# Slow Recoveries, Endogenous Growth and Macro-prudential Policy: Online Appendix *

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## 1 Global Evidence on the Effects of Banking Crises on Productivity and Output

To assess the relationship between banking crises, innovation, and productivity, we estimate local projections à la Jorda (2005) based on panel regressions, using annual data from 1970 to 2018 for 24 advanced economies.

### 1.1 Data

**Total Factor Productivity** We collect data on TFP from the Total Economy Database (Conference Board, 2019), henceforth TED, and from Coe et al. (2009), henceforth CHH. The data from TED are available from 1990 until 2018, while those from CHH go back to 1970 until 2004. We splice the TED data with the second dataset for the period $1970-1989$ by using the OLS fitted values (using the OLS parameter estimate for the overlapping period $1990-2004$).

**Research and Development** We take data on annual real 2015 PPP USD Business Enterprise Expenditure on R&D (henceforth BERD) and Gross Domestic Expenditure on R&D (GERD) from the OECD's Main Science and Technology Indicators (OECD, 2019). To extend the time-series dimension, we also take the TFP series from Coe et al. (2009). Similarly as in CHH, for Austria, we fill gaps in the data by using the fitted value of the regression of the BERD on the (at constant prices and PPP USD). For TFP, we then extrapolate backward the BERD data with the CHH R&D data using the OLS fitted values.

**Gross Domestic Product** We take data on annual real per capita GDP in 2018 PPP USD from the Total Economy Database (Conference Board, 2019) which is available from 1950.

**Banking Crisis and Other Recessions** Data on banking crisis dates are taken from Laeven and Valencia (2018), who define a banking crisis as events that were characterised by both heightened financial distress in the banking system as well policy interventions to respond to significant losses in the banking system. We construct the data on recession dates from quarterly real GDP data from the OECD. In particular, we label a year as a recession, if it was characterised by at least two quarters with negative GDP growth. Similarly, as in Queralto, we only take the first year of the banking crisis (or the recession). For example, if country A had a banking crisis or a recession from 1992 until 1994 (included), our dummy would have a one only in 1992 and zeros in 1993 and 1994. If a country experienced a double-dip recession in 2 consecutive years, e.g. negative GDP growth in 1992Q1-1992Q2 and 1993Q2-1993Q3, then both years (1992 and 1993), would appear with a one. We define "Other recessions" by removing the banking crisis dates from the recessions. We report the Banking Crises and other recessions in Table 1.

*The views expressed in this paper are those of the authors and not necessarily those of the Bank of England or its committees.
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TABLE 1: BANKING CRISIS AND RECESSION DATES

| Country | Banking Crises | Other Recessions |
|---|---|---|
| Australia | | 1971, 1975, 1977, 1981, 1982, 1991 |
| Austria | 2008 | 1981, 1982, 1984, 1992, 2001, 2012 |
| Belgium | 2008 | 1974, 1976, 1980, 1992, 2001, 2012 |
| Canada | | 1974, 1980, 1981, 1990, 2008, 2015 |
| Denmark | 2008 | 1973, 1977, 1980, 1986, 1989, 1992, 1997, 2001, 2006 |
| Finland | 1991 | 1971, 1975, 1977, 1980, 1990, 1992, 1995, 2008, 2012, 2013, 2014 |
| France | 2008 | 1974, 1992, 2012 |
| Germany | 2008 | 1974, 1980, 1982, 1991, 1992, 2000, 2001, 2002, 2004, 2012 |
| Greece | 2008 | 1974, 1975, 1977, 1978, 1979, 1980, 1981, 1982, 1983 |
| | | 1984, 1987, 1989, 1990, 1992, 1994, 2004, 2007, 2010 |
| Iceland | 2008 | 1974, 1982, 1988, 1991, 1994, 1999, 2000, 2009 |
| Ireland | 2008 | 1975, 1982, 1985, 2007, 2011 |
| Israel | 1977 | 2000 |
| Italy | 2008 | 1974, 1977, 1982, 1992, 2001, 2003, 2011, 2018 |
| Japan | 1997 | 1993, 1998, 2001, 2008, 2010, 2012, 2015 |
| South Korea | 1997 | 1979 |
| Netherlands | 2008 | 1973, 1974, 1980, 1981, 2011, 2012 |
| New Zealand | | 1974, 1976, 1978, 1985, 1991, 1997, 2008, 2010 |
| Norway | 1991 | 1980, 1981, 1988, 1992, 2009, 2010, 2016 |
| Portugal | 2008 | 1974, 1980, 1983, 1992, 2002, 2010 |
| Spain | 2008 | 1975, 1978, 1981, 1992, 2009, 2011 |
| Sweden | 1991, 2008 | 1971, 1976, 1990, 1992, 2012 |
| Switzerland | 2008 | 1974, 1977, 1981, 1990, 1992, 1996, 1998, 2001, 2002, 2015, 2018 |
| United Kingdom | 2007 | 1973, 1975, 1980, 1990, 2008 |
| United States | 1988, 2007 | 1974, 1980, 1981, 1990, 2008 |

## 1.2 Methodology and Results

**The Local Projection Method** One key feature of the local projection method is its robustness to model misspecifications, as it does not impose dynamic restrictions on impulse responses. The model we use to estimate our local projections is the following fixed-effects regression:

$$Y_{i,t+h} - Y_{i,t-1} = \alpha_{i,h} + \gamma_{i,h}\,\mathrm{D}_{i,t} + \Gamma_{i,h}(L)\,\mathrm{D}_{i,t-1} + \varepsilon_{i,t+h} \qquad \text{for } h = 0,1,2,...,H \tag{1.1}$$

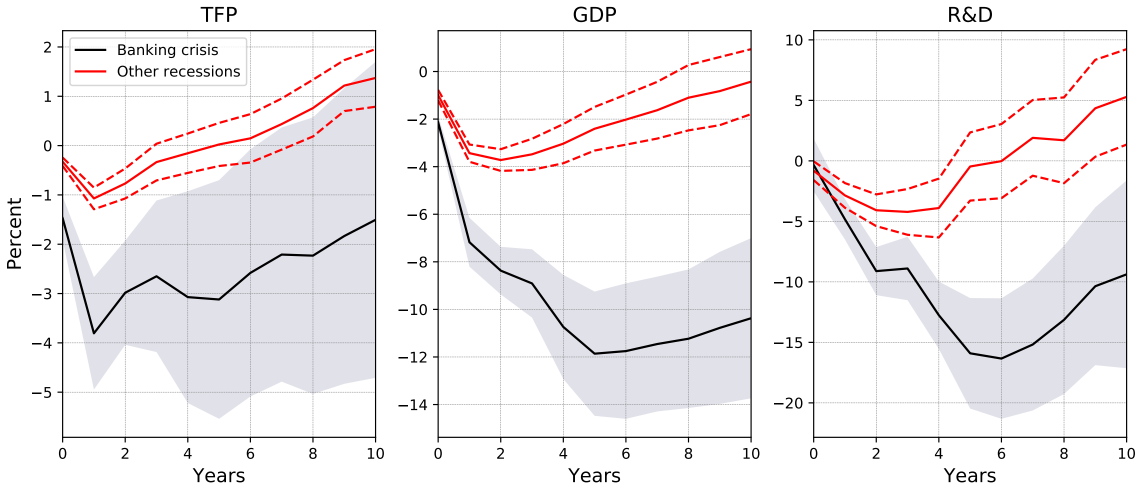
where $\alpha_{i,h}$ are country $i$'s fixed effects, $Y$ is the dependent variable in log-level, $\Gamma_{i,h}(L)$ is a lag polynomial of order 5 and $\mathrm{D}_i$ is the dummy variable of "Banking crisis" or "Other recessions", as defined in Section 1.1. For example, projecting $Y_{t+2}$ onto the variables on the right-hand side, we obtain the estimate $\hat{\gamma}_2$. This is the effect of a banking crisis on $Y$ two years ahead, which is orthogonal to the other variables on the right-hand side of the equation. Estimating $H$ regressions for each response variable $Y$ of interest gives us the sequence of local projections $(\hat{\gamma}_h)_{h=0}^{H}$, where a horizon of $H = 10$ years is considered.

**Evidence based on Local Projections** Figure 1.1 displays the responses of TFP, GDP, and R&D to a banking crisis or a recession. The black and red solid lines represent the local projections for a one-unit increase in the banking crisis dummy or the other recession dummy. The grey shaded areas and red dashed lines are 68% confidence bands. The first panel shows how banking crises are associated with a stronger and more persistent decline in TFP compared to other recessions. TFP declines within the first two years by about 4% and does not revert to its initial trend within the 10-year horizon. Other recessions, instead, induce a 1% decline in TFP, which reverts to its trend within five years and slightly overshoots afterward. The difference in responses between banking crises and other recessions is more striking for GDP. In this case, we find a very persistent and severe decline in GDP by 12% after five years. In normal recessions, GDP declines three times less (about 4%) and recovers within the 10-year horizon. Finally, we find a significant and persistent fall in business expenditure in R&D by more than 15% following a banking crisis. In other recessions, R&D declines by less than 5%, recovering within five years and rebounding afterward. The estimated responses are quantitatively in line with Queralto (2020), except for the drop in R&D after a banking crisis, which is only half as strong in our case, possibly due to the different set of countries under consideration.[1] To make sure our results are not driven by outlier countries, we also re-estimated the local projections dropping one economy at a time from our

[1] We consider our empirical analysis as complementary to that conducted in Queralto (2020), though we differ concerning the methodology and our data. In particular, Queralto (2020) follows the methodology by Cerra and Saxena (2008) and considers both emerging market economies and advanced economies.

sample. We found our results to be robust to this change.

FIGURE 1.1: IMPULSE RESPONSES TO BANKING CRISES AND OTHER RECESSIONS




Note: Black and red solid lines are local projections estimated on a dynamic panel regression with country-fixed effects. Grey shaded areas and red dashed lines are 68% Driscoll-Kraay HAC-robust confidence bands.

# 2 SVAR Analysis

This section of the appendix presents details about the data used to estimate the VAR model, displays the IRFs of all variables included in the model, and shows the results under an alternative Cholesky ordering of the variables.

## 2.1 Data

The table below presents the data series used for the estimation of the VAR model. In particular, we consider the following data transformations:

- $\text{GDP}_t = ln\left(\frac{\text{Gross Domestic Product}_t}{\text{GDP Implicit Price Deflator}_t \times \text{Civilian noninstitutional population}_t}\right)$
- $\text{Consumption}_t = ln\left(\frac{\text{Services Consumption}_t + \text{Nondurables Consumption}_t}{\text{GDP Implicit Price Deflator}_t \times \text{Civilian noninstitutional population}_t}\right)$
- $\text{Investment}_t = ln\left(\frac{\text{Durables Consumption}_t + \text{Private Fixed Investment}_t}{\text{GDP Implicit Price Deflator}_t \times \text{Civilian noninstitutional population}_t}\right)$
- $\text{R\&D}_t = ln\left(\frac{\text{Private Fixed Investment in R\&D}_t}{\text{GDP Implicit Price Deflator}_t \times \text{Civilian noninstitutional population}_t}\right)$
- $TFP_t = \frac{\text{TFP Annualised Growth Rate}}{4}$
- $Prices = ln\left(\text{GDP Implicit Price Deflator}\right)$

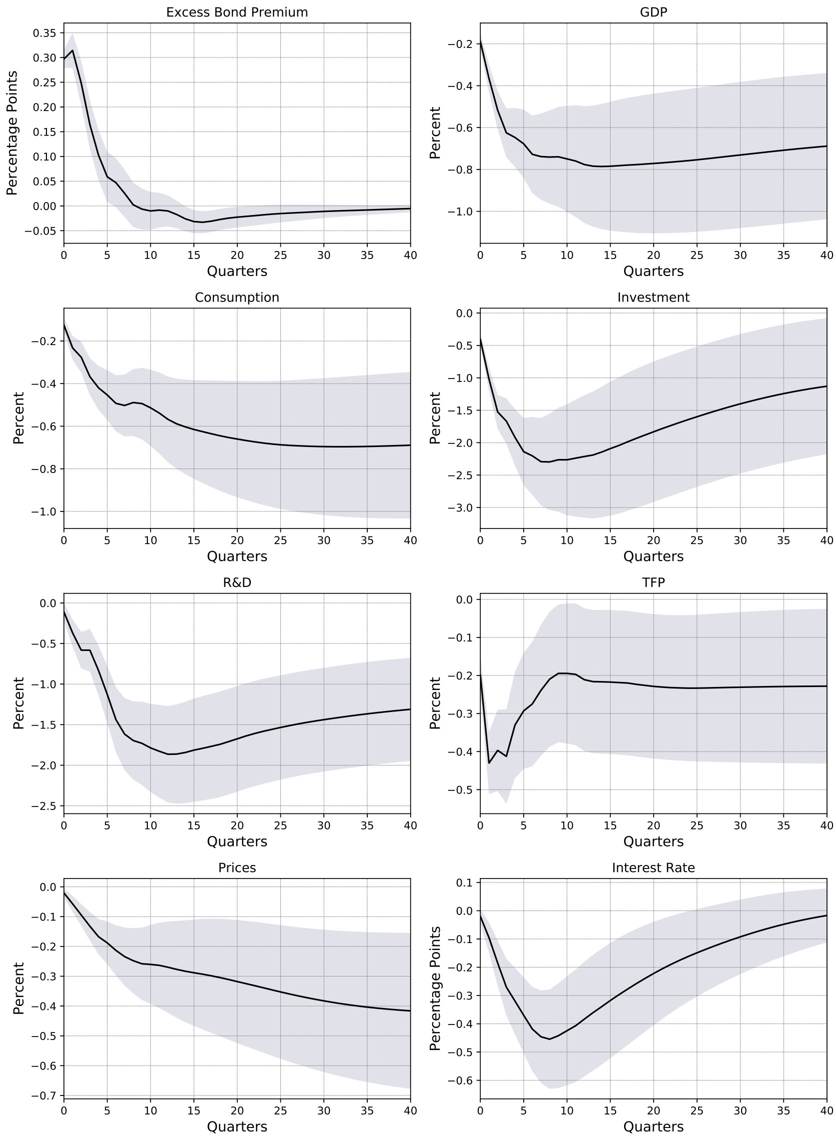
TABLE 2: DATA INCLUDED IN THE VAR MODEL

| Name | Source | Ticker |
|---|---|---|
| Civilian noninstitutional population | FRED (BLS) | CNP16OV |
| Gross Domestic Product | FRED (BEA) | GDP |
| Services Consumption | FRED (BEA) | PCES |
| Nondurables Consumption | FRED (BEA) | PCEND |
| Durables Consumption | FRED (BEA) | PCEDG |
| Private Fixed Investment | FRED (BEA) | FPI |
| Private Fixed Investment in R&D | FRED (BEA) | Y006RC1Q027SBEA |
| GDP Implicit Price Deflator | FRED (BEA) | GDPDEF |
| Federal Funds Rate | FRED | DFF |
| TFP | FRBSF | |
| Excess Bond Premium | FRSB | |

Moreover, we take the first differences of GDP, Consumption, Investment, and R&D. Interest Rate, Excess Bond Premium, and TFP growth rate to enter the VAR as they are.

## 2.2 Complete Set of IRFs

FIGURE 2.2: THE EFFECTS OF FINANCIAL SHOCKS

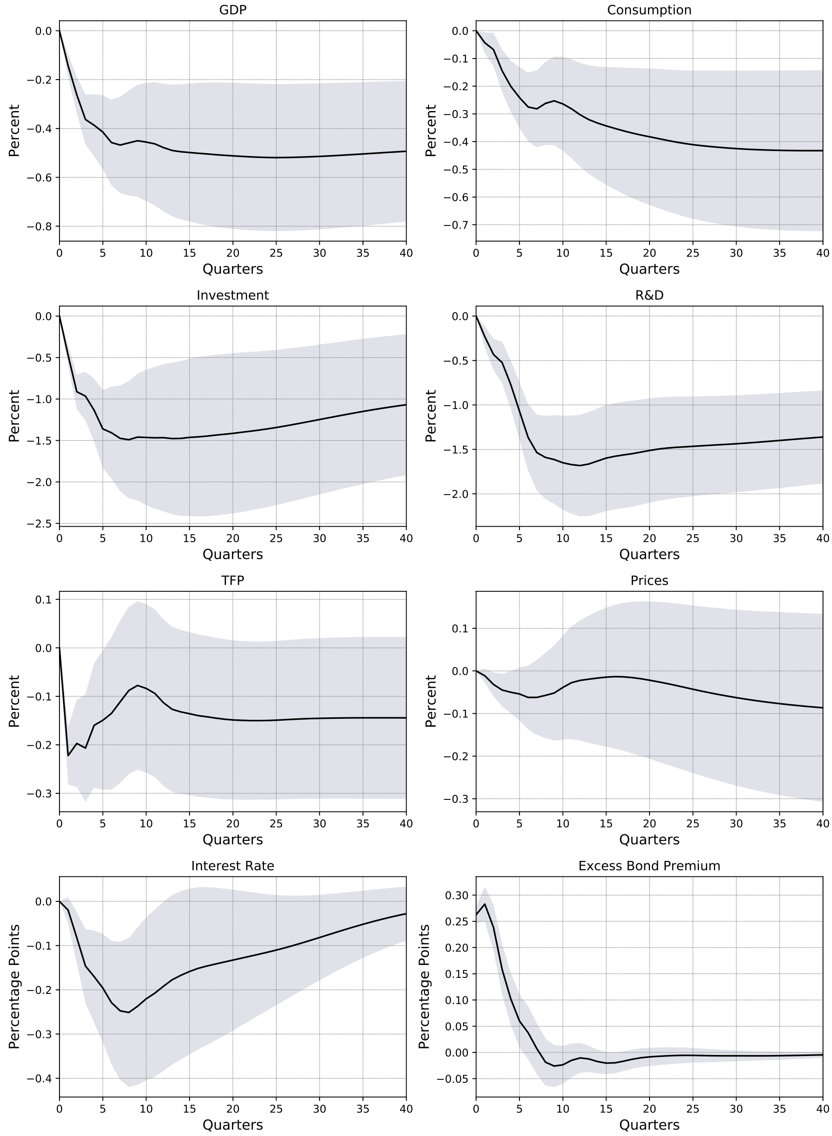



Note: Black lines are the Impulse Responses to a Financial Shock. The IRFs of GDP, R&D, and TFP are cumulated. Grey-shaded areas are 68% confidence bands.

## 2.3 Complete Set of IRFs: Alternative Ordering

The next figure displays the results if we order the Excess Bond Premium last in our SVAR. This means that financial shocks are assumed to not affect the other endogenous variables on impact, but only with a lag.

