Online Appendix to "The Paradox of Price Flexibility in an Open Economy"

1 Model

We describe the baseline two-country model with trade friction (home bias), financial friction, monopolistic competitive markets, and nominal price rigidity. The structure of the appendix follows Groll and Monacelli (2020). There are two countries in the model, each having measure of $\frac{1}{2}$, respectively. Therefore, the total mass of households in the world economy is equal to 1.

1.1 Home Households

A representative household h in the Home country derives positive utility from the consumption basket $C_t(h)$ and negative utility from supplying labor $N_t(h)$. The present discounted value of lifetime utility is given by:

$$U_t(h) = \mathbb{E}_t \sum_{t=0}^{\infty} \left[\beta^t \left(\xi_t \frac{C_t(h)^{1-\rho}}{1-\rho} - \frac{N_t(h)^{1+\eta}}{1+\eta} \right) \right], \tag{1}$$

where β is the discount factor, ρ is the inverse of the elasticity of intertemporal substitution, and η is the inverse of Frisch elasticity of labor supply. ξ_t is a preference shock or a demand shock. We assume that consumption risk is perfectly pooled among households within a country, so we may eliminate the household index h for consumption. The household consumes products from both home and abroad, and its consumption basket at time t is denoted by:

$$C_t = \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where $C_{H,t}$ and $C_{F,t}$ denote Home's composite consumption of Home goods and that of Foreign goods, respectively. θ is the trade elasticity of substitution between these two composite consumption aggregates, and $\nu \in [\frac{1}{2}, 1]$ is the degree of home bias, which measures the degree of trade openness. Consumption aggregates $C_{H,t}$ and $C_{F,t}$ are defined over a range of Home and Foreign differentiated goods, with the elasticity of substitution between goods being σ. That is,

$$C_{H,t} = \left[2^{\frac{1}{\sigma}} \int_0^{1/2} c_{H,t}(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}} \quad \text{and} \quad C_{F,t} = \left[2^{\frac{1}{\sigma}} \int_{1/2}^{1} c_{F,t}(i)^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}},$$

where $c_{j,t}(i)$ is consumption of good i produced in country $j \in \{H, F\}$.

The price index of composite consumption $C_{H,t}$ and $C_{F,t}$ is:

$$P_{H,t} = \left[2 \int_0^{1/2} P_{H,t}(i)^{1-\sigma} \, di\right]^{\frac{1}{1-\sigma}} \quad \text{and} \quad P^*_{F,t} = \left[2 \int_{1/2}^{1} P^*_{F,t}(i)^{1-\sigma} \, di\right]^{\frac{1}{1-\sigma}}, \tag{2}$$

while the price index of the Home consumption basket C_t is:

$$P_t = \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{3}$$

where $P_{j,t}(i)$ is the price of good i produced in country $j \in \{H, F\}$, quoted in the Home currency. $P^*_{j,t}(i)$ is the price of good i produced in country j denominated in Foreign currency. The law of one price holds for each good, so $P_{j,t}(i) = S_t P^*_{j,t}(i)$, where S_t is the nominal exchange rate, which is the Home price of Foreign currency. The Home terms of trade is $T_t = \frac{P_{F,t}}{P_{H,t}} = \frac{S_t P^*_{F,t}}{P_{H,t}}$.

We assume that the return of state contingent security can be taxed by the government à la Devereux and Yetman (2014), meaning that the returns that Home and Foreign consumers face might differ. The flow budget constraint of the representative household h in Home country can be expressed as:

$$\begin{gathered} P_t C_t + (1+\tau_t)\left[\sum_{x^{t+1}} \nu_{t+1,t} D(h, x^{t+1}) - D(h, x^t)\right] + B_{t+1}(h) \leq \\ W_t N_t(h) + (1 + R_{t-1}) B_t(h) + T_t + \Gamma_t, \end{gathered} \tag{4}$$

where τ_t represents the state-contingent tax on securities. Without loss of generality, we assume that, as in Devereux and Yetman (2014), the tax (or transfer) is levied only by the Home government. Household h in Home supplies its labor service $N_t(h)$ and receives the wage rate W_t. $v_{t+1,t}$ is the price of the claim which pays one unit of Home currency contingent upon the realization of state x_{t+1} at time $t+1$, conditional on state x_t. $D(h, x^t)$ represents the nominal balance of state-contingent securities at time t. $B_t(h)$ is the nominal balance of domestic uncontingent bonds at the beginning of period t with the nominal interest rate R_{t-1}. Households also earn a share of aggregate profit Γ_t from firms and receive lump-sum transfers T_t in each period.

Households maximize the lifetime utility subject to the above budget constraint. The usual consumption Euler equation is given by:

$$U_C(C_t, \xi_t) = (1 + R_t)\beta E_t\Big\{U_C(C_{t+1}, \xi_{t+1})\frac{P_t}{P_{t+1}}\Big\} \tag{5}$$

$$U_C(C_t^*, \xi_t^*) = (1 + R_t^*)\beta E_t\Big\{U_C(C_{t+1}^*, \xi_{t+1}^*)\frac{P_t^*}{P_{t+1}^*}\Big\}, \tag{6}$$

where R_t and R_t^* respectively refer to the Home and Foreign nominal interest rates.

The real wage for household h is:

$$\frac{W_t}{P_t} = C_t^{\rho} N_t(h)^{\eta}\frac{1}{\xi_{C,t}} \quad \text{and} \quad \frac{W_t^*}{P_t^*} = C_t^{*\rho} N_t(h)^{*\eta}\frac{1}{\xi_{C^*,t}}. \tag{7}$$

1.2 Risk Sharing and the Real Exchange Rate

Combining the efficiency conditions for holdings of state-contingent securities for Home and Foreign, we obtain the risk-sharing condition that links the real exchange rate to the ratio of the marginal utilities of consumption:

$$U_C(C_t^*, \xi_t^*) = U_C(C_t, \xi_t)\frac{S_t P_t^*}{P_t}(1 + \tau_t) = U_C(C_t, \xi_t)Q_t(1 + \tau_t), \tag{8}$$

where $(1 + \tau_t)$ is the gross tax rate on state-contingent securities and captures the degree of imperfect risk sharing. $Q_t = \frac{S_t P_t^*}{P_t}$ is interpreted as the real exchange rate, which measures the ratio of consumer price indices in the same currency. We assume that the specification for the tax rate $f_t = 1 + \tau_t$ is given by:

$$f_t = \Big(\frac{P_t C_t}{P_{H,t} Y_{H,t}}\Big)^{\frac{\lambda}{1-\lambda}}, \tag{9}$$

where λ captures the degree of capital restriction between the two countries. $\lambda = 0$, equivalently $f_t = 1$, corresponds to complete financial markets, while $\lambda = 1$ corresponds to financial autarky.

All contingent securities and non-contingent bonds are assumed to be in zero supply in the initial period. Within each country, agents have identical preferences, and the asset markets are complete, implying perfect risk-sharing of consumption within each country.

1.3 Producers and Optimal Pricing

Producers set their prices under monopolistic competition. Prices are sticky in the style of Calvo (1983), and the Calvo parameters of Home and Foreign are respectively denoted by α_H and α_F^*. Each firm i employs labor in a country-specific labor market to produce differentiated good according to the following production function:

$$Y_{H,t}(i) = N_t(i).$$

The profit of firm i located in the home country is given by $P_{H,t}(i)Y_{H,t}(i) - W_t N_t(i)\frac{\sigma-1}{\sigma}$, where the term $\frac{\sigma-1}{\sigma}$ is a subsidy financed by lump sum taxation to eliminate first-order inefficiencies in steady state.

The firms that can adjust their prices at period t choose the following optimal price:

$$P_{H,t}^o(h) = \frac{E_t \sum_{k=0} m_{t+k}(\beta\alpha_H)^k W_{t+k} Y_{H,t+k}(h)}{E_t \sum_{k=0} m_{t+k}(\beta\alpha_H)^k Y_{H,t+k}(h)}, \tag{10}$$

where $m_{t+k} = \frac{U_C(C_{t+k},\xi_{t+k})}{P_{t+k}} \frac{P_t}{U_C(C_t,\xi_t)}$ represents the stochastic discount factor.

The price index evolves according to:

$$P_{H,t}^{1-\sigma} = \left[\alpha_H P_{H,t-1}^{1-\sigma} + (1-\alpha_H) {P_{H,t}^o}^{1-\sigma}\right]. \tag{11}$$

The behavior of foreign firms and the foreign good price index may be described analogously.

