi} & \tilde{\Gamma} \end{bmatrix} \begin{bmatrix} z_t \\ cay_t \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ \tilde{\varepsilon}_{t+1} \end{bmatrix} \equiv \Psi \mathbf{z}_t + \varepsilon_t \quad (6)$$

The VECM allows us to use cay as a predictor, while imposing the cross-equation restrictions dictated by the cointegration relationship (see Cochrane (1994)):

$$\begin{aligned} \tilde{\Psi} &= \lambda \Psi_7 - (1 - \bar{\nu})e_1 - \bar{\nu}e_2, \\ \tilde{\Gamma} &= 1 + \Gamma_7 - (1 - \bar{\nu})\Gamma_1 - \bar{\nu}\Gamma_2, \\ \tilde{\varepsilon} &= (e_7 - (1 - \bar{\nu})e_1 - \bar{\nu}e_2)\varepsilon, \end{aligned}$$

where Ψ_i denotes the i^{th} row of the matrix Ψ , and e_k as the k^{th} column of an identity matrix of the same dimension as Ψ . The other variables are defined similarly. The cointegration coefficient vector is $(1, 1 - \bar{\nu}, \bar{\nu})$, where $\bar{\nu}$ is the long-run average human wealth share.

This procedure to estimate (6) consists of four steps. First, we fix a long-run average labor

income share $\ell = .80$. We scale up our quarterly consumption series C by a constant to generate this average labor income share. Dividends on broadly-defined financial wealth equal $D^a = C - Y$. Together with the broad financial wealth data, we obtain the average log price-dividend ratio on broadly defined financial wealth, $E[pd^a] = 4.60$. Throughout, we set the average price-dividend ratio on human wealth equal to that on financial wealth. We do this for consistency with our benchmark model for human wealth returns discussed below. It immediately follows from this assumption that the average wealth-consumption ratio $E[wc] = A_0 = 4.60$, and that the average human wealth share equals the average labor income share $\bar{\nu} = \ell$. The value for A_0 translates into an average annual price-dividend ratio in levels of $e^{A_0 - \log(4)} = 25$.⁵ Second, we estimate Ψ , Γ , and Σ , the covariance matrix of the innovations ε . Third, we construct the VECM according to equation (6). Fourth, we redefine the state vector to include the demeaned cay_t as its tenth element. We denote the augmented vector, companion matrix, and innovation vector in bold.

1.2 Models of Human Wealth Returns

Next, we set up three different models which differ only in the vector C that links expected human wealth returns to the state vector:

$$E_t[r_{t+1}^y] = C' \mathbf{z}_t.$$

The first model, which we will end up using as our benchmark case, is the model of Campbell (1996). It assumes that expected human wealth returns are equal to expected financial asset returns: $E_t[r_{t+1}^y] = E_t[r_{t+1}^a]$, $\forall t$. Parallel to the total wealth return expression in (1), the log return on broad financial wealth can be written as

$$r_{t+1}^a = \kappa_0^a + \Delta d_{t+1}^a + pd_{t+1}^a - \kappa_1^a pd_t^a = \kappa_0^a + \Delta p_{t+1}^a + (\kappa_1^a - 1)dp_t^a \quad (7)$$

where the linearization constants are defined similarly to equation (2), and Δd_{t+1}^a denotes dividend growth on broadly defined financial wealth.⁶ The second equality comes from algebraic manipulation. Because Δp^a is the first element of the VECM and using equation (7), the vector C in the Campbell model amounts to $C' = e_1' \Psi + (\kappa_1 - 1)e_3'$. Campbell's model is the right one if financial wealth is a claim to a constant fraction of aggregate consumption. It will be our benchmark model.

The second model of Shiller (1995), models a constant discount rate on human wealth: $E_t[r_{t+1}^y] = 0$, $\forall t$, and therefore $C' = 0$. The third model, due to Jagannathan and Wang (1996), assumes that

⁵This assumption is made because it is a restriction implied by the Campbell model for expected returns on human wealth. Since this model will serve as our benchmark, we impose the same restriction on the Shiller and Jagannathan-Wang models for consistency. This assumption is defensible on empirical grounds as well, because it implies a human wealth share of 80%. The last section of the paper provides independent evidence from the cross-section of stock and bond returns that justifies this choice.

⁶By virtue of the equalization of the average price-dividend ratios on broad financial and human wealth, it follows that $\kappa_0^a = \kappa_0$ and $\kappa_1^a = \kappa_1$.

expected returns on human wealth equal the expected labor income growth rate. Because labor income growth is the second element of the VAR, the corresponding vector is $C' = e_2'\Psi$.⁷

1.3 Constructing the wc Ratio

In the simplest case, the human wealth share is constant: $\nu_t = \bar{\nu}$. The wealth-consumption ratio is then computed from (4), with (demeaned) expected future cash-flows and discount rates that are related to the state $\mathbf{z}_t$ by the vectors CF^c and DR^c :

$$\Delta c_t^H = CF^c \mathbf{z}_t, \quad CF^c = e_7' \Psi \kappa_1^{-1} (I - \kappa_1^{-1} \Psi)^{-1} \quad (8)$$

$$r_t^H = DR^c \mathbf{z}_t, \quad DR^c = [\bar{\nu} C' \Psi + (1 - \bar{\nu})(e_1' \Psi + (\kappa_1 - 1)e_3')] \kappa_1^{-1} (I - \kappa_1^{-1} \Psi)^{-1}. \quad (9)$$

Different models for human wealth result in different time series for r_t^H because they differ by their vector C . Appendix B shows how to deal with the more general case where the human wealth share moves over time. This time-variation makes no difference in the Campbell model, because expected returns on human and financial wealth are equated anyways. In the other models, the difference is quantitatively small.

The distinction between the log consumption-wealth ratio ($-wc_t$) and cay_t is important. The construction of cay imposes only that log human wealth and log labor income are cointegrated. It is silent on the relative volatilities of log human wealth and log labor income. In other words, human wealth may be more volatile than labor income, yet the two can be cointegrated. Since we are interested in the volatility of wc , we cannot simply use $-cay_t$ as our measure for the log wealth-consumption ratio wc_t .⁸

Table 1 shows the volatility of the various wealth-consumption ratios as well as their autocorrelation properties. The wc ratio is a persistent process; its autocorrelation is around .94 at the 1-quarter horizon, .83 at the 4-quarter horizon, and .74 at the 8-quarter horizon. There is somewhat more persistence in the first column than in the others. While the correlation properties are relatively similar across methods, the volatilities are not. The Campbell model implies a wealth consumption ratio that is four times more volatile than the other measures. It has a standard deviation of 8.45%, compared to 2% for the other measures.⁹ Figure 1 shows the time-

⁷Based on the work on Lustig and Van Nieuwerburgh (2007), we have studied a fourth model where the vector C is chosen so as to minimize the distance between (1) the volatility of consumption growth and (2) the correlation between consumption growth and financial returns between model and data. That C vector implies a strong negative correlation between human and financial wealth returns. The negative correlation generates a wc ratio that is even smoother: its volatility is 1.4% per year. In the interest of space, we will only study the robustness to the Jagannathan-Wang case. The conclusions for the Shiller case and the reverse engineering case are similar.

⁸Our cay measure differs somewhat from the Lettau-Ludvigson measure; see Appendix A for details.

⁹The standard deviation of the wc ratio is directly comparable for annual and quarterly frequencies, because wealth is a stock variable and the log annual consumption flow is approximately equal to the log quarterly consumption flow plus $\log(4)$. Because the latter is a constant, it does not affect the standard deviation.

series for the three measures of wc and $-cay$. The Jagannathan-Wang model, which implies that the dividend-price ratio on human wealth is constant, delivers a wc ratio that is almost identical to $-cay$.

[Table 1 about here.]

The main takeaway is that the wc ratio is not very volatile, compared to the log price-dividend ratio on stocks. The latter has a standard deviation of 42%, which is 5 to 20 times larger than that of the log wealth-consumption ratio. Its autocorrelation coefficients at the 1-, 4-, and 8-quarter horizon are very similar to those in Column 1: .95, .88, and .79.

[Figure 1 about here.]

1.4 Predictability of Total Wealth Returns

Why study the predictability of total wealth returns? A standard manipulation of the Euler equation for total wealth (see equations (41)-(43) in Appendix C.2) shows that the conditional Sharpe ratio on total wealth is approximately equal to the maximal Sharpe ratio, the conditional volatility of the log SDF m :

$$\frac{E_t[r_{t+1}^e]}{Std_t[r_{t+1}^e]} \approx Std_t[m_{t+1}],$$

where r^e denotes the expected return on total wealth in excess of the risk-free rate and corrected for a Jensen term. This is because the correlation between the log stochastic discount factor and the log total wealth return is close to -1, regardless of the state of the economy. This is not surprising, since both m and r will turn out to be linear in consumption growth and the change in the log wealth-consumption ratio.¹⁰ Furthermore, $Std_t[r_{t+1}^e]$ and $Std_t[m_{t+1}]$ move in lock step, so that, if there is substantial time variation in $Var_t[m_{t+1}]$, it should show up as variation in $E_t[r_{t+1}^e]$. Because $E_t[r_{t+1}^e]$ captures the predictability of the total wealth return, its variation is a direct measure of the variation in the conditional market price of risk.

We use the Campbell-Shiller variance decomposition to disentangle the cash-flow and discount rate components of variation in the wc ratio. The decomposition in equation (4) for the infinite horizon implies that:

$$V[w c_t] = Cov[w c_t, \Delta c_t^H] + Cov[w c_t, -r_t^H], \quad (10)$$

$$= Var[\Delta c_t^H] + Var[r_t^H] - 2Cov[r_t^H, \Delta c_t^H], \quad (11)$$

¹⁰That correlation $Corr_t[m_{t+1}, r_{t+1}^e]$ is exactly -1 in the external habit model of Section 3 and -0.89 in the benchmark calibration of the long-run risk model of Section 2.

Fluctuations in the wealth-consumption ratio indicate either consumption growth predictability or total wealth return predictability by the wc ratio. We use the VECM model from Section 1.1 to form the variances and covariances in equation (10).

$$\begin{aligned} Cov[wc_t, -r_t^H] &= DR^c \Omega DR^c - DR^c \Omega CF^c \\ Cov[wc_t, \Delta c_t^H] &= CF^c \Omega CF^c - DR^c \Omega CF^c \end{aligned}$$

where $\Omega = E[\mathbf{z}_t \mathbf{z}_t']$.¹¹ The first three columns in Table 2 report estimates of these covariance terms divided by the variance of the wc ratio. These are slope coefficients in the regression of discounted future log total wealth returns and log consumption growth rates on the current log wealth-consumption ratio. The slope coefficients have the expected sign: a higher wc ratio predicts higher future consumption growth rates and lower future returns. They are statistically significant for returns, but not for discounted consumption growth. Since these coefficients sum to one (100%), they also represent a variance decomposition of the wc ratio. At the infinite horizon, 96% is accounted by the return covariance and only 4% by the consumption growth covariance.

The variance of the wc ratio can also be decomposed into the sum of the variance of the cash flow component, the variance of the discount rate component, and their covariance (equation 11). The second panel of Table 2 shows that, at long horizons, the contribution of the first component is about .5% (column 4), the contribution from the cash-flow component is 93.3% (column 5), and their covariance accounts for the remainder.

Finally, we can decompose the covariance of wc_t with r_t^H into its covariance with future risk-free rates:

$$r_t^H = E_t \left[\sum_{j=1}^{\infty} (\kappa_1)^{-j} r_{t+j-1}^f \right] + E_t \left[\sum_{j=1}^{\infty} (\kappa_1)^{-j} (r_{t+j} - r_{t+j-1}^f) \right] \equiv r_t^{H,f} + r_t^{H,p}. \quad (12)$$

The last column reports the covariance of wc_t and $-r_t^{H,f}$. When scaled by the variance of the wc ratio, that covariance is 23% compared to 96% for the covariance with returns. There is some evidence for predictability of the risk-free rate by the wealth-consumption ratio, but the bulk of the predictability of the total wealth return comes from variation in future risk premia.

In conclusion, (1) the low variation in wc implies that there is not a lot of predictability in either total wealth returns or consumption growth, (2) most of the predictability we find is predictability in returns and not in consumption growth, (3) most of the predictability in returns comes from predictability of excess returns; future risk-free rates are only mildly forecastable.

[Table 2 about here.]

¹¹Similar closed-form formulae exist for finite-horizon sums of expected future returns and consumption growth rates.

1.5 Predictability of Stock Returns

In similar fashion, we can form a system to back out the predictability of stock returns and dividend growth. We extend the approach of Larrain and Yogo (2007) and form a system that contains log stock returns on CRSP (r^m), log dividend growth on CRSP (Δd^m), the log dividend-price ratio on CRSP (dp^m), the log real risk-free rate, and the same return forecasters as before: y_{sp_t} , def_t , and val_t . Denote that state vector by $\tilde{z}_t$:

$$\tilde{z}_t = \left[r_t^m, \Delta d_t^m, dp_t^m, r_t^f, y_{sp_t}, def_t, val_t \right],$$

with covariance matrix $\tilde{\Omega}$. We posit an AR(1) structure for this system, with companion matrix $\tilde{\Psi}$, and impose the log-linear definition of the log stock return on the estimation

$$r_{t+1}^m = \kappa_0^m + \Delta d_{t+1}^m + pd_{t+1}^m - \kappa_1^m pd_t^m. \quad (13)$$

The linearization constants κ_0^m and κ_1^m relate to the long-run average price-dividend ratio on stocks, A_0^m , in the same way as κ_0 and κ_1 relate to the long-run average *wc* ratio, A_0 (equation 2). It is crucial to allow A_0^m to differ from A_0 because we want to investigate the possibility of different risk premia on stocks and total wealth. Based on the long-run log price-dividend ratio on stocks, we estimate $A_0^m = 4.83$, which translates into an annual log price-dividend ratio in levels of 31.

We can then construct expected future stock returns and expected future dividend growth on the CRSP value-weighted portfolio as a function of the state vector. At the infinite horizon ($H = \infty$), that relationship is :

$$\begin{aligned} r_t^{H,m} &\equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^m)^{-j} r_{t+j}^m \right] = DR^m \tilde{z}_t, & DR^m &= (\kappa_1^m)^{-1} \left(e_1' \tilde{\Psi} \right) (I - (\kappa_1^m)^{-1} \tilde{\Psi})^{-1} \\ \Delta d_t^{H,m} &\equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^m)^{-j} \Delta d_{t+j}^m \right] = CF^m \tilde{z}_t, & CF^m &= (\kappa_1^m)^{-1} \left(e_2' \tilde{\Psi} \right) (I - (\kappa_1^m)^{-1} \tilde{\Psi})^{-1} \end{aligned}$$

The (demeaned) log price-dividend ratio satisfies $pd_t^m = \Delta d_t^{H,m} - r_t^{H,m}$ and has variance decompositions parallel to (10) and (11):

$$Cov \left[pd_t, \Delta d_t^{H,m} \right] + Cov \left[pd_t, -r_t^{H,m} \right] = V[pd_t] = Var \left[\Delta d_t^{H,m} \right] + Var \left[r_t^{H,m} \right] - 2Cov \left[r_t^{H,m}, \Delta d_t^{H,m} \right],$$

Fluctuations in the price-dividend ratio indicate either dividend growth predictability or stock

return predictability by the pd ratio. The covariances are given by

$$\begin{aligned} Cov \left[pd_t, -r_t^{H,m} \right] &= DR^m \tilde{\Omega} DR^m - DR^m \tilde{\Omega} CF^m \\ Cov \left[pd_t, \Delta d_t^{H,m} \right] &= CF^m \tilde{\Omega} CF^m - DR^m \tilde{\Omega} CF^m \end{aligned}$$

Table 3 reports the variance decomposition for the pd ratio. The decomposition shows that 80% of that variability is due to discount rates (returns) rather than to cash-flows (dividend growth). This is in line with what the literature has found (Campbell (1991) and Cochrane (1991)). The last column shows that almost none of the predictability in equity returns comes from predictability in risk-free rates. Rather, equity risk premia are strongly time-varying.

[Table 3 about here.]

In a world where expected returns and expected dividend growth rates are i.i.d., the price-dividend ratio is constant and there is no predictability for returns nor dividend growth. The literature has long established that this view of the world is at odds with the data. The same is true for the wealth-consumption ratio. Its moderate variability implies moderate predictability of total wealth returns, but not of consumption growth. The higher variability of the pd ratio, compared to the wc ratio, indicates more variability in stock returns than in total wealth returns. The next two sections investigate what the two leading, non-i.i.d. asset pricing models imply for the relative variability of wc and pd , and hence for the patterns of return and cash-flow predictability in the economy and in the stock market.

2 The Long-Run Risk Model

2.1 Setup

The long-run risk literature works off the class of preferences due to Kreps and Porteus (1978), Epstein and Zin (1989, 1991), and Duffie and Epstein (1992). These preferences impute a concern for the timing of the resolution of uncertainty. A first parameter α governs risk aversion and a second parameter ρ governs the willingness to substitute consumption inter-temporally. In particular, ρ is the inverse of the inter-temporal elasticity of substitution (EIS). In the special case where $\rho = \alpha$, the preferences collapse to the standard power utility preferences, used in Breeden (1979) and Lucas (1978).

We adopt the consumption growth specification of Bansal and Yaron (2004):

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \quad (14)$$

$$x_{t+1} = \rho_x x_t + \varphi_e \sigma_t e_{t+1}, \quad (15)$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1}, \quad (16)$$

where (η_t, e_t, w_t) are i.i.d. mean-zero, variance-one, normally distributed innovations. Consumption growth contains a low-frequency component x_t and is heteroscedastic, with conditional variance σ_t^2 . These two state variables capture time-varying growth rates and time-varying economic uncertainty.

The first proposition shows that the wealth-consumption ratio is a function of the two state variables x and σ_t^2 , as noted in Bansal and Yaron (2004). What is new is that the log SDF is a linear function of the growth rate of consumption and the growth rate of the log wealth-consumption ratio.

Proposition 1. *For $\rho \neq 1$, the log SDF in the long-run risk model can be stated as*

$$m_{t+1} = \left\{ \frac{1-\alpha}{1-\rho} \log \beta + \frac{\rho-\alpha}{1-\rho} \kappa_0 \right\} - \alpha \Delta c_{t+1} - \frac{\alpha-\rho}{1-\rho} (w c_{t+1} - \kappa_1 w c_t) \quad (17)$$

where the log wealth-consumption ratio is linear in the two state variables x_t and $\sigma_t^2 - \bar{\sigma}^2$:

$$w c_t = A_0 + A_1 x_t + A_2 (\sigma_t^2 - \bar{\sigma}^2). \quad (18)$$

Appendix C.2 proves this proposition. This result relies on the Campbell approximation of returns and the joint log-normality of consumption growth and the two state variables.^{12,13} The same appendix also details the (non-linear) system of equations that solves for the coefficients A_0 , A_1 , and A_2 in equation (18) as a function of the structural parameters of the model. This system imposes the non-linear dependence of κ_1 and κ_0 on A_0 (see equation 2) when solving for the long-run wealth-consumption ratio A_0 . This proposition highlights how central the properties of the wealth-consumption ratio are for the LRR model's asset pricing implications. For long-run consumption risk to matter for asset prices, it needs to affect the wealth-consumption ratio in a meaningful way. The data on the wealth-consumption ratio impose new discipline on how much long-run risk to allow for, and hence on the asset pricing implications of the LRR model.

¹²Appendix C.1 shows that the ability to write the SDF in the LRR model as a function of consumption growth and the consumption-wealth ratio is general. It does not depend on the linearization of returns, nor on the assumptions on the stochastic process for consumption growth in equations (14)-(16).

¹³When ρ equals 1, the wealth-consumption ratio is constant, and the SDF does not satisfy (17). However, Appendix E.1 shows that the total wealth risk premium equals the risk premium in a model without long-run risk when $\rho = 1$. Appendix E.1 also discusses the implications for the dividend claim.

Calibration We calibrate the long-run risk model choosing the benchmark parameter values of Bansal and Yaron (2004). Since their model is calibrated at monthly frequency but the data are quarterly, we work with a quarterly calibration instead. Appendix C.6 describes the mapping from monthly to quarterly parameters. We use $\rho = 2/3$, $\alpha = 10$, and $\beta = .997$ for preferences in (17); and $\mu = .45e^{-2}$, $\bar{\sigma} = 1.35e^{-2}$, $\rho_x = .938$, $\varphi_e = .126$, $\nu_1 = .962$, and $\sigma_w = .39 * 10^{-5}$ for the cash-flow processes in (14)-(16). The vector $\Theta^{LR} = (\alpha, \rho, \beta, \mu, \bar{\sigma}, \varphi_e, \rho_x, \nu_1, \sigma_w)$ stores these parameters.¹⁴ We then solve for the loadings of the state variables in the log wealth-consumption ratio expression (18) and find: $A_0 = 5.85$, $A_1 = 5.16$, and $A_2 = -175.10$. The corresponding linearization constants are $\kappa_0 = .0198$ and $\kappa_1 = 1.0029$.

Simulation We run 5,000 simulations of the model for 232 quarters each, corresponding to the period 1948-2005. In each simulation we draw a 232×3 matrix of mutually uncorrelated standard normal random variables for the cash-flow innovations (η, e, w) in (14)-(16). We start off each run at the steady-state ($x_0 = 0$ and $\sigma_t^2 = \bar{\sigma}^2$). For each run, we form log consumption growth Δc_t , the persistent component of consumption growth x_t , the variance of consumption growth σ_t^2 , the log wealth-consumption ratio wc_t and its first difference, the log total wealth return r_t , etc. We compute their first and second moments.¹⁵ These moments are based on the last 216 quarters only, for consistency with the length for our data for consumption growth and the growth rate of the wealth-consumption ratio (1952.I-2005.IV).¹⁶ Column 1 of Table 4 reports the moments for the long-run risk model under the benchmark calibration. All reported moments are averages of the statistics across the 5,000 simulations. The standard deviation of these statistics across the 5,000 simulations can be interpreted as a small-sample bootstrap standard error on the moments, and is reported it in parentheses below the point estimate.

Consumption Growth The first and second row show that consumption grows at a quarterly rate of 0.45%, close to the 0.52% in the data (last column). The standard deviation of consumption growth is 1.43%, or three times higher than in the data.¹⁷ The third row displays the first-order autocorrelation of quarterly consumption growth. The fourth and fifth rows show the 4-quarter and 8-quarter auto-correlations. None of the autocorrelations are statistically different from zero. In

¹⁴The corresponding monthly values are $\Theta^{LR} = (10, .6666, .998985, .0015, .0078, .044, .979, .987, .23 * 10^{-5})$.

¹⁵Most population moments are known in closed-form, so that we do not have to simulate. However, the simulation approach has the advantages of generating small-sample biases that may also exist in the data and delivering (bootstrap) standard errors.

¹⁶This has the added benefit that the first 16 quarters are “burn-in,” so that the first observation we use for the state vector is different in each run.

¹⁷We have also computed annualized consumption growth in the model by aggregating consumption growth across the quarters in the year. The volatility of annual consumption growth in the model is still more than two times higher than in the data. Our empirical measure of consumption growth excludes durable consumption, but this makes little difference for the volatility measure.

the data, there seems to be some positive serial correlation in consumption growth at the 1-quarter horizon, but most of it disappears at the annual horizon.

[Table 4 about here.]

2.2 Properties of the Wealth-Consumption Ratio

Rows 6-9 report the standard deviation, and autocorrelation properties of the log change in the wealth-consumption ratio Δwc . This is the second asset pricing factor in the long-run risk model. It has a mean of zero in model and data. Its standard deviation is 0.90% in the long-run risk model, much below the 4.57% in the data. The first difference of wc has near-zero autocorrelation in the model, as it does in the data. The correlation between the two asset pricing factors Δc and Δwc is -.06 in the model (Row 10), close to the data where it is -.04.

