

Labor-Market Implications of Contracts Under Moral Hazard

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Abstract

The optimal contract under moral hazard is embedded in a standard Mortensen-Pissarides matching model. Under standard assumptions, we show that when firms cannot perfectly observe workers' productivity the optimal contract can take the form of a debt contract exhibiting almost a fixed wage along the business cycle. When this contract is embedded in the standard matching model, the calibrated model generates a more stable wage and more volatile employment than the model with Nash bargaining.

Keywords: Searching Friction, Moral Hazard, Wage Rigidity, Labor Volatility

JEL-Classification: E24, J21, J41, J64

1. Introduction

One of the stylized facts in aggregate labor-market fluctuations is that hours worked move a lot without a corresponding movement in wages. To match this fact, a standard neoclassical model (e.g., Lucas and Rapping (1969), Kydland and Prescott (1982), King, Plosser, and Rebelo (1988)) requires an high intertemporal substitution elasticity of labor supply that is not supported by estimates from the micro data (MacCurdy (1981), Altonji (1986)).¹

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¹Moreover, some argue that the optimization behavior of a worker is not consistent with the movement of wage or hours (Mankiw, Rotemberg and Summers (1985))

An economy with search frictions has been developed (e.g., Mortensen and Pissarides (1994)) as an alternative to the Walrasian neoclassical construct. While these models proved successful in addressing various issues in the labor market, they are not capable of generating the observed volatility in labor-market quantities, such as unemployment and vacancy, in response to plausible productivity shifts. In particular, Shimer (2005) shows that the inability to generate volatile quantities in these models is mainly due to the flexible movement of wages in response to a productivity shock. In the standard matching model, wages are determined by bargaining between the firm and the worker.² As aggregate productivity improves, wages increase rapidly (owing to the increased value of the outside option) and absorb most of the rent from the match, eliminating the incentive to create more vacancies. Shimer (2004), Hall (2005), and Gertler and Trigari (2006) introduced wage rigidity (from the Nash equilibrium of bilateral auction or staggered bargaining) and showed that the volatility of unemployment and vacancies increases as wages become more rigid. In these studies, however, the sticky wage assumption is rather ad hoc.

In this paper, we study the determination of wage as a contract problem where a risk-neutral worker has private information about his effort level. The optimal contract can be characterized by a standard debt contract that exhibits almost a fixed wage (with a bonus for an extremely good performance). The debt contract gives all of the surplus minus a base wage to the firm as long as it is below some predetermined level; then the worker takes any extra rent over that level as a variable wage. When this contract is embedded in the standard MP model, the economy generates much more volatile unemployment and vacancies in response to aggregate productivity shifts than the standard models do. Moreover, we show that the own property of the optimal contract tends to strengthen wage rigidity, thus amplifying the fluctuations of labor quantities.

Previous research with a contracting approach to wage determination includes Boldrin and Horvath (1995), Kennan (2003), Menzio (2004), and Shimer and Wright (2004). Boldrin and Horvath embedded an optimal labor contract in the standard RBC model and explored its quantitative implications toward labor market fluctuation. Nonetheless, they are mostly concerned with optimal risk-sharing between risk-averse workers and risk-neutral (or less risk-averse) firms where there is no informational asymmetry. Kennan and Menzio investigated the wage determination and its effects on employment fluctuation for an economy where the firm, not the worker, has private information on productivity. Shimer and Wright derived an optimal contract under two-sided asymmetric information—that is, only the firm has information regarding job productivity and only the worker knows his effort level. However, they did not explore the dynamic implications of that contract. While the possibility of

²Early research that embedded search frictions into an otherwise standard RBC model includes Andolfatto (1996) and Merz (1999).

wage rigidity in the moral hazard setting was suggested in Shapiro and Stiglitz (1984), most literatures have not fully investigated the macroeconomic implications of the moral hazard problem in labor contracts.

By replacing the Nash bargaining process with explicit contract arrangement, this model has some distinguished contributions to explaining labor market dynamics. First, without risk-averseness, this model can incur real wage rigidity under aggregate shocks unlike many standard implicit contract models. Typical implicit contract models are relevant only for insurable, idiosyncratic shocks, not for uninsurable, aggregate ones. This model found a new mechanism of real rigidity in the face of aggregate shocks, not depending on the risk attitude of agents. Second, few studies (for example, Uhlig and Xu(1996), Costain and Jansen(2006)) investigating the cyclical implications of Shapiro-Stiglitz shirking model have failed to replicate high output and employment volatility in the data. One of main reason was very high counter-cyclical effort typical in the shirking model. In a boom, unemployment as a discipline mechanism gets less unfavorable because labor market is tight so that workers can expect to find another job more easily and earn high wage. That's why workers make less efforts in a boom in the shirking model. Introduction of output-contingent contract wage and incentive provision alleviates this effect and allows only a weak counter-cyclical effort. Finally, this model's performance is robust irrespective of the size of firm's profit. Hagedorn and Manovskii(2006) argued that, if firm's profit is small, the percentage change in profit is large for the same productivity shock, and firms have more incentives to post vacancies in a boom. So, they found that the size of firm's profit is highly correlated with the response of vacancy-unemployment to shocks. In this model, a firm's incentive to post vacancies varies according to the firm's contractual share among marginal surplus from improved productivity and the magnitude of this share depends on the incentive scheme of wage contract. Consequently, macroeconomic implications of this model — including volatility amplifications — do not rely on the size of firm's profit as in Hagedorn and Manovskii(2006).

This paper is organized as follows. Section 2 describes the MP model augmented with a contract arrangement under asymmetric information about the worker's effort. Section 3 derives the general form of an optimal contract in a search environment. In Section 4, the model is calibrated to the data. We show that our model can generate a much more realistic movement of unemployment and vacancies (compared to the standard matching model). Section 5 is the conclusion.

2. Model

This framework is a variation of the Mortensen and Pissarides stochastic matching model (Mortensen and Pissarides (1994), Pissarides (2000)). But, in this setting, marginal produc-

tivity relies on a worker's effort, and information about the effort level is known only to the worker, so that the firm can know only ex-post gross productivity. This information asymmetry leads firms to induce optimal effort from the worker by designing a proper labor contract, while workers and firms negotiate wages based on the Nash bargaining process in a standard MP model.

A distinguishing feature from the standard model is that a temporary layoff is possible as an option for the firm and the worker when negotiation fails. This assumption is not arbitrary in the sense that firms and workers would voluntarily choose that option in order to maximize their expected payoff, as will be proven below. This temporary layoff is assumed to be costless and enforceable, which means that once the worker gets laid off, he should go back to the original job in the next period even though he has another job offering. This enforceability seems to be justified when we consider some of the evidence: Katz and Meyer (1990) reported that the recall rate of temporary layoff is about 80 percent. Allowing a temporary layoff plays an important role in characterizing different wage dynamics from a Nash bargaining wage. This chapter describes the whole aspect of the model except for the design of wage contract. How wage is determined will be discussed in detail in the next chapter.

2.1. Firms and Workers

In the model economy, there are risk-neutral, infinitely lived workers and a continuum of risk-neutral, infinitely-lived firms, each with unit mass. All agents discount future payoffs at the rate $0 < \beta < 1$. Firms are risk-neutral. Firms have constant returns to scale technology in labor as in the standard case. But the main difference is that marginal productivity depends also on the level of a worker's effort. Gross productivity (y) is a function of a stochastic productivity shock and a worker's effort level.

$$y(e, a, x) = f(e) + ax \quad (2.1)$$

where $f(e)$ is an increasing concave function of effort level e and a, x are an aggregate productivity shock and an idiosyncratic (job-specific) shock each. The stochastic properties of each shocks will be described later. This technology means that a worker's effort level shifts the mean of the gross productivity distribution without affecting the dispersion of distribution. This separability is only for simplicity, and there is no difficulty in extending this setting to a more general case.

A matched firm operates production and pays a contract wage based on the gross productivity (y). Then, it may be separated from worker at a constant rate δ . The operating firm's value at the current period is given by,

$$J(a, x) = y(e(a), a, x) - w(y(e(a), a, x)) + \beta(1 - \delta) \int_{a'} \int_{x'} J(a', x') dH(x'|x) dP(a'|a) \quad (2.2)$$

where $w(y)$ is a contract wage contingent on gross productivity y . $H(x'|x)$ and $P(a'|a)$ are cumulative distribution functions of idiosyncratic shock x and aggregate shock a (conditional on a previous shock). To hire a worker, a firm must maintain an open vacancy at flow cost c . An open vacancy is matched to a searching worker with a probability $q(\theta)$ depending on the vacancy-unemployment rate $\theta_t = \frac{v_t}{u_t}$. Free entry drives the expected present value of an open vacancy (V) to zero.

$$c = \beta q(\theta) \int_{a'} \int_{x'} J(a', x') dH(x'|x) dP(a'|a) \quad (2.3)$$

Workers are also risk-neutral. But, Unlike in the standard framework, a worker's utility u^W relies not only on the compensation from work $w(y)$ but also on the effort level e made in production.

$$u^W(w(y(e, a, x)), e) = w(y(e, a, x)) - \phi(e) \quad (2.4)$$

where the disutility function $\phi(e)$ satisfies $\phi'(e) > 0$, $\phi''(e) > 0$ and $w(y)$ is a contract wage contingent on gross productivity y . Workers can either be unemployed or employed. An unemployed worker gets flow utility z from non-market activity and searches for a job. A searching worker is matched to an open vacancy with a probability $p(\theta)$. The value of the unemployed worker (U) in the current period is given by,

$$U(a) = z + \beta[p(\theta) \int_{a'} \int_{x'} W(a', x') dH(x'|x) dP(a'|a) + (1 - p(\theta)) \int_{a'} U(a') dP(a'|a)] \quad (2.5)$$

The value of the employed worker (W) in the current period is then,

$$W(a, x) = w(y(e(a), a, x)) - \phi(e(a)) + \beta[(1 - \delta) \int_{a'} \int_{x'} W(a', x') dH(x'|x) dP(a'|a) + \delta \int_{a'} U(a') dP(a'|a)] \quad (2.6)$$

An employed or newly matched worker chooses an effort level that maximizes his expected utility in each period.

2.2. Vacancy, Unemployment and Matching technology

The fluctuation of this economy depends on two kinds of exogenous shocks: an aggregate productivity shock $a \in [\underline{a}, \bar{a}]$, and an idiosyncratic (job-specific) shock $x \in [\underline{x}, \bar{x}]$. The aggregate shock follows first-order Markov Process and the idiosyncratic shocks are an i.i.d. random process. To be concrete, the logarithm of a follows an AR(1) process with normal innovation.

$$\log a_t = (1 - \rho_a)\overline{\log a} + \rho_a \log a_t + \epsilon_a \quad (2.7)$$

where $\epsilon_a \sim N(0, \sigma_a)$. Suppose that the conditional distribution of a' given a is characterized by $P(a'|a)$. x is an i.i.d. log-normal random variable. $\log x_t \sim N(\mu_x, \sigma_x)$, $\forall t$.

