Online Appendix for "Money, Growth, and Welfare in a Schumpeterian Model with the Spirit of Capitalism" (Not for Publication)*

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Online Appendix A: Inelastic Labor Supply and CIA on R&D Only

The inelastic labor supply is captured by $\eta = 0$. For simplicity, we assume $\beta = 1$.[1] Repeating similar steps as in the main text, we have the free labor mobility condition as:

$$(\gamma - 1)\, l_{x,t} = (1+i)\left(l_{r,t} + \rho/\varphi - \frac{\theta c_t}{\varphi a_t}\right). \tag{1}$$

The CIA constraint binds: $b_t = m_t$. The output market clearing condition gives $c_t = y_t/L$. Given $e_t = v_t/L$, the consumption wealth ratio is:

$$\frac{c_t}{a_t} = \frac{y_t}{v_t + b_t L} = \frac{\gamma w_t L_{x,t}}{(1+i)\, w_t L_{r,t}/\lambda + w_t L_{r,t}} = \frac{\gamma l_{x,t}}{(1+i)/\varphi + l_{r,t}}. \tag{2}$$

Plugging the labor market clearing condition $l_{r,t} + l_{x,t} = 1$ and (2) into (1), we get a univariate quadratic equation for manufacturing labor l_x:

$$\Phi_1 l_x^2 - \Phi_2 l_x + \Phi_3 = 0, \tag{3}$$

*The views expressed here are the opinions of the authors only and do not necessarily represent those of the Federal Reserve Bank of Kansas City or the Federal Reserve System. All remaining errors are our responsibility.

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[1] We also discuss the case of elastica labor supply and $\beta < 1$.

where $\Phi_1 = (\gamma + i)\,\varphi$, $\Phi_2 = (1+i)\,(\varphi+\rho) + (\gamma+i)\,(1+i+\varphi) + (1+i)\,\theta\gamma$ and $\Phi_3 = (1+i)\left(1+\frac{\rho}{\varphi}\right)(1+i+\varphi)$. We can rewrite (3) as

$$\left(l_x - \frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i}\right)\left(l_x - \frac{\varphi+1+i}{\varphi}\right) = \frac{(1+i)\,\theta\gamma}{\varphi\,(\gamma+i)}l_x. \tag{4}$$

Proposition 1 *Under inelastic labor supply, CIA on R&D only and* $\frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i} \leq 1$, *the effect of a higher nominal interest rate on growth depends on the degree of the SOC* (θ), *whereas it is negative without the SOC. When the growth-enhancing effect of the SOC dominates (more likely with a larger* θ*), a higher nominal interest rate increases growth; when the growth-reducing effect from the CIA on R&D dominates, a higher nominal interest rate reduces growth.*

Proof. Using (4), when $\theta = 0$, the solutions are two positive numbers, one is smaller than one ($\frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i} \leq 1$) and the other is larger than one ($\frac{\varphi+1+i}{\varphi} > 1$). Because manufacturing labor cannot be larger than 1, the only admissible solution is the smaller positive root $l_x|_{\theta=0} = \frac{(1+i)\left(1+\frac{\rho}{\varphi}\right)}{\gamma+i}$ (point A in Figure 1). When $\theta = 0$, $\frac{\partial l_x|_{\theta=0}}{\partial i} > 0$. That is, when the nominal interest rate increases, point A moves to the right along the horizontal axis to B, yielding a larger manufacturing labor l_x and thereby a lower R&D labor l_r (i.e., lower long-run growth).

When $\theta > 0$, the solutions to (4) are the intersections of the parabola on the LHS (left-hand side) of (4) and the positively-sloped straightline passing through the origin of coordinates (the RHS (right-hand side) of (4)): the smaller positive root becomes smaller and the larger positive root becomes larger. The admissible solution is the smaller positive root (the equilibrium is point C in Figure 1).

When the nominal interest rate increases, the parabola on the LHS of (4) shifts to the right. Meanwhile, the straightline on the RHS of (4) rotates counter-clockwise. The new equilibrium will be point D. Obviously, whether point D is to the left or right of point C depends on the size of θ. Therefore, whether a higher nominal interest rate increases/decreases manufacturing labor l_x depends on the size of θ. ■

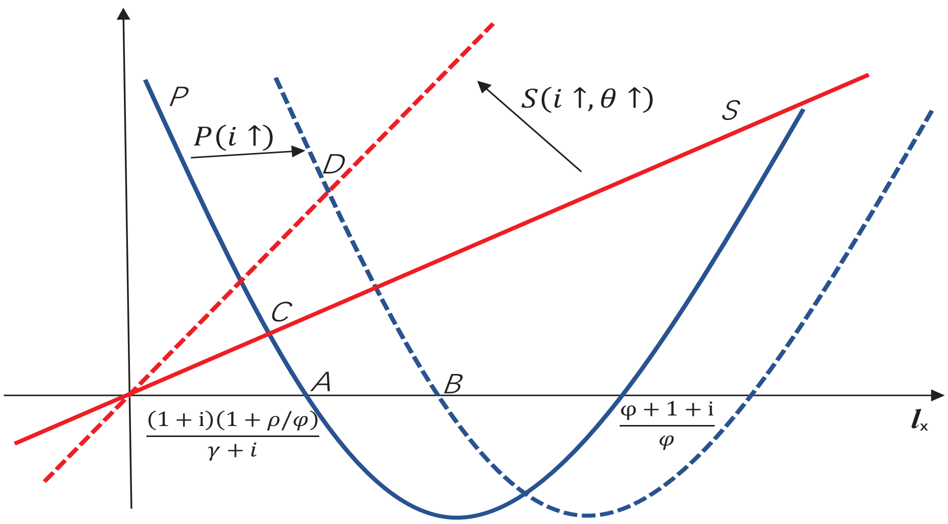
When labor supply is elastic and $\beta < 1$, the proof can be shown similarly using the two conditions that pin down the equilibrium labor allocation: the labor market clearing condition and the free labor mobility condition (as in our proof of Proposition 2 in the main text).

In this case, the labor market clearing condition becomes

$$l_{r,t} + l_{x,t} = \left(\frac{1}{\eta\gamma l_{x,t}}\right)^{\sigma}. \tag{5}$$

Graphically, the L curve remains the same as in Figure 1 in the main text.

Figure 1: Equilibrium Labor Allocation under CIA on R&D and Inelastic Labor Supply



Repeating similar steps, the free labor mobility condition becomes

$$(\gamma - 1)\, l_{x,t} = (1 + \beta i) \left(l_{r,t} + \rho/\varphi - \frac{\theta c_t}{\varphi a_t} \right). \tag{6}$$

We now have $\frac{c_t}{a_t} = \frac{\gamma l_{x,t}}{(1+\beta i)/\varphi + \beta l_{r,t}}$. Taken together, the free labor mobility condition becomes

$$(\gamma - 1)\, l_{x,t} = (1 + \beta i) \left(l_{r,t} + \rho/\varphi - \frac{\theta \gamma l_{x,t}}{(1 + \beta i) + \beta \varphi l_{r,t}} \right), \tag{7}$$

which gives the upward-sloping M curve as in Figure 1 in the main text.

Observing (5), an increase in the nominal interest rate will not shift the L curve in Figure 1 in the main text. That is, the nominal interest rate has no effect on the consumption-leisure choice and thereby labor supply when the CIA on consumption is absent. By contrast, given a change in the nominal interest rate, both the SOC and the CIA on R&D will affect growth through the free labor mobility condition (i.e., through shifting the M curve).

On the one hand, under the CIA on R&D, an increase in the nominal interest rate raises the borrowing cost of entrepreneurs, shifting labor away from R&D to manufacturing. This effect comes from the $(1 + \beta i)$ term in the first bracket on the RHS of (7). Graphically, an increase in the nominal interest rate leads to a rightward shift in the M curve and the magnitude of the shift increases with the strength of the CIA on R&D β (using Figure 1 in the main text), thereby decreasing R&D labor l_r and increasing manufacturing labor l_x, all else equal.

