

Sovereign Debt and The Tragedy of the Commons

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(beta version)

Abstract

Emerging markets borrow substantial amounts of foreign debt. Most of what they borrow, they eventually repay even though there are no clear punishments available to creditors. This paper provides a possible answer to why countries repay foreigners in the absence of direct punishments. The paper studies a simple tragedy of the commons model. The model assumes a country composed of different groups (or regions) that compete for access to government resources. The government can save and borrow from foreigners. This tragedy of the commons model has been used to explain why countries overspend and undersave. The contribution of the present paper is to show that the same economic forces that generate overspending can guarantee as well that a small open economy repays its sovereign obligations. Hence, the model can explain why emerging markets do not save and why they repay their sovereign obligations. A country repays to avoid having its assets inefficiently spent by the groups. However, the basic reason for repayment is the same as in Eaton and Gersovitz: countries repay because they would like to borrow again.

Introduction

Emerging markets borrow substantial amounts of foreign debt. Most of what they borrow, they eventually repay even though there are no clear punishments available to creditors. This paper provides an answer to why these countries repay their debts.

A simple dynamic “tragedy of the commons” model is analyzed: a country is composed of different groups with common access to a savings technology. Dynamic common-pool models of this type have been used in the public debt literature to explain why countries over-spend and under-save. The economic logic behind is as follows: different groups inside a country tend to demand too much spending from the government because each of them enjoys the benefits privately but they all share the costs. A free-rider problem ensues and as a result, countries will tend to spend too much and save too little.

The main contribution of this paper is to show that the same economic forces that generate overspending in a tragedy of the commons model can guarantee that a small open economy repays its sovereign obligations. It is also shown that the reason why it repays is because the country would like to borrow again. Hence, a tragedy of the commons model can explain why emerging markets do not save and why they repay their debts.

This reputational argument for debt repayment was first formalized by Eaton and Gertsovitz (1981) in the context of a small country subject to income shocks: a defaulting country loses access to international credit markets and won't be able to smooth its consumption profile after default. This threat of financial exclusion might be strong enough to enforce repayment. However, Bulow and Rogoff (1989) demonstrated that this argument for repayment breaks down if countries could save. In particular, by defaulting and using the asset markets to self-insure, a country achieves a higher consumption at any point after default. Hence default is an arbitrage.

In the model presented in this paper, the country has the assets available for savings. A weak form of the Bulow and Rogoff result holds: for any debt contract there exists a feasible allocation after default generated only with savings whose payoffs to all domestic groups Pareto dominate the allocation under repayment. In other words, after default there exists a feasible allocation that improves the utility to all parties. The existence of this allocation is usually enough to show that a domestic government would prefer it and default will result. However, in a dynamic tragedy of the commons game where multiple groups inside the country are involved in the decision making, those allocations might be feasible but not necessarily the result of equilibrium.

Why does the country repay? The domestic groups realize that if they were to default, they will inefficiently overspend and save too little even though they might all benefit from savings. External debt reduces this inefficiency. By granting access to funds in states where the country really needs them, while curtailing them in times when they are not, the foreigners in effect generate a commitment technology that at all times can be preferred to default. It is also in the interest of the foreigners to generate this commitment, because it is what guarantees that they get repaid.

Related Literature

Tornell and Velasco (1992) and Lane and Tornell (1999) studied a tragedy of the commons model in a capital accumulation game. The basic model of the present paper is based on these models. However, the issues and techniques differ significantly. The issue of international debt repayment, the issue of interest here, is not a concern in those cited papers. Also, these

papers characterize Markov equilibria, whereas subgame perfect equilibria are the object of study in the present paper.

Several explanations of why countries repay their debts have been proposed in the literature. Researchers have studied the possibility that reputation spillovers to other valuable relationships might be costly enough to enforce repayment (Cole and Kehoe (1995), (1996)). Another approach looks at the assets available to the country after default: technological restrictions (Kletzer and Wright (2000)) or collusion among banks (Wright (2002)) might reduce the range of savings mechanisms available to the country after default. Another branch of the literature studies the punishments available to creditors, from military intervention to trade embargoes.¹²

The paper relates to the political economy literature on fiscal deficits (Alesina and Tabellini (1990), Persson and Svensson (1989)). However, these papers do not consider the possibility of default. Tabellini (1991), and Dixit and Londregan (2000) present models of sustainability of domestic debt. In these models, the lenders are citizens and thus have political rights (they can vote). I analyze a model of *sovereign* debt, where lenders reside outside the country and have no political rights.

The paper is related to recent work by Gul and Pesendorfer (2002). In their work, the authors develop a theory of preferences for commitment, providing a modeling alternative to the standard hyperbolic discounting framework. They study a consumer with these preferences and show how Bulow and Rogoff (1989)'s result might be overturned. Their model does not connect to political economy, as this paper does; and hence does not have clear empirical predictions for sovereign debt.

The closest paper to the present is Amador (2004). That paper also argues that political considerations might restrain a country from implementing the saving sequence that the Bulow and Rogoff argument requires. However, that paper focused in a model of political turnover and only considered cases where the domestic savings use Markov strategies. This paper generalizes that intuition to a very different type of political economy model: a tragedy of the commons model. Also, and importantly, the assumption of Markov domestic agent is completely relaxed in this paper, and a full blown model of cooperation is studied: strategies are not restricted in an ad-hoc way. In particular, this paper studies a tragedy of the commons model where domestic groups can develop reputation in a repeated game context and save for themselves.³ It is shown, however, that this *domestic reputation* might not generate sufficient savings. This under-savings result generates the possibility that the

¹Rose (2002) has shown that after a country defaults, its international trade is significantly reduced, identifying a channel through which external creditors might be punishing the defaulting country.

²Sandleris (2005) presents a different argument for repayment, based on asymmetric information.

³Most of the literature studies the commons problem game under Markov strategies

domestic agents pay back their debts even in the absence of punishments or political costs.

The sequence of the paper is as follows. First, I setup the model without debt in section 1. The model consists of a small economy with different political groups. I define the equilibrium and characterize some of its main properties. In section 2, I introduce the possibility of borrowing from foreigners and characterize issue of debt sustainability: I characterize under which conditions the country repays its debts. I show that the argument of Bulow-Rogoff does not in general hold except in the particular case when there is one party, or the parties are infinitely patient. The final section concludes.

