Supplementary Online Appendix for: “The Risk-Premium Channel of Uncertainty: Implications for Unemployment and Inflation”*

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December 2022

This online appendix contains supplemental material for the article “‘The Risk-Premium Channel of Uncertainty: Implications for Unemployment and Inflation.”

Any references to equations, figures, tables or sections that are not preceded by a capital letter refer to the main article.

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Appendix A

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A.1 Empirical evidence on the effects of uncertainty shocks

To provide context for the theoretical analysis in the main text, here we briefly summarize the empirical effects on key macroeconomic and financial variables of an increase in perceived uncertainty. Methodologically, we follow a vast literature, including Leduc and Liu (2016), whose specification we marginally extend by including evidence on vacancy-posting by firms and a measure of the equity risk premium. Specifically, we estimate a simple vector autoregression (VAR) model that includes a forward-looking measure of economic uncertainty ordered first, and identify the uncertainty shock recursively. We rely on monthly US data spanning 1978:1-2016:12 and include the following six variables in the given order: a measure of perceived economic uncertainty;[A.1] the U-3 unemployment rate from the Current Population Survey; the composite help-wanted index constructed in Barnichon (2010); the one-year ahead equity risk premium (ERP) from Duarte and Rosa (2015); the year-on-year CPI inflation rate; and the 3-month Treasury bill rate. The model includes six lags and is estimated using standard Bayesian methods.

Figure A.1 describes the resulting impulse responses to a one standard-deviation uncertainty shock. We make three observations. First, elevated uncertainty leads to a sizeable contraction in economic activity. In particular, the shock causes the unemployment rate to rise in a persistent, hump-shaped manner, with a sizeable peak effect of more than 0.2 percentage points. Moreover, vacancy posting activity as captured by help-wanted advertising is negatively affected. Second, despite these unambiguous adverse consequences for real economic activity, the response of the inflation rate is small and at no point is the zero not included in the 95 percent credible interval. Finally, and turning to financial variables, while the essentially risk-free Treasury bill rate declines, the risk premium increases by up to 30 basis points.

[A.1] Specifically, this measure is given by the fraction of respondents in the Michigan Consumer Survey pointing to an "uncertain future" as negatively affecting their their spending on durable goods over the coming year. The results are highly comparable when using various alternative uncertainty proxies, as for instance the Equity Market Volatility Tracker of Baker *et al.* (2019).

