

Optimal Unemployment Insurance with Hidden Trade

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Abstract This paper studies the optimal design of unemployment insurance in an environment where the insurance agency could monitor neither the searching efforts nor the asset holding of an unemployed worker. Previous results with no hidden trade violate the Euler equation of a worker and thus can not be implementable if he has private access to asset market. We abandon the traditional first-order approach and solve the incentive problem directly. With CARA utility functions, we obtain the optimal contract in closed form. We find that, counter-intuitively, an unemployed worker's consumption will decline faster than that implied by Hopenhayn and Nicolini (1997).

1 Introduction

Hopenhayn and Nicolini's (1997) paper on optimal unemployment insurance lays out a framework for studying the efficient unemployment contract when job search effort is unobservable. They assume the planner can control the agent's savings in each period. It is well known that, in their setup, the agent would like to save if they could. We allow for hidden savings under the special case of CARA utility, and solve for the optimal contract explicitly. We show features of the contract that are in contrast with both Hopenhayn and Nicolini (1997), as well as other papers that study similar problems such as Kocherlakota (2004). On the one hand, we recover declining consumption for the unemployed over the duration of the spell which is lost in Kocherlakota (2004). In fact, in our setup, the decline is eventually faster than in the benchmark case of observable savings.

Our setup mirrors that of Hopenhayn and Nicolini (1997) and Kocherlakota (2004): an individual begins unemployed, and must search for a job. Search effort is not observed. Utility is separable in consumption and effort. We use a CARA utility function for consumption, which enables us to solve for the agent's consumption-saving decisions and the optimal contract in closed form.

Kocherlakota (2004) shows that the incentive constraints in the problem with hidden savings can not easily be replaced with first order conditions, and that typically to solve the problem recursively the set of state variables would be infinite, making even numerical computation intractable. We overcome this problem by conjecturing and verifying the countable set of constraints that do bind. With this in hand, it is straightforward to explicitly solve for the planner's optimum.

A key difference between our problem and the one that Kocherlakota (2004) studies is the effort levels that we implement. Kocherlakota focuses on interior effort levels. In that case he shows that the contract takes a very simple form: one amount paid while the agent is unemployed, and a different one when the agent is employed, independent of the timing of employment.

We study an environment where the agent has access to both hidden borrowing and savings. Although borrowing is never used by the agent for the effort levels we implement, interior effort levels are not incentive feasible with hidden trade. As a result we study the situation where effort is always at the upper or lower bound. We find that the payment to unemployed agents falls with the length of unemployment. Moreover, the time path for utility is exponential, in contrast to the linear decline when savings are observable. Eventually, with hidden savings, unemployed agents face a more severe penalty for staying unemployed when savings are hidden, the opposite of Kocherlakota's result that hidden savings removed

any change in the payment as a function of time unemployed.

Kocherlakota (2004) is interested in studying the properties of contracts where the principal needs to implement interior efforts. He assumes that an agent could privately save, but not privately borrow, and interior efforts are actually implementable. However, implementing interior efforts is never optimal in his environment. Therefore we could probably say that, that consumption of an unemployed worker does not fall with unemployment duration is because the contract is not designed efficiently in Kocherlakota (2004).

Kocherlakota justifies the interior efforts by search externalities. The disutility of a representative worker is $p\Psi(\bar{p})$, where $\bar{p}$ is the average efforts by other agents. However, instead of every agent searching with interior $p^* \in (0, 1)$, the principal would randomize the efforts, asking p^* fraction of people search with full effort and the rest with 0 effort. These two types are identifiable because the first type finds a job with probability 1, and the second type never finds a job. This randomization will still generate the same congestion effects. Now the consumptions of both types are deterministic, and in particular, the consumption of the searching type is not smoothed, which generates inefficiency.

Our paper is also related to Werning (2002) and Abraham and Pavoni (2004). These papers use the first order condition approach to attack models with hidden savings and borrowing. Briefly speaking, the first order approach studies a relaxed problem, which replaces the incentive constraints in the original problem with some first order conditions of the agent. In these papers, in period $t > 1$, they impose the first order condition for the type that has never deviated in previous periods and thus has 0 hidden wealth. However, we show that at the beginning of period $t > 1$, the binding incentive constraint for exerting effort is that of an out-of-equilibrium type. This is the type that has shirked in all the previous periods and accumulated the most secret wealth. Thus the incentive constraint of the truth-teller is not binding and should not present in a relaxed problem. This suggests that the first order approach is invalid in general¹.

This paper is organized as follows. Section 2 lays out the economic environment and sets up the principal's contracting problem. In section 3, we solve the consumption saving problem of a potentially shirking agent. Thanks to CARA utility function, we obtained closed-form solution. The resulting solutions are put to use in Section 4 to characterize the optimal contract. In particular, we first focus on a certain subset of incentive constraints in a relaxed problem. And then we verify that our relaxed problem provide a contract that is optimal in the original problem. In Section 5, we show the difference between our results

¹Abraham and Pavoni (2004) implement an ex-post verification procedure, which shows that given a contract obtained by the first order approach, the potential gain for an agent to secretly save is small. This suggests that the solution from the first order approach could be close to the true constrained efficient contract.

and that of Hopenhayn and Nicolini (1997). The appendix justifies our assumption that the principal would like to implement high efforts.

2 The model

In this section we describe an unemployment insurance model similar to that of Kocherlakota (2004). There is a risk-neutral principal and a risk-averse agent. The preferences of the agent are

$$E \sum_{t=0}^{\infty} \beta^t [-\exp(-\gamma c_t) - a_t]$$

where c_t and $a_t \in [0, 1]$ are consumption and search effort at time t , $\beta < 1$ is the discount factor, and E is the expectation operator. An agent can be employed or unemployed; he begins life unemployed. The choice of a_t affects the probability of becoming employed for an unemployed agent. Specifically, if an agent is unemployed in period t , then the probability of his becoming employed in period $t + 1$ is πa_t , and the probability of his staying unemployed is $1 - \pi a_t$. We assume $\pi \in (0, 1)$, which implies that the agent may find no job even if he exerts full effort in the previous period². If an agent is employed in period t , he stays employed in period $t + 1$ with probability one. Thus, being employed is an absorbing state.

The agent's employment status is observable to others, but his choice of a_t is unobservable. As well, the agent can secretly trade risk-free bond in a hidden asset market. The principal can observe neither the consumption nor the asset holding of the agent. A contract σ in this environment specifies three sequences $(\{c_t^U\}_{t=0}^{\infty}, \{c_t^E\}_{t=1}^{\infty}, \{a_t\}_{t=0}^{\infty})$. Given such a contract, an agent who is unemployed in period t receives compensation c_t^U from the principal. If an agent became employed for the first time in period t , then his compensation from the principal in period $s \geq t$ is c_t^E . Thus, once an agent is employed, his compensation is constant over time. (It is simple to show that because the principal and agent have the same discount factor, this smooth compensation is efficient in this economy.) The contract will also recommend an effort level a_t , and given that principal can not observe a_t , a_t will be designed to satisfy incentive constraints. To simplify notation, we will let $U_t = -u(c_t^U)$, $E_t = -u(c_t^E)$.

Without loss of generality, we will require that there is no incentive for the agent to trade if he follows the *recommended* strategy $\{a_t\}_{t=0}^{\infty}$. This is true if and only if all the first order conditions are satisfied,

$$U_t = (a_t \pi) E_{t+1} + (1 - a_t \pi) U_{t+1}, t \geq 0.$$

²Kocherlakota (2004) assumed $\pi = 1$.

Note that once the agent chooses a different search effort $\tilde{a}_t \neq a_t$, this first order condition is typically violated, $U_t \neq (\tilde{a}_t \pi)E_{t+1} + (1 - \tilde{a}_t \pi)U_{t+1}$, and the agent would find it in his interest to either borrow or lend risk-free bond in the private trading market. In the literature, cheating and hidden trade afterwards are called joint deviation by the agent. Hidden trade is an additional friction that makes the contract more difficult to solve than a traditional dynamic contracting problems without hidden trade.

Suppose $\{b_t\}_{t=0}^\infty$ is the agent's search strategy. If there is a finite $\bar{t}$, such that $b_t = a_t, \forall t \geq \bar{t}$, then $\{b_t\}_{t=0}^\infty$ is called a *finite-deviation* strategy. Otherwise it is called an *infinite-deviation* strategy. Let $V(\{b_t\}_{t=0}^\infty)$ be an agent's ex-ante utility if he uses strategy $\{b_t\}_{t=0}^\infty$ and privately trades. The incentive constraints will be written as

$$V(\{a_t\}_{t=0}^\infty) \geq V(\{b_t\}_{t=0}^\infty), \forall \{b_t\}_{t=0}^\infty.$$

However, in much of the following discussion, we will look at only incentive constraints for finite-deviation strategies. Actually we first find the optimal contract where only the finite-deviation incentive constraints are imposed. Secondly we show that contract also satisfies the infinite-deviation incentive constraints.

