Online Appendix to "Inflation Gap Persistence, Indeterminacy, and Monetary Policy"

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This appendix is organized as follows. Section A presents the complete set of equilibrium conditions of the model presented in Section 2 of the paper. Section B compares the empirical performance of the model and its variant embedded with indexation to lagged inflation as in Christiano et al. (2005) and Smets and Wouters (2007). Section C empirically compares the model and its counterpart with widely used CES aggregators of goods and labor.

A Equilibrium Conditions of the Model

The representative composite-good producer's zero-profit condition yields

$$1 = \frac{1}{1+\epsilon_p}\, d_t + \frac{\epsilon_p}{1+\epsilon_p}\, e_t, \tag{A1}$$

where

$$e_t \equiv \int_0^1 \frac{P_t(f)}{P_t}\, df. \tag{A2}$$

Firms' cost-minimization implies that all firms choose identical ratios of the three production factors, so that we have

$$\frac{u_t K_{t-1}}{l_t} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_{k,t}}, \tag{A3}$$

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$$\frac{O_t}{l_t} = \frac{\phi}{(1-\alpha)(1-\phi)} W_t. \tag{A4}$$

Using the equilibrium condition $M_{t,t+j} = \beta^j (\Lambda_{t+j}/\Lambda_t)(P_t/P_{t+j})$, where Λ_t is the marginal utility of consumption, the first-order condition for profit maximization can be written as

$$0 = E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \frac{\Lambda_{t+j}}{\Lambda_t} X_{t+j} \Bigg[\left(\frac{P_t^*/P_t}{d_{t+j}} \right)^{-\gamma_p} \prod_{k=1}^{j} \pi_{t+k}^{\gamma_p} \left(\frac{P_t^*}{P_t} \prod_{k=1}^{j} \pi_{t+k}^{-1} - \frac{\gamma_p}{\gamma_p - 1} mc_{t+j} \exp \tilde{z}_{p,t+j} \right) \\ - \frac{\epsilon_p}{\gamma_p - 1} \frac{P_t^*}{P_t} \prod_{k=1}^{j} \pi_{t+k}^{-1} \Bigg], \tag{A5}$$

where P_t^* is the price set by firms that can adjust prices in period t. Moreover, under staggered price-setting, eq. (3) for the real marginal cost d_t of aggregating individual goods and eq. (A2) can be rewritten as, respectively,

$$d_t^{1-\gamma_p} = \xi_p \, \pi_t^{\gamma_p - 1} \, d_{t-1}^{1-\gamma_p} + (1-\xi_p) \left(\frac{P_t^*}{P_t} \right)^{1-\gamma_p}, \tag{A6}$$

$$e_t = \xi_p \, \pi_t^{-1} \, e_{t-1} + (1-\xi_p) \, \frac{P_t^*}{P_t}. \tag{A7}$$

Aggregating the production technology (4) over firms $f \in [0,1]$ and combining the resulting equation with the goods demand curve (2) leads to

$$X_t \Delta_t = \left[(A_t l_t)^{1-\alpha} \, (u_t K_{t-1})^{\alpha} \right]^{1-\phi} O_t^{\phi} - \omega \Upsilon_t, \tag{A8}$$

where

$$\Delta_t \equiv \frac{s_t + \epsilon_p}{1 + \epsilon_p} \tag{A9}$$

represents the relative price distortion and s_t is defined as

$$s_t \equiv \int_0^1 \left(\frac{P_t(f)}{P_t \, d_t} \right)^{-\gamma_p} df,$$

which can be rewritten, under staggered price-setting, as

$$d_t^{-\gamma_p} s_t = \xi_p \, \pi_t^{\gamma_p} \, d_{t-1}^{-\gamma_p} s_{t-1} + (1-\xi_p) \left(\frac{P_t^*}{P_t} \right)^{-\gamma_p}. \tag{A10}$$