1.4 Market Clearing Conditions

The market clearing condition for domestic variety i is:

$$\begin{aligned} Y_{H,t}(i) &= \frac{1}{2}C_{H,t}(i) + \frac{1}{2}C^*_{H,t}(i) \\ &= \Big(\frac{P_{H,t}(i)}{P_{H,t}}\Big)^{-\sigma}\left[\nu\Big(\frac{P_{H,t}}{P_t}\Big)^{-\theta}C_t + (1-\nu)\Big(\frac{P^*_{H,t}}{P^*_t}\Big)^{-\theta}C^*_t\right] \\ &= \Big(\frac{P_{H,t}(i)}{P_{H,t}}\Big)^{-\sigma}\Big(\frac{P_{H,t}}{P_t}\Big)^{-\theta}\left[\nu C_t + (1-\nu)Q_t^{\theta}C^*_t\right]. \end{aligned} \tag{12}$$

The market clearing condition for foreign variety i is similarly defined as:

$$\begin{aligned} Y_{F,t}(i) &= \frac{1}{2}C_{F,t}(i) + \frac{1}{2}C^*_{F,t}(i) \\ &= \Big(\frac{P_{F,t}(i)}{P_{F,t}}\Big)^{-\sigma}\Big(\frac{P_{F,t}}{P_t}\Big)^{-\theta}\left[(1-\nu)C_t + \nu Q_t^{\theta}C^*_t\right]. \end{aligned} \tag{13}$$

Inserting (12) and (13) into the following two equations, respectively,

$$Y_{H,t} = \left[2^{\frac{1}{\sigma}}\int_0^{1/2} Y_{H,t}(i)^{\frac{\sigma-1}{\sigma}}di\right]^{\frac{\sigma}{\sigma-1}} \quad and \quad Y_{F,t} = \left[2^{\frac{1}{\sigma}}\int_{1/2}^{1} Y_{F,t}(i)^{\frac{\sigma-1}{\sigma}}di\right]^{\frac{\sigma}{\sigma-1}}$$

yields the following aggregate demand for each country:

$$Y_{H,t} = (\frac{P_{H,t}}{P_t})^{-\theta}\left[\nu C_t + (1-\nu)Q_t^{\theta}C^*_t\right] \tag{14}$$

$$Y_{F,t} = (\frac{P_{F,t}}{P_t})^{-\theta}\left[(1-\nu)C_t + \nu Q_t^{\theta}C^*_t\right]. \tag{15}$$

1.5 First Best Allocation

The first-best allocation is the equilibrium in which prices are fully flexible, capital markets are complete, and the steady state markups are neutralized at all times with an appropriate subsidy. This efficient allocation provides a useful benchmark in order to assess the welfare implications of the two exchange rate regimes. In this subsection, a variable with hat denotes its log-linearized version, and a variable with superscribt fb indicates its first-best allocation.

The log-linearized risk sharing condition (8) under perfect risk sharing is:

$$\hat{Q}_t^{fb} = \rho(\hat{C}_t^{fb} - \hat{C}_t^{*fb}) + (\epsilon_t^* - \epsilon_t), \tag{16}$$

where $\epsilon_t \equiv \ln \xi_t$, $\epsilon_t^* \equiv \ln \xi_t^*$. Furthermore, log-linearzing the definition of real exchange rate, we have:

$$\hat{Q}_t^{fb} = (2\nu - 1)\hat{T}_t^{fb}. \tag{17}$$

Combining the previous two results with the log-linearized aggregate market clearing conditions (14), we obtain the following:

$$\rho\hat{Y}_{H,t}^{fb} = \rho\hat{C}_t^{fb} + (1-\nu)\left[4\nu(\rho\theta - 1) + 1\right]\hat{T}_t^{fb} + (1-\nu)(\epsilon_t^* - \epsilon_t). \tag{18}$$

Log-linearzing the home optimal pricing equation under flexible prices yields:

$$\eta\hat{Y}_{H,t}^{fb} = -(1-\nu)\hat{T}_t^{fb} - \rho\hat{C}_t^{fb} + \epsilon_t. \tag{19}$$

Combining the previous two equations to eliminate $\hat{C}_t^{fb}$ yields:

$$(\rho + \eta)\hat{Y}_{H,t}^{fb} = 2\nu(1-\nu)(\rho\theta - 1)\hat{T}_t^{fb} - (1-\nu)(\epsilon_t - \epsilon_t^*) + \epsilon_t \tag{20}$$

The corresponding equation for Foreign is derived in a completely analogous way. Thus,

$$(\rho + \eta)\hat{Y}_{F,t}^{fb} = -2\nu(1-\nu)(\rho\theta - 1)\hat{T}_t^{fb} + (1-\nu)(\epsilon_t - \epsilon_t^*) + \epsilon_t^*$$

The efficient terms of trade is given by:

$$\left[4\nu(1-\nu)\rho\theta + (2\nu-1)^2\right]\hat{T}_t^{fb} = \rho(\hat{Y}_{H,t}^{fb} - \hat{Y}_{F,t}^{fb}) - (2\nu - 1)(\epsilon_t - \epsilon_t^*), \tag{21}$$

which is obtained by combining the two log-linearized aggregate market clearing conditions with the risk sharing condition (16) and Equation (17).

2 Full Set of Linearized Equilibrium

We outline the linearized equilibrium conditions that form a set of constraints for the optimal monetary problem. Define $\hat{X}_t \equiv \log\left(\frac{X_t}{X}\right)$ and $\tilde{X}_t \equiv \log\left(\frac{X_t}{X_t^{fb}}\right)$, where X and X_t^{fb} are the steady state of X_t and its efficient level, respectively. Moreover, let $\pi_{H,t} \equiv \ln(P_{H,t}/P_{H,t-1})$ and $\pi_{F,t}^* \equiv \ln(P_{F,t}^*/P_{F,t-1}^*)$.

Equilibrium conditions under flexible exchange rates Given the path of the two policy instruments $\{\hat{R}_t,\ \hat{R}_t^*\}$, the equilibrium under flexible exchange rates consists of endogenous variables $\{\hat{Y}_{H,t},\ \hat{Y}_{F,t},\ \pi_{H,t},\ \pi_{F,t}^*,\ \hat{C}_t,\ \hat{C}_t^*,\ \hat{T}_t,\ \hat{Q}_t,\ \hat{f}_t\}$ and exogenous variables $\{\epsilon_t,\ \epsilon_t^*\}$ that satisfy:

- *Aggregate demand:*

$$\mathbb{E}_t\hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho}\Big\{\hat{R}_t - \Big[\mathbb{E}_t\pi_{H,t+1} + (1-\nu)(\mathbb{E}_t\hat{T}_{t+1} - \hat{T}_t)\Big] + \mathbb{E}_t\epsilon_{t+1} - \epsilon_t\Big\} \tag{22}$$

$$\mathbb{E}_t\hat{C}_{t+1}^* = \hat{C}_t^* + \frac{1}{\rho}\Big\{\hat{R}_t^* - \Big[\mathbb{E}_t\pi_{F,t+1}^* - (1-\nu)(\mathbb{E}_t\hat{T}_{t+1} - \hat{T}_t)\Big] + \mathbb{E}_t\epsilon_{t+1}^* - \epsilon_t^*\Big\} \tag{23}$$

- *Market clearing:*

$$\hat{Y}_{H,t} = 2\nu(1-\nu)\theta\hat{T}_t + \nu\hat{C}_t + (1-\nu)\hat{C}_t^* \tag{24}$$

$$\hat{Y}_{F,t} = -2\nu(1-\nu)\theta\hat{T}_t + (1-\nu)\hat{C}_t + \nu\hat{C}_t^* \tag{25}$$

- *Aggregate supply:*

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa_H\Big\{(\rho+\eta)\tilde{Y}_{H,t} - (1-\nu)\Big[2\nu(\rho\theta-1)\tilde{T}_t - \hat{f}_t\Big]\Big\} \tag{26}$$

$$\pi_{F,t}^* = \beta\mathbb{E}_t\pi_{F,t+1}^* + \kappa_F\Big\{(\rho+\eta)\tilde{Y}_{F,t} + (1-\nu)\Big[2\nu(\rho\theta-1)\tilde{T}_t - \hat{f}_t\Big]\Big\} \tag{27}$$

$$\kappa_H \equiv \frac{(1-\alpha_H\beta)(1-\alpha_H)}{\alpha_H}; \quad \kappa_F = \frac{(1-\alpha_F^*\beta)(1-\alpha_F^*)}{\alpha_F^*}$$

- *Risk sharing and demand imbalances:*

$$\rho(\hat{C}_t - \hat{C}_t^*) + (\epsilon_t^* - \epsilon_t) - \hat{Q}_t = \hat{f}_t \tag{28}$$

$$\hat{Q}_t = (2\nu-1)\hat{T}_t \tag{29}$$

$$\begin{aligned}\hat{f}_t = &\frac{\lambda(2-2\nu)}{\lambda(2-2\nu)(2\theta\nu-2\nu+1)+2(1-\lambda)\Delta}(\rho(2\theta\nu-1)+1-2\nu)(\hat{Y}_{H,t}-\hat{Y}_{F,t})\\ &-\frac{\lambda(2-2\nu)}{\lambda(2-2\nu)(2\theta\nu-2\nu+1)+2(1-\lambda)\Delta}(2\theta\nu-2\nu+1)(\epsilon_t-\epsilon_t^*),\end{aligned} \tag{30}$$

$$\Delta \equiv (2\nu-1)^2 + 4\rho\theta\nu(1-\nu).$$

Once the solution for $\pi_{H,t}$, $\pi_{F,t}^*$, and $\hat{T}_t$ is obtained from the set of equations above, one can derive the dynamic equation for nominal exchange rates using the definition of the terms of trade, expressed in log-deviations:

$$\hat{S}_t - \hat{S}_{t-1} = \hat{T}_t - \hat{T}_{t-1} + \pi_{H,t} - \pi_{F,t}^*. \tag{31}$$

We define the demand shocks in Home as $\epsilon_t = \ln \xi_t$. The shock follows a two-state Markov process, as in Eggertsson and Woodford (2003). In period $t = 0$, the world economy is at steady state. In period $t = 1$, there is an unanticipated fall of ϵ_t to $\bar{\epsilon}_L < 0$. The shock remains constant in each period with probability μ and reverts back to zero with probability $1 - \mu$. Once the shock reverts back to zero, it stays there forever. We assume that both Home and Foreign are hit by the same shock, i.e., $\epsilon_t = \epsilon_t^*$.