FIGURE 2.3: THE EFFECTS OF FINANCIAL SHOCKS: EBP LAST




Note: Black lines are the Impulse Responses to a Financial Shock. The IRFs of GDP, R&D, and TFP are cumulated. Grey-shaded areas are 68% confidence bands.

# 3 The Model

## 3.1 Key Intuitions

In a segmented credit market, lenders provide funding to borrowers who conduct risky capital investments either via debt or state-contingent equity. Equity provides insurance value since it shields the borrower's profits and thus stabilises the profit-based borrowing capacity. However, an increase in the equity-to-debt ratio is assumed to facilitate absconding by the debtor and thereby tighten the borrowing capacity. A pecuniary externality warrants a macro-prudential intervention which increases the borrower's reliance on equity. The increased intermediation capacity leads to higher investment in a risk-adjusted steady state and to a higher growth rate in an endogenous growth framework. Macro-prudential policy thus stabilises fluctuations around a balanced growth path, but also stabilises and steepens the path itself. The welfare gains are much larger once one accounting for endogenous growth.

**Lender** Households consume, supply labour, and lend to borrowers either via non-state-contingent deposits or via state-contingent equity.

**Borrowers** Atomistic borrowers operate a firm and maximise profits. A firm's profit in period is the difference between its return on assets and the cost of financing capital investment, which is financed either externally via debt or state-contingent equity or internally via retained profits. The firm is subject to the law of motion for capital accumulation and to a credit constraint that limits the amount it can borrow to invest to a fraction of its expected sum of discounted profits.

**Equity shields Profits** The return on equity is state-contingent and co-moves with the return on capital. Equity financing thus shields firm profits and hence stabilises the borrowing capacity.

**Equity increases the Absconding Rate** Creditors can only recover a fraction $1 - \Theta_t$ of the debtor's assets in the case of a default. Following Calomiris and Kahn (1991) and GKQ, we argue that the more (outside) equity a borrower uses to finance herself, the more difficult it is to monitor her balance sheet. The absconding rate $\Theta_t$ is thus modelled as an increasing function of equity.

**Pecuniary Externality** Atomistic borrowers take the price of capital as given. They fail to internalise that additional equity would shield their profits, allow them to increase their demand for capital, and hence boost asset prices. The pecuniary externality warrants a macro-prudential policy intervention to increase equity.

**Endogenous Growth** Firms produce output according to a production function which takes as inputs (i) physical capital, (ii) R&D and (iii) labour. As will be described in detail below in Section 3.2, the presence of aggregate R&D in this production function represents a knowledge spillover and gives rise to endogenous productivity growth. We induce stationarity and divide all quantities by the aggregate stock of R&D. Crucially, in our model productivity growth depends on R&D, which in turn depends on financial intermediation.

**Risk-adjusted Steady State** In a risk-adjusted steady state one accounts for the possibility that shocks may hit the economy. A higher reliance on equity finance cushions the effects of adverse shocks on firm profits. A macro-prudential subsidy on equity issuance can increase the risk-adjusted steady-state level of equity finance and stabilise firm profits, enhancing the borrowing capacity and capital investment. However, at the same time, the increase in equity also tightens the borrowing capacity as it increases the absconding rate $\Theta$. As we describe in detail in Section 4, solving the model around a risk-adjusted steady state will allow us to capture the costs and benefits of macro-prudential policy by accounting for the reduced volatility and its impact on intermediation and investment. Note that we will refer to the steady state as the balanced growth path since our model features growth.

**Optimal Simple Rules for Macro-prudential Policy** We study macro-prudential policy interventions that subsidize the use of equity. The subsidy is financed in a budget-neutral manner via a tax on assets. We assume that the subsidy on equity follows a simple rule. The more aggressively the policymaker subsidises equity the higher the ratio of equity to assets in the risk-adjusted steady state. While initially, the beneficial effects from the reduced riskiness and the enhanced borrowing capacity dominate, the adverse effects of an increased absconding rate will curtail intermediation for very high subsidy levels.

**Welfare** Our welfare metric is the lender's lifetime utility. Under endogenous growth, welfare depends not only on the period utility but also on the effective discount factor. In our model, an optimal simple rule for macro-prudential policy can increase the amount of equity borrowers use to finance themselves. Along a risk-adjusted balanced growth path, the increased borrower resilience will give rise to a higher financial intermediation capacity, higher investment, and hence a higher growth rate. The higher growth rate increases welfare. Moreover, it can be shown that the sensitivity of welfare to changes in the growth rate is much higher than to changes in the period utility. A small increase in the growth rate can lead to substantial welfare gains. A model without endogenous growth would not capture these gains.

**Balanced Growth Path Stabilisation** A key message from our model is that macro-prudential policy can stabilise fluctuations *and* the balanced growth path of the economy, not just fluctuations *around* a balanced growth path which is determined by exogenous factors and independent of policy.

## 3.2 Model Derivation

### Households

$$\max_{C_t,L_t,D_t,E_t,B_t^{G,TIPS}} \sum_{t=0}^{\infty} \beta^t \mathcal{U}(C_t, C_{t-1}, L_t) \quad \Leftrightarrow \quad \mathcal{W}_t = \mathcal{U}_t + \beta \mathbf{E}_t \left[\mathcal{W}_{t+1}\right] \tag{3.2.1}$$

$$\text{where } \mathcal{U}_t = \frac{1}{1-\gamma}\left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \tag{3.2.2}$$

$$\text{and } \vartheta_t = \chi \left(N_t\right)^{\iota} \left(\vartheta_{t-1}\right)^{1-\iota} \tag{3.2.3}$$

s.t.
$$P_t C_t + P_t \hat{Q}_t^E E_t + D_t = W_t L_t + \Xi_t - T_t + R_t^E P_{t-1} \hat{Q}_{t-1} E_{t-1} + R_{t-1}^D D_{t-1}$$

$$\mathcal{L}_t = \sum_{s^t} \sum_{t=0}^{\infty} \beta^t \left[\mathcal{U}(C_t, L_t) + \lambda_t \left(-P_t C_t - P_t \hat{Q}_t^E E_t - D_t + W_t L_t + \Xi_t - T_t + R_t^E P_{t-1} \hat{Q}_{t-1}^E E_{t-1} + R_{t-1}^D D_{t-1}\right)\right]$$

$$\frac{\partial \mathcal{L}_t}{\partial L_t} = \mathcal{U}_L + \lambda_t W_t = 0 \quad \Leftrightarrow \lambda_t = -\frac{\mathcal{U}_{L,t}}{W_t}, \text{ note } W_t \equiv \frac{W_t}{P_t}, \ \frac{\partial \mathcal{L}_t}{\partial C_t} = \mathbf{E}_t \mathcal{U}_C - \lambda_t P_t = 0 \quad \Leftrightarrow \lambda_t = \frac{\mathbf{E}_t \mathcal{U}_{C,t}}{P_t}$$

$$\frac{\partial \mathcal{L}_t}{\partial D_t} = -\lambda_t + \beta \mathbf{E}_t \left[\lambda_{t+1} R_t^D\right] = 0 \ \Leftrightarrow 1 = \beta \mathbf{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} R_t^D\right], \ \frac{\partial \mathcal{L}_t}{\partial E_t} = -\lambda_t + \beta \mathbf{E}_t \left[\lambda_{t+1} R_{t+1}^E\right] = 0$$

$$-\mathcal{U}_L = \mathbf{E}_t \mathcal{U}_C W_t = \mathbf{E}_t \mathcal{U}_C \frac{W_t}{P_t} \tag{3.2.4}$$

$$1 = \mathbf{E}_t \left[\Lambda_{t,t+1} \frac{R_t^D}{\Pi_{t+1}}\right] \tag{3.2.5}$$

$$1 = \mathbf{E}_t \left[\Lambda_{t,t+1} \frac{R_{t+1}^E}{\Pi_{t+1}}\right] \tag{3.2.6}$$

$$\Lambda_{t,t+1} \equiv \beta \frac{\mathcal{U}_{C,t+1}}{\mathcal{U}_{C,t}} \tag{3.2.7}$$

$$\mathcal{U}_t^L \equiv -\vartheta_t L_t^{\varphi} \left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{3.2.8}$$

$$\mathcal{U}_t^C \equiv \left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1} \left(C_{t+1} - h\Gamma_{t+1} C_t - \vartheta_{t+1} \frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{3.2.9}$$

### Production

$$Y_{m,t} = \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^{\alpha} \left(\mathcal{X}_{m,t}^{LAP} L_{m,t}\right)^{1-\alpha}. \tag{3.2.10}$$

$$\mathcal{X}_{m,t}^{LAP} = \left(U_{m,t}^N N_{m,t}\right)^{\eta} \left(U_t^N N_t\right)^{1-\eta}. \tag{3.2.11}$$

$$K_{m,t} = B_{m,t-1}^K \tag{3.2.12}$$

$$N_{m,t} = B_{m,t-1}^N. \tag{3.2.13}$$

$$\min \left\{ \tau_t^{\mathcal{M}} \left( P_t \hat{W}_t L_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^K R_t^K - P_t \hat{Q}_t^K (1-\delta_{m,t}^K)\right) \varepsilon_t^K K_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^N R_t^N - P_t \hat{Q}_t^N (1-\delta_{m,t}^N)\right) N_{m,t} \right) \right\}$$

$$s.t. \quad \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^{\alpha} \left(\left(U_{m,t}^N N_{m,t}\right)^{\eta} \left(U_t^N N_t\right)^{1-\eta} L_{m,t}\right)^{1-\alpha} \geq \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_t.$$

$$\mathcal{L}_{m,t} = -\tau_t^{\mathcal{M}} \left( P_t \hat{W}_t L_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^K R_t^K - P_t \hat{Q}_t^K (1-\delta_{m,t}^K)\right) \varepsilon_t^K K_{m,t} + \left(P_{t-1} \hat{Q}_{t-1}^N R_t^N - P_t \hat{Q}_t^N (1-\delta_{m,t}^N)\right) N_{m,t} \right)$$

$$+ P_t \widehat{MC}_{m,t} \left( \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^{\alpha} \left(\left(U_{m,t}^N N_{m,t}\right)^{\eta} \left(U_t^N N_t\right)^{1-\eta} L_{m,t}\right)^{1-\alpha} - \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_t \right)$$

$$\frac{\partial\mathcal{L}}{\partial L_{m,t}} = 0 \Leftrightarrow P_t\hat{W}_t = \frac{P_t\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\frac{Y_{m,t}}{L_{m,t}} \tag{3.2.14}$$

$$\frac{\partial\mathcal{L}}{\partial K_{m,t}} = 0 \Leftrightarrow R_t^K = \frac{\frac{P_t\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}\alpha\frac{Y_{m,t}}{K_{m,t}} + (1-\delta_{m,t}^K)P_t\hat{Q}_t^K\varepsilon_t^K}{P_{t-1}\hat{Q}_{t-1}^K} \tag{3.2.15}$$

$$\frac{\partial\mathcal{L}}{\partial K_{m,t}} = 0 \Leftrightarrow R_t^N = \frac{\frac{P_t\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\eta\frac{Y_{m,t}}{N_{m,t}} + (1-\delta_{m,t}^N)P_t\hat{Q}_t^N}{P_{t-1}\hat{Q}_{t-1}^N} \tag{3.2.16}$$

$$\hat{\mathcal{R}}_t^K \equiv \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}\alpha\frac{Y_{m,t}}{K_{m,t}} \tag{3.2.17}$$

$$\hat{\mathcal{R}}_t^N \equiv \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\eta\frac{Y_{m,t}}{N_{m,t}} \tag{3.2.18}$$

$$\frac{\partial\mathcal{L}}{\partial U_{m,t}^K} = 0 \Leftrightarrow \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}\alpha\frac{Y_{m,t}}{U_{m,t}^K} = \left(\frac{\partial\delta_{m,t}^K}{\partial U_{m,t}^K}\right)\hat{Q}_t^K \tag{3.2.19}$$

$$\frac{\partial\mathcal{L}}{\partial U_{m,t}^N} = 0 \Leftrightarrow \frac{\widehat{MC}_{m,t}}{\tau_t^{\mathcal{M}}}(1-\alpha)\eta\frac{Y_{m,t}}{U_{m,t}^N} = \left(\frac{\partial\delta_{m,t}^N}{\partial U_{m,t}^N}\right)\hat{Q}_t^N. \tag{3.2.20}$$

$$\delta_{m,t}^K = \bar{\delta}^K + \frac{b^K}{1+\zeta^K}\left(U_{m,t}^K\right)^{1+\zeta^K} \tag{3.2.21}$$

$$\delta_{m,t}^N = \bar{\delta}^N + \frac{b^N}{1+\zeta^N}\left(U_{m,t}^N\right)^{1+\zeta^N}. \tag{3.2.22}$$

**Inter-temporal Pricing Problem of Final Output Producers** The objective of each final output producer is to maximise its profits: $Profit_{m,t} = P_{m,t}Y_{m,t} - \tau_t^{\mathcal{M}}P_t\left\{\hat{W}_tL_{m,t} + \hat{\mathcal{R}}_t^KK_{m,t} + \hat{\mathcal{R}}_t^NN_{m,t}\right\}$. With probability $\phi_P$ a firm cannot re-set its desired price $P_{m,t}^*$. In this case the firm is stuck with its previous-period price indexed to $\bar{\Pi}^P$

$$P_{m,t} = \begin{cases} P_{m,t}^* & \textit{with probability:} \quad 1-\phi_P \\ P_{m,t-1}\left(\left(\Pi_{ss}^P\right)^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}\right) & \textit{with probability:} \quad \phi_P \end{cases}$$

where $ind_P = 0$ is the weight attached to previous-period inflation. Consider a firm who can reset its price in the current period $P_{m,t} = P_{m,t}^*$ and who is then stuck with its price until future period $t+s$. The price in this case would be $P_{m,t+s} = P_{m,t}^*\left[(\bar{\Pi})^{s(1-ind_P)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{ind_P}\right]$. The final output producing firms solve the following optimisation problem $\max_{P_{m,t}^*} E_t\sum_{s=0}^{\infty}(\phi_P\beta)^s\frac{\mathcal{U}_{t+s}^C}{\mathcal{U}_t^C}\frac{P_t}{P_{t+s}}\left[P_{m,t+s}Y_{m,t+s|t} - (1+(1-\alpha)\eta)P_t\widehat{MC}_{t+s}Y_{m,t+s|t}\right]$ subject to the above derived demand constraint and assuming that a firm $i$ always meets the demand for its good at the current price $Y_{m,t+s} = \left(\frac{P_{m,t+s}}{P_{t+s}}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}Y_{t+s}$.

Substitute the demand schedule and the relation for $P_{m,t+s}$ into the objective function to get

$$\max_{P_{m,t}^*} E_t\sum_{s=0}^{\infty}(\beta\phi_P)^s\frac{\mathcal{U}_{t+s}^C}{\mathcal{U}_t^C}\frac{P_t}{P_{t+s}}\left[(P_{m,t}^*)^{1-\frac{\mathcal{M}}{\mathcal{M}-1}}\left((\bar{\Pi})^{s(1-ind_P)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{ind_P}\right)^{1-\frac{\mathcal{M}}{\mathcal{M}-1}}\left(\frac{1}{P_{t+s}}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}Y_{t+s}\right.$$
$$\left.-(1+(1-\alpha)\eta)P_{t+s}\widehat{MC}_{t+s}\left[(P_{m,t}^*)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}\left(\frac{\left[(\bar{\Pi})^{s(1-ind_P)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{ind_P}\right]}{P_{t+s}}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}}Y_{t+s}\right]\right], \quad \frac{P_{t+s-1}}{P_{t-1}} = \prod_{g=0}^{s-1}\Pi_{t+g}^P.$$

$$\mathcal{F}_{1,t} = (1+(1-\alpha)\eta)P_t\widehat{MC}_tY_t + \phi_PE_t\left[\Lambda_{t,t+1}\Pi_{t+1}^{-1}\left(\zeta_{t+1}^{\Pi}\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}}\mathcal{F}_{1,t+1}\right] \tag{3.2.23}$$

$$\mathcal{F}_{2,t} = Y_t + \phi_PE_t\left[\Lambda_{t,t+1}\left(\zeta_{t+1}^{\Pi}\right)^{\frac{1}{\mathcal{M}-1}}\mathcal{F}_{2,t+1}\right] \tag{3.2.24}$$

Taking the derivative with respect to $P^*_{m,t}$ and rearranging delivers $P^*_{m,t} = P^*_t = \frac{\mathcal{F}_{1,t}\mathcal{M}}{\mathcal{F}_{2,t}}$, where $\zeta^{\Pi}_t$ is defined as

$$\zeta^{\Pi}_t \equiv \Pi_t \left[ (\bar{\Pi})^{(1-ind_P)} (\Pi_{t-1})^{ind_P} \right]^{-1}. \tag{3.2.25}$$

An expression for the aggregate final output price index $P_t$ can be derived from

$$P_t = \left( \int_F \left(P^*_{m,t}\right)^{\frac{1}{1-\mathcal{M}}} di + \int_{\bar{F}} \left( \left[ (\bar{\Pi})^{(1-ind_P)} \left(\Pi^P_{t-1}\right)^{ind_P} \right] P_{m,t-1} \right)^{\frac{1}{1-\mathcal{M}}} di \right)^{1-\mathcal{M}}$$

where $F$ is the set of those final output producers who can reoptimise their price. The Phillips Curve is given by

$$\frac{P^*_t}{P_t} = \left[ \frac{1-(\phi_P)\left(\zeta^{\Pi}_t\right)^{\frac{-1}{1-\mathcal{M}}}}{1-\phi_P} \right]^{1-\mathcal{M}} = \frac{\mathcal{F}_{1,t}\mathcal{M}}{\mathcal{F}_{2,t}}. \tag{3.2.26}$$

Capital good production is given by

$$I^K_t = K_{t+1} - \left[1-\delta^K_t\right] K_t \varepsilon^K_t \tag{3.2.27}$$

$$Q^K_t = 1 + \frac{\psi_{I^K}}{2}\left(\frac{I^K_t}{I^K_{t-1}} - \bar{\Gamma}\right)^2 + \left(\frac{I^K_t}{I^K_{t-1}}\right)\psi_{I^K}\left(\frac{I^K_t}{I^K_{t-1}} - \bar{\Gamma}\right) - \mathbf{E}_t\left[\Lambda_{t,t+1}\left(\frac{I^K_{t+1}}{I^K_t}\right)^2 \psi_{I^K}\left(\frac{I^K_{t+1}}{I_t} - \bar{\Gamma}\right)\right] \tag{3.2.28}$$

$$I^N_t = N_{t+1} - \left[1-\delta^N_t\right] N_t. \tag{3.2.29}$$

$$Q^N_t = 1 + \frac{\psi_{I^N}}{2}\left(\frac{I^N_t}{I^N_{t-1}} - \bar{\Gamma}\right)^2 + \left(\frac{I^N_t}{I^N_{t-1}}\right)\psi_{I^N}\left(\frac{I^N_t}{I^N_{t-1}} - \bar{\Gamma}\right) - \mathbf{E}_t\left[\Lambda_{t,t+1}\left(\frac{I^N_{t+1}}{I^N_t}\right)^2 \psi_{I^N}\left(\frac{I^N_{t+1}}{I^N_t} - \bar{\Gamma}\right)\right]. \tag{3.2.30}$$

