Online Appendix

Aggregate Fluctuations and the Role of Trade Credit

A Empirical appendix

A.1 Trade credit in the cross section

This appendix section A.1 provides additional results to support evidence documented in section 2.1 of the paper.

A.1.1 Summary statistics.

Table A.1 displays the summary statistics of the Compustat sample.

Table A.1: Summary statistics

Variables	mean	sd	min	max
Total assets	1,486.21	5,947.84	0.02	78,942.00
Firm age	13.65	10.02	1.00	47.00
AR receivable - trade	176.19	710.51	0.00	8,547.22
AP payable - trade	114.20	476.23	0.01	6,274.78
Sales	350.90	1,361.96	0.00	15,495.43
AR to sales ratio	0.57	0.28	0.00	1.36
Net AR to sales ratio	0.10	0.63	-4.37	0.89
AP to sales ratio	0.48	0.65	0.06	5.34

A.1.2 Evidence of relationship between firm size/age and financial constraint

Previous studies have documented that firm size and age are reliable indicators of a firm's financial constraints. To provide some evidence for this relationship in the Compustat data, I performed an analysis to investigate the investment sensitivity to cash flow, stratified by the age and size quartiles of firms. The regression equation used for this analysis is as follows:

$$I/K_{i,t} = \beta_1 \text{Cash Flow}/K_{i,t} + \beta_2 Q_{i,t} + \chi_i + \gamma_t + \varepsilon_{i,t},$$

In this equation, $I/K_{i,t}$ represents the investment rate, Cash Flow$/K_{i,t}$ denotes the firm's cash flow normalized by the capital stock, and $Q_{i,t}$ corresponds to the measured Tobin's Q, which captures the firm's growth opportunities. The terms χ_i and γ_t are firm fixed effects and time fixed effects, respectively. The coefficient of primary interest, $\hat{\beta}_1$, is associated with Cash Flow$/K_{i,t}$ and measures the responsiveness of investment to cash flow.

The equation is estimated across eight subsamples of firms, divided into quartiles based on size and age, in addition to the full sample. Table A.2 displays the estimated coefficient $\hat{\beta}_1$ for these categories. Panel (a) shows that, in the full sample (column 1), both cash flow and Tobin's Q are positively associated with investment rates. This is commonly interpreted in the literature to indicate that financial frictions and future investment opportunities are positively linked with a firm's propensity to invest. Crucially, there is a notable decrease in investment sensitivity to cash flows, from 0.023 (highly significant) to -0.001 (not significant), as we move from the smallest to the largest firms. Likewise, Panel (b) reveals that the estimated investment sensitivity to cash flow diminishes from 0.027 to 0.008 when comparing the youngest to the oldest firms. The results from this table generally support the conclusion that younger and smaller firms are subject to more financial constraints than their older and larger counterparts, which is reflected in their greater investment sensitivity to cash flows.

Table A.2: Investment sensitivity to cash flows

Panel (a): Size groups

	(1)	(2)	(3)	(4)	(5)
cash flow/K	0.021***	0.023***	0.014***	0.009*	-0.001
	(0.001)	(0.002)	(0.003)	(0.005)	(0.011)
Tobin's Q	0.068***	0.040***	0.143***	0.146***	0.078***
	(0.004)	(0.008)	(0.008)	(0.008)	(0.008)
Sample	All	Q1 (smallest)	Q2	Q3	Q4 (largest)
N	123735	30934	30935	30933	30933
R^2	0.230	0.272	0.395	0.382	0.275

Panel (b): Age groups

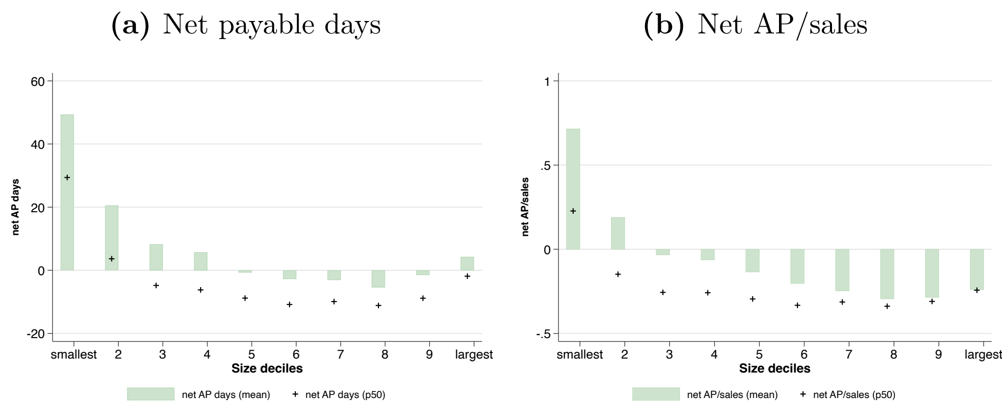
	(1)	(2)	(3)	(4)	(5)
cash flow/K	0.021***	0.027***	0.029***	0.022***	0.008*
	(0.001)	(0.003)	(0.003)	(0.003)	(0.004)
Tobin's Q	0.068***	0.016	-0.004	0.028***	0.043***
	(0.004)	(0.010)	(0.011)	(0.011)	(0.010)
Sample	All	Q1 (youngest)	Q2	Q3	Q4 (oldest)
N	123735	33767	32108	27937	29923
R^2	0.230	0.483	0.411	0.298	0.202

Notes: The sample includes all but financial firms in the Compustat dataset for the period 1960-2007 at the annual frequency. Cash flow of a firm is the net cash flows (`oancf`). Tobin's Q is calculated as `(prcc_f*csho+at-ceq)/at`, where `prcc_f*csho` is the market value, `at` is total asset and `ceq` is the common equity.

A.1.3 Net accounts payable among largest firms

In this section, I examine in detail the trade credit activities of the top two size deciles firms in the Compustat dataset, motivated by the findings in Murfin and Njoroge (2015). In particular, Murfin and Njoroge (2015) finds that net payable days decrease with firm size, except for the largest two deciles of firms. In panel (a) of figure A.1, I replicate their result using our Compustat sample, where net payable days is defined as payable days (AP/cogs $\times$ 365) minus receivable days (AR/sales $\times$ 365).[26] In addition, in panel (b), I present the relationship with firm size using our preferred measure of net accounts payable (i.e., net AP/sales).

Figure A.1: Net accounts payable and firm size: two measures



Notes: The sample includes all but financial firms in the Compustat dataset for 2000-2007. The figures plot net payable days and net AP/sales within each decile of size distribution. Net payable days is defined as payable days (AP/cogs $\times$ 365) minus receivable days (AR/sales $\times$ 365). Panel (a) plots the mean and median net payable days in each size decile. Panel (b) plots each size decile's mean and median net AP/sales.

The relationship between median net payables days and size deciles are very similar in panel (a) and table 1 of Murfin and Njoroge (2015). Specifically, the net payable days of the top two deciles of firms are in fact higher than some of the smaller firms. For instance, the median net payable days of the largest decile firms are even slightly higher than that of the third decile. Additionally, I show in panel (a), that this reversal, albeit not as strong, is also present for the average net payable days. The average net payable days of the largest firms, for instance, is shorter than that of the 4th decile. Similarly, there is also a slight reversal in the net AP/sales among the top two size deciles, as shown in panel (b), but the magnitude of the reversal is even less significant than net payable days. The difference between these two panels is due to the fact that in panel (a), AP is normalized by cogs (cost of goods sold) and in panel (b) it is normalized by sales.

A few remarks regarding these findings are in order. First, the decline of net payable or payable in firm size is fast when firms are small, and the decline slows down as firms grow larger. In other words, the overall decline in net payable is driven by the difference between the smallest versus larger firms. This same pattern is also present in the model (see, for instance, figure C.11). Partly due to this, I note that the negative relationship between net payables and firm size is rather robust, especially comparing the smallest firms versus the rest, despite the reversal among the largest firms.

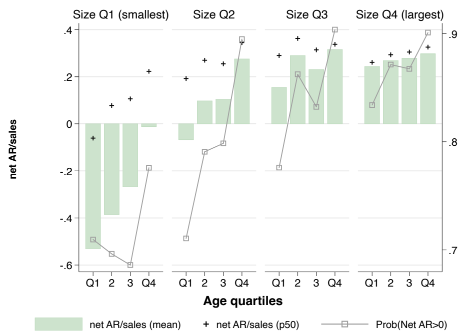
Second, as argued by Murfin and Njoroge (2015), there is a non-financial motive at play when it comes to the trade credit choices of the largest firms in Compustat. Using a sample of large retailers and their suppliers, Murfin and Njoroge (2015) show these largest buyers use a substantial amount of trade credit from their small suppliers. Similarly, Klapper et al. (2012) document that large buyers tend to get long trade credit terms from smaller suppliers. Both papers argue that these patterns are consistent with a non-financial motive to countervail frictions between suppliers

[26] Our Compustat sample excludes financial-sector firms, same as Murfin and Njoroge (2015).