The representative labor packer's zero-profit condition yields

$$1 = \frac{1}{1+\epsilon_w}\, d_{w,t} + \frac{\epsilon_w}{1+\epsilon_w}\, e_{w,t}, \tag{A11}$$

where

$$e_{w,t} \equiv \int_0^1 \frac{P_t\, W_t(h)}{P_t\, W_t}\, dh. \tag{A12}$$

The representative household's utility maximization with respect to consumption, bond holdings, and the wage implies

$$\Lambda_t = \frac{\exp z_{c,t}}{C_t - b\, C_{t-1}}, \tag{A13}$$

$$1 = E_t\left(\frac{\beta\, \Lambda_{t+1}}{\Lambda_t}\frac{r_t}{\pi_{t+1}}\right), \tag{A14}$$

$$\begin{aligned} 0 = E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Lambda_{t+j}}{\Lambda_t}\, l_{t+j} \Bigg[\left(\frac{W_t^*/W_t}{d_{w,t+j}}\right)^{-\gamma_w} \prod_{k=1}^{j} \pi_{w,t+k}^{\gamma_w} \Bigg(\frac{W_t^*}{W_t} \prod_{k=1}^{j} \pi_{t+k}^{-1} \\ - \frac{\gamma_w}{\gamma_w - 1} \frac{\tilde{l}_{t+j}^{\chi} \exp \tilde{z}_{w,t+j}}{\Lambda_{t+j} W_{t+j}} \prod_{k=1}^{j} \frac{W_{t+k}}{W_{t+k-1}} \Bigg) - \frac{\epsilon_w}{\gamma_w - 1} \frac{W_t^*}{W_t} \prod_{k=1}^{j} \pi_{t+k}^{-1} \Bigg], \end{aligned} \tag{A15}$$

where W_t^* is the real wage that is determined in period t. Moreover, under staggered wage-setting, eq. (10) for the real marginal cost $d_{w,t}$ of aggregating individual labor services and eq. (A12) can be rewritten as, respectively,

$$d_{w,t}^{1-\gamma_w} = \xi_w\, \pi_{w,t}^{\gamma_w - 1}\, d_{w,t-1}^{1-\gamma_w} + (1-\xi_w)\left(\frac{W_t^*}{W_t}\right)^{1-\gamma_w}, \tag{A16}$$

$$e_{w,t} = \xi_w\, \pi_{w,t}^{-1}\, e_{w,t-1} + (1-\xi_w)\, \frac{W_t^*}{W_t}. \tag{A17}$$

Combining the labor resource constraint (7) with the labor demand curve (9) leads to

$$\tilde{l}_t = l_t\, \Delta_{w,t}, \tag{A18}$$

where

$$\Delta_{w,t} \equiv \frac{s_{w,t} + \epsilon_w}{1+\epsilon_w} \tag{A19}$$

represents the relative wage distortion and $s_{w,t}$ is defined as

$$s_{w,t} \equiv \int_0^1 \left(\frac{P_t\, W_t(h)}{P_t\, W_t\, d_{w,t}} \right)^{-\gamma_w} dh,$$

which can be rewritten, under staggered wage-setting, as

$$d_{w,t}^{-\gamma_w} s_{w,t} = \xi_w\, \pi_{w,t}^{\gamma_w}\, d_{w,t-1}^{-\gamma_w} s_{w,t-1} + (1-\xi_w) \left(\frac{W_t^*}{W_t} \right)^{-\gamma_w}. \tag{A20}$$

The representative capital-service provider's profit maximization with respect to the utilization rate, investment, and capital implies

$$R_{k,t} = Q_t\, \delta'(u_t), \tag{A21}$$

$$\begin{aligned} \frac{1}{\Psi_t} &= Q_t \left[1 - S\left(\frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right) - S'\left(\frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right) \frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right] \exp z_{i,t} \\ &\quad + E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t}\, Q_{t+1}\, S'\left(\frac{I_t/I_{t-1}}{g_\Upsilon g_\Psi} \right) \frac{(I_t/I_{t-1})^2}{g_\Upsilon g_\Psi} \exp z_{i,t+1} \right], \end{aligned} \tag{A22}$$

$$1 = E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{R_{k,t+1} u_{t+1} + Q_{t+1}\,(1 - \delta(u_{t+1}))}{Q_t} \right], \tag{A23}$$

where Q_t is the real price of capital.

The equilibrium conditions of the model consist of eqs. (6), (14)–(17), (19), (A1), (A3)–(A11), (A13)–(A23), and their counterparts that would be obtained under flexible prices and wages in the absence of price and wage markup shocks, along with the neutral and investment-specific technological change processes (5) and (13), the normally distributed i.i.d. monetary policy rate shock, and the other seven shock processes (24).

