Product Innovation, Diffusion and Endogenous Growth
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S.1 Equilibrium Investment Distortions and Welfare Effects

In Section 4 of the main text, we examined the use of R&D policy as a single policy instrument to achieve a second-best resource allocation. We showed that the optimality condition associated with this allocation can be understood as a weighted combination of the two dynamic distortions present in the model. Specifically, we derived the optimality condition $\mathscr{L}_I^{SB} = 0$, where

$$\mathscr{L}_I^{SB} = \underbrace{\frac{\partial n_B}{\partial I}\Big[\frac{c\delta ln(\lambda)}{\rho\lambda} + \frac{c\phi ln(\lambda)}{\lambda}\Big] - \Big[\kappa_I + \kappa_\delta \delta \frac{\partial n_B}{\partial I}\Big]}_{\mathscr{L}_I^{FB}}$$
$$+ \underbrace{\left(\frac{\frac{\partial H(c,I)}{\partial I}\frac{\partial n_B}{\partial \delta} - \lambda\Big[\kappa_I + \kappa_\delta \delta \frac{\partial n_B}{\partial I}\Big]\frac{\partial H(c,I)}{\partial c}\frac{\partial n_B}{\partial \delta}}{\Big[\lambda \kappa_\delta \frac{\partial H(c,I)}{\partial c}\frac{\partial n_B}{\partial \delta} - \frac{1}{I}\Big]}\right)}_{s_\delta} \cdot \Big(\underbrace{\frac{cln(\lambda)}{\lambda\rho} - \frac{c\phi ln(\lambda)}{\lambda I} - \kappa_\delta}_{\mathscr{L}_\delta^{FB}}\Big). \quad (S.1)$$

In this section, we show that the market equilibrium values of $\mathscr{L}_I^{FB}$ and $\mathscr{L}_\delta^{FB}$ are exactly equal to the sum of all the named welfare effects described in Section 3 that determine the size of the distortion in R&D and advertising respectively.

Since the free-entry condition must hold in equilibrium, the market return to R&D must equal its cost. Rearranging the free-entry condition, equation (2.19) in the main text, we have

$$\frac{c\phi(\lambda-1)}{\lambda(I+\rho)} - (1-\sigma_I)\kappa_I = 0. \quad (S.2)$$

Subtracting this expression, which equals zero in equilibrium, from $\mathscr{L}_I^{FB}$ as defined in (S.1) yields

$$\mathscr{L}_I^{FB} = \underbrace{\frac{\partial n_B}{\partial I}\Big[\frac{c\delta ln(\lambda)}{\rho\lambda} + \frac{c\phi ln(\lambda)}{\lambda}\Big] - \frac{c\phi(\lambda-1)}{\lambda(I+\rho)}}_{w_1} + \underbrace{(1-\sigma_I)\kappa_I - \Big[\kappa_I + \kappa_\delta \delta \frac{\partial n_B}{\partial I}\Big]}_{w_2} \quad (S.3)$$

The first term, labeled w_1, is equal to the social return to R&D from the first-best solution minus the market determined return. It is equal to the sum of the four effects that characterize this difference: the standard *monopoly distortion effect* and *intertemporal spillover effect*, along with two novel effects the *dynamic effect of stochastic diffusion* and the *early adopter market effect*. The second term, labeled w_2, is equal to the difference between the market determined and social cost of R&D. It is equal to the *diffusion resource burden of innovation*.

Similarly, the equilibrium optimality condition for advertising (with $\sigma_\delta = 0$) can be written

$$\frac{c(1-\phi)(\lambda-1)]}{\lambda}\Big[\frac{\delta+I+\rho}{(I+\rho)(\delta+\rho)}\Big] - \kappa_\delta = 0. \quad (S.4)$$

Subtracting this expression from $\mathscr{L}_\delta^{FB}$ yields

$$\mathscr{L}_\delta^{FB} = \frac{cln(\lambda)}{\lambda\rho} - \frac{c\phi ln(\lambda)}{\lambda I} - \frac{c(1-\phi)(\lambda-1)]}{\lambda}\Big[\frac{\delta+I+\rho}{(I+\rho)(\delta+\rho)}\Big] \tag{S.5}$$

Thus, $\mathscr{L}_\delta^{FB}$ is equal to the sum of the three effects that determine the difference between the social and private benefit of diffusion in equilibrium: the *dynamic effect of life-cycle replacement*, the *effect of early adopters on the benefit of diffusion*, and the *mainstream market effect*.