On the other hand, due to the existence of the SOC, an increase in the nominal interest rate leads to a leftward shift in the M curve in the Figure 1 in the main text, thereby increasing

R&D labor l_r and decreasing manufacturing labor l_x, all else equal. This effect comes from the $\frac{(1+\beta i)\theta c_t}{\varphi a_t}$ term on the RHS of (6), which is the $\frac{(1+\beta i)\theta\gamma l_{x,t}}{(1+\beta i)+\beta\varphi l_{r,t}}$ term on the RHS of (7). This term is increasing in the nominal interest rate because $\frac{(1+\beta i)\theta\gamma l_{x,t}}{(1+\beta i)+\beta\varphi l_{r,t}} = \frac{\theta\gamma l_{x,t}}{1+\beta\varphi l_{r,t}/(1+\beta i)}$ when the CIA on R&D decreases R&D labor l_r and increases manufacturing labor l_x, all else equal, following an increase in the nominal interest rate (shown above). Given the same increase in the nominal interest rate, a higher degree of the SOC θ leads to a larger leftward shift in the M curve in Figure 1 in the main text, thereby increasing R&D labor l_r and decreasing manufacturing labor l_x, all else equal.

Therefore, the total effect of the nominal interest rate on R&D labor (and thereby long-run growth) depends on the relative magnitude of the degree of the SOC θ versus the strength of the CIA on R&D (i.e., β). Without the SOC, the effect of a higher nominal interest rate on R&D labor is negative. When the SOC is strong enough, the overall effect could be positive.

Online Appendix B: Inelastic Labor Supply and CIA on Consumption Only

In this section, inelastic labor supply is captured by $\eta = 0$, which yields $l_{r,t} + l_{x,t} = 1$. The CIA constraint becomes $c_t \leq m_t$.

Proposition 2 *Under inelastic labor supply and CIA on consumption, the steady state growth increases with the nominal interest rate i as long as $\theta > 0$ and $\frac{1+\rho/\varphi}{\gamma} \leq 1$. Moreover, the higher the degree of the SOC (i.e., a larger θ), the larger the effect of the nominal interest rate on growth, all else equal.*

Proof. Using $l_{r,t} + l_{x,t} = 1$, imposing $\beta = 0$ and repeating similar steps, we end up with a univariate quadratic equation for manufacturing labor l_x:

$$\gamma^2 \varphi l_x^2 + \Lambda_2^1 l_x - \Lambda_3 = 0, \tag{8}$$

where $\Lambda_2^1 = \gamma \left[1 - (\varphi + \rho) + \theta (1 + i)\right]$ and $\Lambda_3 = 1 + \rho/\varphi$. Because $\gamma^2\varphi > 0$ and $\Lambda_3 > 0$, (8) always has one negative root and one positive root. Because manufacturing labor cannot be negative, the only admissible solution is the positive root.

When $\theta = 0$ (i.e., there is no SOC), using (8), we have the standard Schumpeterian model with the CIA constraint on consumption. l_x is determined by

$$(\gamma l_x - \Lambda_3)(\gamma \varphi l_x + 1) = 0, \tag{9}$$

where the two roots are $l_x|_{\theta=0} = \frac{\Lambda_3}{\gamma} = \frac{1+\rho/\varphi}{\gamma}$ and $l_x|_{\theta=0} = -\frac{1}{\gamma\varphi}$. The only admissible solution is the positive root $l_x|_{\theta=0} = \frac{\Lambda_3}{\gamma} = \frac{1+\rho/\varphi}{\gamma}$. Therefore, we assume $\frac{1+\rho/\varphi}{\gamma} \leq 1$ to ensure $l_x \leq 1$.

When the nominal interest rate increases, the y-intercept of the quadratic function in (8) $-\Lambda_3$ remains constant, so does the coefficient of the quadratic term $\gamma^2\varphi$ (i.e., the shape of the parabola remains unchanged). The axis of symmetry of the quadratic function in (8) is $-\frac{\Lambda_2^1}{2\gamma^2\varphi}$, which shifts to the left as the nominal interest rate increases. Taken together, the graph of the quadratic function in (8) (i.e., the parabola) remains unchanged and wholly shifts to the left as the nominal interest rate increases. Therefore, the positive root of the quadratic equation (i.e., manufacturing labor l_x) decreases. As a result, R&D labor l_r and thereby long-run growth would increase as the nominal interest rate i increases. One can observe from (8) that the increase in θ and that in i tend to impact l_x similarly, which proves the second part of the Proposition. ■

When $\theta = 0$ (i.e., there is no SOC), l_r and l_x (thereby growth and welfare) are independent of the nominal interest rate under inelastic labor supply and CIA on consumption.

The intuition is as follows. Under inelastic labor supply, the effect of the nominal interest rate on labor supply through the consumption-leisure choice is absent. Without the SOC, the free labor mobility condition is not affected by the nominal interest rate under the CIA constraint on consumption. Therefore, without the SOC, both the R&D labor l_r and manufacturing labor l_x (thereby growth and welfare) are not functions of the nominal interest rate (i.e., money is superneutral). By contrast, the existence of the SOC will yield the same labor reallocation effect that shifts labor away from manufacturing to R&D when the nominal interest rate increases. As a result, long-run growth increases and a higher degree of the SOC increases the effect of the nominal interest rate on growth.

Under inelastic labor supply and CIA on consumption, Corollary 1 still holds. Concerning welfare, imposing $\eta = 0$ yields

$$U = \frac{1}{\rho}\left[\ln\left(l_{x,0}\right) + \theta\ln\left(\frac{1}{\varphi} + \gamma l_{x,0}\right) + \frac{(1+\theta)\,g}{\rho} + \Omega\right], \tag{10}$$

where Ω is a constant.

Proposition 3 *Under inelastic labor supply and CIA on consumption, the steady state welfare increases with the nominal interest rate i when $\theta \in (0, \underline{\theta})$ and $\frac{\rho}{\varphi\ln\gamma} < \frac{1+\rho/\varphi}{\gamma} \leq 1$, and it decreases with i when $\theta \in \left(\overline{\theta}, \infty\right)$ and $\rho > \frac{\ln\gamma}{\gamma}$.*

Proof. Taking derivative of U in (10) with respect to i, using $\partial l_{x,0}/\partial i = -\partial l_{r,0}/\partial i$ under inelastic labor supply and $\partial g/\partial i = \varphi\ln\gamma\partial l_{r,0}/\partial i$, we have

$$\frac{\partial U}{\partial i} = \frac{\partial U}{\partial l_{x,0}}\frac{\partial l_{x,0}}{\partial i} + \frac{\partial U}{\partial g}\frac{\partial g}{\partial i} = \frac{1}{\rho}\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\,\varphi\ln\gamma}{\rho}\right]\underset{(-)}{\frac{\partial l_{x,0}}{\partial i}}, \tag{11}$$

where $\partial l_{x,0}/\partial i < 0$ when $\theta > 0$, according to the previous Proposition. Therefore, $sign\left(\frac{\partial U}{\partial i}\right) > 0$ if $\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\varphi\ln\gamma}{\rho}\right] < 0$. We have

$$\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\,\varphi\ln\gamma}{\rho}\right] < \left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{\varphi\gamma l_{x,0}} - \frac{(1+\theta)\,\varphi\ln\gamma}{\rho}\right] \tag{12}$$

$$= (1+\theta)\left[\frac{1}{l_{x,0}} - \frac{\varphi\ln\gamma}{\rho}\right]. \tag{13}$$