Most importantly, the wealth-consumption ratio is not very volatile in the long-run risk model. Its standard deviation is 2.35%, compared to 8.45% in the data (Row 11). The variance of the wealth-consumption ratio equals:

$$V[wc_t] = A_1^2 V[x] + A_2^2 V[\sigma^2 - \bar{\sigma}^2] = \frac{(1 - \rho)^2}{(\kappa_1 - \rho_x)^2} V[x] + A_2^2 V[\sigma^2 - \bar{\sigma}^2]$$

Almost all of the variability in wc comes from the first term; A_1 is about 5 and the volatility of x is about 0.5%. The inter-temporal elasticity of substitution of 1.5 ($\rho = .66$) is close enough to 1 to keep the volatility of wc down. The LRR model induces substantial persistence in the wc ratio: its auto-correlation coefficient is 0.91 at the 1-quarter horizon, 0.69 at the 4-quarter horizon, and 0.47 at the 8-quarter horizon (Rows 12-14). The auto-correlation at the 13-quarter horizon is the first that is no longer different from zero at the 95% level according to the small-sample bootstrap standard error. This persistence is somewhat lower in the data at the 4- and 8-quarter horizon. As in the data, the wc ratio in the LRR model has a positive correlation with both asset pricing factors (Rows 15-16).

The last panel of Table 4 reports asset pricing moments that are determined by the growth rates of consumption and the wc ratio. The log risk-free rate r^f in the LRR model is 0.35% per quarter on average (Row 17), close to the 0.27% per quarter in the data. The T-bill return is somewhat smoother in the model than in the data (.30% standard deviation compared to .55%). The log total wealth return, $r_t = r_0 + \Delta c_t + \Delta wc_t$, has a volatility of 1.64% per quarter in the LRR model, which is lower than in the data. The low autocorrelation in Δwc and Δc generate low autocorrelation in total wealth returns. This is a first hint of low predictability in total wealth returns, which we discuss in detail below. The total wealth return is strongly positively correlated with consumption growth (+.84). This happens because most of the action in the total wealth return comes from consumption growth. This correlation is only .07 in the data.

We conclude that the benchmark calibration of LRR model mostly captures the persistence, and correlation properties of the wealth-consumption ratio well, but understates its volatility.¹⁸

2.3 Predictability of Total Wealth Returns

In the LRR model, the (demeaned) log wealth-consumption ratio can be decomposed into a discount rate and a cash-flow component:

$$wc_t = \underbrace{\frac{1}{\kappa_1 - \rho_x} x_t}_{\Delta c_t^H} - \underbrace{\frac{\rho}{\kappa_1 - \rho_x} x_t - A_2 (\sigma_t^2 - \bar{\sigma}^2)}_{r_t^H}.$$

See Appendix C.3. The discount rate component itself contains a risk-free rate component and a risk premium component. The persistent component of consumption growth x_t drives only the risk-free rate effect (first term in r_t^H). It is governed by ρ . In the log case ($\rho = 1$), the cash flow loading on x and the risk-free rate loading on x exactly offset each other. The risk premium component is driven by the heteroscedastic component of consumption growth.¹⁹

The top panel of Table 5 shows the variance decomposition of the wc ratio. It shows the theoretical (population) moments from equations (10) and (11) in Column 2, and the sample moments that result from the simulation under the benchmark calibration (Column 3). The expressions for the theoretical covariances of wc_t with Δc_t^H and $-r_t^H$ show that both cannot simultaneously be positive. When $\rho < 1$, the sign on the regression coefficient of future consumption growth on the log wealth-consumption ratio is positive, but the sign on the return predictability equation is negative unless the heteroscedasticity mechanism is very strong. The opposite is true for $\rho > 1$. We recall that in the data, both signs are positive and the return predictability effect is the strongest. In the benchmark calibration of the LRR model, we are in the first case: The EIS is high ($\rho < 1$) and the heteroscedasticity channel is not very powerful. As a result, most of the volatility in the wealth-consumption ratio arises from consumption growth. The last row of the top panel shows that cash flows account for 300% of the variance of wc_t . The other -200% are accounted for by the covariance with future returns. Despite the large standard errors on the infinite horizon variance decomposition, this decomposition is rejected by the data.²⁰ Furthermore, virtually all predictability in discount rates arises from predictability in risk-free rates in the benchmark LRR model. In the data, less than one-third came from the risk-free rate channel.

¹⁸The LRR model's volatility is closer to the volatility implied by the Shiller and Jagannathan-Wang methods of computing the wealth-consumption ratio.

¹⁹The heteroscedasticity also affects the risk-free rate component, see equation (51) in the Appendix. But without heteroscedasticity, there would be no time-variation in risk premia.

²⁰The discounted cash-flow slope beta $Cov[w c_t, \Delta c_t^H] / V[w c_t]$ equals $(1 - \rho + \frac{(A_2)^2 (\kappa_1 - \rho_x)^2}{1 - \rho} \frac{V[\sigma^2 - \bar{\sigma}^2]}{V[x]})^{-1}$. In the absence of the heteroscedasticity effect, that slope coefficient is equal to $1/(1 - \rho)$. A small cash-flow effect arises for a ρ far from 1. In the case with heteroscedasticity, $1/(1 - \rho)$ is an upper bound on the cash-flow beta.

To sum up, (i) the LRR model delivers a low volatility of the wealth-consumption ratio, mostly by imputing predictability into the risk-free rate and (ii) the slope coefficient in the return predictability regression has the opposite sign as in the data.

[Table 5 about here.]

Total Wealth Risk Premium Table 6 reports on the total wealth risk premium and the average wealth-consumption ratio. These moments are not immediately observable in the data. Equation (19) is useful for understanding the source and magnitude of the total wealth risk premium:

$$E_t[r_{t+1}^e] \equiv E_t[r_{t+1} - r_t^f] + \frac{1}{2}V[r_{t+1}] = b'\Sigma_t^{ff}[1, 1]' = \ell_{1t}^{LRR} + \ell_{2t}^{LRR}, \quad (19)$$

The first equality defines the expected excess return on the consumption claim, including a Jensen term. The second equality shows that the total wealth risk premium is the product of the asset pricing factor loadings b and the covariance matrix of the two asset pricing factors, consumption growth and the change in the log wealth-consumption ratio $\Sigma_t^{ff} \equiv Cov_t[f_{t+1}, f_{t+1}]$, $f_{t+1} = [\Delta c_{t+1}, \Delta wc_{t+1}]$. The last equality shows that the total wealth risk premium equals the sum of the market prices of risk of aggregate consumption growth (ℓ_{1t}^{LRR}) and the change in the wealth-consumption ratio (ℓ_{2t}^{LRR}). The benchmark calibration of the long-run risk model implies an unconditional risk premium on the consumption claim of 40 basis points per quarter, or 1.60% per year (Row 1). Each market price of risk accounts for about half of the total wealth risk premium in the benchmark LRR model (Rows 3 and 4). A problem with the model is that the total wealth risk premium is too smooth; its volatility is only 0.3% per quarter (Row 2). This is a different reflection of the fact that most of predictability of returns comes from a risk-free rate channel rather than from a risk premium channel. The mean total wealth return is the sum of the total wealth risk premium and the risk-free rate; it equals .74% per quarter (Row 5). This relatively low total wealth return corresponds to a high mean wealth-consumption ratio. In the benchmark LRR calibration $A_0 = 5.85$ (Row 6). The quarterly mean wealth-consumption ratio of 5.85 in logs implies an annual mean wealth-consumption ratio of 87 in levels.

[Table 6 about here.]

2.4 Predictability of Equity Returns

Finally, we study the LRR model's implications for equity returns. The equity risk premium is the expected excess return on a claim to aggregate dividends in excess of the risk-free rate. We follow the specification and the calibration of dividend growth in Bansal and Yaron (2004):

$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1}$$

The shock u_t is orthogonal to the other cash-flow innovations (η, e, w) . Just like the log wealth-consumption ratio, the log price-dividend ratio on stocks pd^m is linear in the same two state variables x_t and $(\sigma_t^2 - \bar{\sigma}^2)$. The coefficients (A_0^m, A_1^m, A_2^m) solve a system of three non-linear equations. Dividend growth has the same mean as consumption growth in the model, but is more volatile (6.25% per quarter versus 1.45%). This greater volatility comes from a larger loading on the long-run risk component x_t ($\phi = 3 > 1$) as well as from a larger loading on the heteroscedasticity component $\sigma_t^2 - \bar{\sigma}^2$. Appendix C.5 proves the linearity, provides expressions for the coefficients, and describes the parameter choices in detail. Its logic is similar to the pricing of the consumption claim.²¹ Row 7 of Table 6 shows that the equity risk premium is 1.39% per quarter or 5.6% per year. This is within the range of empirical estimates for the average excess returns on stocks in CRSP. The equity premium is 4%, 3.5 times higher than the total wealth risk premium. More long-run risk translates into a higher risk premium on stocks. It also corresponds to a lower average price-dividend ratio $A_0^m < A_0$.

The middle panel of Table 5 shows the variance decomposition of the pd^m ratio. The pd ratio in the LRR model is 6 times more volatile than the wc ratio, close to the ratio in the data. Its volatility is only 16%, however, compared to 42% in the data. As was the case for the wc ratio, the pd ratio strongly predicts future cash-flows, but predicts future returns with the wrong sign. The variance decomposition of the pd ratio shows that 128% of the long-run variance comes from its covariance with dividends and -28% from its covariance with returns. The data shows much more predictability of stock returns and less predictability of dividend growth.

3 The External Habit Model

Our second goal is to explore the connection between the external habit (EH) model and the wealth-consumption ratio.

3.1 Setup

We use the specification of preferences proposed by Campbell and Cochrane (1999), henceforth CC. The log SDF is

$$m_{t+1} = \log \beta - \alpha \Delta c_{t+1} - \alpha (s_{t+1} - s_t),$$

²¹The dividend growth specification in Bansal and Yaron (2004) does not impose cointegration with consumption growth. Appendix E.2 derives stock returns in a world with cointegration. For our purposes, the results are similar with and without cointegration. The main text focuses on the case without cointegration. Bekaert, Engstrom, and Grenadier (2005) and Bekaert, Engstrom, and Xing (2005) also consider an extension of the LRR model that imposes cointegration.

where X_t is the external habit, the log surplus-consumption ratio $s_t = \log(S_t) = \log\left(\frac{C_t - X_t}{C_t}\right)$ measures the deviation of consumption from the habit, and has the following law of motion:

$$s_{t+1} - \bar{s} = \rho_s(s_t - \bar{s}) + \lambda_t(\Delta c_{t+1} - \mu).$$

The steady-state log surplus-consumption ratio is $\bar{s} = \log(\bar{S})$. The parameter α continues to capture risk aversion. The “sensitivity” function λ_t governs the conditional covariance between consumption innovations and the surplus-consumption ratio and is defined below in (23). To stay with the spirit of the CC exercise, we assume an i.i.d. consumption growth process:

$$\Delta c_{t+1} = \mu + \bar{\sigma}\eta_{t+1}, \quad (20)$$

where η is mean zero, variance one, i.i.d., and normally distributed. It is the only shock in this model. The following proposition shows that the log SDF in the EH model is a linear function of the same two asset pricing factors as in the LRR model: the growth rate of consumption and the growth rate of the consumption-wealth ratio.

Proposition 2. *The log SDF in the external habit model can be stated as*

$$m_{t+1} = \log \beta - \alpha \Delta c_{t+1} - \frac{\alpha}{A_1} (wc_{t+1} - wc_t) \quad (21)$$

where the log wealth-consumption ratio is linear in the sole state variable $s_t - \bar{s}$,

$$wc_t = A_0 + A_1(s_t - \bar{s}), \quad (22)$$

and the sensitivity function takes the following form

$$\lambda_t = \frac{\bar{S}^{-1} \sqrt{1 - 2(s_t - \bar{s})} + 1 - \alpha}{\alpha - A_1} \quad (23)$$

Appendix D.1 proofs this proposition. The result relies on three assumptions: (1) the Campbell approximation of returns, (2) the joint log-normality of consumption growth and the state variable, and (3) the particular form of the sensitivity function in equation (23). Just like CC’s sensitivity function delivers a risk-free rate that is linear in the state $s_t - \bar{s}$, our sensitivity function deliver a log wealth-consumption ratio that is linear in $s_t - \bar{s}$. To minimize the deviations with the CC model, we pin down the steady-state surplus-consumption level $\bar{S}$ by matching the steady-state risk-free rate to the one in the CC model. Taken together with the expressions for A_0 and A_1 , this restriction amounts to a system of three equations in three unknowns $(A_0, A_1, \bar{S})$. The details are

in Appendix D.2. Appendix D.6 discusses an alternative way to pin down $\bar{S}$.²² The formulation of SDF in function of the wealth-consumption ratio suggests that, for the EH model to matter for asset prices, it needs to alter the properties of the wc ratio in the right way. Different assumptions on the surplus-consumption ratio dynamics translate into different processes for the wealth-consumption ratio.

Calibration We calibrate the long-run risk model choosing the benchmark parameter values of Campbell and Cochrane (1999). Since their model is calibrated at monthly frequency but our data are quarterly, we work with a quarterly calibration instead. Appendix D.7 describes the mapping from monthly to quarterly parameters. We use $\alpha = 2$, $\rho_s = .9658$, and $\beta = .971$ for preferences, and $\mu = .47e^{-2}$ and $\bar{\sigma} = .75e^{-2}$ for the cash-flow process (20), and summarize the parameters in the vector $\Theta^{EH} = (\alpha, \rho_s, \beta, \mu, \bar{\sigma})$.²³ After having found the quarterly parameter values, we solve for the loadings of the state variables in the log wealth-consumption ratio and find: $A_0 = 3.86$, $A_1 = 0.778$, and $\bar{S} = .0474$. The corresponding Campbell-Shiller linearization constants are $\kappa_0 = .1046$ and $\kappa_1 = 1.021583$. The simulation method is parallel to the one described for the LRR model.

Consumption Growth Column 3 of Table 4 reports the moments for the external habit model under the benchmark calibration. The first and second row show that consumption grows at a quarterly rate of .47%, somewhat lower than the .57% in our sample. The standard deviation of consumption growth is .75%, or 50% higher than in the data, but only half as large as in the BY model in Column 1. None of the autocorrelations in rows 3-5 are statistically different from zero.

3.2 Properties of the Wealth-Consumption Ratio

First and foremost, the wc ratio is too volatile in the EH model: it has a standard deviation of 29.3%, which is 3.5 times larger than in the data and 12.5 times larger than in the LRR model (Row 11). The auto-correlation properties of the wc ratio are a reasonably good fit with the data, similar to those of the LRR model. Second, rows 6-9 show that the standard deviation of Δwc is 9.5%, two times higher than in the data, and that Δwc has near-zero autocorrelation in the model, as in the data. The high volatility of the change in the wc ratio translates into a highly volatile total wealth return. Its standard deviation is 10.3% per quarter, more than two times higher than in the data (Row 19). Third, the correlation between the two asset pricing factors in the habit model is .90 (Row 10), whereas in the data it is only .07. Likewise, the total wealth

²²Appendix E.4 shows how to relax the Campbell-Shiller approximation of returns by including a second-order term in the approximation of $\log(\exp(wc_t) - 1)$. The proposition remains unchanged, and the coefficients A_0 and A_1 are unchanged as well for all practical purposes. This suggests that our arguments does not hinge on the accuracy of the Campbell-Shiller approximation.

²³The corresponding monthly values are $\Theta^{EH} = (\alpha, \rho_s, \beta, \mu, \bar{\sigma}) = (2, .9885, .990336, .1575e^{-2}, .433e^{-2})$.

return too strongly positively correlated with consumption growth (.91 versus .08), more so than in the data (Row 20). In the habit model this happens because most of the action in the total wealth return comes from changes in the wc ratio, and the latter has a correlation of .90 with consumption growth.²⁴ These high correlations arise from the fact that a single shock η ultimately drives both asset pricing factors. Fourth, the risk-free rate is 0.44% per quarter, close to the 0.27% in the data (Row 17). It is almost constant in the model (.03% in Row 18), confirming that our sensitivity function does not lead to a volatile risk-free rate.

3.3 Predictability of Total Wealth Returns

In contrast to the LRR model, the EH model asserts that all variability in returns arises from variability in risk premia. The wealth-consumption ratio only has a discount rate component, because aggregate consumption growth is assumed to be i.i.d.:

$$wc_t = \underbrace{\frac{(1 - \rho_s)}{\kappa_1 - \rho_s} \alpha (s_t - s) - \frac{\sigma^2 \bar{S}^{-2}}{\kappa_1 - \rho_s} (s_t - s)}_{r_t^H}$$

The bottom panel of Table 5 lists the components of the variance decomposition of the wc ratio in the EH model and their values arising from a simulation under the benchmark parametrization. Since there is no cash flow predictability, 100% of the variability of wc is variability of the discount rate component. The covariance between the wealth-consumption ratio and returns has the right sign: it is positive by construction. In the EH model, the discounted cash flow slope coefficient $Cov[wc_t, \Delta c_t^H] / var[wc_t]$ equals 0. This variance decomposition is close to the data. However, by overstating the variability of wc , the benchmark CC model overstates the predictability of the total wealth return.

Total Wealth Risk Premium Table 6 shows that the total wealth risk premium in the EH model is 267 basis points per quarter or 10.7% per year (Row 1). Recall that in the LRR model, the total wealth risk premium was only 1.6% per year. The factor representation in equation (19) decomposes the reward for risk, and Appendix D.4 provides the corresponding expressions. The factor loadings b are equal to [2,2.57]. Together with the covariance matrix of the factors, the loadings imply market prices of risk for consumption of ℓ_1^{EH} of .18% and ℓ_2^{EH} of 2.50% (Rows 3 and 4). Most of the risk compensation in the EH model is for bearing Δwc risk. The model implies strong time-variation in risk premia (Row 2). The mean total wealth return is 2.60% per year (Row 5). The high mean total wealth return translates into a low mean wealth-consumption

²⁴This correlation does not diminish much when we time-aggregate quarterly data. The corresponding correlation between the annualized series is .87.

ratio. The quarterly mean wealth-consumption ratio of 3.86 in logs implies an annual mean wealth-consumption ratio of only $e^{A_0 - \log(4)} = 12$ in levels. This compares to a value of 87 in the LRR model.

3.4 Predictability of Stock Returns

Finally, we look at the implications of the EH model for the equity risk premium. In Campbell and Cochrane (1999), dividend growth is i.i.d., with the same mean μ as consumption growth, and innovations that are correlated with the innovations in consumption growth. To make the dividend growth process more directly comparable across models, we write it as a function of innovations to consumption growth η and innovations u that are orthogonal to η :

$$\Delta d_{t+1} = \mu_d + \varphi_d \bar{\sigma} u_{t+1} + \varphi_d \bar{\sigma} \chi \eta_{t+1}. \quad (24)$$

We choose parameters φ_d and χ to match the volatility of dividend growth and its correlation with consumption growth to those in CC. The volatility of quarterly dividend growth is 5.6%, compared to .75% for consumption growth.

We lose the linearity of the log price-dividend ratio in the state variables and solve for pd^m using the numerical algorithm developed by Wachter (2005). Appendix D.5 contains the details of the dividend growth specification, the calibration, and the computation of the price-dividend ratio.²⁵ The last row of Table 6 shows that the equity risk premium in the EH model is somewhat higher than the total wealth risk premium: 3.30% per quarter, or 13.2% per year.²⁶ The EH model's predictions for stock return predictability are the same as for total wealth return predictability: all variability in the pd^m ratio comes from the discount rate channel. Variability in expected future stock returns itself is driven by risk premia rather than by risk-free rates. This characterization of stock return predictability is close to the one we found in the data.

4 Simulated Method of Moments

The result that both leading asset pricing models have a factor representation in terms of consumption growth and the change in the wealth-consumption ratio, and the fact that the asset pricing

²⁵The dividend growth specification in Campbell and Cochrane (1999) does not impose cointegration with consumption growth. Wachter (2006) and others assume that dividends are a levered-up version of consumption: $\Delta d_{t+1} = \phi \Delta c_{t+1}$, which also does not impose cointegration. In Appendix E.3, we develop a model with cointegration. Because the equity returns are similar in both cases, we focus on the case without cointegration.

²⁶We use the average excess return, corrected for the Jensen term, as a proxy because we do not have a closed form expression for the expected excess return in the EH model. For the same reason, we cannot compute the variability of the expected excess return. However, given the similarity between the consumption claim and the dividend claim in the EH model, it seems safe to assume that there is substantial time variation in the equity risk premium as well.

implications of the Lucas (1978)-Breedon (1979) model with only consumption growth are counter-factual suggest that successes of the LRR and the EH model must be linked to the properties of the wealth-consumption ratio that they generate. While both models match the risk-free rate and generate substantial equity risk premia, we found that they generate dramatically different wealth-consumption ratios. In the benchmark calibration of the LRR model, the wealth-consumption ratio is very high on average, persistent, and not very volatile. In the benchmark calibration of the EH model, the wealth-consumption ratio is much lower on average, also persistent, and very volatile. In the LRR model, most predictability is situated in cash-flows. The coefficient in a regression of future returns on the wc ratio generates the wrong sign. Furthermore, risk-free rates are highly predictable and drive most predictability in returns. In sharp contrast, in the EH model, all variability is attributable to excess returns. The model implies no cash-flow predictability, and no risk-free rate predictability.

This logic prompts the question whether these models can be re-calibrated to generate wealth-consumption ratios that are more similar to the data. In Sections 4.1 and 4.2, we estimate the structural parameters of the LRR and the EH model, respectively, by minimizing the distance between the moments of the wealth-consumption ratio in the model and in the data. In both models we choose to fix the mean and standard deviation of consumption growth to those in the data. That puts both models on equal footing and ensures that the results are not driving by excessive consumption growth volatility. Our strategy is to only match features of the joint distribution of the wealth-consumption ratio, and to treat the pricing of equity (the dividend claim) as an *out-of-sample* test. We attempt to match fourteen moments, all moments in Table 4 except those indicated with a ^{*ni*} superscript. The last and fifteenth moment is the slope coefficient in a regression of expected future returns r_t^H on the log wealth-consumption ratio: $Cov[r_t^H, wc_t]/Var[wc_t]$. The latter is .964 in the data.²⁷

4.1 Estimation Exercise for the LRR Model

In the LRR model, we fix μ to match mean consumption growth and set $\sigma = .47 \left(1 + \frac{\varphi_e^2}{1-\rho_x^2}\right)^{-0.5}$ to match the standard deviation of consumption growth. We search over the remaining seven parameters to match the remaining thirteen moments. Once we find the distance-minimizing parameter vector, we simulate the model for 5,000 iterations to construct standard errors.²⁸ The

²⁷All the moments whose absolute value is below one receive a weight of two in the estimation, the other moments a weight of one. Equal weighting would artificially reduce the importance of the correlation moments.

²⁸We impose one last constraint which guarantees that when we simulate equation (16) under the final parameter estimates, σ_t^2 hits its reflecting boundary of 0 no more than 2% of the time. This constraint is $\bar{\sigma}^2 > 2 \left(\frac{\sigma_w^2}{1-\nu_1^2}\right)^{0.5}$. The left-hand side is the unconditional mean of σ_t^2 , the right-hand side is twice its standard deviation. The estimation algorithm itself uses population moments rather than simulation, because all moments are known in closed-form in the LRR model.

resulting moments are reported in the second column of Table 4. The minimum distance parameter vector is

$$\hat{\Theta}^{LRR} = (\alpha, \rho, \beta, \mu, \bar{\sigma}, \varphi_e, \rho_x, \nu_1, \sigma_w) = (13.7, 2.57, .996, .52e^{-2}, .30e^{-2}, .409, .942, .931, .17e^{-5}).$$

The coefficients in the wealth-consumption ratio are $(A_0, A_1, A_2) = (4.81, -23.67, 5008)$. Θ^{LRR} features a slightly higher risk aversion parameter as in the benchmark calibration (13.7 versus 10), but a much lower inter-temporal elasticity of substitution ($\rho^{-1} = 0.39$ versus 1.5).²⁹ As a result, the signs on A_1 and A_2 are reversed compared to the benchmark calibration.

The SMM exercise is successful in that the squared distance is reduced from 3,554 in Column 1 to 21.4 in Column 2 (last row). The volatility of the wc ratio is now in line with the data. It is four times higher than in the benchmark model, despite the lower consumption growth volatility. This is due to a much higher $|A_1|$ and $|A_2|$ because the EIS is farther from 1. The unconditional covariance between consumption growth and returns drops from .84 to .28, as returns reflect predominantly changes in wc and no longer changes in consumption. This low correlation is one of the major successes of this calibration of the LRR model. The other key improvement, and one of the main reasons that the estimation chooses $\rho > 1$, is that the model now generates the correct return predictability sign (see last column of Table 5): a higher wc ratio forecasts lower total wealth returns. The SMM calibration increases the variability of expected future returns relative to expected future consumption growth. This comes at the expense of generating the wrong sign on the consumption growth predictability regression. Given that a choice of ρ is bound to match only one of these two moments, the estimation chooses to match the more salient feature of the data.