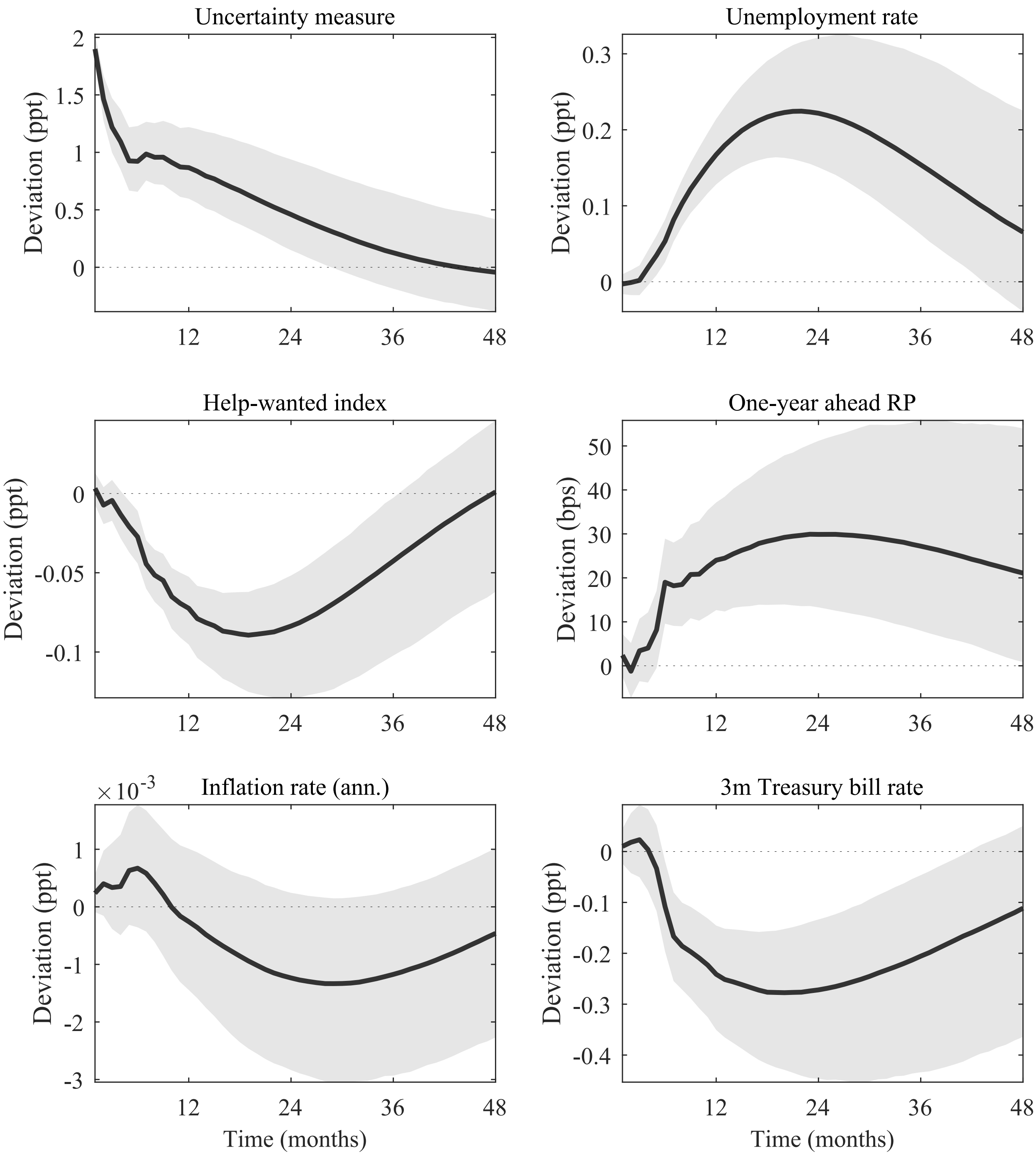

Figure A.1: VAR evidence on the effects of uncertainty shocks

Notes: The figure describes the empirical effects of a one standard-deviation uncertainty shock identified from a Cholesky decomposition. The data included are described in the main text. The VAR is estimated using Markov Chain Monte Carlo Methods and employing a normal-diffuse prior for the coefficient matrix and the covariance matrix of the reduced-form innovations, respectively. Solid lines indicate the median posterior density of impulse responses, while the shaded area represents the highest posterior density (HPD) interval.

Appendix B

Please note that proofs of Propositions 1 and 2 are contained in the primary appendix included alongside the main manuscript.

B.1 Derivation of the resource constraint

Since all firms use a constant returns to scale technology – alongside with the fact that intermediate goods use only labor as an input, retail firms use only intermediate goods, and final goods firms use only retail goods – aggregate output is given by $y_t = z_t n_t$.

As mentioned in section 2.1, the household makes additional profits, $\tilde{d}_t$. These profits are in excess of the dividends arising from the ownership of intermediate firms, and instead include per-period profits from retailers, vacancy-posting agencies, and price adjusting firms. Aggregate profits arising from vacancy-posting agencies are equal to κv_t. Moreover, the aggregate profit arising from retailers net of price adjustment costs is

$$\frac{p_t(i)}{P_t} y_t(i) - x_t y_t(i).$$

Using the fact that in a symmetric equilibrium $p_t(i) = p_t(j) = P_t$, alongside with the demand relation in equation (10) together with $y_t = z_t n_t$, reveals that these profits amount to $z_t n_t (1 - x_t)$. Thus,