Equilibrium conditions under a monetary union Two main differences exist between the equilibrium under a monetary union and the equilibrium under flexible exchange rates. Because the monetary authority sets the common policy instrument under a monetary union, we only need one aggregate demand equation, which is (22). Moreover, one needs to impose the fixed exchange rate condition, which is Equation (31) with $\hat{S}_t = 0$:

$$\hat{T}_t - \hat{T}_{t-1} = \pi_{F,t}^* - \pi_{H,t}. \tag{32}$$

Hence, given the path of common policy instrument ($\hat{R}_t = \hat{R}_t^*$) and shock processes, the equilibrium under a monetary union consists of a set of endogenous variables $\{\hat{Y}_{H,t}$, $\hat{Y}_{F,t}$, $\pi_{H,t}$, $\pi_{F,t}^*$, $\hat{C}_t$, $\hat{C}_t^*$, $\hat{T}_t$, $\hat{Q}_t$, $\hat{f}_t\}$ and exogenous variables $\{\epsilon_t, \epsilon_t^*\}$, satisfying conditions (22), (24)-(30) along with (32).

3 Derivation of Loss Function

In this section, we derive the quadratic welfare loss function in the main text. The derivation closely follows the steps outlined in the appendix of Corsetti et al. (2020).

3.1 Useful First-Order Conditions

The equations outlined in this subsection will be extensively used in the next subsections. Rearranging the risk sharing condition (28) and using the fact that $\hat{C}_t + \hat{C}_t^* = \hat{Y}_{H,t} + \hat{Y}_{F,t}$, we obtain:

$$\hat{C}_t = \frac{1}{2}\Big\{\hat{Y}_{H,t} + \hat{Y}_{F,t} + \frac{1}{\rho}\left[\hat{Q}_t + \hat{f}_t + (\epsilon_t - \epsilon_t^*)\right]\Big\}. \tag{33}$$

Subtracting $\hat{Y}_{H,t}$ from the equation above and utilizing the fact that $\hat{C}_t + \hat{C}_t^* = \hat{Y}_{H,t} + \hat{Y}_{F,t}$, we obtain:

$$-(\hat{C}_t - \hat{Y}_{H,t}) = \hat{C}_t^* - \hat{Y}_{F,t} = \frac{1}{2}\Big\{\hat{Y}_{H,t} - \hat{Y}_{F,t} - \frac{1}{\rho}\left[\hat{Q}_t + \hat{f}_t + (\epsilon_t - \epsilon_t^*)\right]\Big\}. \tag{34}$$

Combining (34) with the market clearing conditions (24) and (25) yields:

$$\hat{C}_t = \hat{Y}_{H,t} - (1-\nu)\frac{1}{\rho}\left[\rho\theta\hat{T}_t + (\rho\theta - 1)\hat{Q}_t - \hat{f}_t - (\epsilon_t - \epsilon_t^*)\right] \tag{35}$$

$$\hat{C}_t^* = \hat{Y}_{F,t} + (1-\nu)\frac{1}{\rho}\left[\rho\theta\hat{T}_t + (\rho\theta - 1)\hat{Q}_t - \hat{f}_t - (\epsilon_t - \epsilon_t^*)\right]. \tag{36}$$

Subtract (25) from (24), then combining it with (28) and (29), we obtain:

$$\left[4\nu(1-\nu)(\rho\theta - 1) + 1\right]\hat{T}_t = \rho(\hat{Y}_{H,t} - \hat{Y}_{F,t}) - (2\nu - 1)(\hat{f}_t + (\epsilon_t - \epsilon_t^*)). \tag{37}$$

In addition, rearranging (20), we obtain:

$$\epsilon_t = (\eta + \rho)\hat{Y}_{H,t}^{fb} - \left[2\nu(1-\nu)(\rho\theta - 1)\right]\hat{T}_t^{fb} + (1-\nu)(\epsilon_t - \epsilon_t^*). \tag{38}$$

Now, log-linearizing (7), we obtain the real marginal cost:

$$\hat{W}_t - \hat{P}_t = \rho\hat{C}_t + \eta\hat{Y}_{H,t} - \epsilon_t. \tag{39}$$

Replacing the CPI price index $\hat{P}_t$ with PPI price index $\hat{P}_{H,t}$, we can rewrite (39) as:

$$\hat{W}_t - \hat{P}_{H,t} = \rho\hat{C}_t - \epsilon_t + \eta\hat{Y}_{H,t} + (1-\nu)\hat{T}_t. \tag{40}$$

Now, multiply both sides of (35) by ρ and then use the resulting equation to rewrite the right-hand side of (40). The right-hand side of (40) is then:

$$\begin{gathered}\rho\hat{C}_t - \epsilon_t + \eta\hat{Y}_{H,t} + (1-\nu)\hat{T}_t = \\ (\eta + \rho)\hat{Y}_{H,t} - (1-\nu)\left[(\rho\theta - 1)(\hat{T}_t + \tilde{Q}_t) - \hat{f}_t\right] + (1-\nu)(\epsilon_t - \epsilon_t^*) - \epsilon_t.\end{gathered} \tag{41}$$

Combining (38) and the above equation, we get:

$$\rho\hat{C}_t - \epsilon_t + \eta\hat{Y}_{H,t} + (1-\nu)\hat{T}_t = (\eta + \rho)\tilde{Y}_{H,t} - (1-\nu)\left[(\rho\theta - 1)(\tilde{T}_t + \tilde{Q}_t) - \hat{f}_t\right].$$

Multiplying both sides by $\frac{1}{2}$ and subtracting $\frac{\epsilon_t}{2}$ to both sides, we obtain:

$$\begin{aligned} \frac{\rho}{2}\hat{C}_t - \epsilon_t + \frac{\eta}{2}\hat{Y}_{H,t} + \frac{1}{2}(1-\nu)\hat{T}_t = \\ \frac{1}{2}(\eta+\rho)\tilde{Y}_{H,t} - \frac{1}{2}(1-\nu)\left[(\rho\theta-1)(\tilde{T}_t+\tilde{Q}_t) - \hat{f}_t\right] - \frac{\epsilon_t}{2}. \end{aligned} \tag{42}$$

Substitute out ϵ using (38) to obtain:

$$\begin{aligned} \frac{\rho}{2}\hat{C}_t - \epsilon_t + \frac{\eta}{2}\hat{Y}_{H,t} + \frac{1}{2}(1-\nu)\hat{T}_t = \\ \frac{1}{2}(\eta+\rho)\tilde{Y}_{H,t} - \frac{1}{2}(1-\nu)\left[(\rho\theta-1)(\tilde{T}_t+\tilde{Q}_t) - \hat{f}_t\right] \\ -\frac{1}{2}\left\{(\eta+\rho)\hat{Y}^{fb}_{H,t} - [2\nu(1-\nu)(\rho\theta-1)]\,\hat{T}^{fb}_t + (1-\nu)(\epsilon_t-\epsilon^*_t)\right\} \\ = (\eta+\rho)(\frac{1}{2}\hat{Y}_{H,t} - \hat{Y}^{fb}_{H,t}) - 2\nu(1-\nu)(\rho\theta-1)(\frac{1}{2}\hat{T}_t - \hat{T}^{fb}_t) + \frac{1}{2}(1-\nu)(\hat{f}_t - (\epsilon_t-\epsilon^*_t)). \end{aligned} \tag{43}$$

3.2 Second-Order Approximation of Utility

The period utility of agents living in Home is given by:

$$U(C) - V(N) = \xi\frac{C^{1-\rho}}{1-\rho} - \frac{N^{1+\eta}}{1+\eta}. \tag{44}$$

The period utility of agents living in Foreign is defined similarly.