These improvements come at the expense of a risk-free rate that is too high at 8% per year, compared to 1% per year in the data. The risk-free rate is both higher and more variable because ρ is higher. The total wealth risk premium turns negative (Column 2 in Table 6). Because $\rho > 1$, the market price of Δwc risk turns negative. The average total wealth return actually increases because the risk-free rate goes up by so much. This lowers the average wealth-consumption ratio to 4.81, which corresponds to an annual wealth-consumption ratio in levels of 31. Despite the negative total wealth risk premium, the model still generates a positive equity risk premium. In order to maintain the same dividend growth volatility as in the data (5.6% per quarter) despite the lower consumption growth volatility, we scale up ϕ and φ_d by a factor of 3. As a result, we make the equity claim substantially riskier, and the model generates a large equity risk premium. However, this introduces an enormous amount of dividend growth predictability (panel two in Table 5), which is not a feature of the data. Because $\phi = 9 > \rho = 2.57$, the LRR model gets the

²⁹In recent work, Bansal, Kiku, and Yaron (2006) estimate an EIS smaller than one in the LRR model, but they argue that “after accounting for time-averaging effects, the most likely value for the population EIS is closer to 2.”

sign on the stock return predictability by the pd^m ratio wrong. Fixing this would require setting $\phi < 2.57$, which in turn would make the equity risk premium all but vanish.³⁰

As a robustness check, we have repeated the SMM exercise with the Jagannathan-Wang wc series as the target to match. The results are similar: the best-fitting parameter vector features a low equity premium and a high risk-free rate.

4.2 Estimation Exercise for the EH Model

In the EH model, we do not have closed-form expressions for the moments and we use a simulation technique. As for the LRR model, we fix μ and σ to match the first two moments of consumption growth. The SMM algorithm starts with an initial guess for the remaining three parameters in Θ^{EH} , simulates, and computes the weighted squared distance between the same fifteen moments in the model and the data. The SMM algorithm focuses its efforts on reducing the volatility of the wc ratio and its first difference. Column 4 of Table 4 reports the best-fitting moments for consumption growth and the wc ratio. The minimum distance parameter vector

$$\hat{\Theta}^{EH} = (\alpha, \rho_s, \beta, \mu, \bar{\sigma}) = (15.3, .973, .869, .52e^{-2}, .47e^{-2})$$

and the coefficients in the wealth-consumption ratio are $(A_0, A_1, \bar{S}) = (3.61, .771, .008)$. Θ^{EH} features a much higher risk aversion parameter as in the benchmark calibration (15.3 versus 2), a somewhat more persistent surplus-consumption ratio ($\rho_s = .973$ versus .966), and a much lower time discount factor ($\beta = .869$ versus .971). On average, consumption is less than 1% away from the habit versus 4.7% in the benchmark case.

The distance between the model's moments and the data falls from 1,084 to 18. The latter is slightly closer than the best-fitting LRR calibration even though the EH model is more parsimoniously parameterized (three fewer parameters). Increasing the persistence of the surplus-consumption ratio ρ_s and decreasing the volatility of consumption growth help to lower the volatility of wc , Δwc , and r . This volatility drop arises from a large decrease in the volatility of the log surplus-consumption ratio (from 38% in the benchmark parametrization to 13%), while the sensitivity of the wc ratio to the surplus-consumption ratio stays the same ($A_1 = .771$ versus .778). The autocorrelations in wc are closer to the data as well.

Improving on these volatility moments comes at the expense of worsening the correlations between Δc on the one hand and r (.98) and Δwc (.98) on the other hand. It also comes at the expense of a higher risk-free rate: it increases from 1.47% per year in the benchmark to 4.56% in the SMM calibration. The lower time discount factor is partially to blame for the higher risk-free

³⁰For example, if we only scale up φ_d and leave $\phi = 3$ at its benchmark value, then the equity risk premium is only 18 basis points per quarter.

rate. However, given the higher risk aversion and persistence in the surplus-consumption ratio, a higher β is needed to keep A_1 low and, with it, the volatility of wc .

Column 4 of Table 6 shows that the total wealth risk premium is still 2.17% per quarter. It is lower than in the benchmark, despite the higher risk aversion α , because the compensation for Δwc -risk is much lower. Because the total wealth risk premium remains relatively high and the risk-free rate increased, the average wealth-consumption ratio is very low ($A_0 = 3.62$). Finally, the equity risk premium remains high at 3.26% per year, after we adjust the dividend growth volatility so that it matches the one in the data (5.6% per quarter).³¹

The alternative models we discussed in Section 1, due to Shiller and Jagannathan-Wang imply a volatility of 2% for the wc ratio. As a robustness check, we have redone the SMM exercise with those wc ratios as our target and found that the equity risk premium puzzle and risk-free rate puzzle reappear in the EH model as well. In other words, for the EH model to match a wc ratio with a 2% volatility, most time-variation in risk premia needs to be shut down. This not only affects the consumption claim, but also the dividend claim.

5 Discussion: The Human Wealth Share

We conclude that both models can be parameterized such that their wealth-consumption ratios are broadly consistent with the wealth-consumption ratio in the data. In the LRR model, the EIS needs to be below 1 instead of above one to get the predictability properties of the total wealth return right. However, this calibration has its drawbacks: It leads to a substantial reduction in the total wealth risk premium and a substantial increase in the risk-free rate. The only way to save the equity risk premium is by increasing the long-run risk exposure of the dividend claim. This comes at the expense of counter-factually high dividend growth predictability. Fixing the dividend growth predictability problem drives the equity risk premium down to zero. In sum, the lack of consumption growth or dividend growth predictability and the strong evidence for return predictability -they are two sides of the same coin as Cochrane (2006) points out- limits the effectiveness of the cash-flow channel in the LRR model.

The external habit model delivers a large amount of time-variation in equity risk premia. It predicts that all variation in the price-dividend and wealth-consumption ratios comes from a discount rate channel instead of a cash-flow channel, a view that cannot be rejected by the data. The SMM exercise is able to bring the volatility in the EH model down to 10%, close to the 8.4% data. The main drawback with the SMM parametrization of the EH model is that it produces an

³¹As a robustness check, we tried to reduce the correlation between consumption growth and the total wealth return by giving that moment a weight of 20 in the estimation compared to a weight of 1 for the other moments. This attempt was unsuccessful insofar that the SMM algorithm never yielded a correlation below 0.92, and only at the expense of a very high risk-free rate and a volatile wc ratio.

average wealth-consumption ratio that is too low. A simple calculation illustrates.

In the last quarter of 2005, US per capita consumption was \$32,000 in nominal dollars. An average wealth-consumption ratio of 9.33 implies that nominal per capita wealth was \$298,000 in that quarter. We observe broad financial wealth (including housing wealth, etc.); it was \$168,000 in that same quarter. That leaves only \$130,000 for human wealth, or less than 44% of total wealth. The corresponding annual human wealth to labor income ratio, the price-dividend ratio on human wealth, is only $A_0^y = 5.56$. The corresponding expected return on human wealth is 20% per year.³² Given a 1% risk-free rate, this translates into a 19% risk premium on human wealth. This is twice the risk premium on equity observed in the data. In other words, the version of the EH model that is consistent with the volatility of the wc ratio and with the equity premium implies that the risk premium/discount rate on human wealth is so high that there is less than 50% human wealth in the economy. In the standard calibration of a real business cycle model, the human wealth share is two-thirds. Jorgenson and Fraumeni (1989) argue that it is 90%.

To generate a human wealth share of 80%, a reasonable value in this range, the wealth-consumption ratio needs to equal 25. The corresponding human wealth-to-labor income ratio is also 25, or 25 years worth of average labor income earnings. It implies a plausible expected return on human wealth of 4% per year. The external habit model can be parameterized to deliver an average wealth-consumption ratio of 25, but only at the expense of reducing the volatility of the price-dividend ratio (See Appendix D.6). The SMM calibration of the LRR model generates a high human wealth share (84% on average) and a low expected human wealth return (3%). We consider it one of its main strengths.

To conclude this discussion, we present evidence for a high wealth-consumption ratio from cross-sectional asset pricing data. These results suggest that 25 is a lower bound for the wealth-consumption ratio.

5.1 Cross-Sectional Asset Pricing

We use two sets of 32 equity and bond portfolios as test assets. The first set consists of 25 Fama-French portfolios sorted on book-to-market and size, 6 Fama bond portfolios sorted on maturity, and the CRSP value-weighted market return. The second set consists of 25 Fama-French portfolios sorted on size and long-term reversal, and the same bond portfolios and the CRSP market portfolio. We use log excess returns, corrected for a Jensen term.

We estimate market prices of risk following Fama and MacBeth (1973), and consider both unconditional and conditional versions of the Euler equation. For the latter, we use the lagged value of the wealth-consumption ratio as conditioning variable (re-scaled between 0 and 1); the

³²To see this, recall that the unconditional expected return is given by $E[r^y] = \kappa_0^y + A_0^y(1 - \kappa_1^y) + \mu_y$, where $\mu_y = 1.98\%$ is annual labor income growth and κ_0^y and κ_1^y are given by the analogous formulae to (2).

estimation contains portfolio returns and the same returns times the lagged wealth-consumption ratio. Asset pricing factors are growth rates of consumption and wealth-consumption ratio. Table 7 reports the corresponding market prices of risk, and shows that the fit is good (cross-sectional R^2 in excess of 67% and pricing errors below 50 basis points per year).

Once we have estimated average market prices of risk from the cross-section, we can back out the long-run average log wealth-consumption ratio, A_0 . This is possible because the average expected excess return on total wealth is simply the sum of the average market prices of risk on the two risk factors. In particular, we use the average risk-free rate, the average consumption growth rate, and the variance of the total wealth return to solve for the mean wealth-consumption ratio A_0 . Recalling that that κ_0 and κ_1 are non-linear functions of A_0 (equation 2), we can rewrite equation (19) as

$$A_0 - \log(e^{A_0} - 1) = E[\ell_{1t}] + E[\ell_{2t}] - \mu + E[r_t^f] - \frac{1}{2}[1, -1]\Sigma^{ff}[1, -1]'. \quad (25)$$

We obtain an annual total wealth risk premium that ranges between 2.4 and 4.2% per year (Column $RP^{\Delta c}$ of Table 7). Because we include the market portfolio as a test asset, we can also compute the difference between the equity risk premium and the total wealth risk premium (Column $RP^M - RP^{\Delta c}$). The latter is substantially lower: between 2.4 and 5.6% per year. These estimates imply average annual wealth-consumption ratios between 33 and 76 (last column). The corresponding human wealth shares are between 85% and 94%.

[Table 7 about here.]

6 Conclusion

Equity risk premia display substantial time-variation. In recognition of this salient feature of the data, modern asset pricing has developed mechanisms to generate such time-variation, mostly through variation in the conditional market price of risk. The external habit (EH) and the long-run risk model (LRR) are the two best-known and most successful examples. They rely on very different mechanisms to generate that time-variation: in the EH model all variation is driven by variation in risk premia (a discount rate mechanism), while in the LRR model most variation comes from a small predictable component in consumption and dividend growth (a cash-flow mechanism).

We find that the benchmark calibrations of these two models cannot match the predictability evidence we have presented for total wealth returns and equity returns. The LRR model can match the relative volatility of the wealth-consumption and price-dividend ratios, but relies on too much cash-flow predictability in doing so. It does not generate enough equity return predictability. On the other hand, the EH model can match the predictability of the equity returns but it overstates

the volatility in the wealth-consumption ratio. Using simulated method of moments estimation, we find the calibration that matches the moments of the observed wc ratio. The EH model, on the one hand, shows a close fit for a higher risk aversion parameter. However, that best-fitting calibration implies that the risk premium on human wealth is implausibly large, and the human wealth share implausibly low. The LRR model, on the other hand, can only be made consistent with the observed wc ratio at the expense of reintroducing the risk-free rate and equity premium puzzle, but it implies a more reasonable human wealth share. We conclude that both models provide important and separate pieces to the puzzle, but that the wealth-consumption ratio is a litmus test that either model considered in isolation ultimately fails.

Our findings have broad implications. The evidence on equity return predictability, large variation in $k - period$ ahead expected equity returns $E_t[r_{t+k}^{m,e}]$, has been commonly interpreted to mean that the conditional market price of risk, $\sigma_t[m_{t,t+k}]$, is highly volatile. Equity is a small asset class. It accounts for only 20% of broadly-defined financial household wealth, which itself only accounts for 20% of total wealth. If this common interpretation is accurate, large variations in market prices of risk should affect other assets classes, such as real estate, as well. The low volatility of the wealth-consumption ratio suggests that these other asset classes display much less predictability. This is true even if we assume that human wealth is as risky as financial wealth. These findings suggests there is less time variation in the market price of risk than commonly assumed.

How then to make sense of the discrepancy between equity return and total wealth return predictability (and the lack of cash-flow predictability)? The only possible answer is that the correlation between equity returns and the stochastic discount factor, $Corr_t[m_{t,t+k}, r_{t+k}^{m,e}]$, must change substantially over time. Many researchers have documented large cross-sectional differences in this correlation across *types* of equity (e.g., value portfolios). If the *composition* of total traded equity (the share of each type) were sensitive to small changes in the total wealth risk premium over time, this could have large effects on expected equity returns without introducing too much total wealth predictability. In future work, we plan to model such equity composition risk.

References

- ABEL, A. (1990): “Asset Prices under Habit Formation and Catching Up with the Joneses,” *American Economic Review*, 80, 38–42.
- ABEL, A. B. (1999): “Risk Premia and Term Premia in General Equilibrium,” *Journal of Monetary Economics*, 43, 3–33.
- BANSAL, R., R. F. DITTMAR, AND C. LUNDBLAD (2005): “Consumption Dividends and the Cross-Section of Equity Returns,” *Journal of Finance*, 60(4), 1639 – 1672.

- BANSAL, R., R. GALLANT, AND G. TAUCHEN (2007): “Rational Pessimism, Rational Exuberance, and Asset Pricing Models,” *Review of Economic Studies*, forthcoming.
- BANSAL, R., D. KIKU, AND A. YARON (2006): “Risks for the Long Run: Estimation and Inference,” Working Paper Duke University.
- BANSAL, R., AND A. YARON (2004): “Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles,” *The Journal of Finance*, 59, 1481–1509.
- BEKAERT, G., E. ENGSTROM, AND S. GRENADIER (2005): “Stock and Bond Returns with Moody Investors,” Working Paper Columbia University.
- BEKAERT, G., E. ENGSTROM, AND S. XING (2005): “Risk, Uncertainty and Asset Prices,” Working Paper Columbia University.
- BENZONI, L., R. GOLDSTEIN, AND P. COLLIN-DUFRESNE (2005): “Portfolio Choice over the Life Cycle in the Presence of ‘Trickle Down’ Labor Income,” University of Minnesota and University of California at Berkeley.
- BREEDEN, D. T. (1979): “An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities,” *Journal of Financial Economics*, 7, 265–296.
- CAMPBELL, J. Y. (1991): “A Variance Decomposition for Stock Returns,” *Economic Journal*, 101, 157–179.
- (1996): “Understanding Risk and Return,” *Journal of Political economy*, 104(2), 298–345.
- CAMPBELL, J. Y., AND J. H. COCHRANE (1999): “By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior,” *Journal of Political Economy*, 107(2), 205–251.
- CAMPBELL, J. Y., C. POLK, AND T. VUOLTEENAHO (2007): “Growth or Glamour? Fundamentals and Systematic Risk in Stock Returns,” Working Paper Harvard University and London School of Economics.
- CAMPBELL, J. Y., AND R. J. SHILLER (1988): “The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors,” *Review of Financial Studies*, 1, 195–228.
- CHEN, X., AND S. C. LUDVIGSON (2007): “Land of Addicts? An Empirical Investigation of Habit-Based Asset Pricing Models,” Working Paper New York University.
- COCHRANE, J. H. (1991): “Explaining the Variance of Price-Dividend Ratios,” *Review of Financial Studies*, 5(2), 243–280.
- (1994): “Permanent and Transitory Components of GNP and Stock Prices,” *The Quarterly Journal of Economics*, 109, 241–265.

- (2006): “The Dog That Did Not Bark: A Defense of Return Predictability,” Unpublished paper, University of Chicago Graduate School of Business.
- CONSTANTINIDES, G. M. (1990): “Habit-formation: A Resolution of the Equity Premium Puzzle,” *Journal of Political Economy*, 98, 519–543.
- DUFFIE, D., AND L. G. EPSTEIN (1992): “Stochastic Differential Utility,” *Econometrica*, 60(2), 353–394.
- EPSTEIN, L. G., AND S. ZIN (1989): “Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” *Econometrica*, 57, 937–969.
- (1991): “Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: an Empirical Investigation,” *Journal of Political Economy*, 99(6), 263–286.
- FAMA, E. F., AND J. D. MACBETH (1973): “Risk, Return, and Equilibrium: Empirical Tests,” *The Journal of Political Economy*, 81, 607–636.
- FERSON, W. E., AND G. M. CONSTANTINIDES (1991): “Habit Persistence and Durability in Aggregate Consumption,” *Journal of Financial Economics*, 29, 199–240.
- GABAIX, X. (2007): “Linearity-Generating Processes: A Modelling Tool Yielding Closed Forms for Asset Prices,” Working Paper New York University.
- HAHN, J., AND H. LEE (2006): “Interpreting the predictive power of the consumption wealth ratio,” *Journal of Empirical Finance*, 13, 183–202.
- HANSEN, L. P., J. C. HEATON, AND N. LI (2006): “Consumption Strikes Back,” Working Paper University of Chicago.
- JAGANNATHAN, R., AND Z. WANG (1996): “The Conditional CAPM and the Cross-Section of Expected Returns,” *The Journal of Finance*, 51(1), 3–53.
- JOHANSEN, S. (1991): “Estimation and hypothesis testing of cointegrating vector in Gaussian vector autoregressive models,” *Econometrica*, 59, 1551–1580.
- JORGENSON, D. M., AND B. FRAUMENI (1989): “The Accumulation of Human and Non-Human Capital,” in *The Measurement of Saving, Investment and Wealth*, ed. by R. Lipsey, and H. Tice, pp. 227–282. University of Chicago press, University of Chicago.
- KREPS, D., AND E. L. PORTEUS (1978): “Temporal Resolution of Uncertainty and Dynamic Choice Theory,” *Econometrica*, 46, 185–200.
- LARRAIN, B., AND M. YOGO (2007): “Does Firm Value Move Too Much to be Justified by Subsequent Changes in Cash Flow?,” Forthcoming Journal of Financial Economics.
- LETTAU, M., AND S. C. LUDVIGSON (2001a): “Consumption, Aggregate Wealth and Expected Stock Returns,” *The Journal of Finance*, 56(3), 815–849.

- (2001b): “Resurrecting the (C)CAPM: A Cross-Sectional Test When Risk Premia Are Time-Varying,” *The Journal of Political Economy*, 109(6), 1238–1287.
- LETTAU, M., AND S. C. LUDVIGSON (2005): “Expected Returns and Expected Dividend Growth,” *Journal of Financial Economics*, 76, 583–626.
- LUCAS, R. (1978): “Asset Prices in an Exchange Economy,” *Econometrica*, 46(6), 1429–54.
- LUSTIG, H., AND S. VAN NIEUWERBURGH (2007): “The Returns on Human Capital: Good News on Wall Street is Bad News on Main Street,” *Forthcoming Review of Financial Studies*.
- MALLOY, C., T. MOSKOWITZ, AND A. VISSING-JORGENSEN (2005): “Job Risk and Asset Returns,” *Working Paper GSB Chicago*.
- MENZLY, L., T. SANTOS, AND P. VERONESI (2004): “Understanding Predictability,” *Journal of Political Economy*, 112(1), 1–47.
- PARK, J. Y. (1990): “Testing for Unit Roots and Cointegration by Variable Addition,” *Advances in Econometrics*, pp. 107–133.
- (1992): “Canonical Cointegrating Regressions,” *Econometrica*, 60, 119–143.
- PARKER, J., AND C. JULLIARD (2005): “Consumption Risk and The Cross-Section of Expected Returns,” *The Journal of Political Economy*, 113(1), 185–222.
- PHILLIPS, P., AND S. OULIARIS (1990): “Asymptotic Properties of Residual Based Tests for Cointegration,” *Econometrica*, 58, 165–193.
- PIAZZESI, M., AND M. SCHNEIDER (2006): “Equilibrium Yield Curves,” *National Bureau of Economic Analysis Macroeconomics Annual*.
- RUDD, J., AND K. WHELAN (2006): “Empirical Proxies for the Consumption-Wealth Ratio,” *Review of Economic Dynamics*, 9, 34–51.
- SHILLER, R. J. (1995): “Aggregate Income Risks and Hedging Mechanisms,” *Quarterly Review of Economics and Finance*.
- WACHTER, J. (2005): “Solving Models with External Habit,” *Finance Research Letters*, 2, 210–226.
- (2006): “A Consumption-Based Model of the Term Structure of Interest Rates,” *Journal of Financial Economics*, 79, 365–399.
- YANG, W. (2007): “Time-Varying Exposure to Long -Run Consumption Risk,” Working Paper University of Rochester.
- YOGO, M. (2006): “A Consumption-Based Explanation of Expected Stock Returns,” *Journal of Finance*, 61(2), 539–580.

A Data Appendix

A.1 Macroeconomic Series

This appendix describes our data sources. We work with quarterly data and we use the longest available sample, 1952:I-2005:IV. Along the way, we point out minor differences with the construction of the consumption-wealth ratio from Lettau and Ludvigson (2001a) (LL). Some of these differences have already been implemented by Rudd and Whelan (2006).

Non-Durable and Services Consumption Consumption data are taken from Table 2.3.5. from the National Income and Product Accounts (NIPA). Values are in billions of dollars and seasonally adjusted. Nominal non-durable and services consumption is the sum of line 6 (non-durable goods $\tilde{E}^{ND}$), and line 13 (services $\tilde{E}^S$). We define consumption as expenditures in non-durables and services. LL define consumption as expenditures on non durables and services excluding clothing and shoes.

Conceptually, LL use total consumption as their measure of consumption. Instead of using a proxy for durable consumption, they assume that *log* total nominal consumption is proportional to *log* nominal non-durable and services consumption. Therefore they scale up their nominal nondurable and services consumption measure to obtain a total consumption series. Conceptually, we use non-durable and services consumption instead.

Financial Wealth Data The financial wealth data are from the balance sheet of households and non-profit organizations, Flow of Funds Accounts Table B-100, produced by the Federal Reserve Board System. Quarterly series start in 1952:I, annual series start in 1945. Flow of Funds wealth measures are expressed on an end-of-period basis. We therefore associate the $t - 1$ value of the data with period t wealth in order to obtain a start-of-period measure.

Our measure of household financial wealth is: household real estate (line 4) + household financial assets (line 8) - household financial liabilities (line 30 - line 34 - line 37 - line 39) + consumer durable goods (line 7). First, this measure takes out all items that explicitly refer to non-profit organizations instead of households. Second, this measure *includes* the net stock of consumer durable goods. Third, this measure *includes* net worth of non-corporate business and owners' equity in farm business. LL use total household net worth (line 41). This includes the non-profit sector items and the value of the stock of consumer durable goods.

Labor Income Our labor income measure is computed from NIPA Table 2.1 as wage and salary disbursements (line 3) + employer contributions for employee pension and insurance funds (line 7) + government social benefits to persons (line 17) - contributions for government social insurance (line 24) + employer contributions for government social insurance (line 8) - labor taxes. As in LL, labor taxes are defined by imputing a share of personal current taxes (line 25) to labor income, with the share calculated as the ratio of wage and salary disbursements to the sum of wage and salary disbursements, proprietors' income (line 9), and rental income of persons with capital consumption adjustment (line 12), personal interest income (line 14) and personal dividend income (line 15). We denote the corresponding nominal labor income measure by $\tilde{Y}$.

Because net worth of non-corporate business and owners' equity in farm business is part of financial wealth, it cannot also be part of human wealth. Consequently, labor income excludes proprietors' income.

Population We interpolate linearly the US CENSUS population series.

