Online Appendix to "The Fiscal Roots of Inflation"

John H. Cochrane

Hoover Institution, Stanford University

Appendix A. Derivation of the linearized identities

In this appendix I derive the linearized identities (1), (2), and (3),

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1} \tag{A.1}$$

$$v_t = \sum_{j=1}^{\infty} \rho^{j-1} s_{t+j} + \sum_{j=1}^{\infty} \rho^{j-1} g_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \left(r^n_{t+j} - \pi_{t+j} \right)$$

and

$$\begin{aligned} \Delta E_{t+1}\pi_{t+1} - \Delta E_{t+1} r^n_{t+1} = &- \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} s_{t+1+j} - \\ &- \sum_{j=0}^{\infty} \rho^j \Delta E_{t+1} g_{t+1+j} + \sum_{j=1}^{\infty} \rho^j \Delta E_{t+1} \left(r^n_{t+1+j} - \pi_{t+1+j} \right). \end{aligned} \tag{A.2}$$

I also define the variables more carefully.

The symbols are as follows:

$$V_t = M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}$$

is the nominal end-of-period market value of debt, where M_t is non-interest-bearing money, $B_t^{(t+j)}$ is zero-coupon nominal debt outstanding at the end of period t and due at the beginning of period $t+j$, and $Q_t^{(t+j)}$ is the time t price of that bond, with $Q_t^{(t)} = 1$. Taking logs,

$$v_t \equiv \log \left(\frac{V_t}{Y_t P_t} \right)$$

is log market value of the debt divided by GDP, where P_t is the price level and Y_t is real GDP or another stationarity-inducing divisor such as consumption, potential GDP, etc. I use consumption times the average GDP to consumption ratio in the empirical work, but I will call Y and ratios to Y "GDP" for brevity. The quantity

$$R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}} \tag{A.3}$$

is the nominal return on the portfolio of government debt, i.e. how the change in prices from the end of t to the beginning of $t+1$ affects the value of debt held between periods. The quantity

$$r_{t+1}^n \equiv \log(R_{t+1}^n)$$

is the log nominal return on that portfolio. The symbols

$$\pi_t \equiv \log\left(\frac{P_t}{P_{t-1}}\right),\ g_t \equiv \log\left(\frac{Y_t}{Y_{t-1}}\right)$$

are log inflation and GDP growth rate.

We can accommodate explicit default, so the formulas can also apply to countries that borrow in foreign currency such as the members of the Euro. An explicit default is a reduction in the nominal quantity of debt between periods. The $B_t^{(t+j)}$ in the numerator of (A.3) represents the post-default number of bonds outstanding, i.e. at the beginning of period $t+1$, while the $B_t^{(t+j)}$ in the denominator represents the pre-default number of bonds outstanding, i.e. at the end of period t. A partial default then shows up as a low return. To handle default one would, of course, add notation distinguishing the pre- and post-default quantity of debt in the definition of return.

We start with the nonlinear flow identity,

$$M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} = P_{t+1} sp_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}. \tag{A.4}$$

Here, sp_{t+1} denotes the real primary (not including interest payments) surplus or deficit. At the beginning of period $t+1$, money M_t and bonds $B_t^{(t+1+j)}$ are outstanding. Money M_{t+1} at the end of period $t+1$ and beginning of period $t+2$ then equals money M_t, money printed up to redeem bonds $B_{t+1}^{(t+1)}$, less money soaked up by a primary surplus $P_{t+1} sp_{t+1}$, or conversely printed to finance a primary deficit, and less money soaked up by net new bond sales, or printed to finance long-term bond purchases, $\sum_{j=1}^{\infty} Q_{t+1}^{t+1+j}(B_{t+1}^{(t+1+j)} - B_{t}^{(t+1+j)})$.

Using the definition of return, (A.4) becomes

$$\left(M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}\right) R_{t+1}^n = P_{t+1} sp_{t+1} + \left(M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}\right),$$

or,

$$V_t R_{t+1}^n = P_{t+1} sp_{t+1} + V_{t+1}.$$

The nominal value of government debt is increased by the nominal rate of return, and decreased by primary surpluses. This seems easy. The algebra all comes from properly defining the return on the portfolio of government debt.

Expressing the result as ratios to GDP, we have a flow identity

$$\frac{V_t}{P_t Y_t} \times \frac{R_{t+1}^n}{G_{t+1}} \frac{P_t}{P_{t+1}} = \frac{sp_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}, \tag{A.5}$$

where $G_{t+1} \equiv Y_{t+1}/Y_t$.

We can iterate this flow identity (A.5) forward to express the nonlinear government debt valuation identity as

$$\frac{V_t}{P_t Y_t} = \sum_{j=1}^{\infty} \prod_{k=1}^{j} \frac{1}{R_{t+k}^n/(\Pi_{t+k} G_{t+k})} \frac{sp_{t+j}}{Y_{t+j}}. \tag{A.6}$$

where $\Pi_{t+1} \equiv P_{t+1}/P_t$. The market value of government debt at the end of period t, as a fraction of GDP, equals the present value of primary surplus to GDP ratios, discounted at the government debt rate of return less the GDP growth rate.

The nonlinear present value identities (A.5) and (A.6) are cumbersome, and as explained below they may not converge even when the true present value and linearized identities do converge. I linearize the flow equation (A.5) and then iterate forward to obtain a linearized version of (A.6). Taking logs of (A.5), we have

$$v_t + r_{t+1}^n - \pi_{t+1} - g_{t+1} = \log\left(\frac{sp_{t+1}}{Y_{t+1}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}}\right). \tag{A.7}$$

I linearize this equation in the *level* of the surplus, not its log as one conventionally does in asset pricing, since the surplus is often negative. To linearize in terms of the surplus/GDP ratio, Taylor expand the last term,

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log(e^v + sy) + \frac{e^v}{e^v + sy}(v_{t+1} - v) + \frac{1}{e^v + sy}(sy_{t+1} - sy)$$

where

$$sy_{t+1} \equiv \frac{sp_{t+1}}{Y_{t+1}} \tag{A.8}$$

denotes the surplus to GDP ratio, and variables without subscripts denote a steady state of (A.7). With $r \equiv r^n - \pi$,

$$r - g = \log \frac{e^v + sy}{e^v}.$$

Then,

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \left[\log(e^v + sy) - \frac{e^v}{e^v + sy}\left(v + \frac{sy}{e^v}\right)\right] + \frac{e^v}{e^v + sy} v_{t+1} + \frac{e^v}{e^v + sy}\frac{sy_{t+1}}{e^v}$$

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \left[v + r - g - \frac{e^v}{e^v + sy}\left(v + \frac{e^v + sy}{e^v} - 1\right)\right] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^v}$$

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \left[r - g + (1 - \rho)(v - 1)\right] + \rho v_{t+1} + \rho \frac{sy_{t+1}}{e^v} \tag{A.9}$$

where

$$\rho \equiv e^{-(r-g)}. \tag{A.10}$$

Suppressing the small constant, and thus interpreting variables as deviations from means, the linearized flow identity is

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \rho \frac{sy_{t+1}}{e^v} + \rho v_{t+1}. \tag{A.11}$$

Iterating forward, the present value identity is

$$v_t = \sum_{j=1}^{T} \rho^{j-1} \left[\rho \frac{sy_{t+j}}{e^v} - \left(r^n_{t+j} - \pi_{t+j} - g_{t+j}\right)\right] + \rho^T v_T. \tag{A.12}$$

If we linearize around $r - g = 0$, then the constant in (A.11) is zero ($sy = 0$), and we obtain the linearized flow and present value identities (A.1) and (A.2), with the symbol s_t representing sy_t/e^v. There is nothing wrong with expanding about $r = g$. The point of expansion need not be the sample mean.