Let D_t denotes the expected value of disutility(discounted to period 0) of using efforts $\{a_s\}_{s=t}^\infty$ conditional on that an agent has not found a job at the beginning of period t ,

$$D_t = \sum_{s=t}^{\infty} \beta^s (\Pi_{k=t}^{s-1} (1 - a_k \pi)) a_s.$$

It is easily seen that $D_t = \beta^t a_t + (1 - a_t \pi) D_{t+1}$. D_0 would be the total discounted disutility for an agent. Since disutility is linear and 1 unit of labor is compensated with $\frac{\beta \pi w}{1 - \beta}$ units of wage income, $\frac{\beta \pi w}{1 - \beta} D_0$ is the expected wage income that the principal can obtain. The expected cost for the principal is equal to the discounted value of consumption goods delivered to the agent, minus the expected wage income,

$$C(\sigma) = \sum_{t=0}^{\infty} \beta^t (\Pi_{s=0}^{t-1} (1 - a_s \pi)) [c_t^U + \beta a_t \pi \frac{c_{t+1}^E}{1 - \beta}] - \frac{\beta \pi w}{1 - \beta} D_0.$$

For later purposes, we will decompose the cost and write it in an alternative way. Introduce new notations $x_t = c_{t-1}^U - c_t^U = \log(U_t/U_{t-1})/\gamma$. Then $c_{t+1}^U = c_0^U - \sum_{s=1}^{t+1} x_s$. Since Euler equations imply $\exp(-\gamma c_t^U) = a_t \pi \exp(-\gamma c_{t+1}^E) + (1 - a_t \pi) \exp(-\gamma c_{t+1}^U)$, we have

$$\begin{aligned} c_t^U - c_{t+1}^E &= \frac{1}{\gamma} \log \left(\frac{1 - (1 - a_t \pi) \exp(\gamma x_{t+1})}{a_t \pi} \right), \\ c_{t+1}^E &= c_0^U - \sum_{s=1}^t x_s - \frac{1}{\gamma} \log \left(\frac{1 - (1 - a_t \pi) \exp(\gamma x_{t+1})}{a_t \pi} \right). \end{aligned}$$

The cost for the principal is

$$C(\sigma) = \frac{c_0^U}{1-\beta} - \frac{\beta}{1-\beta} [a_0\pi \frac{1}{\gamma} \log \left(\frac{1 - (1 - a_0\pi) \exp(\gamma x_1)}{a_0\pi} \right) + (1 - a_0\pi)x_1] - \\ \frac{\beta^2(1 - a_0\pi)}{1-\beta} [a_1\pi \frac{1}{\gamma} \log \left(\frac{1 - (1 - a_1\pi) \exp(\gamma x_2)}{a_1\pi} \right) + (1 - a_1\pi)x_2] - \dots - \frac{\beta\pi w}{1-\beta} D_0.$$

3 Solve the hidden-trade problem in closed form

Let $\{\tilde{a}_t\}_{t=0}^\infty$ be a finite-deviation strategy and $\tilde{a}_t = a_t, \forall t \geq \bar{t}$ for some $\bar{t}$. Then $V(\{\tilde{a}_t\}_{t=0}^\infty)$ can be solved by backward induction. Denote the agent's after-trade consumption by $(\{\tilde{c}_t^U\}_{t=0}^\infty, \{\tilde{c}_t^E\}_{t=1}^\infty)$. At the beginning of period $\bar{t}$, if the agent has additional wealth W , since he will follow strategy $\{a_t\}_{t=\bar{t}}^\infty$, the optimal consumption is the sum of the goods claimed from the principal and the interest payment of wealth W . For $t \geq \bar{t}$,

$$\begin{aligned} \tilde{c}_t^U &= c_t^U + rW \\ \tilde{c}_t^E &= c_t^E + rW \end{aligned}$$

An unemployed agent's period $\bar{t}$ value function taking W as a state variable is

$$V_{\bar{t}}(W) = -\frac{\exp(-\gamma c_{\bar{t}}^U)}{1-\beta} \exp(-\gamma rW) = -\frac{U_{\bar{t}}}{1-\beta} \exp(-\gamma rW).$$

Now suppose we know an unemployed agent's value function at period $t < \bar{t}$ is $V_t(W) = -\frac{x}{1-\beta} \exp(-\gamma rW)$ where x is a constant. We can calculate the value function $V_{t-1}(W)$ as follows. The agent will use effort $\tilde{a}_{t-1}$, and privately knows that he will find a job next period with probability $\tilde{a}_{t-1}\pi$, and has a value function $-\frac{E_t}{1-\beta} \exp(-\gamma rW)$, or he will be unemployed with probability $(1 - \tilde{a}_{t-1}\pi)$, and has a value function $-\frac{x}{1-\beta} \exp(-\gamma rW)$. The agent has to solve an optimal saving problem at period $t-1$,

$$\begin{aligned} V_{t-1}(W) &= \max_{c_{t-1}, W_t} -\exp(-\gamma c_{t-1}) + \\ &\quad \beta \left[\tilde{a}_{t-1}\pi \left(-\frac{E_t}{1-\beta} \exp(-\gamma rW_t) \right) + (1 - \tilde{a}_{t-1}\pi) \left(-\frac{x}{1-\beta} \exp(-\gamma rW_t) \right) \right] \\ s.t. \quad &c_{t-1} + W_t = (1+r)W + c_{t-1}^U \end{aligned}$$

First order condition would be $-\exp(-\gamma c_{t-1}) = -[\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)x] \exp(-\gamma r W_t)$, which implies

$$\begin{aligned} c_{t-1} &= rW + (1 - \beta)c_{t-1}^U + \beta \frac{\log [\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)x]}{-\gamma} \\ V_{t-1}(W) &= \frac{-\exp(-\gamma c_{t-1})}{1 - \beta} \\ &= -\exp(-\gamma r W)(\exp(-\gamma c_{t-1}^U))^{1-\beta} [\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)x]^\beta / (1 - \beta) \\ &= -\frac{U_{t-1}^{1-\beta} [\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)x]^\beta}{1 - \beta} \exp(-\gamma r W). \end{aligned}$$

Using the recursive formula, we have the agent's utility in the initial period (assuming the initial wealth is 0),

$$\frac{-U_0^{1-\beta} [\tilde{a}_0\pi E_1 + (1 - \tilde{a}_0\pi)U_1^{1-\beta} [\tilde{a}_1\pi E_2 + (1 - \tilde{a}_1\pi)U_2^{1-\beta} [\dots [\tilde{a}_{\bar{t}-1}\pi E_{\bar{t}} + (1 - \tilde{a}_{\bar{t}-1}\pi)U_{\bar{t}}]^\beta \dots]^\beta]^\beta}{1 - \beta}$$

The above closed-form value function helps to show that the principal can not implement any interior efforts. That is, for any incentive compatible contract with efforts $\{a_t\}_{t=0}^\infty$, a_t is either 0 or 1. This is in contrast with a model without hidden trade. In Hopenhayn and Nicolini (1997) with linear disutility, the agent's value function is linear in a_t . In order to induce interior effort, first order condition would be both sufficient and necessary. In our model, if the principal still wants to implement interior effort, the first order condition would still be necessary. However, with hidden trade, there is a second order effect. Whenever the agent deviates from $a_t \in (0, 1)$, he would either borrow or lend to smooth consumption. This second order effect bends the value function upward and thus makes it a convex function of a_t . Although his disutility is still linear and the first order condition may be satisfied, we show below that the second order condition is always violated³.

LEMMA 1 *For any $\bar{t}$, if $E_{\bar{t}} \neq U_{\bar{t}}$, then*

$$-U_0^{1-\beta} [\tilde{a}_0\pi E_1 + (1 - \tilde{a}_0\pi)U_1^{1-\beta} [\tilde{a}_1\pi E_2 + (1 - \tilde{a}_1\pi)U_2^{1-\beta} [\dots [\tilde{a}_{\bar{t}-1}\pi E_{\bar{t}} + (1 - \tilde{a}_{\bar{t}-1}\pi)U_{\bar{t}}]^\beta \dots]^\beta]^\beta$$

is a strictly convex function of $\tilde{a}_{\bar{t}-1}$.

Proof. It is easy to see the strict concavity of functions $g(x) = x^\beta$ and $f(x) = U_t^{1-\beta} [\tilde{a}_t\pi E_{t+1} + (1 - \tilde{a}_t\pi)x]^\beta$. The result then follows from the fact that composition of two strictly concave functions is also strictly concave. ■

³Kocherlakota (2004) studies hidden savings but not hidden borrowing. In that case it is feasible to implement interior effort levels.