When we consider the labor disutility specification of Erceg et al. (2000) instead of that of Schmitt-Grohé and Uribe (2006) as presented in Appendix B of the paper, the model's four equilibrium conditions (A15), (A18), (A19), and (A20) are replaced with

$$\begin{aligned} 0 = E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \frac{\Lambda_{t+j}}{\Lambda_t}\, l_{t+j} \Bigg[& \left(\frac{W_t^*/W_t}{d_{w,t+j}} \right)^{-\gamma_w} \prod_{k=1}^{j} \pi_{w,t+k}^{\gamma_w} \Bigg(\frac{W_t^*}{W_t} \prod_{k=1}^{j} \pi_{t+k}^{-1} - \frac{\gamma_w}{\gamma_w - 1} \Bigg\{ \frac{l_{t+j}}{1+\epsilon_w} \\ & \times \left[\left(\frac{W_t^*/W_t}{d_{w,t+j}} \right)^{-\gamma_w} \prod_{k=1}^{j} \pi_{w,t+k}^{\gamma_w} + \epsilon_w \right] \Bigg\}^{\chi} \frac{\exp \tilde{z}_{w,t+j}}{\Lambda_{t+j} W_{t+j}} \prod_{k=1}^{j} \frac{W_{t+k}}{W_{t+k-1}} \Bigg) - \frac{\epsilon_w}{\gamma_w - 1} \frac{W_t^*}{W_t} \prod_{k=1}^{j} \pi_{t+k}^{-1} \Bigg]. \end{aligned}$$

This implies that the relative wage distortion $\Delta_{w,t}$ has no influence on wage inflation dynamics as long as labor disutility follows the specification of Erceg et al. (2000). The other equilibrium conditions of the model are unchanged.

B Comparison of the Model and Its Variant with Indexation to Lagged Inflation

When we embed indexation to lagged inflation in the model along the lines of Christiano et al. (2005) and Smets and Wouters (2007), firms that can adjust prices in period t set the price $P_t(f)$ so as to maximize the relevant profit

$$E_t \sum_{j=0}^{\infty} \xi_p^j \, M_{t,t+j} \left[P_t(f) \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p} \right) - P_{t+j} mc_{t+j} \exp \tilde{z}_{p,t+j} \right] X_{t+j|t}(f)$$

subject to the goods demand curve

$$X_{t+j|t}(f) = \frac{X_{t+j}}{1+\epsilon_p} \left\{ \left[\frac{P_t(f)}{P_{t+j}\, d_{t+j}} \prod_{k=1}^{j} \left(\pi_{t+k-1}^{\iota_p} \pi^{1-\iota_p} \right) \right]^{-\gamma_p} + \epsilon_p \right\},$$

where $\iota_p \in [0,1]$ is the relative weight on lagged price inflation in price indexation. Likewise, wages that can be adjusted in period t are chosen so as to maximize the relevant utility function

$$E_t \sum_{j=0}^{\infty} (\beta \xi_w)^j \left[-\frac{1}{1+\chi} \left(\int_0^1 l_{t+j|t}(h)\, dh \right)^{1+\chi} \exp \tilde{z}_{w,t+j} + \Lambda_{t+j} \frac{P_t\, W_t(h)}{P_{t+j}} \prod_{k=1}^{j} \left(\pi_{w,t+k-1}^{\iota_w} \pi_w^{1-\iota_w} \right) l_{t+j|t}(h) \right]$$

subject to the labor demand curve

$$l_{t+j|t}(h) = \frac{l_{t+j}}{1+\epsilon_w} \left\{ \left[\frac{P_t\, W_t(h)}{P_{t+j}\, W_{t+j}\, d_{w,t+j}} \prod_{k=1}^{j} \left(\pi_{w,t+k-1}^{\iota_w} \pi_w^{1-\iota_w} \right) \right]^{-\gamma_w} + \epsilon_w \right\},$$

where $\iota_w \in [0,1]$ is the relative weight on lagged wage inflation in wage indexation.