**Banks** Recall the flow of funds relation $RE_{j,t} + (1+\tau^N_t)Q^N_t B^N_{j,t} + (1+\tau^K_t)Q^K_t B^K_{j,t} - NW_{j,t} - (1+\tau^E_t)Q^E_t E_{j,t} = D_{j,t}$ and consider that a policymakers wants to subsidise the issuance of outside equity via a tax on assets such that the budget is always balanced $\tau^E_t Q^E_t E_{j,t} = \tau^N_t Q^N_t B^N_{j,t} + \tau^K_t Q^K_t B^K_{j,t}$. Moreover, we assume that the risk profile associated with each type of asset affects the tax rate charged so that $\tau^K_t = \Delta^K \tau^N_t$. Since claims associated with physical capital have a higher liquidation value, they are less risky, and the tax rate charged on them is thus lower than for R&D so that $\tau^K_t < \tau^N_t$. Combine this with the accumulation of retained earnings

$$\begin{aligned} NW_{j,t+1} = & \left[\phi^N_{j,t}\left(R^N_{t+1} - R^D_t\right) + \phi^K_{j,t}\left(R^K_{t+1} - R^D_t\right) + X_{j,t}\phi_{j,t}\left(R^D_t - R^E_{t+1}\right)\right. \\ & \left. + R^D_t\left(1 - \tau^K_t\phi^K_{j,t} - \tau^N_t\phi^N_{j,t} + \tau^E_t X_{j,t}\phi_{j,t}\right) + \frac{RE_{j,t}}{NW_{j,t}}\left(R^{RE}_t - R^D_t\right)\right] NW_{j,t}. \end{aligned}$$

We introduce the definition of the risk-weighted equity-to-asset ratio and the leverage ratios

$$\Theta_t = \theta\left(1 + \omega_1 X_{j,t} + \frac{\omega_2}{2}X^2_{j,t}\right), \;\; X_{j,t} \equiv \frac{Q^E_t E_{j,t}}{\Delta^K Q^K_t B^K_{j,t} + Q^N_t B^N_{j,t}}, \phi_{j,t} \equiv \frac{\Delta^K Q^K_t B^K_{j,t} + Q^N_t B^N_{j,t}}{NW_{j,t}}, \phi^K_{j,t} \equiv \frac{Q^K_t B^K_{j,t}}{NW_{j,t}}, \; \phi^{B^N}_{j,t} \equiv \frac{Q^N_t B^N_{j,t}}{NW_{j,t}}.$$

The value of the bank in period $t$, $V_{j,t}$, is the expected payout from the terminal net worth

$$V_{j,t} = \mathbf{E}_t\left[\sum_{\tau=t+1}^{\infty} (\sigma)^{\tau-t-1}\Lambda_{t,\tau}\Pi^{-1}_{t,\tau}\left((1-\sigma)NW_{b,\tau}\right)\right].$$

Recall the LoM derived for net worth by combining the balance sheet identity with the retained earnings expression

$$\begin{aligned} \mathcal{G}_{j,t} \equiv \frac{NW_{j,t+1}}{NW_t} = & \left[\phi^N_{j,t}\left(R^N_{t+1} - R^D_t\right) + \phi^K_{j,t}\left(R^K_{t+1} - R^D_t\right) + X_{j,t}\phi_{j,t}\left(R^D_t - R^E_{t+1}\right)\right. \\ & \left. + R^D_t\left(1 - \tau^K_t\phi^K_{j,t} - \tau^N_t\phi^N_{j,t} + \tau^E_t X_{j,t}\phi_{j,t}\right) + \frac{RE_{j,t}}{NW_{j,t}}\left(R^{RE}_t - R^D_t\right)\right] \end{aligned}$$

so that

$$g(NW_{j,t+1}, NW_{j,t}, X_{j,t}, \phi_{j,t}, \phi^K_{j,t}, \phi^N_{j,t}) \equiv \mathcal{G}_{j,t} NW_{j,t} - NW_{j,t+1} \quad = \quad 0$$

$$\max V_{j,t} \quad s.t. \quad g(NW_{j,t+1}, NW_{j,t}, X_{j,t}, \phi_{j,t}, \phi^K_{j,t}, \phi^N_{j,t})$$

where $\Omega_{b,\tau}$ is the Lagrange multiplier associated with the net worth accumulation budget constraint.

$$\begin{aligned}
\mathcal{L}_{j,t} &= \sum_{\mathcal{S}_t} \pi^{\mathcal{S}_t} \left[ \sum_{\tau=t+1}^{\infty} \sigma^{\tau-t-1} \Lambda_{t,\tau} \Pi^{-1}_{t,\tau} \left\{ (1-\sigma) NW_{b,\tau} + \Omega_{b,\tau} g(NW_{b,\tau}, NW_{b,\tau-1}, X_{b,\tau-1}, B^K_{b,\tau-1}, B^N_{b,\tau-1}) \right\} \right] \\
\frac{\partial \mathcal{L}_{j,t}}{\partial NW_{b,\tau=t+1}} &= \pi^{\mathcal{S}_t} \sigma^0 \Lambda_{t,t+1} \Pi^{-1}_{t,t+1} \left\{ (1-\sigma) + \Omega_{j,t+1} \frac{\partial g_{t+1}}{\partial NW_{j,t+1}} \right\} + \sum_{\mathcal{S}_{t+1}|\mathcal{S}_t} \pi^{\mathcal{S}_{t+1}} \sigma^1 \Lambda_{t,t+2} \Pi^{-1}_{t,t+2} \left\{ \Omega_{j,t+2} \frac{\partial g_{t+2}}{\partial NW_{j,t+1}} \right\} = 0 \\
0 &= \Lambda_{t,t+1} \Pi^{-1}_{t,t+1} \left\{ (1-\sigma) + \Omega_{j,t+1} (-1) \right\} + \sigma \mathbf{E}_{t+1} \Lambda_{t,t+2} \Pi^{-1}_{t,t+2} \left\{ \Omega_{j,t+2} \left( \mathcal{G}_{j,t+1} \right) \right\} \\
\Omega_{j,t+1} &= (1-\sigma) + \sigma \mathbf{E}_{t+1} \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \left\{ \Omega_{j,t+2} \left( \mathcal{G}_{j,t+1} \right) \right\} \\
\mathcal{G}_{j,t} &\equiv \frac{NW_{j,t+1}}{NW_{j,t}} = \Bigg[ \phi^N_{j,t} \left( R^N_{t+1} - R^D_t \right) + \phi^K_{j,t} \left( R^K_{t+1} - R^D_t \right) + X_{j,t} \phi_{j,t} \left( R^D_t - R^E_{t+1} \right) \\
&\quad + R^D_t \left( 1 - \tau^K_t \phi^K_{j,t} - \tau^N_t \phi^N_{j,t} + \tau^E_t X_{j,t} \phi_{j,t} \right) + \frac{RE_{j,t}}{NW_{j,t}} \left( R^{RE}_t - R^D_t \right) \Bigg] \\
\Omega_{j,t+1} &= (1-\sigma) + \sigma \Bigg\{ \mathbf{E}_{t+1} \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \Omega_{j,t+2} \phi^K_{j,t+1} (R^K_{t+2} - R^D_{t+1}) \\
&\quad + \mathbf{E}_{t+1} \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \Omega_{j,t+2} \phi^N_{j,t+1} (R^N_{t+2} - R^D_{t+1}) \\
&\quad + \mathbf{E}_{t+1} \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \Omega_{j,t+2} \phi_{j,t+1} X_{j,t+1} \left( R_{t+1} - R^E_{t+2} \right) \\
&\quad + \mathbf{E}_{t+1} \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \Omega_{j,t+2} R^D_{t+1} \left( 1 - \tau^K_t \phi^K_{j,t} - \tau^N_t \phi^N_{j,t} + \tau^E_t X_{j,t} \phi_{j,t} \right) \\
&\quad + \mathbf{E}_{t+1} \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \Omega_{j,t+2} \frac{RE_{j,t+1}}{NW_{j,t+1}} \left( R^{RE}_t - R^D_t \right) \Bigg\}
\end{aligned}$$

Using the auxiliary definitions

$$\begin{aligned}
\nu_{j,t} &\equiv \mathbf{E}_t \left[ \Lambda_{t,t+1} \Omega_{j,t+1} \Pi^{-1}_{t,t+1} \left( R^D_t \right) \right] \\
\mu^{B^K}_{j,t} &\equiv \mathbf{E}_t \left[ \Lambda_{t,t+1} \Omega_{j,t+1} \Pi^{-1}_{t,t+1} \left( R^K_{t+1} - R^D_t \right) \right], \quad \mu^{B^N}_{j,t} \equiv \mathbf{E}_t \left[ \Lambda_{t,t+1} \Omega_{j,t+1} \Pi^{-1}_{t,t+1} \left( R^N_{t+1} - R^D_t \right) \right] \\
\mu^{RE}_{j,t} &\equiv \mathbf{E}_t \left[ \Lambda_{t,t+1} \Omega_{j,t+1} \Pi^{-1}_{t,t+1} \left( R^{RE}_t - R^D_t \right) \right], \quad \mu^E_{j,t} \equiv \mathbf{E}_t \left[ \Lambda_{t,t+1} \Omega_{j,t+1} \Pi^{-1}_{t,t+1} \left( R^D_t - R^E_{t+1} \right) \right]
\end{aligned}$$

## Guess the Bank's Franchise Value and Verify

$$\begin{aligned}
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \Delta_{t+1} NW_{j,t+1} \\
V_t(B^G_{j,t}, B^K_{j,t}, B^N_{j,t}, X_{j,t}, NW_{j,t}) &= \mathbf{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \left\{ (1-\sigma) NW_{j,t+1} + \sigma \max_{B_{j,t+1}, X_{j,t+1}} \mathbf{E}_t V_{t+1}(B^G_{j,t+1}, B^K_{j,t+1}, B^N_{j,t+1}, X_{j,t+1}, NW_{j,t+1}) \right\} \\
\mathbf{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \Delta_{t+1} NW_{j,t+1} &= \mathbf{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \left\{ (1-\sigma) NW_{j,t+1} + \sigma \mathbf{E}_t \left( \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \Delta_{t+2} NW_{j,t+2} \right) \right\} \\
\mathbf{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \Delta_{t+1} NW_{j,t+1} &= \mathbf{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \left\{ (1-\sigma) NW_{j,t+1} + \sigma \mathbf{E}_t \left( \Lambda_{t+1,t+2} \Pi^{-1}_{t+1,t+2} \Delta_{t+2} \mathcal{G}_{j,t+1} NW_{j,t+1} \right) \right\} \\
\Omega_{j,t+1} &= \Delta_{t+1} = \left\{ (1-\sigma) + \sigma \mathbf{E}_t \left( \Lambda_{t+1,t+2} \Delta_{t+2} \mathcal{G}_{j,t+1} \right) \right\} \\
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t \left[ \Lambda_{t,t+1} \Pi^{-1}_{t+1} \Omega_{t+1} NW_{j,t+1} \right].
\end{aligned}$$

Recall

$$\Lambda_{t,t+1} \Omega_{j,t+1} \quad = \quad \Lambda_{t,t+1}(1-\sigma) + \sigma \mathbf{E}_{t+1} \Lambda_{t,t+2} \left\{ \Omega_{j,t+2} \left( \mathcal{G}_{j,t+1} \right) \right\}$$

and

$$V_{j,t}(NW_{j,t}) \quad = \quad \mathbf{E}_t \Lambda_{t,t+1} \Pi^{-1}_{t+1} \Omega_{j,t+1} NW_{j,t+1}$$

$$
\begin{aligned}
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\left[\left(\Lambda_{t,t+1}\Pi_{t+1}^{-1}(1-\sigma)NW_{j,t+1}+\sigma\mathbf{E}_{t+1}\Lambda_{t,t+2}\Pi_{t+2}^{-1}\left\{\Omega_{j,t+2}\left(\mathcal{G}_{j,t+1}NW_{j,t+1}\right)\right\}\right)\right]\\
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\left[\left(\Lambda_{t,t+1}\Pi_{t+1}^{-1}(1-\sigma)NW_{j,t+1}+\sigma\underbrace{\mathbf{E}_{t+1}\left\{\Lambda_{t,t+2}\Pi_{t+2}^{-1}\Omega_{j,t+2}NW_{j,t+2}\right\}}_{V_{t+1}}\right)\right]\\
V_{j,t}(NW_{j,t}) &= \mathbf{E}_t\Omega_{j,t+1}\Lambda_{t,t+1}\Pi_{t+1}^{-1}NW_{j,t+1}\\
V_{j,t}(NW_{j,t}) &= \left[\mu_{j,t}^{RE}\frac{RE_{j,t}}{NW_{j,t}}+\mu_{j,t}^{N}\phi_{j,t}^{N}+\mu_{j,t}^{K}\phi_{j,t}^{K}+\mu_{j,t}^{E}\phi_{j,t}X_{j,t}+\nu_{j,t}\left(1-\tau_t^K\phi_{j,t}^K-\tau_t^N\phi_{j,t}^N+\tau_t^E\phi_{j,t}X_{j,t}\right)\right]NW_{j,t}.
\end{aligned}
$$

Next, we maximise

$$
\begin{aligned}
V_{j,t}(NW_{j,t}) &= \left[\mu_{j,t}^{RE}\frac{RE_{j,t}}{NW_{j,t}}+\mu_{j,t}^{N}\phi_{j,t}^{N}+\mu_{j,t}^{K}\phi_{j,t}^{K}+\mu_{j,t}^{E}\phi_{j,t}X_{j,t}+\nu_{j,t}\left(1-\tau_t^K\phi_{j,t}^K-\tau_t^N\phi_{j,t}^N+\tau_t^E\phi_{j,t}X_{j,t}\right)\right]NW_{j,t}\\
s.t. \qquad V_t &\geq \Theta(X_{j,t})\left(\Delta^K Q_t^K B_{j,t}^K+Q_t^N B_{j,t}^N\right)
\end{aligned}
$$

over $X_{j,t}$, $RE_{j,t}$, $B_{j,t}^K$ and $B_{j,t}^N$, so that the Lagrangien is given by

$$
\begin{aligned}
\mathcal{L}_t &= \left\{\left[\mu_{j,t}^{RE}\frac{RE_{j,t}}{NW_{j,t}}+\mu_{j,t}^{N}\phi_{j,t}^{N}+\mu_{j,t}^{K}\phi_{j,t}^{K}+\mu_{j,t}^{E}\phi_{j,t}X_{j,t}+\nu_{j,t}\left(1-\tau_t^K\phi_{j,t}^K-\tau_t^N\phi_{j,t}^N+\tau_t^E\phi_{j,t}X_{j,t}\right)\right]NW_{j,t}\right\}\\
&\quad +LM_1\left(V_t-\Theta(X_{j,t})\left(\Delta^K Q_t^K B_{j,t}^K+Q_t^N B_{j,t}^N\right)\right)\\
\frac{\partial\mathcal{L}}{\partial X_{j,t}} &= 0:\ \left[\mu_{j,t}^{E}\phi_{j,t}+\nu_{j,t}(\tau_t^E\phi_{j,t})\right]NW_{j,t}-LM_1\Theta'(X_{j,t})\left(\Delta^K Q_t^K B_{j,t}^K+Q_t^N B_{j,t}^N\right)=!0,\left[\mu_{j,t}^{E}+\nu_{j,t}\tau_t^E\right]=LM_1\Theta'(X_{j,t})\\
\frac{\partial\mathcal{L}}{\partial RE_{j,t}} &= 0:\ \mu_{j,t}^{RE}=0\\
\frac{\partial\mathcal{L}}{\partial B_{j,t}^K} &= 0:\ \mu_{j,t}^{K}Q_t^K+\mu_{j,t}^{E}X_{j,t}\Delta^K Q_t^K+\nu_{j,t}\underbrace{\left(-\tau_t^K Q_t^K+\tau_t^E X_{j,t}\Delta^K Q_t^K\right)}_{=0\text{, under tax assumption 3.2}}-LM_1\Theta(X_{j,t})\Delta^K Q_t^K!=0,\frac{\mu_{j,t}^K}{\Delta^K}+\mu_{j,t}^{E}X_{j,t}=LM_1\Theta(X_{j,t})
\end{aligned}
$$

$$
\frac{\partial\mathcal{L}}{\partial B_{j,t}^N}=0:\ \mu_{j,t}^{N}Q_t^N+\mu_{j,t}^{E}X_{j,t}Q_t^N+\nu_{j,t}\underbrace{\left(-\tau_t^N Q_t^N+\tau_t^E X_{j,t}Q_t^N\right)}_{=0\text{, under tax assumption 3.2}}-LM_1\Theta(X_{j,t})Q_t^N!=0\ \Leftrightarrow \mu_{j,t}^{N}+\mu_{j,t}^{E}X_{j,t}=LM_1\Theta(X_{j,t})
$$

Combine the optimality conditions to get

$$
\frac{\left[\mu_{j,t}^{E}+\nu_{j,t}\tau_t^E\right]}{\mu_{j,t}^{N}+\mu_{j,t}^{E}X_{j,t}} = \frac{\Theta'(X_{j,t})}{\Theta(X_{j,t})},\qquad \mu_{j,t}^K=\Delta^K\mu_{j,t}^N
$$

Combine the closed-form expression of the banks' franchise value $V_{j,t}$ with the incentive compatibility constraint

$$
\begin{aligned}
&\Theta(X_t)\left(\Delta^K Q_t^K B_{j,t}^K+Q_{j,t}^N B_{j,t}^N\right)=\left[\overbrace{\mu_{j,t}^{RE}}^{=0}\frac{RE_{j,t}}{NW_{j,t}}+\mu_{j,t}^{N}\phi_{j,t}^{N}+\overbrace{\mu_{j,t}^{K}}^{\Delta^K\mu_{j,t}^N}\phi_{j,t}^{K}+\mu_{j,t}^{E}\phi_{j,t}X_{j,t}\right.\\
&\left.+\nu_{j,t}\underbrace{\left(1-\tau_t^K\phi_{j,t}^K-\tau_t^N\phi_{j,t}^N+\tau_t^E\phi_{j,t}X_{j,t}\right)}_{=1\text{, under tax assumption, 3.2}}\right]NW_{j,t}\\
&\Theta(X_t)\phi_{j,t}=\mu_{j,t}^N\left(\Delta^K\phi_{j,t}^K+\phi_{j,t}^N\right)+\mu_{j,t}^E\phi_{j,t}X_{j,t}+\nu_{j,t},\quad \phi_{j,t}=\frac{\nu_{j,t}}{\Theta_t-\left(\mu_{j,t}^{BN}+\mu_{j,t}^{E}X_{j,t}\right)}.
\end{aligned}
$$