To approximate in terms of the surplus to value ratio, write (A.7) as

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left(\frac{V_t}{P_t Y_t} \frac{\frac{sp_{t+1}}{Y_{t+1}}}{\frac{V_t}{P_t Y_t}} + \frac{V_{t+1}}{P_{t+1} Y_{t+1}} \right)$$

$$r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left(\frac{\frac{sp_{t+1}}{Y_{t+1}}}{\frac{V_t}{P_t Y_t}} + \frac{\frac{V_{t+1}}{P_{t+1} Y_{t+1}}}{\frac{V_t}{P_t Y_t}} \right)$$

$$r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log \left(sv_{t+1} + e^{v_{t+1} - v_t} \right).$$

At a steady state

$$r - g = \log\left(1 + sv\right). \tag{A.13}$$

$$e^{r-g} = 1 + sv.$$

Taylor expanding around a steady state,

$$r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(1 + sv\right) + \frac{1}{(1 + sv)} \left(sv_{t+1} - sv + v_{t+1} - v_t \right)$$

$$v_t + (1 + sv) \left[r^n_{t+1} - \pi_{t+1} - g_{t+1} \right] = \left[(1 + sv) \log (1 + sv) - sv \right] + sv_{t+1} + v_{t+1}. \tag{A.14}$$

The linearized flow identity (A.1) follows, with the symbol s_t representing the surplus to value ratio $s_t = sv_t$, if we suppress the constant, using deviations from means in the analysis, or if we use $r = g$ or $sv = 0$, as a point of expansion.

The linearizations in terms of the surplus to value ratio sv_t are more accurate. The units of the flow identities (A.1), (A.11) are rates of return. Dividing the surplus by the previous period's value gives a better approximation to the growth in value, when the value of debt is far from the steady state.

A constant ratio of surplus to market value of debt for any price level path leads to a passive fiscal policy: An unexpected deflation raises the real value of debt. If surpluses always rise in response, they validate the lower price level. Thus, although on the equilibrium path one can describe dynamics via either linearization, if one wants to think about how fiscal-theory equilibria are formed, it is better to describe a surplus that does not react to price level changes, so only one value v_t emerges, as is the case in (A.12). For such purposes, the surplus to GDP definition is appropriate, as well as adopting a linearization point $r > g$ and $\rho < 1$. It's also better to use the nonlinear versions of the identities for determinacy issues. The analysis of this paper is about what happens in equilibrium, and does not require an active-fiscal assumption, so the difference is irrelevant here.

I infer the surplus from the linearized flow identity (A.1) so which concept the surplus corresponds to makes no difference to the analysis. The difference is only the accuracy of approximation, how close the surplus recovered from the linearized flow identity corresponds to a surplus recovered from the nonlinear exact identity (A.7).

Appendix B. Linearizing the bond return formula

Here I derive the linearized identity

$$r^n_{t+1} \approx \omega q_{t+1} - q_t,$$

which leads to (4),

$$\Delta E_{t+1} r^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[(r^n_{t+1+j} - \pi_{t+1+j}) + \pi_{t+1+j} \right].$$

I also derive expectations-hypothesis bond-pricing equations.

$$E_t r^n_{t+1} = i_t$$

$$\omega E_t q_{t+1} - q_t = i_t.$$

These equations are used in the sticky-price model Cochrane (2020a).

Denote the maturity structure by

$$\omega_{j,t} \equiv \frac{B_t^{(t+j)}}{B_t^{(t+1)}}$$

and $B_t \equiv B_t^{(t+1)}$. Then the end of period t nominal market value of debt is

$$\sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)} = B_t \sum_{j=1}^{\infty} \omega_{j,t} Q_t^{(t+j)}.$$

(I ignore money to keep the formulas simple.) Define the price of the government debt portfolio

$$Q_t = \sum_{j=1}^{\infty} \omega_{j,t} Q_t^{(t+j)}.$$

The return on the government debt portfolio is then

$$R_{t+1}^n = \frac{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_{t+1}^{(t+j)}}{\sum_{j=1}^{\infty} B_t^{(t+j)} Q_t^{(t+j)}} = \frac{\sum_{j=1}^{\infty} \omega_{j,t} Q_{t+1}^{(t+j)}}{\sum_{j=1}^{\infty} \omega_{j,t} Q_t^{(t+j)}} = \frac{1 + \sum_{j=1}^{\infty} \omega_{j+1,t} Q_{t+1}^{(t+1+j)}}{Q_t}. \tag{B.1}$$

I loglinearize around a geometric maturity structure, $B_t^{(t+j)} = B_t \omega^{j-1}$, or equivalently $\omega_{j,t} = \omega^{j-1}$. I use variables with no subscripts to denote the linearization points, and tildes to denote deviations from those points.

When we linearize, we move bond prices holding the maturity structure at its steady-state, geometric value, and then we move the maturity structure while holding bond prices at their steady-state value. As a result, changes in maturity structure have no first-order effect on the linearized bond return. At the steady state $Q_t^{t+j} = 1/(1+i)^j$,

$$R_{t+1}^n = \frac{\sum_{j=1}^{\infty} \omega_{j,t}/(1+i)^{j-1}}{\sum_{j=1}^{\infty} \omega_{j,t}/(1+i)^j} = (1+i)$$

independently of $\{w_{j,t}\}$. Intuitively, at the steady state bond prices, all bonds give the same return, so all portfolios of bonds give the same return. Moreover, maturity structure is a time-t variable in the definition of return R_{t+1}^n. The return from t to $t+1$ is not affected by the time $t+1$ maturity structure. (Changes in maturity structure might affect returns if there is price pressure in bond markets. These are formulas for measurement, however, and such effects would show up as changes in measured prices coincident with changes in quantities.)

Maturity structure only has a second-order *interaction* effect on the bond portfolio return. For example, a longer maturity structure at t raises the bond portfolio return at $t+1$ if there is also a level shock, raising long-maturity bond returns at $t+1$. A longer maturity structure at t raises the expected return if the yield curve at t is also temporarily upward sloping. But a linear VAR and a linear decomposition do not include interaction effects.

To be clear, I measure the bond portfolio return r^n_{t+1} directly, and exactly, and this measure includes effects of maturity structure, as well as liquidity and other effects. With a longer maturity structure, the same movement in interest rates will give a larger return. The linearization only affects the decomposition of the bond portfolio return to future inflation and future expected returns. A second-order approximation would effectively use a different ω in the decomposition formula for different dates, as well as estimate a VAR with parameters that depend on the maturity structure or interaction terms. But variation in the geometric maturity structure parameter ω makes little difference to the results. And the sample is too short to add more variables, interaction terms, or time-varying parameters.

The term of the linearization with steady-state bond prices and shocks to maturity thus adds nothing. The linearization only includes a linearization with steady-state, geometric maturity structure and changing bond prices. Linearizing (B.1) then, we have

$$r^n_{t+1} = \log\left(1+\omega e^{q_{t+1}}\right) - q_t \approx \log\left(\frac{1+\omega Q}{Q}\right) + \frac{\omega Q}{1+\omega Q}\tilde{q}_{t+1} - \tilde{q}_t \tag{B.2}$$

where as usual variables without subscripts are steady state values and tildes are deviations from steady state. In a steady state,

$$Q = \sum_{j=1}^{\infty}\omega^{j-1}\frac{1}{(1+i)^j} = \left(\frac{1}{1+i}\right)\left(\frac{1}{1-\frac{\omega}{1+i}}\right) = \frac{1}{1+i-\omega}. \tag{B.3}$$

The limits are $\omega = 0$ for one-period bonds, which gives $Q = 1/(1+i)$, and $\omega = 1$ for perpetuities, which gives $Q = 1/i$. The terms of the approximation (B.2) are then

$$\frac{1+\omega Q}{Q} = 1+i$$
$$\frac{\omega Q}{1+\omega Q} = \frac{\omega}{1+i}$$

so we can write (B.2) as

$$r^n_{t+1} \approx i + \frac{\omega}{1+i}\tilde{q}_{t+1} - \tilde{q}_t.$$

Since $i < 0.05$ and $\omega \approx 0.7$, I further approximate to

$$r^n_{t+1} \approx i + \omega \tilde{q}_{t+1} - \tilde{q}_t. \tag{B.4}$$

I find the value of ω that best fits the return identity, rather than measure the maturity structure directly, so the difference between ω and $\omega/(1+i)$ makes no practical difference.