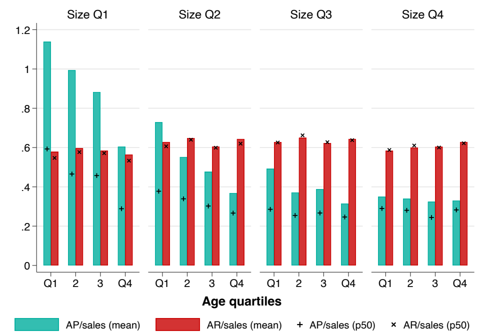
Figure A.2: Trade credit within each age/size quartile

Within each size quartile

(a) Net AR

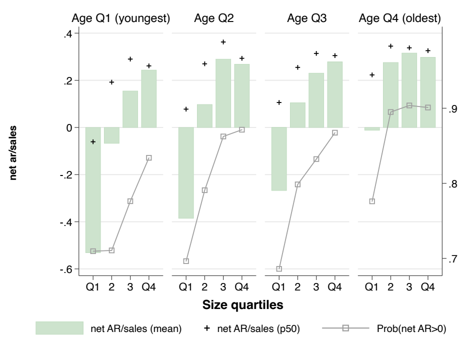


(b) AR and AP

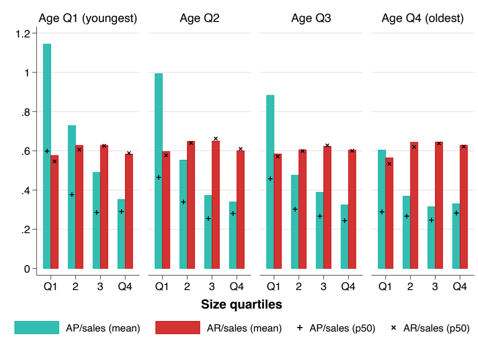


Within each age quartile

(c) Net AR



(d) AR and AP



Notes: The sample includes all but financial firms in the Compustat dataset for the period 2000-2007. The two figures in the upper panel plot net AR/sales (panel a), AP/sales and AR/sales (panel b) within each quartile of firm size distribution. The two figures in the lower panel plot net AR/sales (panel c), AP/sales and AR/sales (panel d) within each quartile of firm age distribution.

and buyers related to product quality. As a caveat to our analysis, I focus on the financial motives that I view as particularly important and relevant.

A.1.4 Conditional on firm age or firm size

Section 2.1 of the paper document that older and larger firms in net terms lend more trade credit, and are more likely to be trade credit lenders. Here, Figure A.2 show that these patterns still hold even conditional on firm size or age. The age gradients of net trade credit lending hold separately within each size quartile (panels a and b); similarly, the size gradients hold separately within each age quartile (panels c and d). Further, the age gradients are less steep among large firms than small firms; and the size gradients are less steep among old than young firms. For instance, as shown in panel (a), within the smallest quartile of firms (size Q1), the difference in average net AR/sales between the youngest (age Q1) and oldest firms (age Q4) is -0.51. In contrast, within the largest quartile (size Q4), the difference is only -0.05, comparing the same two age groups. These patterns are consistent with the fact that firm size and age both contain relevant information about financing frictions.

A.1.5 Controlling for inventory and ROA

Table A.3 results from the regressing equation 1 while controlling for firms' return on assets (ROA) or inventory to sale ratios. Due to data limitations, these regressions are carried at using Compustat data at the annual frequency. The baseline results carry through with these additional controls with the magnitude of the estimates slightly lower than the baseline regressions.

Table A.3: Net AR and firm characteristics, controlling for ROA & inventory

	(1)	(2)	(3)	(4)	(5)	(6)
Firm size (log size)	0.028***	0.007***	0.008***	0.055***	0.021***	0.022***
	(0.004)	(0.002)	(0.002)	(0.006)	(0.005)	(0.005)
Firm age (log years)	0.022***	0.021***	0.015***	0.066***	0.060***	0.052***
	(0.006)	(0.005)	(0.005)	(0.010)	(0.011)	(0.010)
ROA		0.207***	0.184***		0.355***	0.342***
		(0.035)	(0.033)		(0.036)	(0.035)
inventory/sales	0.193		0.181*	0.317		0.275
	(0.123)		(0.101)	(0.254)		(0.220)
Dependent variable	NetAR/Sales	NetAR/Sales	NetAR/Sales	$\mathbb{I}_{\text{NetAR}>0}$	$\mathbb{I}_{\text{NetAR}>0}$	$\mathbb{I}_{\text{NetAR}>0}$
N	14052	13262	12688	14052	13262	12688
R^2	0.191	0.285	0.283	0.213	0.280	0.275

Notes: The table displays results from regression equation 1 while controlling for firms' ROA (returns on assets) and ratio of inventory to sales. The sample includes all non-financial firms in the Compustat dataset for the period 2000-2007. All regressions include a set of industry-quarter fixed effects. Standard errors are clustered two way at the industry and time level and shown in parentheses.

A.1.6 Corporate tax return data

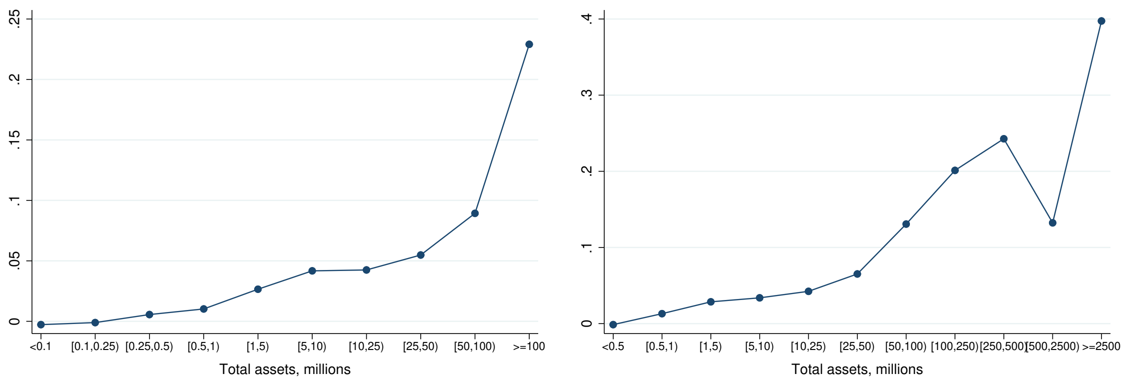
Section 2.1 provides empirical evidence of the financial motives behind firms' choice of trade credit using the Compustat data. Previous papers such as Petersen and Rajan (1997) utilize data from the Survey of Small Business Finances (SSBF). Both datasets have the limitation that they are not representative of the firms in the US economy. In this appendix section, we utilize the Business Tax Statistics dataset, an *aggregate* dataset that covers a broader spectrum of firms. It categorizes firms into different groups based on asset sizes and reports aggregate statistics on balance sheets and income information. A key strength of this dataset is its extensive coverage across all US corporations.

As the first pass of the data, Figure A.3 displays net AR/sales (with sales measured using business receipts) for each firm size group for S and C corporations, respectively. For both types of firms, it is evident that net AR/sales increases with firm size, indicating that, in net terms, larger firms extend more trade credit than smaller firms. This finding aligns with the observations in both

Compustat and SSBF datasets. While C corporations include very large firms within the economy, hence their asset size groupings are relatively broad, S corporations are generally smaller in size, resulting in more detailed size groupings, especially at the lower end. Nevertheless, regardless of the firm type or the different size grouping schemes, both figures reveal a consistent pattern of increasing net trade credit lending with firm size, supporting the financial motive behind trade credit.

Figure A.3: Relationship between net AR/sales and firm size

(a) S Corporations **(b)** C Corporations



Notes: Data in these figures are taken from the Business Tax Statistics from IRS for year 2005. Panel (a) plots the statistics for all S corporations by different size groups. Panel (b) plots the statistics for all C corporations by different asset size groups. Sales in these figures are measured using total business receipts.

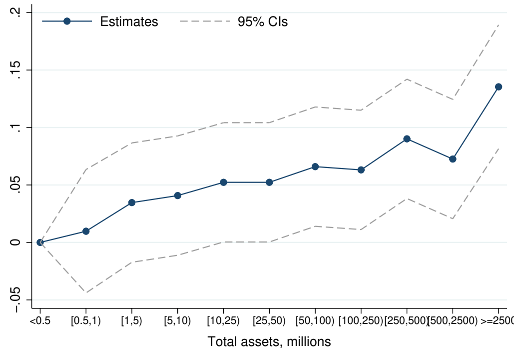
One concern with the findings from the previous figures is that they do not control for the industrial composition of firms within each size category. Consequently, the observed increase with firm size might also reflect differences in industry composition. For C corporations, data are available separately for major sectors of the economy. This allows me to carry out a simple regression of net AR/sales against firm-size group indicators, while including sector fixed effects as controls:

$$\text{net AR/Sales}_{i,f} = \alpha + \left(\sum_{f \in \mathcal{S}} \beta_f \mathbb{I}_f \right) + \chi_i + \varepsilon_{i,f}, \tag{A.1}$$

where net AR/Sales$_{i,f}$ represents the net AR/sales ratio for firm size group f in sector i. The term $\mathbb{I}_f$ is a dummy indicator for firm size group f, and $\mathcal{S}$ denotes the set of these firm size groups $\{$(0,0.5), [0,5,1), [1,5), [5,10), [10,25), [25,50), [50,100), [100,250), [250,500), [500,2500), [2500,∞)$\}$. The variable χ_i is the sector fixed effects, and $\varepsilon_{i,f}$ is the error term. The object of interest is the coefficients in front of the firm-size groups, $\hat{\beta}_f$. Figure A.4 shows estimated coefficients $\hat{\beta}_f$ and the 95% confidence intervals, with $\hat{\beta}_f$ of the smallest group normalized to zero. It shows that larger firms have higher net trade credit lending, with the largest firms' net AR/sales about 15 percentage points higher than the smallest firms.

To summarize, these analyses utilizes Business Tax Return data to corroborate the financial rationale behind firms' trade credit decisions. Despite its caveat of not being at the firm-level, results from this dataset complement the evidence provided by Compustat.