$$\tilde{d}_t = \kappa v_t + z_t n_t (1 - x_t). \tag{B.1}$$

Using the equilibrium relations $B_t = 0$ and $n_t = a_t$, the household's budget constraint is

$$c_t + n_t (J_t - d_t) = w_t n_t + \tilde{d}_t + n_{t-1}(1 - \delta) J_t.$$

Rearranging and using that fact that $d_t = x_t z_t - w_t$ gives

$$c_t + J_t (n_t - n_{t-1}(1 - \delta)) - = n_t x_t z_t + \tilde{d}_t.$$

Using the law of motion for employment in equation (19), and the definition of $\tilde{d}_t$ above reveals that

$$c_t + \kappa v_t - = n_t x_t z_t + \kappa v_t + z_t n_t (1 - x_t),$$

or simply $y_t = c_t = z_t n_t$.

Notice that aggregate consumptions is therefore not affected by the amount of vacancies created, nor the costs associated with price adjustments. This is indeed intentional; as we are exploring the role of uncertainty on behavior, any resource draining activity, such as price adjustments, may, somewhat mechanically, alter the marginal utility of consumption. We explore the role of such activities in appendix section C.4.

B.2 Derivation of the wage-setting rule

Wage setting based on alternating offers stems from the observation that severing a match is not a credible threat; indeed the worker and the firm will always reach an agreement within the period the meeting occurs. Common knowledge of this feature implies that future variables bear no consequence on the currently agreed wage.

The alternating-offers game takes place in fictional time, in which each time-period is of length Δ. If the worker has the opportunity of proposing a wage, w_t, she will offer the highest possible value that the firm will accept. That is, the wage will yield the worker a maximum value of $\overline{v}_w = w_t$, and the firm a minimum value of $\underline{v}_f = x_t z_t - w_t$. However, as the firm can reject the wage proposal and wait to the next (fictional) time-period to make a counteroffer, the minimum value must also satisfy $\underline{v}_f = e^{-\Delta\omega} \times \overline{v}_f$, where $\overline{v}_f$ denotes the firm's maximum value, and $e^{-\Delta\omega}$ the discount factor.

Conversely, if the firm has the opportunity of proposing a wage, w'_t, it will yield the firm a maximum value of $\overline{v}_f = x_t z_t - w'_t$, and the worker a minimum value of $\underline{v}_w = w'$. Again, as the worker can reject the wage proposal and wait to the next (fictional) time-period to make a counteroffer, the worker's minimum value must also satisfy $\underline{v}_w = \Delta\hat{\chi} + e^{-\Delta(1-\omega)} \times \overline{v}_w$, where $\Delta\hat{\chi}$ represents the flow utility the worker receives by not working.[B.1] Notice that the worker and the firm discounts fictional time differently; a higher value of ω renders workers more patient which will play to the worker's advantage, and vice versa.

The above set-up provides six (linear) equations in six unknowns. Solving this system and letting Δ approach zero gives rise to a unique (subgame perfect) wage that is agreed upon immediately

$$w_t = \omega x_t z_t + (1-\omega)\chi, \tag{B.2}$$

with $\chi = \hat{\chi}/(1-\omega)$. That is, the wage agreed by alternating offers is a combination of the firm's revenues and the flow consumption-value the worker receives by delaying agreement.

Appendix C

C.1 Sensitivity analysis: the role of χ

This appendix section shows that the extent to which uncertainty shocks cause mild or severe recessions in the model is shaped by the value of the parameter χ, which in the alternating offer bargaining game

[B.1] We did not explicitly introduce the parameter χ into the utility function but doing so would be straightforward.