1 The Savings Game

In this section a simple model is introduced. It is assumed throughout this section that the country cannot borrow, but it can save. This assumption will be relaxed and the issue of borrowing and debt repayment studied in the following section.

Times is discrete and runs to infinity. There are n symmetric groups that live forever and a government. The government in the country can save using a linear technology that returns a risk-free rate of R . Denote by a_t the amount saved by the government at the end of period t . The government cannot borrow, $a_t \geq 0$, and it is assumed to have no assets initially, $a_0 = 0$. Additionally to its income from previous savings, the government receives an endowment e_t at the beginning of every period t .

The groups receive fiscal provisions from the government. Let the instantaneous utility flow of group i of receiving d_t^i from the government be given by the following linear function,

$$u_t^i(d_t^i) = \phi_t d_t^i$$

The aggregate parameter ϕ_t captures the benefit to the domestic groups of government provision at any time t . Gains from savings are introduced in this linear model through variation in ϕ_t . Note however, that it is assumed that all domestic groups are affected by ϕ in the same way: they all agree on the periods where government spending is relatively more valuable. The parameter ϕ characterizes events that affect all parties in the country in similar fashion: it can represent a natural disaster, a low aggregate productivity state, or a war.

All groups discount the utility flows generate by government provision exponentially at the same rate β . The following restriction on β and R is used to bound the payoffs to the parties below infinity,

Condition 1. *The parameters β and R are such that $\beta R < 1$.*

A fiscal allocation is defined to be a non-negative and non-stochastic sequence of fiscal demand vectors, $d = \{(d_t^1, \dots, d_t^n)\}_{t=1}^\infty$. Let $a(d) = \{a_{t-1}(d)\}_{t=1}^\infty$ be a sequence of assets positions generate by d with $a_0 = 0$, so that

$$a_t(d) = Ra_{t-1}(d) + e_t - \sum_{i=1}^n d_t^i$$

for all $t > 0$. An allocation is feasible is the asset positions generated by it are always positive,

Definition 1. *We say that a fiscal demand allocation d is **feasible** if the asset positions generated by d are positive, $a_t(d) \geq 0$ for all t .*

A desire for savings

A desire for savings in this linear set up is introduced by specifying particular processes for ϕ_t and e_t . We assume that ϕ_t and e_t behave according to the following deterministic rules:

$$\phi_t = \begin{cases} 1 & ; \text{for } t \bmod 3 = 1 \\ \phi & ; \text{for } t \bmod 3 = 2 \\ \phi^2 & ; \text{for } t \bmod 3 = 0 \end{cases}$$

and

$$e_t = \begin{cases} 1 & ; \text{for } t \bmod 3 = 1 \\ 0 & ; \text{otherwise} \end{cases}$$

for $\phi > 1$. The government receives the endowment in the state where consumption is less valuable to all parties: when $\phi_t = 1$. The following condition guarantees that to save the endowment is efficient.

Condition 2. *The parameters are such that $\beta R\phi > 1$.*

Let a first best allocation be an allocation that is Pareto optimal in the feasible set: there does not exists another feasible allocation that weakly Pareto dominates it,

Proposition 1. *Suppose that conditions (1) and (2) hold. Then, a first best allocation will have*

$$a_t = \begin{cases} 0 & ; \text{for } t \bmod 3 = 0 \\ Ra_{t-1} & ; \text{otherwise.} \end{cases}$$

The result relies on two conditions. Condition (1) rules out the need to save when the state is ϕ^2 , while condition (2) guarantees that it is efficient to save in all other periods.

1.1 Definition of Equilibria

The political structure is as follows. The groups compete for government funds. At the beginning of every period, every group makes fiscal demands to the government every period, d_t^i , and receives a fiscal provision denoted by $\hat{d}_t^i$. It is assumed that the domestic groups cannot save on their own.

The government only objective is to satisfy the demands of the parties. At any time, if the fiscal demands are such that the government has enough wealth to satisfy them, $Ra_{t-1} + e_t \leq \sum_{i=1}^n d_t^i$, then, $\hat{d}_t^i = d_t^i$ and assets evolve according to the following budget constraint $a_t = Ra_{t-1} + e_t - \sum_{i=1}^n d_t^i$. However, if the fiscal demands are excessive, $\sum_{i=1}^n d_t^i > Ra_{t-1} + e_t$, the government provides each group with a provision equal to $\hat{d}_t^i = (Ra_{t-1} + e_t) / n$. The importance of this particular division rule will be discussed later on.

At any point in time, the history of the game is given by all the previous fiscal demands made by the parties. A (pure) strategy for a party is a mapping from all possible histories to fiscal demands. A subgame perfect equilibrium is such that at any point in time and after any possible history, any party's fiscal demand is optimal given the other parties' strategies. Given a subgame perfect equilibrium, a subgame perfect allocation is the fiscal demand allocation that results from the equilibrium play.

1.2 Worst Equilibrium

This section characterizes the worst equilibrium of the savings game.

Let the total-exhaustion strategy profile be given by infinite fiscal demands, $d_t^i = \infty$, after any history. Then the following holds.

Proposition 2. *The total-exhaustion strategy profile is a subgame perfect equilibrium of the game. Even more, after any history, this strategy generates the lowest possible payoff to all parties.*

Proof. Total-exhaustion strategies are minimax. □

In the worst equilibrium there is no cooperation among the groups, each of them fears that what is not demanded will be consumed by the other parties. Given this simple characterization of the worst equilibrium, it can now be used to characterize the best subgame perfect equilibria of the game.

1.3 Efficient Subgame Perfect Equilibria

This subsection characterizes the best subgame perfect equilibrium of the savings game that has symmetric payoffs to all domestic groups. It is shown that such a symmetric equilibrium

is stationary and can be characterized by the solution to a linear program. The reasons why a country might not save at first best levels are analyzed.

Given the characterization of the worst equilibrium, incentive compatible allocations are defined as follows.

Definition 2. A fiscal demand allocation d is ***incentive compatible*** if at all t and for all i

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^i \geq \phi_t \left(Ra_{t-1}(d) + e_t - \sum_{j \neq i} d_t^j \right) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \phi_{\tau} \frac{e_{\tau}}{n} \quad (\text{ICa})$$

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^i \geq \phi_t \frac{Ra_{t-1}(d) + e_t}{n} + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \phi_{\tau} \frac{e_{\tau}}{n} \quad (\text{ICb})$$

An allocation is ***a sustainable savings allocation*** if it is both incentive compatible and feasible.