Figure A.4: Relationship between net AR/sales and firm size, $\hat{\beta}_f$



Notes: This figure reports the estimation results from equation A.1. The sample includes 16 industries for the year 2005. These industries are: ASWM services, Accommodation and food service, Agriculture etc, Art, entertainment, and recreation, Construction, Education services, Health care and social assistance, Information, Manufacturing, Mining, Other services, PST services, Real Estate Rental and Leasing, Utility, Wholesale and retail trade, and Transportation and warehouse.

A.2 Trade credit and financial shocks

This appendix section A.2 provides additional results to support evidence documented in section 2.2 of the paper.

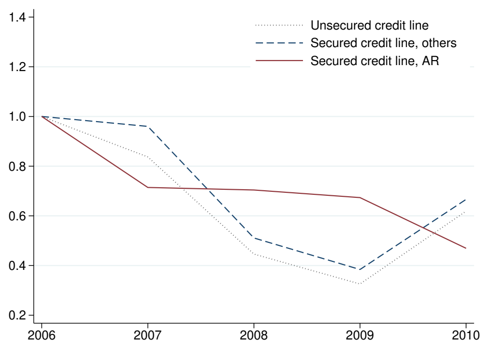
A.2.1 Syndicated market around Lehman bankruptcy

In this section, I examine the changes in the syndicated loan market during the 2007–09 financial crisis. Figure A.5 illustrates the changes in various types of new credit line facilities. A key observation from these figures is that credit facilities secured by accounts receivable (AR) display dynamics similar to those of unsecured credit lines, as well as facilities secured by other types of assets.

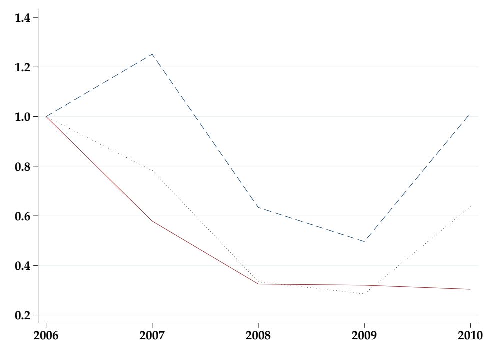
Additionally, Figure A.6 plots the number and the total size of facilities on a quarterly basis. Panel (a), plotting the characteristics of new facilities, reveals that the decline began around 2007Q3, coinciding with the collapse of Bear Stearns, which also marked the onset of the recession designated by the NBER. The downturn continued to deepen after the Lehman bankruptcy in 2008Q3 and reached its nadir a year later. Conversely, the number and size of all open facilities in the market, as shown in Panel (b), started their decline slightly later, around 2008Q1, reaching their lowest point in 2011Q1.

Figure A.5: Characteristics of new credit line facilities by type

(a) Number of new facilities

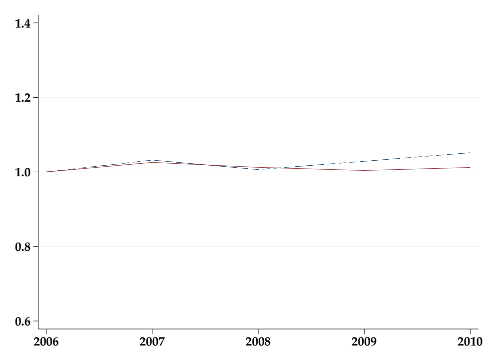


(b) Total facility size



(c) Maturity

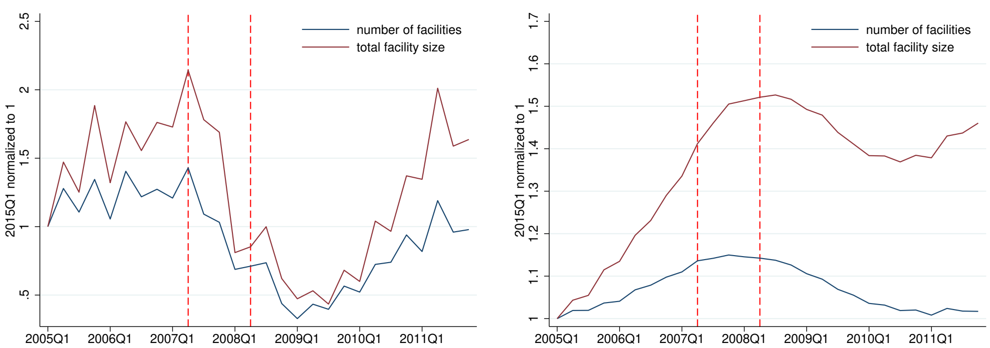
(d) Borrowing base percentage



Notes: I compute the characteristics of the newly opened credit line facilities of each year as the average of all new credit line facilities with non-financial borrowing firms using Thomson Reuters DealScan dataset. The solid lines in these figures are credit line facilities that require accounts receivable as collateral. The dashed lines are credit line facilities that require other types of assets as collateral. The dotted lines are unsecured credit line facilities. The time series are normalized such that they are 1 in year 2006.

Figure A.6: Syndicated market during the crisis, quarterly data

(a) New facilities **(b)** All facilities



Notes: These figures plot the number and total size of new facilities (panel a) and all currently open facilities (panel b) to non-financial borrowing firms in the syndicated loan market during the 2007-09 crisis. The two red vertical lines in each panel mark the quarters of 2007Q2 and 2008Q2, which are the quarters preceding the collapses of Bear Stearns (also the start of the NBER recession) and the Lehman bankruptcy, respectively.

A.2.2 Changes in trade credit as dependent variables

Table A.4 presents another method for estimating how changes in trade credit correlate with firms' exposure to Lehman, where the dependent variables are the changes in trade credit from pre- to post-Lehman shock. Formally, I run the following regression:

$$\Delta y_i = \gamma \text{Exposure}_i + X_i + \varepsilon_i, \tag{A.2}$$

where the dependent variable represents the changes in the median value of AR/sales or AP/sales between the pre-Lehman and post-Lehman periods. This approach removes permanent level differences between firms. The controls for firm characteristics in X_i, identical to those used in regression equation 2, account for variations that could be attributed to factors such as industry affiliation or access to bond and commercial paper markets. What remains, captured by the Exposure$_i$ indicator, are therefore changes in trade credit attributable to the different exposure to Lehman. Reassuringly, results from table A.4 shows a very similar pattern as the baseline estimation in table 2.

Table A.4: Changes in trade credit and exposure to Lehman

	(1)	(2)	(3)	(4)	(5)	(6)
Top quartile of exposure	-0.032**	0.015*				
	(0.014)	(0.008)				
Top 50% of exposure			-0.022*	0.014*		
			(0.012)	(0.007)		
Exposure to Lehman					-0.005	0.013**
					(0.009)	(0.005)
Dependent variable	Δ AR/Sales	Δ AP/Sales	Δ AR/Sales	Δ AP/Sales	Δ AR/Sales	Δ AP/Sales
N	617	613	617	613	617	613
R^2	0.384	0.349	0.379	0.349	0.374	0.352

Notes: This table presents estimates $\hat{\gamma}$ from estimating equation A.2. Firm's exposure to Lehman, Exposure$_i$, is measured in three different ways. Columns (1)–(2) use a dummy variable indicating whether firm i belongs to the top quartile in terms of pre-shock exposure to the Lehman shock. Columns (3)–(4) use a dummy indicating if firms belong to the top 50% exposure to Lehman, and columns (5)–(6) utilize the raw measure of exposure to Lehman, defined as the fraction of a firm's relationship bank's syndication portfolio in which Lehman Brothers had a lead role. Standard errors are clustered at the ultimate lender level and shown in parentheses.

A.2.3 Controlling for differential trends

One potential factor that could lead to differential changes in trade credit following the Lehman shock is the presence of distinct long-term trends in trade credit choices among these groups. Although figure 2 indicate no significant pre-shock trend differences, it remains prudent to formally control for such possibilities in the regression analysis.

To address this, I incorporated linear trends into the estimation equation 2, allowing them to vary among firms with different levels of exposure to the Lehman shock. Specifically, I introduced two linear trend terms into the estimation equation: (i) a general linear trend, represented as $t - t_0$, where t_0 marks the beginning of the sample period and t denotes the current period, and (ii) an interaction term between the linear trend $t - t_0$ and the indicator variable Exposure_i, which signifies whether a firm is in the top quartile or half in terms of exposure to Lehman. This latter interaction term is designed to capture the potential differential trend between firm groups. More formally, I estimate the following equation:

$$\begin{aligned} y_{it} = \quad & \alpha \text{PostLehman}_t + \beta \text{Exposure}_i + \gamma \text{Exposure}_i \times \text{PostLehman}_t \\ & + (t - t_0) + \phi \text{Exposure}_i \times (t - t_0) + X_i + \Lambda_{it} + \varepsilon_{it}, \end{aligned} \tag{A.3}$$

Table A.5 shows that, after controlling for differential linear trends, the patterns previously observed still hold. Notably, firms with greater exposure to Lehman experience a steeper decline in their trade credit lending, by 2.5 to 2.9 percentage points (pps) more than their less exposed counterparts (columns 1 and 3). They also experience a more pronounced increase in their trade credit borrowing by 0.6 to 0.9 pps more (columns 2 and 4) than less exposed firms.

The regressions also reveal that there is no significant differences in these long-term trends across firm groups. The coefficients in front of the interaction between the linear trends and Exposure_i are small and mostly insignificant. Column 3 suggest that firms in the top half of exposure might actually experience significantly faster growth in AR/sales compared to those with lower exposure. This suggests that the notable decline observed in these groups post-Lehman shock is even more striking as it represents a reversal of the long-run trends.