S.2 Welfare in the Combative Advertising Model

In this section, we examine the social return to innovation and diffusion in the presence of defensive advertising by old firms. As in Section 3 of the main text, we consider a social planner who chooses c, I, and δ to maximize social welfare, subject to the corresponding aggregate resource constraint. In addition, we assume that the social planner cannot directly control the resources allocated to defensive advertising.[1] Rather, $L_{\delta,o}$ is determined by the optimality condition for old firm advertising $L_{\delta,o} = \delta V_o$ as in the competitive equilibrium. We note that this specification most closely mirrors the welfare analysis of Section 3 in the main text, which considered the first-best allocation in the informative model. Although the social planner is partially constrained here, she still maintains full control over the resources devoted to R&D and diffusion promoting advertising by young firms.

Total employment in advertising is $L_\delta = n_B[L_{\delta,o}(\kappa_\delta\delta+1)]$. Using the expression for $L_{\delta,o}$ from equation (6.3) in the main text, we express total employment in advertising as

$$L_\delta = n_B\kappa_\delta\delta D(c,\delta) \quad \text{where} \quad D(c,\delta) = \frac{(1-\phi)(\lambda-1)c}{\lambda(\rho+2\delta)}\big(\delta+\frac{1}{\kappa_\delta}\big). \tag{S.6}$$

The function $D(c,\delta)$ determines the total resource cost of diffusion, given the endogenous use of defensive advertising by old firms. Since defensive advertising incentives are strictly increasing in equilibrium consumption expenditure and the diffusion rate, $\frac{\partial D(c,\delta)}{\partial\delta} > 0$ and $\frac{\partial D(c,\delta)}{\partial c} > 0$. We note that the baseline model with only informative advertising corresponds to a fixed $D(c,\delta) = 1$, since $L_\delta = L_{\delta,y} = n_B\kappa_\delta\delta$ in this case.

The Lagrangian associated with the social planner's problem is

$$\mathscr{L}(c,I,\delta,\mu) = \rho U + \mu\Big[1 - \frac{c}{\lambda} - \kappa_I I - n_B\kappa_\delta\delta D(c,\delta)\Big], \tag{S.7}$$

where ρU is unchanged from (3.1) in the main text. The welfare maximizing solution for I yields

[1] Otherwise, it would be possible for the social planner to choose $L_{\delta,o} \to 0$, which implies that $\delta \to \infty$ for any $L_{\delta,y} > 0$ since $\delta = L_{\delta,y}/(\kappa_\delta L_{\delta,o})$. In other words, the social planner could achieve instantaneous new prototype diffusion at zero labor resource cost.

the following expression that equates the social cost and return to R&D:

$$\frac{\kappa_I + \kappa_\delta D(c,\delta)\delta\frac{\partial n_B}{\partial I}}{1+\lambda\kappa_\delta n_B\frac{\partial D(c,\delta)}{\partial c}} = \frac{c}{\lambda}\frac{ln(\lambda)\delta}{\rho}\frac{\partial n_B}{\partial I} + \phi ln(\lambda)\frac{c}{\lambda}\frac{\partial n_B}{\partial I}. \tag{S.8}$$

The market determined cost and return to R&D is unchanged from (2.19),

$$(1-\sigma_I)\kappa_I = \frac{c\phi(\lambda-1)}{\lambda(I+\rho)}. \tag{S.9}$$

Since the presence of defensive advertising does not impact R&D incentives, we can understand how defensive advertising changes the welfare implications of R&D by comparing (S.8) to (3.3) from the main text. First, observe that the social benefit of R&D (the right hand side of (S.8)) is unchanged. This implies that *the monopoly distortion effect*, *the intertemporal spillover effect*, *the dynamic effect of stochastic diffusion*, and *the early adopter market effect* remain present in the combative advertising model.

Indeed, the only change enters through the size of *the diffusion resource burden of innovation.* Specifically, the numerator of the left hand side of (S.8)) captures the resource cost of an increase in the innovation rate through the direct resource cost κ_I and the indirect cost from the associated increase in the proportion of industries that conduct advertising $\kappa_\delta D(c,\delta)\delta\frac{\partial n_B}{\partial I}$. However, the denominator scales this resource burden down according to the resource savings from the implied decrease in defensive advertising incentives. That is, since increasing the innovation rate takes labor resources away from the manufacture of consumption goods, it indirectly decreases defensive advertising incentives and reduces the resources needed to maintain a constant diffusion rate δ. The size of this resource savings is captured by the $\lambda\kappa_\delta n_B\frac{\partial D(c,\delta)}{\partial c} > 0$ term. Overall, the relative size of *the diffusion resource burden of innovation* in the informative and combative advertising versions of the model is ambiguous, and depends on the use of defensive advertising by old firms through $D(c,\delta)$ and the sensitivity of this advertising to changes in c through $\frac{\partial D(c,\delta)}{\partial c}$.