Price Indices We build composite Fisher price indices following closely the BEA guidelines. The price index is the geometric mean of a Laspeyres index and a Paasche index:

$$P_t^F = \sqrt{\frac{\sum P_t Q_{t-1}}{\sum P_{t-1} Q_{t-1}}} \sqrt{\frac{\sum P_t Q_t}{\sum P_{t-1} Q_t}},$$

where P_t and Q_t respectively are the prices and quantities of the index components. The price index data are from NIPA Table 2.3.4 and the quantity index data are from NIPA Table 2.3.3. Both indices are normalized to 100 in 2000.

Rudd and Whelan (2006) note that going from the nominal budget constraint to the real one implies that the consumption deflator should be the same as the wealth deflator. We deflate consumption, financial wealth and labor income by the Fisher price index corresponding to our definition of consumption. To construct the price index for non-durables and services, P^{NDS} , we select the non-durables component (line 6) and the services (line 13) component. LL deflate their consumption measure by the non-durable and services price index, but deflate the household net worth and labor income using the total personal consumption expenditures price index.

Real Per Capita Variables We consider two components of real per capita consumption. The first one is measured as nominal expenditures on non-durables and services divided by the price index for non-durables and services and divided by the population:

$$c = \log \left(\frac{\tilde{E}_t^{ND} + \tilde{E}_t^S}{P_t^{NDS} Pop_t} \right).$$

The second component is real per capita consumption in durables d . Following Yogo (2006), we assume that it is proportional to the per capita stock of durables.

The corresponding measure for real per capita financial wealth is

$$a_t = \log \left(\frac{\tilde{A}_t}{P_t^{NDS} Pop_t} \right).$$

Finally, the corresponding real per capita labor income measure is:

$$y_t = \log \left(\frac{\tilde{Y}_t}{P_t^{NDS} Pop_t} \right).$$

A.2 Cointegration Analysis

LL argue that the right-hand side of equation (4) is stationary, and as a result, log consumption c_t , wealth a_t and labor income y should be cointegrated. We run Johansen (1991), Phillips and Ouliaris (1990) cointegration tests on real values of consumption, wealth and labor income (all per capita and in logs). The null hypothesis is here the absence of cointegration. These tests are known to have low power and often fail to reject the null of no cointegration even though the variables are actually cointegrated. Therefore, we also report results obtained with Park (1992) cointegration tests. The null hypothesis is here the presence of cointegration.

Phillips-Ouliaris cointegration tests We first regress log consumption on log financial wealth and log labor income. We then run a Dickey-Fuller test on the first step residuals using 1 to 4 lags. Table 8 reports the t-statistics and the critical values, obtained from Phillips and Ouliaris (1990). The test statistic can reject the null of no cointegrating relation between c , a and y at the 5% confidence level for all lags considered.

Johansen cointegration tests We run Johansen (1991) L-max and Trace cointegration tests on log consumption, log financial wealth and log labor income. These tests use a 3-dimensional vector error-correction model (VECM) model estimated under different assumptions on the number of cointegrating relationships. The L-max test is conducted sequentially for cointegration rank r equal 0, 1 and 2, and is based on the log-likelihood ratio $\log[L_{max}(r)/L_{max}(r+1)]$ of the VECM model. It tests the null hypothesis that the cointegration rank is equal to r against the alternative that the cointegration rank is equal to $r+1$. The Trace test is based on the log-likelihood ratio $\log[L_{max}(r)/L_{max}(3)]$, and is conducted sequentially for $r = 2, 1$ and 0. It tests the null hypothesis that the cointegration rank is equal to r against the alternative that the cointegration rank is 3. Both tests are run using 1 to 4 lags in the VECM specification. Panel B in table 8 reports the Trace and L-max test statistics and the corresponding 1%, 5% and 10% critical values.

The absence of cointegration is rejected at the 10% confidence level with the L-max test (when using either one or four lags), but not with the trace test.

Park cointegration test Park (1992)'s cointegration tests are based on the framework presented in Park (1990). In order to estimate coefficients and standard errors on potentially cointegrated variables, Park (1990) form 'canonical regressions' by adding time polynomials to the usual regression. Such canonical regressions are used to test deterministic and stochastic cointegration. To illustrate the difference between these two, let us assume that $x_{1,t}$ and $x_{2,t}$ are individually integrated processes of order one with the corresponding first-difference series having nonzero drifts:

$$\begin{aligned}\Delta x_{1,t} &= \pi_1 + \nu_{1,t}, \\ \Delta x_{2,t} &= \pi_2 + \nu_{2,t},\end{aligned}$$

where $\nu_{1,t}$ and $\nu_{2,t}$ are mean zero stationary processes. Assuming that both cointegrated processes start from zero, one obtains:

$$\begin{aligned}x_{1,t} &= \pi_1 t + x_{1,t}^S, \\ x_{2,t} &= \pi_2 t + x_{2,t}^S,\end{aligned}$$

where $x_{i,t}^S = \sum_{s=1}^t \nu_{i,s}$ for $i = 1, 2$. The time-series $x_{1,t}$ and $x_{2,t}$ are driven by the linear trend t and the stochastic components $x_{1,t}^S$ and $x_{2,t}^S$. Suppose that β is the cointegrating vector between the stochastic components:

$$x_{1,t}^S = \beta' x_{2,t}^S + u_t,$$

where u_t is a stationary process. It follows that:

$$x_{1,t} = (\pi_1 - \beta' \pi_2)t + \beta' x_{2,t} + u_t.$$

If $\pi_1 - \beta' \pi_2 = 0$, $x_{1,t}$ and $x_{2,t}$ are stochastically and deterministically cointegrated, while if this condition does not hold, they are only stochastically cointegrated. Park (1992) tests whether the same cointegrating vector applies to both deterministic and stochastic components. Hahn and Lee (2006) reject deterministic cointegration between LL's consumption, wealth and labor income measures. The results of Park (1992)'s tests are reported in panel C, table 8. The test statistics fail to reject the null of deterministic and stochastic cointegration between c , a and y .

[Table 8 about here.]

A.3 Financial Series

Market return and Dividend Process We use CRSP value-weighted quarterly returns excluding and including dividends (respectively $R_t^{m,x}$ and $R_t^{m,i}$) and CRSP quarterly total market value Mkt_t on the NYSE, AMEX and NASDAQ to compute dividend growth rates and dividend price ratios:

$$\begin{aligned} d_t &= \log[(R_t^{m,i} - R_t^{m,x})Mkt_{t-1}], \\ \Delta d_t &= d_t - d_{t-1}, \\ dp_t &= d_t - p_t. \end{aligned}$$

Quarterly dividend growth rates Δd_t contains seasonal components. To remove them, we regress dividend growth rates on a constant and three quarter-specific dummies:

$$\Delta d_t = c_1 + c_2 I_{D2} + c_3 I_{D3} + c_4 I_{D4} + \epsilon_t,$$

where I_{Di} equals 1 in quarter i and zero elsewhere. Our adjusted dividend growth rate is then computed as $\Delta d_t = \epsilon_t + c_1 + \overline{I_{D2}}(c_2 + c_3 + c_4)$, where $\overline{I_{D2}}$ represents the mean of this dummy variable. The adjusted growth rate retains the same mean as the original series.

Return Predictors The yield spread, ysp , is the difference between the nominal yield on a ten-year constant maturity Treasury bond and the yield on a three-month Treasury bill. Both series are from the FRED II system of the Federal Reserve Bank of Saint Louis. The default spread, def is the difference between the Baa and Aaa Moody's seasoned corporate bond yields, also from FRED. The small value spread is the difference between the S1B5 and S1B1 equity portfolios from Kenneth French' web site. This difference reflects the returns spread between small value stocks and small growth stocks.

Cross-Sectional Asset Pricing For the cross-sectional exercise in section 5, we use equity portfolios obtained from Kenneth French' web site and Fama bond portfolios obtained from WRDS. Real returns are obtained by subtracting the inflation rate computed using the price index of consumption in nondurables and services, and returns are corrected from Jensen terms. Excess returns are computed using 3-month Fama risk-free rates.

B VECM with Time-Varying Human Wealth Shares

We write the human wealth share as $\nu_t = \bar{\nu} + \tilde{\nu}_t$, where $\tilde{\nu}_t$ is a mean-zero random variable, and $\bar{\nu}$ is the average human wealth share. Time-variation in the human wealth share adds the following term to the right-hand side of equation (9):

$$E_t \left[\sum_{j=1}^{\infty} (\kappa_1)^{-j} \tilde{\nu}_{t+j-1} r_{t+j}^y \right] - E_t \left[\sum_{j=1}^{\infty} (\kappa_1)^{-j} \tilde{\nu}_{t+j-1} r_{t+j}^a \right].$$

This appendix shows that both components can be written as quadratic functions of the state z_t . Much of this material follows Appendix A.3 in Lustig and Van Nieuwerburgh (2007).

When the expected return on human wealth is a linear function of the state (with loading vector C), the log dividend-price ratio on human wealth dp^y is also linear in the state. In particular, the demeaned log dividend-price

ratio on human wealth is a linear function of the $N \times 1$ state z with a $N \times 1$ loading vector B :

$$dp_t^y - E[dp_t^y] = E_t \sum_{j=1}^{\infty} \rho^j (r_{t+j}^y - \Delta y_{t+j}) = \rho(C' - e_2' \Psi)(I - \rho \Psi)^{-1} \mathbf{z}_t \equiv B' \mathbf{z}_t. \quad (26)$$

The demeaned log dividend-price ratio on financial assets is also a linear function of the state, because it is simply the third element in the VAR: $dp_t^a - E[dp_t^a] = e_3' \mathbf{z}_t$. The price-dividend ratio for all wealth is the wealth-consumption ratio; it is a weighted average of the price-dividend ratio for human wealth and for financial wealth:

$$\frac{W}{C} C = \frac{P^y}{Y} Y + \frac{P^a}{D} D,$$

where $Y = C + D$. Finally, the human wealth to total wealth ratio is given by:

$$\nu_t = \frac{\frac{P^y}{Y} Y}{\frac{W}{C} C} = \frac{e^{-dp_t^y} lis_t}{e^{-dp_t^y} lis_t + e^{-dp_t^a} (1 - lis_t)} = \frac{1}{1 + e^{X_t}}, \quad (27)$$

which is a logistic function of $X_t = dp_t^y - dp_t^a + \log\left(\frac{1-lis_t}{lis_t}\right)$, where $dp_t^y = -\log\left(\frac{P^y}{Y}\right)$. We recall that lis denotes the labor income share $lis_t = Y_t/C_t$ with mean ℓ . When $dp_t^a = dp_t^y$, the human wealth share equals the labor income share $\nu_t = lis_t$. In general, ν_t moves around not only when the labor income share changes, but also when the difference between the log dividend price ratios on human and financial wealth changes. It is increasing in the former, and decreasing in the latter.

The human wealth share ν_t is not a linear, but a logistic function of the state. We use a linear specification $\tilde{\nu}_t \equiv \nu_t - \bar{\nu} = D' \mathbf{z}_t$ and we pin down D ($N \times 1$) using a first order Taylor approximation around $(lis_t = \ell, X_t = 0)$. We obtain:

$$\begin{aligned} \nu_t(lis_t, X_t) &\approx \nu_t(\ell, 0) + \frac{\partial \nu_t}{\partial lis_t} \Big|_{lis_t=\ell, X_t=0} (lis_t - \ell) + \frac{\partial \nu_t}{\partial X_t} \Big|_{lis_t=\ell, X_t=0} (X_t), \\ &\approx \ell + (lis_t - \ell) - (\ell(1 - \ell)) X_t = lis_t - \ell(1 - \ell) dp_t^y + \ell(1 - \ell) dp_t \end{aligned} \quad (28)$$

Because dp_t is the third element of the VECM, $dp_t = e_3' \mathbf{z}_t$, and $lis_t - \ell$ the sixth, and if $dp_t^y = B' \mathbf{z}_t$, then we can solve for D from equation (28) and $\tilde{\nu}_t = D' \mathbf{z}_t$:

$$D = e_6 - \ell(1 - \ell)B + \ell(1 - \ell)e_3. \quad (29)$$

This approximation assumes that the long-run log price-dividend ratio on financial and human wealth are equal. A similar approximation exists when this assumption is relaxed. The coefficients in equation (29) then take on different values.

The next step is to compute the product terms of the demeaned human wealth share $\tilde{\nu}_{t+1-j}$ and returns r_{t+j}^a or

r_{t+j}^y . First, we define expected weighted future asset returns $\widetilde{DR}_t^{w,a}$ as

$$\begin{aligned}
\widetilde{DR}^{w,a}(z_t) &= E_t \sum_{j=1}^{\infty} (\kappa_1)^{-j} \tilde{\nu}_{t+j-1} r_{t+j}^a \\
&= \tilde{\nu}_t E_t (\kappa_1)^{-1} r_{t+1}^a + E_t \sum_{j=2}^{\infty} \tilde{\nu}_{t-1+j} (\kappa_1)^{-j} E_{t-1+j} r_{t+j}^a \\
&= \tilde{\nu}_t' (\kappa_1)^{-1} (e_1' \Psi + (\kappa_1 - 1) e_3') \mathbf{z}_t + (\kappa_1)^{-1} E_t \sum_{j=2}^{\infty} \tilde{\nu}_{t-1+j} (\kappa_1)^{j-1} E_{t-1+j} r_{t+j}^a \\
&= \mathbf{z}_t' D (\kappa_1)^{-1} (e_1' \Psi + (\kappa_1 - 1) e_3') \mathbf{z}_t + (\kappa_1)^{-1} E_t \widetilde{DR}^{w,a}(\mathbf{z}_{t+1}),
\end{aligned} \tag{30}$$

and similarly for $\widetilde{DR}_t^{w,y}$. In a second step, we exploit the recursive structure of $\widetilde{DR}_t^{w,a}$ and $\widetilde{DR}_t^{w,y}$ to show that $\widetilde{DR}^{w,a}$ can be stated as a quadratic function of the state:

$$\widetilde{DR}^{w,a}(\mathbf{z}_t) = \mathbf{z}_t' P \mathbf{z}_t + \frac{1}{\kappa_1 - 1} \text{trace}(P \Sigma)$$

where P solves a matrix Sylvester equation, whose fixed point is found by iterating on:

$$P_{j+1} = R + (\kappa_1)^{-1} \Psi' P_j \Psi, \tag{31}$$

starting from $P_0 = 0$, and $R = D((\kappa_1)^{-1} e_1' \Psi + (1 - (\kappa_1)^{-1}) e_3')$. In the same manner we calculate $\widetilde{DR}^{w,y}$, replacing R in equation (31) by $(\kappa_1)^{-1} D C'$. C takes on different values for the three canonical models.

C The Long-Run Risk Model

C.1 The General Case

Let $V_t(C_t)$ denote the utility derived from consuming C_t , then the value function of the representative agent takes the following recursive form:

$$V_t(C_t) = \left[(1 - \beta) C_t^{1-\rho} + \beta (\mathcal{R}_t V_{t+1})^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{32}$$

where the risk-adjusted expectation operator is defined as:

$$\mathcal{R}_t V_{t+1} = (E_t V_{t+1}^{1-\alpha})^{\frac{1}{1-\alpha}}.$$

For these preferences, α governs risk aversion and ρ governs the willingness to substitute consumption intertemporally. It is the inverse of the inter-temporal elasticity of substitution. In the special case where $\rho = \alpha$, they collapse to the standard power utility preferences, used in Breeden (1979) and Lucas (1978). Epstein and Zin (1989) show that the stochastic discount factor can be written as:

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t+1}}{\mathcal{R}_t V_{t+1}} \right)^{\rho-\alpha} \tag{33}$$

The next proposition shows that the ability to write the SDF in the long-run risk model as a function of consumption growth and the consumption-wealth ratio is general. It does not depend on the linearization of returns, nor on the assumptions on the stochastic process for consumption growth.

Proposition 3. *The log SDF in the non-linear version of the long-run risk model can be stated as*

$$m_{t+1} = \frac{1-\alpha}{1-\rho} \log \beta - \alpha \Delta c_{t+1} + \frac{\rho-\alpha}{1-\rho} \log \left(\frac{e^{-cw_{t+1}}}{e^{-cw_t} - 1} \right) \quad (34)$$

Proof. We start from the value function definition in equation (32) and raise both sides to the power $1-\rho$, and subsequently divide through by $(1-\beta)C_t^{-\rho}$ to obtain:

$$\frac{V_t^{1-\rho}}{(1-\beta)C_t^{-\rho}} = C_t + \beta \frac{(E_t V_{t+1}^{1-\alpha})^{\frac{1-\rho}{1-\alpha}}}{(1-\beta)C_t^{-\rho}} \quad (35)$$

Some algebra and the definition of the risk-adjusted expectation operator imply that

$$E_t(V_{t+1}^{1-\alpha})^{\frac{1-\rho}{1-\alpha}} = E_t(V_{t+1}^{1-\alpha})^{1-\frac{\rho-\alpha}{1-\alpha}} = \frac{E_t(V_{t+1}^{1-\alpha})}{E_t(V_{t+1}^{1-\alpha})^{\frac{\rho-\alpha}{1-\alpha}}} = \frac{E_t(V_{t+1}^{1-\alpha})}{(\mathcal{R}_t V_{t+1})^{\rho-\alpha}} = E_t \left[\frac{V_{t+1}^{\rho-\alpha} V_{t+1}^{1-\rho}}{(\mathcal{R}_t V_{t+1})^{\rho-\alpha}} \right]$$

Substituting this expression into the previous one, and multiplying and dividing inside the expectation operator by $C_{t+1}^{-\rho}$, we get:

$$\frac{V_t^{1-\rho}}{(1-\beta)C_t^{-\rho}} = C_t + E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{V_{t+1}}{\mathcal{R}_t V_{t+1}} \right)^{\rho-\alpha} \frac{V_{t+1}^{1-\rho}}{(1-\beta)C_{t+1}^{-\rho}} \right]$$

Note that the first three terms inside the expectation are equal to the stochastic discount factor in equation (33). This is a no-arbitrage asset pricing equation of an asset with dividend equal to aggregate consumption. The price of this asset is W_t . Hence,

$$W_t = C_t + E_t [M_{t+1} W_{t+1}] \quad \text{and} \quad W_t = \frac{V_t^{1-\rho}}{(1-\beta)C_t^{-\rho}}. \quad (36)$$

This equation, together with $E[M_{t+1} R_{t+1}] = 1$ delivers the return on the total wealth portfolio:

$$R_{t+1} = \frac{W_{t+1}}{(W_t - C_t)} = \frac{\frac{V_{t+1}^{1-\rho}}{(1-\beta)C_{t+1}^{-\rho}}}{\frac{V_t^{1-\rho}}{(1-\beta)C_t^{-\rho}} - C_t} = \frac{\frac{V_{t+1}^{1-\rho}}{(1-\beta)C_{t+1}^{-\rho}}}{\frac{\beta}{(1-\beta)C_t^{-\rho}} (\mathcal{R}_t V_{t+1})^{1-\rho}} = \beta^{-1} \left(\frac{C_{t+1}}{C_t} \right)^{\rho} \left(\frac{V_{t+1}}{\mathcal{R}_t V_{t+1}} \right)^{1-\rho}, \quad (37)$$

where the first equality is a definition, the second one follows from the homogeneity of the value function, the third equality follows from equation (35), and the last one from algebraic manipulation.

Typically, one would rearrange this equation (after raising both sides to the power $\frac{\rho-\alpha}{1-\rho}$)

$$\left(\frac{V_{t+1}}{\mathcal{R}_t V_{t+1}} \right)^{\rho-\alpha} = \beta^{\frac{\rho-\alpha}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{\rho-\alpha}{1-\rho}} R_{t+1}^{\frac{\rho-\alpha}{1-\rho}},$$

and substitute it into the stochastic discount factor expression (33) to obtain an expression that depends only on consumption growth and the return to the wealth portfolio:

$$M_{t+1} = \beta^{\frac{1-\alpha}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho \frac{1-\alpha}{1-\rho}} R_{t+1}^{\frac{\rho-\alpha}{1-\rho}} \quad (38)$$

Instead, we rewrite the return on the wealth portfolio in terms of the wealth-consumption ratio WC

$$R_{t+1} = \frac{WC_{t+1}}{WC_t - 1} \frac{C_{t+1}}{C_t},$$

and use equation (37) to obtain

$$\left(\frac{V_{t+1}}{\mathcal{R}_t V_{t+1}} \right)^{\rho-\alpha} = \beta^{\frac{\rho-\alpha}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{\rho-\alpha} \left(\frac{WC_{t+1}}{WC_t - 1} \right)^{\frac{\rho-\alpha}{1-\rho}},$$

and substitute it into the stochastic discount factor expression (33) to obtain an expression that depends only on consumption growth and the wealth-consumption ratio:

$$M_{t+1} = \beta^{\frac{1-\alpha}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{WC_{t+1}}{WC_t - 1} \right)^{\frac{\rho-\alpha}{1-\rho}} = \beta^{\frac{1-\alpha}{1-\rho}} \left(\frac{C_{t+1}}{C_t} \right)^{-\alpha} \left(\frac{e^{-cw_{t+1}}}{e^{-cw_t} - 1} \right)^{\frac{\rho-\alpha}{1-\rho}} \quad (39)$$

□

The log SDF expression in the BY model is a first special case of this general, non-linear formulation. Indeed, one recovers equation (17) by using a first-order Taylor approximation of wc_t in equation (34) around A_0 .

One obtains a second special case by approximating the last term in (34) using a first-order Taylor expansion of wc_{t+1} around wc_t instead. In that case, we obtain a three-factor model:

$$m_{t+1} \approx \frac{1-\alpha}{1-\rho} \log \beta - \alpha \Delta c_{t+1} + \frac{\rho-\alpha}{1-\rho} \log \left(\frac{e^{-cw_t}}{e^{-cw_t} - 1} \right) - \frac{\rho-\alpha}{1-\rho} \Delta cw_{t+1}. \quad (40)$$

Expressions (40) and (17) are functionally similar because κ_1 is close to 1 and κ_0 equals $\frac{e^{-cw_t}}{e^{-cw_t} - 1}$ when cw_t is evaluated at its long-run mean $-A_0$.

C.2 Proof of Proposition 1

Setting Up Some Notation The starting point of the analysis is the Euler equation $E_t[M_{t+1}R_{t+1}^i] = 1$, where R_{t+1}^i denotes a gross return between dates t and $t+1$ on some asset i and M_{t+1} is the SDF. In logs:

$$E_t[m_{t+1}] + E_t[r_{t+1}^i] + \frac{1}{2}Var_t[m_{t+1}] + \frac{1}{2}Var_t[r_{t+1}^i] + Cov_t[m_{t+1}, r_{t+1}^i] = 0. \quad (41)$$

The same equation holds for the risk-free rate r_t^f , so that

$$r_t^f = -E_t[m_{t+1}] - \frac{1}{2}Var_t[m_{t+1}] = 0. \quad (42)$$

The expected excess return becomes:

$$E_t[r_{t+1}^{e,i}] = E_t[r_{t+1}^i - r_t^f] + \frac{1}{2}Var_t[r_{t+1}^i] = -Cov_t[m_{t+1}, r_{t+1}^i] = -Cov_t[m_{t+1}, r_{t+1}^{e,i}], \quad (43)$$

where $r_{t+1}^{e,i}$ denotes the excess return on asset i corrected for the Jensen term.

Proof of Linearity The proof closely follows the proof in Bansal and Yaron (2004), but adjusts all expressions for our timing of returns.

Proof. In what follows we focus on the return on a claim to aggregate consumption, denoted r , where

$$r_{t+1} = \kappa_0 + \Delta c_{t+1} + wc_{t+1} - \kappa_1 wc_t,$$

and derive the five terms in equation (41) for this asset.