Then, the model's eight equilibrium conditions (A5)–(A7), (A10), (A15)–(A17), and

(A20) are replaced with, respectively,

$$
\begin{aligned}
0 &= E_t \sum_{j=0}^{\infty} (\beta\xi_p)^j \frac{\Lambda_{t+j}}{\Lambda_t} X_{t+j} \left\{ \left(\frac{P_t^*/P_t}{d_{t+j}} \right)^{-\gamma_p} \prod_{k=1}^{j} \left(\frac{\pi_{t+k}}{\pi} \right)^{\gamma_p} \left(\frac{\pi_{t+k-1}}{\pi} \right)^{-\iota_p \gamma_p} \right. \\
&\quad \times \left[\frac{P_t^*}{P_t} \prod_{k=1}^{j} \left(\frac{\pi_{t+k}}{\pi} \right)^{-1} \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\iota_p} - \frac{\gamma_p}{\gamma_p - 1} mc_{t+j} \exp \tilde{z}_{p,t+j} \right] \\
&\quad \left. - \frac{\epsilon_p}{\gamma_p - 1} \frac{P_t^*}{P_t} \prod_{k=1}^{j} \left(\frac{\pi_{t+k}}{\pi} \right)^{-1} \left(\frac{\pi_{t+k-1}}{\pi} \right)^{\iota_p} \right\}, \\
d_t^{1-\gamma_p} &= \xi_p \left(\frac{\pi_t}{\pi} \right)^{\gamma_p - 1} \left(\frac{\pi_{t-1}}{\pi} \right)^{\iota_p(1-\gamma_p)} d_{t-1}^{1-\gamma_p} + (1-\xi_p) \left(\frac{P_t^*}{P_t} \right)^{1-\gamma_p}, \\
e_t &= \xi_p \left(\frac{\pi_t}{\pi} \right)^{-1} \left(\frac{\pi_{t-1}}{\pi} \right)^{\iota_p} e_{t-1} + (1-\xi_p) \frac{P_t^*}{P_t}, \\
d_t^{-\gamma_p} s_t &= \xi_p \left(\frac{\pi_t}{\pi} \right)^{\gamma_p} \left(\frac{\pi_{t-1}}{\pi} \right)^{-\iota_p \gamma_p} d_{t-1}^{-\gamma_p} s_{t-1} + (1-\xi_p) \left(\frac{P_t^*}{P_t} \right)^{-\gamma_p}, \\
0 &= E_t \sum_{j=0}^{\infty} (\beta\xi_w)^j \frac{\Lambda_{t+j}}{\Lambda_t} l_{t+j} \left\{ \left(\frac{W_t^*/W_t}{d_{w,t+j}} \right)^{-\gamma_w} \prod_{k=1}^{j} \left(\frac{\pi_{w,t+k}}{\pi_w} \right)^{\gamma_w} \left(\frac{\pi_{w,t+k-1}}{\pi_w} \right)^{-\iota_w \gamma_w} \right. \\
&\quad \times \left[g_\Upsilon^j \frac{W_t^*}{W_t} \prod_{k=1}^{j} \left(\frac{\pi_{t+k}}{\pi} \right)^{-1} \left(\frac{\pi_{w,t+k-1}}{\pi_w} \right)^{\iota_w} - \frac{\gamma_w}{\gamma_w - 1} \frac{\tilde{l}_{t+j}^{\chi} \exp \tilde{z}_{w,t+j}}{\Lambda_{t+j} W_{t+j}} \prod_{k=1}^{j} \frac{W_{t+k}}{W_{t+k-1}} \right] \\
&\quad \left. - \frac{\epsilon_w}{\gamma_w - 1} g_\Upsilon^j \frac{W_t^*}{W_t} \prod_{k=1}^{j} \left(\frac{\pi_{t+k}}{\pi} \right)^{-1} \left(\frac{\pi_{w,t+k-1}}{\pi_w} \right)^{\iota_w} \right\}, \\
d_{w,t}^{1-\gamma_w} &= \xi_w \left(\frac{\pi_{w,t}}{\pi_w} \right)^{\gamma_w - 1} \left(\frac{\pi_{w,t-1}}{\pi_w} \right)^{\iota_w(1-\gamma_w)} d_{w,t-1}^{1-\gamma_w} + (1-\xi_w) \left(\frac{W_t^*}{W_t} \right)^{1-\gamma_w}, \\
e_{w,t} &= \xi_w \left(\frac{\pi_{w,t}}{\pi_w} \right)^{-1} \left(\frac{\pi_{w,t-1}}{\pi_w} \right)^{\iota_w} e_{w,t-1} + (1-\xi_w) \frac{W_t^*}{W_t}, \\
d_{w,t}^{-\gamma_w} s_{w,t} &= \xi_w \left(\frac{\pi_{w,t}}{\pi_w} \right)^{\gamma_w} \left(\frac{\pi_{w,t-1}}{\pi_w} \right)^{-\iota_w \gamma_w} d_{w,t-1}^{-\gamma_w} s_{w,t-1} + (1-\xi_w) \left(\frac{W_t^*}{W_t} \right)^{-\gamma_w},
\end{aligned}
$$