The welfare maximizing solution for δ yields the following expression that equates the social cost and benefit to diffusion promoting advertising:

$$\frac{\kappa_\delta D(c,\delta) + \frac{\kappa_\delta\delta}{n_B}\frac{\partial D(c,\delta)}{\partial\delta}}{1+\lambda\kappa_\delta n_B\frac{\partial D(c,\delta)}{\partial c}} = \frac{cln(\lambda)}{\rho\lambda} - \frac{\phi cln(\lambda)}{\lambda I}. \tag{S.10}$$

Since defensive advertising determines the difficulty of diffusion, its presence does impact the market determined cost and return to diffusion promoting advertising by young firms. Using the optimality condition $L_{\delta,y} = \delta(V_a - V_y)$, we have

$$\frac{\kappa_\delta D(c,\delta)}{1+\frac{1}{\kappa_\delta\delta}} = \frac{c(1-\phi)(\lambda-1)}{\lambda}\Big(\frac{2\delta+I+\rho}{(I+\rho)(2\delta+\rho)}\Big). \tag{S.11}$$

We begin by noting that the social benefit of diffusion (the right hand side of (S.10)) is unchanged from the informative advertising version of the model (the right hand side of (3.4)). The market benefit (the right hand side of (S.11)) is also unchanged from (2.20) except that 2δ replaces δ in the final term. This is because the reward for diffusion, $V_a - V_y$, incorporates the value of an old firm, V_o, which depends on defensive advertising expenditure when the firm eventually transitions to its old firm stage. Thus, the presence of defensive advertising increases the size of the *dynamic effect of life-cycle replacement*, which implies greater underinvestment in diffusion promoting advertising as described in the main text. However, the *effect of early adopters on the benefit of diffusion* and the *mainstream market effect* remain unchanged.

In addition, the presence of defensive advertising creates a difference between the social and market cost of diffusion promoting advertising as given by the left hand side of (S.10) and (S.11) respectively. First, young firms consider only the direct difficulty of diffusion, and the left hand side of (S.11) is equal $\kappa_\delta L_{\delta,o}(c,\delta)$. In contrast, the social planner considers both the total resource cost of diffusion $\kappa_\delta D(c,\delta) > \kappa_\delta L_{\delta,o}(c,\delta)$ and the effect that increasing the diffusion rate has on old firms' defensive advertising incentives. The $\frac{\kappa_\delta \delta}{n_B}\frac{\partial D(c,\delta)}{\partial \delta} > 0$ term in the numerator captures the increase in total labor resources devoted to advertising through this effect. However, just as in (S.8), the decrease in labor available for consumption goods generates a labor savings effect through its effect on defensive advertising incentives. Thus, the social cost of diffusion is scaled down by the same $1 + \lambda \kappa_\delta n_B \frac{\partial D(c,\delta)}{\partial c} > 0$ term on the left hand side of (S.10). Overall, we again find that the relative size of the market versus social cost of diffusion is ambiguous in the general case, and depends on the sensitivity of defensive advertising to changes in δ and c.

S.2.1 Optimal R&D Subsidies in the Combative Advertising Model

As in the informative advertising version of the model, Proposition 2 establishes that subsidizing R&D investment always decreases the equilibrium diffusion rate. Thus, the total welfare effect of an R&D subsidy again depends on the effect of stimulating innovation against the effect of the associated decrease in diffusion. To better understand how this trade-off differs in the combative advertising version of the model, we compute optimal R&D subsidies in the combative advertising model numerically for the same three initial failure rate values examined in the main text, $f_0 = \{0.01, 0.25, 0.45\}$. In each case, we calibrate κ_I and κ_δ to match the associated target failure rate and an initial growth rate of $g = 1.5\%$. The resulting parameter values are $\kappa_I = \{0.3450, 0.2891, 0.2345\}$ and $\kappa_\delta = \{15.786, 17.086, 17.670\}$ for $f_0 = \{0.01, 0.25, 0.45\}$ respectively. We report results in Table S.1 and Figure S.1

Compared to our baseline results in the informative advertising model, we find a similar qualitative pattern of results in the presence of defensive advertising. As displayed in Figure S.1, we continue to find that the relationship between the R&D subsidy level and welfare and that between the R&D subsidy and growth are characterized by inverted U-shaped curves for each initial equilibrium failure rate. Moreover, we still find that it is optimal to subsidize R&D when the initial failure rate is low and tax R&D when the initial failure rate is high. However, closer inspection of the growth and welfare maximizing policies do highlight important differences between the two

