

# A Note on Borrowing Limits and Welfare

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Robert Aumann and Bezalel Peleg (1974) elaborate on an example provided in Gale (1974) about the fact that, in a 2-trader, 2-commodity economy, one of the traders can gain by throwing away part of her endowment of one of the goods. This result is striking because welfare increases in spite of the fact that wasting endowments clearly limits the possibilities of trade in goods that are valuable in equilibrium. The idea behind Gale (1974) and Aumann and Peleg (1974) (GAP from now on) is that the change in the price of the good may more than compensate for the waste of it. Of course, in the GAP setup the gains of one agent come at the expense of the welfare of the other. Hence, it is impossible that both agents agree on a limitation of the amount of trade. We show that this may not be the case when markets are incomplete and the amount of trade is limited by a more stringent borrowing constraint.

In this note we incorporate uninsurable uncertainty about endowments into an otherwise standard example of inter-temporal trade. Specifically, in our environment agents face idiosyncratic uncertainty about their endowment of labor income, and there is only a riskless bond to smooth out consumption over time. We show in a two period version of the model that a limitation of the extent of trade due to an exogenous reduction of the borrowing limit can only increase the welfare of one type of agents (the borrowers, as predicted in GAP). However, we then add uncertainty about the initial endowment of wealth, and find that before this uncertainty is resolved *all* agents may agree on the reduction of the borrowing limit. That is, we find conditions under which ex ante welfare increases by reducing the amount of trade. We complete this analysis by providing the results of a numerical simulation from a multi period economy similar to that in Huggett (1993, 1997) and Aiyagari (1994), in which the distribution of agents over states is endogenous. Our

results suggest that aggregate welfare, which coincides with ex ante welfare of any given agent, also decreases when the borrowing limit is excessively large. This result is contrary to the widespread wisdom that facilitating borrowing and lending should always help to increase welfare because it helps to smooth out consumption.

An issue that is worth emphasizing is that in the last class of economies welfare gains do not come at the expense of *other* agent's welfare (as in GAP), but at the expense of the future welfare of the *same* agent. Last, but not least, our results suggest an inverted U-shape relationship between the borrowing limit and welfare: a too large amount of borrowing can be as bad as a too small amount of it. This possibility has received very little attention in the related literature, and we think it deserves to be explored in more detail in future research.<sup>1</sup>

To keep matters as simple as possible we assume there is a continuum, mass one, of ex ante identical agents. These agents live for two periods, and they face uncertainty about their endowments: in the first period the endowment can be either  $0 < y_1^1$  or  $y_1^2$  ( $y_1^1 < y_1^2$ ) with probability  $1/2$ ; in the second period the endowment takes the value  $y_h$  with probability  $\phi \in (0, 1)$  and the value  $y_l < y_h$  with probability  $1 - \phi$ . We think of the endowment in the first period as determining the "type" of an agent, so that agents receiving  $y_1^1$  ( $y_1^2$ ) in the first period are poor (rich) relative the expected endowment in the second period.<sup>2</sup>

Agents derive utility from consuming in both periods, and once the initial endowment is known, they can trade in a riskless bond which is the only available asset in the economy. The price of a bond is  $q$  (in terms of the consumption good of the first period) and it entitles one unit of consumption goods in the second period. Borrowing is allowed up to an exogenous limit  $-B$  (as usual, negative bonds increase consumption in the first period and convey the obligation to deliver consumption goods in the second). Notice that the market arrangement and the borrowing limit preclude having access to complete insurance markets. Furthermore, since no action can take place

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<sup>1</sup>The literature about the effects of various forms of borrowing constraints is too large to be reviewed here. See, among many others, Bencivenga and Smith 1991, Marcet and Marimon 1992, Cox and Japelli 1993, Japelli and Pagano 1994, and more recently, Alvarez and Jermann 2000, Monge 2001, Chatterjee et al. 2002, Athreya 2002, Dey and Mumy 2005, and Mateos-Planas and Seccia 2005 and the many references therein.