LEMMA 2 *If a contract recommends efforts $\{a_t\}_{t=0}^\infty$, and for some $\bar{t}$, $a_{\bar{t}} \in (0, 1)$, then the contract is not incentive compatible.*

Proof. Consider a deviation strategy with $\tilde{a}_{\bar{t}} \neq a_{\bar{t}}$. The disutility is

$$D_0 = \sum_{t=0}^{\bar{t}-1} \beta^t (\Pi_{s=0}^{t-1} (1 - a_s \pi)) a_t + (\Pi_{s=0}^{\bar{t}-1} (1 - a_s \pi)) (\beta^{\bar{t}} \tilde{a}_{\bar{t}} + (1 - \pi \tilde{a}_{\bar{t}}) D_{\bar{t}+1}),$$

which is linear in $\tilde{a}_{\bar{t}}$. So the value function is the sum of a strictly convex function and a linear function; it is still strictly convex. Since a strictly convex function can not obtain a global maximum in the interior of an interval, we know that an agent will increase his utility by setting $\tilde{a}_{\bar{t}}$ to be either 0 or 1. ■

4 Implement high effort

We will assume that the principal wants to implement high efforts $\{a_t\}_{t=0}^\infty$, where $a_t = 1$, for all t . In the appendix, we will justify this assumption by showing that implementing the high effort sequence is better than other possible sequences when the initial promised utility is sufficiently low. Although the proof there is technical, the main intuition is straightforward. When an agent starts with low promised utility, his marginal utility of consumption is high. Thus it is cheap for the principal to compensate disutility incurred from effort with consumption.

There is a continuum of finite-deviation strategies in the model, and our first task is to find the binding incentive constraints. We focus on the following cheating strategy: for $\bar{t} \geq 0$, the strategy specifies 0 effort from periods 0 to $\bar{t}$ and full effort from period $\bar{t} + 1$ onward. From our previous calculation, the utility from such a deviation is

$$-\frac{\Pi_{t=0}^{\bar{t}} U_t^{(1-\beta)\beta^t} U_{\bar{t}+1}^{\beta^{\bar{t}+1}}}{1 - \beta}.$$

The disutility is

$$-\frac{\beta^{\bar{t}+1}}{1 - \beta(1 - \pi)}.$$

Let $V = -\frac{U_0}{1-\beta} - \frac{1}{1-\beta(1-\pi)}$ be the initial promised utility for the agent, then the incentive constraints for the above deviation strategies are

$$V \geq -\frac{\Pi_{t=0}^{\bar{t}} U_t^{(1-\beta)\beta^t} U_{\bar{t}+1}^{\beta^{\bar{t}+1}}}{1 - \beta} - \frac{\beta^{\bar{t}+1}}{1 - \beta(1 - \pi)}, \forall \bar{t} \geq 0.$$

Our guess is that the above are all of the binding incentive constraints in the model. To verify our guess, we will first study a relaxed problem with these constraints only. We then show that the solution to the relaxed problem satisfies all the other finite-deviation incentive constraints.

A relaxed problem:

$$\min_{\sigma} \quad C(\sigma) \tag{1}$$

$$s.t. \quad U_t = \pi E_{t+1} + (1 - \pi)U_{t+1}, \forall t \geq 0, \tag{2}$$

$$V = -\frac{U_0}{1 - \beta} - \frac{1}{1 - \beta(1 - \pi)}, \tag{3}$$

$$V \geq -\frac{\prod_{s=0}^t U_s^{(1-\beta)\beta^s} U_{t+1}^{\beta^{t+1}}}{1 - \beta} - \frac{\beta^{t+1}}{1 - \beta(1 - \pi)}, \forall t \geq 0. \tag{4}$$

Since U_0 is pinned down by (3) and E_{t+1} can be solved as a function of U_t, U_{t+1} in (2), the relaxed problem has a equal number of unknowns $\{U_t\}_{t=1}^{\infty}$ and incentive constraints. If all the constraints bind, we can solve for U_t recursively. We verify this in the following lemma.

LEMMA 3 *In the relaxed problem, all the constraints bind.*

Proof. Recall our definition $x_t = c_{t-1}^U - c_t^U$, the relaxed problem can be rewritten as

$$\begin{aligned} \max_{x_t} \quad & \sum_{t=1}^{\infty} \frac{\beta^t (1 - \pi)^{t-1}}{1 - \beta} f(x_t) \\ s.t. \quad & \sum_{s=1}^t \beta^s x_s \geq \frac{1}{\gamma} \log(-V - \frac{\beta^t}{1 - \beta(1 - \pi)}) - \frac{1}{\gamma} \log(-V - \frac{1}{1 - \beta(1 - \pi)}), \forall t \geq 0, \end{aligned}$$

where function $f(x) = \frac{1}{\gamma} \pi \log[\frac{1 - (1 - \pi) \exp(\gamma x)}{\pi}] + (1 - \pi)x$ is strictly concave⁴ and has a unique maximizer at $x = 0$. We show first that in optimal solution, $x_t \geq 0$. Suppose for some $t \geq 1$, $x_t < 0$, then setting $x_t = 0$ would increase the objective function and still be feasible. Then we prove that

$$\sum_{t=1}^{\infty} \beta^t x_t = \frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1 - \beta(1 - \pi)}).$$

By contradiction, let $\bar{t}$ be the smallest t , such that

$$\sum_{s=1}^{\bar{t}} \beta^s x_s > \frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1 - \beta(1 - \pi)}).$$

⁴We can verify that $f(0) = 0$, $f'(x) = 1 - \frac{\pi}{1 - (1 - \pi) \exp(\gamma x)}$, $f'(0) = 0$, $f''(x) = -\pi(1 - \pi)\gamma(1 - (1 - \pi) \exp(\gamma x))^{-2} \exp(\gamma x) < 0$.

Since $\sum_{s=1}^{\bar{t}-1} \beta^s x_s \leq \frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)})$,

$$\beta^{\bar{t}} x_{\bar{t}} > \frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)}) - \sum_{s=1}^{\bar{t}-1} \beta^s x_s \geq 0.$$

Then reducing $\beta^{\bar{t}} x_{\bar{t}}$ to $\frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)}) - \sum_{s=1}^{\bar{t}-1} \beta^s x_s$ would still be feasible and increase the objective function. Last we will show that for all $t \geq 1$,

$$\sum_{s=1}^t \beta^s x_s = \frac{1}{\gamma} \log(-V - \frac{\beta^t}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)}).$$

By contradiction, let t^* be the smallest t such that

$$\begin{aligned} \beta^t x_t &> \frac{1}{\gamma} \log(-V - \frac{\beta^t}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{\beta^{t-1}}{1-\beta(1-\pi)}), \\ \beta^s x_s &= \frac{1}{\gamma} \log(-V - \frac{\beta^s}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{\beta^{s-1}}{1-\beta(1-\pi)}), s < t. \end{aligned}$$

We will show that $\beta^{t^*+1} x_{t^*+1} > \frac{1}{\gamma} \log(-V - \frac{\beta^{t^*+1}}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{\beta^{t^*}}{1-\beta(1-\pi)})$ by the following argument. For any small number $\epsilon \neq 0$, changing x_{t^*} by ϵ and x_{t^*+1} by $-\epsilon/\beta$ will keep $\beta^{t^*} x_{t^*} + \beta^{t^*+1} x_{t^*+1}$ unchanged, and the change in objective function should be 0, which is

$$\frac{\beta^t (1-\pi)^{t-1}}{1-\beta} (f'(x_{t^*}) + (1-\pi)f'(x_{t^*+1})) = 0.$$

The concavity of f implies that $x_{t^*+1} > x_{t^*}$, which is

$$\begin{aligned} x_{t^*+1} &> x_{t^*} \\ &> \frac{\frac{1}{\gamma} \log(-V - \frac{\beta^{t^*}}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{\beta^{t^*-1}}{1-\beta(1-\pi)})}{\beta^{t^*}} \\ &> \frac{\frac{1}{\gamma} \log(-V - \frac{\beta^{t^*+1}}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{\beta^{t^*}}{1-\beta(1-\pi)})}{\beta^{t^*+1}}. \end{aligned}$$

By induction and following the same argument, we can show that for all $t \geq t^* + 1$, $\beta^t x_t > \frac{1}{\gamma} \log(-V - \frac{\beta^t}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{\beta^{t-1}}{1-\beta(1-\pi)})$, which contradicts our previous conclusion that

$$\sum_{t=1}^{\infty} \beta^t x_t = \frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)}).$$

■

The above optimization problem has a equal number of variables and constraints, the constraints are independent, and they all bind. To understand why all the constraints bind, consider the following simple

example.

$$\begin{aligned} \max_{x_1, x_2} \quad & -x_1^2 - x_2^2 \\ \text{s.t.} \quad & x_1 \geq a, \\ & x_1 + x_2 \geq 2. \end{aligned}$$

If the first constraint does not bind, x_1 would take the value of 1; the first constraint starts to bind when a exceeds one. The constraints in the relaxed problem have a similar structure. Starting from the “last” constraint⁵(for the infinite sum of $\beta^t x_t$), the otherwise-unconstrained solution implies the other constraints are violated.

