Labor adjustment and productivity in the OECD
– Online Appendix –

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May 27, 2021

Contents

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1 Data

We list here the data sources and coverage; we summarize the computation of hours per worker, and we explain the variable transformations and filtering methods to obtain the business cycle statistics underlying the scatter plots.

1.1 Data sources and transformations

Coverage. The frequency of the data is quarterly. The sample period underlying the statistics shown on the graphs – excluding the graphs on the Great Recession – is 1984q1-2019q4. We compute business cycle statistics for 23 OECD countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Switzerland (CHE), Germany (DEU), Denmark (DNK), Finland (FIN), France (FRA), Ireland (IRE), Iceland (ISL), Italy (ITA), South Korea (KOR), Luxembourg (LUX), Mexico (MEX), Netherlands (NLD), New Zealand (NZL), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), United Kingkom (GBR), United States (USA).

Data sources. In the following, we list the time series collected. We provide the frequency, units, source and the coverage per series, per country.

- *Pop*: Working-age population aged 15 to 64 (quarterly, number of persons). Source: OECD, Oxford Economics, and International Labour Organization (for ISL and LUX). Data range: 1984q1-2019q4 (2003q1-2019q4 for ISL and LUX). Data were seasonally adjusted using the Census X12 routine.

- *Real output*: Real Gross Domestic Product (quarterly, in constant US dollars). Source: OECD. Data range: 1984q1 to 2019q4.

- *URate*: Unemployment rate (quarterly, percent of labor force). Source: OECD Economic Outlook, Deutsche Bundesbank (DEU) and Eurostat (IRL and LUX). Data range: 1984q1 to 2019q4 (MEX: 1991q1-2019q4).

- *Empl*: Total employment (quarterly, number of persons). Source: OECD Economic Outlook, Ohanian and Raffo (2012) (DEU: 1984q1-1990q4, IRL: 1984q1-1989q4) and Haver Analytics (DESETE@GERMANY: 1991q1-2019q4). Data range: 1984q1 to 2019q4 (LUX: 1985q1-2019q4, MEX: 1995q1-2019q4).

- *Hours*: Hours worked per worker (annualized, number). Source: Eurostat Labour Force Survey (`https://ec.europa.eu/eurostat/web/lfs/data/database`), ILO, National Offices of Statistics. See more details in Table 1 below.

- *EPL*: Employment protection legislation (annual, index). Source: OECD. Strictness of employment protection: individual and collective dismissals (regular contracts). Source: OECD. Data for the year 2019.

- *RPay*: Redundancy pay (annual, weeks of salary). Source: World Bank. Data for the year 2018.

Computation of hours per worker series. To compute the series for hours per worker, we follow a four-step procedure. First, we transform all the series to express them in the same units, namely quarterly hours worked per worker. For the data series given in average weekly hours worked, we multiply by 52/4. For the data series given in quarterly total hours worked, we divide by quarterly total employment. Second, we need to seasonally adjust the hours series that are available only in unadjusted form. For this purpose, we use the Census X12 method. Third, to annualize the adjusted series obtained in the third step, we multiply by four the adjusted quarterly hours worked per worker series. Finally, Ohanian and Raffo (2012) provide an extended data set going up to 2016q4.[1] We extend their data series up to 2019q4.

Transformations. To compute total hours, we first compute the product of employment and (annualized) hours per worker. The variable '*Total hours*' is expressed as a rate, i.e. it is divided by the total yearly time endowment, which is set to 365 days times 14 hours/day, multiplied by the working-age population.

$$\textit{Total hours} = \frac{\textit{Empl} \cdot \textit{Hours}}{\textit{Pop} \cdot 365 \cdot 14}.$$

Labor productivity is computed as real output divided by total hours,

$$\textit{Prod} = \frac{\textit{Real output}}{\textit{Total hours}}.$$

The cyclicality of labor productivity and the relative volatility of employment are computed as:

$$\text{Cyclicality of labor productivity} = corr(\mathcal{P}_t, y_t),$$

$$\text{Relative employment volatility} = \frac{s.d.(n_t)}{s.d.(y_t)},$$

where $\mathcal{P}_t$ is the cyclical component of $\log(\textit{Prod})$, n_t is the cyclical component of $\log(\textit{Empl})$, y_t is the cyclical component of $\log(\textit{Real output})$; *s.d.* denotes the standard deviation. Labor productivity is real output per hour worked. Unemployment u_t is expressed as a rate, i.e. it is

[1]See `http://andrearaffo.com/araffo/Research.html`.