According to the previous Proposition, we have proved that $l_x|_{\theta=0} = \frac{1+\rho/\varphi}{\gamma} \leq 1$ for any i under inelastic labor supply. Therefore, we first need $\frac{1}{l_x|_{\theta=0}} < \frac{\varphi\ln\gamma}{\rho}$, which is equivalent to $\frac{\rho}{\varphi\ln\gamma} < \frac{1+\rho/\varphi}{\gamma}$. Moreover, the previous Proposition shows that $l_{x,0}$ is decreasing in both θ and i. Therefore, for a given i, when θ increases from zero, $l_{x,0}$ decreases from $\frac{1+\rho/\varphi}{\gamma}$. Therefore, as long as $\theta < \underline{\theta}$ ($\underline{\theta}$ is pinned down by $l_{x,0}\left(\underline{\theta}\right) = \frac{\rho}{\varphi\ln\gamma}$), we have $sign\left(\frac{\partial U}{\partial i}\right) > 0$. However, when θ continues to increase, it is possible that $\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\varphi\ln\gamma}{\rho}\right] > 0$. We can find the $\overline{\theta} > \underline{\theta}$ by solving $\left[\frac{1}{l_{x,0}} + \frac{\varphi\gamma\theta}{1+\varphi\gamma l_{x,0}} - \frac{(1+\theta)\varphi\ln\gamma}{\rho}\right] = 0$ ($l_{x,0}$ is a function of θ). If $\rho > \frac{\ln\gamma}{\gamma}$, the middle term in the square bracket increases at a rate of $\varphi\gamma\theta$ as θ approaches infinity, whereas the third term increases at a rate of $\frac{\theta\varphi\ln\gamma}{\rho}$. The middle term increases faster if $\rho > \frac{\ln\gamma}{\gamma}$, and $sign\left(\frac{\partial U}{\partial i}\right) < 0$ when $\theta \in \left(\overline{\theta}, \infty\right)$. ■

The intuition is as follows. According to (10), an increase in the nominal interest rate has two opposing effects on the welfare: (1) it reduces welfare by decreasing manufacturing labor $l_{x,0}$ (thereby initial consumption and wealth) and (2) it increases welfare through raising the growth rate g. When the nominal interest rate increases, the SOC (i.e., households enjoy direct utility from holding wealth) makes households change their consumption-portfolio choice by reducing initial consumption: substituting consumption with more savings in the form of equity shares. The value of innovations increases due to lower borrowing costs, which generates *the labor reallocation effect* that increases R&D labor and thereby growth. When θ is below a threshold, the welfare gain from higher future consumption and wealth dominates the welfare loss from lower initial consumption and wealth, and total welfare increases with the nominal interest rate. However, when θ is beyond a threshold, the welfare loss from lower initial consumption and wealth dominates the welfare gain from higher future consumption and wealth, and total welfare decreases with the nominal interest rate.

The reason to impose $\frac{\rho}{\varphi\ln\gamma} < \frac{1+\rho/\varphi}{\gamma} \leq 1$ is as follows. For instance, when $\gamma = 1$, we have $sign\left(\frac{\partial U}{\partial i}\right) \leq 0$. That is, in an economy without growth (when the step-size of innovation is 1), welfare cannot be increasing in the nominal interest rate even when $\theta > 0$. This is understandable because the trade-off between lower initial consumption and wealth and higher future consumption and wealth is possible only when there is growth. When $\rho > \frac{\ln\gamma}{\gamma}$ (a larger ρ means households are more impatient and discount future consumption more heavily), it is more likely that the welfare loss from lower initial consumption/wealth dominates the welfare gain from higher future consumption/wealth.

Under inelastic labor supply and CIA on consumption, we show that higher nominal interest rates always increase growth when $\theta > 0$. By contrast, our results also indicate that higher nominal interest rates increase welfare when $\theta > 0$ and θ is below a threshold; beyond a threshold, the effect of a higher nominal interest rate on welfare depends on structural parameters. Therefore, concerning optimal monetary policy, the Friedman rule (Friedman, 1969) of keeping the nominal interest rate at zero is suboptimal when $\theta > 0$ and θ is below a threshold; the Friedman may be optimal when θ is above a threshold (this is possible in theory, but in real world situations, θ may not be that large). We summarize this point in the following Corollary.

Corollary 2 *Under inelastic labor supply and CIA on consumption, the Friedman rule is suboptimal when $\theta > 0$ and θ is below a threshold; depending on structural parameters, the Friedman may be optimal when θ is above the threshold.*

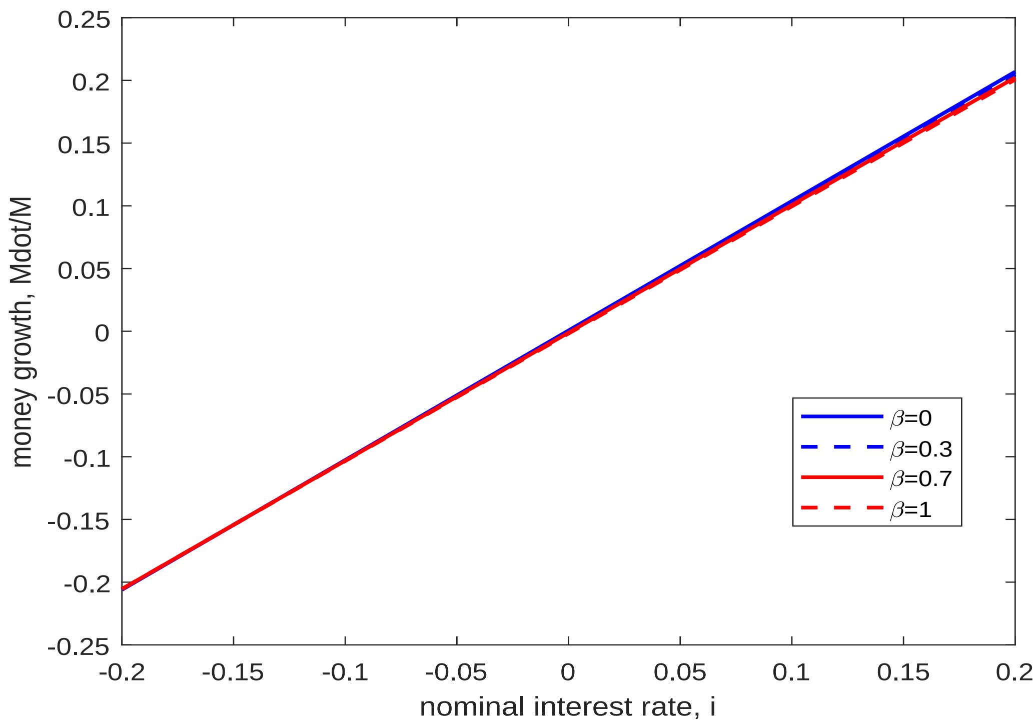
Online Appendix C: Money Supply and Nominal Interest Rate

In this online appendix, we show that under our calibrated parameter values, an increase in money supply also leads to an increase in the nominal interest rate based on the following equation:

$$\frac{\dot{M}_t}{M_t} = i_t - \rho + \theta \left(1 + i_t\right) \frac{c_t}{a_t}. \tag{14}$$

The existence of the SOC slightly complicates the relationship between the nominal interest rate i_t and the money supply growth rate $\dot{M}_t/M_t$ by introducing an extra term on the right side of (14). However, as Figure 2 shows, the positive relationship is well maintained under the calibrated parameter values and different values of β (the tightness of the CIA constraint on R&D).

Figure 2: Relationship between Money Supply and Nominal Interest Rate



Online Appendix D: A Non-monetary Schumpeterian Model with the SOC

Without money, the new asset-accumulation equation becomes

$$\dot{e}_t = r_t e_t + w_t l_t - c_t. \tag{15}$$

Per capita wealth a_t becomes $a_t = e_t$. The optimal condition for labor supply can thus be written as:

$$l_t = \left(\frac{w_t}{\eta c_t}\right)^{\sigma}. \tag{16}$$

The Euler equation is

$$-\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho + \frac{\theta}{\mu_t a_t} = r_t - \rho + \frac{\theta c_t}{a_t}. \tag{17}$$

The dynamics of the model remain the same: it immediately jumps to the (saddle-point stable) balanced growth path on which the labor allocations are stationary and unique.