Taking logs on both sides of the non-linear SDF expression in equation (38) of Appendix C.1 delivers an expression of the log SDF as a function of log consumption changes and the log total wealth return

$$m_{t+1} = \frac{1-\alpha}{1-\rho} \log \beta - \frac{1-\alpha}{1-\rho} \rho \Delta c_{t+1} + \left(\frac{1-\alpha}{1-\rho} - 1 \right) r_{t+1}. \quad (44)$$

We conjecture that the log wealth-consumption ratio is linear in the two states x_t and $\sigma_t^2 - \bar{\sigma}^2$,

$$wc_t = A_0 + A_1 x_t + A_2 (\sigma_t^2 - \bar{\sigma}^2).$$

As BY, we assume joint conditional normality of consumption growth, x , and the variance of consumption growth. We verify this conjecture from the Euler equation

$$E_t[e^{m_{t+1}+r_{t+1}}] = 1 \Leftrightarrow E_t[m_{t+1}] + E_t[r_{t+1}] + \frac{1}{2} V_t[m_{t+1}] + \frac{1}{2} V_t[r_{t+1}] + Cov_t[m_{t+1}, r_{t+1}] = 0 \quad (45)$$

Substituting in the expression for the log total wealth return r into the log SDF, we compute innovations, and the conditional mean and variance of the log SDF:

$$\begin{aligned} m_{t+1} - E_t[m_{t+1}] &= \lambda_{m,\eta} \sigma_t \eta_{t+1} - \lambda_{m,e} \sigma_t e_{t+1} - \lambda_{m,w} \sigma_w w_{t+1}, \\ E_t[m_{t+1}] &= m_0 - \rho x_t + \frac{\alpha - \rho}{1 - \rho} (\kappa_1 - \nu_1) A_2 (\sigma_t^2 - \bar{\sigma}^2) \\ V_t[m_{t+1}] &= (\lambda_{m,\eta}^2 + \lambda_{m,e}^2) \sigma_t^2 + \lambda_{m,w}^2 \sigma_w^2 \\ m_0 &= \frac{1-\alpha}{1-\rho} \log \beta - \frac{\alpha - \rho}{1 - \rho} [\kappa_0 + A_0(1 - \kappa_1)] - \alpha \mu \end{aligned} \quad (46)$$

These expressions correspond to equations (A.10) and (A.27) in the Appendix of BY, but with slightly different definitions for: $\lambda_{m,\eta} = -\alpha$, $\lambda_{m,e} = \frac{\alpha - \rho}{1 - \rho} B$, $\lambda_{m,w} = \frac{\alpha - \rho}{1 - \rho} A_2$, and $B = A_1 \varphi_e$. The differences are only due to the timing in returns.

Likewise, we compute innovations in the consumption claim return, and its conditional mean and variance:

$$\begin{aligned} r_{t+1} - E_t[r_{t+1}] &= \sigma_t \eta_{t+1} + B \sigma_t e_{t+1} + A_2 \sigma_w w_{t+1} \\ E_t[r_{t+1}] &= r_0 + \rho x_t - A_2 (\kappa_1 - \nu_1) (\sigma_t^2 - \bar{\sigma}^2) \\ V_t[r_{t+1}] &= (1 + B^2) \sigma_t^2 + A_2^2 \sigma_w^2 \\ r_0 &= \kappa_0 + A_0(1 - \kappa_1) + \mu \end{aligned}$$

These equations correspond to (A.8) and (A.9).

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations

$$Cov_t[m_{t+1}, r_{t+1}] = (\lambda_{m,\eta} - \lambda_{m,e} B) \sigma_t^2 - A_2 \lambda_{m,w} \sigma_w^2$$

Using the method of undetermined coefficients and the five components of equation (41), we can solve for the

constants A_0 , A_1 , and A_2 :

$$A_1 = \frac{1 - \rho}{\kappa_1 - \rho_x}, \quad (47)$$

$$A_2 = \frac{(1 - \rho)(1 - \alpha)}{2(\kappa_1 - \nu_1)} \left[1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right], \quad (48)$$

$$0 = \frac{1 - \alpha}{1 - \rho} [\log \beta + \kappa_0 + (1 - \kappa_1)A_0] + (1 - \alpha)\mu + \frac{1}{2}(1 - \alpha)^2 \left[1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right] \bar{\sigma}^2 + \frac{1}{2} \left(\frac{1 - \alpha}{1 - \rho} \right)^2 A_2^2 \sigma_w^2 \quad (49)$$

The first two correspond to equations (A.5) and (A.7) in BY. The last equation implicitly solves A_0 as a function of all parameters of the model. Because κ_0 and κ_1 are non-linear functions of A_0 , this system of three equations needs to be solved simultaneously and numerically. Our computations indicate that the system has a unique solution. This verifies the conjecture that the log wealth-consumption ratio is linear in the two state variables.

It follows immediately from the above that the log SDF can be written as:

$$m_{t+1} = \frac{1 - \alpha}{1 - \rho} [\log \beta + \kappa_0] - \kappa_0 - \alpha \Delta c_{t+1} - \frac{\alpha - \rho}{1 - \rho} (wc_{t+1} - \kappa_1 wc_t)$$

This is the expression that arises in the proposition. $\square$

According to (43), the risk premium on the consumption claim is given by

$$E_t [r_{t+1}^e] = E_t [r_{t+1} - r_t^f] + .5V_t[r_{t+1}] = -\lambda_{m,\eta}\sigma_t^2 + \lambda_{m,e}B\sigma_t^2 + \lambda_{m,w}A_2\sigma_w^2, \quad (50)$$

This corresponds to equation (A.11) in BY.

According to equation (42), the expression for the risk-free rate is given by

$$\begin{aligned} r_t^f &= h_0 + \rho x_t + h_1(\sigma_t^2 - \bar{\sigma}^2) \\ h_0 &= -m_0 - .5\lambda_{m,w}^2\sigma_w^2 - .5(\lambda_{m,\eta}^2 + \lambda_{m,e}^2)\bar{\sigma}^2 \\ h_1 &= -\frac{\alpha - \rho}{1 - \rho}(\kappa_1 - \nu_1)A_2 - .5(\lambda_{m,\eta}^2 + \lambda_{m,e}^2) \\ &= .5(\rho - \alpha) \left(1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right) - .5 \left(\alpha^2 + (\alpha - \rho)^2 \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right) \end{aligned} \quad (51)$$

This corresponds to equation (A.25-A.27) in BY. Its unconditional mean is simply h_0 (see A.28).

C.3 Campbell-Shiller Decomposition

Expected discounted future returns and consumption growth rates are given by:

$$r_t^H \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1)^{-j} r_{t+j} \right] = \frac{r_0}{\kappa_1 - 1} + \frac{\rho}{\kappa_1 - \rho_x} x_t - A_2(\sigma_t^2 - \bar{\sigma}^2) \quad (52)$$

$$\Delta c_t^H \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1)^{-j} \Delta c_{t+j} \right] = \frac{\mu}{\kappa_1 - 1} + \frac{1}{\kappa_1 - \rho_x} x_t \quad (53)$$

These expressions enable us to go back and forth between the log wealth-consumption ratio expression in (18) and the Campbell-Shiller equation in (4). Starting from (18)

$$\begin{aligned}
wc_t &= A_0 + A_1 x_t + A_2(\sigma_t^2 - \bar{\sigma}^2) \\
&= A_0 + \frac{1}{\kappa_1 - \rho_x} x_t - \left(\frac{\rho}{\kappa_1 - \rho_x} x_t - A_2(\sigma_t^2 - \bar{\sigma}^2) \right) \\
&= A_0 + \left(\Delta c_t^H - \frac{\mu}{\kappa_1 - 1} \right) - \left(r_t^H - \frac{r_0}{\kappa_1 - 1} \right) \\
&= \frac{\kappa_0}{\kappa_1 - 1} + \Delta c_t^H - r_t^H
\end{aligned}$$

we arrive at equation (4). The second equality uses the definition of A_1 . The third equality uses the definition of r_t^H and Δc_t^H . The fourth equality uses the definition of r_0 .

The variance of the log wealth-consumption ratio can be written in two equivalent ways:

$$V[\Delta c_t^H] + V[r_t^H] - 2Cov[r_t^H, \Delta c_t^H] = V[wc_t] = Cov[wc_t, \Delta c_t^H] + Cov[wc_t, -r_t^H]$$

In the LRR model, the terms in this expression are given by

$$\begin{aligned}
V[\Delta c_t^H] &= \frac{1}{(\kappa_1 - \rho_x)^2} \frac{\varphi_e^2}{1 - \rho_x^2} \bar{\sigma}^2 > 0 \\
V[r_t^H] &= \frac{\rho^2}{(\kappa_1 - \rho_x)^2} \frac{\varphi_e^2}{1 - \rho_x^2} \bar{\sigma}^2 + A_2^2 \frac{\sigma_w^2}{1 - \nu_1^2} > 0 \\
Cov[r_t^H, \Delta c_t^H] &= \frac{\rho}{(\kappa_1 - \rho_x)^2} \frac{\varphi_e^2}{1 - \rho_x^2} \bar{\sigma}^2 > 0 \\
Cov[wc_t, \Delta c_t^H] &= \frac{1 - \rho}{(\kappa_1 - \rho_x)^2} \frac{\varphi_e^2}{1 - \rho_x^2} \bar{\sigma}^2 > 0 \Leftrightarrow \rho < 1 \\
Cov[wc_t, -r_t^H] &= \frac{\rho^2 - \rho}{(\kappa_1 - \rho_x)^2} \frac{\varphi_e^2}{1 - \rho_x^2} \bar{\sigma}^2 + A_2^2 \frac{\sigma_w^2}{1 - \nu_1^2} > 0 \Leftrightarrow \rho > 1
\end{aligned}$$

We can break up expected future returns into a risk-free rate component and a risk premium component. The former is equal to

$$E_t \left[\sum_{j=1}^{\infty} (\kappa_1)^{-j} r_{t+j-1}^f \right] = \frac{h_0}{\kappa_1 - 1} + \frac{\rho}{\kappa_1 - \rho_x} x_t + \frac{h_1}{\kappa_1 - \nu_1} (\sigma_t^2 - \bar{\sigma}^2), \quad (54)$$

where the second equation uses the expression for the risk-free rate in equation (51) to compute future risk-free rates and takes their time- t expectations. The risk premium component is simply the difference between r_t^H and the second expression.

C.4 Risk Factor Representation

We can further rewrite the log SDF in terms of our two demeaned risk factors (denoted with a tilde):

$$m_{t+1} = m_0 - \alpha \widetilde{\Delta c}_{t+1} - \frac{\alpha - \rho}{1 - \rho} \widetilde{\Delta wc}_{t+1} = m_0 - b f_{t+1},$$

where m_0 is defined in (46), the factor loadings are

$$b = \left[\alpha, \frac{\alpha - \rho}{1 - \rho} \right],$$

and where the demeaned risk factors are defined as

$$f_{t+1} = \left[\widetilde{\Delta c}_{t+1}, \widetilde{\Delta wc}_{t+1} \right]' = [\Delta c_{t+1} - \mu, (wc_{t+1} - A_0) - \kappa_1(wc_t - A_0)]'.$$

In the LRR model, the conditional and unconditional first and second moments of the two risk factors are

$$\begin{aligned} E_t \left[\widetilde{\Delta c}_{t+1} \right] &= x_t & E \left[\widetilde{\Delta c}_{t+1} \right] &= 0 \\ V_t \left[\widetilde{\Delta c}_{t+1} \right] &= \sigma_t^2 & V \left[\widetilde{\Delta c}_{t+1} \right] &= \left(1 + \frac{\varphi_e^2}{1 - \rho_x^2} \right) \bar{\sigma}^2 \\ E_t \left[\widetilde{\Delta wc}_{t+1} \right] &= (\rho - 1)x_t + A_2(\nu_1 - \kappa_1)(\sigma_t^2 - \bar{\sigma}^2) & E \left[\widetilde{\Delta wc}_{t+1} \right] &= 0 \\ V_t \left[\widetilde{\Delta wc}_{t+1} \right] &= A_1^2 \varphi_e^2 \sigma_t^2 + A_2^2 \sigma_w^2 & V \left[\widetilde{\Delta wc}_{t+1} \right] &= A_1^2 \varphi_e^2 \left(1 + \frac{(\rho_x - \kappa_1)^2}{1 - \rho_x^2} \right) \bar{\sigma}^2 + A_2^2 \left(1 + \frac{(\nu_1 - \kappa_1)^2}{1 - \nu_1^2} \right) \sigma_w^2 \\ Cov_t \left[\widetilde{\Delta c}_{t+1}, \widetilde{\Delta wc}_{t+1} \right] &= 0 & Cov \left[\widetilde{\Delta c}_{t+1}, \widetilde{\Delta wc}_{t+1} \right] &= (\rho - 1) \frac{\varphi_e^2}{1 - \rho_x^2} \bar{\sigma}^2 \end{aligned}$$

The two risk factors are conditionally uncorrelated and have a positive unconditional correlation only if $\rho > 1$.

Equation (19) in the main text implies that the expected excess return on the consumption claim can be written as the sum of the market prices of risk on the two risk factors.

$$E_t [r_{t+1}^e] = \ell_{1t}^{LRR} + \ell_{2t}^{LRR} = \left\{ b_1 V_t \left[\widetilde{\Delta c}_{t+1} \right] + b_2 Cov_t \left[\widetilde{\Delta c}_{t+1}, \widetilde{\Delta wc}_{t+1} \right] \right\} + \left\{ b_1 Cov_t \left[\widetilde{\Delta c}_{t+1}, \widetilde{\Delta wc}_{t+1} \right] + b_2 V_t \left[\widetilde{\Delta wc}_{t+1} \right] \right\}.$$

After some algebra, we obtain expressions for the conditional market prices of risk that are only a function of the structural parameters of the LRR model:

$$\ell_{1t}^{LRR} = \alpha \sigma_t^2 \tag{55}$$

$$\ell_{2t}^{LRR} = (\alpha - \rho)(1 - \rho) \left\{ \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \sigma_t^2 + \frac{(\alpha - 1)^2}{4(\kappa_1 - \nu_1)^2} \left[1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right]^2 \sigma_w^2 \right\} \tag{56}$$

The unconditional market prices of risk are the unconditional means of the conditional market prices of risk: $\ell_i^{LRR} = E[\ell_{it}^{LRR}]$, for $i = 1, 2$. This amounts to setting $\sigma_t^2 = \bar{\sigma}^2$ in the above equations.

C.5 Pricing Stocks in the LRR Model

We discuss the case where dividends on equity and aggregate consumption are not cointegrated. Section E.2 discusses the case where cointegration is imposed.

Dividend Growth Process We start by pricing a claim to aggregate dividends, where the dividend process follows the specification in Bansal and Yaron (2004):

$$\Delta d_{t+1} = \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \tag{57}$$

The shock u_t is orthogonal to (η, e, w) . This specification does not impose cointegration between consumption and dividends.

Defining returns ex-dividend and using the Campbell (1991) linearization, the log return on a claim to the aggregate dividend can be written as:

$$r_{t+1}^m = \Delta d_{t+1} + p d_{t+1} + \kappa_0^m - \kappa_1^m p d_t,$$

with coefficients

$$\kappa_1^m = \frac{e^{A_0^m}}{e^{A_0^m} - 1} > 1, \quad \text{and} \quad \kappa_0^m = -\log(e^{A_0^m} - 1) + \frac{e^{A_0^m}}{e^{A_0^m} - 1} A_0^m$$

which depend on the long-run log price-dividend ratio A_0^m . We denote the return on financial wealth by a superscript m .

Proof of Linearity We conjecture, as we did for the wealth-consumption ratio, that the log price dividend ratio is linear in the two state variables:

$$p d_t^m = A_0^m + A_1^m x_t + A_2^m (\sigma_t^2 - \bar{\sigma}^2).$$

As we did for the return on the consumption claim, we compute innovations in the dividend claim return, and its conditional mean and variance:

$$\begin{aligned} r_{t+1}^m - E_t[r_{t+1}^m] &= \varphi_d \sigma_t u_{t+1} + \beta_{m,e} \sigma_t e_{t+1} + \beta_{m,w} \sigma_w w_{t+1} \\ E_t[r_{t+1}^m] &= r_0^m + [\phi + A_1^m (\rho_x - \kappa_1^m)] x_t - A_2^m (\kappa_1^m - \nu_1) (\sigma_t^2 - \bar{\sigma}^2) \\ V_t[r_{t+1}^m] &= (\varphi_d^2 + \beta_{m,e}^2) \sigma_t^2 + \beta_{m,w}^2 \sigma_w^2, \\ r_0^m &= \kappa_0^m + A_0^m (1 - \kappa_1^m) + \mu_d \end{aligned}$$

where $\beta_{m,e} = A_1^m \varphi_e$ and $\beta_{m,w} = A_2^m$. These equations correspond to (A.12) and (A.13) in the Appendix of Bansal and Yaron (2004). Finally, the conditional covariance between the log SDF and the log dividend claim return is

$$Cov_t[m_{t+1}, r_{t+1}^m] = -\lambda_{m,e} \beta_{m,e} \sigma_t^2 - \lambda_{m,w} \beta_{m,w} \sigma_w^2$$

From the Euler equation for this return $E_t[m_{t+1}] + E_t[r_{t+1}^m] + \frac{1}{2} V_t[m_{t+1}] + \frac{1}{2} V_t[r_{t+1}^m] + Cov_t[m_{t+1}, r_{t+1}^m] = 0$ and the method of undetermined coefficients, we can use the same procedure as described in C.2, and solve for the constants A_0^m , A_1^m , and A_2^m :

$$\begin{aligned} A_1^m &= \frac{\phi - \rho}{\kappa_1^m - \rho_x}, \\ A_2^m &= \frac{\left[\frac{\alpha - \rho}{1 - \rho} A_2 (\kappa_1 - \nu_1) + .5 H_m \right]}{\kappa_1^m - \nu_1}, \\ 0 &= m_0 + \kappa_0^m + (1 - \kappa_1^m) A_0^m + \mu_d + \frac{1}{2} H_m \bar{\sigma}^2 + \frac{1}{2} \left(A_2^m - A_2 \frac{\alpha - \rho}{1 - \rho} \right)^2 \sigma_w^2 \end{aligned}$$

where

$$\begin{aligned} H_m &= \lambda_{m,\eta}^2 + (\beta_{m,e} - \lambda_{m,e})^2 + \varphi_d^2 \\ &= \alpha^2 + (\phi - \alpha)^2 \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} + \varphi_d^2 \end{aligned}$$

Again, this is a non-linear system in three equations and three unknowns, which we solve numerically. The first two equations correspond to (A.16) and (A.20).

Equity Risk premium and CS Decomposition The equity risk premium on the dividend claim (adjusted for a Jensen term) becomes:

$$E_t [r_{t+1}^{e,m}] \equiv E_t [r_{t+1}^m - r_t^f] + .5V_t[r_{t+1}^m] = \lambda_{m,e}\beta_{m,e}\sigma_t^2 + \lambda_{m,w}\beta_{m,w}\sigma_w^2 \quad (58)$$

This corresponds to equation (A.14) in BY.

Expected discounted future equity returns and dividend growth rates are given by:

$$r_t^{m,H} \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^m)^{-j} r_{t+j}^m \right] = \frac{r_0^m}{\kappa_1^m - 1} + \frac{\rho}{\kappa_1^m - \rho_x} x_t - A_2^m (\sigma_t^2 - \bar{\sigma}^2) \quad (59)$$

$$\Delta d_t^H \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^m)^{-j} \Delta d_{t+j} \right] = \frac{\mu_d}{\kappa_1^m - 1} + \frac{\phi}{\kappa_1^m - \rho_x} x_t \quad (60)$$

From these expressions, it is easy to see that

$$pd_t = \frac{\kappa_0^m}{\kappa_1^m - 1} + \Delta d_t^H - r_t^{m,H},$$

and to compute the elements of the variance-decomposition:

$$V[pd_t^m] = Cov[pd_t^m, \Delta d_t^H] + Cov[pd_t^m, -r_t^{m,H}] = V[\Delta d_t^H] + V[r_t^{m,H}] - 2Cov[\Delta d_t^H, r_t^{m,H}].$$

C.6 Quarterly Calibration LRR Model

The Bansal-Yaron model is calibrated and parameterized to monthly data. Since we want to use data on quarterly consumption and dividend growth, and a quarterly series for the consumption-wealth ratio, we recast the model at quarterly frequencies. We assume that the quarterly process for consumption growth, dividend growth, the low frequency component and the variance has the exact same structure than at the monthly frequency, with mean zero, mean 1 innovations, but with different parameters. This appendix explains how the monthly parameters map into quarterly parameters. We denote all variables, shocks, and all parameters of the quarterly system with a tilde superscript.

Preference Parameters Obviously, the preference parameters do not depend on the horizon ($\tilde{\alpha} = \alpha$ and $\tilde{\rho} = \rho$), except for the time discount factor $\tilde{\beta} = \beta^3$. Also, the long-run average log wealth-consumption ratio at the quarterly frequency is lower than at the monthly frequency by approximately $\log(3)$, because log of quarterly consumption is the log of three times monthly consumption. When we simulate the quarterly model, we solve for the corresponding A_0 , A_1 , and A_2 from the system (47)-(49), but with the quarterly parameter values described in this appendix.

Cash-flow Parameters We accomplish this by matching the conditional and unconditional mean and variance of log consumption and dividend growth. Log quarterly consumption growth is the sum of log consumption

growth of three consecutive months. We obtain $\Delta\tilde{c}_{t+1} \equiv \Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}$

$$\Delta\tilde{c}_{t+1} = 3\mu + (1 + \rho_x + \rho_x^2)x_t + \sigma_t\eta_{t+1} + \sigma_{t+1}\eta_{t+2} + \sigma_{t+2}\eta_{t+3} + (1 + \rho_x)\varphi_e\sigma_te_{t+1} + \varphi_e\sigma_{t+1}e_{t+2} \quad (61)$$

Log quarterly dividend growth looks similar:

$$\Delta\tilde{d}_{t+1} = 3\mu_d + \phi(1 + \rho_x + \rho_x^2)x_t + \varphi_d\sigma_tu_{t+1} + \varphi_d\sigma_{t+1}u_{t+2} + \varphi_d\sigma_{t+2}u_{t+3} + \phi(1 + \rho_x)\varphi_e\sigma_te_{t+1} + \phi\varphi_e\sigma_{t+1}e_{t+2} \quad (62)$$

First, we rescale the long-run component in the quarterly system, so that the coefficient on it in the consumption growth equation is still 1:

$$\tilde{x}_t = (1 + \rho_x + \rho_x^2)x_t.$$

Second, we equate the unconditional mean of consumption and dividend growth :

$$\tilde{\mu} = 3\mu, \quad \tilde{\mu}_d = 3\mu_d.$$

These imply that we also match the the conditional mean of consumption growth :

$$E_t[\Delta c_{t+3} + \Delta c_{t+2} + \Delta c_{t+1}] = 3\mu + (1 + \rho_x + \rho_x^2)x_t = \tilde{\mu} + \tilde{x}_t = E_t[\Delta\tilde{c}_{t+1}]$$

Third, we also match the conditional mean of dividend growth by setting the quarterly leverage parameter

$$\tilde{\phi} = \phi.$$

Fourth, we match the unconditional variance of quarterly consumption growth:

$$\begin{aligned} V[\Delta\tilde{c}_{t+1}] &= (1 + \rho_x + \rho_x^2)^2 V[x_t] + \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\ &= (1 + \rho_x + \rho_x^2)^2 \frac{\varphi_e^2 \sigma^2}{1 - \rho_x^2} + \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\ &= \frac{\tilde{\varphi}_e^2 \tilde{\sigma}^2}{1 - \tilde{\rho}_x^2} + \tilde{\sigma}^2 \end{aligned}$$

The first and second equalities use the law of iterated expectations to show that

$$V[\sigma_{t+j}\eta_{t+j+1}] \equiv E[E_{t+j}\{\sigma_{t+j}^2\eta_{t+j+1}^2\}] - (E[E_{t+j}\{\sigma_{t+j}\eta_{t+j+1}\}])^2 = E[\sigma_{t+j}^2] - 0 = \sigma^2$$

and the same argument applies to terms of type $V[\sigma_{t+j}e_{t+j+1}]$. Coefficient matching on the variance of consumption expression delivers expressions for $\tilde{\sigma}^2$ and $\tilde{\varphi}_e$:

$$\begin{aligned} \tilde{\sigma}^2 &= \sigma^2 [3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2] \\ \tilde{\varphi}_e^2 &= \varphi_e^2 \frac{(1 - \tilde{\rho}_x^2)(1 + \rho_x + \rho_x^2)^2 \sigma^2}{1 - \rho_x^2} \frac{1}{\tilde{\sigma}^2} \\ &= \frac{(1 - \rho_x^6)(1 + \rho_x + \rho_x^2)^2}{1 - \rho_x^2} \frac{\varphi_e^2}{3 + (1 + \rho_x)^2 \varphi_e^2 + \varphi_e^2}, \end{aligned}$$

where the third equality uses the first equality. Note that we imposed

$$\tilde{\rho}_x = \rho_x^3$$

which follows from a desire to match the persistence of the long-run cash-flow component. Recursively substituting, we find that the three-month ahead x process has the following relationship to the current value:

$$x_{t+3} = \rho_x^3 x_t + \varphi_e \sigma_{t+2} e_{t+3} + \rho_x \varphi_e \sigma_{t+1} e_{t+2} + \rho_x^2 \varphi_e \sigma_t e_{t+1}$$

which compares to the quarterly equation

$$\tilde{x}_{t+1} = \tilde{\rho}_x \tilde{x}_t + \tilde{\varphi}_e^2 \tilde{\sigma}_t \tilde{e}_{t+1}$$

The two processes now have the same auto-correlation and unconditional variance.