To derive the bond return identity (4), iterate (B.4) forward to express the bond price in terms of future returns,

$$\tilde{q}_t = -\sum_{j=1}^{\infty} \omega^j \tilde{r}^n_{t+j}.$$

Take innovations, move the first term to the left hand side, and divide by ω,

$$\Delta E_{t+1} \tilde{r}^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \tilde{r}^n_{t+1+j}. \tag{B.5}$$

Then add and subtract inflation to get (4),

$$\Delta E_{t+1} \tilde{r}^n_{t+1} = -\sum_{j=1}^{\infty} \omega^j \Delta E_{t+1} \left[(\tilde{r}^n_{t+1+j} - \tilde{\pi}_{t+1+j}) + \tilde{\pi}_{t+1+j} \right]. \tag{B.6}$$

The expectations hypothesis states that expected returns on bonds of all maturities are the same,

$$\begin{aligned} E_t r^n_{t+1} &= i_t \\ i + \omega E_t \tilde{q}_{t+1} - \tilde{q}_t &= i_t \\ \omega E_t \tilde{q}_{t+1} - \tilde{q}_t &= \tilde{\imath}_t. \end{aligned}$$

In the text, all variables are deviations from steady state, so I drop the tilde notation.

Appendix C. What if r is less than g?

Does $r < g$ imply that the present value of debt is infinite, so deficits do not have to be repaid by subsequent surpluses, and the forward-looking present value is fundamentally wrong? I give here a deeper treatment of the issue.

Again, the present value of debt can be well defined, properly using the stochastic discount factor, contingent claim price, or marginal utility to discount, and large deficits still need to be repaid by subsequent surpluses, yet the economy displays $E(r) < E(g)$.

In essence, though debt is grown out of on many paths and on average, there are paths with low growth and high contingent claim value where growing out of debt fails. The lesson is that one should not take the mean values of r and g from a stochastic economy and use them in perfect certainty discounting formulas. (Bohn (1995), Bassetto and Cui (2018), Bassetto and Sargent (2020), Reis (2021), Cochrane (2021))

The approach here, which discounts using the stochastic ex-post return, is still not out of the woods. While one can always use the ex-post return to discount a finite stream of payoffs – $1 = E_t(R_{t+1}^{-1} R_{t+1})$ – that assurance does not extend to an infinite stream. Discounting using returns can fail, even stochastic returns, while the true present value formula holds.

Here is what can happen. Suppose the value of debt is well-defined using the stochastic discount factor, contingent claims price, or marginal utility Λ_t. Suppose

$$\frac{V_t}{Y_t} = E_t \sum_{j=0}^{T} \frac{\Lambda_{t+j}}{\Lambda_t} \frac{Y_{t+j}}{Y_t} \frac{s_{t+j}}{Y_{t+j}} + E_t \frac{\Lambda_{t+T}}{\Lambda_t} \frac{Y_{t+T}}{Y_t} \frac{V_{t+T}}{Y_{t+T}} \tag{C.1}$$

is well defined, and the right hand term tends to zero. I include GDP terms which complicate the formula but you can see they just multiply and divide the basic present value formula.

Now, try to discount using ex post returns rather than the discount factor. Start with the identity

$$V_{t+1} = R_{t+1} \left(V_t - s_t \right).$$

Rearrange to

$$\frac{V_t}{Y_t} = \frac{s_t}{Y_t} + \frac{1}{R_{t+1}} \frac{Y_{t+1}}{Y_t} \frac{V_{t+1}}{Y_{t+1}}$$

and iterate forward

$$V_t = \sum_{j=0}^{T} \prod_{k=0}^{j} \frac{1}{R_{t+k}} \frac{Y_{t+k}}{Y_{t+k-1}} s_{t+j} + \prod_{k=0}^{T} \frac{1}{R_{t+k}} \frac{Y_{t+k}}{Y_{t+k-1}} \frac{V_{t+T}}{Y_{t+T}}. \tag{C.2}$$

This is the exact nonlinear version of the present value identity (2). It can happen that the expected value of *this* limiting term explodes, and the present value term explodes in the opposite direction, even though the true present value formula (C.1) is well-behaved. And, with stationary debt to GDP ratio, you can see that a nonlinear stochastic version of $r < g$ is exactly the condition for this pathological case.

In sum, by $1 = E_t(R_{t+1}^{-1} R_{t+1})$, the inverse return is a discount factor under mild conditions, basically that moments exist. The inverse return is only an infinite period discount factor if in addition the sums and terminal condition converge.

This result seems to put the whole project of discounting by returns in jeapordy. But that is not necessarily so. For the terms of the *linearized* identity (6) and those of the true present value formula (C.1) may converge, while the terms of the the *nonlinear* identity (C.2) do not converge. I linearize the return formula and then iterate forward. I do not take a direct Taylor approximation of the return-based identity (C.2). The condition for a substitute discount factor to work is, first, that it prices one-period claims, and second, that the sums converge. The linearized return-based present value formula can work when the nonlinear one does not. "Work" means that the formula gives a good approximation to the true, discount-factor, based result.

For example, if $E(r^n - \pi) < E(g)$, the linearized identity (2) indicates that the government can finance a small steady deficit/GDP ratio $E(s) < 0$, while maintaining a steady debt/GDP ratio. But any large additional deficits must still be financed by subsequent surpluses.

If the government issues only non-interest-bearing money, then $r^n = 0$. This is the familiar case of seignorage, which can finance a small steady deficit, but large additional spending must still be financed by borrowing and repayment. The identity indicates that the same principle applies for other sources of $r < g$.

The linearized identity captures this fact of the true present value formula, that the nonlinear return-based present value formula may not capture.

Appendix D. A variance decomposition

I use the elements of the impulse response function and their sums to calculate the terms of the unexpected inflation identity (3). We can interpret this calculation as an decomposition of the variance of unexpected inflation. Multiply both sides of (3) by $\Delta E_{t+1}\pi_{t+1}$ and take expectations, giving

$$
\begin{aligned}
& var\left(\Delta E_{t+1}\pi_{t+1}\right) - cov\left[\Delta E_{t+1}\pi_{t+1}, \Delta E_{t+1}\left(r^n_{t+1} - g_{t+1}\right)\right] = \\
& = -\sum_{j=0}^{\infty} cov\left[\Delta E_{t+1}\pi_{t+1}, \Delta E_{t+1}s_{t+1+j}\right] + \\
& + \sum_{j=1}^{\infty} cov\left[\Delta E_{t+1}\pi_{t+1}, \Delta E_{t+1}\left(r^n_{t+1+j} - \pi_{t+1+j} - g_{t+1+j}\right)\right].
\end{aligned}
\tag{D.1}
$$

Unexpected inflation may only vary to the extent that it covaries with current bond returns, or if it forecasts surpluses or real discount rates.

Dividing by $var\left(\Delta E_{t+1}\pi_{t+1}\right)$, we can express each term as a fraction of the variance of unexpected inflation coming from that term. This decomposition adds up to 100%, within the accuracy of approximation, but it is not an orthogonal decomposition, nor are all the elements necessarily positive. Each term is also a regression coefficient of future long-run variables on unexpected inflation.

The two approaches give exactly the same result – the terms of (D.1) are exactly the terms of the impulse-response function, to an inflation shock orthogonalized last, i.e. a shock that moves all variables at time 1 including $\Delta E_1\pi_1$.

To see this fact, write the VAR in standard notation

$$x_{t+1} = Ax_t + \varepsilon_{t+1} \tag{D.2}$$

so

$$\Delta E_{t+1}\sum_{j=1}^{\infty} x_{t+j} = (I-A)^{-1}\varepsilon_{t+1}.$$

Let a denote vectors which pull out each variable, i.e.

$$\pi_t = a'_\pi x_t, \ s_t = a'_s x_t, \tag{D.3}$$

etc. Then the present value identity (3) reads and may be calculated as

$$a'_\pi\varepsilon_{t+1} - (a_{r^n} - a_g)'\varepsilon_{t+1} = -a'_s(I-A)^{-1}\varepsilon_{t+1} + a'_{rg}(I-A)^{-1}A\varepsilon_{t+1} \tag{D.4}$$

where

$$a_{rg} \equiv a_{r^n} - a_\pi - a_g.$$

We can calculate the variance decomposition (D.1) by

$$a_\pi' \Omega a_\pi - (a_{r^n} - a_g)' \Omega a_\pi = -a_s'(I-A)^{-1}\Omega a_\pi + a_{rg}'(I-A)^{-1}A\Omega a_\pi$$

where $\Omega = cov(\varepsilon_{t+1}, \varepsilon_{t+1}')$, and then divide by $a_\pi' \Omega a_\pi$ to express the result as a fraction,

$$1 - (a_{r^n} - a_g)' \frac{\Omega a_\pi}{a_\pi' \Omega a_\pi} = -a_s'(I-A)^{-1} \frac{\Omega a_\pi}{a_\pi' \Omega a_\pi} + a_{rg}'(I-A)^{-1} A \frac{\Omega a_\pi}{a_\pi' \Omega a_\pi}. \quad \text{(D.5)}$$

To show that this variance decomposition is the same as the elements and sum of elements of the impulse-response function to an inflation shock, orthogonalized last, note that the regression coefficient of any other shock ε^z on the inflation shock is

$$b_{\varepsilon^z,\varepsilon^\pi} = \frac{cov(\varepsilon_{t+1}^z, \varepsilon_{t+1}^\pi)}{var(\varepsilon_{t+1}^\pi)} = \frac{a_z' \Omega a_\pi}{a_\pi' \Omega a_\pi},$$

so the VAR shock, consisting of a unit movement in inflation $\varepsilon_1^\pi = 1$ and movements $\varepsilon_1^z = b_{\varepsilon^z,\varepsilon^\pi}$ in each of the other variables is given by

$$\varepsilon_1 = \frac{\Omega a_\pi}{a_\pi' \Omega a_\pi}.$$

We recognize in (D.5) the responses and sums of responses to this shock. Dividing (D.1) by the variance of unexpected inflation, or examining the terms of (D.5), we recognize that each term is also the coefficient in a single regression of each quantity on unexpected inflation.