We assume efficient steady state so that the subsidy satisfies $\frac{(\sigma-1)(1-\bar{\tau})}{\sigma} = 1$. Furthermore, in the efficient steady state, $U'(C) = -V'(N)$ holds. The second-order approximation of the utility (44) yields

$$v_t = \hat{C}_t - \hat{Y}_{H,t} + (\frac{1-\rho}{2}\hat{C}_t + \epsilon_t)\hat{C}_t - \frac{1}{2}(1+\eta)\hat{Y}^2_{H,t} - \frac{1}{2}\frac{\sigma}{\kappa_H}\pi^2_{H,t} + t.i.p. + O(||Z||^3), \tag{45}$$

where the term $t.i.p.$ collects all the terms that are independent of monetary policy and the term $O(||Z||^3)$ groups all the terms that are of third or higher order. To derive the above expression, the log-linear approximation of the aggregate production function is used, which is given by $\hat{Y}_{H,t} = \hat{N}_t$. The inflation rates appear because the second-order approximation of the labor service is proportional to the price dispersion. Please refer to Woodford (2003) chapter 6 for more detail.

The previous steps can be repeated completely analogously to obtain the corresponding expression for Foreign. The world welfare loss function is given by the weighted average of the country-specific welfare loss functions:

$$\mathcal{L}_t^W = \frac{1}{2}\upsilon_t + \frac{1}{2}\upsilon_t^*,$$

which can be reexpressed as:

$$\begin{aligned}
2\mathcal{L}_t^W = \upsilon_t + \upsilon_t^* = (\hat{C}_t + \hat{C}_t^*) - (\hat{Y}_{H,t} + \hat{Y}_{F,t}) + (\frac{1-\rho}{2}\hat{C}_t + \epsilon_t)\hat{C}_t + (\frac{1-\rho}{2}\hat{C}_t^* + \epsilon_t^*)\hat{C}_t^* \quad (46)\\
-\frac{1}{2}(1+\eta)\hat{Y}_{H,t}^2 - \frac{1}{2}(1+\eta)\hat{Y}_{F,t}^2 \\
-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + o(||Z||^3).
\end{aligned}$$

We shall utilize the equations outlined in Section 3.1 to express the loss function in terms of gaps.

3.3 Expressing Loss Function in Terms of Gaps

To eliminate the linear terms in $\mathcal{L}_t^W$, we first sum up the budget constraints of each country to obtain the world resource constraint. The world resource constraint, expressed in the Home currency, is:

$$P_t C_t + S_t P_t^* C_t^* = P_{H,t} Y_{H,t} + P_{F,t} Y_{F,t}.$$

Dividing both sides by P_t, we have:

$$C_t + Q_t C_t^* = \frac{P_{H,t}}{P_t} Y_{H,t} + \frac{P_{F,t}}{P_t} Y_{F,t}. \quad (47)$$

Using the relationship

$$\frac{P_{H,t}}{P_t} = \left[\nu + (1-\nu)T_t^{1-\theta}\right]^{-\frac{1}{1-\theta}} \quad (48)$$

$$\frac{P_{F,t}}{P_t} = \frac{P_{F,t}}{S_t P_t^*}\frac{S_t P_t^*}{P_t} = \left[\nu + (1-\nu)T_t^{\theta-1}\right]^{-\frac{1}{1-\theta}} Q_t \quad (49)$$

to substitute out the two relative prices in (47), we obtain:

$$C_t + Q_t C_t^* = \left[\nu + (1-\nu)T_t^{1-\theta}\right]^{-\frac{1}{1-\theta}} Y_{H,t} + \left[\nu + (1-\nu)T_t^{\theta-1}\right]^{-\frac{1}{1-\theta}} Q_t Y_{F,t}. \quad (50)$$

Performing a second-order approximation to the above expression and rearranging the linear terms to the left yields:

$$\begin{aligned}\hat{C}_t + \hat{C}_t^* - \hat{Y}_{H,t} - \hat{Y}_{F,t} = &-\frac{1}{2}(\hat{C}_t^2 + {\hat{C}_t^*}^2) + \frac{1}{2}(\hat{Y}_{H,t}^2 + \hat{Y}_{F,t}^2) \\ &+(\hat{Y}_{F,t} - \hat{C}_t^*)\hat{Q}_t - (1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\hat{T}_t \\ &+\nu(1-\nu)\theta\hat{T}_t^2 + t.i.p. + O(||Z||^3).\end{aligned} \tag{51}$$

Inserting this equation into (46) and then combining the like terms yields:

$$\begin{aligned}2\mathcal{L}_t^W = &-(\frac{\rho}{2}\hat{C}_t - \epsilon_t)\hat{C}_t - (\frac{\rho}{2}\hat{C}_t^* - \epsilon_t^*)\hat{C}_t^* + (\hat{Y}_{F,t} - \hat{C}_t^*)\hat{Q}_t \\ &-\frac{1}{2}(1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\hat{T}_t + (1-\nu)\nu\theta\hat{T}_t^2 \\ &-(\frac{\eta}{2}\hat{Y}_{H,t} + \frac{1}{2}(1-\nu)\hat{T}_t)\hat{Y}_{H,t} - (\frac{\eta}{2}\hat{Y}_{F,t} - \frac{1}{2}(1-\nu)\hat{T}_t)\hat{Y}_{F,t} \\ &-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).\end{aligned}$$

Adding and subtracting $\frac{1}{2}(1-\nu)\hat{T}_t(\hat{Y}_{H,t} - \hat{Y}_{F,t})$, $(\frac{\rho}{2}\hat{C}_t - \epsilon_t)\hat{Y}_{H,t}$ and $(\frac{\rho}{2}\hat{C}_t^* - \epsilon_t^*)\hat{Y}_{F,t}$ to the above equation and combining the like term yields:

$$\begin{aligned}2\mathcal{L}_t^W = &-(\frac{\rho}{2}\hat{C}_t - \epsilon_t)(\hat{C}_t - \hat{Y}_{H,t}) - (\frac{\rho}{2}\hat{C}_t^* - \epsilon_t^* + \hat{Q}_t)(\hat{C}_t^* - \hat{Y}_{F,t}) \\ &-\frac{1}{2}(1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\hat{T}_t + (1-\nu)\nu\theta\hat{T}_t^2 \\ &-(\frac{\rho}{2}\hat{C}_t - \epsilon_t + \frac{\eta}{2}\hat{Y}_{H,t} + \frac{1}{2}(1-\nu)\hat{T}_t)\hat{Y}_{H,t} \\ &-(\frac{\rho}{2}\hat{C}_t^* - \epsilon_t^* + \frac{\eta}{2}\hat{Y}_{F,t} - \frac{1}{2}(1-\nu)\hat{T}_t)\hat{Y}_{F,t} \\ &-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).\end{aligned}$$

Using the expressions (38) and (41), we can express the above loss in terms of gaps and the demand imbalance:

$$
\begin{aligned}
2\mathcal{L}_t^W = &-(\frac{\rho}{2}\hat{C}_t - \epsilon_t)(\hat{C}_t - \hat{Y}_{H,t}) - (\frac{\rho}{2}\hat{C}_t^* - \epsilon_t^* + \hat{Q}_t)(\hat{C}_t^* - \hat{Y}_{F,t}) \\
&-\frac{1}{2}(1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\left[\hat{T}_t + \hat{f}_t - (\epsilon_t - \epsilon_t^*)\right] + (1-\nu)\nu\theta\hat{T}_t^2 \\
&- \left[(\rho+\eta)(\frac{1}{2}\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}) - 2\nu(1-\nu)(\rho\theta - 1)(\frac{1}{2}\hat{T}_t - \hat{T}_t^{fb})\right]\hat{Y}_{H,t} \\
&- \left[(\rho+\eta)(\frac{1}{2}\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}) + 2\nu(1-\nu)(\rho\theta - 1)(\frac{1}{2}\hat{T}_t - \hat{T}_t^{fb})\right]\hat{Y}_{F,t} \\
&-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).
\end{aligned}
$$

Next, collecting the terms in output gaps and the terms multiplied by output differentials yields:

$$
\begin{aligned}
2\mathcal{L}_t^W = &-(\frac{\rho}{2}\hat{C}_t - \epsilon_t)(\hat{C}_t - \hat{Y}_{H,t}) - (\frac{\rho}{2}\hat{C}_t^* - \epsilon_t^* + \hat{Q}_t)(\hat{C}_t^* - \hat{Y}_{F,t}) \\
&-\frac{1}{2}(1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\left[\hat{T}_t + \hat{f}_t - (\epsilon_t - \epsilon_t^*)\right] + (1-\nu)\nu\theta\hat{T}_t^2 \\
&+2\nu(1-\nu)(\rho\theta - 1)(\frac{1}{2}\hat{T}_t - \hat{T}_t^{fb})(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
&-(\rho+\eta)(\frac{1}{2}\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb})\hat{Y}_{H,t} - (\rho+\eta)(\frac{1}{2}\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb})\hat{Y}_{F,t} \\
&-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).
\end{aligned}
$$