**Collecting Terms**

$$\Theta_t = \theta\left(1+\omega_1 X_{j,t}+\frac{\omega_2}{2}X_{j,t}^2\right) \tag{3.2.31}$$

$$\phi_{j,t} \equiv \frac{\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N}{NW_{j,t}} \tag{3.2.32}$$

$$\nu_{j,t} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D\right)\right] \tag{3.2.33}$$

$$\Delta^K \mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^K - R_t^D\right)\right] \tag{3.2.34}$$

$$\mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^N - R_t^D\right)\right] \tag{3.2.35}$$

$$\mu_{j,t}^{E} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D - R_{t+1}^E\right)\right] \tag{3.2.36}$$

$$\Omega_{j,t+1} \equiv (1-\sigma)+\sigma\left[\nu_{j,t+1}+\phi_{j,t+1}\left(\mu_{j,t+1}^{B^N}+X_{j,t+1}\mu_{j,t+1}^E\right)\right)\Bigg] \tag{3.2.37}$$

$$\left(\mu_{j,t}^E+\nu_{j,t}\tau_t^E\right)/\left(\mu_{j,t}^N+\mu_{j,t}^E X_{j,t}\right) = \Theta'(X_{j,t})/\Theta(X_{j,t}) \tag{3.2.38}$$

$$\phi_{j,t} = \nu_{j,t}/\left(\Theta_t - \left(\mu_{j,t}^{B^N}+\mu_{j,t}^E X_{j,t}\right)\right). \tag{3.2.39}$$

**Return on Bank's Outside Equity** In order to find the flow return on equity via the flow return on total capital $\mathcal{K}$, the production function can be rewritten in terms of total capital as follows

$$\begin{aligned}
Y_{m,t} &= \left(\varepsilon_t^K U_{m,t}^K K_{m,t}\right)^\alpha \left(\left(U_{m,t}^N N_{m,t}\right)^\eta \left(U_t^N N_t\right)^{1-\eta} L_{m,t}\right)^{1-\alpha} \\
Y_{m,t} &= \left(\varepsilon_t^K\right)^\alpha \underbrace{(K_{m,t})^\alpha (N_{m,t})^{\eta(1-\alpha)}}_{\equiv \mathcal{K}_{m,t}^{\alpha+\eta(1-\alpha)}} \underbrace{\left(U_{m,t}^K\right)^\alpha \left(U_{m,t}^N\right)^{\eta(1-\alpha)}}_{\equiv \left(U_{m,t}^{\mathcal{K}}\right)^{\alpha+\eta(1-\alpha)}} \left(U_t^N N_t\right)^{(1-\eta)(1-\alpha)} (L_{m,t})^{1-\alpha} \\
Y_{m,t} &= \left(\varepsilon_t^E U_{m,t}^{\mathcal{K}} \mathcal{K}_{m,t}\right)^{\alpha+\eta(1-\alpha)} \left(U_t^N N_t\right)^{(1-\eta)(1-\alpha)} (L_{m,t})^{1-\alpha}
\end{aligned}$$

where

$$\mathcal{K}_{m,t} = \left((K_{m,t})^\alpha (N_{m,t})^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}}, \quad U_{m,t}^{\mathcal{K}} = \left(\left(U_{m,t}^K\right)^\alpha \left(U_{m,t}^N\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}}, \quad \varepsilon_t^E \equiv \left(\varepsilon_t^K\right)^{\frac{\alpha}{\alpha+\eta(1-\alpha)}}.$$

We can derive the flow return $\mathcal{R}_t^{\mathcal{K}}$ by taking the derivative of output with respect to total capital

$$\frac{\partial Y_{m,t}}{\partial \mathcal{K}_{m,t}} = (\alpha+\eta(1-\alpha))\frac{Y_{m,t}}{\mathcal{K}_{m,t}} \equiv \mathcal{R}_{m,t}^{\mathcal{K}} = R_{m,t}^K \frac{K_{m,t}}{\mathcal{K}_{m,t}} + R_{m,t}^N \frac{N_{m,t}}{\mathcal{K}_{m,t}}.$$

We can still detrend the production function by dividing by $N_t$

$$\begin{aligned}
\frac{Y_t}{\mathcal{K}_t} &= \left(\varepsilon_t^K\right)^\alpha \left(U_t^{\mathcal{K}}\right)^{\alpha+\eta(1-\alpha)} (\mathcal{K}_t)^{\alpha-1+\eta(1-\alpha)} \left(U_t^N N_t\right)^{(1-\eta)(1-\alpha)} (L_t)^{1-\alpha} \\
\frac{Y_t}{\mathcal{K}_t} &= \left(\varepsilon_t^K\right)^\alpha \left(U_t^{\mathcal{K}}\right)^{\alpha+\eta(1-\alpha)} \left(\frac{\mathcal{K}_t}{N_t}\right)^{(\eta-1)(1-\alpha)} \left(U_t^N\right)^{(1-\eta)(1-\alpha)} (L_t)^{1-\alpha}
\end{aligned}$$

so that the ratio $\frac{Y_t}{\mathcal{K}_t}$ is not trending. We can express total aggregate capital in detrended terms as follows

$$\begin{aligned}
\mathcal{K}_t &= \left((K_t)^\alpha (N_t)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} \\
\hat{\mathcal{K}}_t \equiv \frac{\mathcal{K}_t}{N_t} &= \frac{1}{N_t^{\frac{\alpha+\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}}}\left((K_t)^\alpha (N_t)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} = \left(\frac{1}{N_t^{\alpha+\eta(1-\alpha)}}(K_t)^\alpha (N_t)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha+\eta(1-\alpha)}} \\
\hat{\mathcal{K}}_t &\equiv \left(\hat{K}_t\right)^{\frac{\alpha}{\alpha+\eta(1-\alpha)}}.
\end{aligned}$$

This implies that the flow return and gross return on outside equity can be written as

$$\mathcal{R}_t^E \equiv \mathcal{R}_t^{\mathcal{K}} = R_t^K \hat{K}_t^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} + R_t^N \hat{K}_t^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}} \tag{3.2.40}$$

$$R_t^E = \frac{\mathcal{R}_t^E + Q_t^E \varepsilon_t^E}{Q_{t-1}^E}. \tag{3.2.41}$$

**Monetary Policy**

$$\frac{R_t^{TR}}{\bar{R}^{TR}} = \left(\frac{R_{t-1}^{TR}}{\bar{R}^{TR}}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\kappa_\Pi} \left(\frac{MC_t}{\overline{MC}}\right)^{\kappa_Y}\right]^{1-\rho_R} \varepsilon_t^{MP} \tag{3.2.42}$$

$$R_t^D = \max\left(1, R_t^{TR}\right). \tag{3.2.43}$$

**Macro-prudential Policy** We assume that the risk profile associated with each type of capital affects the tax rate charged so that $\tau_t^K = \Delta^K \tau_t^N$. This implies

$$\tau_t^E Q_t^E E_{j,t} = \tau_t^K Q_t^K B_{j,t}^K + \tau_t^N Q_t^N B_{j,t}^N, \quad \tau_t^E Q_t^E E_{j,t} = \tau_t^N \left(\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N\right), \quad \tau_t^E X_{j,t} = \tau_t^N \quad \Leftrightarrow \quad \Delta^K \tau_t^E X_{j,t} = \tau_t^K.$$

We assume the macro-prudential policy follows a simple rule

$$\tau_t^E = \kappa_\nu \nu_t^{-1} \tag{3.2.44}$$

**Final Output Goods Market Clearing and Price Dispersion** Equating the aggregate supply of final output goods $Y_t^s$ with the aggregate demand $Y_t$ we get $\int_0^1 Y_{m,t}^s di = \int_0^1 \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} Y_t di, \quad Y_t^s = Y_t \int_0^1 \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di$. We define price dispersion $Disp_t^P$ and write it recursively, to get

$$Disp_t^P \equiv \int_0^1 \left(\frac{P_{m,t}}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di = \left(\frac{1}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} \int_0^1 (P_{m,t})^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di = \left(\frac{1}{P_t}\right)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} \left[\int_{i\in F} (P_{m,t}^*)^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di + \int_{i\in \bar{F}} (P_{m,t})^{-\frac{\mathcal{M}}{\mathcal{M}-1}} di\right]$$

where $F$ is the set of those firms that can reoptimise their price. Recall that

$$P_{m,t} = \begin{cases} P_{m,t}^* & \textit{with probability: } 1-\phi_P \\ P_{m,t-1}\left(\left(\Pi_{ss}^P\right)^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}\right) & \textit{with probability: } \phi_P \end{cases}$$

so that the detrended recursive expression of price dispersion is given by

$$Disp_t^P = (1-\phi_P)\left(\frac{1-\phi_P\left(\zeta_t^\Pi\right)^{\frac{1}{\mathcal{M}-1}}}{1-\phi_P}\right)^{\mathcal{M}} + \phi_P\left(\zeta_t^\Pi\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} Disp_{t-1}^P. \tag{3.2.45}$$

This implies $Y_t^s = Y_t Disp_t^P$. We will use the latter relation to replace $Y_t^s$.

**Aggregate Resource Constraint** By combining the household budget constraint, factor prices, capital accumulation equations, the profits of the perfectly competitive capital goods producers, the aggregate bank balance sheet identity, and the net worth accumulation equation one can obtain the aggregate market clearing relationship

$$P_t Y_t = P_t C_t + \left[1+\Psi_t^{I^K}\right] P_t I_t^K + \left[1+\Psi_t^{I^N}\right] P_t I_t^N \tag{3.2.46}$$

$$\Xi_t = \left(P_t - \frac{MC_t}{\tau_t^M}(1+(1-\alpha)\eta) Disp_t^P\right) Y_t - \xi + (1-\sigma) NW_t^o$$

## 3.3 Stationarisation

$$\mathcal{W}_t = \max_{C_t, C_{t-1}, L_t} \{\mathcal{U}_t + \beta \mathbf{E}_t \mathcal{W}_{t+1}\} \quad \Leftrightarrow \quad \hat{\mathcal{W}}_t \equiv \frac{\mathcal{W}_t}{N_t^{1-\gamma}} = \left\{\hat{\mathcal{U}}_t + \beta \mathbf{E}_t \Gamma_{t+1}^{1-\gamma} \hat{\mathcal{W}}_{t+1}\right\} \tag{3.3.1}$$

$$\mathcal{U}_t = \frac{1}{1-\gamma}\left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \Leftrightarrow \hat{\mathcal{U}}_t = \frac{\mathcal{U}_t}{N_t^{1-\gamma}} = \frac{1}{1-\gamma}\left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \tag{3.3.2}$$

$$\vartheta_t = \chi(N_t) \quad \Leftrightarrow \quad \hat{\vartheta}_t \equiv \frac{\vartheta_t}{N_t} = \chi \tag{3.3.3}$$

$$-\frac{\mathcal{U}_t^L}{{N_t}^{1-\gamma}} = \mathbf{E}_t\left[\frac{\mathcal{U}_t^C}{N_t^{-\gamma}} \frac{W_t}{P_t N_t}\right] \quad \Leftrightarrow \quad \mathbf{E}_t \hat{\mathcal{U}}_t^C \hat{W}_t = -\hat{\mathcal{U}}_t^L \tag{3.3.4}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \frac{R_t^D}{\Pi_{t+1}}\right] \tag{3.3.5}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \frac{R_{t+1}^E}{\Pi_{t+1}}\right] \tag{3.3.6}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} R_t^R\right] \tag{3.3.7}$$

$$R_t^E = \frac{P_t}{P_{t-1}} \frac{\hat{\mathcal{R}}_t^E + \hat{Q}_t^E}{\hat{Q}_{t-1}^E} \quad \Leftrightarrow \quad \frac{R_t^E}{\Pi_t} = \frac{\hat{\mathcal{R}}_t^E + \hat{Q}_t^E}{\hat{Q}_{t-1}^E} \tag{3.3.8}$$

$$\Lambda_{t,t+1} = \beta \frac{\mathcal{U}_{C,t+1}}{\mathcal{U}_{C,t}} = \beta \frac{\hat{\mathcal{U}}_{C,t+1}}{\hat{\mathcal{U}}_{C,t}} \frac{{N_{t+1}}^{-\gamma}}{{N_t}^{-\gamma}} \quad \Leftrightarrow \quad \Lambda_{t,t+1} = \beta \frac{\hat{\mathcal{U}}_{C,t+1}}{\hat{\mathcal{U}}_{C,t}} \frac{1}{\Gamma_{t+1}^{\gamma}} \tag{3.3.9}$$

$$\frac{\mathcal{U}_t^C}{N_t^{-\gamma}} = \left(\frac{C_t}{N_t} - h\frac{C_{t-1}}{N_{t-1}} - \frac{\vartheta_t}{N_t}\frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1}\left(\frac{C_{t+1}}{N_t} - h\Gamma_{t+1}\frac{C_t}{N_t} - \frac{\vartheta_{t+1}}{N_t}\frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma}$$

$$\hat{\mathcal{U}}_t^C = \left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1}^{1-\gamma}\left(\hat{C}_{t+1} - h\hat{C}_t - \hat{\vartheta}_{t+1}\frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{3.3.10}$$

$$\mathcal{U}_t^L \equiv -\vartheta_t L_t^{\varphi}\left(C_t - h\Gamma_t C_{t-1} - \vartheta_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \quad \Leftrightarrow \quad \frac{\mathcal{U}_t^L}{N_t^{1-\gamma}} = -\hat{\vartheta}_t L_t^{\varphi}\left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{3.3.11}$$

## 3.4 Summary of Baseline Model Equations

**Households**

$$\hat{\mathcal{W}}_t = \hat{\mathcal{U}}_t + \beta \mathbf{E}_t\left[\Gamma_{t+1}^{1-\gamma} \hat{\mathcal{W}}_{t+1}\right] \tag{3.4.1}$$

$$\hat{\mathcal{U}}_t = \frac{1}{1-\gamma}\left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \tag{3.4.2}$$

$$\hat{\vartheta}_t = \chi\left(\frac{\hat{\vartheta}_{t-1}}{\Gamma_t}\right)^{1-\iota} \tag{3.4.3}$$

$$\mathbf{E}_t \hat{\mathcal{U}}_t^C \hat{W}_t = -\hat{\mathcal{U}}_t^L \tag{3.4.4}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \Pi_{t+1}^{-1} R_t^D\right] \tag{3.4.5}$$

$$1 = \mathbf{E}_t\left[\Lambda_{t,t+1} \Pi_{t+1}^{-1} R_{t+1}^E\right] \tag{3.4.6}$$

$$\hat{\mathcal{U}}_t^L \equiv -\hat{\vartheta}_t L_t^{\varphi}\left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{3.4.7}$$

$$\hat{\mathcal{U}}_t^C \equiv \left(\hat{C}_t - h\hat{C}_{t-1} - \hat{\vartheta}_t \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{-\gamma} - \beta h \Gamma_{t+1}^{1-\gamma}\left(\hat{C}_{t+1} - h\hat{C}_t - \hat{\vartheta}_{t+1}\frac{L_{t+1}^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{3.4.8}$$

$$\Lambda_{t,t+1} = \beta \frac{\hat{\mathcal{U}}_{t+1}^C}{\hat{\mathcal{U}}_t^C} \frac{1}{\Gamma_{t+1}^{\gamma}} \tag{3.4.9}$$

$$
\begin{aligned}
\textbf{Firms} \quad Disp_t^P \hat{Y}_t &= \left(\varepsilon_t^K U_t^K \hat{K}_t\right)^{\alpha} \left(\hat{\mathcal{X}}_t^{LAP} L_t\right)^{1-\alpha} && (3.4.10)\\
\hat{\mathcal{X}}_t^{LAP} &= U_t^N && (3.4.11)\\
\hat{K}_t &= \hat{B}_{t-1}^K \frac{1}{\Gamma_t^N} && (3.4.12)\\
1 &= \hat{B}_{t-1}^N \frac{1}{\Gamma_t^N} && (3.4.13)\\
\hat{W}_t &= \frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} \left((1-\alpha)\frac{Disp_t^P \hat{Y}_t}{L_t}\right) && (3.4.14)\\
R_t^K \Pi_t^{-1} &= \left[\hat{\mathcal{R}}_t^K + \hat{Q}_t^K (1-\delta_{m,t}^K)\varepsilon_t^K\right] / \hat{Q}_{t-1}^K && (3.4.15)\\
R_t^N \Pi_t^{-1} &= \left[\hat{\mathcal{R}}_t^N + \hat{Q}_t^N (1-\delta_{m,t}^N)\right] / \hat{Q}_{t-1}^N && (3.4.16)\\
\hat{\mathcal{R}}_t^K &= \frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} \alpha \frac{Disp_t^P \hat{Y}_t}{\hat{K}_t} && (3.4.17)\\
\hat{\mathcal{R}}_t^N &= \frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} (1-\alpha)\eta Disp_t^P \hat{Y}_t && (3.4.18)\\
\frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} \alpha \frac{Disp_t^P \hat{Y}_t}{U_t^K} &= \hat{Q}_t^K \hat{K}_t b^K \left(U_t^K\right)^{\zeta^K} && (3.4.19)\\
\frac{\widehat{MC}_t}{\tau^{\mathcal{M}}} (1-\alpha)\eta \frac{Disp_t^P \hat{Y}_t}{U_t^N} &= \hat{Q}_t^N b^N \left(U_t^N\right)^{\zeta^N} && (3.4.20)\\
\delta_t^K &= \delta_c^K + \frac{b^K}{1+\zeta^K}\left(U_t^K\right)^{1+\zeta^K} && (3.4.21)\\
\delta_t^N &= \delta_c^N + \frac{b^N}{1+\zeta^N}\left(U_t^N\right)^{1+\zeta^N} && (3.4.22)\\
\frac{\hat{\mathcal{F}}_{1,t}\mathcal{M}}{\hat{\mathcal{F}}_{2,t}} &= \left[\frac{1-(\phi_P)\left(\zeta_t^{\Pi}\right)^{\frac{-1}{1-\mathcal{M}}}}{1-\phi_P}\right]^{1-\mathcal{M}} && (3.4.23)\\
\hat{\mathcal{F}}_{1,t} &= \widehat{MC}_t \hat{Y}_t + \phi_P E_t\left[\Lambda_{t,t+1}\Gamma_{t+1}\left(\zeta_{t+1}^{\Pi}\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} \hat{\mathcal{F}}_{1,t+1}\right] && (3.4.24)\\
\hat{\mathcal{F}}_{2,t} &= \hat{Y}_t + \phi_P E_t\left[\Lambda_{t,t+1}\Gamma_{t+1}\left(\zeta_{t+1}^{\Pi}\right)^{\frac{1}{\mathcal{M}-1}} \hat{\mathcal{F}}_{2,t+1}\right] && (3.4.25)
\end{aligned}
$$

$$
\begin{aligned}
\zeta_t^{\Pi} &= \Pi_t^P \left[(\bar{\Pi})^{1-ind_P}\left(\Pi_{t-1}^P\right)^{ind_P}\right]^{-1} && (3.4.26)\\
\hat{I}_t^K &= \Gamma_{t+1}\hat{K}_{t+1} - \left[1-\delta_{m,t}^K\right]\hat{K}_t && (3.4.27)\\
\hat{Q}_t^K &= 1 + \frac{\psi_{I^K}}{2}\left(\frac{\hat{I}_t^K}{\hat{I}_{t-1}^K}\Gamma_t - \bar{\Gamma}\right)^2 + \frac{\hat{I}_t^K}{\hat{I}_{t-1}^K}\Gamma_t \psi_{I^K}\left(\frac{\hat{I}_t^K}{\hat{I}_{t-1}^K}\Gamma_t - \bar{\Gamma}\right) - E_t\Lambda_{t,t+1}\left(\frac{\hat{I}_{t+1}^K}{\hat{I}_t^K}\Gamma_{t+1}\right)^2 \psi_{I^K}\left(\frac{\hat{I}_{t+1}^K}{\hat{I}_t}\Gamma_{t+1} - \bar{\Gamma}\right) && (3.4.28)\\
\hat{I}_t^N &= \Gamma_{t+1} - \left[1-\delta_{m,t}^N\right] && (3.4.29)\\
\hat{Q}_t^N &= 1 + \frac{\psi_{I^N}}{2}\left(\frac{\hat{I}_t^N}{\hat{I}_{t-1}^N}\Gamma_t - \bar{\Gamma}\right)^2 + \frac{\hat{I}_t^N}{\hat{I}_{t-1}^N}\Gamma_t \psi_{I^N}\left(\frac{\hat{I}_t^N}{\hat{I}_{t-1}^N}\Gamma_t - \bar{\Gamma}\right) - E_t\Lambda_{t,t+1}\left(\frac{\hat{I}_{t+1}^N}{\hat{I}_t^N}\Gamma_{t+1}\right)^2 \psi_{I^N}\left(\frac{\hat{I}_{t+1}^N}{\hat{I}_t}\Gamma_{t+1} - \bar{\Gamma}\right) && (3.4.30)
\end{aligned}
$$