<sup>2</sup>These assumptions essentially represent a set of convenient normalizations that help to show the results when there is inter-temporal trade among the two types of agents. Furthermore, we develop the story in the context of a two-period model, but nothing would change in a purely static interpretation.

before the endowment in the first period is known, the objective of the consumers can be written as  $v^i = u(c_1^i) + \beta[\phi u(c_h^i) + (1 - \phi)u(c_l^i)]$ ,  $\beta \in (0, 1)$  for  $i = 1, 2$ . The utility function  $u$  is strictly increasing, strictly concave, and it is differentiable. Hence, given initial endowments and the price  $q$ , the problem of a given (type of) agent reduces to choosing how much to save or dissave in bonds. Formally, the utility maximization problem as a function of  $B$  for  $i = 1, 2$  is given by:

$$\begin{aligned} \max_{b^i} v_i(B) = & u(c_1^i) + \beta[\phi u(c_h^i) + (1 - \phi)u(c_l^i)] \\ \text{s. to } & c_1^i = y_1^i - qb^i \\ & c_s^i = y_s^i + b^i, \text{ for } s = h, l \\ & c^i \geq 0, b^i \geq -B. \end{aligned}$$

The solution of the previous problem is characterized by the usual first order condition:

$$\begin{aligned} u'(c_1^i)q &= \beta[\phi u'(c_h^i) + (1 - \phi)u'(c_l^i)] & \text{if } b^i > -B \\ u'(c_1^i)q &\geq \beta[\phi u'(c_h^i) + (1 - \phi)u'(c_l^i)] & \text{otherwise.} \end{aligned} \quad (1)$$

A competitive equilibrium (CE) for the previous economy is a price  $q > 0$  such that: (i) the conditions in (1) are satisfied for  $i = 1, 2$  and such that (ii)  $b^1 + b^2 = 0$ .

We are interested in CE such that the borrowing constraint binds for the initially poor agents. In any of such equilibria,  $b^2 = B$ , hence the condition for optimality in (1) holds with equality for  $i = 2$ . Thus, the condition implicitly defines the equilibrium bonds price  $q$  as a function of  $B$ . It is straightforward to show that the assumptions on  $u$  and the implicit function theorem imply that

*Lemma 1:* Let  $q$  be a CE such that  $b^2 = B$ , then  $q'(B) < 0$ .

The intuition behind Lemma 1 is that in equilibrium larger borrowing must come together with larger saving, which can only happen if the return on saving is also larger (the price  $q$  decreases). With this result at hand we are ready to evaluate the effect of changes in  $B$  on the expected equilibrium level of utility once types are known (i.e., once uncertainty in the first period has been realized).

*Lemma 2:* Let  $q$  be a CE such that  $b^2 = B$ , then: 1) if  $q'(B)B + q(B) \leq 0$  then  $v_1'(B) < 0$ ; 2)  $v_2'(B) > 0$ .

Lemma 2.1 simply states that increases in the borrowing limit do not necessarily increase welfare of the initially poor agents.<sup>3</sup> Perhaps surprisingly,

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<sup>3</sup>Let  $I(B) = q(B)B$ . Then the condition in Lemma 2.1 simply requires  $I'(B) < 0$ , which

Lemma 2.2 states that a larger borrowing limit *always* benefit the lenders. Taken together, these results imply that once the uncertainty about types has been resolved, then *some* borrowing may be beneficial for both types of agents. Nevertheless, if  $B$  is sufficiently large, then a marginal change in  $B$  will only benefit one of the types. If we think of the borrowing limit as a device that governs the availability of resources for inter-temporal trade, then Lemma 2 can be seen as an implication of the GAP result (here, reducing the amount of trade may benefit type-one agents).

We now ask a slightly different question. Suppose that before the uncertainty about types is realized we ask the agents whether they would like to reduce their ability to borrow. The answer to this question is the content of the following lemma.

*Lemma 3:* Let  $q$  be a CE such that  $b^2 = B$  and  $q'(B)B + q(B) \leq 0$ . Then there is  $B' < B$  such that  $E[v(B')] > E[v(B)]$ .