Using equations (33) and (34) to rearrange the above expression:

$$
\begin{aligned}
2\mathcal{L}_t^W = &\frac{1}{2}\left[\frac{\rho}{2}(\hat{C}_t - \hat{C}_t^*) - \hat{Q}_t - (\epsilon_t - \epsilon_t^*)\right]\left\{\hat{Y}_{H,t} - \hat{Y}_{F,t} - \frac{1}{\rho}\left[\hat{Q}_t + \hat{f}_t + (\epsilon_t - \epsilon_t^*)\right]\right\} \\
&-\frac{1}{2}(1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\left[\hat{T}_t + \hat{f}_t - (\epsilon_t - \epsilon_t^*)\right] + (1-\nu)\nu\theta\hat{T}_t^2 \\
&+2\nu(1-\nu)(\rho\theta - 1)(\frac{1}{2}\hat{T}_t - \hat{T}_t^{fb})(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
&-(\rho+\eta)(\frac{1}{2}\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb})\hat{Y}_{H,t} - (\rho+\eta)(\frac{1}{2}\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb})\hat{Y}_{F,t} \\
&-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).
\end{aligned}
$$

Now, using the definition of the demand imbalance

$$
\hat{f}_t = \rho(\hat{C}_t - \hat{C}_t^*) - \hat{Q}_t - (\epsilon_t - \epsilon_t^*)
$$

to eliminate the terms expressed in consumption, we obtain:

$$
\begin{aligned}
2\mathcal{L}_t^W =& -\frac{1}{4\rho}\left[\hat{f}_t^2 - (\hat{Q}_t + (\epsilon_t - \epsilon_t^*))^2\right] + \frac{1}{4}\left[\hat{f}_t - (\hat{Q}_t + (\epsilon_t - \epsilon_t^*))\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
& -\frac{1}{2}(1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\left[\hat{T}_t + (\hat{C}_t - \hat{C}_t^*) - (\epsilon_t - \epsilon_t^*)\right] + (1-\nu)\nu\theta\hat{T}_t^2 \\
& +2\nu(1-\nu)(\rho\theta - 1)(\frac{1}{2}\hat{T}_t - \hat{T}_t^{fb})(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
& -(\rho+\eta)(\frac{1}{2}\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb})\hat{Y}_{H,t} - (\rho+\eta)(\frac{1}{2}\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb})\hat{Y}_{F,t} \\
& -\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).
\end{aligned}
$$

Now, collecting the terms in output differentials:

$$
\begin{aligned}
2\mathcal{L}_t^W =& -\frac{1}{4\rho}\left[\hat{f}_t^2 - (\hat{Q}_t + (\epsilon_t - \epsilon_t^*))^2\right] \\
& +\frac{1}{4}\left[(2\nu-1)(\hat{f}_t - (\epsilon_t - \epsilon_t^*)) - \hat{T}_t\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
& +(1-\nu)\nu\theta\hat{T}_t^2 + 2\nu(1-\nu)(\rho\theta - 1)(\frac{1}{2}\hat{T}_t - \hat{T}_t^{fb})(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
& -(\rho+\eta)(\frac{1}{2}\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb})\hat{Y}_{H,t} - (\rho+\eta)(\frac{1}{2}\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb})\hat{Y}_{F,t} \\
& -\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).
\end{aligned}
$$

At the third line of the previous equation, substitute $\hat{T}_t$ and $\hat{T}_t^{fb}$ out using the relationship (37) and (21) to obtain:

$$
\begin{aligned}
2\mathcal{L}_t^W =& -\frac{1}{4\rho}\left[\hat{f}_t^2 - (\hat{Q}_t + (\epsilon_t - \epsilon_t^*))^2\right] \\
& +\frac{1}{4}\left[(2\nu-1)(\hat{f}_t - (\epsilon_t - \epsilon_t^*)) - \hat{T}_t\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) + (1-\nu)\nu\theta\hat{T}_t^2 \\
& -\frac{\nu(1-\nu)(\rho\theta-1)}{4\nu(1-\nu)(\rho\theta-1)+1}(2\nu-1)\left[\hat{f}_t - (\epsilon_t - \epsilon_t^*)\right](\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
& +\frac{2\nu(1-\nu)(\rho\theta-1)\rho}{4\nu(1-\nu)(\rho\theta-1)+1}(\frac{1}{2}(\hat{Y}_{H,t} - \hat{Y}_{F,t}) - (\hat{Y}_{H,t}^{fb} - \hat{Y}_{F,t}^{fb}))(\hat{Y}_{H,t} - \hat{Y}_{F,t}) \\
& -(\rho+\eta)(\frac{1}{2}\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb})\hat{Y}_{H,t} - (\rho+\eta)(\frac{1}{2}\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb})\hat{Y}_{F,t} \\
& -\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).
\end{aligned} \tag{52}
$$

Note that the last three lines of the above expression is the loss function under complete markets. Let us focus on the first three lines. Use the definition of the real exchange rate

(29) to substitute out $\hat{Q}_t$:

$$
\begin{aligned}
&-\frac{1}{4\rho}\left[\hat{f}_t^2-((2\nu-1)\hat{T}_t+(\epsilon_t-\epsilon_t^*))^2\right]\\
&+\frac{1}{4}\left[(2\nu-1)(\hat{f}_t-(\epsilon_t-\epsilon_t^*))-\hat{T}_t\right](\hat{Y}_{H,t}-\hat{Y}_{F,t})+(1-\nu)\nu\theta\hat{T}_t^2\\
&-\frac{\nu(1-\nu)(\rho\theta-1)}{4\nu(1-\nu)(\rho\theta-1)+1}(2\nu-1)\left[\hat{f}_t-(\epsilon_t-\epsilon_t^*)\right](\hat{Y}_{H,t}-\hat{Y}_{F,t}).
\end{aligned}
$$

Again, use (37) to substitute out $\hat{Y}_{H,t}-\hat{Y}_{F,t}$ in the expression above:

$$
\begin{aligned}
&-\frac{1}{4\rho}\left[\hat{f}_t^2-((2\nu-1)\hat{T}_t+(\epsilon_t-\epsilon_t^*))^2\right]\\
&+\frac{1}{4\rho}\left[(2\nu-1)(\hat{f}_t-(\epsilon_t-\epsilon_t^*))-\hat{T}_t\right]\cdot\\
&\left\{[4\nu(1-\nu)(\rho\theta-1)+1]\,\hat{T}_t+(2\nu-1)\left[\hat{f}_t+(\epsilon_t-\epsilon_t^*)\right]\right\}\\
&-\frac{\rho^{-1}(2\nu-1)\nu(1-\nu)(\rho\theta-1)}{4\nu(1-\nu)(\rho\theta-1)+1}\left[\hat{f}_t-(\epsilon_t-\epsilon_t^*)\right]\cdot\\
&\left\{[4\nu(1-\nu)(\rho\theta-1)+1]\,\hat{T}_t+(2\nu-1)\left[\hat{f}_t+(\epsilon_t-\epsilon_t^*)\right]\right\}\\
&+(1-\nu)\nu\theta\hat{T}_t^2.
\end{aligned}
$$

By rearranging, we can finally simplify the expression above as:

$$
-\frac{\nu(1-\nu)\theta}{4\nu(1-\nu)(\rho\theta-1)+1}\hat{f}_t{}^2. \tag{53}
$$

Inserting (53) into equation (52), we get the loss function in deviations from the first best:

$$
\begin{aligned}
2\left[\mathcal{L}_t^W-(\mathcal{L}_t^W)^{fb}\right]&=-\frac{\nu(1-\nu)\theta}{4\nu(1-\nu)(\rho\theta-1)+1}\hat{f}_t{}^2\\
&+\frac{2\nu(1-\nu)(\rho\theta-1)\rho}{4\nu(1-\nu)(\rho\theta-1)+1}(\frac{1}{2}(\hat{Y}_{H,t}-\hat{Y}_{F,t})-(\hat{Y}_{H,t}^{fb}-\hat{Y}_{F,t}^{fb}))(\hat{Y}_{H,t}-\hat{Y}_{F,t})\\
&-(\rho+\eta)(\frac{1}{2}\hat{Y}_{H,t}-\hat{Y}_{H,t}^{fb})\hat{Y}_{H,t}-(\rho+\eta)(\frac{1}{2}\hat{Y}_{F,t}-\hat{Y}_{F,t}^{fb})\hat{Y}_{F,t}\\
&-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2-\frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2+t.i.p.+O(||Z||^3).
\end{aligned} \tag{54}
$$