**Banks**

$$\Theta_t = \theta\left(1+\omega_1 X_{j,t}+\frac{\omega_2}{2}X_{j,t}^2\right) \tag{3.4.31}$$

$$\phi_{j,t} \equiv \left(\Delta^K Q_t^K B_{j,t}^K + Q_t^N B_{j,t}^N\right)/NW_{j,t} \tag{3.4.32}$$

$$\nu_{j,t} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D\right)\right] \tag{3.4.33}$$

$$\Delta^K \mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^K - R_t^D\right)\right] \tag{3.4.34}$$

$$\mu_{j,t}^{B^N} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_{t+1}^N - R_t^D\right)\right] \tag{3.4.35}$$

$$\mu_{j,t}^{E} \equiv \mathbf{E}_t\left[\Lambda_{t,t+1}\Omega_{j,t+1}\Pi_{t,t+1}^{-1}\left(R_t^D - R_{t+1}^E\right)\right] \tag{3.4.36}$$

$$\Omega_{j,t+1} \equiv (1-\sigma)+\sigma\left[\nu_{j,t+1}+\phi_{j,t+1}\left(\mu_{j,t+1}^{B^N}+X_{j,t+1}\mu_{j,t+1}^E\right)\right)\right] \tag{3.4.37}$$

$$\frac{\left[\mu_{j,t}^E+\nu_{j,t}\tau_t^E\right]}{\mu_{j,t}^N+\mu_{j,t}^E X_{j,t}} = \frac{\Theta'(X_{j,t})}{\Theta(X_{j,t})} \tag{3.4.38}$$

$$\phi_{j,t} = \nu_{j,t}/\left(\Theta_t - \left(\mu_{j,t}^{B^N}+\mu_{j,t}^E X_{j,t}\right)\right) \tag{3.4.39}$$

$$R_t^E\Pi_t^{-1} = \left(\hat{R}_t^E + \hat{Q}_t^E\left(\varepsilon_t^K\right)^{\alpha/(\alpha+\eta(1-\alpha))}\right)/\hat{Q}_{t-1}^E \tag{3.4.40}$$

$$\mathcal{R}_t^E \equiv \mathcal{R}_t^{\mathcal{K}} = R_t^K \hat{K}_t^{\frac{\eta(1-\alpha)}{\alpha+\eta(1-\alpha)}} + R_t^N \hat{K}_t^{\frac{-\alpha}{\alpha+\eta(1-\alpha)}} \tag{3.4.41}$$

**Policy**

$$\frac{R_t^{TR}}{\bar{R}^{TR}} = \left(\frac{R_{t-1}^{TR}}{\bar{R}^{TR}}\right)^{\rho_R}\left[\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\kappa_\Pi}\left(\frac{MC_t}{\overline{MC}}\right)^{\kappa_Y}\right]^{1-\rho_R}\varepsilon_t^{MP} \tag{3.4.42}$$

$$R_t^D = \max\left(1, R_t^{TR}\right) \tag{3.4.43}$$

$$\tau_t^E = \kappa_\nu \nu_t^{-1} \tag{3.4.44}$$

**Market Clearing**

$$Disp_t^P = (1-\phi_P)\left(\frac{1-\phi_P\left(\zeta_t^\Pi\right)^{\frac{1}{\mathcal{M}-1}}}{1-\phi_P}\right)^{\mathcal{M}} + \phi_P\left(\zeta_t^\Pi\right)^{\frac{\mathcal{M}}{\mathcal{M}-1}} Disp_{t-1}^P \tag{3.4.45}$$

$$Y_t = C_t + \left(1+\frac{\psi_{I^K}}{2}\left(\frac{I_t^K}{I_{t-1}^K}\Gamma_t^N - \bar{\Gamma}^{I^K}\right)^2\right)I_t^K + \left(1+\frac{\psi_{I^N}}{2}\left(\frac{I_t^N}{I_{t-1}^N}\Gamma_t^N - \bar{\Gamma}^{I^N}\right)^2\right)I_t^N \tag{3.4.46}$$

$$\begin{aligned}\Gamma_t\Pi_t\widehat{NW}_t = \sigma\Big[&\hat{Q}_{t-1}^K\hat{B}_{t-1}^K\left(R_t^K - R_{t-1}^D\right) + \hat{Q}_{t-1}^N\hat{B}_{t-1}^N\left(R_t^N - R_{t-1}^D\right)\\ &+X_{t-1}\phi_{t-1}\left(R_{t-1}^D - R_{t-1}^E\right)\widehat{NW}_{t-1} + R_{t-1}^D\widehat{NW}_{t-1}\Big] + \xi\end{aligned} \tag{3.4.47}$$

**Exogenous Processes**

$$\log\varepsilon_t^K = \varsigma_K\eta_t^K \tag{3.4.48}$$

$$\log\varepsilon_t^{\mathcal{M}} = \varsigma_{\mathcal{M}}\eta_t^{\mathcal{M}} \tag{3.4.49}$$

$$\log\varepsilon_t^{MP} = \varsigma_{MP}\eta_t^{MP} \tag{3.4.50}$$

We define the spreads, the total equity-to-asset ratio and the inside-to-outside equity ratio as

$$Spread_t^K \equiv \left(R_t^K - R_{t-1}^D\right), \quad Spread_t^N \equiv \left(R_t^N - R_{t-1}^D\right), \quad Spread_t^E \equiv \left(R_{t-1}^D - R_t^E\right) \tag{3.4.51}$$

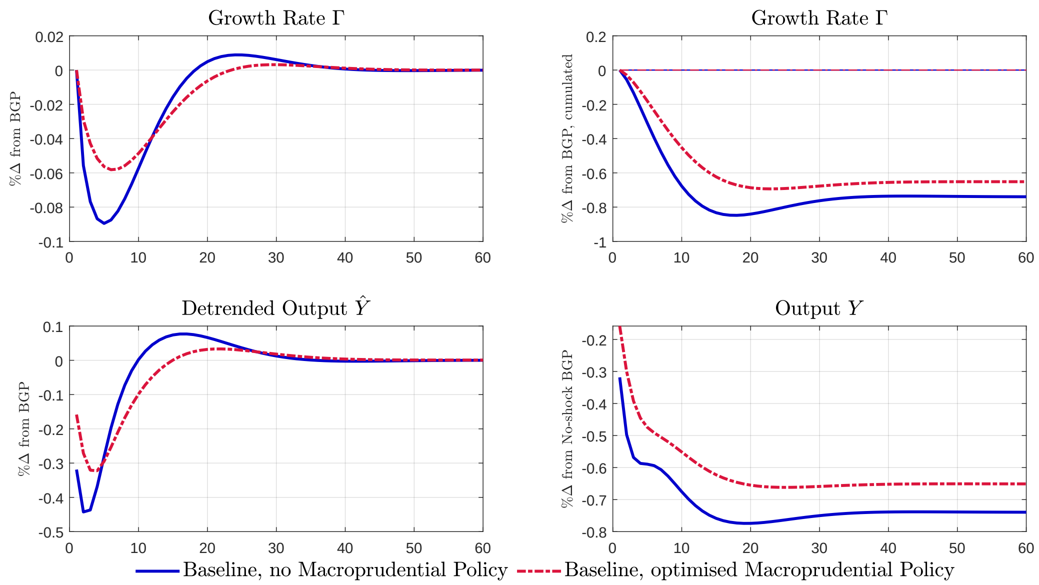
$$TETA_t \equiv \frac{NW_t + Q_t^E E_t}{Q_t^K B_t^K + Q_t^N B_t^N}, \quad INOUTE_t \equiv \frac{NW_t}{X(\Delta^K Q_t^K B_t^K + Q^N B^N)} \tag{3.4.52}$$

### 3.5 Exogenous Growth + FF and Endogenous Growth +noFF

In the main text above we compare the baseline model, Equations (3.4.1) - (3.4.52) to a model with exogenous growth and financial frictions, and to a model with endogenous growth but without financial frictions. In the case of exogenous growth, it holds that $\Gamma_t = \bar{\Gamma}$, which can be thought of as a limiting case in which the investment adjustment cost for research is infinitely high $\psi_{I^N} \to \infty$. All variables associated with R&D become constants. In the model with endogenous growth and without financial frictions, we can disregard the Equations associated to the banking block and all spreads become zero.

### 3.6 IRFs for Detrended Output versus Output in Deviation from Initial BGP

FIGURE 3.4: IRFS FOR GROWTH, THE BGP, DETRENDED OUTPUT AND OUTPUT IN DEVIATION FROM INIT. BGP




Note: The chart depicts the IRFs for a (-1stdev) capital $K$ quality shock. The blue-straight line depicts the responses of the baseline model, with endogenous growth and financial frictions, in the absence of macro-prudential policy. The red dashed line depicts the responses for the baseline model with the optimal simple macro-prudential policy rule.

**From Growth Rate to Trend** Recall the definition of the gross growth rate of knowledge capital $\Gamma_t \equiv \frac{N_t}{N_{t-1}}$. Normalise $N_{t-1} = 1$ so that

$$\begin{aligned} N_t &= \Gamma_t N_{t-1} = \Gamma_t, \quad N_{t+1} = \Gamma_{t+1} N_t = \Gamma_{t+1} \cdot \Gamma_t \\ &\vdots \\ N_{t+h} &= \prod_{s=0}^{h} \Gamma_{t+s} \end{aligned}$$

Then, recall that for each trending variable, we detrended as follows $\hat{Y}_t \equiv \frac{Y_t}{N_t}$. So once we have $\hat{Y}_t$ and $N_t$ (via $cumprod(\Gamma_t)$) we can 're-trend' to obtain $Y_t \equiv \hat{Y}_t N_t$

## 3.7 Discussion of Assumptions on Financial Frictions and Endogenous Growth

Our paper's key result is that macro-prudential policy is associated with large welfare gains since it stabilises fluctuations around the balanced growth path *and the path itself*. Fundamentally, this result does not depend on the specific type of financial friction or endogenous growth framework.

**Modelling Financial Frictions** With regard to the financial friction, our work builds on the GKQ framework, which in turn extends earlier 'costly enforcement'[2] models with a role for bank (outside) equity and thus provides a clear rationale for macro-prudential policy aimed at increasing bank equity issuance.

An alternative to the costly enforcement approach is the 'costly state verification' friction[3]. In these models, the emphasis of the financial friction is on nonfinancial borrowers. This is an important reason why we chose to work with a costly enforcement model since it facilitates the modelling of a financial intermediary that relies on too much debt and too little equity. Both approaches to modelling financial frictions, costly state verification and costly enforcement, essentially introduce a risk premium associated with external financing, which may give rise to an inefficiently low level of capital investment. This low level of investment maps into low productivity growth in endogenous growth models of vertical innovation as well as in models of horizontal innovation.

The framework by GKQ relies on a second-order perturbation instead of a global solution method. Moreover, in a transparent and tractable manner, GKQ capture a lack of self-insurance of financial intermediaries in the context of an endogenous liability choice between debt and equity finance. GKQ thus combine *feasibility*[4] and *transparency*[5].

A key element of the GKQ framework is the assumption that increased reliance on outside equity finance increases the absconding rate and therefore tightens the borrowing capacity. This assumption, based on the seminal work by Calomiris and Kahn (1991), is necessary in order to introduce a limit on the desirability of equity finance. Calomiris and Kahn (1991) show that demandable debt (deposits) has an important advantage as part of an incentive scheme for disciplining the financial intermediary. As the share of debt (deposits) declines relative to equity, the balance sheet monitoring capability of the remaining depositors declines. The moral hazard problem between creditors and debtors is thus aggravated, which is captured by specifying the absconding rate as an increasing (decreasing) function of equity (debt). An alternative approach to imposing a limit on the desirability of equity finance would be to introduce an equity adjustment cost[6].

The absconding rate as an increasing function of equity also gives rise to the possibility that macro-prudential policy can be too tight by incentivising too much equity finance. Alternative approaches to capturing the cost of too much macro-prudential regulation are developed in Gertler et al. (2020b), Ma (2020) and Begenau and Landvoigt (2022). In Gertler et al. (2020b), the macro-prudential regulator may 'stifle good booms'. Belief-driven booms may or may not result in a bust depending on whether the anticipated increase in asset returns materialises and whether the build-up in leverage and vulnerability is followed by a panic-inducing sunspot shock. A macro-prudential regulator that leans very strongly against the expansion in credit will reduce welfare in the stochastic steady state. The benefit of a reduced crisis frequency does not outweigh the cost of stifling many good booms. Our key result of large welfare gains from cycle *and* trend stabilisation would also hold if one were to enrich the framework by Gertler et al. (2020b) with an endogenous growth mechanism[7].

In a similar vein, the paper by Ma (2020) is very closely related to ours since it studies the effects of macro-prudential policy under endogenous growth. In line with our key result, Ma (2020) also finds that changes in the growth rate have very substantial welfare implications. However, in contrast to our paper, macro-prudential policy in his small open economy model is implemented as a tax on capital inflows. In Ma (2020) macro-prudential policy leads to a *quantity* restriction on credit; it reduces financial intermediation and lowers the available amount

[2]The 'costly enforcement' or 'limited enforcement' type of financial friction was first introduced by Hart and Moore (1994), Kiyotaki and Moore (1997) and later incorporated into macro models of financial intermediation by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011).

[3]The 'costly state verification' approach was pioneered by Townsend (1979) and later incorporated into macro models by Carlstrom and Fuerst (1997) and Bernanke et al. (1999) (BGG).

[4]A generally desirable feature of a model of macro-prudential policy is a non-linear crisis event that the policymaker aims to prevent or mitigate. In the literature, this is often achieved via incorporating occasionally binding constraints (often in the context of a sudden stop in a small open economy model as in Mendoza (2010), Bianchi (2011), Fornaro (2015), Ma (2020)) or multiple equilibria (often in the context of bank runs as in Gertler et al. (2020a), Gertler et al. (2020b), Begenau and Landvoigt (2022)). However, it is hardly feasible to expand the state space of models that require global solution techniques and incorporate capital, investment, and R&D.

[5]Models that feature macro-prudential policy but that lack an explicit externality to which the policymaker responds are unable to credibly capture the cost and benefits of macro-prudential policy. While models in this category, such as Angelini et al. (2014), can be estimated and fitted to the data, it is precisely the first-order perturbation-based solution method that inhibits the incorporation of risk and self-insurance motives.

[6]An example of such a convex equity injection cost can be found in Gertler et al. (2020a). The more the equity injection deviates from a certain reference point, the more costly it becomes for households to inject equity into banks.

[7]Given the complexities of global numerical solution methods, it is difficult to enhance the state space of this model. Gertler et al. (2020b) develop an endowment economy model that abstracts from capital and investment.

of resources that can be dedicated to enhancing productivity and growth. In our model, macro-prudential policy can be thought of as a collection of rules and regulations affecting the *quality* of the liability composition of financial intermediaries and hence the safety of the financial system. In our model, optimal macro-prudential policy leads to more, not less, intermediation in the steady state.

Another approach to motivate a cost from too much macro-prudential policy is the potential 'leakage' of intermediation from the safer and more regulated commercial banking system to riskier and less-well-regulated shadow banks, as highlighted by Begenau and Landvoigt (2022).

All three alternative models of macro-prudential policy discussed here feature a competitive equilibrium with too much risk (a crisis frequency that is inefficiently high). Macro-prudential policy can reduce risk and increase welfare, but it can become excessively tight and reduce welfare. Even though our model is not globally solved, we capture this policy trade-off in the context of the GKQ framework and the risk-adjusted steady state around which we solve the model. Our key result of cycle *and* trend stabilisation should be replicable in any model in which the regulated (stochastic steady state) equilibrium with macro-prudential policy is associated with an increase of available funding and resource allocation to productivity-enhancing investment, relative to the competitive (stochastic steady state) equilibrium.