Table A.5: Trade credit and exposure to Lehman, controlling for differential trends

	(1)	(2)	(3)	(4)
Top quartile exposure × trend	0.000	0.001		
	(0.000)	(0.000)		
Top quartile exposure × PostLehman	-0.029***	0.006		
	(0.010)	(0.005)		
Top 50% exposure × trend			0.001**	0.001
			(0.000)	(0.000)
Top 50% exposure × PostLehman			-0.025**	0.009*
			(0.009)	(0.005)
Depdent variable:	AR/Sales	AP/Sales	AR/Sales	AP/Sales
N	16338	16186	16338	16186
R^2	0.625	0.372	0.627	0.372

Notes: This table shows results from estimating equation A.3. Estimates displayed here are the $\hat{\phi}$ and $\hat{\gamma}$. Firm's exposure to Lehman, Exposure_i, is measured in three different ways. Columns (1)–(2) use a dummy variable indicating whether firm i belongs to the top quartile in terms of pre-shock exposure to the Lehman shock. Columns (3)–(4) use a dummy indicating if firms belong to the top 50% exposure to Lehman. Standard errors are clustered at the industry level and shown in parentheses.

B Model appendix

B.1 An alternative model of trade credit

In this section, I present an alternative model in which I treat trade credit as a delay in payment. To do so, I need to adopt a different timing in which output is carried over and sold at the beginning of the next period. The output can be sold on the spot market to generate immediate cash flow, or it can be extended as a trade credit loan.

Timing. Entrepreneurs carry over their wealth a and output y from the previous period. After the idiosyncratic productivity shock z is realized, entrepreneurs sell their output to generate cash flow: they can choose to extend some goods as trade credit loans $\text{AR} \in [0, y]$, and the remaining goods will generate an immediate cash flow $y - \text{AR}$, which can be used to finance working capital. Additionally, entrepreneurs make decisions about their current period production (k, l, x) and whether they will borrow trade credit $\text{AP} \in [0, x]$. Based on these choices, entrepreneurs obtain the required intra-temporal working capital bank loan m. At that point, entrepreneurs decide whether to default on their bank loans. If an entrepreneur decides to default, a renegotiation process occurs between the entrepreneur and the bank, with the ultimate settlement determined by the bank's expected proceeds from liquidating the entrepreneur's collateral. Using the cash flow generated at the beginning of the period and the bank loan, entrepreneurs invest in working capital and production occurs. After entrepreneurs settle the bank loan, collect AR, and repay AP, they consume c and invest in their wealth a'. This period's output is carried over into the next period as y'.

Financial frictions and the existence of trade credit Without loss of generality, assume that working capital includes interest rk, wage bills wl, and inputs x. The entrepreneurs can finance working capital using: i) bank loans, ii) cash flow generated by selling goods on the spot market, and iii) trade credit. As discussed in the benchmark model, the bank loan limit is equal to the expected liquidation value of the collateral $\gamma_1 a + \gamma_2 \text{AR}$.[27] On the other hand, I assume that the repayment of trade credit can be enforced perfectly.

Therefore, I can write the working capital constraint of the entrepreneurs as follows:

$$\begin{aligned} rk + wl + x - \text{AP} &\le \underbrace{\gamma_1 a + \gamma_2 \text{AR}}_{\textit{bank loan}} + \underbrace{y - \text{AR}}_{\textit{cash flow}}, \\ \Longrightarrow rk + wl + x + (\text{AR} - \text{AP}) - y &\le \gamma_1 a + \gamma_2 \text{AR}. \end{aligned}$$

It captures the impact of trade credit on entrepreneurs' liquidity position similar to the benchmark model. Consider the case without trade credit, and the working capital constraint can be written as follows:

$$rk + wl + x - y \le \gamma_1 a.$$

By comparing the two inequalities, I see that the introduction of trade credit changes the entrepreneurs' liquidity position in two ways: the entrepreneurs' needs for bank loans increase by $\text{AR} - \text{AP}$, while their bank loan limit increases by $\gamma_2 \text{AR}$, as is the case in the benchmark model.

[27] Note that due to the difference in timing, the collateral available at the time of default is this period's wealth a and not a' as in the benchmark model.

Although the two models capture a similar mechanism, compared to the benchmark model, this alternative model of trade credit is less tractable computationally as it introduces another state variable y (output carried over from the previous period).

Recursive representation of entrepreneurs' problem. To summarize, in this alternative model, entrepreneurs are characterized by three state variables (a, z, y). I write their problem recursively as follows:

$$\begin{aligned}
V(a,z,y) = &\max u(c) + \beta \mathbb{E}_{z'|z}(a', z', y'),\\
s.t. \quad & c + a' + rk + wl + x = (1+r)a + r^{tc}(\mathrm{AR} - \mathrm{AP}) + y,\\
& rk + wl + x + (\mathrm{AR} - \mathrm{AP}) - y \leq \gamma_1 a + \gamma_2 \mathrm{AR},\\
& 0 \leq \mathrm{AR} \leq y,\\
& 0 \leq \mathrm{AP} \leq x,\\
& y' = AzF(k,l,x),\\
& a' \geq 0.
\end{aligned}$$

B.2 First-order conditions

Here, I present the entrepreneur's optimization problem and derive the first-order conditions (FOCs). The value function of entrepreneurs is

$$\begin{aligned}
V(a,z) \quad = \quad & \max_{c,k,l,AR,AP,a'} \mathbb{E}_{z'|z} log(c) + \beta \mathbb{E}_{z'|z} V(a', z')\\
s.t. \quad\quad & c + a' = (1+r)a + Az\left((k^{\alpha} l^{1-\alpha})^{1-\chi} x^{\chi}\right)^{\mu} - (r+\delta)k - wl - x\\
& + r^{tc}(AR - AP), \qquad (B.1)\\
& Az\left((k^{\alpha} l^{1-\alpha})^{1-\chi} x^{\chi}\right)^{\mu} + (1+r^{tc})(AR - AP) \leq \gamma_1 a' + \gamma_2 AR, \qquad (B.2)\\
& 0 \leq AR \leq Az\left((k^{\alpha} l^{1-\alpha})^{1-\chi} x^{\chi}\right)^{\mu},\\
& 0 \leq AP \leq x,\\
& a' \geq 0.
\end{aligned}$$

Denote $F(k,l,x) = \left((k^{\alpha} l^{1-\alpha})^{1-\chi} x^{\chi}\right)^{\mu}$ as the production function. The Lagrangian of the problem can be written as,

$$\begin{aligned}
\mathcal{L} \quad = \quad & log((1+r)a + AzF(k,l,x) - (r+\delta)k - wl - x + r^{tc}(AR - AP) - a') \qquad (B.3)\\
& + \beta \mathbb{E}_{z'|z} V(a', z') + \xi(\gamma_1 a' + \gamma_2 AR - AzF(k,l,x) - (1+r^{tc})(AR - AP))\\
& + \chi_1(AzF(k,l,x) - AR) + \chi_2 AR\\
& + \chi_3(x - AP) + \chi_4 AP\\
& + \tau a'.
\end{aligned}$$

The FOCs are:

$$
\begin{aligned}
k: &\qquad AzF_k = \frac{r+\delta}{1-c\xi+c\chi_1} \\
l: &\qquad AzF_l = \frac{w}{1-c\xi+c\chi_1} \\
x: &\qquad AzF_x = \frac{1-c\chi_3}{1-c\xi+c\chi_1} \\
AR: &\qquad \frac{1}{c}r^{tc} = \xi(1+r^{tc}-\gamma_2)+\chi_1-\chi_2 \\
AP: &\qquad \frac{1}{c}r^{tc} = \xi(1+r^{tc})-\chi_3+\chi_4 \\
a': &\qquad \frac{1}{c} = \beta\mathbb{E}_{z'|z}V_{a'}(a',z')+\xi\gamma_1+\tau
\end{aligned}
$$

The envelope theorem is

$$
V_a(a,z) = \frac{1}{c}(1+r)
$$

That gives the Euler equation

$$
\frac{1}{c} = \beta\mathbb{E}_{z'|z}\frac{1}{c'}(1+r')+\xi\gamma_1+\tau \tag{B.4}
$$

In addition, according to the Kuhn-Tucker condition, the Lagrangian multipliers and the constraints have the following properties:

$$
\begin{aligned}
&\xi \geq 0, \gamma_1 a' + \gamma_2 AR - AzF(k,l,x) - (1+r^{tc})(AR-AP) \geq 0, \\
&\chi_1 \geq 0, AzF(k,l,x) - AR \geq 0, \\
&\chi_2 \geq 0, AR \geq 0, \\
&\chi_3 \geq 0, x \geq AR, \\
&\chi_4 \geq 0, AP \geq 0, \\
&\tau \geq 0, a' \geq 0,
\end{aligned}
$$

with complementary slackness.

B.3 Proofs of propositions 1 and 2

Before proceeding to the proofs of the propositions, I first prove the monotonicity of the optimal policy function in a lemma. To do this, I rewrite the value function as

$$
\begin{aligned}
V(a,z) &= \max_{c,a'} u((1+r)a+\pi(z,a')-a')+\beta\int_{z'}V(a',z')d\lambda(z',z) \\
s.t. &\qquad a' \geq 0.
\end{aligned} \tag{B.5}
$$

where given a',

$$\begin{aligned}
\pi(z,a') &= \max_{k,l,x,AR,AP} Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} - (r+\delta)k - wl - x + r^{tc}(AR-AP) \\
s.t. \quad & Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} + (1+r^{tc})(AR-AP) \leq \gamma_1 a' + \gamma_2 AR, \\
& 0 \leq AR \leq Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu}, \\
& 0 \leq AP \leq x. \qquad \text{(B.6)}
\end{aligned}$$

Essentially, I are breaking down this dynamic problem into two problems and proving the monotonicity property in each one. First, given a', the static production optimization problem is established in B.6, which gives the optimal profit function $\pi(z,a')$ for a given z. Second, the dynamic problem of choosing next period wealth a', taken as given the function $\pi(z,a')$ as shown in B.5.