In an analogous way, we can interpret the responses to other shocks as a decomposition of the *co*variance of unexpected inflation with that shock, based on

$$\begin{aligned} &cov\left(\Delta E_{t+1}\pi_{t+1}\varepsilon_{t+1}\right) - cov\left[\varepsilon_{t+1}, \Delta E_{t+1}\left(r_{t+1}^n - g_{t+1}\right)\right] \\ &= -\sum_{j=0}^{\infty} cov\left[\varepsilon_{t+1}, \Delta E_{t+1} s_{t+1+j}\right] + \sum_{j=1}^{\infty} cov\left[\varepsilon_{t+1}, \Delta E_{t+1}\left(r_{t+1+j}^n - \pi_{t+1+j} - g_{t+1+j}\right)\right]. \end{aligned}$$

This variance decomposition is similar in style to the decomposition of return variance in Campbell and Ammer (1993). To avoid covariance terms, however, it follows the philosophy of the price/dividend variance decomposition in Cochrane (1992), extended to a multivariate context. With $x = y + z$, I explore $var(x) = cov(x, y) + cov(x, z)$ rather than $var(x) = var(y) + var(z) + 2cov(y, z)$.

Appendix E. Monte Carlo details

To evaluate sampling distributions I run a simple Monte Carlo. I start with the estimated VAR. I find the covariance matrix of the residuals ε_{t+1}. The identity (1) implies

$$\varepsilon_{s,t+1} = \varepsilon_{r^n,t+1} - \varepsilon_{g,t+1} - \varepsilon_{\pi,t+1} - \varepsilon_{v,t+1}. \tag{E.1}$$

Since I infer the surplus data s_t from (1), the data obey this identity and the covariance matrix of residuals is singular. Thus I simulate iid shocks from the covariance matrix of all shocks except the surplus, and then I infer the surplus shock from the identity (E.1).

I initialize the VAR at the first data point, thereby generating the conditional sampling distribution. I simulate forward 50,000 artificial data samples using the estimated VAR parameters. I re-estimate the VAR and I calculate impulse responses and inflation decompositions in each artificial sample. I tabulate the sampling distribution of these quantities and report quantiles.

In a very few artificial samples, the VAR estimate has eigenvalues greater than or equal to one, so $(I - A)^{-1}$ cannot be computed. I omit these 38 out of 50,000 samples. As a result the reported quantiles are slightly smaller than actual quantiles. Avoiding these infinities and beyond is one reason that I report quantiles rather than standard errors. More generally, the distribution of statistics is not normal.

It is also not always possible to find $\omega \in [0, 1]$ to satisfy the return identity, so many Monte Carlo draws use a best fit value of ω in which the return identity does not hold. Weights have little effect on the results however, so this fact seems to have little effect. Since this is what I would have done in sample had I not been able to find an $\omega \in [0, 1]$ that satisfied the return identity, this fact just fills out the correct sampling distribution.

I run the Monte Carlo using sample estimates, and in particular the estimated 0.98 coefficient of debt on lagged debt. Near unit roots are biased down, and one might wish also to run a Monte Carlo with a bias-corrected estimate with eigenvalues closer to one. That procedure would likely lead to somewhat larger sampling distributions.

Between the conditional Monte Carlo – starting at the first data point – the problem of draws with A eigenvalues greater than one, near-unit roots, and non-normal error distributions, one could likely find sampling experiments that produce even larger distributions.

But remember, I am not testing anything, so the point is simply to give a sense of the sampling error of the measurements. My main conclusion is that the sampling distribution of the response functions and decompositions, though narrow enough that the qualitative results are reasonably reliable, is still pretty wide already, steering me away from model complications. Sampling exercises that produce even wider distributions would only emphasize that point.

Appendix F. Sources of sampling variation

Table F.4 includes the regression of other shocks on inflation shock that starts off the main inflation decomposition, and thus determines the instantaneous response in Figures 2 and 9. The table also includes the correlation matrix of the shocks.

To measure the relative contribution of the shock correlation and the long-run response function given the shock identification as sources of variation, Table F.5 includes two other sampling calculations. The "no b" columns resample data using the original regression of shocks ε^z_{t+1} on inflation shocks ε^π_{t+1}, the top row of Table F.4, in each sample. The VAR coefficients still vary across samples, but the identification of the inflation shock does not. The "no A" columns likewise keep constant the VAR regression coefficients, but reestimate the shock regression in each sample. Turning off either source of sampling variation reduces that variation, but not as much as you might think. Sampling

	r^n	g	π	s	v	i	y
	Regression of other shocks on inflation shock						
Coefficient	-0.56	-0.33	1.00	-0.58	-0.65	0.24	0.23
Std. err.	(0.24)	(0.17)	(0.00)	(0.53)	(0.74)	(0.14)	(0.24)
	Correlation matrix of VAR shocks						
r^n	1.00	-0.25	-0.29	-0.27	0.63	-0.74	-0.93
g	-0.25	1.00	-0.24	0.39	-0.56	0.41	0.20
π	-0.29	-0.24	1.00	-0.14	-0.11	0.21	0.31
s	-0.27	0.39	-0.14	1.00	-0.88	0.35	0.26
v	0.63	-0.56	-0.11	-0.88	1.00	-0.63	-0.60
i	-0.74	0.41	0.21	0.35	-0.63	1.00	0.75
y	-0.93	0.20	0.31	0.26	-0.60	0.75	1.00

Table F.4: Regression of other shocks on inflation shock, and correlation matrix of VAR shocks

	Fraction			No b		No A	
Component	Estimate	25%	75%	25%	75%	25%	75%
Inflation π_1	1.00						
Bond return $(r_1^n - g_1)$	-0.23	-0.45	0.00	-0.23	-0.23	-0.45	0.00
Future Σs	-0.06	-0.69	0.23	-0.60	0.14	-0.69	0.23
Future $\Sigma r - g$	1.17	0.42	1.57	0.63	1.37	0.42	1.57

Table F.5: Decomposition of unexpected inflation variance – distribution quantiles. No b holds the initial response constant across trials. No A holds the VAR regression coefficients constant across trials variation is still large in either case, and variances add, not standard deviations. Moreover the sampling variation associated with shock orthogonalization – the "no A" exercise – does not go away no matter how small the shocks. Both left and right hand sides of the shock on shock regressions get smaller at the same rate.

Appendix G. 1980-2018 subsample results

This section presents results using the 1980-2018 subsample. Much monetary macroeconomics isolates this period as having a different set of correlations that the earlier 1970s inflation, 1960s under Bretton woods, etc. Breaking the sample also allows us to see if the results are stable across subsamples.

	r^n_{t+1}	g_{t+1}	π_{t+1}	s_{t+1}	v_{t+1}	i_{t+1}	y_{t+1}
r^n_t	-0.22*	0.05	-0.10**	-0.25*	0.08	-0.04	0.07*
g_t	-0.11	0.13	0.06	0.76	-1.06	0.20*	-0.04
π_t	-0.04	-0.57*	0.67**	-1.55	1.41	-0.08	-0.12
s_t	0.10*	0.07*	-0.02	0.38**	-0.34*	-0.02	-0.04*
v_t	-0.00	0.00	-0.00	0.05	0.95**	0.01	0.00
i_t	-0.12	-0.27*	0.20*	1.14*	-1.19	0.61**	0.31*
y_t	1.61**	0.67**	-0.08	0.05	0.98	0.32*	0.57**
$100 \times std(\varepsilon_{t+1})$	2.44	1.10	0.51	5.17	7.00	1.15	0.93
Corr $\varepsilon, \varepsilon_\pi$	-0.40	0.14	1.00	0.11	-0.31	0.27	0.42
R^2	0.73*	0.54*	0.88*	0.50*	0.94*	0.85*	0.89*
$100 \times std(x)$	4.74	1.63	1.48	7.30	28.88	2.97	2.84

Table G.6: OLS VAR estimate. Sample 1980-2018. One (two) stars means the estimate is one (two) Monte Carlo standard errors away from zero.