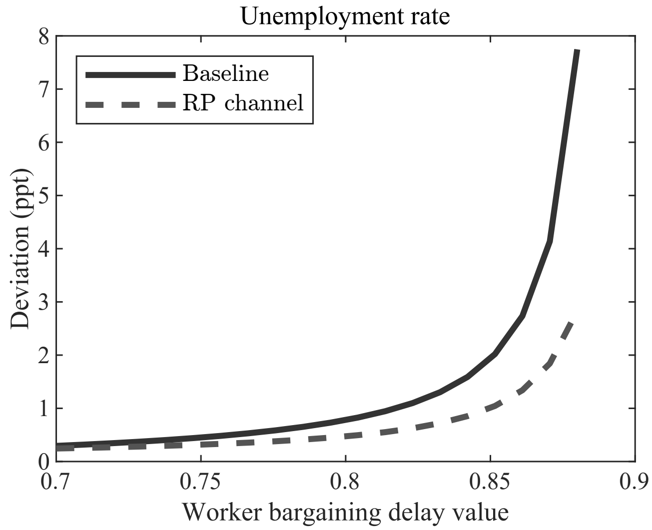

Figure C.1: Sensitivity to χ

Notes: The figure illustrates the cumulative response of unemployment to a one standard-deviation uncertainty shock, varying the flow consumption-value the worker receives by delaying agreement during the alternating offers bargaining process, χ.

over wages represents the worker's outside option.

To illustrate this property, we solve the model repeatedly for a grid of values for χ, holding the other parameters constant, and, each time, compute the cumulative response of unemployment to the uncertainty shock. The solid line in Figure C.1 depicts the result. It can be seen that the cumulative impact of an uncertainty shock on the unemployment rate monotonically increases in χ.[C.1] The reasons for this result relate to the size of the fundamental surplus $xz - \chi$ (Ljungqvist and Sargent, 2017). Intuitively, if the fundamental surplus is small relative to output, then a given percentage change in productivity induces a greater percentage change in investment into vacancies (and, concomitantly, a larger responses of employment). This same logic carries through to the analysis of uncertainty shocks, with a smaller fundamental surplus translating into more unemployment volatility.[C.2]

Lastly, and for completeness, the dashed line in Figure C.1 quantifies the magnitude of the uncertainty effect on the unemployment rate that is due to the risk-premium channel as a function of χ. Unsurprisingly, per the above reasoning, that particular channel is stronger for higher values of χ. The *relative* contribution of the risk-premium channel declines somewhat for still higher values of χ.

[C.1] The same result obtains under flexible prices.

[C.2] An additional nuance is that employment asymmetries become disproportionately more important when employment is more volatile. That is, when χ is high, then following an increase in perceived future exogenous volatility agents to downgrade their expectations for future average employment by a significant amount. Indeed, higher volatility makes it more likely that the model gives rise to what Petrosky-Nadeau *et al.* (2018) describe as "endogenous disasters". The anticipation of such an outcome leads agents to change their behavior in the present, giving rise to strong pure uncertainty effects.