Now the labor market clearing condition becomes

$$l_{r,t} = \left(\frac{1}{\eta\gamma}\right)^{\sigma} l_{x,t}^{-\sigma} - l_{x,t}. \tag{18}$$

The final goods market clearing condition gives $c_t = y_t/L$. We have $e_t = v_t/L$. Therefore, the consumption wealth ratio is

$$\frac{c_t}{a_t} = \frac{y_t}{v_t} = \frac{\gamma w_t L_{x,t}}{w_t L_{r,t}/\lambda} = \varphi\gamma l_{x,t}. \tag{19}$$

The free labor mobility condition is $(\gamma - 1)\, l_{x,t} = l_{r,t} + \rho/\varphi - \theta c_t/(\varphi a_t)$. Combining with (19) yields

$$l_{r,t} = (\gamma - 1)\, l_{x,t} - \rho/\varphi + \theta\gamma l_{x,t}. \tag{20}$$

Proposition 4 *Given $(\eta\gamma)^{-\sigma/(1+\sigma)} \leq 1$, the steady state growth increases with θ (the degree of the SOC), all else equal.*

Proof. For the labor market clearing condition, we still have: $\frac{\partial l_{r,t}}{\partial l_{x,t}} < 0$ and $\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} > 0$. Given $\lim_{l_{x,t}\to 0} l_{r,t} = \infty$, the labor market clearing condition becomes $l_{r,t} + l_{x,t} = 1$ when $l_{x,t}$ is below a threshold. We assume $l_{x,t}|_{l_r=0} = \left(\frac{1}{\eta\gamma}\right)^{\sigma/(1+\sigma)} \leq 1$. Therefore, the labor market clearing condition is described by the L curve in Figure 3. Using the free labor mobility condition, we have $\frac{\partial l_{r,t}}{\partial l_{x,t}} = (\gamma - 1) + \theta\gamma > 0$ and $\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} = 0$. Therefore, the M curve for the free labor mobility condition is a straight upward-sloping line. Using Figure 3, an increase in θ does not shift the L curve, but it does rotate the M curve counter-clockwise around $(0, -\rho/\varphi)$ (i.e., the equilibrium would move from O to E), thereby increasing R&D labor l_r and decreasing manufacturing labor l_x, all else equal. ■

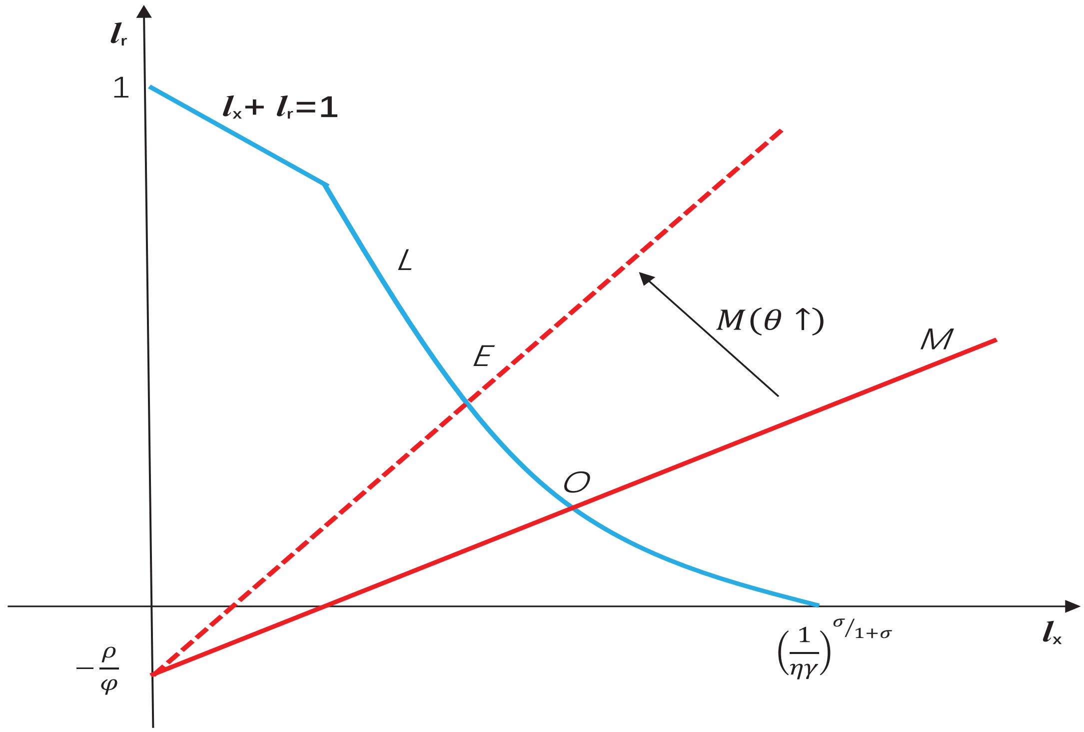


Figure 3: **Effect of the SOC on Labor Allocation in a Non-monetary Schumpeterian Model** This graph shows the determination of equilibrium labor allocation in the R&D sector and the production sector in a Schumpeterian model without money. The L is the labor-market clearing condition and the M curve is the labor mobility condition.

The intuition behind this Proposition can be explained in a similar way. With the SOC, the savings decision will depend on $\theta c_t/a_t + r_t - \rho$. An increase in θ raises the marginal benefit of saving through the direct preference for wealth. This would lead people to substitute more consumption with savings, which drives down the real interest rate and the borrowing cost of entrepreneurs. This increases the return to entrepreneurial investment and attracts more labor into R&D (causing labor reallocation from manufacturing to R&D). However, an increase in θ would not affect the labor supply choice, leaving total labor supply unchanged (i.e., the consumption-leisure choice effect is absent). The larger labor reallocation effect due to a higher θ results in larger R&D labor and thereby higher long-run growth.

This result echoes the finding in the literature that the existence of the SOC promotes growth in the Ramsey-Cass-Koopmans capital accumulation models, the AK model (see Zou, 1994), the Romer's expanding-varieties model (Pan et al., 2018), and Romer's expanding-varieties model with capital accumulation (Hof and Prettner, 2019).

Online Appendix E: Comparison with a Monetary AK Model with the SOC

In this section, we replace the Schumpterian growth model with an AK growth model. To introduce money into the growth model, we continue to assume a CIA constraint on consumption. The utility function remains the same. Each individual is endowed with one unit of labor. There is no population growth and total population is fixed at L. Each household maximizes its lifetime utility (the same as the benchmark model) subject to the asset-accumulation equation:

$$\dot{k}_t + \dot{m}_t = Ak - c_t - \pi_t m_t + \tau_t, \tag{21}$$

where k_t is the real value of capital stock held by each person, and A is the level of technology; the other variables remain the same as before. Here the per capita wealth is $a_t = k_t + m_t$. The CIA constraint is still $c_t \leq m_t$.

Proposition 5 *In the AK model with the SOC and a CIA constraint on consumption, growth in the steady state increases with the nominal interest rate (i.e., money is not superneutral). Without the SOC, money is superneutral.*

Proof. Household's Hamiltonian function in this AK model is given by

$$H_t = \ln c_t + \theta \ln (a_t) + \mu_t (Ak_t - c_t - \pi_t m_t + \tau_t) + \xi_t (m_t - c_t),$$

where μ_t is the co-state variable; ξ_t is the Lagrangian multiplier for the CIA constraint; $a_t = k_t + m_t$.