Table 1: Data on hours worked per worker

Country	S. Adj.	Description	Source (Identifier)	Coverage
AUS	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
AUS	SA	HW (total)	Austral. Bureau of Stat. (A84426298K)	2017q1 to 2019q4
AUS	SA	Employment	Haver Analytics (AUSELTT@ANZ)	2017q1 to 2019q4
AUT	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
AUT	SA	weekly HW p.w.	Eurostat	2017q1 to 2019q4
BEL	SA	HW p.w.	OECD Economic Outlook	1984q1 to 2019q4
CAN	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
CAN	SA	HW (total)	Haver Analytics (v4391505)	2017q1 to 2019q4
CAN	SA	Employment	Haver Analytics (v2062811@CANADA)	2017q1 to 2019q4
CHE	SA	HW p.w.	OECD Economic Outlook	1984q1 to 2019q4
DEU	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
DEU	SA	HW (total)	Haver Analytics (DESNHT@GERMANY)	2017q1 to 2019q4
DEU	SA	Employment	Haver Analytics (DESETE@GERMANY)	2017q1 to 2019q4
DNK	SA	HW p.w.	OECD Economic Outlook	1984q1 to 2019q4
FIN	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
FIN	NSA	HW (total)	Haver Analytics (FINEH@NORDIC)	2017q1 to 2019q4
FIN	NSA	Employment	Haver Analytics (FINET@NORDIC)	2017q1 to 2019q4
FRA	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
FRA	SA	HW p.w.	OECD Economic Outlook	2017q1 to 2019q4
IRL	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
IRL	SA	HW p.w.	OECD Economic Outlook	2017q1 to 2019q4
ISL	SA	HW per worker	Haver Analytics (ISSEWH@NORDIC)	2003q1 to 2019q4
ITA	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
ITA	SA	HW (total)	Haver Analytics (ITSNF@ITALY)	2017q1 to 2019q4
ITA	SA	Employment	Haver Analytics (ITSET@ITALY)	2017q1 to 2019q4
KOR	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
KOR	SA	HW p.w.	Haver Analytics (H542EK@EMERGEPR)	2017q1 to 2019q4
LUX	SA	HW (total)	Haver Analytics (J137WETE@EUDATA)	1995q1 to 2019q4
LUX	SA	Employment	Haver Analytics (J137TETE@EUDATA)	1995q1 to 2019q4
MEX	SA	HW p.w.	OECD Economic Outlook	1991q1 to 2019q4
NLD	SA	HW p.w.	OECD Economic Outlook	1984q1 to 2019q4
NZL	SA	HW p.w.	OECD Economic Outlook	1984q1 to 2019q4
NOR	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
NOR	SA	HW (total)	Haver Analytics (NOSEH@NORDIC)	2017q1 to 2019q4
NOR	SA	Employment	Haver Analytics (NOSET@NORDIC)	2017q1 to 2019q4
PRT	SA	HW p.w.	OECD Economic Outlook	1995q1 to 2019q4
ESP	SA	HW p.w.	OECD Economic Outlook	1984q1 to 2019q4
SWE	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
SWE	SA	HW p.w.	OECD Economic Outlook	2017q1 to 2019q4
GBR	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
GBR	SA	HW (total)	Haver Analytics (YBUSQ@UK)	2017q1 to 2019q4
GBR	SA	Employment	Haver Analytics (MGRZQ@UK)	2017q1 to 2019q4
USA	SA	HW per worker	Ohanian and Raffo 2012	1984q1 to 2016q4
USA	SA	HW (total)	Bureau of Labor Statistics	2017q1 to 2019q4
USA	SA	Employment	Bureau of Labor Statistics	2017q1 to 2019q4

Notes. NSA: not seasonally adjusted, SA: seasonally adjusted, HW: hours worked, p.w.: per worker.

measured in percentage points. The unemployment rate is defined as the number of registered unemployed persons in percent of the labor force. Volatility is measured as the standard deviation, such that

$$\text{Unemployment volatility} = s.d.(u_t).$$

The cyclical component of labor productivity and unemployment is extracted using the Hodrick-Prescott (1997) trend-cycle decomposition with smoothing parameter set to 1600.