What does the definition of incentive compatibility⁴ implies? Given a proposed allocation of fiscal demands, we say that it is incentive compatible if no party at any time will gain by grabbing all the residual income of the government and having as a continuation value the worst equilibrium payoff.

It is possible to show that a sustainable allocation is subgame perfect and viceversa.

Proposition 3. A feasible allocation is subgame perfect if and only if it is sustainable.

Proof. Should follow from standard arguments. □

The next definition defines efficiency in the usual Pareto way.

Definition 3. A savings allocation is ***efficient*** if it is sustainable and there does not exist another sustainable savings allocation that weakly Pareto dominates it at time 0.

To provide a characterization of the best equilibria, we first prove the following symmetry result.

Proposition 4. (Symmetry) There exists an efficient allocation of fiscal demands d such that $d_t^i = d_t$ for all i and t .

⁴The second incentive compatibility constraint depends on the tie-breaking rule of the government when a party makes an unfeasible demand. However, as we will show, when focusing in equilibrium with symmetric payoffs, the second incentive compatibility constraint can be safely ignored.

Proof. Note first that the set of sustainable savings allocations is convex. This follows by the linearity of the feasibility constraints and the incentive constraints. Given the linearity of the objective function, the set of payoffs generated by sustainable allocations is also convex. Given the symmetry of the game, this implies that there exists an efficient allocation d that delivers the same time zero payoff to all parties. Let V be that payoff, so that $V_1^i = V$ for all i . Now, let a new allocation $\bar{d}$ be such that $\bar{d}_t^i = \sum_j d_t^j / n$. Given that $\bar{d}$ is a convex combination of d , the incentive compatibility constraints will be satisfied as well as the feasibility constraints. So, $\bar{d}$ is an incentive compatible equilibrium that delivers V to all players at time 0 (so, it is efficient) and has $\bar{d}_t^i$ independent of i for all t . $\square$

Given the symmetry result, it is safe to ignore the second incentive constraint, **ICb**, when solving for sustainable allocations that generate symmetric payoffs. In a symmetric equilibrium $d_t^i = d_t$, which implies from the budget constraint that $d_t \leq (Re_t + a_{t-1})/n$. This final inequality implies that if **ICa** is satisfied, so will be **ICb**.

The following assumption is used in what follows,

Condition 3. *The parameters are such that $\beta R\phi < n$.*

The symmetry result can be used to show that an efficient allocation with symmetric payoffs do not generate savings at times where $\phi_t = \phi^2$.

Proposition 5. (No Over Savings) *Suppose that conditions (1), (2) and (3) hold. Then, an efficient allocation d that delivers symmetric payoffs to parties at time 1 is such that $a_t(d) = 0$ for all $t \bmod 3 = 0$.*

Proof. In the appendix. $\square$

This result is useful because it allows us now to proof stationarity of the efficient sustainable equilibrium with symmetric payoffs.

Proposition 6. (Stationarity) *Suppose that conditions (1), (2) and (3) hold. Then there exists an efficient allocation with symmetric payoffs that is stationary, i.e. $d_t = d_{t \bmod 3}$ for all t .*

Proof. Let d be an efficient allocation with symmetric payoffs. The previous result showed that for any $\tau \bmod 3 = 1$ we have that $d_\tau + d_{\tau+1}/R + d_{\tau+2}/R^2 = 1$. Now, let $d_{i \bmod 3}^* = (1 - \beta^3) \sum_{j=0}^{\infty} \beta^{3j} d_{3j+i}$ for $i \in \{1, 2, 3\}$. Define the allocation $\hat{d}$ to be such that $\hat{d}_t = d_{t \bmod 3}^*$. Note that $\hat{d}$ achieves the same payoffs as d at time 0. Note as well that the allocation $\hat{d}$ is feasible: for any $\tau \bmod 3 = 1$ it follows that $\hat{d}_\tau + \hat{d}_{\tau+1}/R + \hat{d}_{\tau+2}/R^2 = 1$. Given the linearity of the incentive constraints, we can show as well that $\hat{d}$ is incentive compatible. $\square$

The efficient sustainable allocation with symmetric payoffs can be completely characterized by the fiscal provisions allocated in the first three periods; the rest of time, the cycle of provisions just repeats itself. We can then characterize the best sustainable savings allocation recursively by only two numbers: how much is saved in period 1 and in period 2.

Let V_t represent the income to a party at time t . Let a_t be the savings done at time t . Then the following holds,

Proposition 7. *Suppose assumptions (1), (2) and (3) hold. Then in an efficient allocation with symmetric payoffs, the utilities to the parties at the first two periods, V_1 and V_2 , are given by,*

$$(1 - \beta^3)V_1 = \frac{1 - a_1}{n} + \beta\phi\frac{Ra_1 - a_2}{n} + \beta^2\phi^2\frac{Ra_2}{n} \quad (1)$$

$$(1 - \beta^3)V_2 = \phi\frac{Ra_1 - a_2}{n} + \beta\phi^2\frac{Ra_2}{n} + \beta^2\frac{1 - a_1}{n} \quad (2)$$

for some a_1 and a_2 such that $0 \leq a_2 \leq Ra_1 \leq R$.

Given condition (2), $\beta\phi R > 1$, it is easy to see that the value functions V_1 and V_2 are increasing in both a_1 and a_2 : it is first best to save up to the last period. However, as will be shown below, to save might not be incentive compatible.

Note that in a symmetric allocation there are no savings done in periods where $\phi_t = \phi^2$, and hence the incentive constraint in such periods will be satisfied. There are then, only two incentive constraints to satisfy,

$$V_1 \geq \left(a_1 + \frac{1 - a_1}{n}\right) + \frac{\beta^3}{1 - \beta^3} \frac{1}{n} \quad (3)$$

$$V_2 \geq \phi \left(a_2 + \frac{Ra_1 - a_2}{n}\right) + \frac{\beta^2}{1 - \beta^3} \frac{1}{n} \quad (4)$$

Plugging for the values of V_1 and V_2 from (1) and (2) into these equations, and letting $\theta = \beta\phi R$, and $\tilde{a}_2 = a_2/R$, then the incentive constraints can be shown to be equivalent to:

$$\theta(\theta - 1)\tilde{a}_2 \geq ((1 - \beta^3)(n - 1) + 1 - \theta)a_1 \quad (\text{IC1})$$

$$\beta^3(\theta - 1)a_1 \geq \theta((1 - \beta^3)(n - 1) + 1 - \theta)\tilde{a}_2 \quad (\text{IC2})$$

Solving for the Efficient Allocation

The problem of finding the efficient allocation with symmetric payoffs can now be written as follows,

$$\max_{0 \leq \tilde{a}_2 \leq a_1 \leq 1} \frac{1}{n} \{1 - a_1 + \theta(a_1 - \tilde{a}_2) + \theta^2 \tilde{a}_2\} \quad (\text{P})$$

subject to (IC1) and (IC2). Note that written this way, the problem is independent of R given θ .