To see how this is related to the relaxed problem we study, consider the relaxed problem with only the “last” limit-constraint imposed,

$$\begin{aligned} \max_{x_t} \quad & \sum_{t=1}^{\infty} \frac{\beta^t (1-\pi)^{t-1}}{1-\beta} f(x_t) \\ \text{s.t.} \quad & \sum_{t=1}^{\infty} \beta^t x_t \geq \frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)}). \end{aligned}$$

The solution would be $x_t < x_{t+1}$, for all $t \geq 1$, and $\sum_{t=1}^{\infty} \beta^t x_t = \frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)})$. This implies, for finite t , that

$$\sum_{s=1}^t \beta^s x_s < (1-\beta^t) [\frac{1}{\gamma} \log(-V) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)})].$$

But now consider the constraint for that t ; namely

$$\sum_{s=1}^t \beta^s x_s \geq \frac{1}{\gamma} \log(-V - \frac{\beta^t}{1-\beta(1-\pi)}) - \frac{1}{\gamma} \log(-V - \frac{1}{1-\beta(1-\pi)})$$

Concavity of the log function can be easily shown to imply that

$$\log(-V - \frac{\beta^t}{1-\beta(1-\pi)}) - \log(-V - \frac{1}{1-\beta(1-\pi)}) > (1-\beta^t) [\log(-V) - \log(-V - \frac{1}{1-\beta(1-\pi)})].$$

so the constraint must bind for t . The proof extends this intuition to show that, if any constraint did not bind, the solution would imply an infeasible choice for that t .

Now we need to show that the solution to the relaxed problem with all the constraints binding will satisfy the other incentive constraints as well. We explain the intuition first. Consider the beginning of

⁵The infinite sum constraint is implied by the actual constraints for large t .

period t , when an agent has not been able to find a job in the previous periods. Fix the saving decisions of agents up to t . At this moment, there are potentially many such types of agents. The truth teller has 0 wealth, and the other deviating types have saved various amounts and thus start with positive wealth. Among them, the type that has never exerted any efforts (later called the all-shirking type) is the richest, since he knew he would achieve this state with probability one, and has saved to prepare accordingly. Without further deviation, these agents consumption behavior is simply to consume the claimed goods, plus the interests payment from the secretly accumulated wealth⁶. Now these agents are contemplating on a one-period deviation, which is to lower a_t from 1 to 0 and follow the suggested strategy $a_s = 1$, for all $s \geq t + 1$. Doing this, all the types have reduced the disutility by the same amount, which is 1. However, they value the transition from employment state (with claim c_{t+1}^E) to unemployment state (with claim c_{t+1}^U) differently. Because they have different wealth levels, and their true consumption is the sum of claimed goods and interests payment of wealth, the reduction in terms of utility is inversely proportional to wealth level. The richer the agent is, the less he suffers from this transition from employment to unemployment state. Thus the truth teller suffers the most, while the all-shirking type suffers the least, and has a comparative advantage in taking this deviation. Therefore, if we know that the all-shirking type would not take this one-step deviation, then the other types are not willing either, since they gain even less.

We call the above story an *ex-post* story, because it is as if the agent considered an ex-post deviation, given the optimal savings rules when deviation was not anticipated. Of course, in this kind of problem, deviation combined with revised savings is typically optimal. An agent will increase his saving if he expects to make more deviations in the future. However, to understand why the logic works nonetheless, consider instead a small reduction in effort. Since all the types have saved appropriate amounts to satisfy their intertemporal Euler equations, the envelope theorem tells us that, for such changes, the benefit to revised saving is second order, and the transition from good to bad state is first order and is the dominant force; we can therefore always say that the all-shirking type has a comparative advantage at shirking by a small amount. As a result, it must be the case that the type that always shirks finds additional shirking most attractive. The following lemmas prove this result formally.

LEMMA 4 *If $0 < U_0 < U_1 < \dots < U_{t+1}$ and $\tilde{a}_t < 1$, then*

$$U_0^{1-\beta} [\tilde{a}_0 \pi E_1 + (1 - \tilde{a}_0 \pi) U_1^{1-\beta} [\tilde{a}_1 \pi E_2 + (1 - \tilde{a}_1 \pi) U_2^{1-\beta} [\dots [\tilde{a}_t \pi E_{t+1} + (1 - \tilde{a}_t \pi) U_{t+1}]^\beta \dots]^\beta]^\beta$$

⁶This depends on the assumption of CARA utilities.

$$\begin{aligned}
& -U_0^{1-\beta}[\tilde{a}_0\pi E_1 + (1 - \tilde{a}_0\pi)U_1^{1-\beta}[\tilde{a}_1\pi E_2 + (1 - \tilde{a}_1\pi)U_2^{1-\beta}[\dots[\pi E_{t+1} + (1 - \pi)U_{t+1}]^\beta \dots]^\beta]^\beta \\
& \geq (\Pi_{s=0}^{t-1}(1 - \pi\tilde{a}_s))(1 - \tilde{a}_t)[\Pi_{s=0}^t U_s^{(1-\beta)\beta^s} U_{t+1}^{\beta^{t+1}} - \Pi_{s=0}^{t-1} U_s^{(1-\beta)\beta^s} U_t^{\beta^t}]
\end{aligned}$$

Proof. First we list an inequality that will be used.

$$(x + a(z - y))^\beta \geq x^\beta + a(z^\beta - y^\beta), \text{ for all } 0 < x \leq y \leq z, 0 \leq a \leq 1. \quad (5)$$

This can be easily shown by the concavity of function x^β . By induction, since $\pi E_{t+1} = U_t - (1 - \pi)U_{t+1}$,

$$\begin{aligned}
& [\tilde{a}_t\pi E_{t+1} + (1 - \tilde{a}_t\pi)U_{t+1}]^\beta \\
& = [\tilde{a}_t(U_t - (1 - \pi)U_{t+1}) + (1 - \tilde{a}_t\pi)U_{t+1}]^\beta \\
& = [U_t + (1 - \tilde{a}_t)(U_{t+1} - U_t)]^\beta \\
& \geq U_t^\beta + (1 - \tilde{a}_t)(U_{t+1}^\beta - U_t^\beta).
\end{aligned}$$

The last inequality follows from inequality (5). We can proceed to the second step,

$$\begin{aligned}
& [\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)U_t^{1-\beta}(U_t^\beta + (1 - \tilde{a}_t)(U_{t+1}^\beta - U_t^\beta))]^\beta \\
& \geq [\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)U_t + (1 - \tilde{a}_{t-1}\pi)(1 - \tilde{a}_t)(U_t^{1-\beta}U_{t+1}^\beta - U_t)]^\beta \\
& \geq [\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)U_t]^\beta + (1 - \tilde{a}_{t-1}\pi)(1 - \tilde{a}_t)(U_t^{(1-\beta)\beta}U_{t+1}^{\beta^2} - U_t^\beta)
\end{aligned}$$

Again, since $\tilde{a}_{t-1}\pi E_t + (1 - \tilde{a}_{t-1}\pi)U_t \leq U_t \leq U_{t+1}$, the last inequality can be justified with inequality (5).

We can use the same technique and prove the lemma by induction. ■

Now we verify that if the all-shirking-up-to- $(t - 1)$ type does not want to shirk one more period, the other types want to set $a_t = 1$ too. With the help the above lemma, this is straightforward to show.

LEMMA 5 *If in a contract, for all $t \geq 0$,*

$$-\frac{\Pi_{i=0}^{t-1}U_i^{(1-\beta)\beta^i}U_t^{\beta^t}}{1 - \beta} - \frac{\beta^t}{1 - \beta(1 - \pi)} \geq -\frac{\Pi_{i=0}^tU_i^{(1-\beta)\beta^i}U_{t+1}^{\beta^{t+1}}}{1 - \beta} - \frac{\beta^{t+1}}{1 - \beta(1 - \pi)},$$

then all finite-deviation incentive constraints are satisfied.