1.2 Business cycle statistics

We present cross-country scatter plots showing business cycle statistics per country. The baseline plots show HP-filtered data; below, we compute the statistics using three alternative filters.

Baseline plots. The top left subplot of Figure 1 below and in the main text shows a cross-country scatter plot with the cyclicality of labor productivity on the vertical axis vs. the volatility of employment (relative to output) on the horizontal axis. Figure 2 shows the cyclicality of labor productivity on the vertical axis vs. the volatility of unemployment on the horizontal axis. Figure 3 shows the cyclicality of labor productivity on the vertical axis vs. the volatility of hours per worker relative to employment on the horizontal axis.

Alternative filtering techniques. We use the HP filter (Hodrick and Prescott, 1997) in our baseline plots, as this is the most common filter used in business cycle analysis. Figure 1 also shows business cycle statistics based on alternative filtering methods, more specifically, the band pass filter of Christiano and Fitzgerald (2003), the 4th difference filter and the Hamilton filter (Hamilton, 2018).

Great Recession. Restricting the underlying data series to the Great Recession period from 2008q3 until 2013q2 (Christiano et al., 2015), we obtain the plot in Figure 4.

Alternative proxies for labor market frictions Figures 5 and 6 relate the cyclicality of productivity across countries to two alternative measures of labor market frictions: (a) an index of employment protection legislation in 2019 from the OECD, and (b) redundancy pay in 2018, measured in weeks of salary, obtained from the World Bank's Doing Business report.