Fifth, we match the unconditional variance of dividend growth. Given the assumptions we have made sofar, this pins down φ_d :

$$\tilde{\varphi}_d^2 = \frac{3\varphi_d^2 + \phi^2(1 + \rho_x)^2\varphi_e^2 + \phi^2\varphi_e^2}{3 + (1 + \rho_x)^2\varphi_e^2 + \varphi_e^2}$$

Sixth, we match the autocorrelation and the unconditional variance of economic uncertainty σ_t^2 . Iterating forward, we obtain an expression that relates variance in month t to the one in month $t + 3$:

$$\sigma_{t+3}^2 - \sigma^2 = \nu_1^3(\sigma_t^2 - \sigma^2) + \sigma_w \nu_1^2 w_{t+1} + \sigma_w \nu_1 w_{t+2} + \sigma_w w_{t+3}$$

By setting

$$\tilde{\nu}_1 = \nu_1^3$$

and

$$\tilde{\sigma}_w^2 = \sigma_w^2(1 + \nu_1^2 + \nu_1^4)$$

we match the autocorrelation and variance of the quarterly equation

$$\tilde{\sigma}_{t+1}^2 - \tilde{\sigma}^2 = \tilde{\nu}_1(\tilde{\sigma}_t^2 - \tilde{\sigma}^2) + \tilde{\sigma}_w \tilde{w}_{t+1}$$

A simulation of the quarterly model recovered the annualized cash-flow and asset return moments of the monthly simulation.

D The External Habit Model

D.1 Proof of Proposition 2

The reasoning for the habit model exactly parallels the one in appendix (C.2).

Proof. We conjecture that the log wealth-consumption ratio is linear in the sole state variable $(s_t - \bar{s})$,

$$wc_t = A_0 + A_1(s_t - \bar{s}).$$

As CC, we assume joint conditional normality of consumption growth and the surplus consumption ratio. We verify this conjecture from the Euler equation (45).

We start from the canonical log SDF in the external habit model:

$$m_{t+1} = \log \beta - \alpha \Delta c_{t+1} - \alpha \Delta s_{t+1}.$$

Substituting in the expression for returns into the log SDF, we compute innovations, and the conditional mean and variance of the log SDF:

$$\begin{aligned}
m_{t+1} - E_t[m_{t+1}] &= -\alpha(1 + \lambda_t)\bar{\sigma}\eta_{t+1} \\
E_t[m_{t+1}] &= m_0 + \alpha(1 - \rho_s)(s_t - \bar{s}) \\
V_t[m_{t+1}] &= \alpha^2(1 + \lambda_t)^2\bar{\sigma}^2 \\
m_0 &= \log \beta - \alpha\mu
\end{aligned} \tag{63}$$

Likewise, we compute innovations in the consumption claim return, and its conditional mean and variance:

$$\begin{aligned}
r_{t+1} - E_t[r_{t+1}] &= (1 + A_1\lambda_t)\bar{\sigma}\eta_{t+1} \\
E_t[r_{t+1}] &= r_0 - A_1(\kappa_1 - \rho_s)(s_t - \bar{s}) \\
V_t[r_{t+1}] &= (1 + A_1\lambda_t)^2\bar{\sigma}^2 \\
r_0 &= \kappa_0 + A_0(1 - \kappa_1) + \mu
\end{aligned}$$

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations

$$Cov_t[m_{t+1}, r_{t+1}] = -\alpha(1 + \lambda_t)(1 + A_1\lambda_t)\bar{\sigma}^2$$

We assume that the sensitivity function takes the following form

$$\lambda_t = \frac{\bar{S}^{-1}\sqrt{1 - 2(s_t - \bar{s})} + 1 - \alpha}{\alpha - A_1}$$

Using the method of undetermined coefficients and the five components of equation (45), we can solve for the constants A_0 and A_1 :

$$A_1 = \frac{(1 - \rho_s)\alpha - \bar{\sigma}^2\bar{S}^{-2}}{\kappa_1 - \rho_s}, \tag{64}$$

$$0 = \log \beta + \kappa_0 + (1 - \kappa_1)A_0 + (1 - \alpha)\mu + .5\bar{\sigma}^2\bar{S}^{-2} \tag{65}$$

This verifies that our conjecture was correct. It follows immediately that the log SDF can be written as

$$m_{t+1} = \log \beta - \alpha\Delta c_{t+1} - \frac{\alpha}{A_1}(wc_{t+1} - wc_t),$$

which is the expression in the main text. □

The risk premium on the consumption claim is given by $Cov_t[r_{t+1}, -m_{t+1}]$:

$$E_t[r_{t+1}^e] \equiv E_t[r_{t+1} - r_t^f] + .5V_t[r_{t+1}] = \alpha(1 + \lambda_t)(1 + A_1\lambda_t)\bar{\sigma}^2, \tag{66}$$

where the second term on the left is a Jensen adjustment. The expression for the risk-free rate appears in the next section D.2.

D.2 The Steady-State Habit Level

Campbell and Cochrane (1999) engineer their sensitivity function λ_t to deliver a risk-free rate that is linear in the state $s_t - \bar{s}$. (They mostly study a special case with a constant risk-free rate.) The linearity of the risk-free rate is accomplished by choosing

$$\lambda_t^{CC} = \bar{S}^{-1} \sqrt{1 - 2(s_t - \bar{s})} - 1 \quad (67)$$

Note that if the risk aversion parameter $\alpha = 2$ and $A_1 = 1$, our sensitivity function exactly coincides with CC's. Instead, we engineer our sensitivity function to deliver a log wealth-consumption ratio that is linear in $s_t - \bar{s}$.

As a result of our choice, the risk-free rate, $r_t^f = -E_t[m_{t+1}] - .5V_t[m_{t+1}]$, is no longer linear in the state, but contains an additional square-root term:

$$r_t^f = h_0 + \left[\frac{\bar{\sigma}^2 \alpha^2 \bar{S}^{-2}}{(\alpha - A_1)^2} - \alpha(1 - \rho_s) \right] (s_t - \bar{s}) - \bar{\sigma}^2 \alpha^2 \frac{(1 - A_1) \bar{S}^{-1}}{(\alpha - A_1)^2} \left(\sqrt{1 - 2(s_t - \bar{s})} - 1 \right) \quad (68)$$

$$h_0 = -\log \beta + \alpha \mu - .5 \bar{\sigma}^2 \alpha^2 (1 + \lambda(\bar{s}))^2, \quad \text{where} \quad \lambda(\bar{s}) = \left(\frac{\bar{S}^{-1} + 1 - \alpha}{\alpha - A_1} \right) \quad (69)$$

where $\lambda(\bar{s})$ is obtained from evaluating our sensitivity function at $s_t = \bar{s}$.

CC obtain a similar expression, but without the last term. If $\alpha = 2$ and $A_1 = 1$, the expression collapses to the one in CC. A constant risk-free rate obtains in the CC model when $\bar{S}^{-1} = \bar{\sigma}^{-1} \sqrt{\frac{1 - \rho_s}{\alpha}}$ because this choice makes the linear term vanish. While there is no $\bar{S}$ that guarantees a constant risk-less interest rate under our assumptions, we choose $\bar{S}$ to match the *steady-state* risk-free rate in CC, $\bar{r} = -\log \beta + \alpha \mu - .5 \alpha (1 - \rho_s)$. That is, we set $s_t = \bar{s}$ in the above equation, which then collapses to h_0 . Setting $\bar{r} = h_0$ allows us to solve for $\bar{S}^{-1}$ as a function of A_1 and the structural parameters α , ρ_s , and $\bar{\sigma}$:

$$\bar{S}^{-1} = (\alpha - A_1) \left(\bar{\sigma}^{-1} \sqrt{\frac{1 - \rho_s}{\alpha}} \right) - 1 + A_1. \quad (70)$$

Substituting this expression back into the sensitivity function (23), we find that the steady-state sensitivity level $\lambda(\bar{s}) = \bar{\sigma}^{-1} \sqrt{\frac{1 - \rho_s}{\alpha}} - 1$. This implies that we generate the same steady-state conditional covariance between the surplus consumption ratio and consumption growth as in CC.

As in CC, we define a maximum value for the log surplus consumption ratio s_{max} , as the value at which λ_t runs into zero:

$$s_{max} = \bar{s} + \frac{1}{2} (1 - (\alpha - 1)^2 \bar{S}^2)$$

Note that if $\alpha = 2$, this coincides with equation (11) in CC. It is understood that $\lambda_t = 0$ for $s_t \geq s_{max}$.

D.3 Campbell-Shiller Decomposition

Expected discounted future returns and consumption growth rates are given by:

$$r_t^H \equiv E_t \left[\sum_{j=1}^{\infty} r_{t+j} \right] = \frac{r_0}{\kappa_1 - 1} - A_1 (s_t - \bar{s}) \quad (71)$$

$$\Delta c_t^H \equiv E_t \left[\sum_{j=1}^{\infty} \Delta c_{t+j} \right] = \frac{\mu}{\kappa_1 - 1} \quad (72)$$

These expressions enable us to go back and forth between the log wealth-consumption ratio expression in (22) and the Campbell-Shiller equation in (4). Starting from (22)

$$\begin{aligned}
wc_t &= A_0 + A_1(s_t - \bar{s}) \\
&= A_0 + \left(\Delta c_t^H - \frac{\mu}{\kappa_1 - 1} \right) - \left(r_t^H - \frac{r_0}{\kappa_1 - 1} \right) \\
&= \frac{\kappa_0}{\kappa_1 - 1} + \Delta c_t^H - r_t^H
\end{aligned}$$

we arrive at equation (4). The second equality uses the definitions of r_t^H and Δc_t^H . The third equality uses the definition of r_0 .

The variance of the log wealth-consumption ratio can be written in two equivalent ways:

$$V[\Delta c_t^H] + V[r_t^H] - 2Cov[r_t^H, \Delta c_t^H] = V[wc_t] = Cov[wc_t, \Delta c_t^H] + Cov[wc_t, -r_t^H]$$

In the EH model, the terms in this expression are given by

$$\begin{aligned}
V[\Delta c_t^H] &= 0 \\
V[r_t^H] &= A_1^2 \left(\frac{\bar{S}^{-1} + 1 - \alpha}{\alpha - A_1} \right)^2 \frac{1}{1 - \rho_s^2} \bar{\sigma}^2 > 0 \\
Cov[r_t^H, \Delta c_t^H] &= 0 \\
Cov[wc_t, \Delta c_t^H] &= 0 \\
Cov[wc_t, -r_t^H] &= A_1^2 \left(\frac{\bar{S}^{-1} + 1 - \alpha}{\alpha - A_1} \right)^2 \frac{1}{1 - \rho_s^2} \bar{\sigma}^2 > 0
\end{aligned}$$

Likewise, there is no predictability in dividend growth (see equation 75). Therefore, $V[pd_t] = V[r_t^{H,m}]$, where the latter is the unconditional variance of the expected return on the dividend claim.

D.4 Risk Factor Representation

We can further rewrite the log SDF in terms of our two demeaned risk factors (denoted with a tilde):

$$m_{t+1} = m_0 - \alpha \widetilde{\Delta c}_{t+1} - \frac{\alpha}{A_1} \widetilde{\Delta wc}_{t+1} = m_0 - b f_{t+1},$$

where m_0 is defined in (63), the factor loadings are

$$b = \left[\alpha, \frac{\alpha}{A_1} \right],$$

and where the demeaned risk factors are defined as

$$f_{t+1} = \left[\widetilde{\Delta c}_{t+1}, \widetilde{\Delta wc}_{t+1} \right]' = [\Delta c_{t+1} - \mu, (wc_{t+1} - A_0) - (wc_t - A_0)]'.$$

In the EH model, the conditional and unconditional first and second moments of the two risk factors are

$$\begin{aligned}
E_t [\widetilde{\Delta c}_{t+1}] &= 0 & E [\widetilde{\Delta c}_{t+1}] &= 0 \\
V_t [\widetilde{\Delta c}_{t+1}] &= \bar{\sigma}^2 & V [\widetilde{\Delta c}_{t+1}] &= \bar{\sigma}^2 \\
E_t [\widetilde{\Delta w c}_{t+1}] &= -A_1(1 - \rho_s)(s_t - \bar{s}) & E [\widetilde{\Delta w c}_{t+1}] &= 0 \\
V_t [\widetilde{\Delta w c}_{t+1}] &= A_1^2 \bar{\sigma}^2 \lambda_t^2 & V [\widetilde{\Delta w c}_{t+1}] &= A_1^2 \bar{\sigma}^2 \left(1 + \frac{(1 - \rho_s)(\kappa_1 - \rho_s)}{1 - \rho_s^2}\right) \left(\frac{\bar{S}^{-1} + 1 - \alpha}{\alpha - A_1}\right)^2 \\
Cov_t [\widetilde{\Delta c}_{t+1}, \widetilde{\Delta w c}_{t+1}] &= A_1 \bar{\sigma}^2 \lambda_t & Cov [\widetilde{\Delta c}_{t+1}, \widetilde{\Delta w c}_{t+1}] &= A_1 \bar{\sigma}^2 \left(\frac{\bar{S}^{-1} + 1 - \alpha}{\alpha - A_1}\right)
\end{aligned}$$

The two risk factors are conditionally and unconditionally positively correlated as long as $\lambda_t > 0$ (which is true for our calibrations).

Equation (19) in the main text implies that the expected excess return on the consumption claim can be written as the sum of the market prices of risk on the two risk factors.

$$E_t [r_{t+1}^e] = \ell_{1t}^{EH} + \ell_{2t}^{EH} = \left\{ b_1 V_t [\widetilde{\Delta c}_{t+1}] + b_2 Cov_t [\widetilde{\Delta c}_{t+1}, \widetilde{\Delta w c}_{t+1}] \right\} + \left\{ b_1 Cov_t [\widetilde{\Delta c}_{t+1}, \widetilde{\Delta w c}_{t+1}] + b_2 V_t [\widetilde{\Delta w c}_{t+1}] \right\}.$$

After some algebra, we obtain expressions for these market prices of risk that are only a function of the structural parameters of the EH model:

$$\ell_{1t}^{EH} = \alpha(1 + \lambda_t)\bar{\sigma}^2 \quad (73)$$

$$\ell_{2t}^{EH} = \alpha A_1 \lambda_t (1 + \lambda_t) \bar{\sigma}^2 \quad (74)$$

The unconditional market prices of risk are the unconditional means of the conditional market prices of risk: $\ell_i^{LRR} = E[\ell_{it}^{LRR}]$, for $i = 1, 2$. This amounts to setting $\lambda_t = E[\lambda_t] = \lambda(\bar{s}) = \left(\frac{\bar{S}^{-1} + 1 - \alpha}{\alpha - A_1}\right)$ in the above equations.

D.5 Pricing Stocks in EH Model

The main difference with the analysis in the long-run risk model, and the analysis for the total wealth return in the EH model is that the return to the aggregate dividend claim *cannot* be written as a linear function of the state variables. Our choice of the sensitivity function makes the log wealth-consumption ratio linear in the surplus consumption ratio. But, for that same sensitivity function, the log price-dividend ratio is not linear in the surplus-consumption ratio. As a result, we need to resort to a non-linear computation of the price-dividend ratio on the aggregate dividend claim. We focus here on the case where no cointegration is imposed between consumption and dividends on equity. Section E.3 discusses the case with cointegration.

Dividend Growth Process In Campbell and Cochrane (1999), dividend growth is i.i.d., with the same mean μ as consumption growth, and innovations that are correlated with the innovations in consumption growth. To make the dividend growth process more directly comparable across models, we write it as a function of innovations to consumption growth η and innovations u that are orthogonal to η :

$$\Delta d_{t+1} = \mu_d + \varphi_d \bar{\sigma} u_{t+1} + \varphi_d \bar{\sigma} \chi \eta_{t+1}. \quad (75)$$

It follows immediately that its (un)conditional variance equals $\varphi_d^2 \bar{\sigma}^2 (1 + \chi^2)$ and its (un)conditional covariance with consumption growth is $\varphi_d \bar{\sigma}^2 \chi$. If correlation between consumption and dividend growth is $corr$, then $\chi = \sqrt{corr^2 / (1 - corr^2)}$. We set φ_d and χ to replicate the unconditional variance of dividend growth and the correlation of dividend growth and consumption growth $corr$ in Campbell and Cochrane (1999). We set $\mu_d = \mu$, $\varphi_d = 7.32$, and $\chi = 0.20$.

Computation of Price-Dividend Ratio Wachter (2005) shows that the price-dividend ratio on a claim to aggregate dividends can be written as the sum of the price-dividend ratios on strips to the period- n dividend, for $n = 1, \dots, \infty$:

$$\frac{P_t}{D_t} = \sum_{n=1}^{\infty} \frac{P_{nt}^d}{D_t} \quad (76)$$

We adopt her methodology and show it continues to hold for our slightly different dividend growth process in equation (75).

The Euler equation for the period- n strip delivers the following expression for the price-dividend ratio

$$\frac{P_{nt}^d}{D_t} = E_t \left[M_{t+1} \frac{P_{n-1,t+1}^d}{D_{t+1}} \frac{D_{t+1}}{D_t} \right]$$

We conjecture that the price-dividend ratio on the period- n strip equals a function $F_n(s_t)$, which follows the recursion

$$F_n(s_t) = \beta e^{\mu_d - \alpha\mu + \alpha(1-\rho_s)(s_t - \bar{s}) + \frac{1}{2}\varphi_d^2 \bar{\sigma}^2} E_t \left[e^{[\varphi_d \chi - \alpha(1+\lambda_t)]\bar{\sigma}\eta_{t+1}} F_{n-1}(s_{t+1}) \right],$$

starting at $F_0(s_t) = 1$. We now verify this conjecture.

Proof. Substituting in the conjecture $\frac{P_{nt}^d}{D_t} = F_n(s_t)$ into the Euler equation for the period- n strip, we get

$$F_n(s_t) = E_t \left[M_{t+1} F_{n-1}(s_{t+1}) \frac{D_{t+1}}{D_t} \right].$$

Substituting in for the stochastic discount factor M and the dividend growth process (75), this becomes

$$F_n(s_t) = \beta e^{\mu_d - \alpha\mu + \alpha(1-\rho_s)(s_t - \bar{s})} E_t \left[e^{-\alpha(1+\lambda_t)\bar{\sigma}\eta_{t+1}} F_{n-1}(s_{t+1}) e^{\varphi_d \bar{\sigma} u_{t+1} + \varphi_d \chi \bar{\sigma} \eta_{t+1}} \right].$$

Because u and η are independent, we can write the expectation as a product of expectations. Because u is standard normal, the expectation in the previous expression can be written as

$$e^{\frac{1}{2}\varphi_d^2 \bar{\sigma}^2} E_t \left[e^{[\varphi_d \chi - \alpha(1+\lambda_t)]\bar{\sigma}\eta_{t+1}} F_{n-1}(s_{t+1}) \right].$$

This then verifies the conjecture for $F_n(s_t)$. □

Finally, let $g(\eta)$ be the standard normal pdf, then we can compute this function through numerical integration

$$F_n(s_t) = \beta e^{\mu_d - \alpha\mu + \alpha(1-\rho_s)(s_t - \bar{s}) + \frac{1}{2}\varphi_d^2 \bar{\sigma}^2} \int_{-\infty}^{+\infty} e^{[\varphi_d \chi - \alpha(1+\lambda(s_t))]\bar{\sigma}\eta_{t+1}} F_{n-1}(s_{t+1}) g(\eta_{t+1}) d\eta_{t+1},$$

starting at $F_0(s_t) = 1$. The grid for s_t includes 14 very low values for s_t (-300, -250, -200, -150, -100, -50, -40, -30, -20, -15, -10, -9, -8, -7), 100 linearly spaced points between -6.5 and $\bar{s} * 1.001 = -2.85$, and the log of 100 linearly spaced points between $\bar{S}$ and $\exp(1.0000001 s_{max})$. The function evaluation $F_{n-1}(s_{t+1})$ is done using linear

interpolation (and extrapolation) on the grid for the log surplus-consumption ratio s . The integral is computed in matlab using quad.m. The price dividend ratio is computed as the sum of the price-dividend ratios for the first 500 strips.

D.6 Alternative Way of Pinning Down $\bar{S}$

To conclude the discussion of the EH model, we investigate an alternative way to pin down $\bar{S}$. In our benchmark method, outlined in Appendix D.2, we chose it to match the steady state risk-free rate in Campbell and Cochrane (1999). Here, the alternative is to pin down $\bar{S}$ to match the average wealth-consumption ratio of 26.75 in Campbell and Cochrane (1999).

As before, we solve a system of three equations in $(A_0, A_1, \bar{S})$, only the third of which is different and simply imposes that $e^{A_0 - \log(4)} = 26.75$. We obtain the following solution: $A_0 = 4.673$, $A_1 = 0.447$, and $\bar{S} = 0.0339$. The wealth-consumption ratio is higher and less sensitive to the surplus-consumption ratio than in the benchmark case. The volatility of the surplus-consumption ratio is 41.6%, similar to the benchmark model. Because A_1 is lower, so is the volatility of the wc ratio. It is 18.6% in the model, still much higher than the 8.4% in the data. The volatilities of the change in the wc ratio and of the total wealth return are also lowered, but remain too high. Since, we are no longer pinning $\bar{S}$ down to match the steady-state risk-free rate, the risk-free rate turns negative: -33 basis points per quarter or -1.2% per year. It is also more volatile: .59% versus .03 in the main text and .55 in the data. The consumption risk premium is down from 2.67% per quarter to 1.97% per quarter and the equity premium is down from 3.30% per quarter to 2.23%. The main cost of this calibration is a price-dividend ratio that is too low. The volatility of pd^m is now only 12.45% per quarter compared to 42% in the data.

D.7 Quarterly Calibration EH Model

Preference Parameters Obviously, the preference parameter does not depend on the horizon ($\tilde{\alpha} = \alpha$, except for the time discount factor $\tilde{\beta} = \beta^3$). The surplus consumption ratio has the same law of motion as in the monthly model, but we set its persistence equal to $\tilde{\rho}_s = \rho_s^3$. When we simulate the quarterly model, we solve for the corresponding A_0 , A_1 , and $\bar{S}$ from the system (84), (85), and (70), but with the quarterly parameter values described in this appendix.

Cash-flow Parameters Following a similar logic, we can match mean and variance of quarterly consumption and dividend growth in the CC model. From matching the means we get:

$$\tilde{\mu} = 3\mu, \quad \tilde{\mu}_d = 3\mu_d.$$

From matching the variances we get

$$\tilde{\sigma}^2 = 3\sigma^2, \quad \tilde{\varphi}_d = \varphi_d, \quad \tilde{\chi} = \chi.$$

A simulation of the quarterly model recovered the annualized cash-flow and asset return moments of the monthly simulation.

E Supplementary Material

This section contains additional material that illustrates further details on the theory-side and robustness exercise on the empirical side.

E.1 LRR Model: $\rho \rightarrow 1$

In this appendix we study the LRR model as the inverse intertemporal elasticity of substitution, ρ goes to one.