**Modelling Endogenous Growth** The second key feature of our model is the endogenous growth mechanism. As explained in Section 3.2, and in line with Kung (2015) and Bianchi et al. (2019), we consider a model of vertical innovation, in which aggregate productivity is a function of the stock of (and, hence, investment in) R&D and its utilisation chosen by the intermediate non-financial firm. This approach allows us to capture the hysteresis effect following a fall in demand due to a financial shock in a relatively stylised way. Alternatively, we could have followed the approach of Comin and Gertler (2006), and Anzoategui et al. (2019), consisting of an expanding variety model of technological change, modified to include an endogenous pace of technology adoption. This approach has the advantage of modelling R&D and adoption more explicitly. However, it comes at the cost of adding significant complexity without substantially affecting the main result that *adverse demand disturbances lead to a decline in R&D*, its utilisation (or adoption), and a permanent fall in the level of macroeconomic activity.

# 4 The Risk-adjusted Steady State

In this paper, we evaluate the dynamics of the linearised model around what Coeurdacier et al. (2011) refer to as the risk-adjusted steady state. Loosely speaking, the risk-adjusted steady state is the state of the economy where agents choose to stay when they expect future risk and if the realization of shocks at this period is 0. Opposite to the deterministic steady state where agents anticipate no future shocks, the risk-adjusted steady state incorporates information relative to the stochastic nature of the economy. Such information can be crucial to characterise banks' optimal portfolio choice or household welfare.[8]

## 4.1 Intuition

The reason for solving the model around the risk-adjusted BGP rather than using the standard deterministic BGP is to account for the stabilising properties of banks' equity. When solved around the deterministic BGP, the model implies that banks have no advantage to fund themselves with equity relative to using debt. This is because shocks are expected to be zero in the future. As banks anticipate only one possible future state for the economy, banks prefer to use cheap debt rather than go for expensive state-contingent equity. In contrast, allowing for future risk in the computation of the BGP enables us to express the level of risk as a function of banks' liabilities. The fact that banks do not fully internalise the benefits of funding with equity, as they take asset prices as given, motivates macro-prudential policy in our model. A better-capitalised banking system with a higher intermediation capacity is able to channel more funds from lenders to borrowers and thereby facilitates higher levels of investment in physical capital and R&D. Thus, solving around a risk-adjusted BGP will allow us to capture the benefits of macro-prudential policy by accounting for the reduced volatility.

To further illustrate the implications of the risk-adjusted BGP, recall the household's no-arbitrage relationship between debt and equity (3.1)

$$0 \quad = \quad \mathbf{E}_t \left[ \Lambda_{t,t+1} \Pi_{t+1}^{-1} \left( R_t^D - R_{t+1}^E \right) \right]$$

and the financial intermediaries' optimality condition for choosing debt versus equity finance (3.13)

$$\mu_t^E \quad = \quad E_t \left[ \Lambda_{t,t+1} \Omega_{t+1} \Pi_{t+1}^{-1} \left( R_t^D - R_{t+1}^E \right) \right].$$

In a conventional deterministic BGP equilibrium, these two equations would imply that $\mu^E_{bgp,d} = 0$, so that the excess value of using equity is zero. In the risk-adjusted BGP equilibrium, we would instead have

$$\begin{aligned} 0 \quad &= \quad \left[ \Lambda_{bgp,r} \Pi_{bgp,r}^{-1} \left( R_{bgp,r}^D - R_{bgp,r}^E \right) \right] + M_1 \\ \mu_{bgp,r}^E \quad &= \quad \left[ \Lambda_{bgp,r} \Omega_{bgp,r} \Pi_{bgp,r}^{-1} \left( R_{bgp,r}^D - R_{bgp,r}^E \right) \right] + M_2. \end{aligned}$$

The intermediary's excess value of using equity is then given by $\mu^E_{bgp,r} = -\Omega_{bgp,r} M_1 + M_2$, which is positive as long as $\Omega_{bgp,r} M_1 < M_2$. The risk adjustments $M_1$ and $M_2$ are functions of the covariances between the household's and the financial intermediary's stochastic discount factors and the return on equity. Note that the household discounts the spread between deposits and equity at $\Lambda_{t,t+1}$ while the intermediary discounts at $\Lambda_{t,t+1}\Omega_{t+1}$. Under our calibration, the financial intermediary's augmented SDF is more volatile, so that the intermediary is more risk-averse. $M_2 > \Omega_{bgp,r} M_1$ and $\mu^E_{bgp,r} > 0$ implies that the intermediary receives hedging value by substituting debt with equity. The substitution towards equity is constrained by the increased divertibility ($(\Theta'(X)_{bgp,r} > 0)$), as discussed above. In the unregulated equilibrium, the use of outside equity is inefficiently low since the atomistic intermediary takes asset prices as given.

## 4.2 Description of the Methodology

We provide a brief description of the method used to compute the risk-adjusted steady state. We follow de Groot (2013) who uses an iterative method relying on the second-order approximation of the decision rules to compute the risk-adjusted steady state.[9]

[8] See for instance Devereux and Sutherland (2011) for an example of risk-adjusted steady state applied to solve a portfolio choice problem.

[9] The method is also described in Juillard (2011).

Consider the equilibrium conditions describing the behavior of our model,

$$E_t[f(y_{t+1},y_t,x_{t+1},x_t,z_{t+1},z_t)] = 0, \quad z_{t+1} = \Lambda z_t + \eta\sigma\varepsilon_{t+1}, \tag{4.1}$$

where $y_t$ is an $n_y \times 1$ vector of non-predetermined variables, $x_t$ is an $n_x \times 1$ vector of predetermined variables, $z_t$ is an $n_z \times 1$ vector of exogenous variables, and $\varepsilon_{t+1}$ is an $n_z \times 1$ vector of i.i.d exogenous disturbances. $\Lambda$ and $\eta$ are parameters matrices of size $n_z \times n_z$ and $\sigma$ is the stochastic scale of the model so that if $\sigma = 0$ the model is deterministic.

We define functions $h$ and $g$ as the decision rules that solve the equilibrium conditions defined in (4.1) with $y_t = g(x_t,z_t,\sigma)$ and $x_{t+1} = h(x_t,z_t,\sigma)$. We can now formally define the risk-adjusted steady state as the vector $x^r$ that solves,

$$x^r = h\left(x^r,0,\sigma\right).$$

Computing a Taylor approximation of the decision rules $h$ around the deterministic steady state $x^d$ yields:

$$x^i_{t+1} = x^{d,i} + h^{d,i}_x\left(x_t - x^d\right) + \frac{1}{2}\left(x_t - x^d\right)' h^{d,i}_{xx}\left(x_t - x_d\right) + \frac{1}{2}z_t' h^{d,i}_{zz} z_t + \left(x_t - x^d\right)' h^{d,i}_{xz} z_t + \frac{1}{2}h^{d,i}_{\sigma\sigma}\sigma^2, \tag{4.2}$$

for $i = 1,...,n_x$, where the vector $h^{d,i}_x$ and the matrices $h^{d,i}_{xx}$, $h^{d,i}_{xz}z_t$ and $h^{d,i}_{\sigma\sigma}$ correspond respectively to the jacobian and the hessians of the decision rules evaluated at the deterministic steady state. Because at the risk-adjusted steady state, all shocks are zero, it is possible to write (4.2) as:

$$x^{r,i} = x^{d,i} + h^{d,i}_x\left(x^r - x^d\right) + \frac{1}{2}\left(x^r - x^d\right)' h^{d,i}_{xx}\left(x^r - x^d\right) + \frac{1}{2}h^{d,i}_{\sigma\sigma}\sigma^2. \tag{4.3}$$

Stacking each of the decision rules in (4.3) and defining $x^* \equiv x^r - x^d$, we obtain the following quadratic equation,

$$C + Bx^* + Avec\left(x^* x^{*\prime}\right) = 0, \tag{4.4}$$

where,

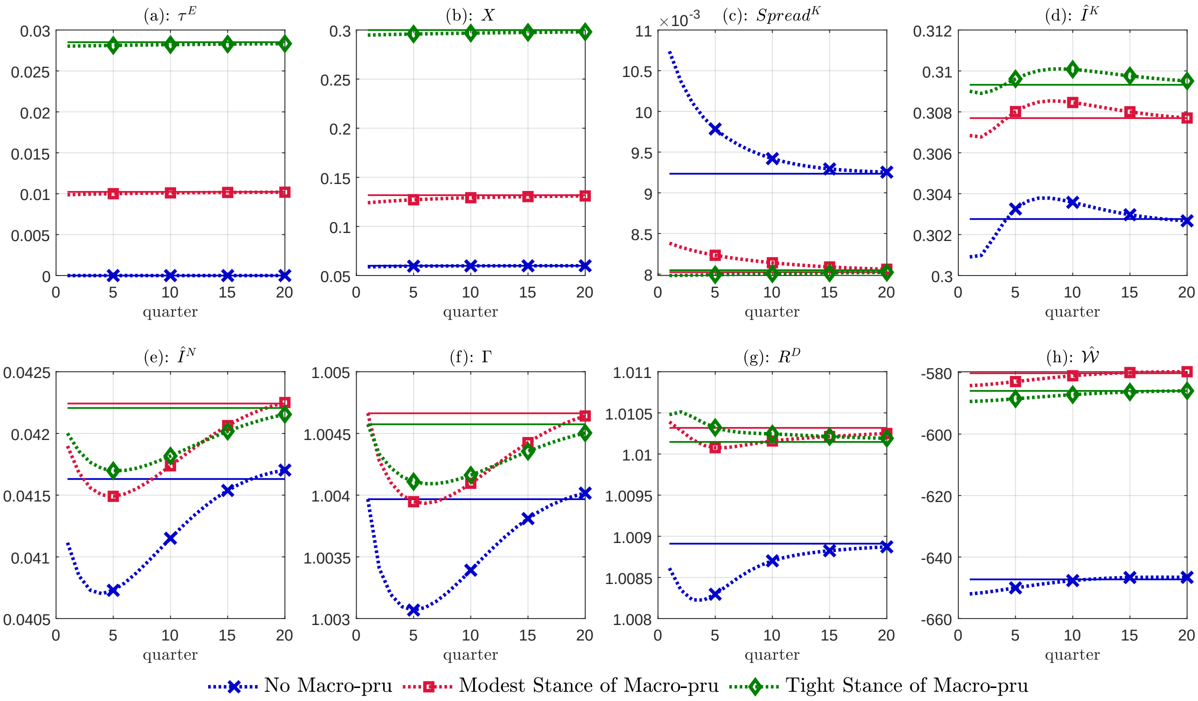
$$C \equiv h^d_{\sigma\sigma}\frac{\sigma^2}{2}, \quad B \equiv h^d_x - I_{n_x}, \quad A \equiv \frac{1}{2}\begin{bmatrix} vec(h^{d,1}_{xx})' \\ \vdots \\ vec(h^{d,n_x}_{xx})' \end{bmatrix}.$$

Finally, we obtain the risk-adjusted steady state using a nonlinear solver to resolve (4.4).

# 5 Dual Effects of Macro-prudential Policy

In this section, we illustrate that macro-prudential policy dampens the fluctuations induced by financial shocks, as more equity shields the net worth of financial intermediaries from shocks.

FIGURE 5.5: IMPULSE RESPONSES WITHOUT AND WITH MACRO-PRUDENTIAL POLICY




Note: The dotted blue line depicts the impulse responses in levels of several variables in the absence of macro-prudential policy ($\kappa_\nu = 0$). The straight blue line depicts the level of the risk-adjusted steady state. The dotted red line depicts the case with a modest and the dotted green line with a very tight macro-prudential policy stance. Under the modest (tight) stance the risk-adjusted steady state / BGP capital ratio is around 13% (30%). The straight red and green lines depict the corresponding risk-adjusted steady state / BGP levels.

In Panel (a) of Figure 5.5 we show that increasing the responsiveness parameter $\kappa_\nu$ in the reaction function of macro-prudential policy leads to a higher risk-adjusted steady-state level of the subsidy on outside equity issuance $\tau^E$. This corresponds to a higher risk-adjusted steady-state level of the equity-to-asset ratio $X$. This in turn, lowers the spread, as can be seen in Panel (c). However, as we discussed above, at some point the adverse effect of a higher equity-to-asset ratio on the absconding rate dominates, the intermediary's borrowing constraint actually starts to tighten again, and we find that for a macro-prudential policy stance that is very tight, the level of the spread actually increases.

A low steady-state level of the spread corresponds to a high amount of intermediated funds and hence high steady-state investment. This in turn corresponds to a higher growth rate $\Gamma$. A high gross growth rate $\Gamma$ increases the equilibrium interest rate (which reduces the probability of hitting the ZLB), and it enters the household lifetime utility and hence increases overall welfare. We will conduct a welfare analysis in the next section.

In Panel (c), (d) and (e) of Figure 5.5 we can also see that the introduction of macro-prudential policy reduced the volatility of variables in the model. While the spread increases and investment falls substantially in the absence of macro-prudential policy (blue line), the increase in the spread and the fall in investment is mitigated in the presence of macro-prudential policy. A very tight macro-prudential policy stance will succeed in reducing the volatility induced by shocks. As can be seen in Panel (e), the fall in R&D investment is mitigated best under the very tight regime (green line). The deviation from the steady state is smaller for the green line compared to the red line. However, the green line corresponds to a very tight regime which reduces the overall intermediation capacity of the financial system and is therefore associated with a lower gross growth rate $\Gamma$. As we will show below, it is not optimal to completely eliminate volatility if the consequence is less intermediation in the risk-adjusted steady state.

# 6 Optimal Macro-prudential Policy and Risk-Adjusted BGP Values

In Table 3, we report the balanced-growth path values of key variables, under endogenous and exogenous growth, without macro-prudential policy and with the optimal simple macro-prudential policy rule. In the baseline model, the outside equity ratio $X^*_{bgp,r}$ associated with the optimal value for $\kappa_\nu$ is around 19.9%. In the exogenous growth model, instead, the optimal outside equity ratio is around 15.7%. In other words, based on this numerical lifetime optimisation approach, a stronger macro-prudential policy stance is warranted once one allows for endogenous productivity growth. Intuitively, the fact that the growth rate of productivity and the BGP are subject to shocks, which are amplified via the balance sheet constraints of financial intermediaries, justifies a stronger macro-prudential response. The optimal macro-prudential policy regime in the endogenous growth model is associated with a consumption-equivalent welfare gain of roughly 8.1% compared to the unregulated regime. This value is one order of magnitude larger than in the exogenous growth model, which is only around 0.7%. The welfare gains are also significantly larger than previously found in the literature. For example, GKQ find welfare improvements of around 0.29%.

TABLE 3: RISK-ADJUSTED BGP VALUES WITHOUT AND WITH MACRO-PRUDENTIAL POLICY

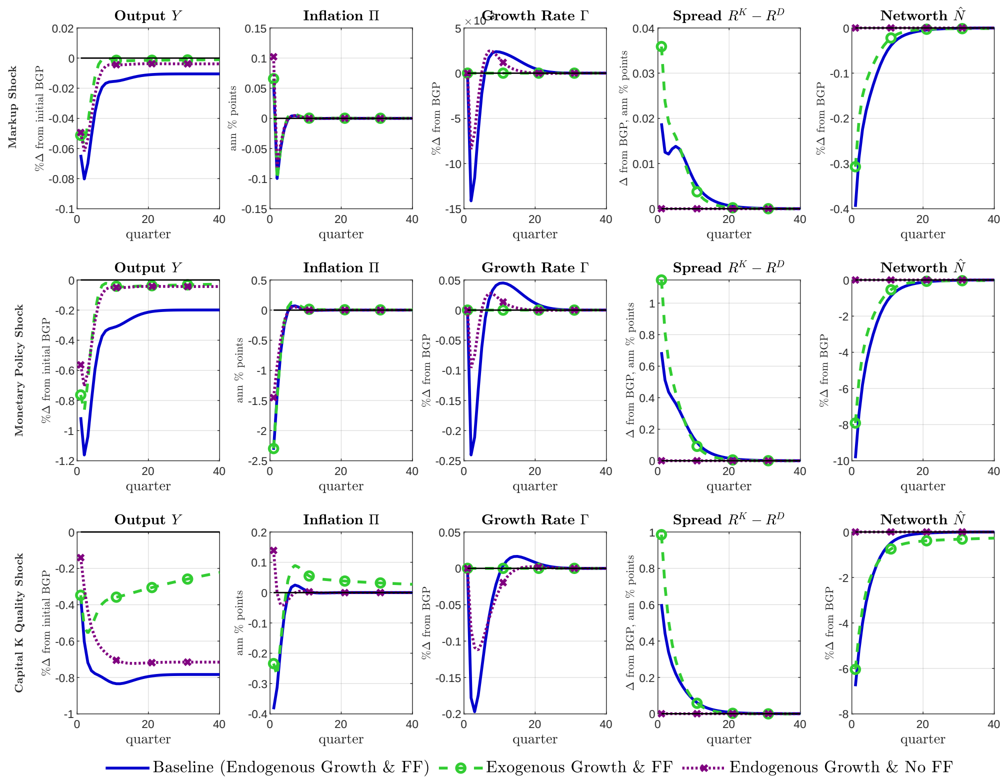
| | | **Endogenous Growth Model** | | **Exogenous Growth Model** | |
|---|---|---|---|---|---|
| | | No MacroPru | With MacroPru | No MacroPru | With MacroPru |
| **Variables** | | | | | |
| $\hat{Y}$ | Output | 1.4605 | 1.4493 | 1.4686 | 1.5003 |
| $\hat{C}$ | Consumption | 1.1161 | 1.0972 | 1.1608 | 1.1793 |
| $L$ | Labor | 0.5031 | 0.5014 | 0.5057 | 0.5111 |
| $\hat{K}$ | K Capital | 12.6539 | 12.4785 | 12.7294 | 13.3148 |
| $\hat{I}^K$ | K Investment | 0.3028 | 0.3096 | 0.3078 | 0.3210 |
| $\hat{I}^N$ | RnD Investment | 0.0416 | 0.0424 | 0.0000 | 0.0000 |
| $\Gamma$ | Growth Rate | 1.0040 | 1.0048 | 1.0040 | 1.0040 |
| $Spread^K$ | SpreadK | 0.0092 | 0.0077 | 0.0089 | 0.0081 |
| $Spread^N$ | SpreadRnD | 0.0142 | 0.0120 | 0.0000 | 0.0000 |
| $\widehat{NW}$ | Net Worth | 2.0479 | 1.7378 | 2.4563 | 2.3715 |
| $X$ | Outside Equity | 0.0600 | 0.1987 | 0.0600 | 0.1570 |
| $\phi$ | Leverage | 4.2104 | 4.9069 | 5.2003 | 5.6360 |
| $\Theta$ | Absconding rate | 0.7255 | 0.8259 | 0.3910 | 0.4141 |
| $\nu$ | Cost Deposits | 2.9146 | 3.8421 | 1.9618 | 2.2416 |
| $\mu^E$ | Exc Val Equity | 0.0017 | 0.0025 | 0.0014 | 0.0010 |
| $R^E$ | Return Equity | 1.0090 | 1.0109 | 1.0091 | 1.0091 |
| $R^K$ | Return K Capital | 1.0181 | 1.0184 | 1.0179 | 1.0171 |
| $R^D$ | Deposit Rate | 1.0089 | 1.0107 | 1.0090 | 1.0090 |
| $\tau^E$ | MPP Subsidy | 0.0000 | 0.0184 | 0.0000 | 0.0071 |
| $\hat{U}$ | Period Utility | -3.2023 | -3.2839 | -3.0010 | -2.9661 |
| $\hat{W}$ | HH Welfare | -647.2517 | -565.7528 | -602.6059 | -595.5995 |
| C-equivalent Welfare Gain in % | | | 8.06 | | 0.68 |

# 7 Robustness Checks

## 7.1 Higher Degree of Knowledge Spillover: $\eta = 0.2$

In our baseline parametrisation discussed above, the degree of knowledge spillovers was $\eta = 0.062$. In this robustness check, we repeat the analysis above for $\eta = 0.2$, a roughly threefold increase. Unsurprisingly, increasing $\eta$ will make the endogenous growth channel relatively more important. In Figure 7.6 we show the IRFs for the three shocks under the calibration with $\eta = 0.2$. In the lower-left panel, we can see that the capital quality shock will give rise to a deviation of output from the initial BGP of around -0.8 % after 40 quarters. Under the standard calibration, the loss at that horizon would have been around -0.75 %.