Proof. Consider a finite-deviation strategy $\{\tilde{a}_t\}_{t=0}^\infty$, where $\tilde{a}_t = 1, \forall t \geq \bar{t} + 1$ for some $\bar{t}$. I first show that

setting $\tilde{a}_{\bar{t}} = 1$ and keeping other efforts unchanged will weakly improve an agent's utility, which is

$$\begin{aligned}
& -U_0^{1-\beta}[\tilde{a}_0\pi E_1 + (1 - \tilde{a}_0\pi)U_1^{1-\beta}[\tilde{a}_1\pi E_2 + (1 - \tilde{a}_1\pi)U_2^{1-\beta}[\dots[\tilde{a}_{\bar{t}}\pi E_{\bar{t}+1} + (1 - \tilde{a}_{\bar{t}}\pi)U_{\bar{t}+1}]^\beta \dots]^\beta]^\beta \\
& -[\tilde{a}_0 + \beta(1 - \pi\tilde{a}_0)\tilde{a}_1 + \dots + \beta^{\bar{t}}(\Pi_{s=0}^{\bar{t}-1}(1 - \pi\tilde{a}_s))(\tilde{a}_{\bar{t}} + \beta(1 - \pi\tilde{a}_{\bar{t}})\frac{1}{1 - \beta(1 - \pi)})] \\
\leq & -U_0^{1-\beta}[\tilde{a}_0\pi E_1 + (1 - \tilde{a}_0\pi)U_1^{1-\beta}[\tilde{a}_1\pi E_2 + (1 - \tilde{a}_1\pi)U_2^{1-\beta}[\dots[\pi E_{\bar{t}+1} + (1 - \pi)U_{\bar{t}+1}]^\beta \dots]^\beta]^\beta \\
& -[\tilde{a}_0 + \beta(1 - \pi\tilde{a}_0)\tilde{a}_1 + \dots + \beta^{\bar{t}}(\Pi_{s=0}^{\bar{t}-1}(1 - \pi\tilde{a}_s))(1 + \beta(1 - \pi)\frac{1}{1 - \beta(1 - \pi)})]
\end{aligned}$$

But we know that

$$\begin{aligned}
& U_0^{1-\beta}[\tilde{a}_0\pi E_1 + (1 - \tilde{a}_0\pi)U_1^{1-\beta}[\tilde{a}_1\pi E_2 + (1 - \tilde{a}_1\pi)U_2^{1-\beta}[\dots[\tilde{a}_{\bar{t}}\pi E_{\bar{t}+1} + (1 - \tilde{a}_{\bar{t}}\pi)U_{\bar{t}+1}]^\beta \dots]^\beta]^\beta \\
& -U_0^{1-\beta}[\tilde{a}_0\pi E_1 + (1 - \tilde{a}_0\pi)U_1^{1-\beta}[\tilde{a}_1\pi E_2 + (1 - \tilde{a}_1\pi)U_2^{1-\beta}[\dots[\pi E_{\bar{t}+1} + (1 - \pi)U_{\bar{t}+1}]^\beta \dots]^\beta]^\beta \\
\geq & (\Pi_{s=0}^{\bar{t}-1}(1 - \pi\tilde{a}_s))(1 - \tilde{a}_{\bar{t}})[\Pi_{s=0}^{\bar{t}}U_s^{(1-\beta)\beta^s}U_{\bar{t}+1}^{\beta^{\bar{t}+1}} - \Pi_{s=0}^{\bar{t}-1}U_s^{(1-\beta)\beta^s}U_{\bar{t}}^{\beta^{\bar{t}}}] \\
\geq & (\Pi_{s=0}^{\bar{t}-1}(1 - \pi\tilde{a}_s))(1 - \tilde{a}_{\bar{t}})\frac{\beta^{\bar{t}} - \beta^{\bar{t}+1}}{1 - \beta(1 - \pi)}
\end{aligned}$$

and it implies the inequality we want to show. By backward induction, we can further set $\tilde{a}_{\bar{t}-1} = 1$ and show that the utility is weakly increased. In this way, we proved that setting $\tilde{a}_t = 1, \forall t \geq 0$ is the optimal strategy. ■

5 Comparison of this paper with Hopenhayn and Nicolini (1997)

This is perhaps the most interesting part, we will show that the utility of an unemployed worker declines geometrically in our model with hidden trade, while linearly in Hopenhayn and Nicolini (1997).

PROPOSITION 1 $\frac{U_t}{U_{t-1}}$ is decreasing in t and $\lim_{t \rightarrow \infty} \frac{U_t}{U_{t-1}} > 1$.

Proof. Since $U_t/U_{t-1} = \exp(\gamma x_t)$,

$$\beta^t x_t = \frac{1}{\gamma} \log\left(-V - \frac{\beta^t}{1 - \beta(1 - \pi)}\right) - \frac{1}{\gamma} \log\left(-V - \frac{\beta^{t-1}}{1 - \beta(1 - \pi)}\right)$$

Therefore $\lim_{t \rightarrow \infty} x_t = \frac{(1-\beta)}{\gamma(-V)\beta(1-\beta(1-\pi))} > 0$, $\lim_{t \rightarrow \infty} U_t/U_{t-1} = \exp(\frac{(1-\beta)}{(-V)\beta(1-\beta(1-\pi))}) > 1$. ■

Let us now move to the model of Hopenhayn and Nicolini (1997), where the agent could not privately save or borrow. We still assume that the principal wants to implement high efforts. Let W_t^U denote the promised utilities if the agent has not found a job at the beginning of period t . The principal's problem

can be written recursively using W_t^U as state variable.

$$\begin{aligned} \min_{c_t^U, c_{t+1}^E, W_{t+1}^U} c(W_t^U) &= c_t^U + \beta \left(\pi \frac{c_{t+1}^E - w}{1 - \beta} + (1 - \pi) c(W_{t+1}^U) \right) \\ \text{s.t.} \quad W_t^U &= -U_t - 1 + \beta \left(\pi \frac{-E_{t+1}}{1 - \beta} + (1 - \pi) W_{t+1}^U \right), \\ \beta \pi \left(\frac{-E_{t+1}}{1 - \beta} - W_{t+1}^U \right) &\geq 1. \end{aligned}$$

Hopenhayn and Nicolini (1997) showed that $\{U_t\}_{t=0}^\infty$ is monotonic increasing, $U_t < U_{t+1}$, for all $t \geq 0$. Since we also have inverse Euler equation,

$$\frac{1}{U_t} = \pi \frac{1}{E_{t+1}} + (1 - \pi) \frac{1}{U_{t+1}}.$$

This would imply that $U_{t+1} > U_t > E_{t+1}$, for all $t \geq 0$.

In the optimal solution, the incentive constraint is binding and an agent is indifferent between exerting full and zero effort. We have

$$\begin{aligned} W_t^U &= \sum_{s=0}^{\infty} \beta^s (-U_{t+s}) = -U_t - 1 + \beta \left(\pi \frac{-E_{t+1}}{1 - \beta} + (1 - \pi) W_{t+1}^U \right), \\ W_t^U &= -U_t - \frac{1}{\pi} + \beta \frac{-E_{t+1}}{1 - \beta}. \end{aligned}$$

Thus by substituting into the incentive constraint, we have

$$\left(\frac{-E_{t+1}}{1 - \beta} + U_{t+1} + \frac{1}{\pi} + \beta \frac{E_{t+2}}{1 - \beta} \right) = \frac{1}{\beta \pi}.$$

Notice that the inverse Euler equation gives $E_{t+1} = \frac{\pi U_t U_{t+1}}{U_{t+1} - (1 - \pi) U_t}$, after substitution, we have a second order difference equation in U_t, U_{t+1}, U_{t+2} .

$$\frac{U_t + (U_{t+1} - U_t)}{\pi U_t + (U_{t+1} - U_t)} (U_{t+1} - U_t) - \beta \frac{(1 - \pi) U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})} (U_{t+2} - U_{t+1}) = \frac{(1 - \beta)^2}{\beta \pi}.$$

This difference equation allows us to show that $\{U_t\}$ is increasing linearly.

PROPOSITION 2 $\lim_{t \rightarrow \infty} U_t = \infty$, $\lim_{t \rightarrow \infty} (U_{t+1} - U_t) = \frac{(1 - \beta)^2}{\beta(1 - \beta(1 - \pi))}$.

Proof. Since $\{U_t\}_{t=0}^\infty$ is monotonic, it converges to either ∞ or a finite number. Assume that for some $B > 0$, $\lim_{t \rightarrow \infty} U_t = B$. Then since an agent could never exert effort from t onward, $\lim_{t \rightarrow \infty} W_t^U \geq \frac{-B}{1 - \beta}$. Also by the inverse Euler equation, $\lim_{t \rightarrow \infty} E_t = B$. Therefore, then incentive constraints $\beta \pi \left(\frac{-E_{t+1}}{1 - \beta} - W_{t+1}^U \right) \geq 1$ would be violated for large t .

Secondly we show that $\limsup_{t \rightarrow \infty} (U_{t+1} - U_t) < \infty$. Since $\sum_{s=0}^{\infty} \beta^s U_{t+1+s} - \frac{E_{t+1}}{1-\beta} = \frac{1}{\beta\pi}$, and $U_t < U_{t+1}$, for all $t > 0$, we have $U_{t+1} - E_{t+1} \leq \frac{1-\beta}{\beta\pi}$. Therefore, $U_{t+1} - U_t \leq U_{t+1} - E_{t+1}$ is bounded.