A first result quickly obtains: if $(1 - \beta^3)(n - 1) + 1 - \theta \leq 0$, then any feasible pair a_1 and a_2 is incentive compatible (this can be seen by observing the incentive constraints and using that $\theta > 1$). Thus the best sustainable equilibrium is a first best allocation in this case.

Assuming instead that $(1 - \beta^3)(n - 1) + 1 - \theta > 0$, the incentive compatibility constraints (IC1 and IC2) can be rewritten as follows,

$$\frac{\beta^3(\theta - 1)/\theta}{(1 - \beta^3)(n - 1) + 1 - \theta} \geq \frac{\tilde{a}_2}{a_1} \geq \frac{(1 - \beta^3)(n - 1) + 1 - \theta}{\theta(\theta - 1)} \quad (5)$$

So, for a positive pair of a_1 and $\tilde{a}_2$ to be incentive compatible it is necessary that

$$(\theta - 1)\beta^3 \geq ((1 - \beta^3)(n - 1) + 1 - \theta)^2$$

If this condition does not hold, then no positive pair of a_1 and $\tilde{a}_2$ is incentive compatible. Note that

$$(\theta - 1)\beta^3 < ((1 - \beta^3)(n - 1) + 1 - \theta)^2 \Rightarrow (1 - \beta^{3/2})(n - 1) + 1 - \theta > 0$$

The following condition will then be used to characterize the efficient savings allocation.

Condition 4. *The parameters are such that $(1 - \beta^{3/2})(n - 1) + 1 - \theta > 0$.*

The following theorem now follows.

Theorem 1. (The Best Equilibrium) *Suppose that conditions (1), (2) and (3) hold. Then,*

- *If condition (4) holds then the worst equilibrium is the unique equilibrium of the game.*
- *If condition (4) does not hold and $(1 - \beta^3)(n - 1) + 1 - \theta \geq 0$, then the efficient allocation with symmetric payoffs is such that $a_1 = 1$ and*

$$a_2 = R \min \left\{ \frac{\beta^3(\theta - 1)}{\theta((1 - \beta^3)(n - 1) + 1 - \theta)}, 1 \right\} \quad (6)$$

- If $(1 - \beta^3)(n - 1) + 1 - \theta \leq 0$ then the efficient sustainable allocation with symmetric payoffs is first best: $Ra_1 = a_2 = R$.

Proof. The proof follows by noticing that if the condition 4 is satisfied, the only feasible and incentive compatible allocation is the one with $a_1 = a_2 = 0$. The second part of the theorem follows directly from the maximization subject to the linear constraints. $\square$

If condition (4) holds, then no cooperation is possible. Even though there are efficiency gains from savings, the temptations that assets generate are too strong to support any savings. When condition (4) does not hold, some level of savings might be sustainable, but not necessarily the first best efficient level. However, note that if any positive amount of savings is done in a sustainable allocation, it is the case that $a_1 = 1$. So, the first period savings are either zero or first best efficient. It is the second period savings, a_2 , that can lie between zero and the first best level R .

Figure 1 displays, given conditions (1), (2), and (3), the result of theorem 1. The incentive constraints are linear in the $(a_1, a_2/R)$ -space and are denoted by IC1 and IC2. The feasibility constraint restricts the choices to lie in the triangle formed by the 45 degree line, the horizontal axis and the vertical line $a_1 = 1$. The first panel illustrates a situation where the first best level of savings is sustainable. The second panel illustrates a situation where some savings are sustainable but lie strictly below the first best level. The third panel illustrates a situation where no savings are sustainable.

This finishes the characterization of the best equilibrium when the country cannot borrow but can save. It has been shown that whether or not the country is able to save at first best levels depends on how many groups there are (n), the discount factor (β) of the players, and the gains from saving (θ). When there are many groups, or the groups are impatient, or the gains from savings are not high, the best equilibrium will entail savings below the first best level.

Understanding the Nature of the Incentive Constraints

Suppose that condition (4) is holding (this corresponds to the third panel of Figure 1). Let us rewrite the incentive constraints one more time:

$$\frac{\beta^3(\theta - 1)/\theta}{(1 - \beta^3)(n - 1) + 1 - \theta} \geq \frac{a_2}{Ra_1} \geq \frac{(1 - \beta^3)(n - 1) + 1 - \theta}{\theta(\theta - 1)}$$

The first inequality corresponds to the incentive constraints in second periods (when $\phi_t = \phi$). This constraint generates an upper bound on a_2 : savings cannot be too high in second periods, otherwise the agents would prefer to deviate, grab the assets and play the worst