A matching technology of this economy is as follows. When the total number of unemployed workers and vacancies is u_t and v_t , we assume that the number of new matches m_t is a function of u_t and v_t as follows:

$$m(v, u) = \omega v^\alpha u^{1-\alpha} \quad (2.8)$$

where α denotes the elasticity of the matched to the vacancy and ω is a matching efficiency parameter. The firm's probability of filling a vacancy $q(\theta_t)$ is,

$$q(\theta_t) = \frac{m_t}{v_t} = \omega \theta_t^{\alpha-1} \quad (2.9)$$

the worker's probability of finding a job $p(\theta_t)$ is,

$$p(\theta_t) = \frac{m_t}{u_t} = \omega \theta_t^\alpha \quad (2.10)$$

A fraction δ of workers separate from their jobs exogenously each period. In this economy, though jobs are (ex-post) heterogeneous, endogenous separation does not occur because productivity is unknown before production is carried out, thus a firm makes a separation decision based on expected productivity, not a realized one, which implies separation does not occur as long as the expected rent from production is above zero. Consequently, the fluctuation of unemployment in this economy is mainly due to the cyclical movement of hiring, not separation. This feature of the model can be justified by the observation that the countercyclicality of the separation rate is weak relative to the strong procyclicality of the job finding rate (or vacancy) (Shimer (2005)). Finally, unemployment evolves following this law of motion.

$$u_{t+1} = \delta(1 - u_t) - p(\theta_t)u_t \quad (2.11)$$

2.3. Information Structure

In this economy, the timing of the event is different from that in the standard framework mainly in that firms and workers decide whether to produce or not and how to allocate outcome before the stochastic part of productivity becomes known to both. Owing to a worker's private information about his own effort level, a typical moral hazard problem arises. The timing of the event is summarized as follows.

- Aggregate productivity a is realized and commonly known.
- A new labor contract is negotiated (when negotiations are broken off, the firm and the worker can choose whether to destroy a job or to lay off (get laid off) temporarily).
- Given the wage scheme, workers set an effort level that maximizes his expected utility. (The worker still cannot observe an job-specific shock x)
- Production is carried out.
- Gross productivity y is known, and outcome is allocated by the predetermined wage rule.
- Given the chosen level of vacancy, new matches and exogenous separations occur.

3. Searching Equilibrium under an Optimal Contract

3.1. Optimal Contract in a Moral Hazard Environment

The optimal contract problem under an agent's moral hazard has been a traditional issue. In the case of risk-neutral agents without limited liability, it is well known that a principal can induce a first-best effort level ³ with fixed rent contract, which means that the entire output minus a fixed rent is given to the agent (Harris and Raviv(1979)). On the contrary, in the case of risk-averse agents, an optimal contract usually cannot achieve a first-best effort level (Holmstrom(1979)). Even in the risk-neutral case, if either agent's wealth constraint is binding or his liability is limited for any other reason, a fixed rent contract is not feasible.

³First-best effort level denotes the optimal level for a firm when there is no moral hazard problem

This kind of information structure was first investigated by Innes (1990), though he focuses on financial contracts between risk-neutral investors and entrepreneurs when the liabilities of both are limited. Then, Kim (1997) shows that there exists a first-best optimal contract (that necessarily contains a bonus contract) even in a limited liability (LL) case under the regular condition on the distribution function.

For the characterization of an optimal contract, the following basic assumptions are needed:

- (A1) $g(y|e)$ is continuously twice differentiable in e .
- (A2) $\frac{g_e(y|e)}{g(y|e)}$ is increasing in y , $\forall e$.
- (A3) $G(y|e)$ is convex in e .

where $G(y|e)$ is a cumulative distribution function of gross productivity y conditioned on effort and $g(y|e)$ is its density. Assumption 1 and Assumption 3 are necessary for the existence and uniqueness of the optimal effort level. Assumption 2 represents the well-known monotone likelihood-ratio property which means that higher output is more likely due to higher effort.

As in Innes (1990), We assume here that the liability of the worker and the firm is limited so that a wage contract should warrant a base pay for a worker and a minimum level of profit for a firm. Base wage here does not necessarily mean legal minimum wage. It can be some wage norm accepted by social convention or consensus. Or, because of the difficulty of verifying productivity and private information on both sides (firm and worker), an up-or-out contract can be optimal, thus it can impose some minimum wage level over the reservation wage (Kahn and Huberman (1988), Shimer and Wright (2004)). Later on, We will investigate the effect of a change in base pay within a comprehensive interval. In addition, there is limited liability for firms so that the outcome net the variable part of wage should be positive. It is not easy to justify the limited liability of a firm in the case of a non-financial contract as compared to that of a worker. But it would not be a strong assumption in the sense that a firm would go bankrupt if it could not cover even variable costs with earned profit. Next, We assume the monotonicity of a contract for a firm, which means a firm's share should be a non-decreasing function in outcome.⁴

Before proceeding, the intuition of a debt contract needs to be explained briefly. With any monotonic contract, some of the benefits of marginal effort are shared with firms. Since the worker bears the total cost of effort, he will choose an effort level that is less than a first-best choice. This implies that the firm will select a contract form that commits the worker

⁴For the rationale of this assumption, see Innes (1990).

to the highest possible effort level, thereby permitting him to reap as much of the first-best surplus as possible. Since higher effort increases the probability weight placed on high-profit outcomes (Assumption A2), with a contract that gives the worker maximal payoffs in high-profit states, the worker is induced to choose maximal effort. Among monotonic contracts subject to liability limits, the debt contract has a "maximal high-profit-state payoff" property as pointed out by Innes and, hence, will be selected.

The derivation of the optimal wage contract here is a labor-market modification of Innes' discussion of financial contracts. In this economy, workers are competitive in the labor market; i.e., a worker participates in production as long as his reservation wage is guaranteed. The firm would try to design a wage contract so as to induce optimal effort and consequently maximize its profit. The firm's problem is given by,

$$\text{Max}_{w(y),e} \int_y (y - w(y)) dG(y|e) \quad (3.1)$$

$$s. t. \int_y w(y) dG(y|e) + \bar{w} - \phi(e) \geq w^r \quad (P.C.)$$

$$\int_y w(y) g_e(y|e) dy - v'(e) = 0 \quad (I.C.)$$

$$0 \leq w(y) \leq y, \quad \forall y \quad (L.L)$$

$$B(y + \epsilon) \geq B(y), \quad \forall y, \quad \forall \epsilon > 0 \quad (\text{Monotone Constraint})$$

where w^r is the reservation utility of the worker and $B(y) = y - w(y)$. Lemma 1 shows that the reservation utility of the worker is the same as the flow utility from non-market activity when a temporary layoff is available

Lemma 1.

$$w^r = z \text{ if } (1 - \delta) > p(\theta)$$

Proof. When negotiation fails, the firm has three choices: destroying a job, posting a new vacancy, or imposing a temporary layoff. The first two choices lead to zero expected profit by the free entry condition. When a firm lays off a worker temporarily, its value is,

$$J^L(a) = \beta(1 - \delta) \int_{a'} \int_{x'} J(a', x') dH(x') dP(a'|a) > 0 \quad (3.2)$$

Thus, when negotiation fails, the firm chooses to lay the worker off temporarily rather than to destroy a job.

When negotiation fails, workers prefer getting laid off to being fired, since the value of getting laid off W^L is higher than that of being fired U if $(1 - \delta) > p(\theta)$.

$$\begin{aligned}
W^L(a) &= z + \beta[(1 - \delta) \int_{a'} \int_{x'} W(a', x') dH(x') dP(a'|a) + \delta \int_{a'} U(a') dP(a'|a)] \quad (3.3) \\
&> z + \beta[p(\theta) \int_{a'} \int_{x'} W(a', x') dH(x') dP(a'|a) + (1 - p(\theta)) \int_{a'} U(a') dP(a'|a)] = U(a)
\end{aligned}$$

so the reservation wage that the firm should guarantee is the same as $z + \phi(e)$ since $W(a, x) = W^L(a) \forall a, x$ only if $w(y) + \bar{w} = z + \phi(e)$. Therefore $w^r = z$

Lemma 1 denotes that what determines the reservation wage of a worker is how much a worker can get from a firm when negotiation fails, not depending on the expected surplus of unemployment as in the standard model. Now, I will show that any solution to (3.1) contains a debt contract (for a firm). Before proceeding, Lemma 2 below from Innes (1990) is helpful in characterizing the effort level induced by a debt contract.

Lemma 2. *Suppose two wage contracts $w^1(y)$ and $w^2(y)$ are satisfying these two conditions,*

$$\begin{aligned}
i) \quad & E_y(w^1(y) - w^2(y)) = 0 \\
ii) \quad & \exists y^0 \in (\underline{y}, \bar{y}), \\
& w^1(y) \geq w^2(y), \text{ for } \forall y \leq y^0 \text{ and } w^1(y) \leq w^2(y), \text{ for } \forall y \geq y^0
\end{aligned} \tag{3.4}$$

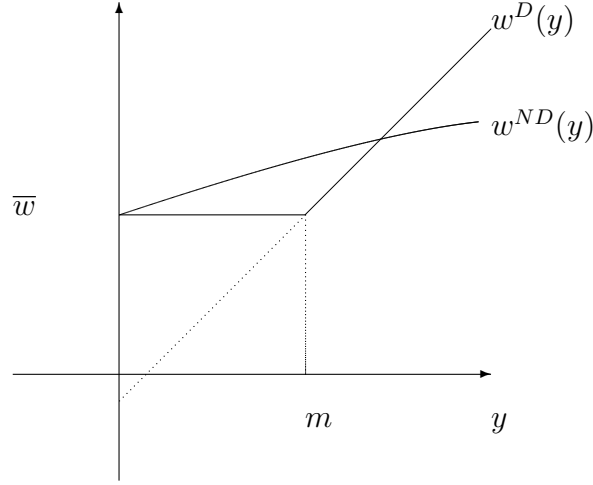
then $e^1 \leq e^2$ where e^i is the optimal effort level under $w^i(y)$.

Proof. See the proof of Lemma 1 and Lemma 2 in Innes (1990)

Lemma 2 means that one contract, which has the same expected value and crosses the other once from below to above, induces a higher effort level. According to Lemma 2, now suppose there is an optimal non-debt contract $w^{ND}(y)$ and consider another debt contract $w^D(y)$ that satisfies the condition below,

$$\begin{aligned}
w^D(y) &= y - B^D(y), \quad B^D(y) = \min(y, m), \quad 0 < m < \bar{y} \\
E_y(w^D(y(e^{ND}, a, x)) - w^{ND}(y(e^{ND}, a, x))) &= 0
\end{aligned} \tag{3.5}$$

Non-debt contract $w^{ND}(y)$ and debt contract $w^D(y)$



which means that given the same effort level e^{ND} , the debt wage contract $w^D(y)$ gives the same expected compensation as the non-debt contract $w^{ND}(y)$. That debt contract gives all of the surplus to the firm as long as it is below some threshold level m , then the worker takes any extra rent over that level as a wage. This is a special case ($\bar{w} = 0$) of the firm's problem (3.1). With no loss of generality, from now on, We assume the case $\bar{w} = 0$. The figure above illustrates the relation between the non-debt optimal contract $w^{ND}(y)$ and the debt contract $w^D(y)$ in general case.

By Lemma 2, it is straightforward to see $e^{ND} \leq e^D$, where e^{ND} and e^D means a unique optimal effort level under the non-debt contract and the debt contract.