The corresponding first-order conditions are given by:

$$\frac{\partial H_t}{\partial c_t} = \frac{1}{c_t} - \mu_t - \xi_t = 0, \tag{22}$$

$$\frac{\partial H_t}{\partial k_t} = \frac{\theta}{a_t} + A\mu_t = \rho\mu_t - \dot{\mu}_t, \tag{23}$$

$$\frac{\partial H_t}{\partial m_t} = \frac{\theta}{a_t} - \mu_t\pi_t + \xi_t = \rho\mu_t - \dot{\mu}_t. \tag{24}$$

On the balanced growth path, we have $\dot{c}_t/c_t = \dot{k}_t/k_t = g_{ak}$. Using the resource constraint, we have $\dot{k}_t = Ak_t - c_t$, which gives

$$\frac{c_t}{k_t} = A - g_{ak}. \tag{25}$$

Combining (23) and (24) yields $\xi_t = (A + \pi_t)\mu_t$. We have $m_t = M_t/(P_t L)$, which yields $\dot{m}_t/m_t = \dot{M}_t/M_t - \pi_t$. The binding CIA constraint yields $\dot{c}_t/c_t = \dot{m}_t/m_t = g_{ak}$. Taken together, we have

$$\xi_t = \left(A + \frac{\dot{M}_t}{M_t} - g_{ak}\right)\mu_t. \tag{26}$$

Using the Fisher equation $i_t = \pi_t + r_t$, we have $i_t = \dot{M}_t/M_t - \dot{m}_t/m_t + A$, where we have used $r_t = A$ in the AK model. Therefore, we have $i_t = \dot{M}_t/M_t - g_{ak} + A$, and combining with (26) yields $\xi_t = i_t\mu_t$. Plugging this condition into (22) yields

$$\frac{1}{c_t} = \mu_t (1 + i_t), \tag{27}$$

which shows that, given $i_t = i$, on the balanced growth path, $\dot{c}_t/c_t = -\dot{\mu}_t/\mu_t = g_{ak}$.

Using (23), we have

$$-\frac{\dot{\mu}_t}{\mu_t} = A - \rho + \frac{\theta}{\mu_t a_t}. \tag{28}$$

Plugging (27) into (28), we have the Euler equation

$$-\frac{\dot{\mu}_t}{\mu_t} = g_{ak} = A - \rho + \frac{\theta c_t (1+i)}{a_t}. \tag{29}$$

In equilibrium, the CIA constraint binds, which gives $c_t = m_t$. Using (25), we can rewrite (29) as

$$g_{ak} = A - \rho + \frac{\theta (1+i)}{1 + \frac{1}{A - g_{ak}}}. \tag{30}$$

Rewriting (30), we have the univariate quadratic equation for g_{ak}:

$$g_{ak}^2 - \Omega_1 g_{ak} + \Omega_2 = 0, \tag{31}$$

where $\Omega_1 = \theta (1+i) + (A+1) + (A-\rho)$ and $\Omega_2 = \theta A (1+i) + (A+1)(A-\rho)$.

When $i \geq -1$, we have $\Omega_1 > 0$ and $\Omega_2 > 0$. Therefore, the quadratic equation (31) has two positive roots when $\Delta = \Omega_1^2 - 4\Omega_2 > 0$ and no solution (in real numbers) when $\Delta < 0$. We assume $\Delta > 0$ because $\Delta|_{\theta=0} > 0$.

When $\theta = 0$ (i.e., there is no SOC), we have

$$g_{ak}^2 - [(A+1) + (A-\rho)] g_{ak} + (A+1)(A-\rho) = 0, \tag{32}$$

where the two roots are $g_{ak}|_{\theta=0} = A + 1$ and $g_{ak}|_{\theta=0} = A - \rho$. The only admissible solution is the smaller root $g_{ak}|_{\theta=0} = A - \rho$. That is, money is superneutral in the AK model with a CIA constraint on consumption (see also Dotsey and Sarte, 2000).

Therefore, when $\theta > 0$, focusing on the unique perfect-foresight equilibrium, the only admissible solution is the smaller root of (31). Thus, the balanced growth rate is

$$g_{ak} = \frac{\Omega_1 - \sqrt{\Omega_1^2 - 4\Omega_2}}{2}. \tag{33}$$

When the nominal interest rate i increases, the y-intercept Ω_2 of the quadratic function increases, and its axis of symmetry $\frac{\Omega_1}{2}$ shifts to the right. Therefore, the smaller positive root of the quadratic function (i.e., the balanced growth rate) increases. ∎

The intuition behind this proposition is similar to that provided in Proposition 2. It is worth noting that Sidrauski (1967) proves the superneutrality of money in a qualitatively equivalent approach which assumes money enters the utility function.[2] As explained by Sidrauski (1967, p. 544), "In the short run, an increase in the rate of monetary expansion is equivalent to a rise in government transfers to the private sector. It therefore results in an increase in consumption and a fall in the rate of capital accumulation." In addition, monetary expansion pushes people to substitute real balance with capital holdings (the Mundell-Tobin portfolio-shift effect that refers to shifts between assets related to inflation changes, see Mundell, 1965; Tobin, 1965), which tends to reduce consumption under the CIA constraint on consumption. The two opposing effects (the positive Sidrauski government transfer effect and the negative Mundell-Tobin portfolio-shift effect) offset each other, leaving the steady state consumption, savings and capital stock unchanged in Ramsey models and the saving rate and thereby long-run growth unchanged in the AK model (see e.g., Dotsey and Sarte, 2000). In other words, with a CIA constraint on consumption, long-run growth in capital accumulation models depends solely on the real return to capital accumulation r_t (i.e., not on labor supply l_t) that is independent of monetary policy because the nominal interest rate does not alter the saving behavior of households nor the marginal productivity of capital.

To recap, in the AK model with the CIA constraint on consumption, money is superneutral without the SOC because the aforementioned two opposing effects (the positive Sidrauski government transfer effect and the negative Mundell-Tobin portfolio-shift effect) on consumption still offset each other, leaving the saving rate and thereby long-run growth unchanged. By contrast, in the presence of the SOC, a higher nominal interest rate yields a larger consumption-portfolio effect that decreases the marginal propensity to consume due to stronger saving motivation. In the AK model, labor supply has no effect on long-run growth (i.e., the negative consumption-leisure choice effect is absent). Taken together, long-run growth is an increasing function of the nominal interest rate.

There is one important difference between our Schumpeterian model and the AK model. That is, a higher nominal interest rate always increases growth in the AK model, while it possibly decreases growth in our Schumpeterian model. The reason is as follows. In the Schumpeterian model with the SOC and elastic labor supply, there exists the consumption-leisure choice channel from a higher nominal interest rate. This consumption-leisure choice effect through decreasing

[2] This is an equivalent way, as using a CIA constraint, to introduce money into growth models.

market size tends to reduce growth when the nominal interest rate increases. When this effect dominates the labor reallocation effect, a higher nominal interest rate reduces long-run growth. However, this consumption-leisure choice effect is absent in the AK model and thus growth always increases with the nominal interest rate in the AK model.

Online Appendix F: Alternative Utility Function Nesting Chu and Cozzi (2014)

Maintaining the utility function in Chu and Cozzi (2014) helps identify the driving force of our main results from a comparative perspective. To do so, we use the following utility function:[3]

$$U = \int_0^{\infty} e^{-\rho t} \left[\ln(c_t) + \theta \ln(a_t) + \eta \frac{(1-l_t)^{1-\sigma}}{1-\sigma} \right] dt, \tag{34}$$

where c_t is per capita real consumption of final goods, a_t is per capita wealth, and l_t is per capita labor supply at time t. ρ is the rate of time preference; $\theta \geqslant 0$ captures the preference for wealth (i.e., the degree of the SOC, and a larger θ means a higher degree of the SOC); $\eta \geqslant 0$ is still the taste for leisure as in Doepke and Zilibotti (2008); $\sigma > 0$ determines the elasticity of labor supply.

This new utility specification has the advantage of nesting the Chu and Cozzi's (2014) log-log utility as a special case with $\sigma = 1$.

Unlike the Chu-Cozzi specification, both the new utility specification and the one used in our main text allow us to separate the elasticity of labor supply versus the taste for leisure. Both parameters are important in the macroeconomic literature (e.g., the taste for leisure is highlighted in Doepke and Zilibotti, 2008, as one fundamental parameter of the SOC, whereas the elasticity of labor supply is highlighted in Aghion, Akcigit and Fernández-Villaverde, 2013, for the optimal labor versus capital taxes in Schumpeterian models).