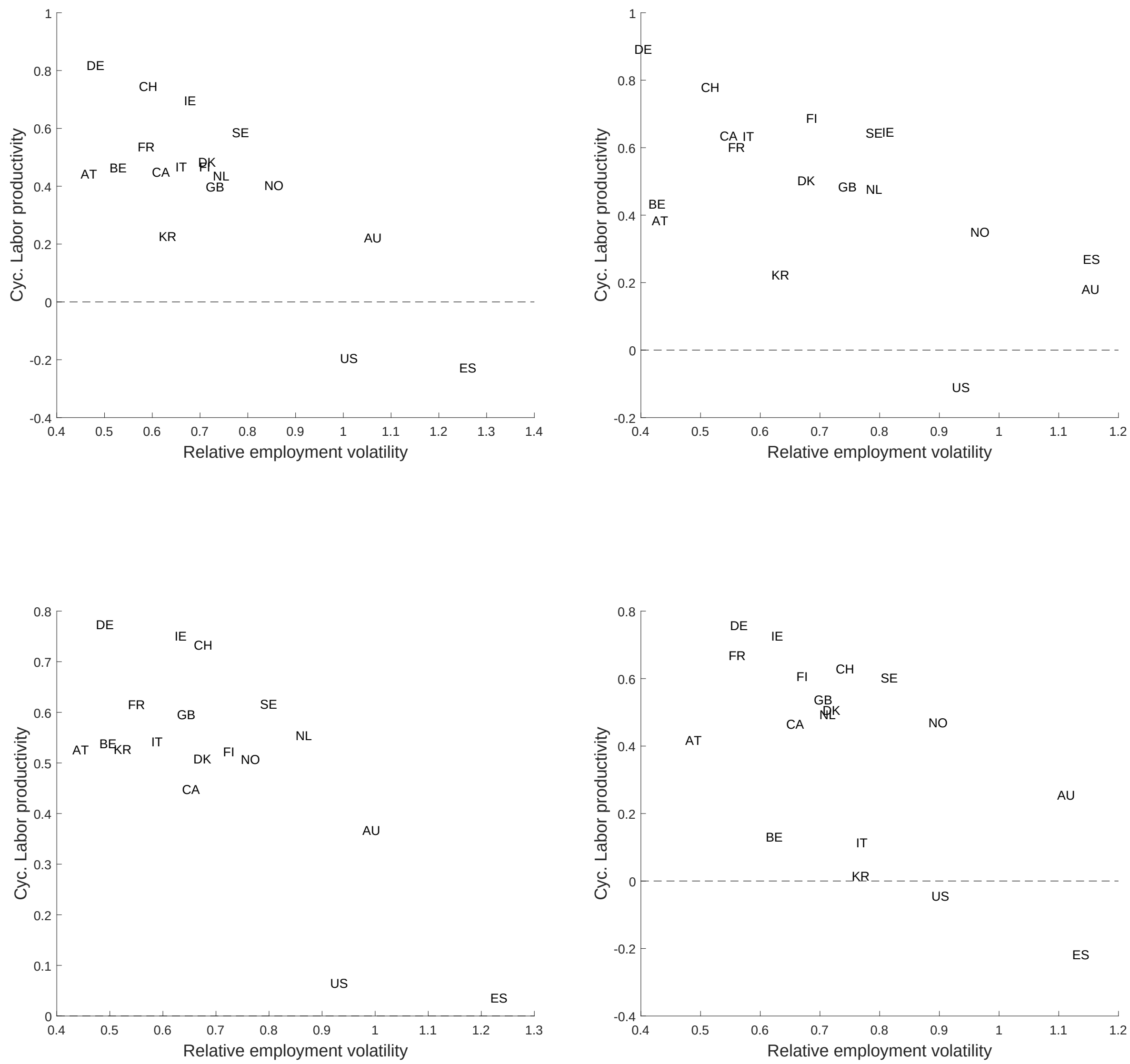

Figure 1: Relative employment volatility and cyclicality of labor productivity

Notes. Sample period 1984q1 to 2019q4. 18 Countries covered: Australia (AU), Austria (AT), Belgium (BE), Canada (CA), Switzerland (CH), Germany (DE), Denmark (DK), Finland (FI), France (FR), Ireland (IE), Italy (IT), South Korea (KR), Netherlands (NL), Norway (NO), Spain (ES), Sweden (SE), United Kingkom (GB), United States (US). Labor productivity measured as quarterly real output per hour. Employment volatility measured relative to output. Volatility measured as standard deviation, cyclicality measured as correlation between cyclical components of output and labor productivity. Clockwise from top left subplot: HP filter, band pass filter, fourth-difference filter, Hamilton filter.