Lemma 3. *Any solution to (3.1) contains a debt contract.*

Proof. The optimal non-debt contract $(w^{ND}(y), e^{ND})$ satisfies all of the constraints of (3.1). Obviously, $(w^D(y), e^D)$ also satisfies all of the constraints of (3.1) except for (P.C.) by definition of $(w^D(y), e^D)$. By Lemma 1 and the definition of $(w^D(y), e^D)$

$$\begin{aligned}
 w^r = z &\leq \int_y w^{ND}(y) dG(y|e^{ND}) - \phi(e^{ND}) \\
 &= \int_y w^D(y) dG(y|e^{ND}) - \phi(e^{ND}) \\
 &\leq \int_y w^D(y) dG(y|e^D) - \phi(e^D)
 \end{aligned} \tag{3.6}$$

Thus, contract $(w^D(y), e^D)$ satisfies constraint (P.C.), too. Next, firms prefer contract

$(w^D(y), e^D)$ to $(w^{ND}(y), e^{ND})$, since, by definition of $w^D(y)$ and Lemma 2,

$$\begin{aligned} \int_y (y - w^{ND}(y)) dG(y|e^{ND}) &= \int_y (y - w^D(y)) dG(y|e^{ND}) \\ &\leq \int_y (y - w^D(y)) dG(y|e^D) \end{aligned} \quad (3.7)$$

The second inequality is due to the fact that $\int_y (y - w^D(y)) dG(y|e)$ is a non-decreasing function of e . It is because the following can be shown by substituting $w^D(y)$.

$$\frac{\partial \int_y (y - w^D(y)) dG(y|e)}{\partial e} = \frac{\partial E_y(y - w^D(y))}{\partial e} = - \int_{\underline{y}}^m G_e(y|e) dy > 0 \quad (3.8)$$

The last inequality is due to the first order stochastic dominance (FOSD) property implied by Assumption (A2).

Now we are ready to derive a debt contract as an optimal contract and characterize its properties. For the existence of an optimal debt contract, an additional assumption is needed.

$$(A4) \quad \exists m, E_y(w^D(y)) - \phi(e^D) \geq z$$

Proposition 1. *A solution to problem (3.1) exists and has the following properties:*

$$i) \quad w(y) = w^D(y; m) \equiv y - \min(y, m), 0 < m < \bar{y} \quad (3.9)$$

$$ii) \quad e^D < e^* \quad (3.10)$$

where e^* is the first-best effort level.

Proof. For i), by Lemma 3, the existence of an optimal debt contract (w^D, e^D) is a necessary and sufficient condition for (3.1) to have a solution, i.e., if and only if the following problem

$$Max_m \int_y (y - w^D(y; m)) dG(y|e^D(m)) \quad (3.11)$$

$$s. t. \quad \int_y w^D(y; m) dG(y|e^D(m)) - \phi(e^D(m)) \geq z$$

has a solution, then (3.1) has a solution. By definition of w^D , the choice set of m is

compact and, by (A4), nonempty. With the continuity of the objective function and $e^D(m)$ ⁵, (3.11) has a solution by the Weierstass Theorem.

For ii), the first order condition for effort level can be rewritten,

$$\frac{\partial E_y(w^D(y) - \phi(e^D))}{\partial e} = \frac{\partial E_y(y - \phi(e^D) - z)}{\partial e} - \frac{\partial E_y(y - w^D(y))}{\partial e} = 0$$

Since the second term of the left side has a positive sign as seen in the inequality (3.8), $\frac{\partial E_y(y - \phi(e^D) - z)}{\partial e} > 0$, which means first-best effort level e^* is higher than e^D .

Property ii) means that the optimal debt contract induces a second-best effort level. In other words, a firm would want its worker to increase his effort level if there is no additional cost to inducing him to do so. With Proposition 1, we can restrain the functional form of the optimal contract to a debt contract. But it does not mean that the debt contract is the only possible optimal contract. All we can say is that choosing the debt contract is quite natural for a firm because whenever an optimal contract exists, there is at least one debt contract among them. Now, the firm's problem is reduced to (3.11).

3.2. Characterization of a Parametric Optimal Contract

Now, we are ready to derive the optimal effort level e and the contract-characterizing variable m with a parameterized functional form. There are two functions to be parameterized: the production function and the disutility function in effort ($f(e)$ and $\phi(e)$).

$$f(e) = \log e \quad \text{and} \quad \phi(e) = Be^2, \quad B > 0.$$

Under the proper domain of x , it is easy to show that assumption (A1)-(A3) is satisfied. By picking up reasonable B , there exists some m satisfying (A4). Furthermore, the convexity of the disutility function is related to the existence and uniqueness of an optimal effort level.

As in the last section, the optimal debt contract is characterized by two constraints (the participation constraint and incentive compatibility constraint). By the second part of Proposition 1, we know that the incentive compatibility constraint is binding.

First, by substituting specified functional forms, the participation constraint can be written as follows.⁶

$$a\left(\int_{m_x}^{\bar{x}} xh(x)dx - m_x(1 - H(m_x))\right) \geq Be^2 - \bar{w} + z \quad (P.C.)$$

⁵It is straightforward to prove the continuity of $e^D(m)$ by the Maximum Theorem

⁶See the Appendix for details.

where m_x is a variable satisfying $m = \log e + am_x$ and $H(x)$ and $h(x)$ is the cumulative and probability distribution function of an idiosyncratic shock x . From the binding incentive compatibility constraint,

$$e = \sqrt{\frac{1 - H(m_x)}{2B}} \quad (3.12)$$

Plugging the second equation into the first one, we can modify the participation constraint to the inequality with respect to m_x only.

$$a \int_{m_x}^{\bar{x}} xh(x)dx - (am_x + B)(1 - H(m_x)) \geq z - \bar{w} \quad (3.13)$$

Now the firm's problem of the firm is reduced to maximization with respect to m_x .

$$\text{Max}_{m_x} \int_y (y - w^D(y; m_x)) dG(y|e(m_x)) \quad (3.14)$$

$$\text{s. t. } a \int_{m_x}^{\bar{x}} xh(x)dx - (am_x + B)(1 - H(m_x)) \geq z - \bar{w} \quad (3.15)$$

From the solution to this problem, the contract-characterizing variable m_x is determined, then the optimal effort level can be picked by (3.12).

Now, let's turn to some comparative statics. The main interest is the effect of the aggregate productivity shock a and base pay $\bar{w}$ on the optimal contract. First look at the case that (3.15) is binding (i.e. satisfying (3.15) as equality).

Totally differentiating (3.15)

$$\frac{dm_x}{da} = \frac{m_x(1 - H(m_x)) - \int_{m_x}^{\bar{x}} xh(x)dx}{(1 - H(m_x))(B \frac{h(m_x)}{1-H(m_x)} - a)} \quad (3.16)$$

$$\frac{dm_x}{d\bar{w}} = -\frac{1}{(1 - H(m_x))(B \frac{h(m_x)}{1-H(m_x)} - a)} \quad (3.17)$$

The numerator of the right-hand side of (3.16) is negative irrespective of the distributional property, but the sign of the denominator depends on it. If $\frac{h(m_x)}{1-H(m_x)} < \frac{a}{B}$ for all m_x and a , $\frac{dm_x}{da} > 0$ and $\frac{dm_x}{d\bar{w}} > 0$

Next look at the case that (3.15) is not binding. From the first order condition of problem (3.14),

$$\frac{h(m_x)}{(1 - H(m_x))^2} = 2a \quad (3.18)$$

The left-hand side of (3.18) is a non-decreasing function of m_x as long as Assumptions (A1)-(A3) are satisfied.⁷ Thus, m_x is increasing in a and not affected by $\bar{w}$ when (3.15) is not binding. To sum up,

Corollary 1. *if $\frac{h(m_x)}{1-H(m_x)} < \frac{a}{B} \forall a, m_x$, then m_x is nondecreasing function of a and $\bar{w}$.*

Corollary 1 means that if $\frac{h(m_x)}{1-H(m_x)}$ is smaller relative to productivity, then the threshold level of the wage contract gets extended (which means more workers would get only base pay) when productivity or base pay is rising. $\frac{h(m_x)}{1-H(m_x)}$, basically the hazard rate in m_x , has some contents about incentive provision because this ratio means the rate of change in the expected income of the worker under the step bonus contract, which gives the worker none if $x < m_x$ and some level of bonus if $x > m_x$ ⁸. In other words, that hazard rate denotes a maximum degree of incentive that any feasible contract involving threshold m_x can provide. The fact that this ratio is small indicates a weak incentive effect of a lower threshold m_x . Thus, when higher productivity occurs or a higher base pay is prepaid (thus any additional wage asked for is lower), firms have good reason to raise the threshold level in order to take extra rent over a worker's reservation utility rather than provide more incentive by lowering it. This is the case as long as it does not harm the worker's incentive too much to the extent of decreasing the firm's after-wage profit. This hazard rate reminds us of a Shimer's(2005) suggestion that dropping some informational assumptions can generate a more rigid wage. In his suggestion, a hazard rate of productivity level affects a worker's wage demand, since there is a trade-off between demanding a higher wage and reducing the risk of unemployment. Here, however, the hazard rate affects the wage scheme, since it is associated with the degree of incentive provision.

The figure below illustrates Corollary 1 by comparing optimal contracts when aggregate productivity goes up from a to a' . The productivity range within which workers get paid only a base wage is extended as aggregate productivity goes up. However, by paying higher wage for an extremely good performance, the reservation wage is guaranteed on a expectation basis.

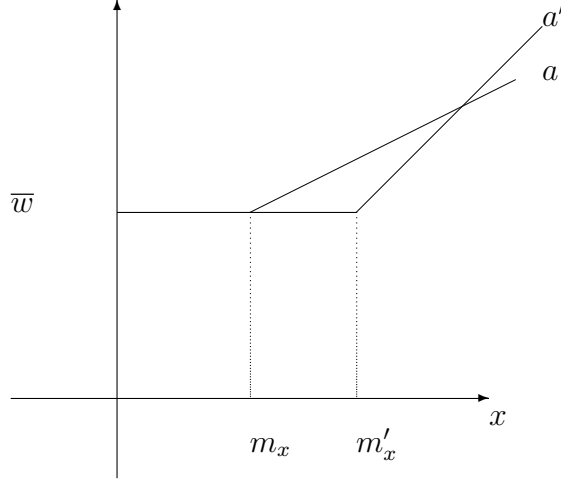
Meanwhile, this property can be understood in terms of an information criterion. Say there are only two aggregate productivity levels, good and bad. Suppose that a firm observes the same outcome in both states. Then, the firm would judge roughly that the worker has made less effort in the good state. Thus, in order to induce higher effort, the firm would provide the same level of compensation only if the outcome in the good state is somewhat

⁷It is straightforward to show that $\frac{1}{ea} \frac{h(m_x)}{(1-H(m_x))} = -\frac{G_e(m|e)}{1-G(m|e)}$. Then, we can show that $-\frac{G_e(m|e)}{(1-G(m|e))}$ is a non-decreasing function of m (or m_x) if (A1)-(A3) is satisfied. See the proof of Lemma 1 in Kim (1997).

⁸See Kim (1997).

higher than that in the bad state. To put it in a more formal manner, gross productivity y is not a sufficient statistic of effort level e , so information about aggregate productivity a is informative in Holmstrom's(1979) notion. Consequently, that information should be considered when the wage contract is designed.

Illustration of Corollary 1



3.3. Searching Equilibrium

The searching equilibrium of the model economy is defined as follows.