Utilizing the fact that, for any given variable X_t,

$$
\begin{aligned}
(\frac{1}{2}\hat{X}_t - \hat{X}_t^{fb})\hat{X}_t &= \frac{1}{2}(\hat{X}_t^2 - 2\hat{X}_t\hat{X}_t^{fb}) \\
&= \frac{1}{2}(\hat{X}_t^2 - 2\hat{X}_t\hat{X}_t^{fb} + (\hat{X}_t^{fb})^2) - \frac{1}{2}(\hat{X}_t^{fb})^2 \\
&= \frac{1}{2}(\hat{X}_t - \hat{X}_t^{fb})^2 + t.i.p.,
\end{aligned} \tag{55}
$$

we can further reduce (54) as:

$$
\begin{aligned}
2\left[\mathcal{L}_t^W - (\mathcal{L}_t^W)^{fb}\right] &= -\frac{\nu(1-\nu)\theta}{4\nu(1-\nu)(\rho\theta-1)+1}\hat{f_t}^2 \\
&+ \frac{\nu(1-\nu)(\rho\theta-1)\rho}{4\nu(1-\nu)(\rho\theta-1)+1}((\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}) - (\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}))^2 \\
&- \frac{1}{2}(\rho+\eta)(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb})^2 - \frac{1}{2}(\rho+\eta)(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb})^2 \\
&- \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_F}{\pi_{F,t}^*}^2 + t.i.p. + O(||Z||^3).
\end{aligned}
$$

Again, rearranging it further, we finally obtain the complete loss function:

$$
\begin{aligned}
\mathcal{L}_t^W - (\mathcal{L}_t^W)^{fb} = -\frac{1}{4}\Big\{&(\rho+\eta)(\tilde{Y}_{H,t}^2 + \tilde{Y}_{F,t}^2) + \frac{\sigma}{\kappa_H}\pi_{H,t}^2 + \frac{\sigma}{\kappa_F}\pi_{F,t}^{*2} + \\
-\frac{2\nu(1-\nu)}{4\nu(1-\nu)(\rho\theta-1)+1}&\left[(\rho\theta-1)\rho(\tilde{Y}_{H,t} - \tilde{Y}_{F,t})^2 - \theta\hat{f}_t^2\right]\Big\} + t.i.p + O(||Z||^3).
\end{aligned} \tag{56}
$$

This completes the derivation of the loss function in the paper.

4 Derivation of Country-Specific Loss Function

In this section, we derive the quadratic country-specific loss function. The derivation closely follows the steps outlined in the appendix of Pappa (2004). Specifically, we focus on the derivation under perfect financial integration and symmetric demand shocks, because the country-specific loss function described in the main text corresponds to this environment. We derive only the Home loss function. The Foreign loss can be obtained analogously.

4.1 Useful Relationship

From the home market clearing condition (24), we can obtain:

$$\begin{aligned}\hat{Y}_{H,t} &= 2\nu(1-\nu)\theta\hat{T}_t + \nu\hat{C}_t + (1-\nu)\hat{C}_t^* \\ &= 2\nu(1-\nu)\theta\hat{T}_t + (\nu-1)(\hat{C}_t - \hat{C}_t^*) + \hat{C}_t. \end{aligned} \tag{57}$$

With our assumption of symmetric demand shocks and complete financial markets, we can rewrite equation (28) as: $\frac{\hat{Q}_t}{\rho} = \hat{C} - \hat{C}^*$. Using this relative consumption relation, we can rewrite equation (57) as:

$$\begin{aligned}\hat{Y}_{H,t} &= 2\nu(1-\nu)\theta\hat{T}_t - \frac{(1-\nu)}{\rho}\hat{Q}_t + \hat{C}_t \\ &= 2\nu(1-\nu)\theta\hat{T}_t - \frac{(1-\nu)}{\rho}(2\nu-1)\hat{T}_t + \hat{C}_t \\ &= (1-\nu)\left[2\nu\theta + \frac{1-2\nu}{\rho}\right]\hat{T}_t + \hat{C}_t \\ &= (1-\nu)\phi\hat{T}_t + \hat{C}_t, \end{aligned} \tag{58}$$

where the second equality follows from (29) and $\phi = 2\nu\theta + \frac{1-2\nu}{\rho}$. Squaring both sides of equation (58), we get:

$$\hat{Y}_{H,t}^2 = \hat{C}_t^2 + (1-\nu)^2\phi^2\hat{T}_t^2 + 2(1-\nu)\phi\hat{C}_t\hat{T}_t. \tag{59}$$

The foreign counterpart of (59) is:

$$\hat{Y}_{F,t}^2 = {\hat{C}_t^*}^2 + (1-\nu)^2\phi^2\hat{T}_t^2 - 2(1-\nu)\phi\hat{C}_t^*\hat{T}_t. \tag{60}$$

Adding (59) and (60) yields:

$$\hat{Y}_{H,t}^2 + \hat{Y}_{F,t}^2 = \hat{C}_t^2 + {\hat{C}_t^*}^2 + 2(1-\nu)^2\phi^2\hat{T}_t^2 + 2(1-\nu)\phi\hat{T}_t(\hat{C}_t - \hat{C}_t^*).$$

Applying (28) and (29) to substitute out relative consumption:

$$\hat{Y}_{H,t}^2 + \hat{Y}_{F,t}^2 = \hat{C}_t^2 + {\hat{C}_t^*}^2 + 2(1-\nu)^2\phi^2\hat{T}_t^2 + 2(1-\nu)\phi\frac{(2\nu-1)}{\rho}\hat{T}_t^2. \tag{61}$$

Rearranging, we obtain:

$$\hat{Y}_{H,t}^2 + \hat{Y}_{F,t}^2 - \hat{C}_t^2 - \hat{C}_t^{*\,2} = 2(1-\nu)\phi\left[\frac{2\nu-1}{\rho} + (1-\nu)\phi\right]\hat{T}_t^2. \tag{62}$$

Rewriting the second-order approximation of the world resource constraint, given by (51), for convenience:

$$\begin{aligned}\hat{C}_t + \hat{C}_t^* - \hat{Y}_{H,t} - \hat{Y}_{F,t} = \frac{1}{2}(\hat{Y}_{H,t}^2 + \hat{Y}_{F,t}^2 - \hat{C}_t^2 + \hat{C}_t^{*\,2})\\ +(\hat{Y}_{F,t} - \hat{C}_t^*)\hat{Q}_t - (1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\hat{T}_t\\ +\nu(1-\nu)\theta\hat{T}_t^2 + O(||Z||^3).\end{aligned}$$

Plugging (62) into the above expression:

$$\begin{aligned}\hat{C}_t + \hat{C}_t^* - \hat{Y}_{H,t} - \hat{Y}_{F,t} = \frac{1}{2}\left[2(1-\nu)\phi(\frac{2\nu-1}{\rho} + (1-\nu)\phi)\right]\hat{T}_t^2\\ +(\hat{Y}_{F,t} - \hat{C}_t^*)\hat{Q}_t - (1-\nu)(\hat{Y}_{H,t} - \hat{Y}_{F,t})\hat{T}_t\\ +\nu(1-\nu)\theta\hat{T}_t^2 + O(||Z||^3).\end{aligned} \tag{63}$$

To express all terms on the right-hand side of (63) as a function of the terms of trade, we use the following relationships. First, the linearized market clearing condition for Foreign implies:

$$\begin{aligned}\hat{Y}_{F,t} - \hat{C}_t^* &= -2\nu(1-\nu)\theta\hat{T}_t + (1-\nu)(\hat{C}_t - \hat{C}_t^*)\\ &= -2\nu(1-\nu)\theta\hat{T}_t + (1-\nu)\frac{(2\nu-1)}{\rho}\hat{T}_t\\ &= \left[\frac{(1-\nu)(2\nu-1)}{\rho} - 2\nu(1-\nu)\theta\right]\hat{T}_t.\end{aligned} \tag{64}$$

Second, subtracting the linearized market clearing conditions for Home from that of Foreign, we obtain:

$$\begin{aligned}\hat{Y}_{H,t} - \hat{Y}_{F,t} &= 4\nu(1-\nu)\theta\hat{T}_t + (2\nu-1)(\hat{C}_t - \hat{C}_t^*)\\ &= 4\nu(1-\nu)\theta\hat{T}_t + \frac{(2\nu-1)^2}{\rho}\hat{T}_t\\ &= \left[4\nu(1-\nu)\theta + \frac{(2\nu-1)^2}{\rho}\right]\hat{T}_t.\end{aligned} \tag{65}$$

Inserting (64) and (65) into (63), we obtain:

$$\hat{C}_t + \hat{C}_t^* - \hat{Y}_{H,t} - \hat{Y}_{F,t} = \frac{1}{2}\left[2(1-\nu)\phi(\frac{2\nu-1}{\rho} + (1-\nu)\phi)\right]\hat{T}_t^2$$
$$+\left[\frac{(1-\nu)(2\nu-1)}{\rho} - 2\nu(1-\nu)\theta\right]\hat{T}_t\hat{Q}_t - (1-\nu)\left[4\nu(1-\nu)\theta + \frac{(2\nu-1)^2}{\rho}\right]\hat{T}_t^2$$
$$+\nu(1-\nu)\theta\hat{T}_t^2 + O(||Z||^3).$$

Combining all of the like terms and using equation (29) to substitute out the real exchange rate, we obtain:

$$\hat{C}_t + \hat{C}_t^* - \hat{Y}_{H,t} - \hat{Y}_{F,t} = \left[-\nu(1-\nu)\theta + (1-\nu)\phi(\frac{2\nu-1}{\rho} + (1-\nu)\phi)\right]\hat{T}_t^2.$$

Equivalently,

$$\hat{C}_t - \hat{Y}_{H,t} = \hat{Y}_{F,t} - \hat{C}_t^* + \left[-\nu(1-\nu)\theta + (1-\nu)\phi(\frac{2\nu-1}{\rho} + (1-\nu)\phi)\right]\hat{T}_t^2. \tag{66}$$

4.2 Second-order Approximation

Rewriting the second-order approximation of the utility (44) for convenience:

$$v_t \approx \hat{C}_t - \hat{Y}_{H,t} + (\frac{1-\rho}{2}\hat{C}_t + \epsilon_t)\hat{C}_t - \frac{1}{2}(1+\eta)\hat{Y}_{H,t}^2 - \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 + t.i.p. + O(||Z||^3), \tag{67}$$

where the term $t.i.p.$ collects all the terms that are independent of monetary policy and the term $O(||Z||^3)$ groups all the terms that are of third or higher order.