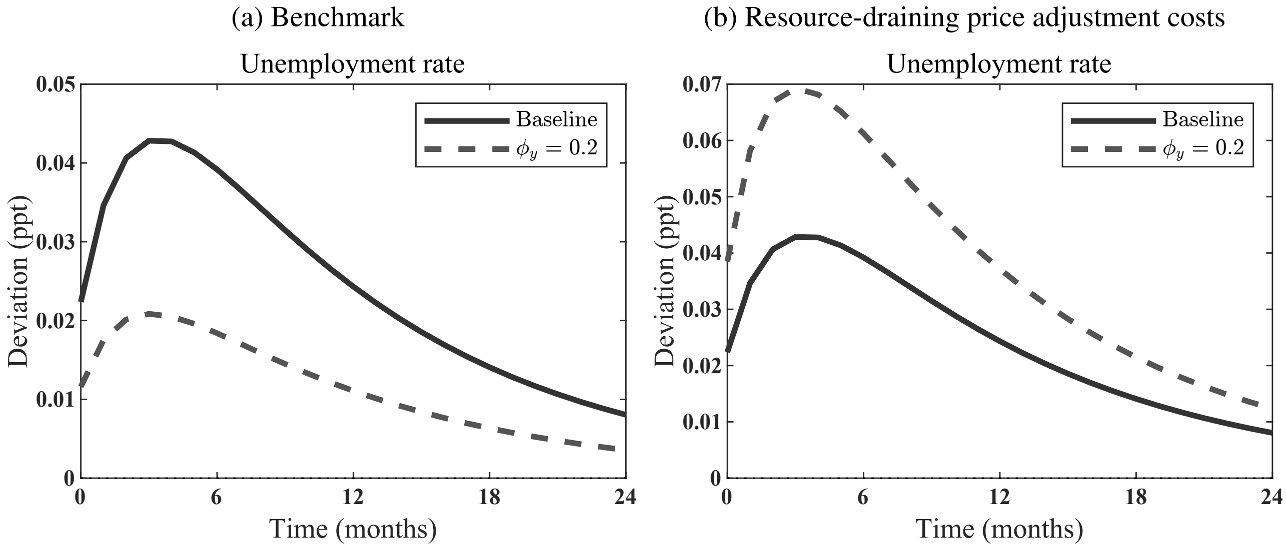

Figure C.2: Taylor rule with active response to output

Notes: The figure illustrates the response of unemployment to an uncertainty shock under sticky prices and log utility, varying the aggressiveness of the monetary authority's response to output. The left-hand panel assumes that all output is available for consumption, whereas the right-hand panel assumes that price-adjustment costs divert real resources.

C.2 A more general Taylor rule

The baseline analysis considered a Taylor rule according to which the central bank responds to inflation only. This section explores the implications of allowing for a more general specification according to which the monetary authority also responds to output fluctuations as it is quite commonly used in the literature (and is, arguably, is of empirical relevance). The correponding coefficient is denoted ϕ_y.[C.3]

Figure C.2a shows that the recessionary impact of elevated uncertainty on unemployment is much less severe if the monetary authority responds not only to inflation but also to output (with $\phi_y = 0.2$). This is for two reasons. First, $\phi_y > 0$ directly implies a more accommodative policy stance given the contraction in output in the present. Second, expectations for future production are stabilized, which mutes not only precautionary savings due to prudence but also the force of employment asymmetries generating a fall in expected consumption that would incentivize saving in the present. To see the latter feature, notice first that the direct impact of, say, an increase in the productivity of the intermediate goods sector is to push down the intermediate price of inputs, x_t. However, the same shock also increases income and stimulates demand, which instead puts upward pressure on the relative price, x_t. The reverse holds true for negative productivity shocks. By responding to output, policymakers can limit the second effect, engineer a relatively more countercyclical relative price, and thus dampen business cycle volatility in hiring (see Lepetit (2020) for a detailed discussion). Given the asymmetric

[C.3] Indeed, both Leduc and Liu (2016) and Basu and Bundick (2017) consider $\phi_y = 0.2$ in their baseline parameterization. Basu and Bundick (2017) assume that ϕ_y applies to output in deviation from its own lag, rather than its deviation from the deterministic steady-state.

employment dynamics of the model, the expected *employment gap* – caused by the anticipation of greater future volatility – and resulting pessimism about the future, is then smaller in absolute magnitude.[C.4]

The implications of setting $\phi_y > 0$ for *inflation* are more intricate in two respects. First, as uncertainty shocks tend to push inflation and output in the same direction, responding more aggressively to either variable helps stabilizing the other one as well. Yet in response to a productivity shock, the inflation rate and output move in *opposite* directions. Accordingly, by stabilizing *future* output the monetary authority also leaves *future* inflation relatively more volatile.[C.5] Thus, the monetary authority faces a tradeoff between stabilizing either future real or nominal dynamics, even when this is not the case with respect to today's economy in which, by construction, only uncertainty shocks materialize.