Table S.1: Optimal R&D Subsidies in the Combative Advertising Model

(a): $f_0 = 0.01$	$g(\%)$	I	δ	c	f	U	$\sigma^\star_{I,g}$	$\sigma^\star_{I,U}$
$\sigma_I = 0$	1.500	0.068	6.636	1.103	0.010	2.090	0.408	0.215
$\sigma^\star_{I,U} = 0.215$	1.673	0.098	0.321	1.069	0.234	2.218	–	–
(b): $f_0 = 0.25$	$g(\%)$	I	δ	c	f	U	$\sigma^\star_{I,g}$	$\sigma^\star_{I,U}$
$\sigma_I = 0$	1.500	0.090	0.269	1.082	0.250	1.940	0.286	0.067
$\sigma^\star_{I,U} = 0.067$	1.528	0.100	0.221	1.074	0.310	1.945	–	–
(c): $f_0 = 0.45$	$g(\%)$	I	δ	c	f	U	$\sigma^\star_{I,g}$	$\sigma^\star_{I,U}$
$\sigma_I = 0$	1.500	0.122	0.149	1.068	0.450	1.886	0.122	-0.117
$\sigma^\star_{I,U} = -0.117$	1.480	0.105	0.181	1.077	0.366	1.903	–	–

Table S.1 presents equilibrium results when $\sigma_I = 0$ and when R&D subsidized at the welfare maximizing level ($\sigma^\star_{I,U}$) when $f_0 = \{0.01, 0.25, 0.45\}$.

Figure S.1: Optimal R&D Subsidies in the Combative Advertising Model

(a) g (b) Welfare

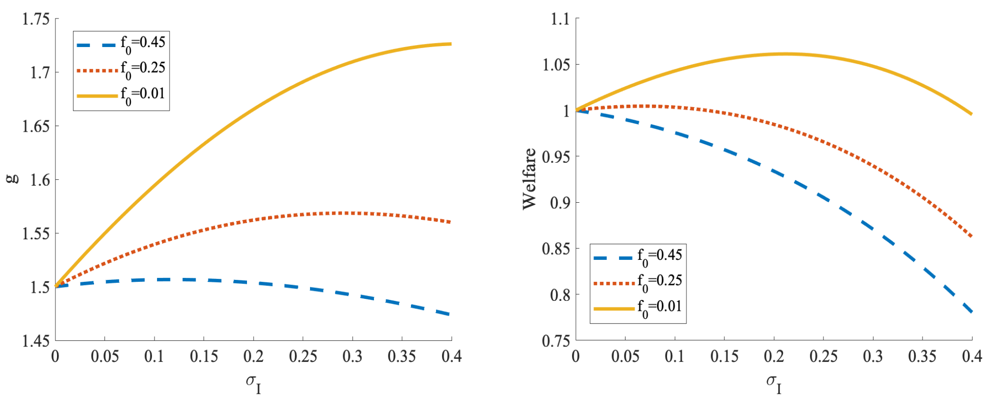


Figure S.1 plots equilibrium values of growth g and welfare against the R&D subsidy rate σ_I for three distinct initial values of the failure rate f_0. Panel (b) plots U after normalizing its value to one when $\sigma_I = 0$.

frameworks.

First, we find that the growth maximizing subsidy $\sigma^\star_{I,g}$ is positive in each case, and uniformly larger than its counterpart in the informative advertising case. This is because the decrease in the equilibrium diffusion rate per unit of R&D subsidy is smaller in the presence of defensive advertising. To see this, note that young firm advertising incentives are increasing in c and decreasing in I. In the informative advertising model, the decrease in c and increase in I from an R&D subsidy combined to generate a relatively large decrease in δ. In the combative advertising model however, old firm advertising incentives also increase in c and are independent of I. Thus, the decrease in defensive advertising associated with the decline in c when R&D is subsidized partially offsets the impact of reduced young firm advertising incentives, and mitigates the equilibrium decrease in δ.

Despite this effect, we find that the welfare maximizing R&D policy in the combative advertising model is either a smaller subsidy (in the $f_0 = 0.01$ and $f_0 = 0.25$ cases) or a larger tax (in the $f_0 = 0.45$ case) compared to the informative advertising model. As discussed in the previous section, the social return to increasing the innovation rate differs in the two versions of the model through the *the diffusion resource burden of innovation.* In our numerical simulations, we find that the presence of defensive advertising implies a greater effective resource cost of innovation, which diminishes the social desirability of R&D subsidies. Intuitively, as R&D is subsidized, the proportion of B industries increases. Since both young and old firms devote labor resources to advertising in the combative advertising model, the increase in n_B implies a greater reallocation of labor resources away from manufacturing. This creates a greater static welfare loss from lower equilibrium consumption per unit of R&D subsidy.

S.3 Jointly Optimal R&D and Advertising Policy

In this section, we numerically examine the optimal σ_I and σ_δ when both policy instruments are available to the policy maker. We focus on the benchmark equilibrium case, where $f_0 = 0.25$, in both the informative and combative advertising versions of the model. Results are reported in Table S.2 and Figure S.2.