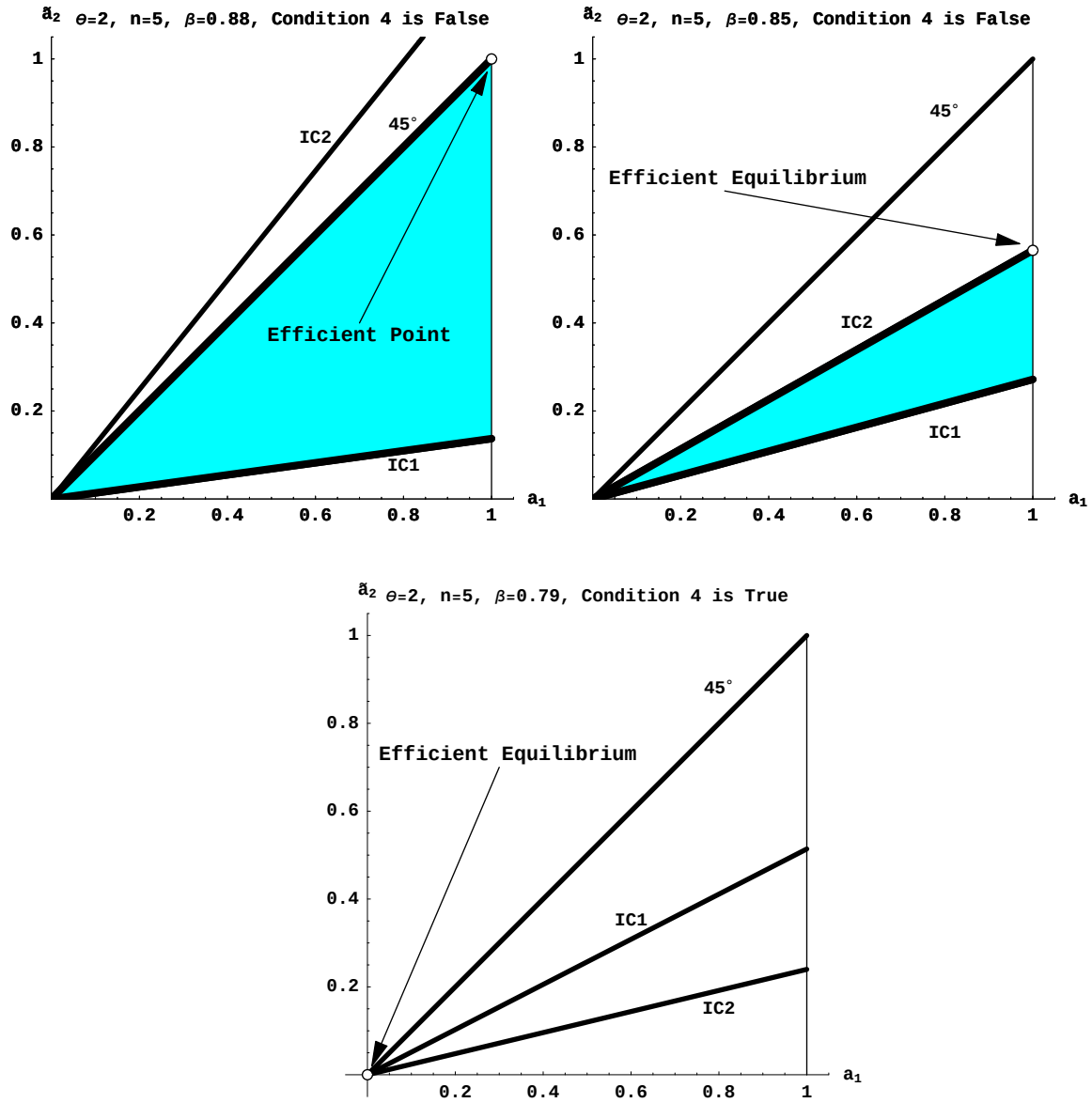


Figure 1: The Efficient Savings Equilibrium

equilibrium. The second inequality corresponds to the incentive constraints in first periods. Suppose that $\frac{(1-\beta^3)(n-1)+1-\theta}{\theta(\theta-1)} < 1$, then there exists positive values of a_2 and a_1 that are feasible and such that the incentive constraint in period 1 is not binding. In other words, if the groups could commit to a given amount a_2 of savings in second periods, they will save as well in period 1. But in equilibrium they don't. The reason is that condition 4 guarantees that any promised level of savings in period 2, that is incentive compatible in period 1, is not incentive compatible in period 2. Even though parties are willing to save in period 1 if they could commit in period 2 to save a fraction of their wealth, the lack of commitment in period 2 impedes such an allocation to be subgame perfect. Note however, if the parties could bypass the incentive compatibility constraint in period 2, then savings could be sustained. As will be shown, this commitment role could be provided by a contingent credit line, and it is the reason why such a credit line might be valuable enough to all the parties in the game not to default out of it. This is what the next section studies.

2 A Contingent Credit Line

The previous section characterized the equilibrium when the country had no access to external borrowing. In this section, the possibility of borrowing from foreign creditors is analyzed. It is assumed, as in Bulow and Rogoff (1989) and most of the subsequent literature on this topic⁵, that the foreign investors have the ability to commit to exclude the country from borrowing again in external markets if the country ever defaults on the credit line. However, the foreign investors cannot stop the country from saving.

We will say that an allocation generated with the use of a credit line is **repayable** if it satisfies the following two requirements:

- (i) No party has an incentive to deviate from the credit-line allocation when the punishment that follows a deviation is the worst savings equilibrium.
- (ii) The credit-line allocation is such that at any time t there does not exist a sustainable allocation of the savings game starting at time t that Pareto dominates it.

The first requirement is subgame perfection. Individually, all parties should be willing not to deviate from the credit line allocation. The deviation considered is, without loss of generality, the one where any given party can demand a provision equal to the total income of the government this period excluding the provisions already granted to the other parties, (but including possible payments to foreigners).

⁵Important exceptions is Kletzer and Wright, and Wright

The second one is a weak coalition proofness requirement. At any point in time, there does not exist a subgame perfect equilibrium of the savings game from that point onwards, that, from the perspective of the parties, Pareto dominates the credit-line allocation. It is reasonable to expect that if such savings allocation exists, then parties would have an incentive to default out of the credit line, and coordinate play in such a savings allocation.

Remark. Bulow and Rogoff (1989) showed under quite general conditions that if countries can save after default, defaulting out any debt contract is always an arbitrage as long as the net interest rate is positive. In particular, pick any debt contract and any allocation that is *feasible* under it. Then, there always exists a state such that we can find a *feasible* savings allocation (that uses no debt and that doesn't pay back any debt) that generates a higher income in all following states. Hence such an allocation would be strictly preferred. Such an argument is also present in this model: there exists *feasible* savings allocations that would Pareto dominate any credit line allocation. The question is whether these savings allocations are *incentive compatible*: they would be the outcome of equilibrium play.

In this section, a particular type of credit line is analyzed. Let $a(d, b)$ be a sequence of asset positions generated by an allocation of fiscal provisions d , $a_0 = 0$ and a credit line b , so that

$$a_t(d, b) = Ra_{t-1}(d, b) + e_t + b_t(b) - \sum_{i=1}^n d_t^i$$

where

$$b_t(b) = \begin{cases} b & \text{if } t \bmod 3 = 0 \\ -Rb & \text{if } t \bmod 3 = 1 \\ 0 & \text{otherwise} \end{cases}$$

The credit line is as follows. In periods where $\phi_t = \phi^2$ the country can access an extra amount of funds equal to b from foreigners. The country is supposed to repay those funds in periods where $\phi_t = 1$, with interest rate R . The credit line is closed in periods where $\phi_t = \phi$. So, the credit line provides funds only in periods where there are most needed.