*Proof:* Notice that  $E[v(B)] = 1/2(v_1(B) + v_2(B))$  and that it suffices to show that  $v'_1(B) + v'_2(B) < 0$ . We have that  $v'_1(B) + v'_2(B) = u'(c_1^1)(q'(B)B + q(B)) - \beta E[u'(c^2)] - q'(B)Bu'(c_1^2)$  where  $E[u'(c^1)] = [\phi u'(c_h^1) + (1 - \phi)u'(c_l^1)]$  and we have used the fact that condition (1) holds with equality for agent 2. It follows that

$$\begin{aligned} v'_1(B) + v'_2(B) &< u'(c_1^1)(q'(B)B + q(B)) - \beta E[u'(c^2)] - q'(B)Bu'(c_1^2) \\ &= u'(c_1^1)(q'(B)B + q(B)) - \beta E[u'(c^2)] - \frac{q'(B)B}{q(B)}\beta E[u'(c^2)] \end{aligned}$$

where the first line uses that  $u'' < 0$  and  $c_s^1 < c_s^2$  for  $s = h, l$ , and the second line uses condition (1) for agent 2. Multiplying and dividing by  $q(B)$  the first term on the right hand side and rearranging we finally get that

$$v'_1(B) + v'_2(B) < \left(1 + \frac{q'(B)B}{q(B)}\right) [u'(c_1^1)q(B) - \beta E[u'(c^2)]] < 0,$$

because the first term is negative by hypothesis, and the second term is positive since  $c_s^1 < c_s^2$  for  $s = h, l$  and  $u$  is strictly concave.

Contrary to the widespread wisdom that facilitating borrowing and lending improves welfare because it helps to smooth consumption over time and states, Lemma 3 states conditions under which a too large borrowing limit

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after tedious algebra, it can be shown that it is equivalent to  $0 > E[u'(c_2)(c_2 - B\sigma)/c_2]$ , which depends only on exogenous parameters:  $B$ , the distribution of endowments in the second period, and  $\sigma$  (the coefficient of relative risk aversion in a CRRA utility function). Hence, it is unlikely that the condition is satisfied for  $B$  arbitrarily close to zero. However, it is straightforward to produce examples, say, by increasing  $B$  or  $\sigma$ .

is in fact undesirable.<sup>4</sup> That is, ex ante welfare would be larger with a smaller borrowing limit.

A serious limitation of the previous analysis is that with only two periods, it seems possible to choose an initial distribution of types such that ex ante welfare increases or decreases in face of a given variation in  $B$ . To overcome this limitation, we use numerical methods to look at a multi period version of the previous environment (such as Huggett 1993, 1997, and Aiyagari 1994) in which the equilibrium is characterized by a price level and a unique, endogenously determined, distribution of agents over states such that markets clear.

To obtain a multi-period version of the previous setting simply assume that agents face an infinite horizon and that the probabilities of transition between the two states  $(y_h, y_l)$  are given by a matrix  $\Pi = [\pi_{y'|y}]$ , with entries  $0 < \pi_{y'|y} < 1$  indicating the probability of reaching state  $y'$  in the following period provided that the current period state is  $y$  (the other assumptions remain as before). Using standard recursive methods, a stationary competitive equilibrium (SRCE) for this environment can be described as: a price  $q$ , a value function  $v(b, y_s)$  describing expected present value of utility from any pair  $(b, y_s)$ , policy functions for consumption and assets (respectively  $c(b, y_s)$  and  $b(b, y_s)$ ),  $s = h, l$ , and a stationary distribution of agents over states  $\psi$  such that: (i) policy functions attain the value function given  $q$ ; (ii)  $\psi$  is consistent with the optimal  $b(b, y_s)$ ,  $s = h, l$ , given  $q$ ; and (iii)  $q$  is such that the bonds market clears  $\int b(b, y)d\psi = 0$ .

We are interested in the distribution  $\psi$  that will enable us to compute ex ante (or average) welfare in equilibrium:  $W = \int u(c(b, y))d\psi$ .<sup>5</sup> To this end we assume  $u(c) = \log c$  and that  $\beta = 0.99$ . These are standard choices in quantitative work. The stochastic process for income is similar to that in Huggett 1993:  $y_h = 1$  and  $y_l = 0.1$ , and the probabilities of transition satisfy  $\pi_{y_h|y_h} = 0.99$  and  $\pi_{y_h|y_l} = 0.01$ . For the illustrative purposes of our examples this choices for the transition matrix are convenient because it helps us to obtain high accuracy with a small computing cost.<sup>6</sup>

<sup>4</sup>Notice that since  $u$  is an arbitrary concave function, then the lemma states conditions for  $B$  to represents an *increase in risk* in the sense of Rothschild and Stiglitz 1970.