Definition 1. *A searching equilibrium of the model economy consists of a wage rule $w(\cdot)$, effort level e , vacancy level v , and unemployment level u such that:*

1. *The wage rule $w(\cdot)$ is optimal policy for (3.1) and that rule induces the effort level e from worker.*
2. *Firms post vacancies to the level of v satisfying (2.3).*
3. *Workers choose the effort level e in order to maximize their expected utility, i.e., satisfying (I.C) of (3.1).*
4. *The unemployment level u evolves following (2.11) given $\theta = \frac{v}{u}$.*

Based on the derived optimal contract, we can find the searching equilibrium of the economy. In this economy, the value of a filled job is represented by the following bellman equation,

$$J(a, x) = y(e(a), a, x) - w^D(y(e(a), a, x)) + \beta(1 - \delta) \int_{a'} \int_{x'} J(a', x') dH(x') dP(a'|a) \quad (3.19)$$

By applying the Contraction Mapping Theorem, we can solve for value function $J(a, x)$. Then, using free entry condition (2.8), we can solve for the equilibrium labor-market $v - u$ ratio θ as a function of aggregate productivity a . This ratio θ renders a general equilibrium in the sense that it represents the outcome of the worker's utility-maximizing effort choice and firm's profit-maximizing policy on vacancy posting and contract design given the unemployment level.

4. Model Evaluation

4.1. Calibration

Table 1 summarizes the calibrated parameters for the model. To begin with, We normalize a time period to be one quarter and the average productivity in the economy to be 3. This scaling of production is consistent with the empirical labor share of output (about 0.6) in the optimal contract economy. Thus, the conventional non-market utility level ($z = 0.4$) is multiplied by three (Shimer (2005)). The exogenous separation rate is set to 0.1 based on Abowd and Zellner's(1985) measurement from 1972-1982 data (3.42 percent per month). This rate is almost the same as that of Job Openings and Labor Turnover survey (JOLT) (3.4 percent per month). It also implies that a job lasts for about 2.5 years. The discount factor β is 0.99, which implies an annual interest rate of about 4 percent. The elasticity of the number of the matched to unemployment ($1-\alpha$) is fixed at 0.6, the median of the range of estimates that Petrongolo and Pissarides (2001) report. For the standard Nash bargaining model, We set a worker's bargaining power parameter η to the same value 0.6. It is roughly consistent with the empirical labor share in aggregate output, and, moreover, it makes the decentralized equilibrium achieve Pareto optimality (Hosios (1990)). For the logged aggregate productivity shock, its persistence (0.95) and standard deviation (0.007) of innovation is chosen following Prescott (1986). Chang (2000) calibrated the distribution of a worker's talent using cross-sectional wage data from the Panel Study of Income Dynamics(PSID). The chosen value 0.58 of the standard deviation for the logged idiosyncratic shock (σ_x) is based on his calibration results with the assumption that the worker's wage difference in the PSID sample mainly reflects differences in job productivity. The matching technology parameter ω and vacancy posting cost c is set in order to pin down the steady-state unemployment level 0.11 for each model. This value is grounded in the Shimer's(2005) observation that the average unemployment rate during 1951-2003 was 5.67 percent and the findings of Blanchard and Diamond (1990) that the magnitude of inflow from unemployment to employment is as large as that of inflow from out-of-the-labor force. The effort-disutility parameter B is fixed at one half. The level of this parameter does not affect the main results.

4.2. Computation

For aggregate and idiosyncratic shocks, the point grid is constructed. The range of the grid is $\pm 3SD$ around its mean. The number of the grid for a and x is 9 and 1001. The reason why there are a large number of x grid points relative to a is that the effect of the optimal debt contract is highly dependent on the choice of its threshold. The law of motion for aggregate shocks is approximated to the discrete state space by the method suggested by Tauchen (1986).

We draw 1000 periods of aggregate shocks following their approximated law of motion and given the initial level. Then, given the aggregate state, solve for the model so that the time series of labor quantities and wage can be generated. By repeating this simulation 100 times, we can generate 100 sample time series. For each sample, we calculated sample moments of interest and then averaged these moments among samples.

4.3. Results

Table 2 summarizes the key statistics of quarterly U.S. data. I used is quarterly data from Shimer(2005) and added real wage data from the Current Employment Statistics(CES) survey. The time period of the data is from the first quarter of 1964 to the last quarter of 2003. As mentioned by Shimer (2005), the response of real wages to a productivity shock is the important factor determining the response of other labor-market quantity variables. Thus, in order to see the model's implication for wage rigidity, We added real wage data for the purpose of comparison. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5 .

As is well known, vacancy is highly pro-cyclical, and unemployment is countercyclical. They are strongly negatively correlated with each other. Relative to the volatility of a productivity shock, vacancy and unemployment are ten times more volatile than a productivity shock. On the contrary, Table 9 shows that real wages are almost acyclical relative to employment or vacancy so that real wages' positive correlation with productivity is much weaker than predicted by the standard RBC model. In addition, wage varies too little compared with other quantity variables. The standard deviation of real wages is almost the same as that of a productivity shock so that no amplification like that in other quantity variables is found. In brief, real wages are too rigid.

Table 3 shows the sample moments of the time series generated by the standard MP model, in which wage is determined by a Nash bargaining process. In response to a productivity change of similar magnitude in the real data, the standard MP model generates too small fluctuations for unemployment, vacancy, and the v-u ratio. Furthermore, there is no propagation of a labor productivity shock, since the correlations between vacancy or

unemployment and a productivity shock are close to one.

Tables 4 through 6 show the corresponding sample moments of the time series generated by the optimal contract model, in which wage is determined by the bilateral contract described in earlier sections. Before proceeding, parameter γ characterizing the base pay level needs to be explained. γ is the ratio of base pay (that is, compensation independent of production outcome) to non-market utility z , that is, $\bar{w} = z\gamma$. The reason why the base pay level is specified relative to non-market utility z is as follows. First, non-market utility z is an opportunity cost for the employed, which should be one of the main ingredients of wages. Second, since z is not dependent on a worker's job history or his effort level, $\bar{w}$ also can be determined time independently. Tables 4 through 6 show the results of the optimal contract model with $\gamma = 0.6, 0.9, 0.985$. The range of γ is set so as to fit the lower bound of the range to the legal minimum wage level. Mankiw et al. (1992) have estimated the share of return to human capital in total labor income by using the fact that the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing in the U.S. With a rough labor share of total output (0.6) and calibrated z (40 percent of average output flow), if we choose median value (0.4) of this minimum wage interval, it leads to the lower bound of γ (about 0.6).⁹ To see how the market operates when the base pay converges to the reservation wage level, we included the case $\gamma = 0.985$.

The volatility of unemployment, vacancy, and the v-u ratio is much bigger in the optimal contract model than in the standard Nash bargaining case for all ranges of γ . Thus, there is a relatively higher amplification of the productivity shock in the optimal contract case. As suggested by Corollary 1, the amplification is more prominent as γ increases; i.e., base pay is going up. When base pay covers 90 percent of non-market utility, the volatility of unemployment, vacancy, and the v-u ratio is almost 2.3 times as big as the standard case. Figure 2 compares the impulse responses of the labor-market quantity variables to 1 SD of the productivity shock between the standard model and the optimal contract model. Figure 3 shows the impulse responses of the labor-market quantity variables by base pay ratio γ . We can confirm that the amplification effect is getting bigger as the base pay ratio increases. As γ approaches one like the case of $\gamma = 0.985$, the optimal wage scheme gets much closer to a fixed wage. Thus, the magnitude of the amplification effect in the model exceeds that of the real data as seen in Table 6. This suggests that the model generates enough (more than enough) amplification when base pay is almost the same level as the reservation wage. Nonetheless, the lack of propagation of the shock is still a problem. In contrast to real data, the contemporary correlation between the productivity shock and quantity variables

⁹The base pay to non-market utility ratio can be calibrated as follows. When y and Y are output per worker and total output, $\frac{\bar{w}}{y} = \frac{\bar{W}L}{pY} = \frac{\bar{W}}{W} * \frac{WL}{pY} = 0.4 \times 0.6$. Then $\gamma = \frac{\bar{w}}{z} = \frac{0.4 \times 0.6}{0.4} = 0.6$

is almost close to one regardless of model setting. This property is mainly due to the risk-neutrality of worker and the static property of optimal contract. This propagation problem might be addressed by allowing multi-period contracts and changing the worker's attitude for risk.

The main reason for the model's improvement on amplification is the more rigid wage movement it induces. When a productivity shock hits, the rigid wage movement gives firms a chance to adjust to more vacancies based on the expected change in future profit. Table 7 compares the real data with the simulated series in terms of real wage volatility and correlation with other variables. Obviously, the optimal contract case generates more rigid wage movements compared to the standard framework, in which wage dynamics are basically the same as shock dynamics. Figure 5 compares the impulse response of the model-generated real wage and labor quantities to 1 SD of productivity shock between the standard model and the optimal contract model. In the optimal contract economy, the real wage responds less to a productivity shock, and consequently vacancy and unemployment are more volatile. This wage rigidity is helpful not only in explaining the amplification of productivity shock but it is also consistent with the real-world relationship between wage and labor-market quantities as shown in Figure 4. Thus, in the optimal contract economy, the wage elasticity of the aggregate labor supply can be much higher than the individual elasticity estimates that many micro-studies (Altonji (1986) and MacCurdy (1981)) suggest.

Figure 8 plots the real wage and the employment growth rate of both the data and the simulated series. For each series, We estimated the following equation to see how much different aggregate labor elasticities each model implies.

$$\log \frac{n_t}{n_{t-1}} = \beta_1 + \beta_2 \log \frac{w_t}{w_{t-1}} \quad (4.1)$$

where n_t is the employment ratio. Considering the endogeneity bias and extracting the technological component of wage fluctuation, we estimated this equation for real data by both OLS and IV estimation with the TFP growth rate as an instrumental variable. OLS coefficient estimates are 0.15 for the data, 0.0037 for the standard MP model, and 0.73 for the optimal contract model. But the IV estimate for the data is 0.857, slightly higher than that of the optimal contract case. Thus, in contrast to the standard MP model, the optimal contract model seems to explain the highly elastic response of employment to a change in wages fairly well.

What factors make wages more rigid in the optimal contract model? The first, as Lemma 1 proves, is the allowance of a temporary layoff. By considering the possibility and enforceability of a temporary layoff, the worker's outside option, which is the important factor determining the reservation wage, becomes independent of the state of the labor market

summarized by the v-u ratio in the MP model. That makes wages' response to a shock more inert. The second factor is the competitive market assumption for the worker. All that firms need to care about in order to induce workers' participation is to guarantee the reservation wage, that is, the worker's bargaining power is zero as opposed to that in the standard case. The difference between the first and second factor will be reviewed below. The final factor is from the optimal debt contract's own property described earlier. As Corollary 1 shows, firms designing the optimal contract adjust the threshold level according to the aggregate shock. When a positive productivity shock hits, firms have leeway to widen the range of paying only the base wage, still guaranteeing the reservation utility for each worker. Certainly, offering only base pay to more and more workers weakens workers' incentive, as (3.12) shows. But among every kind of feasible contract giving the same expected wage, the debt contract is the one that provides the highest incentive for the worker (see Lemma 2). Furthermore, a broad class of distribution satisfies the sufficient condition of Corollary 1 on almost a whole support of it: for example, the normal distribution truncated at zero, the log-normal or the exponential distribution. Thus, the incentive-lessening effect of extending the threshold is relatively small compared to the profit-increasing effect.