The above difference equation could be written as

$$(U_{t+2} - U_{t+1}) = \frac{\frac{U_t + (U_{t+1} - U_t)}{\pi U_t + (U_{t+1} - U_t)} (U_{t+1} - U_t) - \frac{(1-\beta)^2}{\beta\pi}}{\frac{\beta(1-\pi)U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})}},$$

or,

$$\begin{aligned} & (U_{t+2} - U_{t+1}) - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \\ = & \frac{\frac{U_t + (U_{t+1} - U_t)}{\pi U_t + (U_{t+1} - U_t)}}{\frac{\beta(1-\pi)U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})}} \left[(U_{t+1} - U_t) - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right] + \\ & \frac{\left(\frac{U_t + (U_{t+1} - U_t)}{\pi U_t + (U_{t+1} - U_t)} + \frac{\beta(1-\pi)U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})} \right) \left[\frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right] - \frac{(1-\beta)^2}{\beta\pi}}{\frac{\beta(1-\pi)U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})}}. \end{aligned}$$

For any small $\epsilon > 0$, there is a big t , such that U_t is large enough to guarantee that

$$\begin{aligned} & \frac{\frac{U_t + (U_{t+1} - U_t)}{\pi U_t + (U_{t+1} - U_t)}}{\frac{\beta(1-\pi)U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})}} \geq \frac{1}{\beta(1-\pi)} - \epsilon \\ & \left| \frac{\left(\frac{U_t + (U_{t+1} - U_t)}{\pi U_t + (U_{t+1} - U_t)} + \frac{\beta(1-\pi)U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})} \right) \left[\frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right] - \frac{(1-\beta)^2}{\beta\pi}}{\frac{\beta(1-\pi)U_{t+1}}{\pi U_{t+1} + (U_{t+2} - U_{t+1})}} \right| \leq \epsilon^2. \end{aligned}$$

So if $\left| U_{t+1} - U_t - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right| \geq \epsilon$,

$$\begin{aligned} & \left| U_{t+2} - U_{t+1} - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right| \\ \geq & \left(\frac{1}{\beta(1-\pi)} - \epsilon \right) \left| U_{t+1} - U_t - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right| - \epsilon^2 \\ \geq & \frac{\frac{1}{\beta(1-\pi)} + 1}{2} \left| U_{t+1} - U_t - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right| \\ \geq & \epsilon. \end{aligned}$$

By induction, $\left| U_{t+2} - U_{t+1} - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right|$ will grow at an exponential rate and it is a contradiction to the fact that $(U_{t+1} - U_t)$ is bounded. Therefore when U_t is large enough, $\left| U_{t+2} - U_{t+1} - \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))} \right| \leq \epsilon$. Thus $\lim_{t \rightarrow \infty} (U_{t+1} - U_t) = \frac{(1-\beta)^2}{\beta(1-\beta(1-\pi))}$. ■

Both rates depend on $\frac{1-\beta}{1-\beta(1-\pi)}$, which is a decreasing function of the discount factor and the job-finding probability. Since the total discounted disutility is $\frac{1}{1-\beta(1-\pi)}$, we can interpret this as the total amount of

incentives that the principal needs to provide by distortion. The per period distortion is $(1-\beta) \times \frac{1}{1-\beta(1-\beta\pi)}$. The declining rate of utility is a measure of the per period distortion, and thus is positively related to this term.

There are two key differences between the declining patterns in our paper and Hopenhayn and Nicolini (1997). First our model implies geometric decline, rather than linear decline. Second, the rate in our model depends on the level of initial promised utility, while Hopenhayn and Nicolini (1997)'s rate only depends on the discount factor β and job-finding probability π .

With geometric speed, the utility of an unemployed worker will eventually decline faster with hidden trade than without. The result is a bit surprising and counter-intuitive. When a worker can participate in the hidden trade, he has more means of protecting himself, in the sense that he could privately save and prevent his consumption from dropping too fast. Put it differently, when an agent is faced with a contract in Hopenhayn and Nicolini (1997), he is able to make his utility decrease more slowly than what is indicated in the contract. One would be tempted to conjecture that with hidden trade, a worker's utility will therefore decrease less drastically than without hidden trade. In particular, Kocherlakota found that an unemployed worker's utility should not fall at all.

Our results show the opposite. We believe that this comes from the follow consideration of the principal. Since the possibility of the agent's participating in hidden trade is fully understood by the principal, he realizes that if he still offers the same contract as Hopenhayn and Nicolini (1997) does, a cheating agent is able to gain by first shirking and then saving. The cheating type is not punished severe enough to make the contract incentive compatible. Therefore, the principal is forced to punish the unemployed agent more severely than what is required in Hopenhayn and Nicolini (1997). The very fact that hidden trade could protect the agents makes them suffer more, once the principal anticipates and internalizes this effect.

The result contrasts with the case studied by Kocherlakota (2004). In a sense, both papers get their results by mechanical forces. Kocherlakota forces the principal to implement interior effort levels. In that case, it is impossible to move away from constant unemployment benefits; any attempt to make benefits fall with the duration of unemployment would be met by a joint deviation of saving and shirking. In our model, we force the principal to satisfy the Euler equation of the agent by our assumption of hidden trade. Given that a higher consumption after employment is needed for incentives in the previous period, the consumption of an agent after not finding a job has to fall. This imposition of the Euler equation has an even stronger implication, that Kocherlakota's interior effort levels is not implementable. In other words, it seems that we force the principal to implement a corner of effort levels.

However, there is a sense in which the hidden trade assumption is not as strong relative to hidden savings as it first appears. In the appendix, we show that, for non-interior effort levels, if we study a hidden saving environment, we obtain the same optimal contract. In other words, the principal would never distort the Euler equation in a hidden-saving environment, even if he could do so in one direction (as is possible with hidden savings). Only for the case of interior effort levels would the principal deviate from the Euler equation⁷.

The second difference is that our rate $\frac{(1-\beta)}{(-V)\beta(1-\beta(1-\pi))}$ depends on V , which we believe highlights the role played by hidden saving. Recall that

$$\sum_{t=1}^{\infty} \beta^t x_t = \frac{1}{\gamma} \log \left(\frac{-V}{-V - \frac{1}{1-\beta(1-\pi)}} \right).$$

Each term x_t would be zero if the principal does not induce effort in period $t - 1$. Thus x_t is a measure of distortion at period $t - 1$. The sum on the left hand side can be interpreted as the total distortion introduced in the model. As the above equation shows, it would depend positively on promised utility V . The lower the promise is, the more an agent cares about declining consumption rather than exerting effort, since the disutility of effort is fixed and the marginal utility of consumption is high for agents who consume little. Therefore the principal could better manipulate the consumption to compensate searching efforts, thus making it easier for the principal to provide incentives with less distortion introduced.

Our optimal contract also generates interesting consumption patterns for unemployed agents with different utility promises. Consider two agents a and b with the same observable initial wealth 0 but different outside options. The principal provides different initial promised utility V^a and V^b , with $0 > V^a > V^b$. We say that agent b is initially poorer, because the initial consumption goods he receives is lower. However, since it is harder to provide incentive to the rich agent, the contract specifies a faster declining rate for consumption for a , and eventually, agent a 's utility will be below agent b 's. If neither of the agents has found a job after a long period of time, the initially rich agent will eventually be poorer. This interesting pattern is not in a model without hidden trade. There monotonicity is maintained; since all agents' utility declines at the same linear rate, if an agent starts with higher promised utility, his future utility is always higher.

⁷Note that, in terms of resources used to implement the contract, the principal in Kocherlakota (2004) is indifferent between the interior effort and high effort, given the distorted Euler equation. Therefore, by moving to high effort and satisfying the Euler equation, the principal could save resources.

In a model without hidden trade, the rich agent a and poor agent b have the same disutility 1 in each period, and thus to provide incentive of exerting effort, the principal needs to lower both types' utility by the same amount.

$$U_{t+1}^a - U_t^a = U_{t+1}^b - U_t^b > 0.$$

The amount is chosen so that both types feel indifferent between using efforts and shirking. The situation is different when agents have access to hidden trade, where the punishment is stronger for agent a in both the absolute($U_{t+1} - U_t$) term and relative($\frac{U_{t+1}}{U_t}$) term. To keep matters simple, we look at the initial period first. It is easy to see in the relative term, suppose

$$\frac{U_1^a}{U_0^a} = \frac{U_1^b}{U_0^b} > 1.$$

Both agents experience the same reduction in the claim of consumption goods in period 1. But since $U_0^a < U_0^b$, and agent a has lower marginal utility, he suffers less. In order to keep incentive compatibility, we must have $\frac{U_1^a}{U_0^a} > \frac{U_1^b}{U_0^b}$. This stronger punishment is also true in the absolute term,

$$U_1^a - U_0^a > U_1^b - U_0^b > 0.$$

By contradiction, if $U_1^a - U_0^a = U_1^b - U_0^b$, then before they adjust their savings, both agents suffer the same reduction in utility and agent a suffers more in terms of the reduction in consumption. However, a rich agent always gains more benefit from revised saving. This is easy to see in an extreme case, where agent a starts in the first period with $c_0^{U,a} = \infty, U_0^a = 0$. Then any reduction in agent a 's period 1's utility will be fully compensated with a tremendous amount of hidden saving, since he is so rich in period 0 as if he had no resource constraint. Now consider periods $t \geq 1$. At the beginning of t , since an all-shirking-up-to- t type starting with promise V^a has always saved more than an all-shirking-up-to- t type starting with V^b , the above mentioned reasons for stronger punishment for a will only be reinforced.