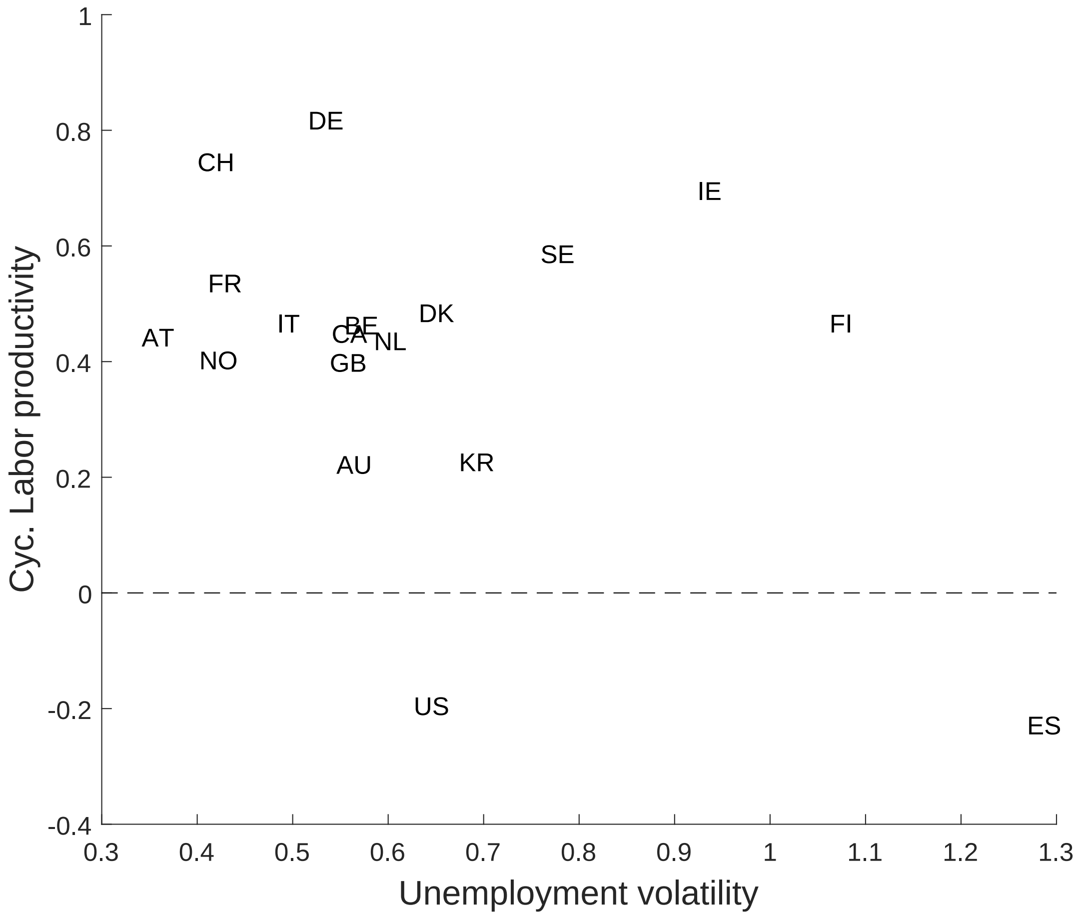

Figure 2: Unemployment volatility and cyclicality of labor productivity

Notes. Sample period 1984q1 to 2019q4. Countries covered are the same as in Figure 1. Labor productivity measured as quarterly real output per hour. Hours per worker volatility measured relative to employment. Volatility measured as standard deviation, cyclicality measured as correlation between cyclical components of output and labor productivity. Cyclical component of log productivity and unemployment extracted with HP filter. Unemployment measured in percentage points.