while the other equilibrium conditions of the model are unchanged. Consequently, the price and wage NKPCs (20) and (21) are replaced with, respectively, the canonical ones

$$
\begin{aligned}
\hat{\pi}_t &= \frac{\iota_p}{1+\beta\iota_p} \hat{\pi}_{t-1} + \frac{\beta}{1+\beta\iota_p} E_t \hat{\pi}_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p(1+\beta\iota_p)[1-\epsilon_p\theta_p/(\theta_p-1)]} \hat{mc}_t + z_{p,t}, \\
\hat{\pi}_{w,t} &= \frac{\iota_w}{1+\beta\iota_w} \hat{\pi}_{w,t-1} + \frac{\beta}{1+\beta\iota_w} E_t \hat{\pi}_{w,t+1} + \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(1+\beta\iota_w)[1-\epsilon_w\theta_w/(\theta_w-1)]} \left[\chi \hat{l}_t - \left(\hat{\lambda}_t + \hat{w}_t \right) \right] + z_{w,t}.
\end{aligned}
$$

We estimate the model's variant with indexation to lagged inflation using the same data as that for the model. We then set the prior distributions for the relative weights on lagged price and wage inflation in price and wage indexation (ι_p, ι_w) to be the beta distributions with mean of 0.5 and standard deviation of 0.15, following Smets and Wouters (2007), and fix the values of the parameters governing the superelasticity of demand for goods and labor (ϵ_p, ϵ_w) at the prior mean of -3.0 to avoid the identification issue pointed out by Eichenbaum and Fisher (2007).

The estimation result shows that the variant's log marginal data density for the pre-1979 period is -409.71, which is slightly greater than the model's counterpart of -410.43. However, the Bayes factor in favor of the variant relative to the model is around $2:1$, which constitutes "evidence not worth more than a bare mention" according to Jeffreys (1961). As for the post-1982 period, the variant's log marginal data density is -621.94, which is much smaller than the model's counterpart of -592.15. Therefore, we conclude that the model performs equally well as the variant with indexation to lagged inflation for the pre-1979 period and outperforms the variant for the post-1982 period.

C Comparison of the Model and Its Counterpart with CES Aggregators

In this section we examine the role of the Kimball (1995)-type non-CES aggregators of goods and labor employed in the model for the estimation results, the decline in inflation gap persistence, and the source of the decline. Before proceeding, it is worth mentioning that our model (with the non-CES aggregators) empirically outperforms its counterpart with CES aggregators for both pre-1979 and post-1982 periods, as noted in Section 4.1 of the paper.

Table A1 reports the posterior estimates of model parameters for the pre-1979 and post-1982 periods in the CES counterpart model, i.e., $\epsilon_p = \epsilon_w = 0$. The last line of the table displays the posterior probability of equilibrium determinacy $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D | \mathbf{Y}^T\}$ in the two samples, and shows the same result as that obtained in our estimated model (with the non-CES aggregators): the US economy was likely in the indeterminacy region of the model parameter space during the pre-1979 period, while the economy likely entered the determinacy region during the post-1982 period. The table also indicates that the posterior estimates of 13