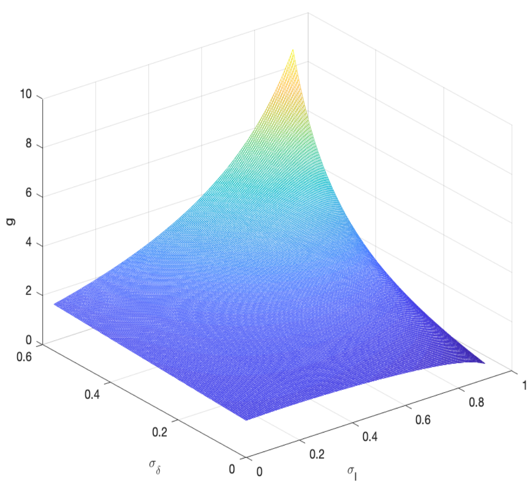
Table S.2: Jointly Optimal Policy ($f = 0.25$ case)

(a): Informative	g	I	δ	c	f	U	L_δ
$\sigma_\delta = \sigma_I = 0$	1.500	0.090	0.269	1.092	0.250	2.106	0.100
$\sigma_\delta^\star \to \sigma_\delta^{sup} = 0.815$, $\sigma_I^\star = 0.765$	6.435	0.294	∞	0.596	0.000	5.891	0.437
(b): Combative	g	I	δ	c	f	U	L_δ
$\sigma_\delta = \sigma_I = 0$	1.500	0.090	0.269	1.082	0.250	1.940	0.109
$\sigma_\delta^\star \to -\infty$, $\sigma_I^\star = 0.095$	1.563	0.124	0.161	1.205	0.436	4.056	0.000

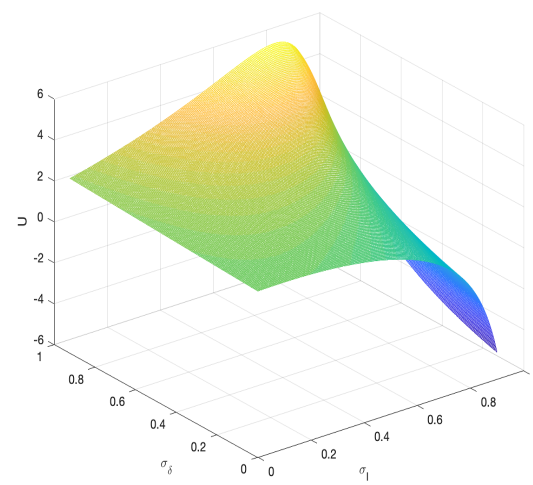
When advertising is purely informative, we continue to find that the optimal advertising policy takes the form of the maximum feasible subsidy, $\sigma_\delta^\star \to \sigma_\delta^{sup}$. Note from equation (6.6) in the main text that σ_δ^{sup} is strictly increasing in the R&D subsidy level. Since the optimal joint policy calls for a subsidy to both R&D and advertising, the optimal advertising subsidy is considerably larger when both policies are available. However, the intuition remains the same; the policy maker uses the advertising subsidy to achieve $\delta \to \infty$ and completely eliminate the friction associated with endogenous diffusion. The corresponding optimal R&D policy is now a far larger subsidy with $\sigma_I^\star = 0.765$, compared to the single policy case from the main text where the optimal subsidy rate is 9.2%. There are two main reasons for this difference. First, Proposition 3 establishes that the innovation rate is strictly decreasing in the advertising subsidy level. This implies that the advertising subsidy magnifies equilibrium underinvestment in R&D, and a larger R&D subsidy is required to eliminate this distortion. Second, when R&D policy is the sole policy instrument, a major social cost of subsidizing R&D is that it magnifies equilibrium underinvestment in diffusion promoting advertising. When both policies are available, the advertising subsidy adjusts to remove this distortion and eliminate this cost.

Figure S.2: Jointly Optimal Policy ($f = 0.25$ case)