Lemma 1. *The value function $v(a,z)$ is supermodular in a and has increasing differences in (a,z). Given z, the policy functions $k(a,z)$, $l(a,z)$, $x(a,z)$, $AR(a,z)$, $-AP(a,z)$ and $a'(a,z)$ increase in a.*

Proof. First, I consider the optimization problem B.6: I intend to show that the optimal policy increase with a' for a given z using Theorem 2.8.1 from Topkis (1998).[28] It is easy to verify that the feasibility set increases strictly with a'; therefore I only need to show that inequality 2.8.1 from Topkis (1998) is satisfied.

Denote $W(k,l,x,AR,AP) = AzF(k,l,x) - (r+\delta)k - wl - x + (r^{tc}(AR-AP))$. Given any $\{k_1,l_1,x_1,AR_1,AP_1\}$ and $\{k_2,l_2,x_2,AR_2,AP_2\}$, I need to show that

$$\begin{aligned}
& W(k_1,l_1,x_1,AR_1,AP_1) + W(k_2,l_2,x_1,AR_2,AP_2) \\
\leq \; & W(k_1\wedge k_2, l_1\wedge l_2, x_1\wedge x_2, AR_1\wedge AR_2, AP_1\wedge AP_2) + \\
& W(k_1\vee k_2, l_1\vee l_2, x_1\vee x_2, AR_1\vee AR_2, AP_1\vee AP_2),
\end{aligned}$$

which reduces to

$$\begin{aligned}
& zAF(k_1,l_1,x_1) + zAF(k_2,l_2,x_2) \\
\leq \; & zAF(k_1\wedge k_2, l_1\wedge l_2, x_1\wedge x_2) + zAF(k_1\vee k_2, l_1\vee l_2, x_1\vee x_2).
\end{aligned}$$

This is straightforward to prove because function $F(\cdot,\cdot,\cdot)$ features strictly increasing differences in its inputs. Following Theorem 2.8.1 of Topkis (1998), I know that the optimal policy increases with a'.

For the next step, I move on to the optimization problem B.5. Following Proposition 2 in Hopenhayn and Prescott (1992), I will show the supermodularity of the value function $V(a,z)$ and policy function $a'(a,z)$ increase with a.

I need to verify the following three conditions using equations B.5 and B.6. First, $u((1+r)a + \pi(z,a') - a')$ is supermodular in (a,a') and has increasing differences in (a,a') given any z. Second, the graph of the feasibility set $\{a'|a' \geq 0\}$ is a sublattice. Third, $\lambda(\cdot,z)$ is increasing in z with respect to the first-order stochastic dominance. The second and third conditions are straightforward in this

[28] Topkis, D. M. (1998). Submodularity and Complementarity. Princeton University Press.

case. I next need to show that the first condition holds, which is equivalent to showing $\frac{\partial^2 u}{\partial a \partial a'} \geq 0$.

$$\frac{\partial^2 u}{\partial a \partial a'} = u''(c)(1+r)(\pi_{a'}(z,a') - 1).$$

Since $u''(\cdot) < 0$, it is sufficient to show that $\underline{\pi_{a'}(z,a') \leq 1}$.

Write the the Lagrangian of the problem B.6 as follows:

$$\begin{aligned}
\mathcal{L} \quad = \quad & AzF(k,l,x) - (r+\delta)k - wl - x + r^{tc}(AR - AP) \\
& + \xi(\gamma_1 a' + \gamma_2 AR - AzF(k,l,x) - (1+r^{tc})(AR - AP)) \\
& + \chi_1(AzF(k,l,x) - AR) + \chi_2 AR + \chi_3(x - AP) + \chi_4 AP.
\end{aligned}$$

Note that this is different from the Lagrangian of the full dynamic problem B.3. With a slight abuse of notation, I use the same $(\xi, \chi_1, \chi_2, \chi_3, \chi_4)$ to represent the Lagrangian multipliers in both problems.

The FOCs are:

$$k: \qquad AzF_k = \frac{r+\delta}{1-\xi+\chi_1} \tag{B.7}$$

$$l: \qquad AzF_l = \frac{w}{1-\xi+\chi_1} \tag{B.8}$$

$$x: \qquad AzF_x = \frac{1-\chi_3}{1-\xi+\chi_1} \tag{B.9}$$

$$AR: \qquad r^{tc} = \xi(1+r^{tc}-\gamma_2) + \chi_1 - \chi_2 \tag{B.10}$$

$$AP: \qquad r^{tc} = \xi(1+r^{tc}) - \chi_3 + \chi_4 \tag{B.11}$$

There are two cases I need to investigate. The first one is if the borrowing constraint is not binding ($\xi(a',z) = 0$) and the second one is if it is binding ($\xi(a',z) > 0$).

(1) $\xi = 0$

If $\xi = 0$, from equation B.10 and B.11, I know that $\chi_1 = r^{tc}$, $\chi_2 = 0$, $\chi_3 = 0$ and $\chi_4 = r^{tc}$. Therefore equation B.7, B.8 and B.9 become

$$\begin{aligned}
k: \qquad & AzF_k = \frac{r+\delta}{1+r^{tc}} \\
l: \qquad & AzF_l = \frac{w}{1+r^{tc}} \\
x: \qquad & AzF_x = \frac{1}{1+r^{tc}}
\end{aligned}$$

Denote the solution to the above system of equations as k^*, l^*, x^* and the corresponding output $y^* = AzF(k^*, l^*, x^*)$. Because $x_1 > 0$ and $x_4 > 0$, the complementary slackness conditions imply that $AR = y^*$ and $AP = 0$. As a result, given z, $\pi(z,a')$ does not change with a', as a result, $\pi_{a'}(z,a') = 0$.

(2) $\xi > 0$

If ξ, I know that the borrowing constraint holds with equality. That is,

$$y + (1 + r^{tc})(AR - AP) = \gamma_1 a' + \gamma_2 AR \Longrightarrow y = \gamma_1 a' - (1 + r^{tc} - \gamma_2)AR + (1 + r^{tc})AP$$

By definition, $\pi(z, a') = y - (r + \delta)k - wl - x + r^{tc}(AR - AP)$. Replacing y using the above equation yields

$$\pi(z, a') = \gamma_1 a' - (1 - \gamma_2)AR + AP - (r + \delta)k - wl - x$$

Since I have shown that $\frac{dk}{da'} \geq 0, \frac{dl}{da'} \geq 0, \frac{dx}{da'} \geq 0$, $\frac{dAR}{da'} \geq 0$ and $\frac{dAP}{da'} \leq 0$, and because $\gamma_1 < 1$, I have $\pi_{a'}(z, a') < 1$. *Q.E.D.* □

B.3.1 Proof of Proposition 1

Cut-off for financial constraint Given z, define set $\mathbf{U}^z = \{a|\xi(a, z) = 0\}$. I intend to show that the set $\mathbf{U}^z$ is in the following form $(\underline{a}, \infty)$.[29] To do this, I first show that $\mathbf{U}^z$ has the following property: if $a \in \mathbf{U}^z$ and $\hat{a} > a$, then $\hat{a} \in \mathbf{U}^z$.

Let $a \in \mathbf{U}^z$. According to the definition of $\mathbf{U}^z$, I know that $\xi(a, z) = 0$. The complementary slackness condition then implies that for entrepreneur (a, z), the working capital constraint is not binding,

$$AzF(k, l, x) + (1 + r^{tc})(AR - AP) < \gamma_1 a' + \gamma_2 AR.$$

According to equation B.4 and B.4, $\xi = 0$ implies that $\chi_2 = 0$, $\chi_1 = \frac{1}{c} r^{tc}$, $\chi_3 = 0$ and $\chi_4 = \frac{1}{c} r^{tc}$. Taking the value of ξ, χ_1, χ_2 back into equations B.4, B.4 and B.4, I get

$$\begin{aligned}
k: &\quad AzF_k = \frac{r + \delta}{1 + r^{tc}}, \\
l: &\quad AzF_l = \frac{w}{1 + r^{tc}}, \\
x: &\quad AzF_x = \frac{1}{1 + r^{tc}}.
\end{aligned}$$

Since production function F is decreasing return to scale, there exist optimal k, l and x that solve the above system of three equations. Denote the solution as k^*, l^* and x^*. Since $\chi_1 > 0$ and $\chi_4 > 0$, the complementary slackness condition implies that $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$.

Let $m = Az\left((k^\alpha l^{1-\alpha})^{1-\chi} x^\chi\right)^\mu - (r + \delta)k - wl - x$, and the budget constraint B.1 can be re-written as,

$$c + a' = (1 + r)a + m.$$

It is clear that m is maximized when $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$. In other words, entrepreneurs will always choose $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$ if they are feasible under the working capital constraint (equation B.2).

[29] This statement is equivalent to the first part of Proposition 1.

Consider the entrepreneur with productivity z and wealth $\hat{a} > a$. According to Lemma 1, $a'(\hat{a}, z) \geq a'(a, z)$. Therefore, since $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$ are feasible for entrepreneur (a, z), they must be feasible for entrepreneur $(\hat{a}, z)$ as well. Following the above analysis, I know that entrepreneurs will choose $k = k^*$, $l = l^*$, $x = x^*$, $AR = AzF(k^*, l^*, x^*)$ and $AP = 0$, and the working capital constraint holds with strict inequality. Using the complementary slackness condition, this implies that $\xi(\hat{a}, z) = 0$.