FIGURE 7.6: IRFS TO MARKUP, MONETARY POLICY AND CAPITAL QUALITY SHOCK FOR $\eta = 0.2$

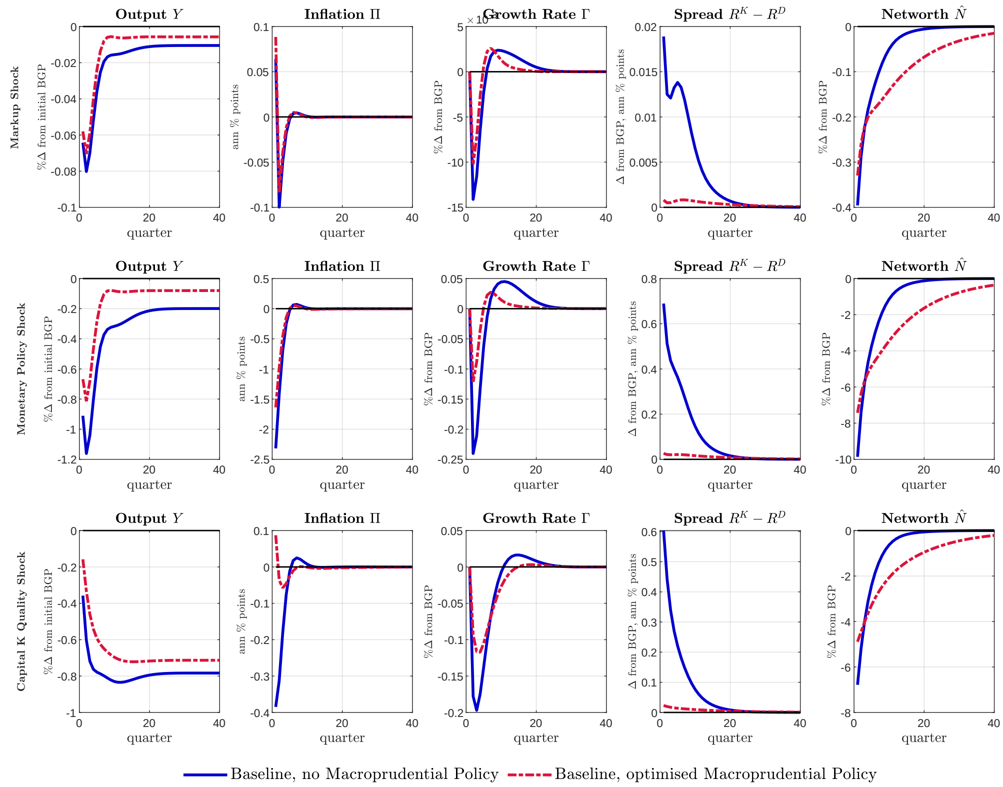



Note: The upper row depicts the IRFs for a (+1stdev) markup shock, the middle row depicts the IRFs for a (+1stdev) monetary policy shock and the lower row for a (-1stdev) capital $K$ quality shock. The blue-straight line depicts the responses of the baseline model, with endogenous growth and financial frictions, the green-circled line depicts the responses for an exogenous growth model with financial frictions and the purple-crossed line depicts the responses for an endogenous growth model without financial frictions.

Under the cross-check with $\eta = 0.2$, the relative importance of financial frictions decreases, the relative importance of the endogenous growth channel increases. The distance between the blue-straight line and the purple-dashed line for output is smaller, as can be seen in the lower left panel in Figure 7.6. Macro-prudential policy under endogenous growth is still much more welfare-enhancing than under exogenous growth. But due to the lower relatively lower weight of the financial frictions, the welfare gain in the case of $\eta = 0.2$ is smaller, 5.81, than in the case of $\eta = 0.062$, 8.06.

The results in our model regarding the importance of endogenous growth in matching the data and our policy analysis for macro-prudential policy are thus robust to changes in the degree of the knowledge spillover $\eta$.

FIGURE 7.7: IRFS TO MARKUP, MONETARY POLICY AND CAPITAL QUALITY SHOCK FOR $\eta = 0.2$ WITH MACRO-PRU




Note: The upper row depicts the IRFs for a (+1stdev) markup shock, the middle row depicts the IRFs for a (+1stdev) monetary policy shock and the lower row for a (-1stdev) capital $K$ quality shock. The blue-straight line depicts the responses of the baseline model, with endogenous growth and financial frictions, in the absence of macro-prudential policy. The red dashed line depicts the responses for the baseline model with the optimal simple macro-prudential policy rule.

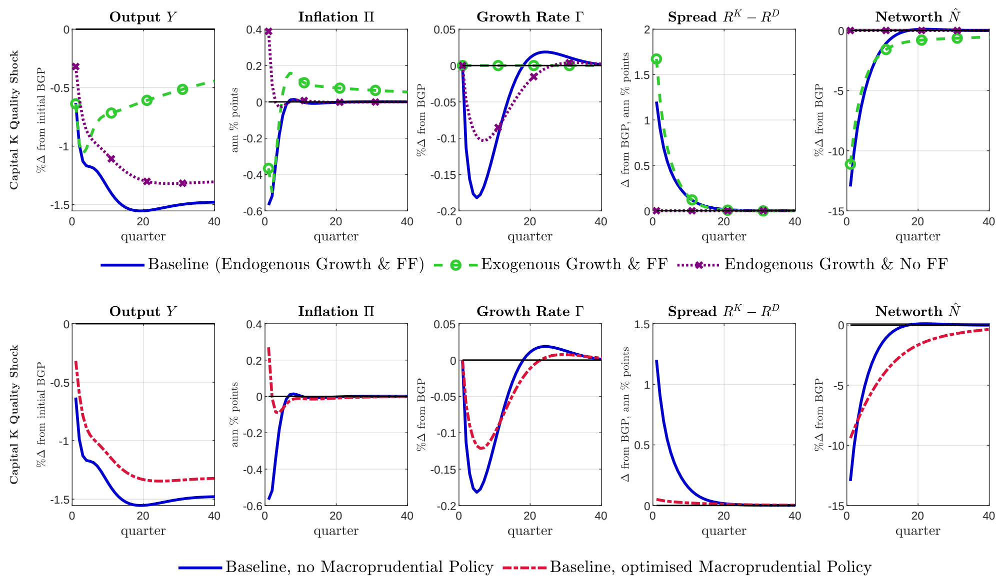
## 7.2 More Risk due to Higher Volatility from Financial Shocks: $\varsigma^K = 0.016$

We double the volatility of financial shocks, $\varsigma^K = 0.016$, and set $\varsigma^{\mathcal{M}} = \varsigma^{MP} = 0$. More risk due to high volatility from financial shocks implies that intermediaries choose a higher equity-to-asset ratio: $X_{bgp,r}$ is higher than in the baseline parametrisation. Moreover, the welfare gains from macro-prudential policy are larger.

TABLE 4: RISK-ADJUSTED BGP VALUES WITHOUT AND WITH MACRO-PRUDENTIAL POLICY

| | | **Endogenous Growth Model, $\eta = 0.062$** | | **Endogenous Growth Model, $\eta = 0.2$** | |
|---|---|---|---|---|---|
| | | No MacroPru | With MacroPru | No MacroPru | With MacroPru |
| **Variables** | | | | | |
| $\hat{Y}$ | Output | 1.4605 | 1.4493 | 1.4488 | 1.4445 |
| $\hat{C}$ | Consumption | 1.1161 | 1.0972 | 0.9565 | 0.9439 |
| $L$ | Labor | 0.5031 | 0.5014 | 0.5011 | 0.5006 |
| $\hat{I}^K$ | K Investment | 0.3028 | 0.3096 | 0.3004 | 0.3081 |
| $\hat{I}^N$ | RnD Investment | 0.0416 | 0.0424 | 0.1919 | 0.1925 |
| $\Gamma$ | Growth Rate | 1.0040 | 1.0048 | 1.0041 | 1.0048 |
| $Spread^K$ | SpreadK | 0.0092 | 0.0077 | 0.0092 | 0.0077 |
| $Spread^N$ | SpreadRnD | 0.0142 | 0.0120 | 0.0144 | 0.0123 |
| $\widehat{NW}$ | Net Worth | 2.0479 | 1.7378 | 2.0225 | 1.7454 |
| $X$ | Outside Equity | 0.0600 | 0.1987 | 0.0589 | 0.1976 |
| $\phi$ | Leverage | 4.2104 | 4.9069 | 4.2126 | 4.8613 |
| $\Theta$ | Absconding rate | 0.7255 | 0.8259 | 0.7255 | 0.8243 |
| $\nu$ | Cost Deposits | 2.9146 | 3.8421 | 2.9157 | 3.7998 |
| $\mu^E$ | Exc Val Equity | 0.0017 | 0.0025 | 0.0012 | 0.0023 |
| $R^E$ | Return Equity | 1.0090 | 1.0109 | 1.0092 | 1.0109 |
| $R^K$ | Return K Capital | 1.0181 | 1.0184 | 1.0183 | 1.0184 |
| $R^D$ | Deposit Rate | 1.0089 | 1.0107 | 1.0092 | 1.0107 |
| $\tau^E$ | MPP Subsidy | 0.0000 | 0.0184 | 0.0000 | 0.0184 |
| $\hat{W}$ | HH Welfare | -647.2517 | -565.7528 | -840.8311 | -751.7634 |
| C-equivalent Welfare Gain in % | | | 8.06 | | 5.81 |

FIGURE 7.8: IRFS TO A CAPITAL QUALITY SHOCK FOR $\varsigma^K = 0.016$ WITH MACRO-PRU




Note: The upper row depicts IRFs for a (-1stdev) capital $K$ quality shock. The blue-straight line depicts the responses of the baseline model, with endogenous growth and financial frictions (FF), the green-circled line depicts the responses for an exogenous growth model with FF and the purple-crossed line depicts the responses for an endogenous growth model without FF. The lower row depicts the IRFs for a (-1stdev) capital $K$ quality shock. The blue-straight line depicts the responses of the baseline model, with endogenous growth and financial frictions, in the absence of macro-prudential policy. The red dashed line depicts the responses for the baseline model with the optimal simple macro-prudential policy rule.

TABLE 5: RISK-ADJUSTED BGP VALUES WITHOUT AND WITH MACRO-PRUDENTIAL POLICY

| Variables | | **Endogenous Growth Model, $\varsigma^K = 0.008$** | | **Endogenous Growth Model, $\varsigma^K = 0.016$** | |
|---|---|---|---|---|---|
| | | No MacroPru | With MacroPru | No MacroPru | With MacroPru |
| $\hat{Y}$ | Output | 1.4605 | 1.4493 | 1.4396 | 1.4343 |
| $\hat{C}$ | Consumption | 1.1161 | 1.0972 | 1.1036 | 1.0881 |
| $L$ | Labor | 0.5031 | 0.5014 | 0.4995 | 0.4987 |
| $\hat{I}^K$ | K Investment | 0.3028 | 0.3096 | 0.2946 | 0.3040 |
| $\hat{I}^N$ | RnD Investment | 0.0416 | 0.0424 | 0.0414 | 0.0423 |
| $\Gamma$ | Growth Rate | 1.0040 | 1.0048 | 1.0039 | 1.0048 |
| $Spread^K$ | SpreadK | 0.0092 | 0.0077 | 0.0098 | 0.0080 |
| $Spread^N$ | SpreadRnD | 0.0142 | 0.0120 | 0.0136 | 0.0115 |
| $\widehat{NW}$ | Net Worth | 2.0479 | 1.7378 | 2.0893 | 1.7527 |
| $X$ | Outside Equity | 0.0600 | 0.1987 | 0.0616 | 0.1945 |
| $\phi$ | Leverage | 4.2104 | 4.9069 | 4.0388 | 4.7911 |
| $\Theta$ | Absconding rate | 0.7255 | 0.8259 | 0.7256 | 0.8201 |
| $\nu$ | Cost Deposits | 2.9146 | 3.8421 | 2.8021 | 3.7294 |
| $\mu^E$ | Exc Val Equity | 0.0017 | 0.0025 | 0.0023 | 0.0028 |
| $R^E$ | Return Equity | 1.0090 | 1.0109 | 1.0089 | 1.0110 |
| $R^K$ | Return K Capital | 1.0181 | 1.0184 | 1.0185 | 1.0186 |
| $R^D$ | Deposit Rate | 1.0089 | 1.0107 | 1.0087 | 1.0106 |
| $\tau^E$ | MPP Subsidy | 0.0000 | 0.0184 | 0.0000 | 0.0179 |
| $\hat{\mathcal{W}}$ | HH Welfare | -647.2517 | -565.7528 | -665.7514 | -573.8060 |
| C-equivalent Welfare Gain in % | | | 8.06 | | 8.99 |

# 8 Alternative Macro-prudential Rules

Our key result of large welfare gains from macro-prudential policy in a framework with endogenous growth does not hinge on the exact specification of the macro-prudential policy rule. We show that our results hold also under alternative macro-prudential rules.

## 8.1 Rule that responds to Equity-to-Asset Ratio $X_t$

Recall the balance sheet identity of a bank

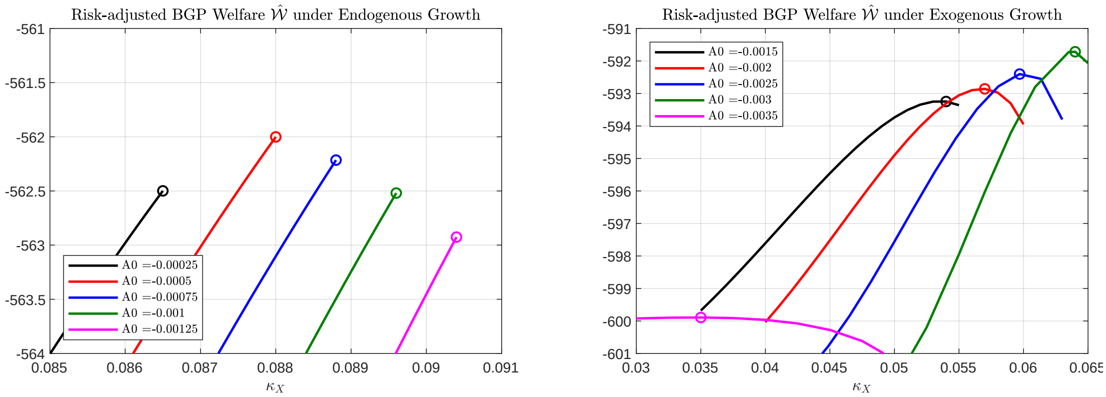
$$RE_{j,t} + \left(1 + \tau_t^N\right) Q_t^N B_{j,t}^N + \left(1 + \tau_t^K\right) Q_t^K B_{j,t}^K = NW_{j,t} + \left(1 + \tau_t^E\right) Q_t^E E_{j,t} + D_{j,t},$$

which states that all assets (LHS) are financed either via retained earnings (inside equity), non-state-contingent debt, or via outside equity. Note also that the macro-prudential regulator subsidises the issuance of outside equity via a tax on assets. In the benchmark model, we used the same rule as in Gertler et al. (2012) $\tau_t^E = \kappa_\nu \nu_t^{-1}$. This GKQ-rule implies that the macro-prudential subsidy on outside equity, $\tau_t^E$ responds inversely to the shadow costs of deposits, $\nu_t$. If $\nu_t$ is low, the incentive for the financial intermediary to rely on debt is high. In this situation, the macro-prudential policymaker would subsidize outside equity stronger. In this regard, the GKQ rule is counter-cyclical. We now provide a cross-check to the welfare analysis above, by studying the implications of a macro-prudential rule that responds to the aggregate outside equity-to-asset ratio $X_t$, as proposed by Liu (2016)

$$\tau_t^E \quad = \quad A_0 + \kappa_X X_t. \tag{8.1.1}$$

As described in Liu (2016), this rule implies that the macro-prudential policymaker will respond directly to the outside equity-to-assets ratio $X_t$ which is arguably easier to observe. The sensitivity with which the policymaker responds to $X_t$ is $\kappa_X > 0$. Following Liu (2016) we allow for a constant term in the rule, $A_0 < 0$. The Liu-rule implies a stronger response of the equity subsidy $\tau_t^E$ to movements in the equity ratio $X_t$. If an adverse shock hits the bank and the equity-to-asset ratio falls, the policymaker will reduce the subsidy more than he would have reduced it under the GKQ rule. This policy rule will therefore allow the Bank to rely more on deposits in the aftermath of an adverse shock. It will thus allow banks to rebuild net worth (and hence their intermediation capacity) faster than under the GKQ rule. In this sense, the Liu rule is more responsive and countercyclical than the GKQ rule since it reduces the incentive towards relying on equity finance more after an adverse shock.

FIGURE 8.9: WELFARE ANALYSIS UNDER LIU (2016)-RULE




Note: The panel on the left (right) depicts the risk-adjusted BGP welfare $\hat{\mathcal{W}}$ under endogenous (exogenous) growth for various combinations of the policy parameters $\{A_0, \kappa_X\}$. Under endogenous growth, the model's risk-adjusted steady state cannot be solved for large values of $\kappa_X$ due to Blanchard-Kahn instability issues. Under exogenous growth, the model's risk-adjusted steady state cannot be solved for values of $\kappa_X > 0.06$ due to Blanchard-Kahn instability issues.