Table A1: Posterior estimates of model parameters in the CES counterpart model

	Pre-1979		Post-1982	
Parameter	Mean	90% interval	Mean	90% interval
$\overline{\pi}$	0.906	[0.680, 1.129]	0.653	[0.389, 0.910]
$\overline{r}$	1.482	[1.331, 1.643]	1.395	[1.184, 1.609]
$\overline{g_\Upsilon}$	0.421	[0.325, 0.513]	0.427	[0.322, 0.522]
$\overline{g_\Psi}$	0.312	[0.226, 0.397]	0.377	[0.291, 0.461]
$\overline{l}$	−0.073	[−0.147, −0.013]	−0.020	[−0.094, 0.052]
b	0.606	[0.533, 0.680]	0.623	[0.530, 0.723]
χ	0.957	[0.493, 1.401]	2.524	[1.716, 3.336]
α	0.234	[0.208, 0.257]	0.235	[0.206, 0.263]
ϕ	0.442	[0.392, 0.492]	0.486	[0.427, 0.542]
δ_2/δ_1	0.872	[0.752, 1.007]	0.788	[0.653, 0.917]
ζ	1.197	[0.804, 1.575]	2.174	[1.313, 2.917]
$\theta_p - 1$	9.826	[8.207, 11.473]	9.651	[7.804, 11.586]
$\theta_w - 1$	9.526	[7.688, 11.310]	9.787	[7.273, 12.413]
ξ_p	0.453	[0.385, 0.519]	0.603	[0.555, 0.650]
ξ_w	0.744	[0.691, 0.799]	0.515	[0.430, 0.608]
ϕ_r	0.796	[0.739, 0.864]	0.793	[0.746, 0.838]
ϕ_π	0.415	[0.190, 0.614]	2.626	[2.103, 3.112]
ϕ_y	0.102	[0.060, 0.145]	0.022	[0.007, 0.036]
ϕ_{gy}	0.141	[0.072, 0.214]	0.137	[0.057, 0.199]
ρ_π	0.946	[0.894, 1.000]	0.976	[0.956, 0.999]
ρ_a	0.099	[0.024, 0.170]	0.040	[0.006, 0.072]
ρ_c	0.507	[0.291, 0.734]	0.741	[0.569, 0.927]
ρ_i	0.587	[0.350, 0.819]	0.758	[0.675, 0.849]
ρ_g	0.848	[0.775, 0.923]	0.964	[0.941, 0.989]
ρ_p	0.628	[0.515, 0.746]	0.973	[0.947, 0.997]
ρ_w	0.624	[0.486, 0.769]	0.964	[0.940, 0.990]
σ_π	0.114	[0.059, 0.165]	0.068	[0.043, 0.090]
σ_a	1.135	[0.968, 1.299]	0.820	[0.702, 0.942]
σ_c	1.357	[1.057, 1.636]	1.685	[1.162, 2.230]
σ_i	0.640	[0.302, 0.994]	1.721	[1.229, 2.225]
σ_g	2.617	[2.339, 2.922]	2.037	[1.825, 2.225]
σ_p	0.450	[0.326, 0.568]	0.184	[0.138, 0.223]
σ_w	0.386	[0.302, 0.466]	0.960	[0.551, 1.307]
σ_r	0.283	[0.233, 0.330]	0.208	[0.171, 0.245]
σ_s	0.536	[0.374, 0.705]	–	–
M_π	0.399	[−0.941, 1.676]	–	–
M_a	0.254	[0.057, 0.476]	–	–
M_c	−0.235	[−0.395, −0.068]	–	–
M_i	−0.124	[−0.704, 0.385]	–	–
M_g	−0.171	[−0.261, −0.085]	–	–
M_p	−0.256	[−0.766, 0.250]	–	–
M_w	−0.360	[−1.081, 0.285]	–	–
M_r	0.789	[−0.087, 1.659]	–	–
$\log p(\mathbf{Y}^T)$	−417.861		−654.435	
$\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D \| \mathbf{Y}^T\}$	0.000		1.000	

Notes: For each estimated parameter of the CES counterpart model (i.e., $\epsilon_p = \epsilon_w = 0$), the table reports the posterior mean and 90 percent HPD intervals based on $10,000$ particles from the final importance sampling in the SMC algorithm. In the last two lines, $\log p(\mathbf{Y}^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\boldsymbol{\theta} \in \boldsymbol{\Theta}^D|\mathbf{Y}^T\}$ denotes the posterior probability of equilibrium determinacy.

parameters—χ, ξ_p, ξ_w, ϕ_π, ϕ_y, ρ_g, ρ_p, ρ_w, σ_a, σ_i, σ_g, σ_p, and σ_w—changed substantially in that each parameter's 90 percent HPD intervals in the two periods are disjoint. Among the changes in the parameters, the following three are worth noting. First, the monetary policy response to the inflation gap ϕ_π exhibits an even larger increase than in our estimated model, and demonstrates a change from a passive to an active policy response in line with the result of the model. Second, the policy response to the GDP gap ϕ_y fell substantially. Third, the probability of no price change ξ_p displayed a smaller increase than in the estimated model, whereas the probability of no wage change ξ_w decreased.