4.4. Decomposition of Wage Rigidity Effect

We have enumerated three factors that make real wages more rigid, consequently inducing more volatile unemployment and vacancy in an optimal contract model. Though the first and second factors look similar, they have different magnitudes and implications for other variables' fluctuations. Now, try to extract the first factor from the second and third factor to see how the introduction of a temporary layoff affects labor-market fluctuations.

Define the value function of those who get laid off in the current period as $W^L(a)$,

$$\begin{aligned} W^L(a) = & z + \beta[(1 - \delta) \int_{a'} \int_{\bar{x}^{r(a')}} W(a', x') dH(x') dP(a'|a) \\ & + (1 - \delta) \int_{a'} \int_{\underline{x}}^{\bar{x}^{r(a')}} U(a') dP(a'|a) + \delta \int_{a'} U(a') dP(a'|a)] \end{aligned} \quad (4.2)$$

where $\bar{x}^{r(a')}$ is the reservation productivity level depending on aggregate productivity. Defining total net surplus in this case as $S^L(a, x)$ ¹⁰,

$$\begin{aligned} S^L(a, x) &= J(a, x) + W(a, x) - W^L(a) \\ &= ax - z + \beta(1 - \delta)(1 - \eta) \int_{a'} \int_{\bar{x}^{r(a')}} S^L(a', x') dH(x') dP(a'|a) \end{aligned} \quad (4.3)$$

¹⁰See the Appendix for the derivation of following equations.

Now, wage is determined as a solution to the following Nash bargaining problem.

$$Max_{w(a,x)} J(a, x)^{1-\eta} (W(a, x) - W^L(a))^\eta \quad (4.4)$$

Solving the problem, the wage determination equation is

$$w(a, x) = \eta \left(ax + \frac{(1-\delta)c}{q(\theta)} \right) + (1-\eta)z \quad (4.5)$$

There are two points of interest. First, a wage responds to the market situation characterized by the v-u ratio θ in contrast to the optimal contract case. This is because, under Nash bargaining, a wage is dependent on the change not only in the worker's bargaining power but also in the firm's relative bargaining power. Thus, even though the worker's bargaining power (depending on the worker's threat point) is not affected by the market situation due to the introduction of a temporary layoff, the change in θ caused by a productivity shock affects the firm's worker finding rate $q(\theta)$ so that it consequently affects the firm's relative bargaining power. That is why wage fluctuates in response to the change in the market situation. Second, in contrast to the standard Nash bargaining case, the magnitude of wage fluctuation depends on the elasticity of the worker finding rate to θ and the survival rate $1 - \delta$. The more elastic the worker finding rate is, the more volatile are wages. The higher the survival rate is, the more sensitive a wage is to the change in market tightness θ .

Table 8 shows the summary statistics of unemployment, vacancy, and the v-u ratio in the case of the Nash bargaining model with temporary layoff. Figure 6 compares the impulse responses of unemployment, vacancy, and the v-u ratio to the 1 SD of the productivity shock among these models: standard, Nash bargaining with temporary layoff, and optimal contract model. The volatility of quantity variables is 1.5 times higher than in the standard Nash bargaining case, but it amounts to only about 40 percent of the volatility increase in the optimal contract case with, for example, $\gamma = 0.9$. Moreover, most of that effect is resulted from the level decrease of firm's surplus $J(a, x)$ due to the increase of reservation wage level when temporary layoff is introduced. This partial increase in volatility is mainly due to the fact that wage fluctuation relies on market tightness θ as opposed to the optimal contract case. In the optimal contract case, because the worker's bargaining power is zero, the firm's relative bargaining power does not affect wages. To sum up, in this economy, wages respond more sensitively to a productivity shock compared to the optimal contract case. This leads to relatively less fluctuation in unemployment, vacancy, and v-u ratio θ . Table 9 and Figure 7 confirm this argument. In the Nash bargaining case with temporary layoff, wage volatility is almost the same level as in the standard case. Furthermore, as in the standard case, the correlation between a productivity shock and wages is close to unit, which means wage adjustment soaks up a considerable portion of the productivity effect. Figure 7 shows the

wage dynamics of different models in response to the positive productivity shock. The wage dynamics of the Nash bargaining model with temporary layoff are almost the same as that of the standard model. In sum, fixing a reservation wage up doesn't contribute much to the wage rigidity.

5. Conclusion

In this paper, we construct a search model augmented with contract arrangements under asymmetric information. Under standard regularity conditions, a simple debt contract form à la Innes (1990) is derived as the optimal contract, which exhibits almost a fixed wage. This contract environment leads to a smaller wage adjustment and a bigger fluctuation of unemployment and vacancy as compared to the standard MP model. Moreover, by Corollary 1, we show that wage rigidity strengthens as aggregate productivity or the level of base pay goes up. When the wage rigidity effect is decomposed, the increased volatility in the model is only partially explained by the introduction of temporary layoff.

There are many limitations to this model. First, though it improves the volatility of quantities, it is not enough compared to volatility in the real data unless base pay is almost equivalent to the reservation wage. In the economy with only one driving force, a productivity shock, it seems to be very difficult to generate a quantity amplification of the data's magnitude unless wage or exogenous labor share moves countercyclically as implied by Shimer (2005) and Hall (2006). Thus, it is very important to find an interaction between productivity shock and another shock so that they can produce higher volatility jointly: for example, a reallocation shock or a labor supply shock. Second, the static property of this model's contract leads to the lack of propagation of the model. A dynamic contract embedded in a searching model could be an interesting area for further research.

Appendix : Derivation of (3.12) and (3.13)

To begin with, the worker's maximization problem is,

$$Max_e \int_y w(y) dG(y|e) + \bar{w} - \phi(e) \quad (5.1)$$

By the assumption (A1), we can find the first order condition of this problem. It becomes the incentive compatibility constraint on the firm's side as can be seen in firm's problem (3.1).

$$\int_y w(y) g_e(y|e) dy - v'(e) = 0 \quad (5.2)$$

By the assumption (A3) and the convexity of $\phi(e)$, this condition has a solution given $w(y)$. By Proposition 1, we can substitute an optimal debt contract $w(y) = w^D(y) \equiv y - \min(y, m) + \bar{w}, m > 0$.

$$\frac{\partial}{\partial e} \left(\int_m^{\bar{y}} (y - m) dG(y|e) + \bar{w} - \phi(e) \right) = 0 \quad (5.3)$$

where $\bar{y} = \log e + a\bar{x}$ is the possible maximum outcome given effort and aggregate productivity. Differentiating with respect to e and by the change of variable technique,

$$\frac{1}{e} \int_{m_x}^{\bar{x}} h(x) dx - v'(e) = 0 \quad (5.4)$$

Note that all of the arguments are translated to an idiosyncratic shock x ($y = \log e + ax$ and $m = \log e + am_x$). Substituting the functional form of $\phi(e)$, this leads to (3.12).

Then, from the participation constraint (P.C.) of (3.1) and by Lemma 1,

$$\int_y w(y) dG(y|e) + \bar{w} - \phi(e) \geq z \quad (5.5)$$

Plugging the optimal debt contract $w^D(y; m)$ in and by the change of variable technique,

$$a \left(\int_{m_x}^{\bar{x}} x h(x) dx - m_x (1 - H(m_x)) \right) - \phi(e) \geq z - \bar{w} \quad (5.6)$$

Substituting (3.12) for e , (3.13) follows.

Appendix : Nash Bargaining Economy with Temporary Layoff

When a temporary layoff is available and enforceable, by Lemma 1, it is a favorable outside option for both the worker and the firm in case negotiation are broken off. It can also be applied to the Nash bargaining model with a temporary layoff. In a Nash bargaining economy with a temporary layoff, the timing of events is as follows:

- Aggregate productivity a is realized and commonly known
- A new labor contract is negotiated (if negotiations are broken off, the firm can lay the worker off or destroy the job.)
- Idiosyncratic productivity x becomes known, and the firm then decides whether to go on to production or not. (The firm can decide to produce or to destroy the job.)
- Production is carried out.
- Flow value from production is allocated by the Nash bargaining solution.
- New matches and exogenous separations occur.

Note that whether to lay off or not is determined before productivity is known or production is executed in this economy as is the case in the optimal contract economy. So, once idiosyncratic productivity x becomes known, the firm can choose only to go ahead to production or to fire the worker. We assumed this for the purposes of comparison with the optimal contract economy, abstracting the effect of information about productivity.

Now in this economy, an operating firm's value is

$$J(a, x) = ax - w(a, x) + \beta(1 - \delta) \int_{a'} \int_{x^r(a')}^{\bar{x}} J(a', x') dH(x') dP(a'|a) \quad (5.7)$$

From the free entry condition,

$$c = \beta q(\theta) \int_{a'} \int_{x^r(a')}^{\bar{x}} J(a', x') dH(x') dP(a'|a) \quad (5.8)$$

The employed worker's value is

$$\begin{aligned}
W(a, x) = & w(a, x) + \beta[(1 - \delta) \int_{a'} \int_{\bar{x}^{r(a')}} W(a', x') dH(x') dP(a' | a) \\
& + (1 - \delta) \int_{a'} \int_{\underline{x}}^{\bar{x}^{r(a')}} U(a') dP(a' | a) + \delta \int_{a'} U(a') dP(a' | a)]
\end{aligned} \tag{5.9}$$

The laid-off worker's value is

$$\begin{aligned}
W^L(a) = & z + \beta[(1 - \delta) \int_{a'} \int_{\bar{x}^{r(a')}} W(a', x') dH(x') dP(a' | a) \\
& + (1 - \delta) \int_{a'} \int_{\underline{x}}^{\bar{x}^{r(a')}} U(a') dP(a' | a) + \delta \int_{a'} U(a') dP(a' | a)]
\end{aligned} \tag{5.10}$$

Adding (5.7) to (5.9) and subtracting (5.10), the net surplus $S^L(a, x)$ is

$$\begin{aligned}
S^L(a, x) = & J(a, x) + W(a, x) - W^L(a) \\
= & ax - z + \beta(1 - \delta) \int_{a'} \int_{\bar{x}^{r(a')}} J(a', x') dH(x') dP(a' | a)
\end{aligned} \tag{5.11}$$

The Nash bargaining wage is set to satisfy this condition.

$$S^L(a, x) = \frac{J(a, x)}{(1 - \eta)} = \frac{W(a, x) - W^L(a)}{\eta} \tag{5.12}$$

Substituting (5.12) for $J(a', x')$ in (5.11) results in (4.3). Then, manipulating (5.7), (5.12) and (4.3), (4.5) is derived.