<sup>5</sup>For the individual decision problem we use iterations on the derivative of the value function on a grid of points, combined with linear interpolation to allow the decision rules to take values not in the grid. Huggett 1993 describes this method in detail, as well as how to determine the ergodic distribution  $\psi$  applying recursions from an arbitrary  $\psi_0$ . The grid we use has 2,000 equally spaced points, with a distance between 0.00001 and 0.025 units.

<sup>6</sup>With less persistent income processes for  $y$  the state space becomes very large even

Table 1 reports aggregate welfare  $W$  and the equilibrium price  $q$  corresponding to several levels for the borrowing limit  $B$ . It is clear from the table that aggregate welfare increases as we allow *some borrowing*, but that it also decreases when the borrowing limit is sufficiently large. For this example we have computed that the optimal borrowing limit (in the welfare sense) is about 6.5 units.

Table 1: Welfare and borrowing limits.

| $B =$ | 0.5     | 1       | 3       | 5       | 7       | 9       | 10      |
|-------|---------|---------|---------|---------|---------|---------|---------|
| $W$   | -1.0672 | -1.0377 | -0.9876 | -0.9714 | -0.9689 | -0.9748 | -0.9804 |
| $q$   | 1.0226  | 1.0148  | 1.0047  | 1.0011  | 0.9991  | 0.9978  | 0.9973  |

It is also clear from this example that the equilibrium price level  $q$  decreases as  $B$  increases. To understand these results it is convenient to remember that in any SRCE: (1) there is always a positive mass of agents for whom the borrowing limit  $B$  is binding; and (2) that  $\beta/q < 1$  (see Huggett 1993 or Aiyagari 1994, for instance). Fact (1) above implies that there will be a fraction of poorer and poorer agents the larger the borrowing limit, hence the intuition from our Lemma 1 applies in the current dynamic setting. However, fact (2) means that the return of saving cannot grow unboundedly. Therefore, the larger the borrowing limit the poorer agents may be and the more they need to save for precautionary reasons, and on top of this, the return to saving is smaller than with a smaller borrowing limit. From this perspective a larger borrowing limit appears to be hardly a welfare enhancing policy. Quite on the contrary, a shorter borrowing limit prevents agents from attaining very low consumption levels and the need of a large buffer stock of savings. As an illustration of this, Figure 1 provides the equilibrium distribution of assets for  $B = -6.5$  (thin lines) and for  $B = -10$  (thick lines).

## References

with moderately large borrowing limits, resulting in either substantial losses of accuracy or in huge computing costs.

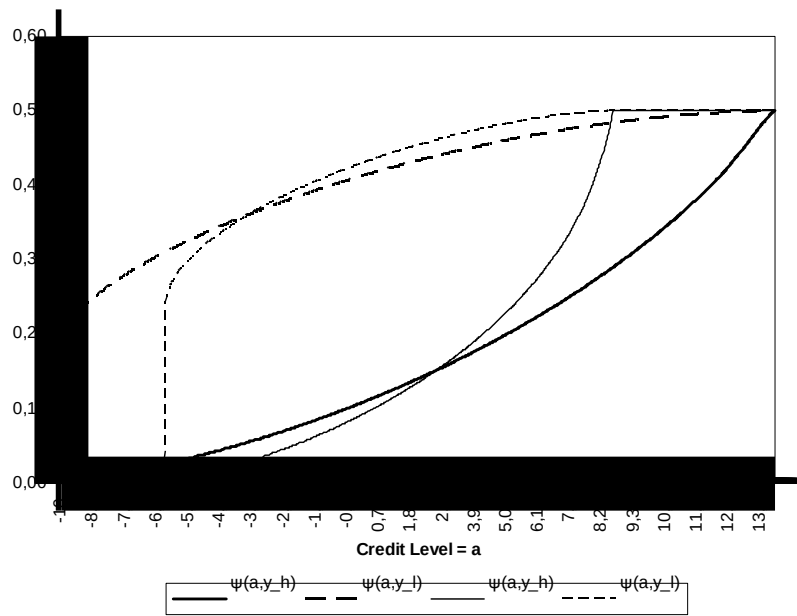


Figure 1: Equilibrium distribution of assets for  $B = 6.5$  (thin lines), and for  $B = 10$  (thick lines).

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