Plugging (59) into (67) results in:

$$v_t \approx \hat{C}_t - \hat{Y}_{H,t} + (\frac{1-\rho}{2}\hat{C}_t + \epsilon_t)\hat{C}_t - \frac{1}{2}(1+\eta)\left[\hat{C}_t^2 + (1-\nu)^2\phi^2\hat{T}_t^2 + 2(1-\nu)\phi\hat{C}_t\hat{T}_t\right]$$
$$-\frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 + t.i.p. + O(||Z||^3).$$

Combining all of the like terms, we obtain:

$$= \hat{C}_t - \hat{Y}_{H,t} + \hat{C}_t\epsilon_t - (\frac{\rho+\eta}{2})\hat{C}_t^2 - \frac{1}{2}(1+\eta)(1-\nu)^2\phi^2\hat{T}_t^2 \tag{68}$$
$$-(1+\eta)(1-\nu)\phi\hat{C}_t\hat{T}_t - \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 + t.i.p. + O(||Z||^3).$$

Substituting out $\hat{C}_t - \hat{Y}_{H,t}$ using (66):

$$\upsilon_t \approx \left[-\nu(1-\nu)\theta + (1-\nu)\phi(\frac{2\nu-1}{\rho} + (1-\nu)\phi)\right]\hat{T}_t^2 + \hat{C}_t\epsilon_t - (\frac{\rho+\eta}{2})\hat{C}_t^2$$
$$-\frac{1}{2}(1+\eta)(1-\nu)^2\phi^2\hat{T}_t^2 - (1+\eta)(1-\nu)\phi\hat{C}_t\hat{T}_t - \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 + t.i.p. + O(||Z||^3), \tag{69}$$

where we have included $\hat{Y}_{F,t} - \hat{C}_t^*$ in the *t.i.p.*, since it is a term independent of domestic monetary policy.

Under symmetric demand shocks, the efficient level is characterized by no movement in terms of trade since the responses of output and consumption between the two countries are identical. Using the fact $\epsilon_t = \epsilon_t^*$ and $\hat{T}_t^{fb} = 0$ under symmetric demand shocks, equation (18) implies $\hat{Y}_{H,t}^{fb} = \hat{C}_t^{fb}$. Inserting this relationship into (20) yields $\hat{C}_{H,t}^{fb} = \frac{\epsilon_t}{(\rho+\eta)}$. Using the fact $\epsilon_t = (\rho+\eta)\hat{C}_t^{fb}$ and the relationship (55), we can organize the above expression as:

$$\begin{aligned}\upsilon_t &\approx \left[-\nu(1-\nu)\theta + (1-\nu)\phi(\frac{2\nu-1}{\rho} + (1-\nu)\phi)\right]\hat{T}_t^2 - \frac{1}{2}(\rho+\eta)\hat{C}_t(\hat{C}_t - \hat{C}_t^{fb}) \\ &-\frac{1}{2}(1+\eta)(1-\nu)^2\phi^2\hat{T}_t^2 - (1+\eta)(1-\nu)\phi\hat{C}_t\hat{T}_t - \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 + t.i.p. + O(||Z||^3) \\ &= \left[-\nu(1-\nu)\theta + (1-\nu)\phi(\frac{2\nu-1}{\rho}) - \frac{1}{2}(1-\nu)^2\phi^2(\eta-1)\right]\hat{T}_t^2 - \frac{1}{2}(\rho+\eta)\tilde{C}_t^2 \\ &\quad -(1+\eta)(1-\nu)\phi\hat{C}_t\hat{T}_t - \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t}^2 + t.i.p. + O(||Z||^3).\end{aligned} \tag{70}$$

Equation (70) is the country-specific loss function for the home country that we use in the paper.

5 Discretionary Policy FOCs

5.1 Floating Exchange Rate Regime

The optimal cooperative policy under discretion is characterized by the solution to the following optimization problem.

$$
\begin{aligned}
\mathcal{L} = U_C C \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ & -\frac{1}{4}\Big((\rho+\eta)(\tilde{Y}_{H,t}^2 + \tilde{Y}_{F,t}^2) + \frac{\sigma}{\kappa_H}\pi_{H,t}^2 + \frac{\sigma}{\kappa_F}\pi_{F,t}^{*2} \\
& - \frac{2\nu(1-\nu)}{4\nu(1-\nu)(\rho\theta-1)+1}\left[(\rho\theta-1)\rho(\tilde{Y}_{H,t}-\tilde{Y}_{F,t})^2 - \theta \hat{f}_t^2\right]\Big) \\
& + \lambda_{1t}\left[-E_t\hat{C}_{t+1} + \hat{C}_t + \frac{1}{\rho}(\hat{R}_t - (E_t\pi_{H,t+1} + (1-\nu)(E_t\hat{T}_{t+1} - \hat{T}_t)) + E_t\epsilon_{t+1} - \epsilon_t)\right] \\
& + \lambda_{2t}\left[-E_t\hat{C}^*_{t+1} + \hat{C}^*_t + \frac{1}{\rho}(\hat{R}^*_t - (E_t\pi^*_{F,t+1} - (1-\nu)(E_t\hat{T}_{t+1} - \hat{T}_t)) + E_t\epsilon^*_{t+1} - \epsilon^*_t)\right] \\
& + \lambda_{3t}\left[-\hat{Y}_{H,t} + 2\nu(1-\nu)\theta\hat{T}_t + \nu\hat{C}_t + (1-\nu)\hat{C}^*_t\right] \\
& + \lambda_{4t}\left[-\hat{Y}_{F,t} - 2\nu(1-\nu)\theta\hat{T}_t + (1-\nu)\hat{C}_t + \nu\hat{C}^*_t\right] \\
& + \lambda_{5t}\left[-\pi_{H,t} + \beta E_t\pi_{H,t+1} + \kappa_H\Big\{(\rho+\eta)\tilde{Y}_{H,t} - (1-\nu)\left[2\nu(\rho\theta-1)\tilde{T}_t - \hat{f}_t\right]\Big\}\right] \\
& + \lambda_{6t}\left[-\pi^*_{F,t} + \beta E_t\pi^*_{F,t+1} + \kappa_F\Big\{(\rho+\eta)\tilde{Y}_{F,t} + (1-\nu)\left[2\nu(\rho\theta-1)\tilde{T}_t - \hat{f}_t\right]\Big\}\right] \\
& + \lambda_{7t}\left[\rho(\hat{C}_t - \hat{C}^*_t) + (\epsilon^*_t - \epsilon_t) - \hat{Q}_t - \hat{f}_t\right] \\
& + \lambda_{8t}\left[\hat{Q}_t - (2\nu-1)\hat{T}_t\right] \\
& + \lambda_{9t}\left[\hat{f}_t - A(\hat{Y}_{H,t} - \hat{Y}_{F,t}) - B(\epsilon_t - \epsilon^*_t)\right]\Big\}
\end{aligned}
\tag{71}
$$

where the coefficients A and B are given by:

$$
\begin{aligned}
A &= \frac{\lambda(1-\nu)}{\lambda(1-\nu)(2\theta\nu - 2\nu + 1) + 2(1-\lambda)\Delta}(\rho(2\theta\nu - 1) + 1 - 2\nu), \\
B &= \frac{\lambda(1-\nu)}{\lambda(1-\nu)(2\theta\nu - 2\nu + 1) + 2(1-\lambda)\Delta}(2\theta\nu - 2\nu + 1).
\end{aligned}
$$