For the Chu-Cozzi utility specification, we have the following

$$\frac{1}{c_t} = \mu_t (1 + i_t) \text{ and } \frac{\eta}{1-l_t} = w_t \mu_t,$$

which gives

$$\frac{\eta c_t}{1-l_t} = \frac{w_t}{1+i_t}. \tag{35}$$

Given η and holding c_t and i_t fixed, the elasticity of leisure with respect to the wage rate is defined as:

$$\frac{d \ln(1-l_t)}{d \ln(w_t)} = -1. \tag{36}$$

To compute the elasticity of labor supply with respect to the wage rate, we approximate $\ln(1-l_t)$ around the steady state $\bar{l}$ as follows:

$$\ln(1-l_t) = \ln(1-\exp(l_t)) \cong \ln\left(1-\bar{l}\right) - \frac{\bar{l}}{1-\bar{l}} \ln\left(l_t - \bar{l}\right) = -1,$$

which means that

$$\frac{d \ln(l_t)}{d \ln(w_t)} = \frac{1-\bar{l}}{\bar{l}}. \tag{37}$$

[3] We thank Angus Chu for helpful discussion.

In other words, in the Chu and Cozzi (2014) setting, the elasticity of labor supply with respect to the wage rate is $\frac{1-\bar{l}}{\bar{l}}$, not 1 or η.

As a comparison, in our alternative additively separable utility function $\ln(c_t) + \theta \ln(a_t) - \eta \frac{l_t^{1+1/\sigma}}{1+1/\sigma}$, combining the two FOCs yields

$$\frac{1}{c_t} = \mu_t (1 + i_t) \text{ and } \eta l_t^{1/\sigma} = w_t \mu_t,$$

which obtains

$$\eta c_t l_t^{1/\sigma} = \frac{w_t}{1 + i_t}. \tag{38}$$

Given η and holding c_t and i_t fixed, the Frisch elasticity of labor supply with respect to the wage rate is defined as:

$$\frac{d \ln(l_t)}{d \ln(w_t)} = \sigma. \tag{39}$$

Separating these two parameters will not change our main predictions, but it does allow us to evaluate the separate roles of the taste for leisure (η) and the Frisch elasticity of labor supply (σ). Our model yields testable predictions concerning the two important macroeconomic parameters.

For our new utility specification that nests the Chu and Cozzi's (2014) log-log utility as a special case with $\sigma = 1$, repeating similar steps, we can calculate the elasticity of labor supply as $\frac{1}{\sigma}\frac{1-\bar{l}}{\bar{l}}$, which means the elasticity of labor supply is proportional to $\frac{1}{\sigma}$ (similar to the intertemporal elasticity of substitution for CRRA utility function). Therefore, the new utility function still allows us to separate and isolate the roles of the elasticity of labor supply versus the taste for leisure in the effect of monetary policy on long-run growth and welfare.

F.1. Solving the Model

In this alternative model, we still use the CIA constraint $c_t \leq m_t$.

Using Hamiltonian, we can derive the optimality condition for consumption

$$\frac{1}{c_t} = \mu_t (1 + i_t), \tag{40}$$

where μ_t is the Hamiltonian co-state variable, and $i_t = \pi_t + r_t$ (the Fisher equation) is the nominal interest rate.

The optimal condition for labor supply is

$$\eta (1 - l_t)^{-\sigma} = w_t \mu_t. \tag{41}$$

Using (40), we rewrite the optimal condition for labor supply as

$$l_t = 1 - \left[\frac{\eta c_t (1 + i_t)}{w_t}\right]^{\frac{1}{\sigma}}. \tag{42}$$

Equations (41) and (42) show that a higher nominal interest rate reduces labor supply through the consumption-leisure choice (i.e., *the negative consumption-leisure choice effect is still present*).

The Euler equation is

$$-\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho + \frac{\theta}{\mu_t a_t} = r_t - \rho + \frac{\theta\,(1+i_t)\,c_t}{a_t}, \tag{43}$$

where the last equality uses (40). When $\theta = 0$, we will have the monetary Schumpeterian model without the SOC. $\theta > 0$ will yield *the positive labor reallocation effect.*

Repeating similar steps, we have the free labor mobility condition as

$$(\gamma - 1)\, l_{x,t} = l_{r,t} + \rho/\varphi - \frac{\theta\,(1+i)\,c_t}{\varphi a_t}. \tag{44}$$

We have $a_t = e_t + m_t$. The CIA constraint binds: $c_t = m_t$. The output market clearing condition gives $c_t = y_t/L$. We have $e_t = v_t/L$. Therefore, the consumption wealth ratio is

$$\frac{c_t}{a_t} = \frac{y_t}{v_t + y_t} = \frac{\gamma w_t L_{x,t}}{w_t L_{r,t}/\lambda + \gamma w_t L_{x,t}} = \frac{\gamma l_{x,t}}{1/\varphi + \gamma l_{x,t}}. \tag{45}$$

Plugging (45) into (44), we have the free labor mobility condition as

$$(\gamma - 1)\, l_x = l_r + \rho/\varphi - \frac{\theta\,(1+i)\,\gamma l_x}{1 + \varphi\gamma l_x}. \tag{46}$$

Using the output market clearing condition $c_t = y_t/L$ and $y_t = \gamma w_t L_{x,t}$, the labor market clearing condition (42) becomes

$$l_{r,t} + l_{x,t} = l_t = 1 - \left[\eta\gamma\,(1+i_t)\,l_{x,t}\right]^{1/\sigma}. \tag{47}$$

The above two conditions pin down the equilibrium labor allocation.

Below we will show Proposition 2 (our key results in the paper) holds for $\sigma = 1$ (Chu and Cozzi 2014) and $\sigma \neq 1$.

We rewrite the labor market clearing condition (47) as

$$l_{r,t} = 1 - \left[\eta\gamma\,(1+i_t)\,l_{x,t}\right]^{1/\sigma} - l_{x,t}. \tag{48}$$

Using (48), we have

$$\frac{\partial l_{r,t}}{\partial l_{x,t}} = -\frac{1}{\sigma}\left[\eta\gamma\,(1+i_t)\right]^{1/\sigma} l_{x,t}^{1/\sigma - 1} - 1 < 0, \tag{49}$$

$$\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} = -\frac{1}{\sigma}\left(\frac{1}{\sigma} - 1\right)\left[\eta\gamma\,(1+i_t)\right]^{1/\sigma} l_{x,t}^{1/\sigma - 2} \gtrless 0. \tag{50}$$

As discussed above, $\frac{1}{\sigma}$ captures the elasticity of labor supply (a smaller σ means a higher elasticity of labor supply). It's easy to see $\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} < 0$ when $\sigma \in (0,1)$, $\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} > 0$ when $\sigma \in (1,\infty)$, and $\frac{\partial^2 l_{r,t}}{\partial l_{x,t}^2} = 0$ when $\sigma = 1$.

Therefore, the labor market clearing condition shows that, with manufacturing labor ($l_{x,t}$) on the horizontal axis, R&D labor share ($l_{r,t}$) is a downward-sloping concave (convex) function of $l_{x,t}$ when $\sigma \in (0,1)$ (when $\sigma \in (1,\infty)$). It is a downward-sloping linear function when $\sigma = 1$ (the case for Chu and Cozzi 2014). We denote this curve as the L curve in Figure 4. A higher elasticity of labor supply (a smaller σ) shifts the L curve leftward.

The free labor mobility condition is unchanged from our paper which is an upward-sloping concave function of manufacturing labor $l_{x,t}$ (with $l_{x,t}$ on the horizontal axis). We denote this curve as the M curve in Figure 4.