Table G.6 presents OLS VAR regression coefficients, parallel to Table 1. Table G.7 compiles inflation decompositions, parallel to Table 2. Figures G.10, G.11 and G.12 plot responses to inflation shocks, paralleling Figures 2, 3, and 4.

The broad pattern of Figure G.10 is similar to the full postwar sample. There are some differences. The surplus and growth shocks are now positively correlated with the inflation shock, seen in the period 1 responses. There is less

$$\sum_{j=0}^{\infty}\omega^j \Delta E_1 \pi_{1+j} = -\sum_{j=0}^{\infty}\Delta E_1 s_{1+j} - \sum_{j=0}^{\infty}\Delta E_1 g_{1+j} + \sum_{j=1}^{\infty}(1-\omega^j)\Delta E_1 r_{1+j}$$

	π	$=$	s	g	r
Inflation	2.32	=	-(0.19)	-(0.52)	+(3.03)
Recession	-2.50	=	-(-0.31)	-(-1.67)	+(-4.49)
Surplus	-0.08	=	-(-0.46)	-(-0.54)	+(-1.08)
Disc. Rate	-0.12	=	-(-0.40)	-(-0.49)	+(-1.00)
Surplus, no i	0.07	=	-(-0.70)	-(-0.30)	+(-0.92)

$$\Delta E_1 \pi_1 - \Delta E_1 r_1^n = -\sum_{j=0}^{\infty}\Delta E_1 s_{1+j} - \sum_{j=0}^{\infty}\Delta E_1 g_{1+j} + \sum_{j=1}^{\infty}\Delta E_1 r_{1+j}$$

	π	r^n	$=$	s	g	r
Inflation	1.00	-(-1.92)	=	-(0.19)	-(0.52)	+(3.63)
Recession	-1.00	-(2.44)	=	-(-0.31)	-(-1.67)	+(-5.42)
Surplus	-0.01	-(0.32)	=	-(-0.46)	-(-0.54)	+(-1.33)
Disc. Rate	-0.02	-(0.35)	=	-(-0.40)	-(-0.49)	+(-1.25)
Surplus, no i	0.07	-(-0.01)	=	-(-0.70)	-(-0.30)	+(-0.92)

$$\Delta E_1 r_1^n = -\sum_{j=1}^{\infty}\omega^j \Delta E_1 r_{1+j} - \sum_{j=1}^{\infty}\omega^j \Delta E_1 \pi_{1+j}$$

	r^n	$=$	r	π
Inflation	-1.92	=	-(0.60)	-(1.32)
Recession	2.44	=	-(-0.94)	-(-1.50)
Surplus	0.32	=	-(-0.25)	-(-0.07)
Disc. Rate	0.35	=	-(-0.25)	-(-0.09)
Surplus, no i	-0.01	=	-(-0.00)	-(0.00)

Table G.7: Terms of the inflation and bond return identities. Sample 1930-2018.

need to isolate a separate growth+inflation shock in this period, dominated by "aggregate demand" rather than "stagflation" episodes.

However, the surplus and growth responses turn negative after one period, as they are in the full sample. Higher inflation strongly forecasts a lower surplus, -1.55 in Table G.6 rather than -0.25 in Table 1, and similarly higher inflation forecasts lower growth -0.57 rather than -0.14. The overall responses are then similar to the full period.

Surpluses then recover and turn positive as before. The sum of the surplus response remains small, 0.19 rather than -0.06.

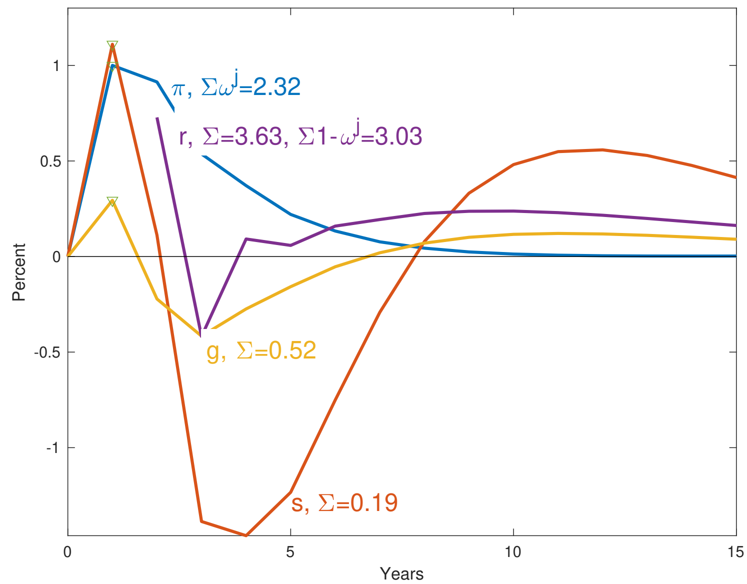

Figure G.10: Response to inflation shocks, sample 1980-2018.

Figure G.11 explores the long-run surplus response, and you can see the same dynamics playing out. Inflation forecasts a rise in debt (1.41 in Table G.6), and the period of deficits also raises debt (-0.34). But the rise in debt leads to a rise in surpluses, which slowly pay down much of that debt.

The expected return also rises in Figure G.10, and accounts for all the inflation and more in this subsample as it does in the main estimate.

Figure G.12 shows the interest rate response in more detail. The wiggly response, which I pointed out in the postwar sample and is a result of slight overfitting there, is even more pronounced here. However, wiggles aside, the basic picture is similar. Interest rates and the expected bond return rise together, and almost permanently in response to the inflation shock. They do not rise as much as inflation, giving a few periods of negative expected returns, but their rise is so much more persistent than that of inflation that we see a very long period of high expected returns on the right side of the graph. As in the full sample, the much greater persistence of yield-curve changes than of inflation generates the long-term discount rate rise which accounts for most of

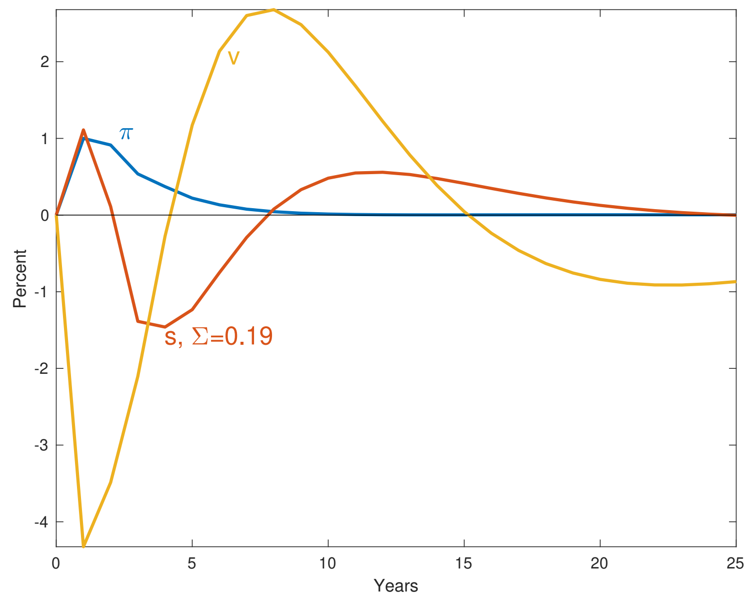

Figure G.11: Response to inflation shocks, sample 1980-2018.

the inflation shock.

The impulse-response quantiles, plotted in Figure G.13, are even larger than those of the full sample, but not so large that the results are meaningless.