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Table 1 : Parameter Calibration

Parameter	Interpretation	Value
z	Utility from non-market activity	0.4×3
δ	Exogenous separation rate	0.1
β	Discount factor	0.99
η	Labor share	0.6
α	Elasticity of the matched to vacancy	0.4
ρ_a	Persistence parameter of aggregate shock	0.95
σ_a	SD of innovation to aggregate shock process	0.007
μ_x	Mean of logged idiosyncratic shock	0.93
σ_x	SD of logged idiosyncratic shock	0.58

Table 2 : Summary Statistics (Quarterly U.S. data (1964 – 2003))

	a	θ	u	v
Standard deviation	0.019	0.371	0.180	0.202
Quarterly autocorrelation	0.881	0.952	0.956	0.946
Correlation matrix	a	1	0.304	-0.319
	θ	0	1	-0.969
	u	0	0	1
	v	0	0	0

Table 3 : Summary Statistics (standard Nash bargaining)

	a	θ	u	v
Standard deviation	0.0192	0.0329	0.0116	0.0235
Quarterly autocorrelation	0.866	0.866	0.886	0.707
Correlation matrix	a	1	0.999	-0.866
	θ	0	1	-0.866
	u	0	0	1
	v	0	0	0

Table 4: Summary Statistics (optimal contract ($\gamma = 0.6$))

		a	θ	u	v
Standard deviation		0.0192	0.0518	0.0182	0.0371
Quarterly autocorrelation		0.865	0.865	0.886	0.707
Correlation matrix	a	1	0.999	-0.866	0.969
	θ	0	1	-0.866	0.970
	u	0	0	1	-0.718
	v	0	0	0	1

Table 5: Summary Statistics (optimal contract ($\gamma = 0.9$))

		a	θ	u	v
Standard deviation		0.0192	0.0750	0.0263	0.0537
Quarterly autocorrelation		0.866	0.866	0.886	0.708
Correlation matrix	a	1	0.999	-0.866	0.969
	θ	0	1	-0.866	0.970
	u	0	0	1	-0.718
	v	0	0	0	1

Table 6: Summary Statistics (optimal contract ($\gamma = 0.985$))

		a	θ	u	v
Standard deviation		0.0192	0.6630	0.1777	0.5103
Quarterly autocorrelation		0.865	0.857	0.913	0.740
Correlation matrix	a	1	0.870	-0.804	0.829
	θ	0	1	-0.757	0.973
	u	0	0	1	-0.629
	v	0	0	0	1

Table 7: Comparison between Model's Real Wage and Real Wage Data

Real Wage		U. S. Data	NB	Optimal Contract
SD		0.0220	0.020	0.0021
Autocorrelation		0.958	0.882	0.636
Correlation with	a	0.425	0.999	0.610
	θ	0.180	0.999	0.612
	u	-0.164	-0.884	-0.918
	v	0.186	0.960	0.405

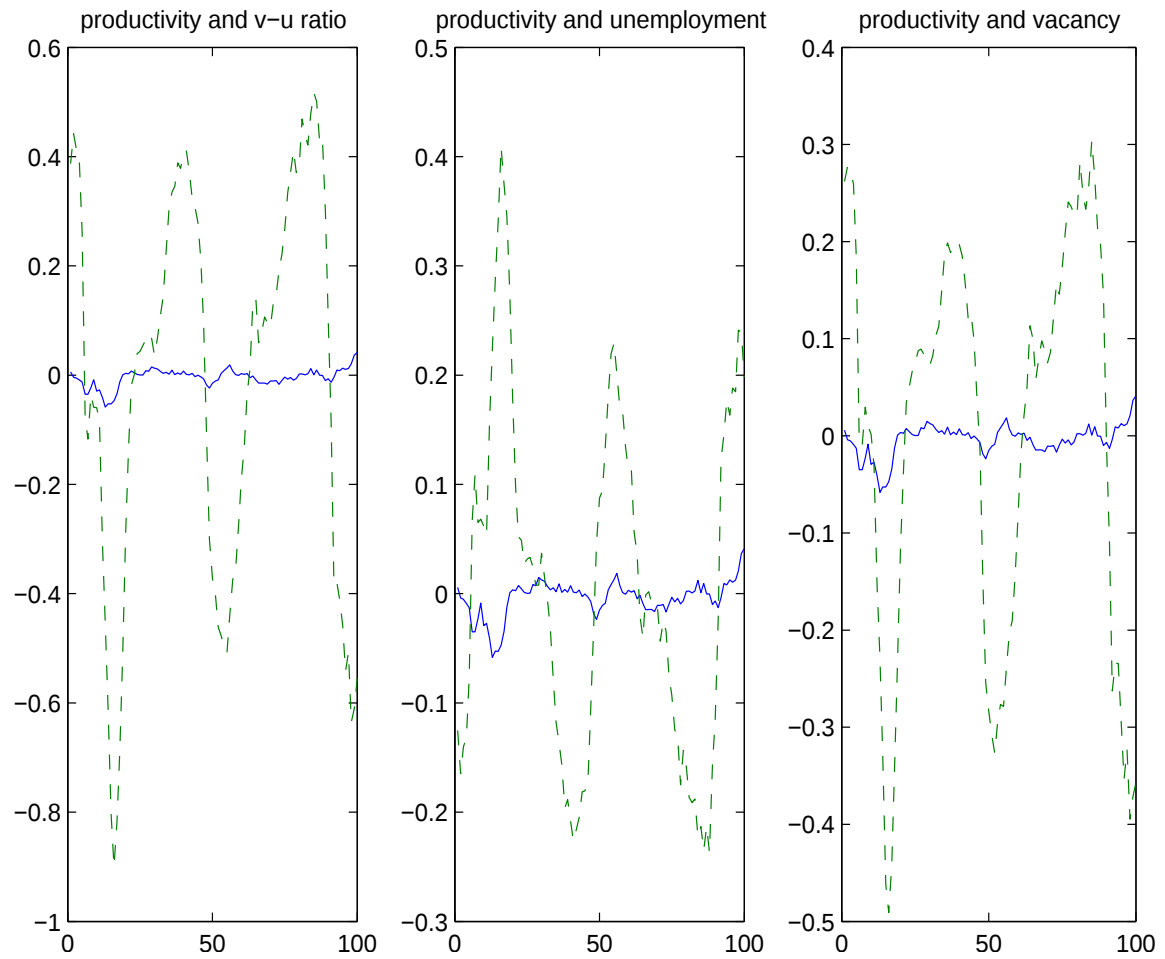
Table 8: Summary Statistics (Nash bargaining w/temporary layoff)

		a	θ	u	v
Standard deviation		0.0192	0.0496	0.0174	0.0355
Quarterly autocorrelation		0.865	0.865	0.886	0.707
Correlation matrix	a	1	0.999	-0.865	0.970
	θ	0	1	-0.865	0.970
	u	0	0	1	-0.717
	v	0	0	0	1

Table 9: Comparison of Model's Real Wage

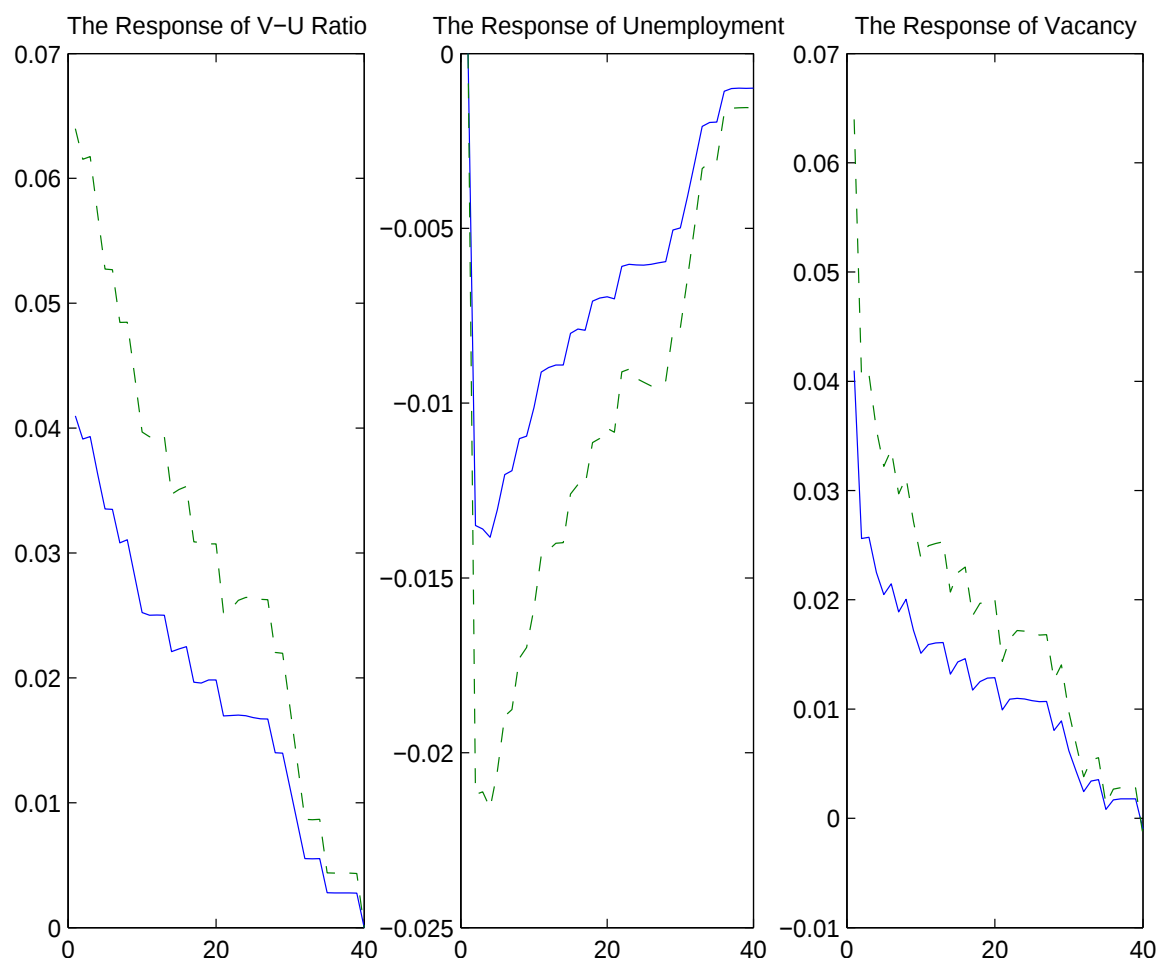
Real Wage		NB	NB(w/layoff)	Optimal Contract
SD		0.020	0.0201	0.0021
Autocorrelation		0.882	0.888	0.636
Correlation with	a	0.999	0.999	0.610
	θ	0.999	0.998	0.612
	u	-0.884	-0.891	-0.918
	v	0.960	0.955	0.405

Figure .1: The Response of Unemployment, Vacancy and v-u Ratio(U.S. data (1964.1-2003.4))