With the help of this property, I show that $\mathbf{U}^z$ is an interval. Suppose that it is not; then there exists $x < w < y$, such that $x, y \in \mathbf{U}^z$ but $w \notin \mathbf{U}^z$. This violates the property, since it means $x \in \mathbf{U}^z$, $w < x$, but $w \notin \mathbf{U}^z$. I can also show that $\mathbf{U}^z$ is unbounded from above. Suppose that it is not; then there exists $w \notin \mathbf{U}^z$ but $w > a$ for all $a \in \mathbf{U}^z$, which violates the property.

Cut-off for AR Define a set $\mathbf{H}^z = \{a | AR(a, z) > 0\}$. I show that $\mathbf{H}^z$ is in the form of $(\underline{a}, \infty)$. The proof is very similar. Essentially, I need to prove that the set $\mathbf{H}^z$ has the following property: if $a \in \mathbf{H}^z$ and $\hat{a} > a$, then $\hat{a} \in \mathbf{H}^z$. It is clear that this property holds since according to Lemma 1, $AR(a, z)$ is an increasing function in a. Therefore, for any $\hat{a} > a$, I have $AR(\hat{a}, z) \geq AR(a, z) > 0$.

Cut-off for AP Similarly, define a set $\mathbf{W}^z = \{a | AP(a, z) = 0\}$. I can show that $\mathbf{W}^z$ is in the form of $(\underline{a}, \infty)$. According to Lemma 1, $AP(a, z)$ is a decreasing function in a. Therefore for any $\hat{a} > a$, I have $0 \leq AP(\hat{a}, z) \leq AP(a, z) = 0$. As a result, $AP(\hat{a}, z) = 0$. *Q.E.D.*

B.3.2 Proof of Proposition 2

Proving this proposition is equivalent to showing that $\mathbf{U}^z \subseteq \mathbf{W}^z \subseteq \mathbf{H}^z$. I do it in two steps.

$\mathbf{U}^z \subseteq \mathbf{W}^z$ Take any $a \in \mathbf{U}^z$ and ξ be the Lagrangian multiplier associated with it; I know that $\xi = 0$ according to the definition of $\mathbf{U}^z$. According to equation B.4, if $\xi = 0$ then $\frac{1}{c(a,z)} r^{tc} = \chi_4(a, z) - \chi_3(a, z)$. Since $\frac{1}{c(a,z)} r^{tc} > 0$, it has to be the case that $\chi_4(a, z) = \frac{1}{c(a,z)} r^{tc}$ and $\chi_3(a, z) = 0$. Apply the complementary slackness condition, I know $AP(a, z) = 0$, which means $a \in \mathbf{W}^z$.

$\mathbf{W}^z \subseteq \mathbf{H}^z$ For any $a \in \mathbf{W}^z$, I know $AP(a, z) = 0$, thus the complementary slackness condition implies that $\chi_4(a, z) > 0$ and $\chi_3(a, z) = 0$. Therefore equation B.4 implies $\frac{1}{c} r^{tc} > \xi(1 + r^{tc})$. As a result, $\frac{1}{c} r^{tc} > \xi(1 + r^{tc} - \gamma)$ because $\xi(a, z) \geq 0$. Take $\frac{1}{c} r^{tc} > \xi(1 + r^{tc} - \gamma)$ back to equation B.4, I get $\chi_1(a, z) > 0$ and $\chi_2(a, z) = 0$. The complementary slackness condition implies that $AR(a, z) > 0$, which means $a \in \mathbf{H}^z$. *Q.E.D.*

B.4 Equilibrium definition of the counterfactual economy

The stationary equilibrium of the counterfactual economy without trade credit is defined as follows:

Definition. The recursive competitive equilibrium consists of interest rate of rental capital r, wage rate w; value function of the entrepreneurs $V(a, z)$; policy functions $c(a, z)$, $k(a, z)$, $l(a, z)$,

$x(a,z)$, and $a'(a,z)$; consumption and hours of the workers (c^h, h); and the CDF of the stationary distribution $\Phi(a,z)$, such that

1. Given prices, the value functions and policy functions solve the entrepreneurs' problem.

$$\begin{aligned} V(a,z) &= \max_{c,k,l,x,AR,AP,a'} log(c) + \beta \mathbb{E}_{z'} V(a', z'), \\ s.t. \quad & c + a' = (1+r)a + Az((k^{\alpha} l^{1-\alpha})^{1-\chi} x^{\chi})^{\mu} - (r+\delta)k - wl - x, \\ & Az((k^{\alpha} l^{1-\alpha})^{1-\chi} x^{\chi})^{\mu} \leq \gamma_1 a', \ a' \geq 0. \end{aligned}$$

2. Given prices, the consumption and hours of the workers solve the workers' problem.
3. Labor market clears

$$\int l(a,z) d\Phi(a,z) = N \cdot h.$$

4. Rental capital market clears

$$\int k(a,z) d\Phi(a,z) = \int a d\Phi(a,z).$$

5. Goods market clear

$$\int y(a,z) d\Phi(a,z) = N \cdot c^h + \int [c(a,z) + a'(a,z) - a + x(a,z)] \Phi(a,z).$$

B.5 Trade credit default

My model assumes that trade credit does not suffer from moral hazard issues, and that it is always repaid. This is a a simplifying assumption that can potentially be relaxed. For instance, Altinoglu (2018), which introduces an interesting trade credit contract structure, where firms can divert trade credit from suppliers. Although the cost of this diversion remains constant, the marginal benefit increases with the proportion of inputs purchased on trade credit. Based on these principles, suppliers are likely to restrict the size of trade credit they offer, ensuring that the marginal cost always exceeds the marginal benefit.

Relaxing the assumption. Incorporating the moral hazard problem described by Altinoglu (2018) into my model is feasible, because these two models share some similar structures. This modification would effectively introduce an additional constraint on the size of trade credit available. Specifically, it would cap the amount of trade credit such that AP $\leq \gamma_3 x$ and AR $\leq \gamma_3 y$, where $\gamma_3 < 1$. For context, under my current setup without moral hazard considerations, trade credit is only limited by its natural feasibility, which dictates that AP $\leq x$ and AR $\leq y$. This is equivalent to the case where $\gamma_3 = 1$ and I will call this the benchmark model in the discussion below.

Next, I will explore the addition of this extra constraint to my model. This will effectively transform the firms' problem into the following:

$$
\begin{aligned}
V(a,z) = & \max_{c,k,l,x,\mathrm{AR},\mathrm{AP},a'} \log(c) + \beta \mathbb{E}_{z'|z} V(a',z'), \\
s.t. \quad & c + a' = (1+r)a + Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} - (r+\delta)k - wl - x \\
& + r^{tc}(\mathrm{AR} - \mathrm{AP}), \\
& Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu} + (1+r^{tc})(\mathrm{AR} - \mathrm{AP}) \leq \gamma_1 a' + \gamma_2 \mathrm{AR}, \\
& 0 \leq \mathrm{AR} \leq \gamma_3 Az\left((k^{\alpha}l^{1-\alpha})^{1-\chi}x^{\chi}\right)^{\mu}, \qquad \text{(B.12)} \\
& 0 \leq \mathrm{AP} \leq \gamma_3 x, \qquad \text{(B.13)} \\
& a' \geq 0.
\end{aligned}
$$

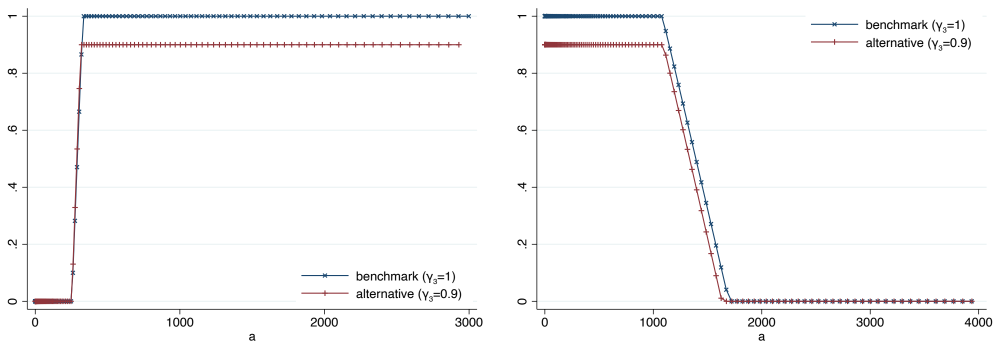
Notice the changes to the two constraints B.12 and B.13 with the addition of γ_3. This problem's FOCs can be written as the following:

$$
\begin{aligned}
k : AzF_k &= \frac{r+\delta}{1-\mu_1+\chi_1\gamma_3}, \\
l : AzF_l &= \frac{w}{1-\mu_1+\chi_1\gamma_3}, \\
x : AzF_x &= \frac{1-\chi_3\gamma_3}{1-\mu_1+\chi_1\gamma_3}, \\
AR : r^{tc} &= \mu_1(1-\gamma_2) + \chi_1 - \chi_2, \\
AP : r^{tc} &= \mu_1 - \chi_3 + \chi_4.
\end{aligned}
$$

Compared to the previous first-order conditions, we observe that the equations with respect to k, l, and x now include the additional γ_3 term, while those with respect to AR and AP remain unchanged. Therefore, the analysis concerning the cut-off values for the AR and AP choices proceeds in the same manner as in the old model. However, the only difference is that the upper bounds of AR and AP, should firms decide to borrow or lend trade credit, will be lower than before, binding at $\gamma_3 y$ and $\gamma_3 x$ instead of y and x.