Holding κ_1 fixed, it is easy to see that

$$A_1 = \frac{1-\rho}{\kappa_1 - \rho_x} \rightarrow 0 \text{ as } \rho \rightarrow 1$$

and

$$A_2 = \frac{(1-\rho)(1-\alpha)}{2(\kappa_1 - \nu_1)} \left[1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right] \rightarrow 0 \text{ as } \rho \rightarrow 1$$

The limit argument is more subtle because κ_1 depends on A_0 which in turn depends on ρ . We have solved the system of three non-linear equations (described in appendix C.2) for a sequence of values of ρ approaching 1 (from above and from below) and verified that $A_1 \rightarrow 0$ and $A_2 \rightarrow 0$. Furthermore, we found that $A_0 \rightarrow \log\left(\frac{1}{1-\beta}\right)$, so that $\kappa_1 \rightarrow \beta^{-1}$ and $\kappa_0 \rightarrow -\log\left(\frac{\beta}{1-\beta}\right) + \frac{1}{\beta} \log\left(\frac{1}{1-\beta}\right)$.

As ρ goes to one, the consumption risk premium converges to the one in the standard Lucas-Breeden economy:

$$E_t[r_{t+1} - r_t^f] + .5V_t[r_{t+1}] \rightarrow \alpha\sigma_t^2 \text{ as } \rho \rightarrow 1.$$

This happens because the other two consumption risk premium components converge to zero. Holding κ_1 fixed, this can be seen in the expressions for these two components:

$$\begin{aligned} \lambda_{m,e}B &= (1-\rho)(\alpha-\rho)\frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \rightarrow 0 \text{ as } \rho \rightarrow 1 \\ \lambda_{m,w}A_2 &= (1-\rho)(\alpha-\rho)\frac{(1-\alpha)^2}{4(\kappa_1 - \nu_1)^2} \left[1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right]^2 \rightarrow 0 \text{ as } \rho \rightarrow 1 \end{aligned}$$

We have confirmed numerically that the two consumption risk premium components $\lambda_{m,e}B$ and $\lambda_{m,w}A_2$ go to zero as ρ goes to one (from above or from below), when solving the system of equations. Explained differently the conditional market price of *wc* risk in equation (56) goes to zero, while the market price of standard consumption growth in equation (55) has a well-defined, non-zero limit $\alpha\sigma_t^2$. So, the only risk with a positive compensation associated to it that remains when $\rho \rightarrow 1$ is the standard high-frequency aggregate consumption growth risk.

The same is not true for the risk premium on the claim to the stream of aggregate dividends. We focus on the case without cointegration in Appendix C.5.1.

$$E_t[r_{t+1}^m - r_t^f] + .5V_t[r_{t+1}^m] \rightarrow \xi_{m,e}\sigma_t^2 + \xi_{m,w}\sigma_w^2$$

where

$$\begin{aligned} \xi_{m,e} &\equiv \lim_{\rho \rightarrow 1} \lambda_{m,e}\beta_{m,e} = \frac{(\alpha-1)(\phi-1)\varphi_e^2}{(\kappa_1 - \rho_x)(\kappa_1^m - \rho_x)} \\ \xi_{m,w} &\equiv \lim_{\rho \rightarrow 1} \lambda_{m,w}\beta_{m,w} = \frac{(\alpha-1)^2}{2(\kappa_1 - \nu_1)} \left(1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right) \left[\frac{(\alpha-1)^2}{2(\kappa_1^m - \nu_1)} \left(1 + \frac{\varphi_e^2}{(\kappa_1 - \rho_x)^2} \right) - .5 \frac{H_m}{(\kappa_1^m - \nu_1)} \right] \end{aligned}$$

As the second expression for H_m in Appendix C.5.1 shows, H_m does not depend on ρ . Clearly, for $\phi \neq 1$ and $\alpha \neq 1$, there are positive *equity* risk premia (on the dividend claim) over and above the ones that would arise in a Lucas-Breeden economy.

E.2 LRR Model: Asset Pricing with Cointegration

Dividend Growth Process In the previous specification, consumption and dividends can drift arbitrarily away from each other. In this section, we follow Bansal, Dittmar, and Lundblad (2005) and modify the dividend growth process to impose cointegration between consumption and dividends. Log dividends are stochastically cointegrated with log consumption, and may have a deterministic trend:

$$\begin{aligned} d_{t+1} &= \varpi + \delta(t+1) + \phi c_{t+1} + q_{t+1} \\ \Delta d_{t+1} &= \delta + \phi \Delta c_{t+1} + \Delta q_{t+1}, \end{aligned} \tag{77}$$

where the second equation is obtained by taking first differences of the first equation. The process $\{q\}$ denotes the dividend-consumption ratio, which we specify as a mean-zero, autoregressive process with heteroscedasticity:

$$q_{t+1} = \rho_q q_t + \varphi_q \sigma_t u_{t+1} \tag{78}$$

This is a generalization from the process in Bansal, Dittmar, and Lundblad (2005), who work with a homoscedastic model ($\sigma_t^2 = \bar{\sigma}^2, \forall t$). Equations (77) and (78) completely specify the dividend growth process in the cointegration case and replace equation (57) in the no cointegration case. The rest of the technology process is unaffected: the processes for Δc_{t+1} , x_{t+1} , and σ_{t+1}^2 remain unchanged from the main text. As a result, the stochastic discount factor, and the consumption-wealth ratio process all remain unaltered.

To facilitate comparison with the no-cointegration case, we use the same values for ϕ and φ_d as in the no cointegration case. We match the unconditional mean and variance of dividend growth in the cases with and without cointegration. I.e., we choose the parameter δ to match the mean:

$$\delta = \mu_d - \phi \mu,$$

with $\mu_d = \mu$, and we choose φ_q to match the variance:

$$\varphi_d^2 = \frac{2}{1 + \rho_q} \varphi_q^2 + \phi^2 \Rightarrow \varphi_q = \sqrt{.5(1 + \rho_q)(\varphi_d^2 - \phi^2)}.$$

We keep the parameter ϕ the same in both cases. Following Bansal and Yaron (2004), we choose $\mu_d = \mu$, $\phi = 3$ and $\varphi_d = 4.5$. The only other parameter is the persistence of the quarterly log dividend-consumption ratio ρ_q , which we set equal to 0.83. This follows Lettau and Ludvigson (2005), who document a persistence of .475 at annual frequency (or .83 at quarterly frequency) for the cointegration vector between log consumption, log stock dividends, and log labor income.

Proof of Linearity The only difference with the no-cointegration case is that q_t becomes an additional state variable for the price-dividend ratio. That is, we conjecture:

$$pd_t^m = A_0^m + A_1^m x_t + A_2^m (\sigma_t^2 - \bar{\sigma}^2) + A_3^m q_t.$$

This leads to different expressions for the innovations in the dividend claim return, and the conditional mean and variance of the dividend claim return:

$$\begin{aligned}
r_{t+1}^m - E_t[r_{t+1}^m] &= \phi \sigma_t \eta_{t+1} + (1 + A_3^m) \varphi_q \sigma_t u_{t+1} + \beta_{m,e} \sigma_t e_{t+1} + \beta_{m,w} \sigma_w w_{t+1} \\
E_t[r_{t+1}^m] &= r_0^m + [\phi + A_1^m(\rho_x - \kappa_1^m)] x_t - A_2^m(\kappa_1^m - \nu_1)(\sigma_t^2 - \bar{\sigma}^2) + (\rho_q - 1 - A_3^m(\kappa_1^m - \rho_q)) q_t \\
V_t[r_{t+1}^m] &= ((1 + A_3^m)^2 \varphi_q^2 + \beta_{m,e}^2 + \phi^2) \sigma_t^2 + \beta_{m,w}^2 \sigma_w^2 \\
r_0^m &= \kappa_0^m + A_0^m(1 - \kappa_1^m) + \delta + \phi \mu
\end{aligned}$$

Finally, the conditional covariance between the log SDF and the log dividend claim return is

$$Cov_t[m_{t+1}, r_{t+1}^m] = (\lambda_{m,\eta} \phi - \lambda_{m,e} \beta_{m,e}) \sigma_t^2 - \lambda_{m,w} \beta_{m,w} \sigma_w^2.$$

Using the method of undetermined coefficients, we obtain expressions for A_0^m , A_1^m , A_2^m , and A_3^m :

$$\begin{aligned}
A_1^m &= \frac{\phi - \rho}{\kappa_1^m - \rho_x}, \\
A_2^m &= \frac{[(1 - \theta)A_2(\kappa_1 - \nu_1) + .5\hat{H}_m]}{\kappa_1^m - \nu_1}, \\
A_3^m &= \frac{\rho_q - 1}{\kappa_1^m - \rho_q}, \\
0 &= m_0 + \kappa_0^m + (1 - \kappa_1^m)A_0^m + \delta + \phi \mu + \frac{1}{2}\hat{H}_m \bar{\sigma}^2 + \frac{1}{2} \left(A_2^m - \frac{\alpha - \rho}{1 - \rho} A_2 \right)^2 \sigma_w^2
\end{aligned}$$

where

$$\hat{H}_m = (\lambda_{m,\eta} + \phi)^2 + (\beta_{m,e} - \lambda_{m,e})^2 + (1 + A_3^m)^2 \varphi_q^2$$

The expressions for A_1^m and A_2^m are functionally identical to the ones in the no cointegration case, except that the definition of $\hat{H}_m$ is slightly different from that of H_m . This is a non-linear system in four equations and four unknowns, which we solve numerically.

Equity Risk premium and CS Decomposition The equity risk premium on the dividend claim (adjusted for a Jensen term) becomes:

$$E_t[r_{t+1}^{e,m}] \equiv E_t[r_{t+1}^m - r_t^f] + .5V_t[r_{t+1}^m] = (-\lambda_{m,\eta} \phi + \lambda_{m,e} \beta_{m,e}) \sigma_t^2 + \lambda_{m,w} \beta_{m,w} \sigma_w^2. \quad (79)$$

Note that q_t does not affect the equity risk premium. Its only driver is the conditional variance of consumption growth $\sigma_t^2 - \bar{\sigma}^2$.

Expected discounted future equity returns and dividend growth rates are given by:

$$r_t^{m,H} \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^m)^{-j} r_{t+j}^m \right] = \frac{r_0^m}{\kappa_1^m - 1} + \frac{\rho}{\kappa_1^m - \rho_x} x_t - A_2^m (\sigma_t^2 - \bar{\sigma}^2) \quad (80)$$

$$\Delta d_t^H \equiv E_t \left[\sum_{j=1}^{\infty} (\kappa_1^m)^{-j} \Delta d_{t+j} \right] = \frac{\delta + \phi \mu}{\kappa_1^m - 1} + \frac{\phi}{\kappa_1^m - \rho_x} x_t + \frac{\rho_q - 1}{\kappa_1^m - \rho_q} q_t \quad (81)$$

The only difference with the no-cointegration case is that expected future dividend growth rates now also depend

on the current dividend-consumption ratio q_t . Discount rates remain unchanged. As before,

$$pd_t^m = \frac{\kappa_0^m}{\kappa_1^m - 1} + \Delta d_t^H - r_t^{m,H},$$

which allows us to compute the elements of the variance-decomposition:

$$V[pd_t^m] = Cov[pd_t^m, \Delta d_t^H] + Cov[pd_t^m, -r_t^{m,H}] = V[\Delta d_t^H] + V[r_t^{m,H}] - 2Cov[\Delta d_t^H, r_t^{m,H}].$$

E.3 EH Model: Asset Pricing with Cointegration

Dividend Growth under Cointegration Just as in the LRR model, we impose cointegration and use the dividend growth specification

$$\Delta d_{t+1} = \delta + \phi \Delta c_{t+1} + \Delta q_{t+1} \quad (82)$$

instead of equation (75) in the case without cointegration. The process $\{q\}$ again denotes the log consumption-dividend ratio. We specify q as an autoregressive process with homoscedastic innovations that are correlated with consumption growth innovations η . Relative to the LRR specification, we loose heteroscedasticity, but we gain correlation between consumption growth innovations and innovations to the dividend-consumption process. Since we prefer to work with independent innovations, we write:

$$q_{t+1} = \rho_q q_t + \varphi_q \bar{\sigma} u_{t+1} + \varphi_q \bar{\sigma} \chi \eta_{t+1}, \quad (83)$$

where, as usual, $\eta_t \perp u_t$.

The parameter choices for δ , ϕ , and φ_q are the same as in the LRR model. The choice for χ is the same as in the no-cointegration case.

Computation of Price-Dividend Ratio Under the assumption of cointegration, the dividend growth process is given by equations (77) and (83). Closely following Appendix A in Wachter (2005), we conjecture that the price-dividend ratio can be written as the product of a function that only depends on the log surplus-consumption ratio and another function that only depends on the log dividend-consumption ratio:

$$\frac{P_{nt}^d}{D_t} = F_n^d(s_t) e^{A_n + B_n q_t}$$

The function that depends on s_t follows a recursion

$$F_n^d(s_t) = E_t [M_{t+1} F_{n-1}^d(s_{t+1}) e^{\phi \mu + X \bar{\sigma} \eta_{t+1}}] = \beta e^{\phi \mu - \alpha \mu + \alpha(1 - \rho_s)(s_t - \bar{s})} E_t [e^{\{X - \alpha(1 + \lambda_t)\} \bar{\sigma} \eta_{t+1}} F_{n-1}^d(s_{t+1})].$$

The verification of this conjecture delivers expressions for the constants X , A_n , and B_n .

Proof. The Euler equation for the period- n strip delivers the following expression for the price-dividend ratio

$$\frac{P_{nt}^d}{D_t} = E_t \left[M_{t+1} \frac{P_{n-1,t+1}^d}{D_{t+1}} \frac{D_{t+1}}{D_t} \right] = E_t [M_{t+1} F_{n-1}^d(s_{t+1}) e^{A_{n-1} + B_{n-1} q_{t+1}} e^{\delta + \phi \Delta c_{t+1} + \Delta q_{t+1}}],$$

where the second equality substituted in the expression for dividend growth, and the conjecture for the price-dividend ratio. Next we substitute in for the expressions for consumption growth, the log dividend-consumption

ratio q , and Δq :

$$\frac{P_{nt}^d}{D_t} = e^{A_{n-1} + \delta + [B_{n-1}\rho_q + \rho_q - 1]q_t} E_t \left[M_{t+1} F_{n-1}^d(s_{t+1}) e^{(B_{n-1}+1)\varphi_q \bar{\sigma} u_{t+1}} e^{[(B_{n-1}+1)\varphi_q \chi + \phi] \bar{\sigma} \eta_{t+1} + \phi \mu} \right].$$

Because u is independent of η and standard normally distributed, we have

$$\frac{P_{nt}^d}{D_t} = e^{A_{n-1} + \delta + \frac{1}{2}(B_{n-1}+1)^2 \varphi_q^2 \bar{\sigma}^2 + [B_{n-1}\rho_q + \rho_q - 1]q_t} E_t \left[M_{t+1} F_{n-1}^d(s_{t+1}) e^{[(B_{n-1}+1)\varphi_q \chi + \phi] \bar{\sigma} \eta_{t+1} + \phi \mu} \right].$$

Recursively define the coefficients A_n and B_n as

$$\begin{aligned} A_n &= A_{n-1} + \delta + \frac{1}{2}(B_{n-1} + 1)^2 \varphi_q^2 \bar{\sigma}^2 \\ B_n &= B_{n-1} \rho_q + \rho_q - 1, \end{aligned}$$

starting at $A_0 = B_0 = 0$, and define the constant X as $X = (B_{n-1} + 1)\varphi_q \chi + \phi$, then we obtain

$$\frac{P_{nt}^d}{D_t} = e^{A_n + B_n q_t} E_t \left[M_{t+1} F_{n-1}^d(s_{t+1}) e^{X \bar{\sigma} \eta_{t+1} + \phi \mu} \right],$$

which verifies the conjecture. □

We use numerical integration to compute the sequence $\{F^d(s_t)\}$:

$$F_n^d(s_t) = e^{\log(\beta) + (\phi - \alpha)\mu - \alpha(1 - \rho_s)(\bar{s} - s_t)} \int_{-\infty}^{+\infty} e^{[X - \alpha(1 + \lambda(s_t))] \bar{\sigma} \eta_{t+1}} F_{n-1}^d(s_{t+1}) g(\eta_{t+1}) d\eta_{t+1},$$

where $g(\eta)$ is the standard normal pdf, and start from $F_0^d(s_t) = 1$.

E.4 EH Model: Improving on the Campbell-Shiller Approximation

We start from the definition of the log total wealth return $r_{t+1} = \Delta c_{t+1} + w c_{t+1} - \log(e^{w c_t} - 1)$. Instead of a first-order Taylor approximation around the mean log wealth-consumption ratio A_0 , we do a second-order approximation:

$$\begin{aligned} \log(e^{w c_t} - 1) &\approx \log(e^{A_0} - 1) + \kappa_1(w c_t - A_0) + .5\kappa_1(1 - \kappa_1)(w c_t - A_0)^2 \\ &= -\kappa_0 + [\kappa_1 - A_0\kappa_1(1 - \kappa_1)] w c_t + .5\kappa_1(1 - \kappa_1)w c_t^2 \end{aligned}$$

where

$$\kappa_1 = \frac{e^{A_0}}{e^{A_0} - 1} \text{ and } \kappa_0 = -\log(e^{A_0} - 1) + \kappa_1 A_0 - .5\kappa_1(1 - \kappa_1)A_0^2$$

This leads to the return approximation

$$r_{t+1} \approx \Delta c_{t+1} + w c_{t+1} + \kappa_0 - \kappa_1 w c_t + \{A_0\kappa_1(1 - \kappa_1)w c_t - .5\kappa_1(1 - \kappa_1)w c_t^2\}$$

The term in accolades comes from adding second-order terms.

We conjecture that the log wealth-consumption ratio is linear in the sole state variable $(s_t - \bar{s})$,

$$w c_t = A_0 + A_1(s_t - \bar{s}).$$

As CC, we assume joint conditional normality of consumption growth and the surplus consumption ratio. We verify

this conjecture from the Euler equation (45).

We slightly modify the preferences:

$$m_{t+1} = \log \beta - \alpha \Delta c_{t+1} - \alpha \Delta s_{t+1} + K (s_t - \bar{s})^2.$$

The term $K (s_t - \bar{s})^2$ is a linearity-inducing term, similar in spirit to Gabaix (2007), whose role will become clear below. As before, we compute innovations, and the conditional mean and variance of the log SDF:

$$\begin{aligned} m_{t+1} - E_t [m_{t+1}] &= -\alpha(1 + \lambda_t) \bar{\sigma} \eta_{t+1}, \\ E_t [m_{t+1}] &= m_0 + \alpha(1 - \rho_s) (s_t - \bar{s}) + K (s_t - \bar{s})^2 \\ V_t [m_{t+1}] &= \alpha^2 (1 + \lambda_t)^2 \bar{\sigma}^2 \\ m_0 &= \log \beta - \alpha \mu \end{aligned}$$

Likewise, we compute innovations in the consumption claim return, and its conditional mean and variance:

$$\begin{aligned} r_{t+1} - E_t [r_{t+1}] &= (1 + A_1 \lambda_t) \bar{\sigma} \eta_{t+1} \\ E_t [r_{t+1}] &= [\kappa_0 + A_0(1 - \kappa_1) + .5A_0^2 \kappa_1(1 - \kappa_1)] + \mu - A_1(\kappa_1 - \rho_s) (s_t - \bar{s}) - .5\kappa_1(1 - \kappa_1)A_1^2 (s_t - \bar{s})^2 \\ V_t [r_{t+1}] &= (1 + A_1 \lambda_t)^2 \bar{\sigma}^2 \end{aligned}$$

Again, the only difference with the previous version is the extra term is in $E_t[r_{t+1}]$; its intercept has an additional term, and it has an additional quadratic term in $(s_t - \bar{s})^2$.

The conditional covariance between the log consumption return and the log SDF is given by the conditional expectation of the product of their innovations

$$Cov_t [m_{t+1}, r_{t+1}] = -\alpha (1 + \lambda_t) (1 + A_1 \lambda_t) \bar{\sigma}^2$$

We assume that the sensitivity function takes the following form

$$\lambda_t = \frac{\bar{S}^{-1} \sqrt{1 - 2(s_t - \bar{s})} + 1 - \alpha}{\alpha - A_1}$$

Using the method of undetermined coefficients and the five components of equation (45), we can solve for the constants A_0 and A_1 :

$$A_1 = \frac{(1 - \rho_s)\alpha - \bar{\sigma}^2 \bar{S}^{-2}}{\kappa_1 - \rho_s}, \quad (84)$$

$$0 = \log \beta + \kappa_0 + (1 - \kappa_1)A_0 + .5A_0^2 \kappa_1(1 - \kappa_1) + (1 - \alpha)\mu + .5\bar{\sigma}^2 \bar{S}^{-2} \quad (85)$$

When we choose the constant $K = .5\kappa_1(1 - \kappa_1)A_1^2$, the terms in $(s_t - \bar{s})^2$ cancel. This verifies that our conjecture was correct. Note that because κ_1 is close to 1, K is close to zero.

Note also that the steady-state risk-free rate is unchanged. Even though $r_t^f = -E_t[m_{t+1}] - .5V_t[m_{t+1}]$ will have the additional term $-.5\kappa_1(1 - \kappa_1)A_1^2(s_t - \bar{s})^2$, this term is zero when evaluated at $s_t = \bar{s}$. That implies that the third equation of the system of three equations in three unknowns is the same as before.

The solution to this system is virtually identical to that of the linear system. I.e., A_0 , A_1 , $\bar{s}$, s_{max} , and κ_1 are identical up to the 9th decimal. Only κ_0 is different, as it should be, because of its changed definition. We conclude that the Campbell-Shiller approximation does an excellent job at approximating the log total wealth return.

Table 1: The Wealth-Consumption Ratio in the Data

This table displays the standard deviation and the autocorrelation properties (at the 1-, 4-, and 8-quarter horizon) of the log wealth-consumption ratio wc measured over the 216 quarters between 1952.1 and 2005.4. The first column is the Campbell model of human wealth returns, which implies that the expected returns on human wealth equal the expected returns on financial wealth. The second column is the Shiller model which assumes that the expected return on human wealth is constant. We report both the case without (no TV) and with time-varying human wealth share (TV). The third column is the Jagannathan-Wang model which assumes that the expected return on human wealth equals the expected labor income growth rate. The fourth column is the cointegration vector between consumption, financial wealth and labor income cay . The cointegration coefficient vector is $(1, .20, .80)$, i.e., the human wealth share is 80%.

moments	Campbell	Shiller		Jag.-Wang		cay
	no TV	no TV	TV	no TV	TV	
$Std[wc]^*$	8.45	1.91	1.90	2.37	2.37	2.14
$AC(1)[wc]^1$	.954	.932	.933	.936	.938	.923
$AC(4)[wc]^1$	.860	.814	.815	.825	.827	.789
$AC(8)[wc]^1$	.759	.713	.707	.759	.759	.712

Table 2: Variance Decomposition - Log Wealth/Consumption Ratio.

The first four columns of this table report the variance decomposition (in percentage points) of the wealth-consumption ratio into three components at different horizons. The next six columns of this table report the variance decomposition (in percentage points) of the wealth-consumption ratio into six components at different horizons. These decomposition stem from the expression of the wealth-consumption ratio wc_t as the sum of the following components: $wc_t = -r_t^H + \Delta c_t^H + wc_t^H$, where $r_t^H = \sum_{s=1}^H \kappa_1^{-s} r_{t+s}$, $\Delta c_t^H = \sum_{s=1}^H \kappa_1^{-s} \Delta c_{t+s}$ and $wc_t^H = \kappa_1^{-s} wc_{t+H}$. The last column reports the regression coefficient of the price-dividend ratio on (minus) the log risk-free rate r_t^f , $H = \sum_{s=1}^H \rho^{s-1} r_{t+s}^f$. Variances and covariances are derived from a one-lag VAR of log change in broadly-defined financial wealth Δp^a , labor income growth Δy , the log dividend-price ratio on broadly-defined financial wealth dp^a , the real risk-free rates r^f , the term spread ysp on the yield curve, the labor income share lis , consumption growth Δc , the spread between Baa bonds and Treasury bills def , the small value spread vsl and log wealth-consumption ratio wc . Standard errors computed from bootstrapping (1000 draws) are reported between parenthesis. Quarterly data span the 1952:2-2005:4 period. Horizons are expressed in quarters.