Figure 4: **Alternative Figure 1 (in the Paper) under the New Utility Function**

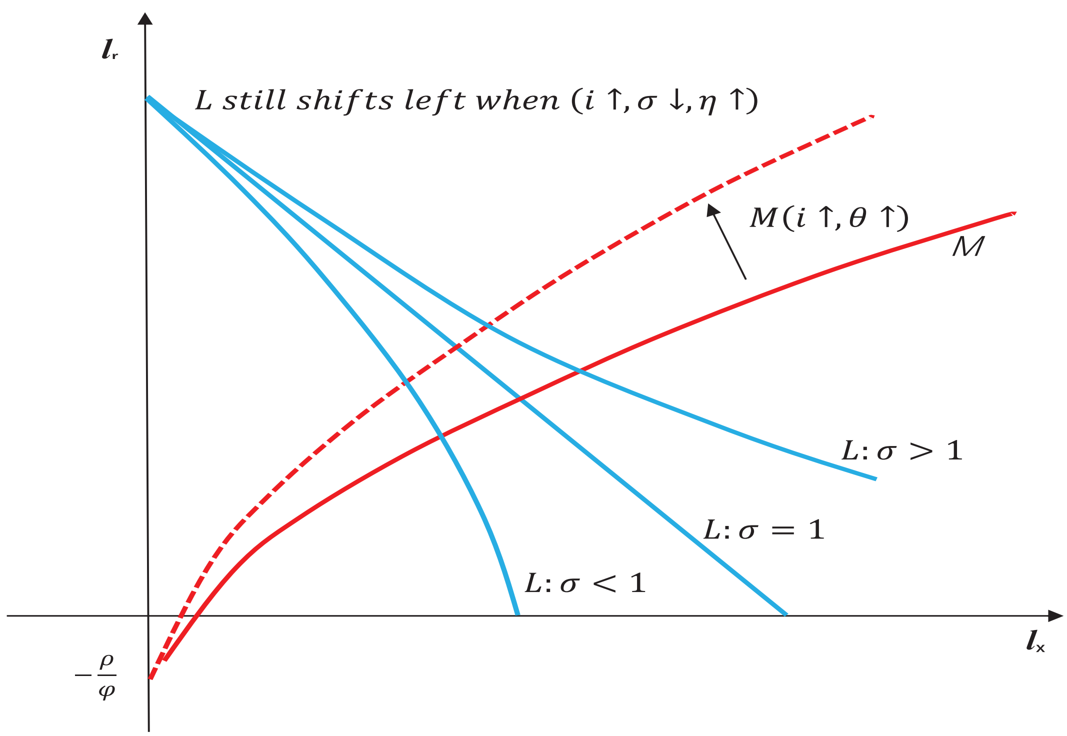


Figure 4 helps illustrate why the key mechanism is robust to $\sigma > 1$, $\sigma < 1$, and $\sigma = 1$. Our key mechanism is that an increase in the nominal interest rate (i) will shift the M curve to the left (or rotate it counterclockwise) while a stronger SOC (θ) amplifies this effect. For the L curve, an increase in the nominal interest rate shifts it to the left (or rotate it clockwise), for all values of σ). Therefore, for any value of σ, the equilibrium labor in the R&D sector (l_r) could rise if the M curve shifts to the left enough, leading to a higher long-run growth rate. In other words, as σ only matters for the shape of the L curve but does not influence the M curve, the key mechanism holds generally at different values of σ.

To briefly summarize, the consumption-leisure choice effect will always decrease labor supply with a higher nominal interest rate, the magnitude of the effect depends on the labor supply parameter σ. However, the growth-promoting effect of the SOC with a higher nominal interest rate does not hinge on the specific value of the labor-supply elasticity.

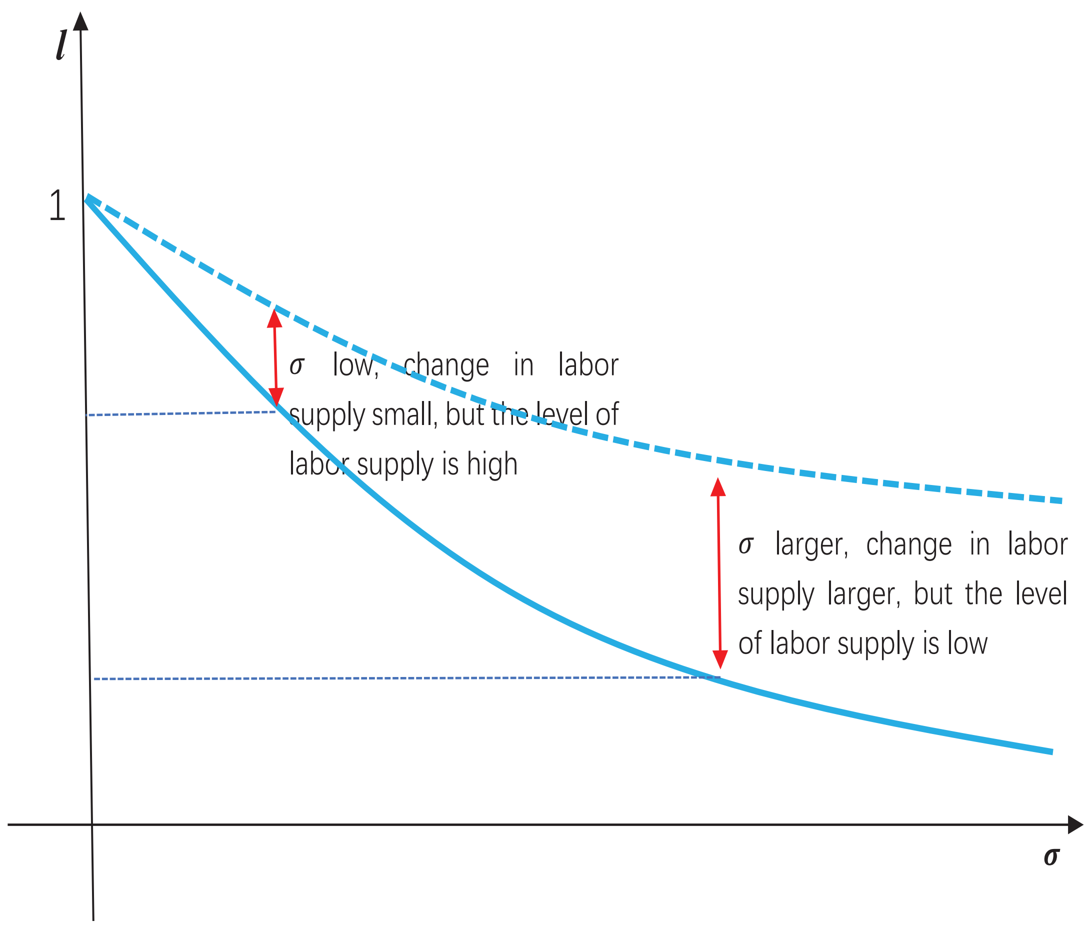
F.2. Further Discussions

It is worth noting that there exists a minimum level of θ under which our main prediction—with the SOC, higher nominal interest rates may promote log-run economic growth (a finding that was not found in the previous literature)—holds true. When $\eta = 0$ (i.e., inelastic labor supply), as long as $\theta > 0$, higher nominal interest rate would promote long-run economic growth. This is because the negative *consumption-leisure choice effect* is absent under inelastic labor supply, while the *labor reallocation effect* is positive when $\theta > 0$. When $\eta > 0$, however, this minimum level of θ obviously depends on other structural parameters, especially the taste for leisure η. When $\eta > 0$, for a given nominal interest rate level, an increase in the nominal interest rate will shift the M curve upwardly and the L curve leftward. So we need to find out the threshold $\underline{\theta}$ such that an upward shift in the M curve will offset the leftward shift of the L in keeping the equilibrium labor in the R&D sector (l_r) unchanged. However, even under the case $\sigma = 1$ which corresponds to Chu and Cozzi's (2014) log-log utility specification, plugging $l_{r,t} + l_{x,t} = 1 - \eta\gamma\,(1 + i_t)\, l_{x,t}$ into (46) to substitute out l_x leaves l_r as a non-linear function of i and θ, which is why an analytical solution for $\underline{\theta}$ is unavailable even for the case of $\sigma = 1$. However, we have quantitatively shown by figure in the paper that when θ increases from 0 to 0.3, the correlation between the nominal interest rate and growth changes from negative to positive.