Overall, we see a comfortingly similar picture, and many signs of weak estimation in a short sample. At least it is comforting not to see the point estimates paint a much different picture, as they do in the prewar sample studied in the next section.

I do not present results for the 1947-1980 subsample to save space, since it too paints about the same picture. The near-term (5 years) response functions are similar. However the point estimate has an eigenvalue of the transition matrix greater than one, so one must either reduce that or make calculations based on the first few responses only, not $(I - A)^{-1}$ calculations.

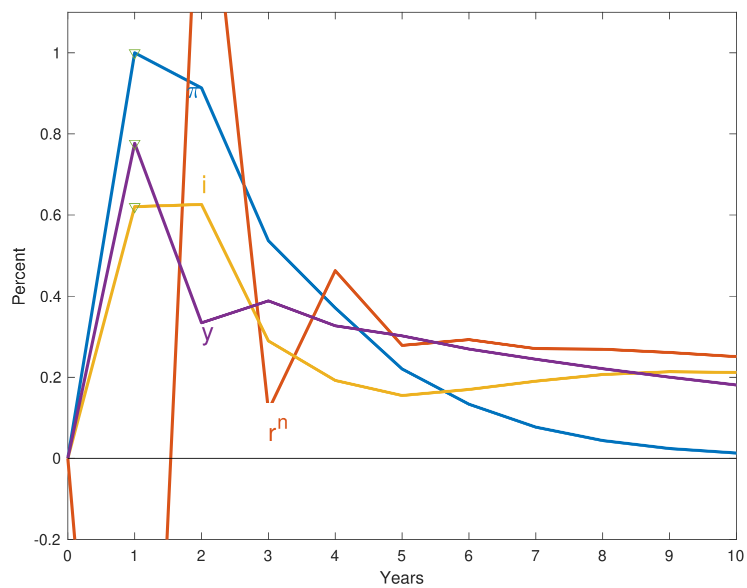

Figure G.12: Response to inflation shocks, sample 1980-2018.

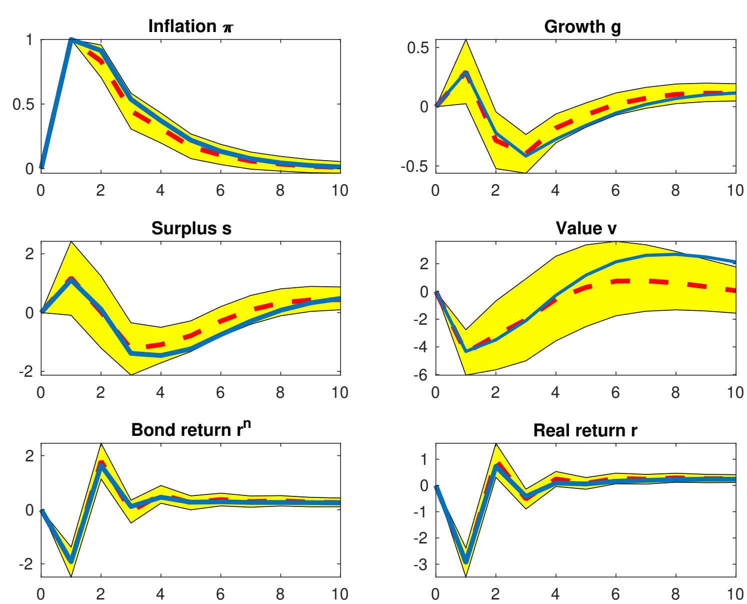

Figure G.13: Inflation shock response quantiles, sample 1980-2018.

Appendix H. Full sample results

This section presents results using the full sample of data that I have been able to collect, 1930-2018.

	r^n_{t+1}	g_{t+1}	π_{t+1}	s_{t+1}	v_{t+1}	i_{t+1}	y_{t+1}
r^n_t	-0.23**	0.06	-0.02	-0.12	-0.14	-0.06*	0.05*
g_t	0.02	0.42**	0.25**	0.52*	-1.17**	0.07*	-0.01
π_t	-0.11*	0.05	0.53**	-0.75**	0.05	0.02	0.02
s_t	0.01	-0.02	-0.02	0.65**	-0.61**	0.00	-0.01*
v_t	0.01	0.01*	0.01	0.08**	0.91**	-0.00	-0.00*
i_t	-0.32*	-0.35*	0.26	0.63	-0.87	0.79**	0.31**
y_t	1.85**	0.40*	-0.05	0.59	0.90	0.14	0.52**
$100 \times std(\varepsilon_{t+1})$	2.22	2.15	2.28	7.34	9.04	1.24	0.77
Corr $\varepsilon, \varepsilon_\pi$	-0.14	0.21	1.00	-0.07	-0.28	0.15	0.17
R^2	0.68*	0.32*	0.56*	0.54*	0.96*	0.84*	0.91*
$100 \times std(x)$	3.92	2.61	3.44	10.80	42.76	3.05	2.60

Table H.8: OLS VAR estimate. Sample 1930-2018. One (two) stars means the estimate is one (two) Monte Carlo standard errors away from zero.

Table H.8 presents OLS VAR regression coefficients, parallel to Table 1. Table H.9 compiles inflation decompositions, parallel to Table 2. Figures H.14, H.15, and H.16 plot responses to inflation shocks, paralleling Figures 2, 3, and 4. Figure H.17 presents sampling quantiles, paralleling Figure 9.

Start with the impulse response function for the inflation shock, Figure H.14, paralleling Figure 2. The general pattern is similar. But the magnitudes are completely different. The 1% inflation shock still corresponds to a prolonged deficit, and the deficit eventually turns to surplus. But the deficit is larger and longer, and following surpluses no longer pay off the accumulated debts. The sum of the surplus responses is -2.59, not -0.06, accounting for more than all of the 1.83% weighted sum of inflation.

Discount rates follow the same general pattern as well. But the decline in discount rate is longer lasting, and the subsequent rise much smaller, so discount rates now account for -0.52% inflation, not +1.004% inflation.

$$\sum_{j=0}^{\infty} \omega^j \Delta E_1 \pi_{1+j} = -\sum_{j=0}^{\infty} \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} (1-\omega^j) \Delta E_1 r_{1+j}$$

	π	=	s	g	r
Inflation	1.83	=	-(-2.59)	-(0.93)	+(0.17)
Recession	-2.00	=	-(2.59)	-(-2.13)	+(-1.54)
Surplus	0.09	=	-(-1.04)	-(0.04)	+(-0.91)
Disc. Rate	-0.05	=	-(-0.89)	-(-0.05)	+(-1.00)
Surplus, no i	0.30	=	-(-1.27)	-(0.27)	+(-0.70)

$$\Delta E_1 \pi_1 - \Delta E_1 r_1^n = -\sum_{j=0}^{\infty} \Delta E_1 s_{1+j} - \sum_{j=0}^{\infty} \Delta E_1 g_{1+j} + \sum_{j=1}^{\infty} \Delta E_1 r_{1+j}$$

	π	r^n	=	s	g	r
Inflation	1.00	-(-0.14)	=	-(-2.59)	-(0.93)	+(-0.52)
Recession	-1.00	-(0.17)	=	-(2.59)	-(-2.13)	+(-0.72)
Surplus	0.07	-(0.13)	=	-(-1.04)	-(0.04)	+(-1.05)
Disc. Rate	-0.01	-(0.16)	=	-(-0.89)	-(-0.05)	+(-1.11)
Surplus, no i	0.26	-(0.01)	=	-(-1.27)	-(0.27)	+(-0.75)

$$\Delta E_1 r_1^n = -\sum_{j=1}^{\infty} \omega^j \Delta E_1 r_{1+j} - \sum_{j=1}^{\infty} \omega^j \Delta E_1 \pi_{1+j}$$

	r^n	=	r	π
Inflation	-0.14	=	-(-0.69)	-(0.83)
Recession	0.17	=	-(0.82)	-(-1.00)
Surplus	0.13	=	-(-0.14)	-(0.02)
Disc. Rate	0.16	=	-(-0.11)	-(-0.05)
Surplus, no i	0.01	=	-(-0.05)	-(0.04)

Table H.9: Terms of the inflation and bond return identities. Sample 1930-2018.

The growth response goes the other way, now rising with inflation rather than declining, and therefore contributes -0.93% inflation rather than +0.49%.