Based on the posterior mean estimates of model parameters for the pre-1979 and post-1982 samples, Table A2 presents the variability of the inflation gap and its predictability at forecasting horizons of one, four, and eight quarters in the CES counterpart model, i.e., $\epsilon_p = \epsilon_w = 0$. This counterpart model also exhibits the decline in inflation gap persistence, although the decline in the eight-quarter-ahead measure of the predictability is much smaller than in our estimated model (with the non-CES aggregators).

Table A2: Inflation gap variability and predictability in the CES counterpart model

	$std(\hat{\pi}_t - z_{\pi,t})$	R_1^2	R_4^2	R_8^2
Pre-1979 period	1.050	0.817	0.489	0.189
Post-1982 period	0.385	0.512	0.222	0.179
Percent change	−63	−37	−55	−5

Note: The statistics are calculated using the posterior mean estimates of parameters in the CES counterpart model (i.e., $\epsilon_p = \epsilon_w = 0$) reported in Table A1.

Table A3 presents the results of counterfactual exercises in which, starting with the pre-1979 posterior mean of all estimated model parameters, the values of parameters indicated in the first column of the table are altered to their post-1982 posterior mean in the CES counterpart model, i.e., $\epsilon_p = \epsilon_w = 0$. In this counterpart model, the changes in all estimated monetary policy parameters—ϕ_r, ϕ_π, ϕ_y, ϕ_{gy}, π, ρ_π, σ_π, and σ_r—to their post-1982 posterior mean can explain more than the entire declines in inflation gap predictability as shown on the first line of the table, and moreover, most of the contribution of the changes in monetary policy parameters is attributed to the change from a passive to an active policy response to the inflation gap—which generates a shift to determinacy—as indicated on the second line. These results are consistent with those obtained in our estimated model (with the non-CES

Table A3: Counterfactual exercises in the CES counterpart model: Accounting for the declines in inflation gap variability and predictability

Model parameter	std($\hat{\pi}_t - z_{\pi,t}$)	R_1^2	R_4^2	R_8^2	determinacy
1. Post-1982 policy	72	173	138	1,121	yes
2. Post-1982 ϕ_π	43	131	116	711	yes
3. Post-1982 ϕ_y	9	56	68	467	no
4. Post-1982 ϕ_π, ϕ_y	65	170	149	1,474	yes
5. Post-1982 nonpolicy	−348	85	146	1,290	no
6. Post-1982 ξ_p	−11	−1	−13	−17	no
7. Post-1982 ξ_w	−34	89	127	1,196	no
8. Post-1982 nonpolicy shocks	−1,220	−25	−78	−3,408	no

Notes: For the CES counterpart model (i.e., $\epsilon_p = \epsilon_w = 0$), the table shows the percentage changes in inflation gap variability and predictability in counterfactual exercises in which, starting with the pre-1979 posterior mean of all estimated model parameters, the values of parameters indicated in the first column are changed to their post-1982 posterior mean reported in Table A1. "Post-1982 policy" refers to the post-1982 posterior mean of ϕ_r, ϕ_π, ϕ_y, ϕ_{gy}, π, ρ_π, σ_π, and σ_r, while "Post-1982 nonpolicy" refers to the post-1982 posterior mean of the other estimated parameters.

aggregators). One difference from the result with the model is that the decrease in the policy response to the GDP gap ϕ_y plays a secondary role in the contribution of the changes in policy parameters as seen on the third to fourth lines. The changes in the other policy parameters—ϕ_r, ϕ_{gy}, π, ρ_π, σ_π, and σ_r—do not contribute meaningfully to the decline in inflation gap predictability. As for the changes in nonpolicy parameters, they can account for the decline in the predictability as well (although they cannot explain the decline in inflation gap variability) as displayed on the fifth line. In addition, lines 6 and 7 demonstrate that most of the contribution of the changes in nonpolicy parameters is attributed to the decrease in the probability of no wage change ξ_w but not to the increase in the probability of no price change ξ_p, which contrasts sharply with the results obtained in our estimated model. Notably, the decrease in ξ_w reduces inflation gap predictability even though it does not bring about a shift to determinacy. The changes in the other nonpolicy parameters, including the parameters governing processes of all nonpolicy shocks (line 8), do not contribute to the decline in the predictability. Thus, the result that the Fed's change from a passive to an active policy response to the inflation gap can fully account for the decline in inflation gap persistence is robust regardless of the CES or non-CES aggregators, while the result pertaining to the probability of no price change is not.

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