Moreover, we can see the advantage of using the two utility functions (the one used in our main text and the new one that nests the Chu-Cozzi log-log utility as a special case with $\sigma = 1$): they both allow us to study the separate roles of the taste for leisure (η) and the elasticity of labor supply in the effect of higher nominal interest rate on long-run growth. For instance, in our main text model, when $\eta > 0$, labor supply is still inelastic when σ approaches zero. Although both cases (σ approaches zero and $\eta = 0$) correspond to inelastic labor supply (i.e., households spend all of time working, $l = 1$), the mechanism is different.

$\eta = 0$ means people love work (a low taste for leisure means a higher degree of the SOC as highlighted Doepke and Zilibotti, 2008). According to Doepke and Zilibotti (2008), an upper class imbued with disdain for work cultivate a refined taste for leisure, whereas a hard-working and frugal middle class develop patience and work ethics: "These class-specific attitudes, which are rooted in the nature of pre-industrial professions, become key determinants of success once industrialization transforms the economic landscape."

By contrast, even with $\eta > 0$, labor supply could also be inelastic when σ approaches zero in our main text model. The elasticity of labor supply measures the responsiveness of households' labor supply with respect to the changes in the wage rate, whereas the taste for leisure has nothing to do the responsiveness of the households' labor supply with respect to the changes in the wage rate. We can separatedly discuss the effect of the Frisch elasticity of labor supply, as shown in the following figure. Obviously, this change in labor supply is totally due to the change in elasticity of labor supply (for any given $\eta > 0$).



Online Appendix G: Introducing Money through Money in the Utility Function

In the following we will show that the same positive effect of a higher nominal interest on savings due to the existence of the SOC still exists if we use the MIUF (money-in-the-utility-function) approach, which indicates that our mechanism is quite robust as long as fiat money enters the households' problem.

At time t, the population size of each household is fixed at L. There is a unit continuum of identical households, which have a lifetime utility function as

$$U = \int_0^\infty e^{-\rho t} \left[\ln\left(c_t\right) + \theta \ln\left(a_t\right) + \xi \ln\left(m_t\right)\right] dt, \tag{51}$$

where m_t is per capita real balance and the other variables remain the same.

Each household maximizes its lifetime utility given in equation (51) subject to the asset-accumulation equation given by

$$\dot{e}_t + \dot{m}_t = r_t e_t + w_t - c_t - \pi_t m_t + \tau_t, \tag{52}$$

where the variables remain the same as in the main text.

Given the budget constraint, per capita wealth a_t is defined as $a_t = e_t + m_t$. The no-arbitrage condition is $i_t = \pi_t + r_t$, where i_t is also the nominal interest rate. Household's Hamiltonian function is

$$H_t = \ln c_t + \theta \ln\left(a_t\right) + \xi \ln\left(m_t\right) + \mu_t\left(r_t e_t + w_t - c_t - \pi_t m_t + \tau_t\right),$$

where μ_t is the co-state variable on (52).

The first-order conditions include:

$$\frac{\partial H_t}{\partial c_t} = \frac{1}{c_t} - \mu_t = 0, \tag{53}$$

$$\frac{\partial H_t}{\partial e_t} = \frac{\theta}{a_t} + \mu_t r_t = \rho\mu_t - \dot{\mu}_t, \tag{54}$$

$$\frac{\partial H_t}{\partial m_t} = \frac{\theta}{a_t} + \frac{\xi}{m_t} - \mu_t \pi_t = \rho\mu_t - \dot{\mu}_t. \tag{55}$$

Combining (54) and (55) yields

$$\frac{\xi}{m_t} = \mu_t\left(r_t + \pi_t\right) = \mu_t i_t. \tag{56}$$

The Euler equation is

$$-\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho + \frac{\theta}{\mu_t a_t}. \tag{57}$$

Using (53), (57) and (56), the consumption Euler equation is

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + \theta \frac{i_t m_t}{\xi a_t}. \tag{58}$$

This consumption Euler equation points to the same mechanism as we highlight under the CIA constraint on consumption. When $\theta = 0$ (i.e., without the SOC), the households' problem shows that the nominal interest rate has no direct effect on the real interest rate and thereby the consumption/savings decision of the households. The result is expected because the cross partial derivatives of the utility function are zeros (i.e., there is neither substitutability nor complementarity between consumption, real balance and leisure). Under inelastic labor supply, money is superneutral under this special MIUF model.

By contrast, when $\theta > 0$ (i.e., with the SOC), the preference for wealth accumulation tends to lower consumption by introducing a third term on the right side of equation (58). In general, this equation shows that, when people have the SOC, consumption-savings choices depend not only on the real return on wealth, $r_t - \rho$, but also on the marginal rate of substitution between wealth and real balance. In other words, as Michaillat and Saez (2019) also point out, people become less forward-looking and future interest rates have less impact on today's consumption than in the standard model. The modification of the Euler equation due to wealth in the utility in equation (58) still reveals that a higher nominal interest rate tends to increase the marginal rate of substitution between wealth and real balance. This makes households even less forward-looking, thereby increasing households' saving.

A couple of results follow. First of all, higher money growth reduces the demand for real balance because the opportunity cost of holding real balance is the foregone nominal interest rate. This is seen by combining (54) and (55) to get (56). That is, the left-hand-side of (56) is the additional marginal benefit of holding real balance, whereas the right-hand-side of (56) is opportunity cost of holding real balance. Therefore, higher money growth reduces the demand for real balance, which tends to generate the Mundell-Tobin effect because households would substitute their money holding with real asset, all else equal.

In addition, the consumption Euler equation indicates that a higher nominal interest rate would generate the consumption-portfolio effect by discouraging consumption and increasing savings. The reason is due to the increased marginal rate of substitution between wealth and real balance, the third term in (58).

References

[1] Aghion, P., Akcigit, U. and Fernández-Villaverde, J. 2013. "Optimal Capital versus Labor Taxation with Innovation-Led Growth." *NBER Working Paper* No. 19086.

[2] Chu, A., and Cozzi, G., 2014. "R&D and Economic Growth in a Cash-in-Advance Economy." *International Economic Review*, 55: 507–524.

[3] Doepke, M., and Zilibotti, F., 2008. "Occupational Choice and the Spirit of Capitalism." *The Quarterly Journal of Economics* 123(2), 747-793.

[4] Dotsey, M., and Sarte, P., 2000. "Inflation Uncertainty and Growth in a Cash-in-Advance Economy." *Journal of Monetary Economics* 45, 631-655.

[5] Friedman, M., 1969. *The Optimum Quantity of Money*. Macmillan.

[6] Hof, F.X., and Prettner, K., 2019. "The Quest for Status and R&D-based Growth." *Journal of Economic Behavior and Organization* 162(C), 290-307.

[7] Michaillat, P., and Saez, E., 2019. "Resolving New Keynesian Anomalies with Wealth in the Utility Function." *Review of Economics and Statistics* (forthcoming).

[8] Mundell, R., 1965. "Growth, Stability and Inflationary Finance." *Journal of Political Economy* 73, 97–109.

[9] Pan, S., Zhang, M., and Zou, H., 2018. "Status Preference and the Effects of Patent Protection: Theory and Evidence." *Macroeconomic Dynamics* 22, 837-863.

[10] Sidrauski, M., 1967. "Rational Choice and Patterns of Growth in a Monetary Economy." *American Economic Review* 57(2), 534–544.

[11] Tobin, J., 1965. "Money and Economic Growth." *Econometrica* 33: 671–684.

[12] Zou, H.-F., 1994. "The Spirit of Capitalism and Long-run Growth." *European Journal of Political Economy* 10(2), 279-293.