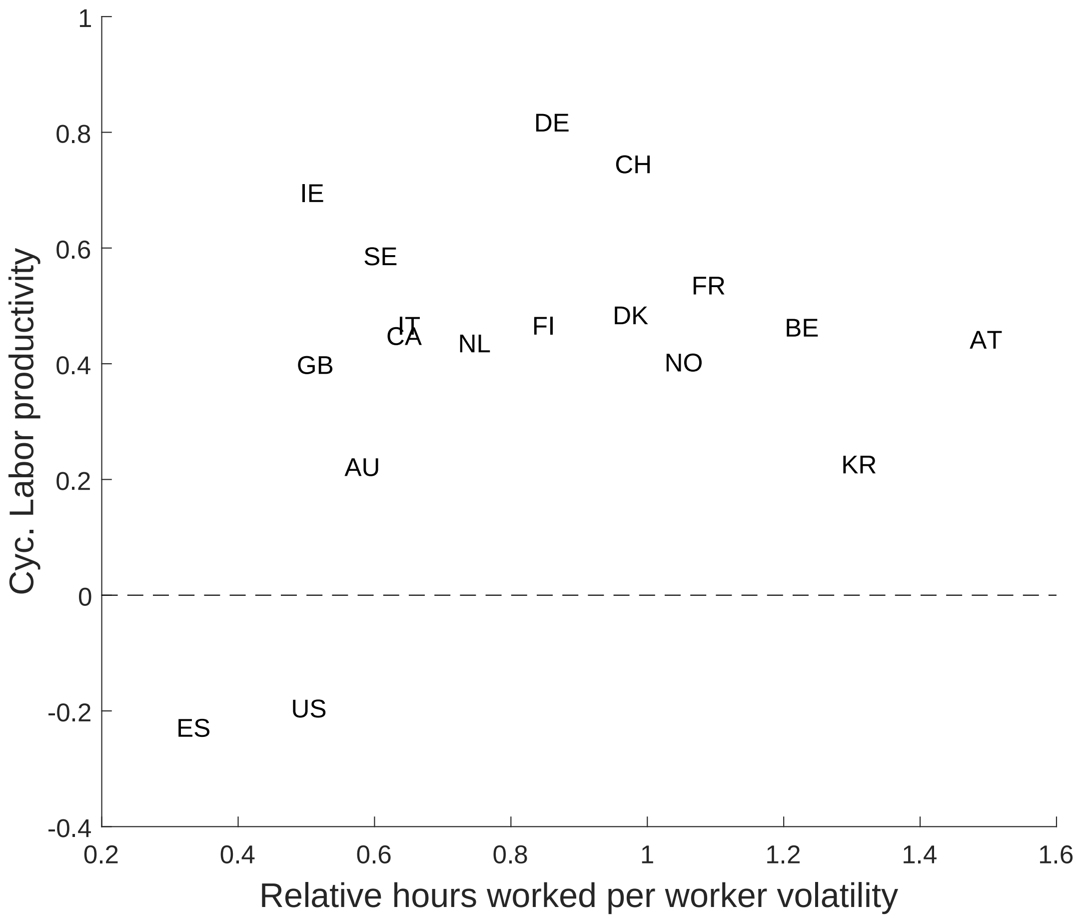

Figure 3: Relative hours volatility and cyclicality of labor productivity

Notes. Sample period 1984q1 to 2019q4. Countries covered are the same as in Figure 1. Labor productivity measured as quarterly real output per hour. Hours per worker volatility measured relative to employment. Volatility measured as standard deviation, cyclicality measured as correlation between cyclical components of output and labor productivity. Cyclical component extracted with the HP filter.

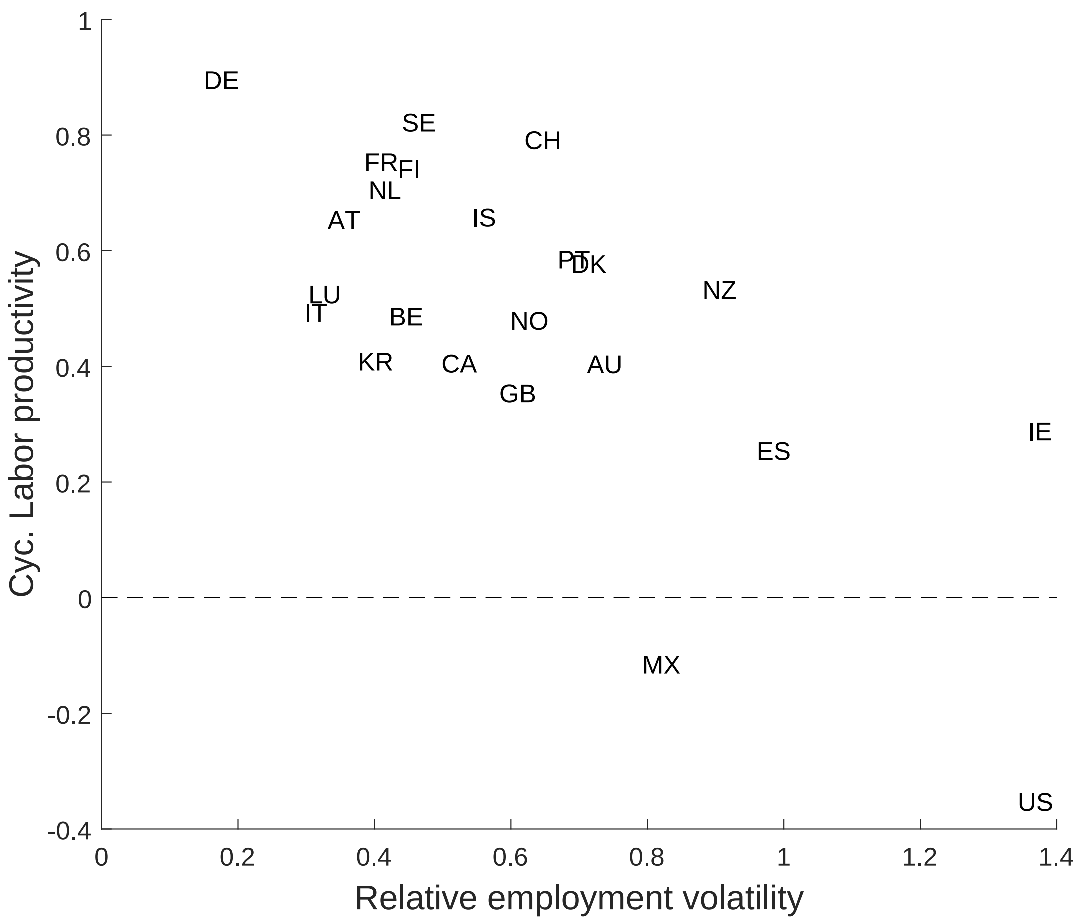

Figure 4: Relative employment volatility and cyclicality of labor productivity: Great Recession

Notes. Sample period 2008q3 to 2013q2. Countries covered are the same ones as in Figure 1, plus Mexico (MX), New Zealand (NZ), Portugal (PT), Iceland (IS), Luxembourg (LU). Labor productivity measured as quarterly real output per hour. Employment volatility measured relative to output. Volatility measured as standard deviation, cyclicality measured as correlation between cyclical components of output and labor productivity. Cyclical component extracted with the HP filter.