INTRODUCE EQUILIBRIUM NOTION, STRATEGIES AND THE WORST EQUILIBRIUM AS IN PREVIOUS SECTION.

Definition 4. We say that an allocation d is ***feasible under a credit line*** b if $d_t^i \geq 0$ for all t and i , and the assets positions generated by (d, b) are always positive: $a_t(d, b) \geq 0$ for all t .

Now we define our notion of repayability of the credit line.

Definition 5. We say that an allocation d is ***sustainable under a credit line*** b if d is *feasible under* b , and

- (i) For all i and t it is the case that

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^i \geq \phi_t \left(Ra_{t-1}(d, b) + e_t + (b_t(b))^+ - \sum_{j \neq i} d_t^j \right) + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \phi_{\tau} \frac{e_{\tau}}{n} \quad (7)$$

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^i \geq \phi_t \frac{Ra_{t-1}(d) + e_t + (b_t(b))^+}{n} + \sum_{\tau=t+1}^{\infty} \beta^{\tau-t} \phi_{\tau} \frac{e_{\tau}}{n} \quad (8)$$

- (ii) For all t and for any sustainable savings allocation $\hat{d}$ starting at t with assets $a_{t-1}(d, b)$, there exists an i such that

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^i > \sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} \hat{d}_{\tau}^i \quad (9)$$

Item (i) in the definition checks that the allocation under a credit line b is subgame perfect, given the commitment of the external investors to punish the country by closing the credit line. Item (ii) refers to coalition proofness: there does not exist an incentive compatible and feasible allocation of the savings game that Pareto dominates the credit line allocation at any time.

As mentioned before, in this economy, a weaker form of the Bulow and Rogoff result holds.

Proposition 8. (Bulow and Rogoff) *For any feasible allocation under a credit line b there exists a savings allocation that is feasible and that Pareto dominates it as long as $R > 1$.*

Proof. An application of Bulow and Rogoff (1989). □

Note that this feasibility result was enough for Bulow and Rogoff to conclude that the country should always default out of the debt contract, and hence, we should not observe any lending in equilibrium. The reason why feasibility is all that is needed is that, in Bulow and Rogoff, the country could always implement any feasible allocation if it chooses so. At some point, defaulting always expands the budget set of the country, and hence the country could in principle achieve a higher objective value. However, this paper emphasizes that feasibility is not enough to guarantee default when the country is composed of groups that lack commitment internally (as well as externally): the allocations that the players consider possible after default have to be incentive compatible as well as feasible.

Notice that the set of sustainable allocations under a credit line b is not convex. The problem is condition (ii). Even though equation (9) is linear, the condition is not. For example, it is possible for there to be two allocations under a credit line b that are sustainable

at some time t and whose payoffs lie outside the Pareto frontier of the savings game, but a convex combination lies inside the frontier of the *savings game*. Hence the convex combination won't be sustainable. However, as our objective is to analyze whether Bulow and Rogoff result holds in this economy, we will focus for now on allocations under a credit line that generate symmetric payoffs and construct examples where Bulow and Rogoff does not hold.

Symmetric Allocations

The following result relaxes the constraints when focusing in symmetric allocations.

Lemma 1. *Suppose that d is a symmetric allocation under a credit line b . Suppose that d satisfies condition (i) of definition 5. Suppose that the efficient sustainable allocation of the savings game d^* starting at t with assets $a_{t-1}(d, b)$ is such that*

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau} > \sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^* \quad (10)$$

for all t . Then the allocation d is sustainable under credit line b .

Proof. Suppose the allocation that satisfies all the conditions is not sustainable under the credit line. Then this means that there exists a time t such that a sustainable allocation $\hat{d}$ starting with assets $a_{t-1}(d, b)$ at time t satisfies

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} \hat{d}_{\tau}^i \geq \sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau} > \sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_{\tau} d_{\tau}^*$$

for all i . But then, this implies that d^* is not efficient. A contradiction. $\square$

The previous result guarantees that to check whether a symmetric allocation under a credit line satisfies the coalition proofness requirement, it suffices to compare its payoffs to the efficient symmetric allocation of the savings game.

We are now ready to state the main theorem of the paper.

Theorem 2. *Suppose that θ , β and n are such that $\theta \in (1, n)$, $(1 - \beta^3)(n - 1) + 1 - \theta > 0$, and*

$$\max \left\{ \frac{\beta^3(\theta - 1)}{\theta((1 - \beta^3)(n - 1) + 1 - \theta)}, \frac{(1 - \beta^3)(n - 1) + 1 - \theta}{\theta(\theta - 1)} \right\} < 1 \quad (11)$$

Then we can find $\bar{R} > 1$ and $b > 0$ such that for all $R \in (1, \bar{R})$, there exists an allocation feasible under the credit line b that is sustainable.

Proof. In the appendix □

The conditions in theorem 1 guarantee that we are in a situation as in the third panel of figure 1: from the perspective of time 1, it would have been incentive compatible to save at first best levels, but the incentive constraint at time 2 restricts the savings to lie below first best. From the perspective of time 1, the groups have like to save more, but it is not incentive compatible to do so. They cannot commit not to overconsume in period 2! The contingent credit lines generates that commitment and that is why it is valued by the groups. The credit line reduces the amount of assets that have to be send to period 2. Basically the credit line is a way (although an inefficient one) of saving from period 1 to period 3 without the country having to hold assets in second periods. By bypassing the second period (where the incentive constraint is tight), the credit line can achieve an allocation that generates higher

Figure 2 plots the region where the conditions for the theorem hold. For parameters values inside that region, it is possible to find values of R close enough to one such that repayment is possible. The shaded area is the region where the Bulow-Rogoff result does not hold. It is important to notice that the graphics plot β against given values of θ . So, to interpret the graphics correctly it is important to remember that θ remains constant as β varies because $R\phi$ is changing in the opposite way. It is easy to see that when $n = 1$, there is only one party, no amount of debt is sustainable. It is also possible to show that when β is one no credit line is sustainable.