One interesting fact is worth mentioning, for large t , although a truth teller from V^a is poorer than that from V^b , the all-shirking-up-to- t type from V^a is always richer. In fact, since an all-shirking-up-to- t type from V^a always obtains an ex-ante utility equivalent to a truth teller, he has secretly saved so much wealth so that even if he faces lower future consumption claims, he is able to live better than an all-shirking-up-to- t type from V^b . In this sense, our results is consistent with the monotonicity in Hopenhayn and Nicolini (1997): monotonicity is maintained for the wealth levels that are most likely to consider one deviation. In Hopenhayn and Nicolini (1997), there is trivially just the zero wealth level, and that agent is indifferent between shirking and giving effort at $t + 1$. Here the critical wealth level belongs to the all-shirking-up-to- t type, who is ex-ante indifferent between his strategy and one more time period of shirking.

6 Comparison of this paper with Shimer and Werning (2005)

At the beginning of period t , we consider 2 types of agents, one finds a job at period t , and the other finds a job at $t+1$. Relative to the first type, the second type has an additional bad unemployment shock. We compare the unemployment benefit(discounted to t) ex-post. The subsidy to the first type is $\frac{c_t^E - w}{1-\beta}$, while it is $c_t^U + \beta \frac{c_{t+1}^E - w}{1-\beta}$ for the second type. The subsidy to the second type is generally higher, and the difference is

$$c_t^U + \beta \frac{c_{t+1}^E - w}{1-\beta} - \frac{c_t^E - w}{1-\beta}.$$

We can think of the above as the insurance for the unemployment risk that an agent face at period t . It is well known that in Shimer and Werning (2005), this is a constant. In our model, it is strictly increasing.

PROPOSITION 3 $c_t^U + \beta \frac{c_{t+1}^E - w}{1-\beta} - \frac{c_t^E - w}{1-\beta}$ is increasing in t .

Proof. We know from previous results that $x_t > x_{t+1} > 1$, for all $t \geq 1$. Since $\frac{U_{t+1}}{U_t} = \exp(\gamma x_{t+1})$ is decreasing in t and we have

$$1 = \pi \frac{E_{t+1}}{U_t} + (1-\pi) \frac{U_{t+1}}{U_t}.$$

We know that $\frac{E_{t+1}}{U_t}$ is increasing. We show first that $c_t^E - c_t^U$ is decreasing, because

$$\begin{aligned} \frac{U_t}{E_t} &= \frac{U_t}{U_{t-1}} \frac{U_{t-1}}{E_t} \text{ is decreasing.} \\ c_t^E - c_t^U &= \frac{1}{\gamma} \log\left(\frac{U_t}{E_t}\right). \end{aligned}$$

Then $c_t^E - c_{t+1}^E$ is decreasing because

$$\begin{aligned} \frac{E_{t+1}}{E_t} &= \frac{U_t - (1-\pi)U_{t+1}}{U_{t-1} - (1-\pi)U_t} \\ &= \frac{\exp(\gamma x_t) - (1-\pi)\exp(\gamma x_t)\exp(\gamma x_{t+1})}{1 - (1-\pi)\exp(\gamma x_t)} \\ &= \exp(\gamma x_t) \left(1 + (1-\pi)\exp(\gamma x_{t+1}) \frac{\exp(\gamma(x_t - x_{t+1})) - 1}{1 - (1-\pi)\exp(\gamma x_t)}\right). \end{aligned}$$

Since $\exp(\gamma x_t)$, $(x_t - x_{t+1})$, $\frac{1}{1-(1-\pi)\exp(\gamma x_t)}$ are all decreasing, we know that $\frac{E_{t+1}}{E_t}$ is decreasing. Now we know that

$$\begin{aligned} &c_t^U + \beta \frac{c_{t+1}^E - w}{1-\beta} - \frac{c_t^E - w}{1-\beta} \\ &= w - (c_t^E - c_t^U) - \frac{\beta(c_t^E - c_{t+1}^E)}{1-\beta} \end{aligned}$$

is increasing in t . ■

7 Concluding Comments

Our paper is a first step toward solving the mechanism design problems with hidden trade. Since it suggests that the richest type plays a role, if this type of problem could be studied recursively, an artificial type needs to be added to the state space in addition to the promised utility of the truth-teller. To figure out how to do this exactly is the task of future research.

8 Appendix I

We will show that implementing high efforts is optimal. In order to do this, we need to consider the cost of implementing an arbitrary effort sequence of 0s and 1s. Let $\{a_t\}_{t=0}^\infty$, $a_t = 0$ or 1 , be an effort sequence that the principal wants to implement. Let $\{n_k\}_{k=1}^\infty$ denote the time periods where high effort is recommended in the previous period, i.e.

$$\begin{aligned} a_{n_k-1} &= 1, k \geq 1, \\ a_t &= 0, n_k - 1 < t < n_{k+1} - 1. \end{aligned}$$

The first order condition would be written as

$$\begin{aligned} U_{n_k-1} &= \pi E_{n_k} + (1 - \pi)U_{n_k}, \\ U_{t-1} &= U_t, n_k < t < n_{k+1}. \end{aligned}$$

In order to prevent the agent from using effort when $t \neq n_k - 1$, the principal can set $E_{t+1} = -\infty$. We will proceed with the same method as in the case of implementing high efforts. Define $x_t = c_{t-1}^U - c_t^U$. Then $x_{n_k} > 0$, and $x_t = 0, n_k < t < n_{k+1}$. The disutility sequences satisfy $D_{n_k} = \beta^{n_{k+1}-1} + (1 - \pi)D_{n_{k+1}}$. The relaxed problem is

$$\begin{aligned} \min_{\sigma} \quad & C(\sigma) \\ s.t \quad & U_{n_k-1} = \pi E_{n_k} + (1 - \pi)U_{n_k}, \forall k \geq 1, \\ & V = -\frac{U_0}{1 - \beta} - D_0, \\ & V \geq -\frac{U_0^{1-\beta^{n_1}} \prod_{s=1}^{k-1} U_{n_s}^{\beta^{n_s} - \beta^{n_{s+1}}} U_{n_k}^{\beta^{n_k}}}{1 - \beta} - D_{n_k}, \forall k \geq 1. \end{aligned}$$

It can be rewritten as

$$\begin{aligned} \max_{x_{n_k}} \quad & \sum_{k=1}^{\infty} \frac{\beta^{n_k} (1-\pi)^{k-1}}{1-\beta} f(x_{n_k}) \\ \text{s.t.} \quad & \sum_{s=1}^k \beta^{n_s} x_{n_s} \geq \log(-V - D_{n_k}) - \log(-V - D_0), \forall k \geq 1. \end{aligned}$$

LEMMA 6 Under assumption ?, all the constraints in the above problem bind.

Proof. The proof is similar to a previous lemma. Let k^* be the smallest k such that

$$\begin{aligned} \beta^{n_k} x_{n_k} &> \log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}}), \\ \beta^{n_s} x_{n_s} &= \log(-V - D_{n_s}) - \log(-V - D_{n_{s-1}}), s < k. \end{aligned}$$

We want to show that $\beta^{n_{k^*+1}} x_{n_{k^*+1}} > \log(-V - D_{n_{k^*+1}}) - \log(-V - D_{n_{k^*}})$. Since we still have the first-order condition

$$f'(x_{n_k}) - (1-\pi)f'(x_{n_{k+1}}) = 0,$$

and x_{n_k} is a small number, we have

$$\begin{aligned} x_{n_{k+1}} &> \frac{1}{1-\beta\pi} x_{n_k} \\ &> \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{(1-\beta\pi)\beta^{n_k}} \\ &= \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{D_{n_{k-1}} - D_{n_k}} \cdot \frac{D_{n_{k-1}} - D_{n_k}}{(1-\beta\pi)\beta^{n_k}} \\ &= \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{D_{n_{k-1}} - D_{n_k}} \cdot \frac{\beta^{n_k-1} - \pi(\beta^{n_{k+1}-1} + (1-\pi)D_{n_{k+1}})}{(1-\beta\pi)\beta^{n_k}} \\ &\geq \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{D_{n_{k-1}} - D_{n_k}} \cdot \frac{\beta^{n_{k+1}-1} - \pi(\beta^{n_{k+1}} + \beta(1-\pi)D_{n_{k+1}})}{(1-\beta\pi)\beta^{n_{k+1}}} \\ &= \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{D_{n_{k-1}} - D_{n_k}} \cdot \frac{1 - \beta\pi(1 + \beta(1-\pi)(\beta^{-n_{k+1}} D_{n_{k+1}}))}{(1-\beta\pi)} \cdot \frac{\beta^{n_{k+1}-1}}{\beta^{n_{k+1}}} \end{aligned}$$

Since $(\beta^{-n_{k+1}} D_{n_{k+1}}) \in [0, \frac{1}{1-\beta(1-\pi)}]$, we know that

$$(1-\beta\pi) \leq \frac{1 - \beta\pi(1 + \beta(1-\pi)(\beta^{-n_{k+1}} D_{n_{k+1}}))}{1 - \beta\pi(\beta^{-n_{k+1}} D_{n_{k+1}})} \leq 1.$$