To illustrate the changes in the trade credit policy function, Figure B.7 below displays the trade credit policy functions AR/y and AP/x, for the benchmark case $\gamma_3 = 1$ and the alternative case $\gamma_3 = 0.9$, under the same prices. This figure clearly shows the changes in policy functions. Notably, the upper bounds for the trade credit choices have been reduced from the benchmark $\gamma_3 = 1$ to the alternative setup $\gamma_3 = 0.9$, however, the threshold value of wealth remains roughly unchanged. Importantly, even with $\gamma_3 = 0.9$, for a given z, the lending of trade credit still increases with a while the borrowing of trade credit decreases with a. In equilibrium, trade credit continues to flow from relatively unconstrained to constrained firms, and the redistributive effects of trade credit are maintained in the model.

In my model, the production function is decreasing returns to scale and there exists a continuum of heterogeneous firms. As a result, the aggregate trade credit size can attain an interior value. Specifically, its size in equilibrium is predominantly determined by two critical factors: the financial conditions of the firms, which are influenced by parameters γ_1 and γ_2, and the prevailing price of trade credit in the market. Incorporating the exogenous constraint γ_3 into the model does not alter the key mechanisms that I have previously highlighted; however, it adds complications to the calibration process. This complexity arises because any changes to γ_3 tend to have offsetting effects



(a) AR/y **(b)** AP/x

Figure B.7: Trade credit choice in the benchmark and alternative models

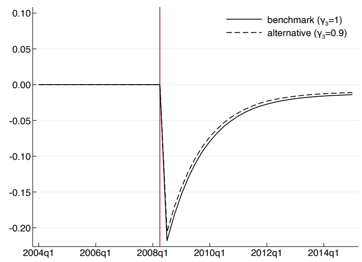
on γ_1 and γ_2. For instance, a reduction in γ_3 would require corresponding increases in γ_1 and γ_2 to ensure the model matches trade credit and bank loan sizes observed in the data. As a result, attempting to calibrate the model can often lead to indeterminate outcomes.

Given the calibration challenges, in the next exercise, I opted to fix γ_3 at 0.9 and recalibrate the remaining parameters. In this recalibrated alternative model, I introduced shocks to γ_1 and γ_2, which produced a drop in bank and trade credit similar to that in the benchmark model (see Figure B.8 panels a-b). The output dynamics following these shocks show a similar magnitude of decline in both models (panel c). A breakdown of different inputs reveals that while the dynamics in total factor productivity (TFP) are very similar, there are slight differences in labor and capital inputs, potentially due to variations in the calibrated discount factor β and Frisch elasticity ψ. In short, this exercise reveals that both models exhibit comparable dynamics following financial shocks. However, due to the complications that the additional constraint of γ_3 introduces, I prefer to retain the benchmark model as is, to maintain transparency and straightforwardness in the calibration process.

Default in the data. Another feature that my model shares with Altinoglu (2018) is the absence of trade credit default in *equilibrium*, as the trade credit contract precludes such occurrences. It is therefore worthwhile to examine the prevalence of trade credit default in the data and to discuss whether this is a reasonable assumption.

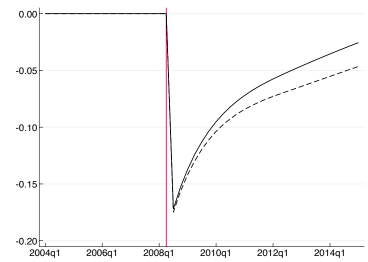
Often in the data, firms are labeled as having 'defaulted on trade credit,' which might imply one of two scenarios: (i) delays in payment beyond pre-agreed dates, or (ii) complete default or non-payment. Essentially, the first type of 'default' serves as a means for firms to extend their borrowing from suppliers by delaying payment. Importantly, my model captures this type of 'default,' as it manifests in an increase in the size of accounts receivable on firms' balance sheets. Through the lenses of the model, the nature of this type of default is that it leads to delayed cash flow for suppliers who extend trade credit, and this delay is costly for suppliers who are financially constrained, but less so for those who are not. The option to delay payments to suppliers is compensated by an interest rate on trade credit, which offset the loss of liquidity for constrained suppliers.

In contrast, the second type of 'default' – non-payment – is not captured by the model, as including it would require allowing trade credit defaults in equilibrium. However, from what I see

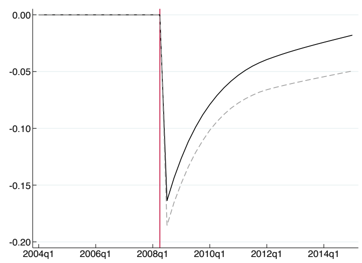


(a) Credit market liability to capital stock ratio

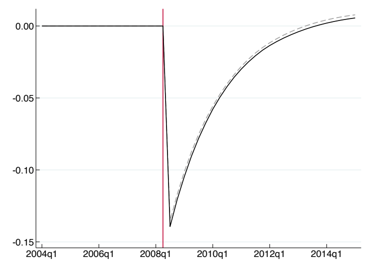
(b) Trade credit to credit market liability ratio



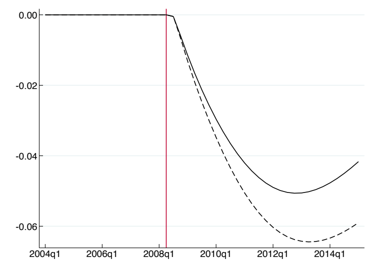
(c) Output



(d) Hours



(e) TFP



(f) Capital

Figure B.8: Model dynamics following financial shocks

from the existing literature, this type of default is not very prevalent in the data. For instance, Boissay and Gropp (2007) report that of all recorded trade credit default cases, only approximately 2% are due to insolvency leading to non-payment. The authors also document that, at the quarterly frequency, the average default-to-payable ratio is approximately 2%, indicating that the loss from true non-payment of trade credit is about 0.04%. The size of this loss is rather small compared to the trade credit interest rate in the model, which is approximately 4%. Considering the relatively minor impact indicated by these figures, the inclusion of equilibrium default in the current model would not substantially modify its quantitative properties.

C Quantitative analysis appendix

C.1 Algorithms

In this section, I describe the algorithms for computing the benchmark model. The algorithms to compute the counterfactual model are very similar to the benchmark model, only with different sets of FOCs, budget constraints, and working capital constraints. Hence they are omitted here.

C.1.1 Stationary equilibrium

- Guess equilibrium prices r, w, r^{tc}.
- Given the prices, solve the household problem.
- Given the prices, solve the entrepreneurs problem as follows:
  - Discretize the state space.
  - Guess policy function $c(a, z)$.
  - For each (a, z), assume that the entrepreneur is unconstrained, i.e., $\mu(a, z) = 0$. Solve for the system of equations that consists of FOCs and budget constraint.
  - Check whether the working capital constraint is satisfied with the solution to the above system of equations.
  - If the working capital constraint is not satisfied, it means that $\mu(a, z) > 0$ and working capital constraint holds with equality. Solve the system of equations that consists of FOCs, budget constraint, and working capital constraint (with equality).
  - Use the Euler equation to update the policy function $c(a, z)$ until it converges.
- Given any arbitrary distribution of (a, z), iterate using the policy functions derived above until a stationary distribution is reached.
- Generate the aggregate statistics of the three markets: capital, labor, and trade credit market.
- Update (r, w, r^{tc}) until the markets clear simultaneously.

C.1.2 Transitional dynamics

To compute the transitional dynamics of the economy, I consider a transition path of $T = 100$ periods. The economy is at the initial stationary equilibrium level in period $t = 1$, and I assume that it converges back to the initial stationary equilibrium at period $t = T$.

- Guess a sequence of prices $\{r_t, w_t, r_t^{tc}\}_{t=2}^{T-1}$.
- Backward induction. For each $t = T-1, T-2, ..., 2$,
  - Discretize the state space.
  - Given prices, solve the household problem for period t.

- Given prices, solve the entrepreneurs' policy functions for period t.
    1. Guess $c_t(a,z)\mu_t(a,z) = 0$, solve the system of equations that consists of FOCs of period t, budget constraint, and Euler equations (with the next period policy function $c_{t+1}(a,z)$ known).
    2. Check whether the working capital constraint is satisfied under the above solution.
    3. If the working capital is not satisfied, $c_t(a,z)\mu_t(a,z) > 0$ and the working capital constraint holds with equality. Solve the system of equations that consists of FOCs of period t, budget constraint, Euler equations (with the next period policy function $c_{t+1}(a,z)$ known), and working capital constraint with equality.

- Forward induction. The first period stationary distribution $\Phi_1(a,z)$ is set to be the stationary equilibrium distribution. Using the policy functions for period $t = 2, ..., T-1$, compute the distribution along the transition path $\Phi_t(a,z)$.

- Generate aggregate statistics for the four markets in every period $t = 2, ..., T-1$ using the policy functions and the distributions.

- Update $\{r_t, w_t, r_t^{tc}\}_{t=2}^{T-1}$ until the four markets clear simultaneously in each period $t = 2, ..., T-1$.

C.2 Firm size distribution in the model and data

This section compares the firm-size distribution between the model-generated data and the Business Dynamics Statistics (BDS) database, which covers the universe of firms in the US. To operationalize the comparison, I focus on four size categories. These size categories are firms with 1 to 0 employees, 10-99 employees, 100-999 employees, and 1000+ employees. According to the BDS, the fraction of firms in these four categories in 2005 was 76, 22, 1.8, and 0.2 percent. Firms in our model are then categorized into four groups to match this distribution, once they are sorted by size. As discussed in Section 4.1, I calibrate the Pareto shape parameter of the firm productivity distribution to match the employment share of the largest firm size categories. Figure C.9 compares the full distribution between the model and the BDS data. It shows that the mapping between the two is reassuring: not only is the firm grouping similar to the data in terms of the percentage of firms in each size category (panel a) but also in terms of how average employment increases over the size categories (panel b).