Second, when price adjustments are costly in resource terms, then stochastic volatility renders the tradeoffs facing the monetary authority more complex. To see this, suppose we relax the assumption, maintained thus far, that Rotemberg adjustment costs subtracted from retail firms' profits are rebated to the household as a lump-sum. Then the GDP identity is instead given by $y_t = c_t + ac_t$, where ac_t represents the convex adjustment cost

$$ac_t = \frac{\Omega_p}{2}\left(\frac{p_t(i)}{p_{t-1}(i)\Pi} - 1\right)^2 y_t.$$

Figure C.2b illustrates that under this assumption the economy may even fall into a *deeper* recession following an uncertainty shock when the monetary authority actively responds to output. This result stands in marked contrast to Figure C.2a. The key insight is that by setting $\phi_y > 0$ the monetary authority may stabilize expected future *production*, but now tolerating greater future inflation variability no longer guarantees stabilizing future *consumption*. The convex nature of adjustment costs implies that greater expected future price adjustments translate into lower expected consumption for any given level of production. As before, expectations for lower future consumption then ripple through the economy, leading to a fall in current demand but also the expectation of lower future asset prices. To summarize, with resource draining price-adjustment costs, an increase in anticipated volatility leaves households more pessimistic about future consumption, either because employment is expected to be significantly more volatile – when the monetary authority ignores output fluctuations – or because inflation is expected to be more variable – when the Taylor rule does give weight to output. The monetary authority thus faces an uncomfortable tradeoff between expected future output volatility and inflation volatility; both of which contribute to exacerbating the effects of an uncertainty.

[C.4] Notice that in a standard New Keynesian model without asymmetric employment dynamics, the expectation for more volatile employment would typically not affect the conditional mean.

[C.5] This statement implicitly rests on the premise that overall dynamics are more strongly influenced by shocks to levels than by shocks to second moments. In practice, this is the case.

C.3 Precautionary pricing

When firms are subject to nominal rigidities, an increase in uncertainty about future demand conditions may give rise to precautionary pricing that renders markups more countercyclical.[C.6] Intuitively, such behavior arises when firms' marginal revenue product exhibits convexity; it is more costly for a given firm to set too low a price relative to its competitors (more units need to be sold at a sub-optimally low price) compared to setting it suboptimally high (the higher price per unit partially compensated for fewer units sold).

To avoid confounding the key mechanisms of interest in our analysis with such effects, all results in the main text are derived by imposing a linearized version of the New Keynesian Phillips curve (NKPC) in equation (13). This approach eliminates non-linear terms that could potentially generate an upward pricing bias (cf. Fernández-Villaverde *et al.* (2015, section VI)). To verify the robustness of our results, Figure C.3 plots the usual pure uncertainty IRFs under the benchmark parameterization (solid line), but illustrating also the outcome when the underlying model is solved such that the nonlinearity of the NKPC is preserved (dashed line)

Figure C.3 illustrates why we consider our baseline implementation not only a useful simplification, in that it strengthens tractability, but also an innocuous one. As can be seen from the figure, nonlinearities in the NKPC play virtually no role when $\phi_\pi = 1.5$ and $\phi_y = 0$. Moreover, even when we allow for a non-zero response to output, which leads to greater volatility in future inflation, the difference in inflation responses is modest. Consistent with the principles of precautionary pricing, to the extent that there do exist nonlinearities in the NKPC, they bias inflation upward and the relative price (i.e., the inverse relative markup) downward; the latter generally exerts a negative effect on job creation. To the extent that nonlinearities in the NKPC *do* exert upward pressure on inflation, this only goes to reinforce our finding that uncertainty shocks are less deflationary than regular demand shocks.