Therefore,

$$\begin{aligned}
x_{n_{k+1}} &> \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{D_{n_{k-1}} - D_{n_k}} \cdot (1 - \beta\pi(\beta^{-n_{k+1}} D_{n_{k+1}})) \cdot \frac{\beta^{n_{k+1}-1}}{\beta^{n_{k+1}}} \\
&= \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{D_{n_{k-1}} - D_{n_k}} \cdot \frac{\beta^{n_{k+1}-1} - \pi D_{n_{k+1}}}{\beta^{n_{k+1}}} \\
&= \frac{\log(-V - D_{n_k}) - \log(-V - D_{n_{k-1}})}{D_{n_{k-1}} - D_{n_k}} \cdot \frac{D_{n_k} - D_{n_{k+1}}}{\beta^{n_{k+1}}} \\
&> \frac{\log(-V - D_{n_{k+1}}) - \log(-V - D_{n_k})}{D_{n_k} - D_{n_{k+1}}} \cdot \frac{D_{n_k} - D_{n_{k+1}}}{\beta^{n_{k+1}}} \\
&> \frac{\log(-V - D_{n_{k+1}}) - \log(-V - D_{n_k})}{\beta^{n_{k+1}}}.
\end{aligned}$$

Therefore, by induction, we have for all $s \geq k^* + 1$, $\beta^{n_s} x_{n_s} > \log(-V - D_{n_s}) - \log(-V - D_{n_{s-1}})$, which would jointly imply

$$\sum_{s=1}^{\infty} \beta^{n_s} x_{n_s} > \log(-V) - \log(-V - D_0).$$

■

Since we show that the solution to the relaxed problem has all the constraints binding, we know that they are incentive compatible in the original problem. All we need to show in the rest is that this solution will have a lower objective than that from implementing high efforts.

Let k^* be the largest k such that $n_k = k$. Since $n_s = s, \forall s \leq k^*$, we have $a_t = 1$, for $t \leq n_{k^*} - 1$ and $a_{n_{k^*}} = 0$. We want to show that setting $a_{n_{k^*}} = 1$ will lower the cost. The old cost is

$$\frac{c_0^U}{1 - \beta} - \sum_{k=1}^{\infty} \frac{\beta^{n_k} (1 - \pi)^{k-1}}{1 - \beta} f(x_{n_k}) - D_0 \pi w$$

We add tildes to denote variables after this change.

$$\begin{aligned}
\tilde{D}_t &= D_t + (1 - \pi)^{k^* - t} (\beta^{k^*} - \pi D_{k^*}), 0 \leq t \leq k^*, \\
\tilde{D}_t &= D_t, t > k^*, \\
\tilde{x}_t &< x_t, t \leq k^*, \\
\tilde{x}_{k^*+1} &> x_{k^*+1} = 0, \\
\tilde{x}_t &= x_t, t > k^* + 1, \\
\tilde{c}_0^U &= -\log((1 - \beta)(-V - D_0))/\gamma, \\
\tilde{c}_0^U &= -\log((1 - \beta)(-V - \tilde{D}_0))/\gamma,
\end{aligned}$$

The new objective is

$$\begin{aligned} \frac{\tilde{c}_0^U}{1-\beta} - \sum_{k=1}^{k^*} \frac{\beta^k(1-\pi)^{k-1}}{1-\beta} f(\tilde{x}_k) - \frac{\beta^{k^*+1}(1-\pi)^{k^*}}{1-\beta} f(\tilde{x}_{k^*+1}) \\ - \sum_{k=k^*+1}^{\infty} \frac{\beta^{n_k}(1-\pi)^k}{1-\beta} f(x_{n_k}) - \tilde{D}_0 \pi w \end{aligned}$$

It is easily seen that when V is low, the new objective is lower. (Details will be added.)

9 Appendix II

We will show that in an environment with hidden saving but not hidden borrowing, if the principal wants to implement positive efforts in all the periods, the optimal contract to achieve this goal is the same as the contract we derived in this paper. If the planner wants to implement $a_t \in (0, 1)$, the agent is indifferent between effort a_t and 1, due to the linear disutility, and the planner is indifferent as well. Without loss of generality, we could assume that the planner wants to implement high efforts always. Of course, hidden saving is closer to reality than hidden trade because burying some potatoes in an agent's backyard is always much harder to monitor than borrowing, where he has to go to a bank to sign a loan contract, or at least go to his neighborhood to take the cash.

With hidden saving, principal has more freedom in designing the contract, for he can easily distort the agent's first order condition and let the agent be borrowing-constrained. We will show that although the principal could do this, it is never optimal.

We first show that after an agent finds a job, it is not optimal to distort the first order condition. For convenience, let $c_{t,s}^E, s \geq t$ be the period- s consumption of an employed agent if he finds a job at period of t . By contradiction, suppose for some s ,

$$c_{t,s}^E < c_{t,s+1}^E$$

Then we could modify the contract by having $\epsilon > 0, \delta > 0, \delta < \beta\epsilon$, such that $\tilde{c}_{t,s}^E = c_{t,s}^E + \delta, \tilde{c}_{t,s+1}^E = c_{t,s+1}^E - \epsilon, u(\tilde{c}_{t,s}^E) + \beta u(\tilde{c}_{t,s+1}^E) = u(c_{t,s}^E) + \beta u(c_{t,s+1}^E)$. This modification will

- (1) Save resource for the planner.
- (2) Truth-teller would not hidden save in the new contract.
- (3) All the deviating types might revise their savings, but will be worse off.

Secondly we show that $E^{t+1} < U^{t+1}$. By contradiction, suppose $-E_{t+1} \leq -U_{t+1}$. Under the hidden saving assumption, $-U_t \leq \pi(-E_{t+1}) + (1 - \pi)(-U_{t+1})$, so we have

$$\frac{-E_{t+1}}{1 - \beta} \leq \sum_{s=t+1}^{\infty} \beta^{s-t-1} (1 - \pi)^{s-t-1} (-U_s + \frac{\beta\pi(-E_{s+1})}{1 - \beta}),$$

where the right hand side is the discounted utility of consumption from history U_{t+1} to the future. Then the contract is not incentive compatible for the agent to use positive efforts, since the gain from one unit of effort is

$$\beta\pi \left(\frac{-E_{t+1}}{1 - \beta} - \sum_{s=t+1}^{\infty} \beta^{s-t-1} (1 - \pi)^{s-t-1} (-U_s + \frac{\beta\pi(-E_{s+1})}{1 - \beta}) \right) + \beta\pi \left(\frac{1}{1 - \beta(1 - \pi)} \right) < 1.$$

Thirdly, we show that the Euler equation is never distorted for the truth-teller. By contradiction, assume that for some t ,

$$-U_t < \pi(-E_{t+1}) + (1 - \pi)(-U_{t+1}).$$

Then we could modify the contract by having $\epsilon > 0, \delta > 0, \delta < \beta\epsilon$, such that $\tilde{c}_t^U = c_t^U + \delta$, $\tilde{c}_{t+1}^E = c_{t+1}^E - \epsilon$, $\tilde{c}_{t+1}^U = c_{t+1}^U - \epsilon$, and ϵ and δ are chosen so that the truth-teller is indifferent,

$$-\tilde{U}_t + \beta[\pi(-\tilde{E}_{t+1}) + (1 - \pi)(-\tilde{U}_{t+1})] = -U_t + \beta[\pi(-E_{t+1}) + (1 - \pi)(-U_{t+1})].$$

This modification will

- (1) Save resource for the planner.
- (2) Truth-teller would not hidden save in the new contract.
- (3) All the deviating types are worse off. We need to imagine a person that potentially start with positive wealth and use some future strategies that involves shirking. Now we fix his effort strategies from period t onward and look at the change in his welfare after the modification. If the agent saves in period t in the old contract, then he suffers because his total income is lowered. If he does not save, then compared to the truth teller,
 - (a) His utility gain in period t is weakly less than the truth-teller because he start with nonnegative wealth.
 - (b) His utility loss conditional on E_{t+1} is the same as the truth-teller. His utility loss conditional on U_{t+1} is weakly bigger because he might save in the old contract.

- (c) Since he might shirk in period t , his probability of moving to U_{t+1} is weakly bigger than $(1 - \pi)$, which makes him suffer more than the truth-teller.

To summarize, if the principal distorts the first order condition, he could lower the degree the distortion and save resource.

Lastly, the first order condition of a deviator is not distorted as well. Since $-U_t = \pi(-E_{t+1}) + (1 - \pi)(-U_{t+1})$ and $-E_{t+1} > -U_{t+1}$. Whenever the agent shirk ($a_t < 1$), we have

$$-U_t > a_t \pi(-E_{t+1}) + (1 - a_t \pi)(-U_{t+1}).$$

Therefore, with hidden saving, the deviator always choose his consumption to satisfy his first order condition.

Thus from the principal's view point, the first order condition should always be satisfied, and the solution is no different from our model where an agent could both save and borrow.

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