3 Extensions and Discussions

3.1 A Non-Contigent Credit Line

3.2 Why would the foreigners offer credit line?

4 Conclusion

This paper presented a simple model of the tragedy of the commons as in Tornell and Velasco: a country is composed of infinitely lived groups that share common access to a savings technology. Differently from previous work, I characterized the best subgame perfect equilibria of the game. It was shown that even though the country have access to rich enough asset markets (as in Bulow and Rogoff 1989), debt contracts with foreigners can be sustained in equilibrium. The Bulow and Rogoff contracts are feasible, but not generally subgame perfect.

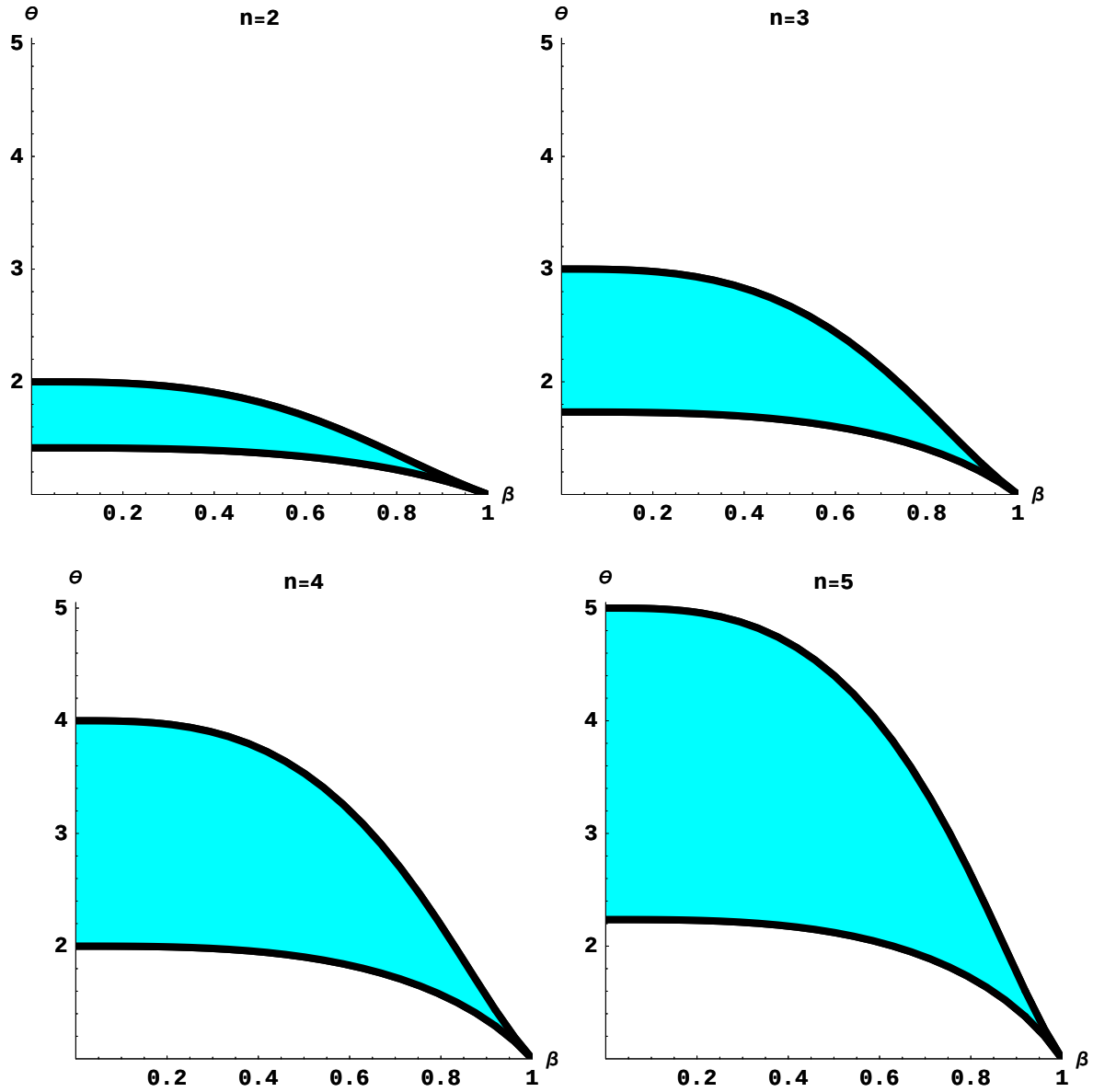


Figure 2: Where Repayment is Possible

The shaded region represents the area where the conditions of theorem 1 hold. Each panel plots a different value for n in the $\beta - \theta$ space.

Appendix

Proof of proposition 5

Proof. By result 4, we can focus on allocations that have equal provisions. Let d be such an efficient allocation. Suppose that for some $\tau \bmod 3 = 0$, $a_\tau(d) > 0$. Then,

1. (i) $d_{\tau+1} = 0$. To see this suppose that it is not true. From the budget constraint, we could reduce $d_{\tau+1}$ by $\epsilon > 0$ and increase d_τ by ϵ/R . We do this while keeping d_t constant for all $t > \tau + 1$, without affecting as well $a_{\tau+1}$. This reduction in $d_{\tau+1}$ does not affect the incentive compatibility constraint at time $\tau + 1$, which is

$$d_{\tau+1} + \sum_{t=1}^{\infty} \phi_{\tau+1+t} \beta^t d_{\tau+1+t} \geq a_{\tau+1} + d_{\tau+1} + \frac{\beta^3}{1 - \beta^3} \frac{1}{n}$$

Note however that for times $t_0 < \tau$, given that $\phi^2 > \beta R$, this change in the allocation strictly increases the utility on the equilibrium path without affecting the deviation value. From the budget constraint, $a_{\tau+1} = Ra_\tau + e_\tau - nd_{\tau+1}$, we have that the reduction in $d_{\tau+1}$ by epsilon reduces a_τ by $n\epsilon/R$. The incentive compatibility at time τ is

$$\phi^2 d_\tau + \sum_{t=1}^{\infty} \phi_{\tau+t} \beta^t d_{\tau+t} \geq \phi^2 (a_\tau + d_\tau) + \frac{\beta}{1 - \beta^3} \frac{1}{n}$$

which becomes relaxed with the change in the allocation given that $\phi^2 n/R - \beta > 0$. So the proposed change is feasible, incentive compatible and generates an strict improvement in the utility at time zero. Hence $d_{\tau+1}$ has to be zero.