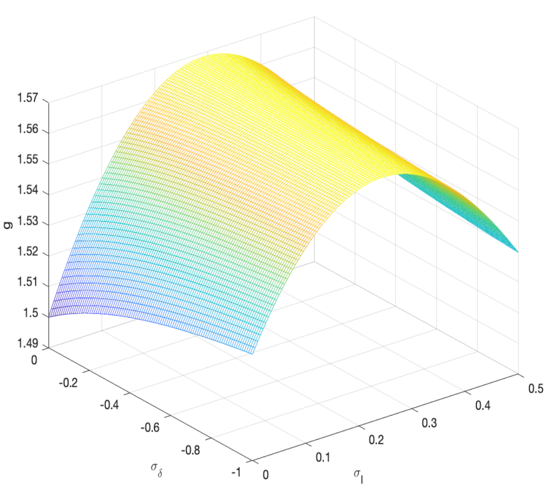
(a) Informative: Growth



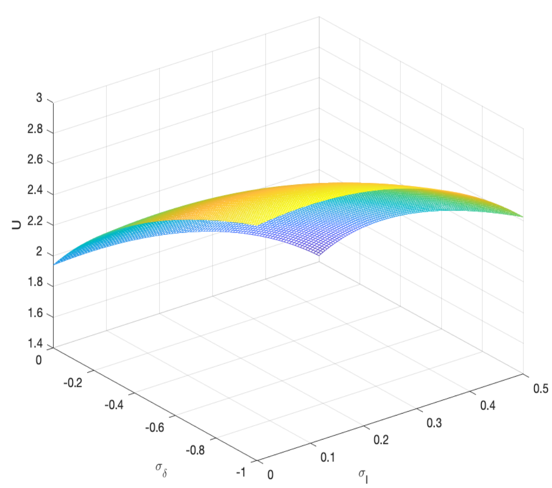
(b) Informative: Welfare



(c) Combative: Growth



(d) Combative: Welfare



When advertising is combative, we continue to find that it is optimal to impose an infinitely large tax on advertising. Once again, the intuition follows from the discussion in the main text; the total labor resources used in advertising goes to zero as a result of the tax, without directly affecting relative investment incentives. The corresponding optimal R&D policy of $\sigma_I^\star = 0.095$, is only slightly higher than the 6.7% optimal subsidy rate in the single policy case. This is because the increase in I associated with R&D subsidies continue to reduce young firm advertising incentives and decrease the equilibrium rate of diffusion. Since the advertising policy impacts both informative and defensive advertising incentives equally, the advertising policy can not be used to counteract this welfare loss.

S.4 Alternate Versions of the Model

S.4.1 Incorporating Population Growth

In the model presented in the main text, we set the rate of population growth to zero and normalized the supply of labor to one. This approach follows much of the recent Schumpeterian growth literature, including Chu and Cozzi (2018), Yang (2018) and Chu et al. (2019), which sidesteps the well-known issue of scale-effects in endogenous growth settings. More specifically, Schumpeterian growth models exhibit scale-effects when the rate of technological progress is assumed to be proportional to the level of R&D employment. This property implies that an economy's per capita growth rate increases in the size of its population. Consequently, scale-effects are inconsistent with a steady-state equilibrium in which both the economy and the economy's population grow at a positive, finite rate.

In an earlier version of our paper that is available upon request, we indeed considered a model with positive population growth. We removed scale-effects by assuming that the difficulty of successful innovation and diffusion were proportional to the economy's population. That is, suppose that the economy's population $L(t)$ grows at an exogenous rate $g_L > 0$. In place of the R&D and advertising technologies assumed in the informative advertising version of the model (equations (2.5) and (2.6)), let

$$I(t) \equiv \frac{L_I(t)}{\kappa_I L(t)} \qquad \text{and} \qquad \delta(t) \equiv \frac{L_{\delta,y}(t)}{\kappa_\delta L(t)}. \tag{S.12}$$

This simple approach removes scale-effects from the model because $I(t)$ and $\delta(t)$ now depend on the *share* of labor employed in R&D and advertising respectively. In the earlier version of our paper, we showed that all of our main results continue to hold in this scale-free setting. In this section, we briefly discuss the economic motivation for this approach.

First, the Schumpeterian growth literature has demonstrated that this R&D technology arises in a framework with both quality improvements and variety expansion. In this setting, the number of varieties increases at the same rate as population growth. Keeping the rate of quality-improvement constant for each variety requires an increasing amount of labor resources to be spread over an expanding set of varieties. Consequently, the model absorbs the expansion in the size of the population without leading to explosive growth. The microfoundations for this approach have been provided by Peretto (1998), Young (1998), Aghion and Howitt (1998), which were further developed by Dinopoulos and Thompson (1998) and Howitt (1999). In their overview of scale-effects, Dinopoulos and Sener (2007) explicitly establish the proportionality between R&D difficulty and the size of the labor force in a parsimonious model with variety and quality expansion.

In addition, a separate line of Schumpeterian growth literature has shown that this R&D technology follows from the introduction of Rent Protection Activities (RPAs). In this setting, incumbent firms with the top-quality product undertake RPAs (such as lobbying and legal action) to increase the difficulty of subsequent innovation and prolong the duration of their market power. Since the gains from RPAs are proportional to market size, the labor resources utilized in these activities are proportional to the size of the labor force. This implies that R&D difficulty becomes proportional to total labor supply as in (S.12). The microfoundations for this approach have been provided by Dinopoulos and Syropoulos (2007), and were further developed in Grieben and Şener (2009) and Davis and Şener (2012). See again Dinopoulos and Sener (2007) for a formalization of the RPA approach in a simple quality-ladders growth setting that yields the proportionality result.