Horizon	$\frac{Cov[w_c, \Delta c^H]}{Var(w_c)}$	$\frac{Cov[w_c, -r^H]}{Var(w_c)}$	$\frac{Cov[w_c, wc^H]}{Var(w_c)}$	$\frac{Var(\Delta c^H)}{Var(w_c)}$	$\frac{Var(r^H)}{Var(w_c)}$	$\frac{-2 * Cov[r^H, \Delta c^H]}{Var(w_c)}$	$\frac{Var(wc^H)}{Var(w_c)}$	$\frac{2 * Cov[w_c^H, \Delta c^H]}{Var(w_c)}$	$\frac{-2 * Cov[w_c^H, r^H]}{Var(w_c)}$	$\frac{Cov[w_c, -r^f H]}{Var(w_c)}$
1	0.49 (0.63)	4.90 (2.77)	94.61 (2.64)	0.05 (0.05)	0.61 (0.89)	-0.18 (0.25)	89.70 (4.62)	1.06 (1.18)	8.76 (4.22)	1.75 (0.95)
4	1.22 (2.38)	18.82 (9.11)	79.95 (8.34)	0.25 (0.30)	4.42 (6.42)	-0.38 (1.94)	64.20 (11.42)	2.33 (3.50)	29.19 (8.05)	6.61 (3.64)
8	1.50 (4.30)	35.07 (13.93)	63.43 (12.11)	0.34 (0.58)	13.24 (14.82)	0.01 (5.03)	40.45 (12.31)	2.30 (4.59)	43.65 (6.98)	11.13 (6.66)
16	1.95 (6.69)	58.40 (16.20)	39.65 (12.80)	0.38 (0.96)	34.79 (23.53)	1.34 (10.36)	15.81 (8.00)	1.79 (4.22)	45.89 (11.09)	16.23 (10.99)
20	2.21 (7.41)	66.44 (15.77)	31.35 (11.79)	0.40 (1.10)	44.74 (24.64)	2.04 (12.21)	9.88 (6.04)	1.58 (3.72)	41.36 (12.29)	17.83 (12.49)
40	3.13 (9.08)	87.14 (12.27)	9.73 (6.07)	0.46 (1.49)	76.36 (22.37)	4.67 (16.91)	0.95 (1.53)	0.67 (1.74)	16.89 (8.95)	21.84 (16.45)
80	3.57 (9.74)	95.49 (10.17)	0.94 (1.52)	0.49 (1.70)	91.55 (19.81)	6.09 (18.93)	0.01 (0.17)	0.07 (0.44)	1.79 (2.52)	23.42 (18.53)
∞	3.62 (9.85)	96.38 (9.85)	0.00 (0.00)	0.49 (1.75)	93.26 (19.41)	6.25 (19.29)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	23.59 (19.17)

Table 3: Variance Decomposition - Log Price/Dividend Ratio.

The first four columns of this table report the variance decomposition (in percentage points) of the wealth-consumption ratio into three components at different horizons. The next six columns of this table report the variance decomposition (in percentage points) of the price-dividend ratio into six components at different horizons. These decompositions stem from the expression of the log price-dividend ratio pd_t^m as the sum of the following components: $pd_t^m = -r_t^{m,H} + \Delta d_t^{H,m} - pd_t^{H,m}$, where $r_t^{H,m} = \sum_{s=1}^H \rho^{s-1} r_{t+s}^m$, $\Delta d_t^{H,m} = \sum_{s=1}^H \rho^{s-1} \Delta d_{t+s}^m$ and $pd_t^{H,m} = \rho^H pd_{t+H}^m$. r^m is the CRSP value-weighted return and Δd^m the real per capita dividend growth rate. The last column reports the regression coefficient of the price-dividend ratio on (minus) the log risk-free rate $r_t^{f,H} = \sum_{s=1}^H \rho^{s-1} r_{t+s}^f$. Variances and covariances are derived from a one-lag VAR of log CRSP value-weighted return r^m , dividend growth on CRSP Δd^m , log dividend-price ratio on broad financial wealth dp^a , real risk-free rates r^f , the term spread y_{sp} on the yield curve, the spread between Baa bonds and Treasury bills def , and the small value spread val . Standard errors computed from bootstrapping (1000 draws) are reported between parenthesis. Quarterly data span the 1952:2-2005:4 period. Horizons are expressed in quarters.

Horizon	$\frac{Cov[pd, \Delta d^H]}{Var(pd)}$	$\frac{Cov[pd, -r^{m,H}]}{Var(pd)}$	$\frac{Cov[pd, pd^H]}{Var(pd)}$	$\frac{Var(\Delta d^H)}{Var(pd)}$	$\frac{Var(r^{m,H})}{Var(pd)}$	$\frac{-2 * Cov[r^{m,H}, \Delta d^H]}{Var(pd)}$	$\frac{Var(pd^H)}{Var(pd)}$	$\frac{2 * Cov[pd^H, \Delta d^H]}{Var(pd)}$	$\frac{-2 * Cov[pd^H, r^{f,H}]}{Var(pd)}$	$\frac{Cov[pd, -r^{f,H}]}{Var(pd)}$
1	4.72 (3.62)	4.10 (2.45)	91.19 (5.56)	3.46 (3.11)	0.67 (0.69)	2.08 (1.97)	88.59 (6.07)	0.44 (2.72)	4.77 (2.86)	0.00 (0.23)
4	6.51 (5.38)	12.22 (6.63)	81.27 (8.97)	1.61 (1.92)	3.21 (3.83)	0.61 (2.42)	67.99 (12.12)	9.19 (6.63)	17.39 (7.73)	0.08 (0.90)
8	10.47 (8.48)	22.14 (10.51)	67.39 (12.69)	2.74 (3.84)	7.83 (8.45)	2.24 (5.39)	47.59 (13.58)	13.22 (8.48)	26.38 (9.87)	0.23 (1.63)
16	16.07 (12.16)	37.27 (14.05)	46.66 (14.06)	4.36 (6.58)	16.91 (14.54)	8.63 (9.79)	23.22 (10.37)	14.79 (8.64)	32.08 (11.62)	0.42 (2.59)
20	18.18 (13.29)	42.92 (14.75)	38.90 (13.40)	5.07 (7.54)	21.19 (16.32)	12.16 (11.27)	16.23 (8.44)	14.06 (8.17)	31.29 (11.93)	0.48 (2.90)
40	24.61 (16.02)	59.54 (15.94)	15.85 (8.17)	7.80 (10.14)	37.46 (20.71)	25.75 (15.30)	2.71 (2.77)	7.87 (5.17)	18.41 (9.70)	0.64 (3.68)
80	28.34 (17.31)	69.01 (17.05)	2.65 (2.65)	9.80 (11.46)	49.42 (24.17)	35.56 (18.14)	0.08 (0.48)	1.53 (1.69)	3.62 (3.86)	0.74 (4.07)
∞	29.09 (17.63)	70.91 (17.63)	0.00 (0.00)	10.23 (11.77)	52.06 (25.89)	37.71 (19.34)	0.00 (0.00)	0.00 (0.00)	0.00 (0.00)	0.76 (4.16)

Table 4: Moments of Consumption Growth and the Wealth-Consumption Ratio

This table displays means and variances of log consumption growth, the log consumption-wealth ratio, the risk-free rate, and the log total wealth return. Each column represents the results from a model simulation at quarterly frequency. Each simulation is ran for 280 quarters and repeated 5,000 times. We form unconditional means and variances of these annual series. All reported moments are averages of the annual statistics across the 5,000 simulations. The numbers reported in a row with a % are multiplied by 100. All moments reported in this table are used in the Simulated Method of Moments estimation exercise of Columns 2 and 4, except for the ones indicated with a ⁿⁱ.

	moments	Long-Run Risk Model		External Habit Model		Data
		benchmark	SMM	benchmark	SMM	
1	$E[\Delta c]\%$ (s.e.)	.45 (.21)	.52 (.14)	.47 (.05)	.52 (.03)	.52
2	$Std[\Delta c]\%$ (s.e.)	1.43 (.08)	.45 (.06)	.75 (.04)	.47 (.02)	.47
3	$AC(1)[\Delta c]^{ni}$ (s.e.)	.09 (.08)	.49 (.11)	-.01 (.07)	-.01 (.07)	.36
4	$AC(4)[\Delta c]^{ni}$ (s.e.)	.07 (.08)	.39 (.12)	-.01 (.07)	-.01 (.07)	.05
5	$AC(8)[\Delta c]^{ni}$ (s.e.)	.05 (.07)	.27 (.12)	-.00 (.07)	-.00 (.07)	-.15
6	$Std[\Delta wc]$ (s.e.)	.90 (.05)	3.11 (.28)	9.46 (2.17)	2.91 (.30)	4.43
7	$AC(1)[\Delta wc]^{ni}$ (s.e.)	-.03 (.07)	-.03 (.07)	-.02 (.08)	-.02 (.07)	-.02
8	$AC(4)[\Delta wc]^{ni}$ (s.e.)	-.02 (.07)	-.02 (.07)	-.02 (.08)	-.01 (.07)	-.01
9	$AC(8)[\Delta wc]^{ni}$ (s.e.)	-.02 (.07)	-.02 (.07)	-.01 (.08)	-.01 (.07)	.00
10	$Corr[\Delta c, \Delta wc]$ (s.e.)	-.06 (.06)	.15 (.05)	.90 (.03)	.98 (.01)	-.04
11	$Std[wc]\%$ (s.e.)	2.35 (.43)	8.20 (1.68)	29.33 (12.75)	10.00 (2.84)	8.45
12	$AC(1)[wc]$ (s.e.)	.91 (.03)	.92 (.03)	.93 (.03)	.94 (.02)	.95
13	$AC(4)[wc]$ (s.e.)	.70 (.10)	.70 (.10)	.74 (.11)	.79 (.09)	.86
14	$AC(8)[wc]$ (s.e.)	.47 (.15)	.48 (.15)	.55 (.17)	.62 (.14)	.76
15	$Corr[wc, \Delta c]$ (s.e.)	.29 (.08)	-.66 (.09)	.27 (.04)	.25 (.03)	.14
16	$Corr[wc, \Delta wc]$ (s.e.)	.19 (.04)	.18 (.04)	.18 (.05)	.16 (.04)	.03
17	$E[r^f]\%$ (s.e.)	.35 (.12)	2.03 (.36)	.44 (.02)	1.14 (.01)	.27
18	$Std[r^f]\%$ (s.e.)	.30 (.05)	.87 (.18)	.03 (.02)	.01 (.00)	.55
19	$Std[r]\%$ (s.e.)	1.64 (.09)	3.21 (.29)	10.26 (2.21)	3.42 (.32)	4.43
20	$Corr[\Delta c, r]$ (s.e.)	.84 (.02)	.28 (.05)	.91 (.03)	.98 (.01)	.07
	Obj. Fcn.	3,554.5	21.39	1,084.3	17.86	

Table 5: Variance Decomposition of the Wealth-Consumption and Price-Dividend Ratios.

This table shows the variance decomposition of the wealth-consumption and price-dividend ratios at the infinite horizon. The variance of the log wealth-consumption ratio can be written in two equivalent ways: $V[\Delta c_t^H] + V[r_t^H] - 2Cov[r_t^H, \Delta c_t^H] = V[wc_t] = Cov[wc_t, \Delta c_t^H] + Cov[wc_t, -r_t^H]$. In this expression, $V[x] = \frac{\varphi_c^2}{1-\rho_x^2} \bar{\sigma}^2$ and $V[\sigma^2 - \bar{\sigma}^2] = \frac{\sigma_w^2}{1-\nu_1^2}$. The first line is the sum of the second, third line minus 2 times the fourth line. In addition, the first line is the sum of the sixth and seventh line. All variances and covariances are multiplied by 10,000. The last two column report the results from a simulation of 5,000 iterations of 232 quarters. The numbers in parentheses are bootstrap standard errors, computed as standard deviation across the 5,000 bootstrap iterations.

	Long-Run Risk Model		benchmark	SMM
$V[wc_t]$	$\frac{(1-\rho)^2}{(\kappa_1-\rho_x)^2} V[x] + A_2^2 V[\sigma^2 - \bar{\sigma}^2]$	> 0	5.72 (2.13)	70.08 (29.65)
$V[\Delta c_t^H]$	$\frac{1}{(\kappa_1-\rho_x)^2} V[x]$	> 0	51.03 (18.93)	26.55 (11.32)
$V[r_t^H]$	$\frac{\rho^2}{(\kappa_1-\rho_x)^2} V[x] + A_2^2 V[\sigma^2 - \bar{\sigma}^2]$	> 0	22.73 (8.43)	180.30 (76.65)
$Cov[r_t^H, \Delta c_t^H]$	$\frac{\rho}{(\kappa_1-\rho_x)^2} V[x]$	> 0	34.02 (12.62)	68.38 (29.30)
$Cov[wc_t, \Delta c_t^H]$	$\frac{1-\rho}{(\kappa_1-\rho_x)^2} V[x]$	$> 0 \Leftrightarrow \rho < 1$	17.01 (6.33)	-41.83 (18.06)
$Cov[wc_t, -r_t^H]$	$\frac{\rho^2-\rho}{(\kappa_1-\rho_x)^2} V[x] + A_2^2 V[\sigma^2 - \bar{\sigma}^2]$	$> 0 \Leftrightarrow \rho > 1$	-11.29 (4.21)	111.92 (47.57)
$Cov[wc_t, \Delta c_t^H]/V[wc_t]$	$(1 - \rho + \frac{(A_2)^2(\kappa_1-\rho_x)^2}{1-\rho} \frac{V[\sigma^2-\bar{\sigma}^2]}{V[x]})^{-1}$	$> 0 \Leftrightarrow \rho < 1$	297.48 (7.94)	-59.57 (4.00)
$V[pd_t]$	$\frac{(\phi-\rho)^2}{(\kappa_1^m-\rho_x)^2} V[x] + (A_2^m)^2 V[\sigma^2 - \bar{\sigma}^2]$	> 0	229.74 (85.41)	786.02 (331.76)
$V[\Delta d_t^H]$	$\frac{\phi^2}{(\kappa_1^m-\rho_x)^2} V[x]$	> 0	377.62 (140.08)	1354.35 (577.33)
$V[r_t^{m,H}]$	$\frac{\rho^2}{(\kappa_1^m-\rho_x)^2} V[x] + (A_2^m)^2 V[\sigma^2 - \bar{\sigma}^2]$	> 0	19.93 (7.50)	155.80 (61.35)
$Cov[r_t^{m,H}, \Delta d_t^H]$	$\frac{\phi\rho}{(\kappa_1^m-\rho_x)^2} V[x]$	> 0	83.91 (31.76)	362.06 (171.42)
$Cov[pd_t, \Delta d_t^H]$	$\frac{(\phi-\rho)\phi}{(\kappa_1^m-\rho_x)^2} V[x]$	$> 0 \Leftrightarrow \rho < \phi$	293.72 (109.19)	992.28 (429.96)
$Cov[pd_t, -r_t^{H,m}]$	$\frac{(\rho-\phi)\rho}{(\kappa_1^m-\rho_x)^2} V[x] + (A_2^m)^2 V[\sigma^2 - \bar{\sigma}^2]$	$> 0 \Leftrightarrow \rho > \phi$	-63.98 (24.46)	-206.26 (118.30)
$Cov[pd_t, \Delta d_t^H]/V[pd_t]$	$(1 - \frac{\rho}{\phi} + \frac{(A_2^m)^2(\kappa_1^m-\rho_x)^2}{(\phi-\rho)\phi} \frac{V[\sigma^2-\bar{\sigma}^2]}{V[x]})^{-1}$	$> 0 \Leftrightarrow \rho < \phi$	127.87 (2.75)	125.94 (9.56)
External Habit Model				
$V[wc_t]$			1023.00 (1013.67)	108.07 (67.10)
$V[\Delta c_t^H]$	0		0 (0)	0 (0)
$V[r_t^H]$	$A_1^2 (\frac{\bar{S}^{-1}+1-\alpha}{\alpha-A_1})^2 V[s - \bar{s}]$	> 0	1023.00 (1013.67)	108.07 (67.10)
$Cov[r_t^H, \Delta c_t^H]$	0		0 (0)	0 (0)
$Cov[wc_t, \Delta c_t^H]$	0		0 (0)	0 (0)
$Cov[wc_t, -r_t^H]$	$A_1^2 (\frac{\bar{S}^{-1}+1-\alpha}{\alpha-A_1})^2 V[s - \bar{s}]$	> 0	1023.00 (1013.67)	108.07 (67.10)
$Cov[wc_t, \Delta c_t^H]/V[wc_t]$			0 (0)	0 (0)

Table 6: The Risk Premium

This table displays the mean total wealth risk premium (Row 1), its standard deviation (Row 2), its decomposition into a market price of aggregate consumption growth risk (Row 3) and a market price of *wc*-growth risk (Row 4), the mean total wealth return (Row 5), the mean wealth-consumption ratio Row 6), the mean equity risk premium (Row 7) and its variability (Row 8). The columns of this table correspond to those of Table 4. For the external habit model, the mean equity risk premium is computed as the average excess return on equity plus Jensen adjustment. Its standard deviation is not available.

	moments	Long-Run Risk Model		External Habit Model	
		benchmark	SMM	benchmark	SMM
1	$E [E_t[r_{t+1}^e]] \%$ (s.e.)	.40 (.01)	-.65 (.09)	2.67 (1.16)	2.17 (.35)
2	$Std [E_t[r_{t+1}^e]] \%$ (s.e.)	.03 (.01)	.26 (.04)	2.00 (0.86)	.55 (.16)
3	$E[\ell_{1t}] \%$ (s.e.)	.18 (.01)	.01 (.00)	.18 (.04)	.30 (.02)
4	$E[\ell_{2t}] \%$ (s.e.)	.22 (.01)	-.66 (.10)	2.50 (1.12)	1.87 (.33)
5	$E[r] \%$ (s.e.)	.74 (.20)	1.34 (.18)	2.60 (.39)	3.25 (.16)
6	$E[wc]$ (s.e.)	5.85 (.01)	4.81 (.03)	3.86 (.17)	3.62 (.06)
7	$E [E_t[r_{t+1}^{e,m}]] \%$ (s.e.)	1.39 (.05)	2.21 (.32)	3.30 (1.78)	3.26 (.64)
8	$Std [E_t[r_{t+1}^{e,m}]] \%$ (s.e.)	.09 (.02)	.89 (.14)	×	×

Table 7: Cross-Sectional Asset Pricing Results

Estimation of linear factor models in $(\Delta c_{t+1}, \Delta wc_{t+1})$ on the cross-section of assets. This table reports Fama-MacBeth estimates of the risk prices λ (in percentage points), the p-value (in percentage points) of a F -test (under the null that the sum in risk prices is zero), the p-value (in percentage points) of a χ^2 test (under the null that the pricing errors are zero), the R^2 , the square root of the mean square pricing errors $RMSE$ (in percentage points) and the mean absolute pricing error MAE (in percentage points). The table also reports the mean total wealth risk premium $RP^{\Delta c}$, the spread between the equity premium and the total wealth risk premium $RP^M - RP^{\Delta c}$, the mean log return on wealth R^W and the annualized mean log wealth-consumption ratio A_0^{Ann} . In panel A, test assets are the Fama-French 25 portfolios (sorted on book-to-market and size), the 6 Fama bond portfolios (sorted on maturities) and the CRSP value-weighted market return. In panel B, test assets are the Fama-French 25 portfolios (sorted on size and long-term reversal), the 6 Fama bond portfolios (sorted on maturities) and the CRSP value-weighted market return. Real returns are obtained by subtracting the inflation rate computed using the price index of consumption in nondurables and services. Real returns are corrected from Jensen terms. Excess returns are computed using 3-month Fama risk-free rates. In both panels, test assets include either actual returns for unconditional asset pricing ('Uncond.') or actual returns and returns times the lagged wealth-consumption ratio rescaled between 0 and 1 for conditional asset pricing ('Cond.'). Risk factors are consumption growth in nondurables and services Δc_{t+1} and the change in the log wealth-consumption ratio Δwc_{t+1} . Data are quarterly and the sample is 1952:II-2005:IV. Standard errors are reported between parenthesis. Shanken-corrected standard errors are reported between brackets. P-values of the F and χ^2 tests are reported for both estimates of the standard errors.

	Factor Prices		Tests		Fit			Returns and Wealth-Consumption			
	λ_c	λ_{wc}	F -test	χ^2	R^2	$RMSE$	MAE	$RP^{\Delta c}$	$RP^M - RP^{\Delta c}$	R^W	A_0^{Ann}
Panel A: Size and book-to-market											
<i>Uncond.</i>	0.61	0.01	0.61		0.67	0.56	0.42	2.45	5.66	3.40	76.25
	(0.17)	(0.35)	(5.68)	(0.00)							
	[0.27]	[0.53]	[18.23]	[8.38]							
<i>Cond.</i>	0.44	0.27	0.71		0.69	0.50	0.38	2.85	2.41	3.80	58.50
	(0.15)	(0.33)	(2.21)	(0.00)							
	[0.20]	[0.42]	[6.50]	[0.00]							
Panel B: Size and long-term reversal											
<i>Uncond.</i>	0.03	0.97	1.01		0.82	0.41	0.31	4.02	5.54	4.97	34.82
	(0.16)	(0.30)	(0.17)	(0.00)							
	[0.17]	[0.31]	[0.25]	[0.00]							
<i>Cond.</i>	-0.06	1.10	1.04		0.86	0.33	0.25	4.17	3.14	5.12	33.10
	(0.15)	(0.30)	(0.07)	(0.00)							
	[0.16]	[0.32]	[0.14]	[0.00]							

Table 8: Cointegration tests.

The table reports cointegration tests for the following variables: log real per capita consumption c_t , log real per capita labor income y_t , and log real per capita financial wealth a_t . Panel A reports reports the ADF t-statistics (and the corresponding 1%, 5% and 10% critical values) for α in regressions of the form $\Delta \hat{v}_t = \alpha \hat{v}_{t-1} + A(L)\Delta \hat{v}_{t-1}$, where $\hat{v}_t$ is the estimated residual of the following regression: $c_t = \beta_0 + \beta_1 a_t + \beta_2 y_t + v_t$. The Dickey-Fuller test is run assuming 1 to 4 lagged values of the residuals' first-differences $\Delta \hat{v}_{t-1}$. Under the null hypothesis, the vector $[c_t \ a_t \ y_t]$ is not cointegrated. Critical values assume trending regressors; see table IIc in Phillips and Ouliaris (1990). Panel B reports the trace and L-max test statistics and the corresponding 1%, 5% and 10% critical values. The procedure estimates a VAR with 1 to 4 lags. We assume that the cointegrating relation has a constant but no trend. The cointegrating rank is r . Significance at 10% level is indicated with a *. Panel C reports results from Park (1992) canonical cointegration regression (CCR) test. The null hypothesis is the presence of cointegration. Park's $H(0, 1)$ tests for deterministic cointegration (with time polynomials of order 1). Park's $H(1, 2)$, $H(1, 3)$, $H(1, 4)$, $H(1, 5)$ tests for stochastic cointegration (with time polynomials of order 2 to 5). The p -value is the probability that the χ^2 is zero, i.e that we cannot reject the null of cointegration. In all panels, the data are quarterly, and the sample spans the first quarter of 1952 to the fourth quarter of 2005.

Panel A: Phillips - Ouliaris cointegration test								
Dickey-Fuller <i>t</i> -Statistics						Critical Values		
Lag=1		Lag=2	Lag=3	Lag=4		1%	5%	10%
−4.48		−4.23	−4.24	−3.97		−4.36	−3.80	−3.52
Panel B: Johansen cointegration test								
<i>L</i> -max Statistics						<i>Trace</i> Statistics		
r	1 lag	2 lags	3 lags	4 lags	1 lag	2 lags	3 lags	4 lags
1	26.43*	23.45	21.85	24.85*	20.28	17.28	16.58	19.48
2	6.15	6.16	5.27	5.37	6.15	6.09	5.04	5.18
3	0.00	0.07	0.23	0.19	0.00	0.07	0.23	0.19
Panel C: Park cointegration test								
Deterministic cointegration			Stochastic cointegration					
$H(0, 1)$			$H(1, 2)$	$H(1, 3)$	$H(1, 4)$	$H(1, 5)$		
χ^2			0.14	0.34	1.22	1.36	1.37	
<i>p</i> − value			0.71	0.56	0.54	0.71	0.85	

Figure 1: The Log Wealth-Consumption Ratio in the Data

The figure plots the demeaned log wealth-consumption ratio (with time-varying wealth shares) for the three different assumptions on expected human wealth returns, as well as $-\widehat{cay}$. The data are quarterly and the sample is 1952.I-2005.IV. Appendix A contains all data definitions.