2. (ii) $d_{\tau+2} = 0$. To show this, again assume it is not true. Suppose now that we decrease $d_{\tau+2} = 0$ by ϵ and increase d_τ by ϵ/R^2 . This change is feasible. Note then that given that $d_t = 0$ for $t > \tau + 2$ has not changed, the incentive constraints are still holding for $t > \tau + 2$. Note again that the change does not affect $a_{\tau+2}$, and hence the incentive constraint at $\tau + 2$ is still holding. However, from the budget we know that $a_{\tau+2} = Ra_{\tau+1} - nd_{\tau+2}$. So $a_{\tau+1}$ has to decrease by $\epsilon n/R$. The incentive constraint at time $\tau + 1$ is

$$\sum_{t=1}^{\infty} \phi_{\tau+1+t} \beta^t d_{\tau+1+t} \geq a_{\tau+1} + \frac{\beta^3}{1 - \beta^3} \frac{1}{n}$$

Given that $\epsilon n/R - \beta \phi \epsilon > 0$ (by condition 2), the change relaxes the time $\tau + 1$ incentive

constraint. What about the constraint at time τ ?

$$\phi^2 d_\tau + \sum_{t=1}^{\infty} \phi_{\tau+t} \beta^t d_{\tau+t} \geq \phi^2 (a_\tau + d_\tau) + \frac{\beta}{1 - \beta^3} \frac{1}{n}$$

Now note that the change in the right hand side is $\phi^2 \epsilon / R^2 - \beta^2 \phi \epsilon$ (which is positive by condition). The left hand side increases by $\phi^2 (\epsilon / R^2 - \epsilon n / R^2)$ (this follows because a_τ is reduced by $\epsilon n / R^2$). So as long as $\phi^2 n / R^2 - \beta^2 \phi$, which follows by $\phi n > 1 > (\beta R)^2$, the change is incentive compatible at time τ . However, the change strictly increased the payoff for all $t < \tau$, without affecting the deviation values, so, the change is an improvement over the old allocation. It has to be the case then that $d_{\tau+2} = 0$.

3. (iii) $d_{\tau+3} = 0$. Again, suppose not. Then we can reduce $d_{\tau+3}$ by ϵ and increase d_τ by ϵ / R^3 . This change is feasible. It is also incentive compatible at all times t such that $t \geq \tau + 3$. For time $\tau + 2$, the incentive constraint is

$$\sum_{t=1}^{\infty} \phi_{\tau+2+t} \beta^t d_{\tau+2+t} \geq \phi a_{\tau+2} + \frac{\beta^2}{1 - \beta^3} \frac{1}{n}$$

Now, the change reduces $a_{\tau+2}$ by $\epsilon n / R$. The incentive constraint then is slacked by $-\beta \phi^2 \epsilon + \phi \epsilon n / R$. So as long as, $\beta R \phi < n$, the change relaxes the constraint. Note as well that the incentive constraint at time $\tau + 1$ is implied by the incentive constraint at time $\tau + 2$ given that $d_{\tau+1} = d_{\tau+2} = 0$. It is possible to show that the change is incentive compatible at time τ , and also for all $t < \tau$. Given that $\beta R < 1$, the change is an improvement over the previous allocation. It is feasible and incentive compatible. So it has to be the case that $d_{\tau+3} = 0$.

4. (iv) $a_{\tau+3} > 0$. Given the previous results, given that no spending is done after time τ , this follows.

Now, the previous results tell us that if an efficient allocation is ever the case that $a_\tau > 0$ for some $\tau \bmod 3 = 0$, then $d_t = 0$ for all $t > \tau$. However, $\beta R < 1$ implies this cannot be incentive compatible – it is dominated by the worst equilibrium payoff. $\square$

Proof of Theorem 1

We will consider two cases.

- (i) Suppose condition (4) holds. Then the unique savings equilibrium is the worst equilibrium of the savings game. Consider the following allocation: the country borrows

$1/R$ in periods where $\phi_t = \phi^2$ and spends all of the income symmetrically across parties. The country is supposed to pay back 1, when $\phi_t = 1$. The value of such an allocation at times when $\phi_t = 1$ is $(1 - \beta^3)V_1^D = \frac{\beta^2\phi^2}{Rn}$. The incentive compatibility constraints and the coalition proofness constraint collapse in this case to $V_1^D \geq 1 + \frac{\beta^3}{1-\beta^3}\frac{1}{n}$. Debt is sustainable as long as $\theta^2 \geq R^3((1 - \beta^3)n + \beta^3)$ which is equivalent to

$$\theta(R^{-3}\theta - 1) \geq ((1 - \beta^3)(n - 1) + 1 - \theta)$$

Note that this last inequality is satisfied with slackness when $R = 1$ given condition (11). So, we can find R close enough to one such that the inequality still holds. Hence debt can be sustained.

- (ii) Suppose that condition (4) fails. In this case we will proceed by construction, but differently from case (i), we modify the efficient sustainable allocation of the savings game to allow for a credit line. The efficient sustainable allocation in the savings game has $a_1 = 1$ and $\frac{a_2}{R} = \frac{\beta^3(\theta-1)}{\theta((1-\beta^3)(n-1)+1-\theta)}$ where $a_2 < Ra_1$ by the premises of the theorem. Suppose now that in the third periods the government borrows an amount $\epsilon/R > 0$ and spends it equally between the parties. The government repays ϵ in first periods. Assume that $\epsilon < a_1 - a_2/R$. Now we construct a new allocation feasible under the credit line $b = \epsilon/R$. Let $\hat{a}_1 = a_1 - \epsilon$ and $\hat{a}_2 = a_2$. Let the new allocation $\hat{d}$ be such that $\hat{d}_1 = 0$, $\hat{d}_2 = \frac{R\hat{a}_1 - \hat{a}_2}{n}$ and $\hat{d}_3 = \frac{R\hat{a}_2 + \epsilon/R}{n}$. The new allocation $\hat{d}$ is feasible under the credit line. When $R = 1$, the new allocation $\hat{d}$ strictly Pareto dominates the old allocation at all times. So it is coalition proof. Note also that the deviations from the new allocation are the same as the deviations from the old one. Given that the new allocation Pareto dominates the old one, the incentive constraints are strictly relaxed at all times. So, when $R = 1$ debt is sustainable: the coalition proofness constraint is strictly slack and so are the incentive constraints. Given that the value functions are continuous in R , we can find R close enough to one such that the new allocation under the credit line b is incentive compatible.

The analysis of the two cases thus completes the proof.