C.4 Resource-draining vacancy posting costs

In the benchmark model we assume that vacancy posting costs are rebated to the household. If one were to suppose, instead, that these expenditures subtract from consumption in the resource constraint – that is, $c_t + \kappa v_t = z_t n_t$ – then this gives rise to an additional transmission channel that operates under both flexible and sticky prices. As vacancies v_t are convex in productivity, a spike in expected volatility lowers the expected level of output available for consumption. Such an expectation then sets off a greater desire for savings in the present. Quantitatively, this channel turns out to be insignificant (figures are available upon request), so that we abstract from it in order to focus on the key transmission channels of interest.

[C.6] For details on precautionary pricing, see Fernández-Villaverde *et al.* (2015), Oh (2019), and Born and Pfeifer (2020).

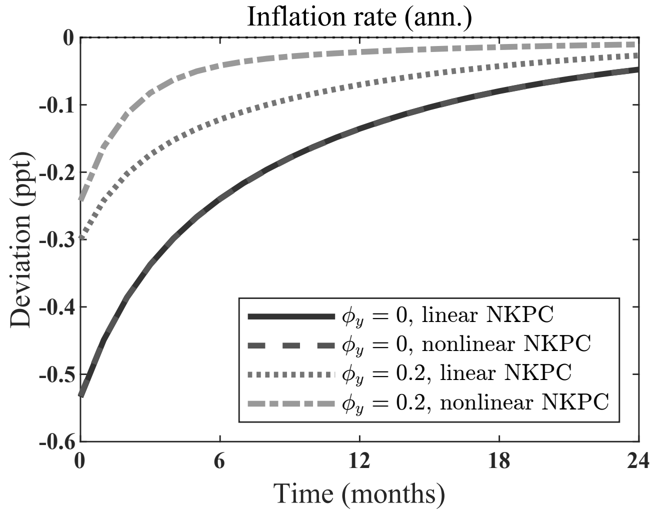

Figure C.3: The limited role of precautionary pricing

Notes: The figure illustrates the pure-uncertainty IRFs for a one standard-deviation shock to volatility. The different IRFs vary according to the coefficient on output in the Taylor rule, ϕ_y and differ in whether the structural New Keynesian Phillips curve equation is linearized or not when solving the model.

References

Baker, S., Bloom, N., Davis, S., and Kost, K. (2019). Policy News and Stock Market Volatility. *SSRN Electronic Journal*.

Barnichon, R. (2010). Building a composite Help-Wanted Index. *Economics Letters*, **109**(3), 175–178.

Basu, S. and Bundick, B. (2017). Uncertainty Shocks in a Model of Effective Demand. *Econometrica*, **85**(3), 937–958.

Born, B. and Pfeifer, J. (2020). Uncertainty-driven Business Cycles: Assessing the Markup Channel. *CEPR Discussion Paper DP11745*.

Duarte, F. and Rosa, C. (2015). The Equity Risk Premium: A Review of Models. *SSRN Electronic Journal*.

Fernández-Villaverde, J., Guerrón-Quintana, P., Kuester, K., and Rubio-Ramírez, J. (2015). Fiscal Volatility Shocks and Economic Activity. *American Economic Review*, **105**(11), 3352–3384.

Leduc, S. and Liu, Z. (2016). Uncertainty Shocks are Aggregate Demand Shocks. *Journal of Monetary Economics*, **82**, 20–35.

Lepetit, A. (2020). Asymmetric Unemployment Fluctuations and Monetary Policy Trade-Offs. *Review of Economic Dynamics*.

Ljungqvist, L. and Sargent, T. J. (2017). The Fundamental Surplus. *American Economic Review*, **107**(9), 2630–2665.

Oh, J. (2019). The Propagation of Uncertainty Shocks: Rotemberg vs. Calvo. *mimeo*.

Petrosky-Nadeau, N., Zhang, L., and Kuehn, L.-A. (2018). Endogenous Disasters. *American Economic Review*, **108**(8), 2212–2245.