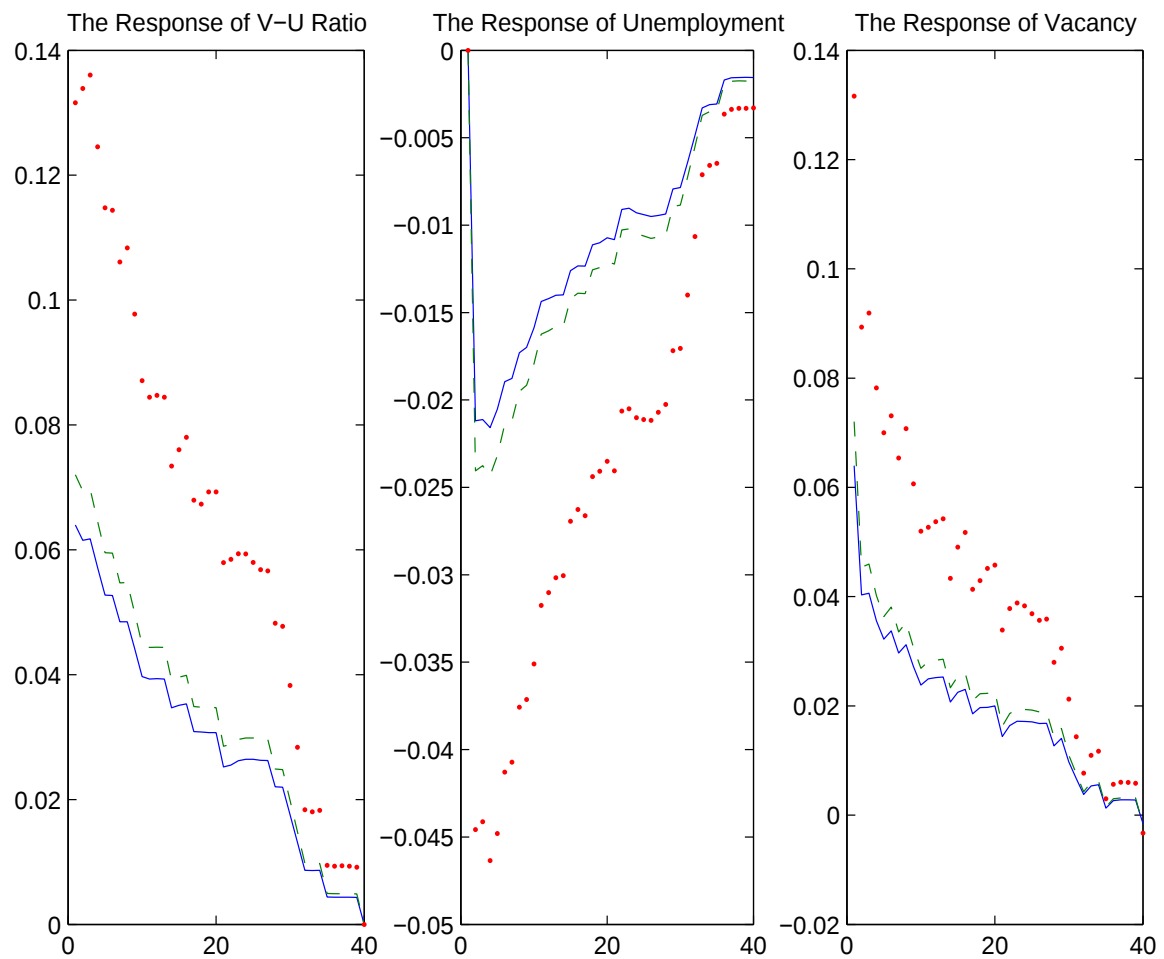
Note: The solid line denotes productivity and the dotted line shows vacancy, unemployment, and the v-u ratio respectively.

Figure .2: The Response of Unemployment, Vacancy and v-u Ratio to 1 SD Productivity Shock



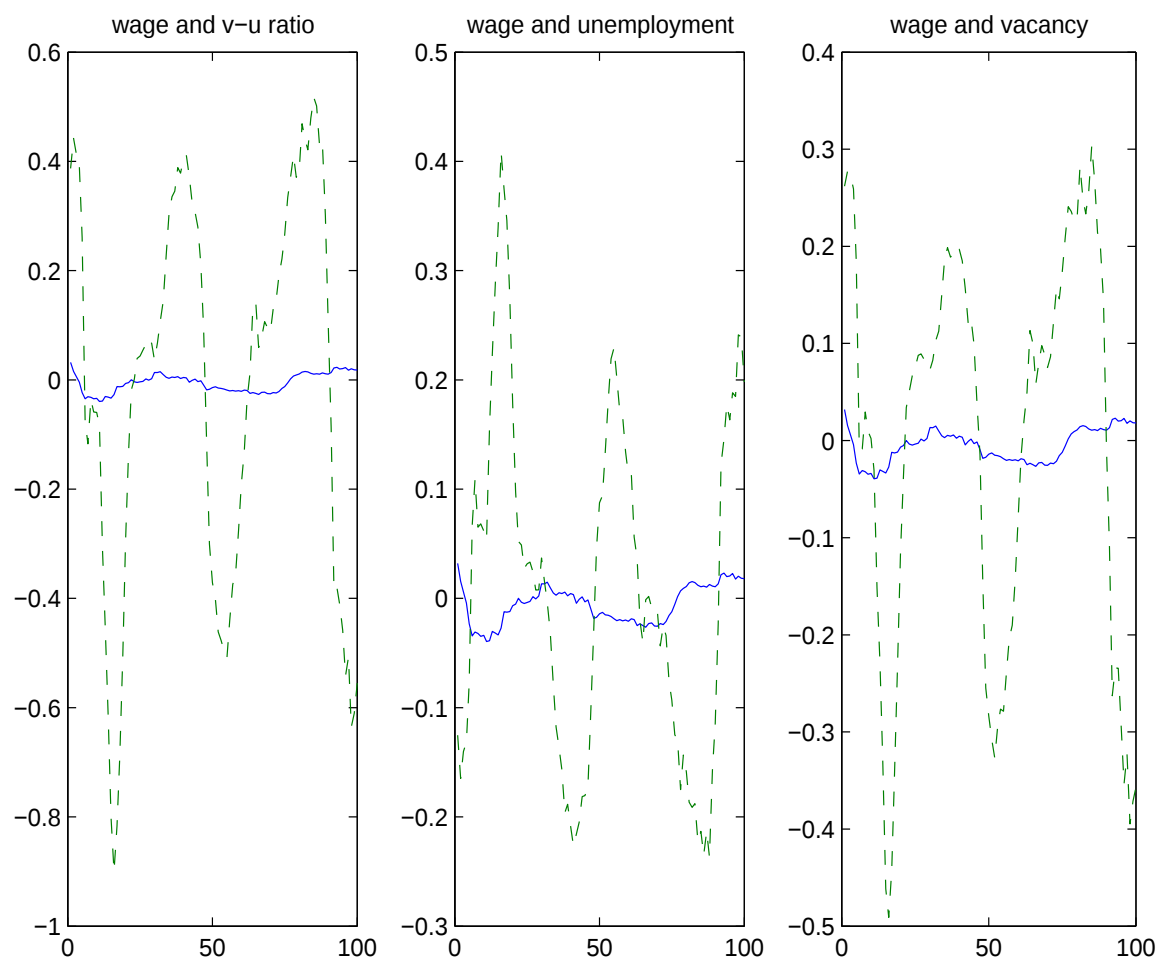
Note: The solid line denotes the impulse response of the standard model and the dotted line shows that of the optimal contract model ($\gamma = 0.6$)

Figure .3: The Response of Unemployment, Vacancy and v-u Ratio to 1 SD Productivity Shock by Base Pay Ratio



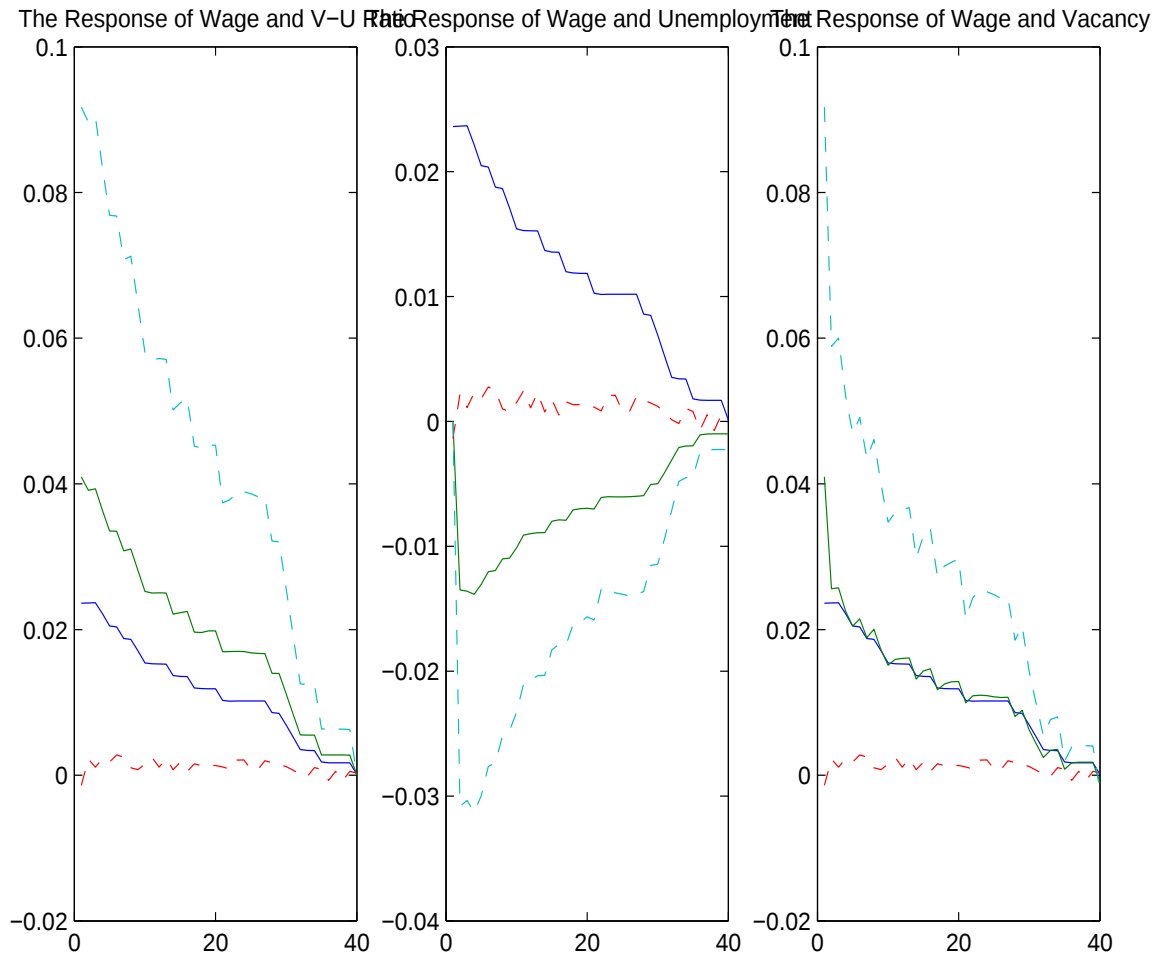
Note: The solid line denotes the impulse response of the optimal contract model ($\gamma = 0.6$) and the dotted line shows that of the optimal contract model($\gamma = 0.8$). The plot is for the case of $\gamma = 0.95$

Figure .4: Real Wage and Labor Market Quantity (U.S. data (1964.1-2003.4))