The advertising technology specified in (S.12) reflects the idea that it is more difficult to successfully commercialize an innovative new product into a larger market. For example, persuading a larger customer base to adopt a new product may require a more varied, and thus costly, advertising strategy. Indeed, Dinopoulos and Segerstrom (1999) reference this exact effect as motivation for their assumption that R&D difficulty is proportional to the size of the market. Since they do not model the commercialization process, all costs associated with introducing a new product are implicitly included in upfront R&D costs. Our framework's endogenous diffusion process allows us to capture this same idea more naturally through our advertising technology. Finally, we note that the specification of combative advertising used in the main text is consistent with scale-free growth as long as the R&D technology is given by (S.12). This is because the diffusion rate is determined by relative advertising employment in this formulation with $\delta(t) = L_{\delta,y}(t)/[\kappa_\delta L_{\delta,o}(t)]$. When the population grows at rate $g_L > 0$, the labor resources devoted to both informative and defensive advertising also grow at rate g_L and the diffusion rate remains constant in the steady-state equilibrium. In essence, combative advertising naturally introduces endogenous advertising difficulty, eliminating the scale-effect associated with diffusion activity.

S.4.2 Removing the General Equilibrium Effect of Resource Reallocation

In the model presented in the main text, the reward for successful innovation, $V_y(t)$, and diffusion, $V_a(t) - V_y(t)$, scale proportionally with consumption expenditure, c. This implies that any change to the equilibrium labor resources devoted to the manufacture of consumption goods, exerts an indirect impact on R&D and advertising incentives. Consequently, changes to the cost of R&D and advertising impact the model's endogenous variables through two distinct channels: (1) their direct effect on investment incentives and the resulting change in I and δ, and (2) the general equilibrium effect of the reallocation of labor resources across R&D, advertising, and manufacturing. In this section, we consider an alternate specification for the cost of R&D and advertising that eliminates this resource reallocation channel. The objective of this exercise is to re-examine the effect of R&D and advertising subsidies when their impact on I and δ is solely driven by their effect on investment incentives.

To accomplish this in a simple way, we consider a version of the informative advertising model in which the difficulty of successful innovation and diffusion also increase proportionally with c. Let the instantaneous probability of innovation and diffusion be defined as

$$I(t) \equiv \frac{L_I(t)}{c(t)\cdot \kappa_I}, \qquad \delta(t) \equiv \frac{L_{\delta,y}(t)}{c(t)\cdot \kappa_\delta}, \tag{S.13}$$

which replace equation (2.5) and (2.6) in the main text. For example, one may imagine that the overall cost of an R&D project includes fixed costs associated with setting up manufacturing facilities, which may depend on the size of the market and the quantity of prototypes to be produced. Similarly, the cost of displacing an incumbent from the mainstream market may depend on the quantity that the incumbent sells to each consumer. Ultimately though, whether the specification is realistic is irrelevant for our purposes. We consider it because it conveniently shuts down a particular channel of general equilibrium effects through labor resource reallocation.

Given (S.13), the free entry condition of (2.19) is replaced by,

$$(1-\sigma_I)\kappa_I c = \frac{c\phi(\lambda-1)}{\lambda(I+\rho)}. \qquad \textbf{[FE]} \quad \text{(S.14)}$$

Similarly, the optimal advertising condition of (2.20) is replaced by,

$$(1-\sigma_\delta)\kappa_\delta c = \frac{c(1-\phi)(\lambda-1)]}{\lambda}\Big[\frac{\delta+I+\rho}{(I+\rho)(\delta+\rho)}\Big]. \qquad \textbf{[AD]} \quad \text{(S.15)}$$

Since both the cost and reward to R&D and advertising investment scale with c, general equilibrium changes to c no longer impact investment incentives. Indeed, rearranging the FE condition yields a closed form solution for I,

$$I = \frac{\phi(\lambda-1)}{\lambda(1-\sigma_I)\kappa_I} - \rho, \tag{S.16}$$

and the AD condition implicitly defines δ as a strictly decreasing function of I.