In sum, the full sample data paint a picture more than diametrically opposite. A 1% inflation shock, drawn out to 1.83% cumulative weighted inflation, is more than accounted for by 2.53% cumulative deficits, and buffered by an 0.52% disinflationary decline in discount rates, and 0.93% disinflationary rise in growth.

The full-sample results appear to support a simple fiscal theory, which would be convenient – inflation comes from persistent deficits. Discount rates only

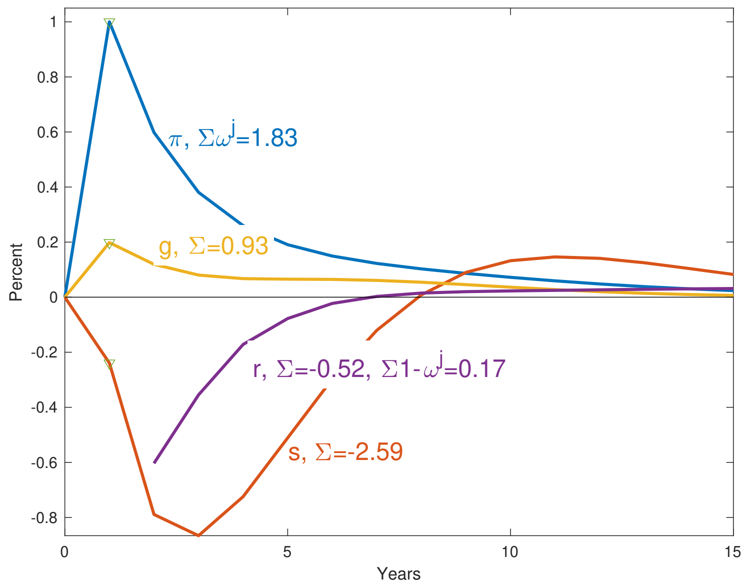

Figure H.14: Response to inflation shocks, sample 1930-2018.

mitigate that result.

Why then do I emphasize the postwar sample in the text, and relegate these to an online appendix? Clearly, the full sample results do not carry through the postwar period to the present. As in essentially all macroeconomics and monetary economics, which studies the post-1947 sample, the post-1959 sample, or, increasingly, the post-1980 sample, the war and prewar data behave differently. My interest in this paper is to characterize the behavior of inflation in postwar recessions, and the peacetime inflation of the 1970s and 1980s. Making an inference about that behavior from war and prewar data, when the central results switch in a postwar-only sample would be hugely misleading.

The nature of the prewar and war regime is interesting. Alas, the 1930-1947 sample is too short for these VAR methods. An investigation of the prewar regime with a long historical time series beckons.

What are the stylized facts and influential data points behind this switch in behavior? As before, long-run forecasts are driven by slow-moving state

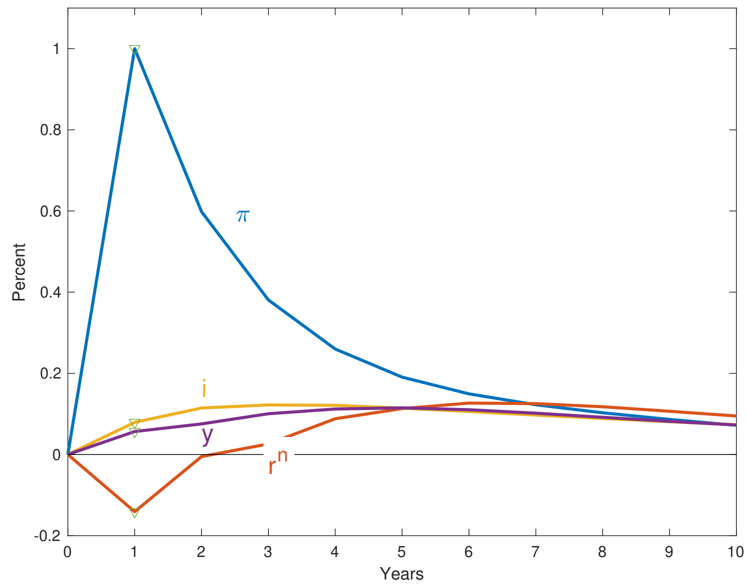

Figure H.15: Response to inflation shocks, sample 1930-2018.

variables. Think of a system

$$x_{t+1} = \alpha y_t + \varepsilon_{x,t+1}$$
$$y_{t+1} = \rho y_t + \beta \varepsilon_{x,t+1} + \varepsilon_{y,t+1}.$$

In the second equation, I express the y shock in terms of a component correlated with the x shock and an orthogonal component. In this system, the variable y is the persistent state variable for long-run responses. The long-response of x to the ε_x shock depends on how much the state variable y moves, β, and the persistence of the y variable. In response to $\varepsilon_{x,1} = 1$, the long-run x response is

$$\Delta E_1 \sum_{j=0}^{\infty} x_{1+j} = 1 + \frac{\alpha\beta}{1-\rho}.$$

With this insight, let us understand the responses of Figure H.14. Three state variables matter most. From Table H.8, inflation basically follows its own AR(1), unaffected by other variables, with a a persistence of 0.53, the same value as the postwar sample. The value of debt is the most important state variable for long-run responses with an 0.91 coefficient on its own lag. However, this

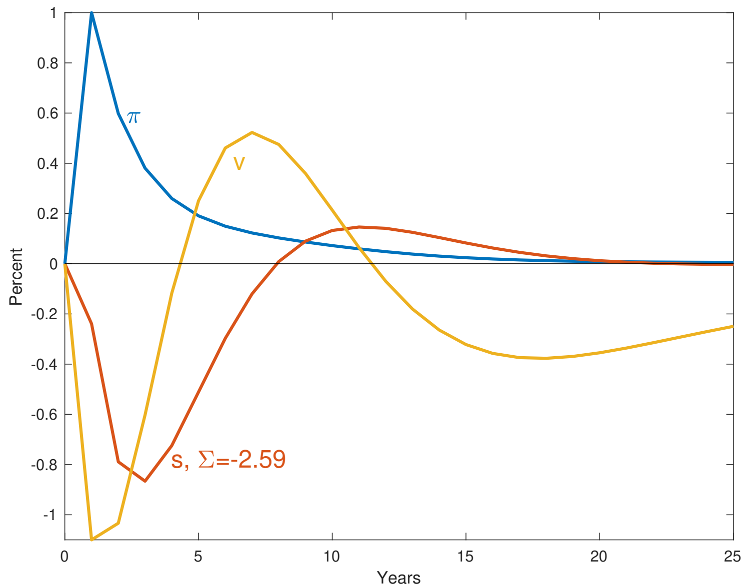

Figure H.16: Response to inflation shocks, sample 1930-2018.

debt to GDP ratio does respond strongly (-0.61) to surpluses, and to lagged growth (-1.1) as we would expect, so at medium runs it evolves jointly with these other variables. The surplus has a strong coefficient on its lag, 0.65, so in part any shock to surpluses coincident with the inflation shock will persist. The surplus also responds positively though with a small value 0.08 to the debt. This coefficient does not account for much of the short run dynamics, as the movements of surplus and debt are roughly the same size, but is the dominant force behind very long run surpluses which repay debts. The surplus responds and negatively -0.75 to inflation. This key coefficient is only -0.25 in the postwar sample. Interest rates also have a persistent response, but they move so little in this estimate that they are not an important state variable.

So, what accounts for the long deficits in Figure H.14? The surplus does not jump down by a large amount with the shock, declining only 0.25, so the surplus' autocorrelation is not a big part of the story. The big decline in surplus follows from its -0.75 coefficient on inflation, and the inflation AR(1) response. If inflation this year forecasts deficits next year, then a very simple fiscal theory

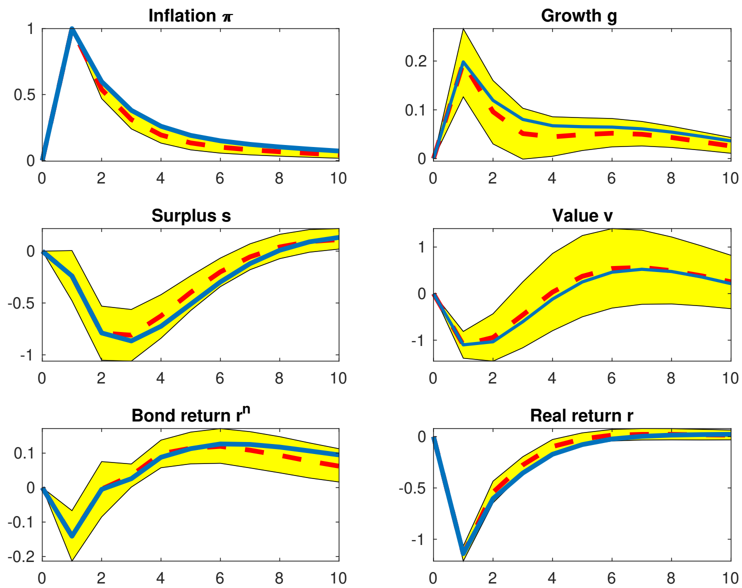

Figure H.17: Inflation shock response quantiles, sample 1930-2018.

story that inflation is accounted for by deficits follows swiftly.