In the discretionary policy, the central bank cannot commit itself to any future action. Therefore, all the expectation terms are considered as given by the central bank. Furthermore, the shock terms in the Lagrangian are given, because they are independent of policy. The first-order conditions are:

$$
\begin{aligned}
[\hat{Y}_{H,t}] : -\frac{1}{2}(\rho+\eta)(\hat{Y}_{H,t} - \hat{Y}^{fb}_{H,t}) + \frac{\nu(1-\nu)}{4\nu(1-\nu)(\rho\theta-1)+1}\left[(\rho\theta-1)\rho(\hat{Y}_{H,t} - \hat{Y}^{fb}_{H,t} - \hat{Y}_{F,t} + \hat{Y}^{fb}_{F,t})\right] \\
-\lambda_{3t} + \kappa_H(\rho+\eta)\lambda_{5t} - A\lambda_{9t} = 0
\end{aligned}
$$

$$[\hat{Y}_{F,t}] : -\frac{1}{2}(\rho+\eta)(\hat{Y}_{F,t} - \hat{Y}^{fb}_{F,t}) - \frac{\nu(1-\nu)}{4\nu(1-\nu)(\rho\theta-1)+1}\left[(\rho\theta-1)\rho(\hat{Y}_{H,t} - \hat{Y}^{fb}_{H,t} - \hat{Y}_{F,t} + \hat{Y}^{fb}_{F,t})\right]$$
$$-\lambda_{4t} + \kappa_F(\rho+\eta)\lambda_{6t} + A\lambda_{9t} = 0$$

$$[\pi_{H,t}] : \lambda_{5,t} + \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t} = 0$$

$$[\pi^*_{F,t}] : \lambda_{6,t} + \frac{1}{2}\frac{\sigma}{\kappa_F}\pi^*_{F,t} = 0$$

$$[\hat{C}_t] : \lambda_{1,t} + \nu\lambda_{3,t} + (1-\nu)\lambda_{4,t} + \rho\lambda_{7,t} = 0$$

$$[\hat{C}^*_t] : \lambda_{2,t} + \nu\lambda_{4,t} + (1-\nu)\lambda_{3,t} - \rho\lambda_{7,t} = 0$$

$$[\hat{R}_t] : \lambda_{1,t} \geq 0, \hat{R}_t \geq 0, \lambda_{1,t}\hat{R}_t = 0$$

$$[\hat{R}^*_t] : \lambda_{2,t} \geq 0, \hat{R}^*_t \geq 0, \lambda_{2,t}\hat{R}^*_t = 0$$

$$[\hat{T}_t] : -\frac{1-\nu}{\rho}\lambda_{1,t} + \frac{1-\nu}{\rho}\lambda_{2,t} - 2\nu(1-\nu)\theta\lambda_{3,t} + 2\nu(1-\nu)\theta\lambda_{4,t}$$
$$+2\kappa_H\nu(1-\nu)(\rho\theta-1)\lambda_{5,t} - 2\kappa_F\nu(1-\nu)(\rho\theta-1)\lambda_{6,t} + (2\nu-1)\lambda_{8,t} = 0$$

$$[\hat{Q}_t] : \lambda_{8,t} - \lambda_{7,t} = 0$$

$$[\hat{f}_t] : \kappa_H(1-\nu)\lambda_{5,t} - \kappa_F(1-\nu)\lambda_{6,t} - \lambda_{7,t} + \lambda_{9,t} - \frac{\nu(1-\nu)\theta}{4\nu(1-\nu)(\rho\theta-1)+1}\hat{f}_t = 0$$

5.2 Monetary Union Regime

The optimal policy in a monetary union under discretion is characterized by the solution to the following optimization problem.

$$
\begin{aligned}
\mathcal{L} = U_C C \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ & -\frac{1}{4}\Big((\rho+\eta)(\tilde{Y}_{H,t}^2 + \tilde{Y}_{F,t}^2) + \frac{\sigma}{\kappa_H}\pi_{H,t}^2 + \frac{\sigma}{\kappa_F}\pi_{F,t}^{*2} \\
& - \frac{2\nu(1-\nu)}{4\nu(1-\nu)(\rho\theta-1)+1}\left[(\rho\theta-1)\rho(\tilde{Y}_{H,t}-\tilde{Y}_{F,t})^2 - \theta \hat{f}_t^2 \right] \Big) \\
& + \lambda_{1t}\left[-E_t\hat{C}_{t+1} + \hat{C}_t + \frac{1}{\rho}(\hat{R}_t^{MU} - (E_t\pi_{H,t+1} + (1-\nu)(E_t\hat{T}_{t+1} - \hat{T}_t)) + E_t\epsilon_{t+1} - \epsilon_t) \right] \\
& + \lambda_{2t}\left[\hat{T}_t - \hat{T}_{t-1} - \pi_{F,t}^* + \pi_{H,t} \right] \\
& + \lambda_{3t}\left[-\hat{Y}_{H,t} + 2\nu(1-\nu)\theta\hat{T} + \nu\hat{C}_t + (1-\nu)\hat{C}_t^* \right] \\
& + \lambda_{4t}\left[-\hat{Y}_{F,t} - 2\nu(1-\nu)\theta\hat{T} + (1-\nu)\hat{C}_t + \nu\hat{C}_t^* \right] \\
& + \lambda_{5t}\left[-\pi_{H,t} + \beta E_t\pi_{H,t+1} + \kappa_H\Big\{ (\rho+\eta)\tilde{Y}_{H,t} - (1-\nu)\left[2\nu(\rho\theta-1)\tilde{T}_t - \hat{f}_t \right] \Big\} \right] \\
& + \lambda_{6t}\left[-\pi_{F,t}^* + \beta E_t\pi_{F,t+1}^* + \kappa_F\Big\{ (\rho+\eta)\tilde{Y}_{F,t} + (1-\nu)\left[2\nu(\rho\theta-1)\tilde{T}_t - \hat{f}_t \right] \Big\} \right] \\
& + \lambda_{7t}\left[\rho(\hat{C}_t - \hat{C}_t^*) + (\epsilon_t^* - \epsilon_t) - \hat{Q}_t - \hat{f}_t \right] \\
& + \lambda_{8t}\left[\hat{Q}_t - (2\nu-1)\hat{T}_t \right] \\
& + \lambda_{9t}\left[\hat{f}_t - A(\hat{Y}_{H,t} - \hat{Y}_{F,t}) - B(\epsilon_t - \epsilon_t^*) \right] \Big\}
\end{aligned}
$$

The first order conditions are given as follows:

$$
\begin{aligned}
[\hat{Y}_{H,t}] : & -\frac{1}{2}(\rho+\eta)(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb}) + \frac{\nu(1-\nu)}{4\nu(1-\nu)(\rho\theta-1)+1}\left[(\rho\theta-1)\rho(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb} - \hat{Y}_{F,t} + \hat{Y}_{F,t}^{fb}) \right] \\
& -\lambda_{3t} + \kappa_H(\rho+\eta)\lambda_{5t} - A\lambda_{9t} = 0
\end{aligned}
$$

$$
\begin{aligned}
[\hat{Y}_{F,t}] : & -\frac{1}{2}(\rho+\eta)(\hat{Y}_{F,t} - \hat{Y}_{F,t}^{fb}) - \frac{\nu(1-\nu)}{4\nu(1-\nu)(\rho\theta-1)+1}\left[(\rho\theta-1)\rho(\hat{Y}_{H,t} - \hat{Y}_{H,t}^{fb} - \hat{Y}_{F,t} + \hat{Y}_{F,t}^{fb}) \right] \\
& -\lambda_{4t} + \kappa_F(\rho+\eta)\lambda_{6t} + A\lambda_{9t} = 0
\end{aligned}
$$

$$
[\pi_{H,t}] : \lambda_{5,t} + \frac{1}{2}\frac{\sigma}{\kappa_H}\pi_{H,t} - \lambda_{2,t} = 0
$$

$$
[\pi_{F,t}^*] : \lambda_{6,t} + \frac{1}{2}\frac{\sigma}{\kappa_F}\pi_{F,t}^* + \lambda_{2,t} = 0
$$

$$[\hat{C}_t] : \lambda_{1,t} + \nu\lambda_{3,t} + (1-\nu)\lambda_{4,t} + \rho\lambda_{7,t} = 0$$

$$[\hat{C}^*_t] : \nu\lambda_{4,t} + (1-\nu)\lambda_{3,t} - \rho\lambda_{7,t} = 0$$

$$[\hat{R}^{MU}_t] : \lambda_{1,t} \geq 0, \hat{R}^{MU}_t \geq 0, \lambda_{1,t}\hat{R}^{MU}_t = 0$$

$$[\hat{T}_t] : -\frac{1-\nu}{\rho}\lambda_{1,t} - \lambda_{2,t} - 2\nu(1-\nu)\theta\lambda_{3,t} + 2\nu(1-\nu)\theta\lambda_{4,t}$$
$$+2\kappa_H\nu(1-\nu)(\rho\theta-1)\lambda_{5,t} - 2\kappa_F\nu(1-\nu)(\rho\theta-1)\lambda_{6,t} + (2\nu-1)\lambda_{8,t} = 0$$

$$[\hat{Q}_t] : \lambda_{8,t} - \lambda_{7,t} = 0$$

$$[\hat{f}_t] : \kappa_H(1-\nu)\lambda_{5,t} - \kappa_F(1-\nu)\lambda_{6,t} - \lambda_{7,t} + \lambda_{9,t} - \frac{\nu(1-\nu)\theta}{4\nu(1-\nu)(\rho\theta-1)+1}\hat{f}_t = 0$$

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