Thus, the equilibrium I, δ, f and g are now completely determined by these two conditions, and c simply adjusts so that the labor market always clears. Specifically, the labor market clearing condition is now,

$$1 = c\Big(\frac{1}{\lambda} + \kappa_I I + \kappa_\delta \frac{I\delta}{I+\delta}\Big). \tag{S.17}$$

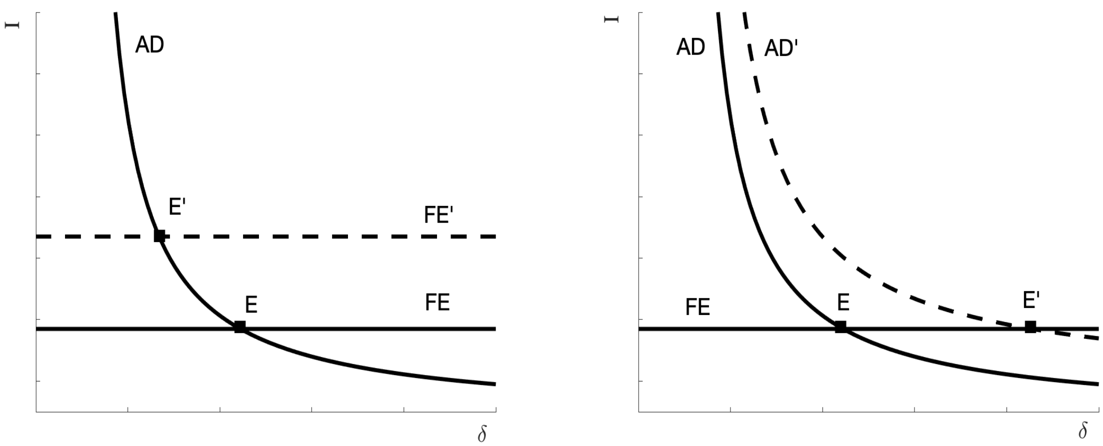
Figure S.3 illustrates the equilibrium impact of an R&D subsidy and a reduction in the cost of informative advertising. The following is immediate.

Proposition S.1. *When the general equilibrium labor resource reallocation effect is removed from the informative advertising model, subsidizing R&D decreases the diffusion rate, increases the innovation rate, increases the product failure rate, and has an ambiguous effect on economic growth. That is,* $\frac{\partial \delta}{\partial \sigma_I} < 0$, $\frac{\partial I}{\partial \sigma_I} > 0$, $\frac{\partial f}{\partial \sigma_I} > 0$, *and* $\frac{\partial g}{\partial \sigma_I} >< 0$. *Subsidizing advertising increases the diffusion rate, does not change the innovation rate, decreases the product failure rate, and increases economic growth. That is,* $\frac{\partial \delta}{\partial \sigma_\delta} > 0$, $\frac{\partial I}{\partial \sigma_\delta} = 0$, $\frac{\partial f}{\partial \sigma_\delta} < 0$, *and* $\frac{\partial g}{\partial \sigma_\delta} > 0$.

Proposition S.1 demonstrates that our central results regarding the impact of R&D subsidies

Figure S.3: Subsidizing R&D and Advertising

(a) R&D Subsidy (b) Advertising Subsidy



do not depend on the labor resource reallocation effect. Rather, they are driven by the direct effect of the innovation rate on advertising incentives. When new innovations arrive more frequently, the reward for successful diffusion into the mainstream market shrinks, and each young firm reduces advertising investment. In contrast, we find that advertising subsidies are always growth enhancing when the labor reallocation channel is absent. This is because R&D incentives do not depend directly on the expected cost of diffusing into the mainstream. We conclude that the decline in the innovation rate resulting from advertising subsidies is entirely driven by the resource reallocation channel.

Finally, it is easy to see how these results carry over when combative advertising is considered. Specifically, suppose that the innovation difficulty continues to scale with c as in (S.13) and diffusion difficulty depends on endogenous defensive advertising as in (6.1) of the main text. Since the inclusion of defensive advertising does not impact the free entry condition, the innovation rate remains independent of δ and c as in (S.16). The relative advertising incentives of young and old firms determine the diffusion rate exactly as in the DC of equation (6.5) of the main text. As discussed in Section 6, the DC is strictly downward sloping in (δ, I) space because young firm advertising incentives decrease in I. Therefore, R&D subsidies continue to imply the same qualitative pattern of results.

In contrast, when the advertising subsidy applies equally to both informative and defensive advertising, the subsidy no longer has *any effect* on the equilibrium I, δ, f and g. This is because the cost reduction generates a proportional increase in both forms of advertising, leaving the DC unchanged. Labor resources devoted to advertising still increase, but consumption expenditure simply adjusts down so that the labor market continues to clear. It is in this sense that the central effect of the subsidy is a socially wasteful increase in the resources devoted to advertising. In the version of the model examined in the main text, the reallocation of labor resources leading to a decrease in c implied a decrease to R&D incentives. The resulting general equilibrium decrease in I impacted relative advertising incentives and led to the increase in δ. When this channel of general equilibrium effects is removed, the only effect of advertising subsidies is a welfare reducing decrease in equilibrium consumption expenditure.

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