But deficits should raise the value of debt, and the rise in the value of debt, which is very persistent, should pull deficits back to surplus, no? Here, another difference in the full sample is key. In the full sample, the value of debt v jumps down by 1.10% when inflation jumps up 1%, where in the postwar sample the value of debt jumps down half as much, 0.65%. Now, a low value of debt does not put into motion additional surpluses. So, the effect seen in the postwar sample of Figure 3, that deficits quickly give rise to higher debt which then triggers surpluses, is absent here because so much debt was wiped out by the inflation shock.

Contrasting Figures H.15 and 3 help to explain the differing behavior of the discount rate. In both cases, the behavior of nominal interest rates is disturbingly disconnected from the behavior of inflation. In the postwar sample, nominal rates rise immediately and very persistently. When inflation declines and passes by the higher nominal rates, real rates are higher. In the full sample, nominal rates move much less, reflecting the zero bound in the great depression and interest rate targets in WWII and the early postwar period. The resulting real rate the inverse of the inflation AR(1), and mostly negative.

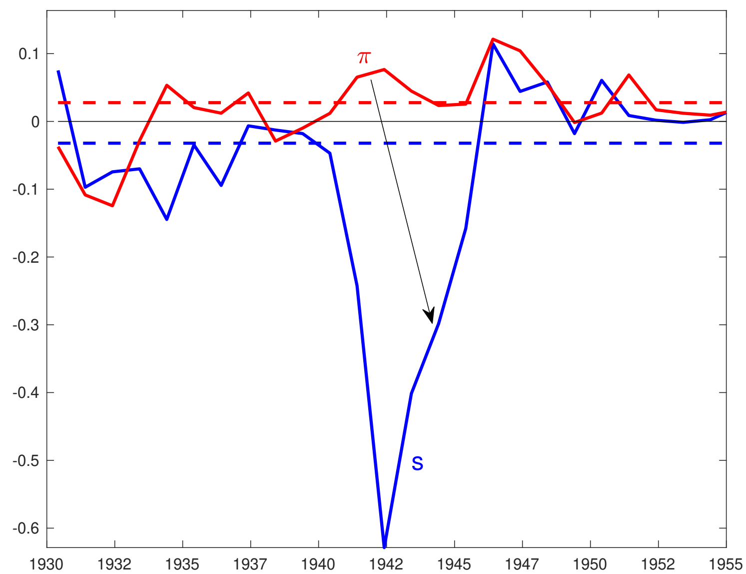

Figure H.18: Surplus and inflation during WWII

The massive deficits of 1943 and 1944 are key influential data points that account for the shift in behavior of the full sample. Estimates from the 1940-2018 sample, not shown, are similar. Figure H.18 plots inflation and surplus during WWII. The WWII deficits are immense. Inflation, more volatile in the pre-1947 period, was above its mean in the years prior to these immense deficits. Thus, this inflation preceding deficits of 1943 and 1944 drives the result that inflation forecasts deficits in the full sample, and thus the result that inflation shocks are accounted for by deficits. This is clearly not a robust result, or one that should be taken as evidence that inflation today is due to deficits.

The strong negative correlation between shocks to inflation and to the value of debt in the full sample comes from a different set of influential observations. The inflation of 1943 and 1944 was largely expected, according to the VAR, and preceded rather than coincided with increased debt. Instead, the sharp and unexpected (by the VAR) postwar inflation of 1947 coincided with a sharp decline in the real value of debt, and the sharp deflation of 1932 coincided with a sharp rise in the real value of debt. These events are conventionally regarded as times in which deflation raised the value of debt, in the first, and inflated it away, in the second. But again, one is loath to let these two observations double our estimate of the correlation between shocks to inflation and the value of debt for the postwar period.

The inflation shock is already positively correlated with a growth shock in the full sample, due to a strong positive correlation in the 1930s. As a result, the response to the inflation + growth shock (not shown) is not much different from the response to the inflation shock. Again, the 1% deflation and 2% cumulative inflation corresponds to 2.6% cumulative rise in surpluses. This time a long-run decline in discount rate contributes to deflation, but an equally large decline in growth contributes to inflation.

Appendix I. Growth and inflation plots

This section plots growth and inflation, to document that responses to a shock that combines 1% lower growth and 1% lower inflation is interesting. Figure I.19 presents GDP growth and CPI inflation.

Growth and inflation move in opposite directions during the 1970s stagflation episodes. By contrast, inflation declined in 1982 along with the comparably sized recession. This decline was permanent, not a u shape, but nonetheless coincident with the recession. Inflation moved a bit less than growth in the 2000 recession, but again moved about one for one with growth in 2008. The late 1940s and early 1950s also show roughly one for one positive comovement.

Figure I.20 presents the growth and inflation VAR residuals. These are not as clear, being annual data, consumption growth, GDP deflator, but tell roughly the same story.

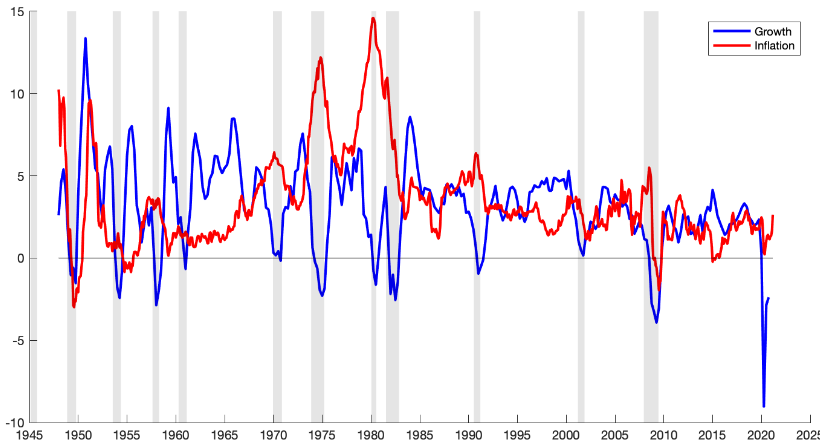

Figure I.19: CPI inflation and real GDP growth; percent changes from a year earlier.

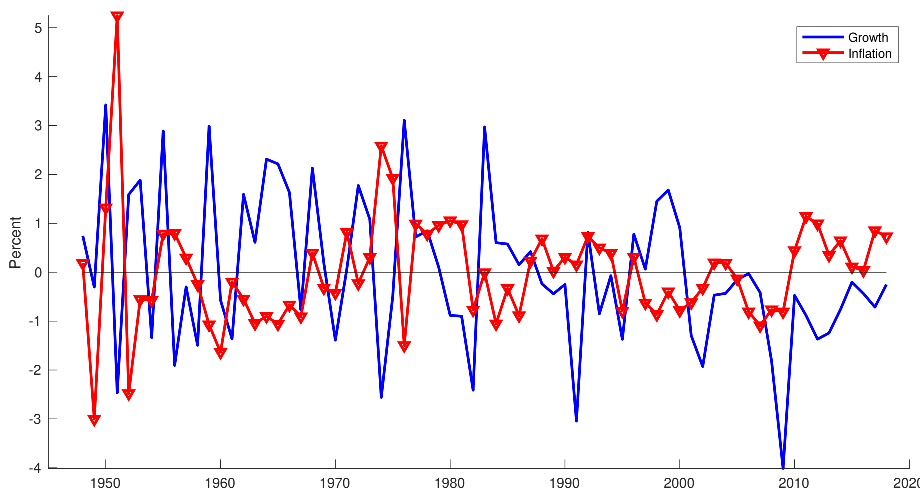

Figure I.20: VAR residuals to growth (consumption growth) and inflation (GDP deflator). Annual data 1948-2018.