In Figure 8.9 we conduct a welfare optimisation to find the values of $\{\kappa_X^*, A_0^*\}$ that maximise welfare in the economy in the case of endogenous and exogenous growth. Under endogenous growth, the model's risk-adjusted steady state cannot be solved for large values of $\kappa_X$ due to Blanchard-Kahn instability issues. As in the paper by Liu (2016) the optimum is found on the boundary of the set of parameter values that guarantee the Blanchard–Kahn condition. In the endogenous growth model we therefore pick $\{\kappa_X^*, A_0^*\} = \{0.088, -0.0005\}$. In the exogenous growth case, we pick $\{\kappa_X^*, A_0^*\} = \{0.064, -0.003\}$.

TABLE 6: RISK-ADJUSTED BGP VALUES WITHOUT AND WITH MACRO-PRU FOLLOWING LIU (2016)-RULE

| Variables | | Endogenous Growth Model | | Exogenous Growth Model | |
|---|---|---|---|---|---|
| | | No MacroPru | With MacroPru | No MacroPru | With MacroPru |
| $\hat{Y}$ | Output | 1.4605 | 1.4473 | 1.4686 | 1.5186 |
| $\hat{C}$ | Consumption | 1.1161 | 1.0953 | 1.1608 | 1.1899 |
| $L$ | Labor | 0.5031 | 0.5011 | 0.5057 | 0.5143 |
| $\hat{K}$ | K Capital | 12.6539 | 12.4488 | 12.7294 | 13.6615 |
| $\hat{I}^K$ | K Investment | 0.3028 | 0.3096 | 0.3078 | 0.3287 |
| $\hat{I}^N$ | RnD Investment | 0.0416 | 0.0424 | 0.0000 | 0.0000 |
| $\Gamma$ | Growth Rate | 1.0040 | 1.0049 | 1.0040 | 1.0040 |
| $Spread^K$ | SpreadK | 0.0092 | 0.0076 | 0.0089 | 0.0076 |
| $Spread^N$ | SpreadRnD | 0.0142 | 0.0118 | 0.0000 | 0.0000 |
| $\widehat{NW}$ | Net Worth | 2.0479 | 1.7228 | 2.4563 | 2.3208 |
| $X$ | Outside Equity | 0.0600 | 0.1583 | 0.0600 | 0.1384 |
| $\phi$ | Leverage | 4.2104 | 4.9394 | 5.2003 | 5.9097 |
| $\Theta$ | Absconding rate | 0.7255 | 0.7770 | 0.3910 | 0.4061 |
| $\nu$ | Cost Deposits | 2.9146 | 3.6427 | 1.9618 | 2.3027 |
| $\mu^E$ | Exc Val Equity | 0.0017 | 0.0024 | 0.0014 | 0.0006 |
| $R^E$ | Return Equity | 1.0090 | 1.0110 | 1.0091 | 1.0092 |
| $R^K$ | Return K Capital | 1.0181 | 1.0184 | 1.0179 | 1.0166 |
| $R^D$ | Deposit Rate | 1.0089 | 1.0108 | 1.0090 | 1.0090 |
| $\tau^E$ | MPP Subsidy | 0.0000 | 0.0134 | 0.0000 | 0.0059 |
| $\hat{U}$ | Period Utility | -3.2023 | -3.2910 | -3.0010 | -2.9468 |
| $\hat{W}$ | HH Welfare | -647.2517 | -562.0011 | -602.6059 | -591.7168 |
| C-equivalent Welfare Gain in % | | | 8.49 | | 1.1 |

In Table 6 we show the values of the risk-adjusted BGP without and with the macro-prudential policy a la Liu, in the case with and without endogenous growth. The consumption welfare equivalent gain of macro-prudential policy is still much higher under endogenous growth, at roughly 8.49 %, compared to the case of exogenous growth, at roughly 1.1%. As in Liu (2016), the BGP level of the equity-to-asset ratio $X$ is not as high as it was under the GKQ-rule (depicted in Table 3).

## 8.2 Constant Subsidy Rule

In this subsection, we analyse the implications of a constant subsidy rule of the following form

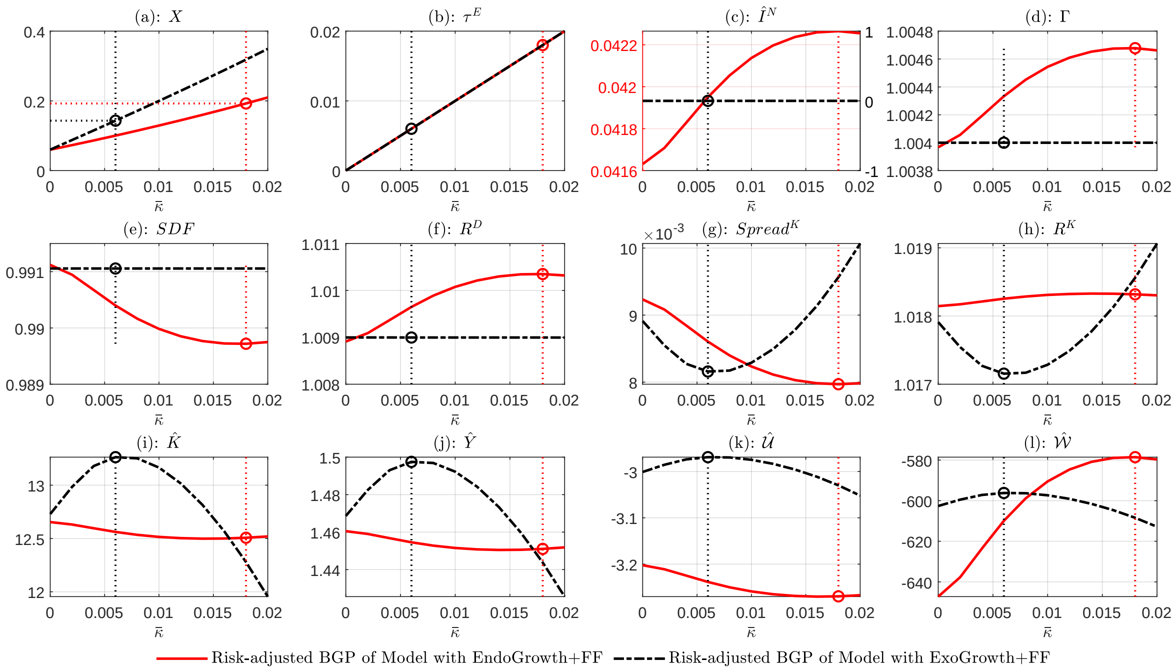
$$\tau_t^E = \bar{\kappa}. \tag{8.2.1}$$

Under the constant subsidy rule (8.2.1), the policymaker would provide a constant subsidy to outside equity issuance, regardless of the state of the economy. This contrasts with the rules considered above, which are both essentially countercyclical capital rules, which provide a stronger incentive to rely on equity whenever the private incentive to do so would be low.

In Figure 8.10 we plot the BGP value of several variables as a function of the degree of the subsidy. The red-straight line depicts the case with endogenous growth. The black-dashed line depicts the case with exogenous growth. As can be inferred from Panel (a), increasing the subsidy will linearly increase the amount of outside equity in the risk-adjusted BGP (steady state). As can be seen in Panel (c) and (d), the key difference between the endogenous and the exogenous growth cases is that the level of R&D investment and consequently the growth rate $\Gamma$ is fixed in the risk-adjusted BGP in the exogenous growth case. In the case of endogenous growth, the SDF declines as the subsidy increases, until the subsidy reaches the welfare-optimal level of around 0.16. A lower SDF implies a higher BGP deposit rate. While the optimal subsidy lowers the spread in both, the exogenous and the endogenous growth cases, the subsidy slightly increases the rate of return on capital, detrended capital is thus lower under the optimal subsidy. As described above, the detrended period utility in the endogenous growth case is lower under the optimal subsidy. But since the growth rate $\Gamma$ is higher, the overall lifetime utility, and hence welfare, is substantially higher under the endogenous growth case.

In Table 7 we show the values of the risk-adjusted BGP without and with macro-prudential policy, in the case with and without endogenous growth. The consumption welfare equivalent gain of macro-prudential policy is still much higher under endogenous growth, at roughly 6.6 %, compared to the case of exogenous growth, at roughly 0.6%. As pointed out by Liu (2016), the reason why a constant subsidy rule performs worse than a countercyclical rule is as follows. Consider that the economy is hit by an adverse shock (i.e., an adverse bank survival rate or capital quality shock). In the case of a constant subsidy on outside equity issuance, the tax on bank assets that is needed to finance the subsidy is much higher than in the case in which a time-varying (counter-cyclical) subsidy/tax scheme

FIGURE 8.10: WELFARE ANALYSIS UNDER CONSTANT-SUBSIDY-RULE




Note: The red-straight line depicts the values of variables in their risk-adjusted balanced growth equilibrium as a function of the macro-prudential sensitivity parameter $\bar{\kappa}$ in the model with endogenous growth. The black-dashed line depicts the variables in their risk-adjusted balanced growth equilibrium in the case with exogenous growth. The vertical-dotted lines indicate at which level of $\bar{\kappa}$ welfare would be optimal.

TABLE 7: RISK-ADJUSTED BGP VALUES WITHOUT AND WITH MACRO-PRU FOLLOWING CONSTANT-SUBSIDY-RULE

| | | **Endogenous Growth Model** | | **Exogenous Growth Model** | |
|---|---|---|---|---|---|
| | | No MacroPru | With MacroPru | No MacroPru | With MacroPru |
| **Variables** | | | | | |
| $\hat{Y}$ | Output | 1.4605 | 1.4510 | 1.4686 | 1.4975 |
| $\hat{C}$ | Consumption | 1.1161 | 1.1005 | 1.1608 | 1.1777 |
| L | Labor | 0.5031 | 0.5017 | 0.5057 | 0.5107 |
| $\hat{K}$ | K Capital | 12.6539 | 12.5068 | 12.7294 | 13.2629 |
| $\hat{I}^K$ | K Investment | 0.3028 | 0.3083 | 0.3078 | 0.3198 |
| $\hat{I}^N$ | RnD Investment | 0.0416 | 0.0423 | 0.0000 | 0.0000 |
| $\Gamma$ | Growth Rate | 1.0040 | 1.0047 | 1.0040 | 1.0040 |
| $Spread^K$ | SpreadK | 0.0092 | 0.0080 | 0.0089 | 0.0082 |
| $Spread^N$ | SpreadRnD | 0.0142 | 0.0123 | 0.0000 | 0.0000 |
| $\widehat{NW}$ | Net Worth | 2.0479 | 1.7935 | 2.4563 | 2.3798 |
| $X$ | Outside Equity | 0.0600 | 0.1926 | 0.0600 | 0.1431 |
| $\phi$ | Leverage | 4.2104 | 4.7630 | 5.2003 | 5.5943 |
| $\Theta$ | Absconding rate | 0.7255 | 0.8174 | 0.3910 | 0.4078 |
| $\nu$ | Cost Deposits | 2.9146 | 3.6942 | 1.9618 | 2.1928 |
| $\mu^E$ | Exc Val Equity | 0.0017 | 0.0024 | 0.0014 | 0.0011 |
| $R^E$ | Return Equity | 1.0090 | 1.0105 | 1.0091 | 1.0091 |
| $R^K$ | Return K Capital | 1.0181 | 1.0183 | 1.0179 | 1.0172 |
| $R^D$ | Deposit Rate | 1.0089 | 1.0103 | 1.0090 | 1.0090 |
| $\tau^E$ | MPP Subsidy | 0.0000 | 0.0180 | 0.0000 | 0.0060 |
| $\hat{\mathcal{U}}$ | Period Utility | -3.2023 | -3.2695 | -3.0010 | -2.9691 |
| $\hat{\mathcal{W}}$ | HH Welfare | -647.2517 | -578.5537 | -602.6059 | -596.1971 |
| C-equivalent Welfare Gain in % | | | 6.64 | | 0.62 |

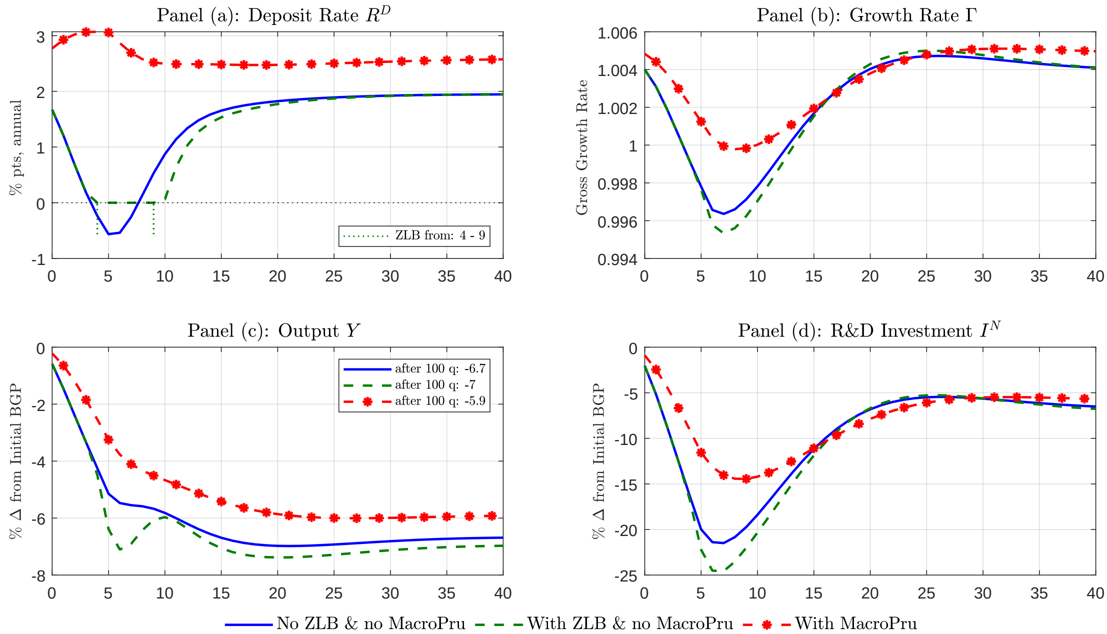
is in place. The constant subsidy rule thus impedes the recovery of bank net worth, low bank net worth impedes banks' intermediation capacity and thus impedes investment.

# 9 Macro-prudential Policy and the ZLB

In this Section, we study the effects of macro-prudential policy when the deposit rate hits the zero lower bound (ZLB). In Figure 9.11, we show the IRFs of several key variables in response to a sequence of consecutive adverse capital quality shocks of 1.5 standard deviations from period 1 to 6[10]. To highlight the implications of the occasionally-binding ZLB constraint and its interaction with macro-prudential policy, we compare three scenarios: one without ZLB and macro-prudential policy (blue-solid line), one with (a binding) ZLB[11] constraint but without macro-prudential policy (green-dashed line) and lastly one scenario with (a non-binding) ZLB constraint and macro-prudential policy (red-dotted line).

To implement a scenario in which the ZLB becomes binding, we lowered the risk-adjusted steady-state interest rate to around 2% by lowering the inflation target accordingly, while keeping the growth rate at 1.004.[12]

FIGURE 9.11: ZLB SCENARIO FOR A SEQUENCE OF CAPITAL QUALITY SHOCKS




Note: The Figure displays a scenario for a sequence of consecutive adverse capital quality shocks of 1.5 stdev from period 1 to 6. The blue-straight line depicts the case in which the deposit rate is not constrained by the ZLB. The green-dashed line depicts the case in which the deposit rate is constrained by the ZLB. The red-dotted line depicts the case under the optimal macro-prudential policy rule.

The sequence of negative financial shocks causes a fall in output and, in the absence of macro-prudential policy, inflation, which calls for a significant cut in the central bank's deposit rate. In the unconstrained case, the deposit rate is allowed to fall below zero. In the constrained case without macro-prudential policy, the deposit rate hits the ZLB in period four and starts to rise again in period 9. The liquidity trap causes a substantial fall in inflation expectations and a rise in the real rate, which amplifies the drop in output, compared to the unconstrained case. Besides the fall in output, the liquidity trap also amplifies the fall in both types of investment and the decline in the productivity growth rate. Due to the stronger fall in the growth rate, the permanent effects on the output level are more severe with a binding ZLB constraint. In particular, output remains 7% below its initial BGP, compared to 6.7% in the unconstrained case.

[10]We follow the approach by Sims and Wu (2021).

[11]We use the occbin toolkit to construct a piece-wise linear solution for the model in which the ZLB on the deposit rate is binding. Occbin was not able to handle our risk adjustment of the steady state. The IRFs for the case with the ZLB are therefore calculated around the deterministic steady state. The first-order dynamics around the deterministic steady state are similar to the first-order dynamics around the risk-adjusted steady state. The experiment is thus still insightful in that it illustrates the additional benefit of macro-prudential policy via avoiding ZLB episodes. However, to correctly calculate the welfare costs and benefits of macro-prudential policy and its interaction with the ZLB we would need to solve for the risk-adjusted steady state with the ZLB, which is challenging numerically. In this regard, the scenario we show here falls short.

[12]Without lowering the risk-adjusted steady-state interest rate we would need very large shocks to drive the model toward the ZLB. For these large shocks, occbin does not produce a solution.

Under the optimal macro-prudential policy the financial system is more resilient, and asset prices fall less, thus mitigating the tightening in credit conditions. Consequently, the fall in investment in physical capital and R&D and output is more muted. Under the optimal macro-prudential policy, inflation increases in response to a capital quality shock. The milder decline in macroeconomic conditions calls for a less drastic cut and a mild increase in the policy rate, which therefore does not reach the ZLB. The presence of macro-prudential policy significantly mitigates the fall in the growth rate and, thus, the permanent output losses amount to only $-5.9\%$, an improvement of 1.1 percentage points compared to the case with the ZLB. Moreover, as discussed above, an additional benefit of macro-prudential policy is not just the stabilisation of fluctuations around the BGP but the effect on the BGP itself. Thanks to the increased steady-state growth rate, the interest rate steady state is also higher. As can be inferred from Table 3, the deposit rate in the unregulated BGP was 1.0089, which corresponds to an annual rate of 3.56 %. Under the optimal simple macro-prudential policy rule the steady state deposit rate would be 4.24%, an increase of roughly 0.7pp. In addition to the welfare gains in normal times shown in Section 5.2, this last analysis highlights the importance of macro-prudential policy in avoiding the short-term and long-term adverse consequences of a liquidity trap. Finally, it is important to note that our exercises assumed the ZLB to bind for only six quarters. In reality, in the US the Federal Funds Rate stayed at the ZLB for more than 20 quarters between 2009 and 2015. Keeping the ZLB binding for such an extended period would further strengthen our results and suggest even larger stabilisation gains from the macro-prudential policy.

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