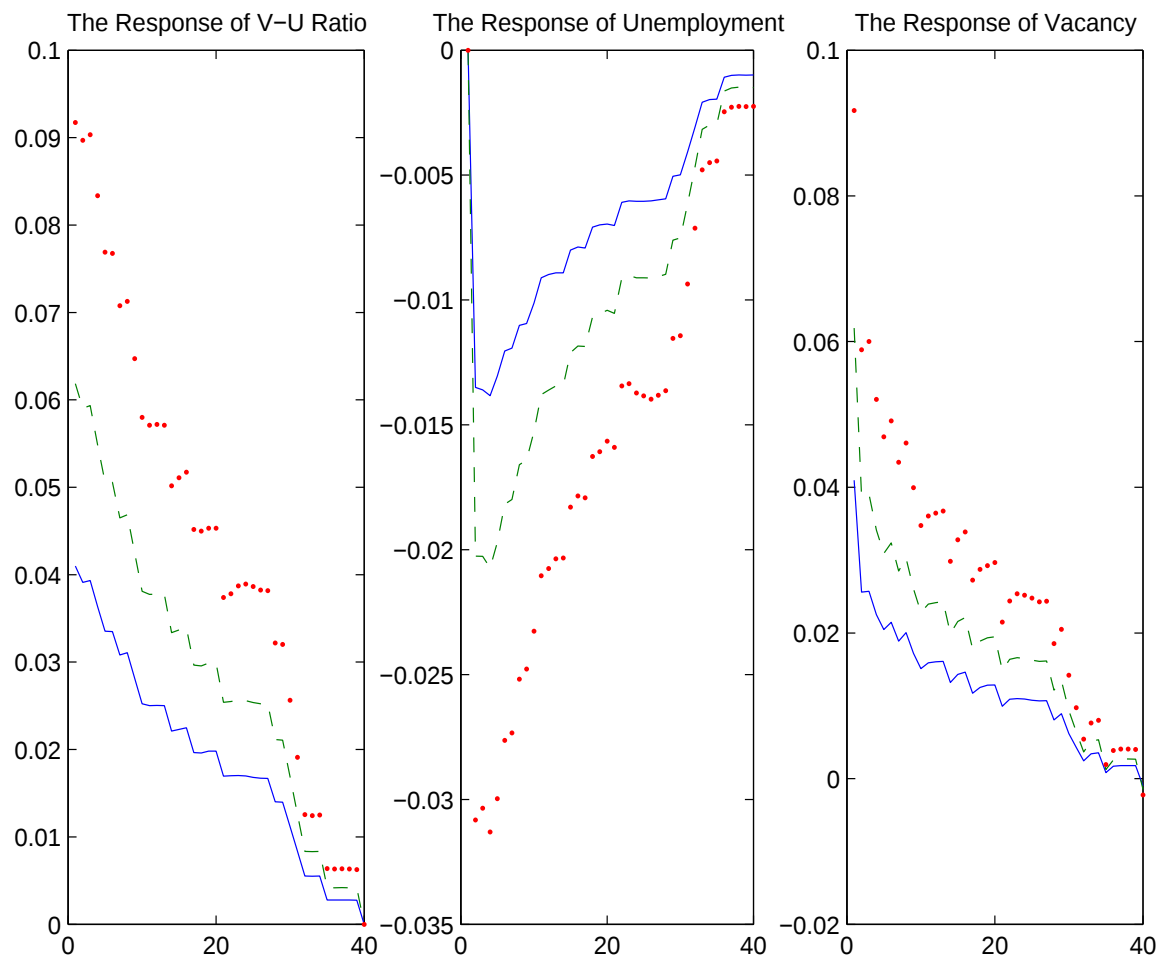
Note: The solid line denotes the real wage and the dotted line shows vacancy, unemployment, and the v-u ratio respectively.

Figure .5: Real Wage and Labor Market Quantity (standard and optimal contract model)



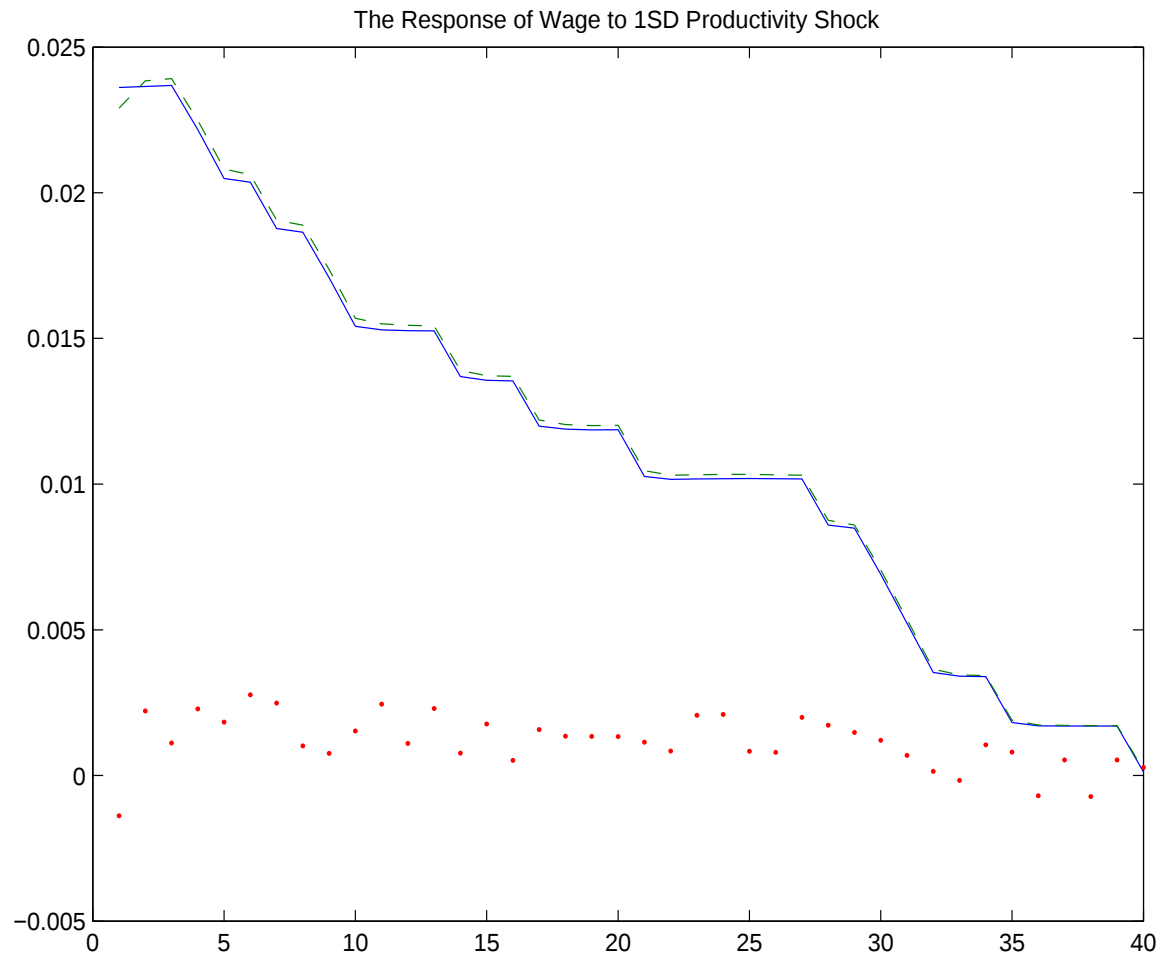
Note: The solid line denotes the real wage (blue line) and labor-market quantities of the standard model. The dotted line shows the real wage (red line) and labor-market quantities of the optimal contract model($\gamma = 0.9$).

Figure .6: The Response of Unemployment, Vacancy and v-u Ratio to 1 SD Productivity Shock



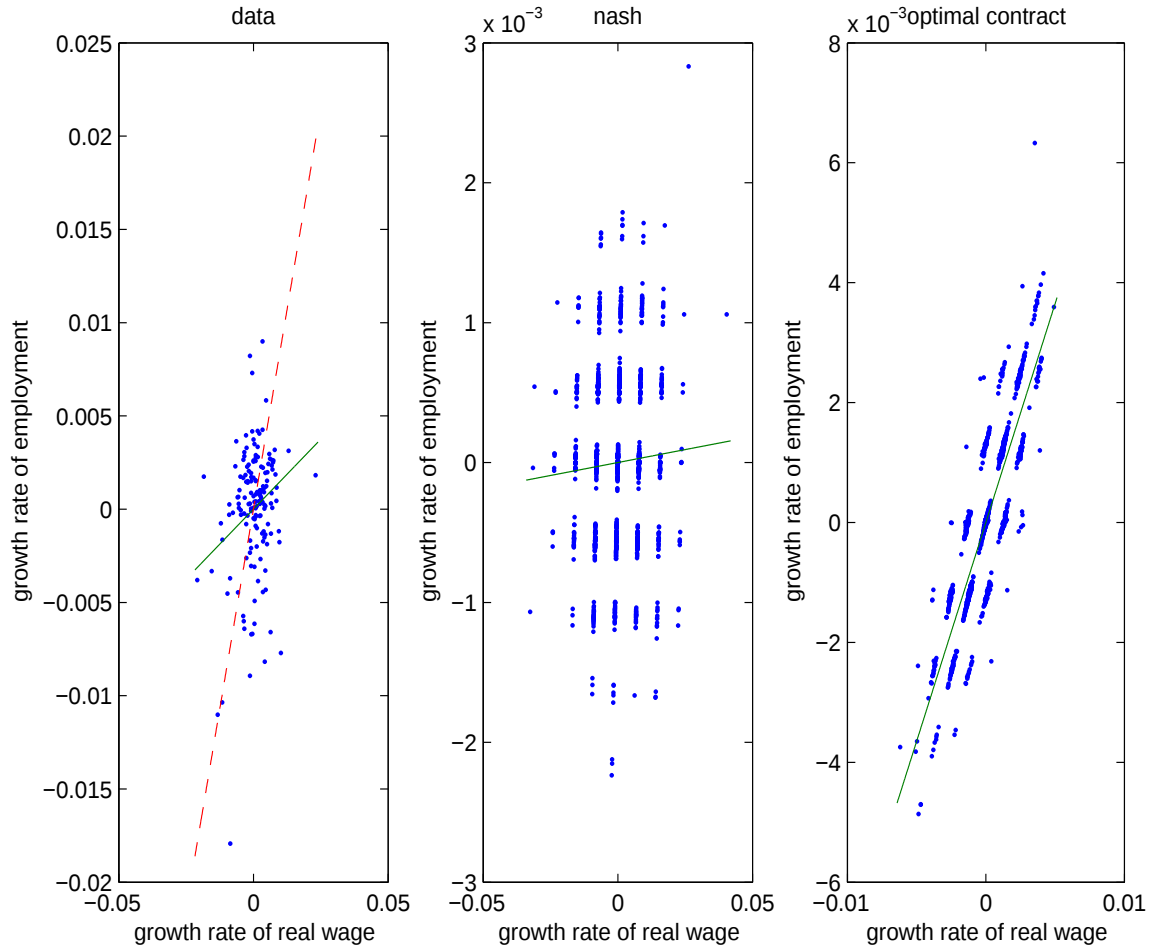
Note: The solid line denotes the impulse response of the standard model and the dotted line shows that of the NB model with temporary layoff. The plot is for the optimal contract model with $\gamma = 0.9$

Figure .7: The Response of Real Wage to 1 SD Productivity Shock



Note: The solid line denotes the impulse response of the standard model and the dotted line shows that of the NB model with temporary layoff. The plot is for the optimal contract model with $\gamma = 0.9$

Figure .8: Aggregate Elasticity of Employment to Real Wage



Note: Estimated equation is $\log \frac{n_t}{n_{t-1}} = \beta_1 + \beta_2 \log \frac{w_t}{w_{t-1}}$. The solid line denotes the OLS fitted line and the dotted line shows the IV fitted line. TFP growth rate is used as an instrumental variable. The number of observations is 160 for real data and 1500 for each simulated series.