C.3 The shadow value of liquidity function

I begin by looking at how the shadow value of liquidity ξ varies with wealth a for a given productivity level. Figure C.10 shows that ξ decreases in a, indicating that producers are less constrained with higher wealth. Notably, ξ is not strictly monotone; the function is divided into six segments by three values of ξ: $\{0,\ r^{tc},\ r^{tc}/(1-\gamma_2)\}$. Each segment is associated with different value ranges for AR and AP.

(a) Firm and employment shares **(b)** Average employment

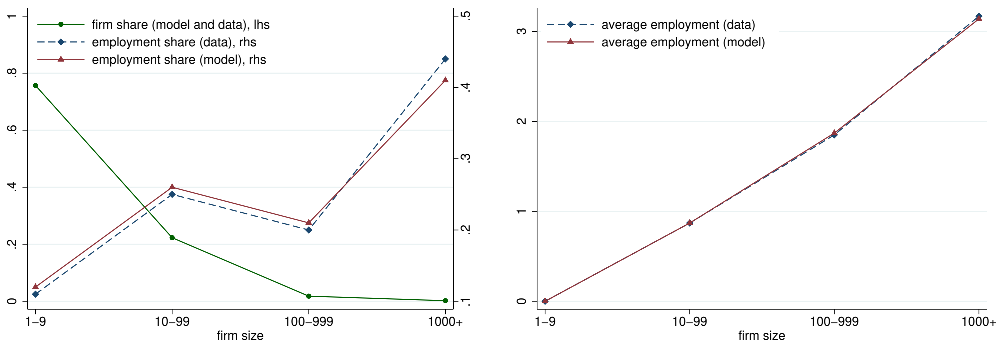


Figure C.9: Firm size distribution in the data and model

Notes: Panel (a) shows the share of firms in each size categories in the model and the Business Dynamics Statistics (BDS) data (left axis) and the corresponding employment shares (right axis). Panel (b) shows the (logarithms of) average employment by size categories, with the smallest category normalized to 0.

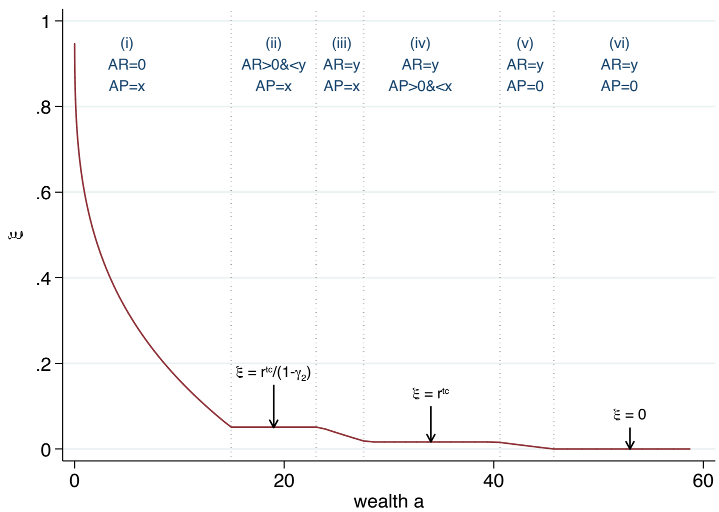


Figure C.10: Shadow value of liquidity and its role in determining trade credit choices

Notes: This figure plots the liquidity value ξ as a function of wealth a for a given z. The function ξ decreases in a, but not strictly decreasing. There are three regions where ξ is constant, marked by $\xi \in \{r^{tc}/(1-\gamma_2), r^{tc}, 0\}$. As such, the function ξ is divided into six segments, each associated with a different value range for AR and AP.

C.4 Policy functions

The policy functions of AR and AP as shown in figure 6 indicates that some entrepreneurs choose not to lend or borrow trade credit (AR $= 0$ or AP $= 0$). In the data, however, it is very rare for firms to have zero AR or AP on their balance sheet. In this section, I show that this discrepancy can be solved by recognizing that measured AR and AP is a snapshot of firms' activities and they likely reflect the averages of AR and AP over several sales and purchases that overlapped in time.

In figure C.11, I plot AR/y and AP/x averaged over three sales/production cycles. More specifically, I track each entrepreneurs over three periods and use average $\{\text{AR}, \text{AP}, x, y, a\}$ to plot this figure. The shapes of AR/y and AP/x can more closely resemble the data patterns. In particular, they no longer take the corner values as before.

(a) AR/y **(b)** AP/x

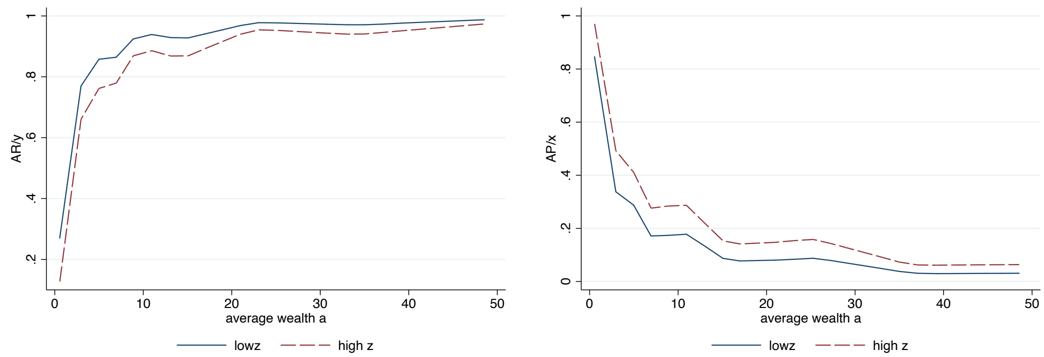


Figure C.11: Trade credit choice, average over three periods

Notes: This figure displays trade credit choices as a function of wealth a for given low and high values of z. Panel (a) shows the average AR/average output over three periods. Panel (b) shows the average AP/average input costs over three periods.

In the data, firms might make multiple sales and purchases that overlap in time; therefore, the observed AR and AP may reflect the average of these sales and purchases. In this exercise, I consider the case where AR, AP, and sales reflect the averages of three cycles of sales/production. Although in the model, I do not explicitly model the *overlap* of these sales, this exercise conveys the simple idea that, in the presence of idiosyncratic productivity randomness (shocks to z), taking averages smooths out the policy functions; hence they can better mimic the data.

C.5 Shifts in policy functions following the Lehman shock

Figure C.12 shows the policy functions for the two groups before and immediately after the collateral shock. Panel (a) shows the ξ function shifts upward for both groups, indicating an increased value for liquidity. Panel (b) shows that, after the shock, there is a leftward shift in AR/y of the low-exposure group and a rightward shift of the high-exposed group. In other words, the low-exposure entrepreneurs increase while the high-exposure ones cut back their lending of trade credit. Interestingly, as shown in panel (c), there is a slightly different pattern for AP/x. The curve of AP/x shifts to the right – the borrowing of trade credit increases – for both groups. Importantly, the increase in AP/x is more significant for the high-exposure than the low-exposure entrepreneurs.

The fact that the low-exposure group increased their their lending of trade credit (panel b, blue line) and their borrowing of trade credit (panel c, blue line) simultaneously reveals the intertwined nature of AR, AP, and production in equilibrium. Following a negative financial shock, trade credit becomes more expensive due to decreased supply and increased demand for trade credit. In turn, the higher trade credit interest rate has two opposing effects on the choice of AP. On the one hand, as trade credit becomes more costly, entrepreneurs would want to borrow less of it. On the other hand, higher trade credit interest rates also raise entrepreneurs' profitability (if they lend trade credit), increasing the optimal production scale. To expand production, some entrepreneurs need to borrow more trade credit.

(a) ξ **(b)** AR/y **(c)** AP/x

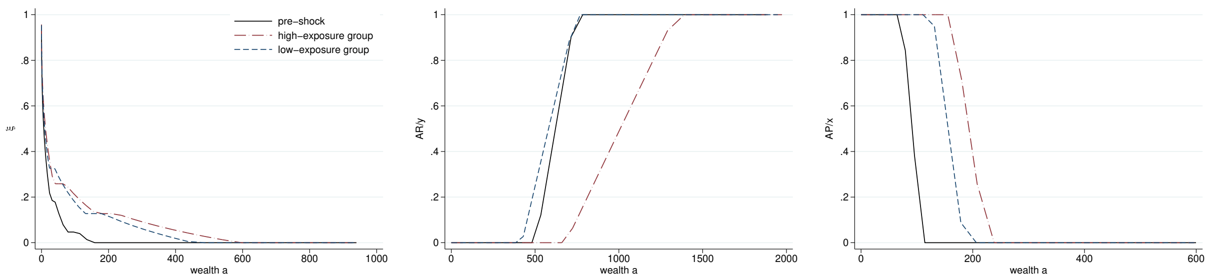


Figure C.12: Shifts in policy functions by severity of the financial shock

Notes: These figures illustrate the shift in policy function after the negative shock for the low-exposure group (blue line) and the high-exposure group (red line). The black line shows the pre-shock policy function. Panel (a) plots the liquidity value as a function of wealth for given productivity. Panels (b) and (c) show the lending and borrowing of trade credit, respectively.