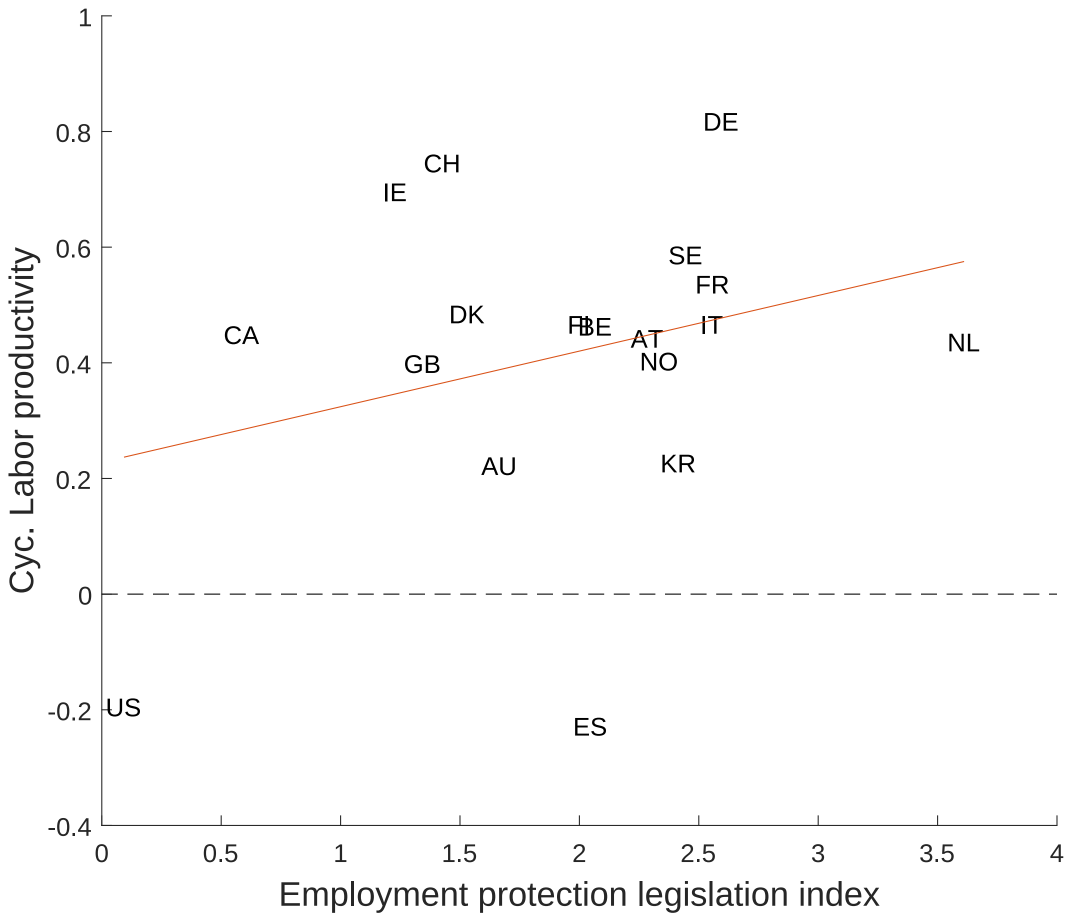

Figure 5: Employment protection and cyclicality of labor productivity

Notes. Sample period for labor productivity cyclicality 1984q1 to 2019q4. Countries covered are the same as in Figure 1. Labor productivity measured as quarterly real output per hour. Cyclicality measured as correlation between cyclical components of output and labor productivity. Cyclical component extracted with the HP filter. Employment Protection Legislation (EPL) index refers to the value in 2019.

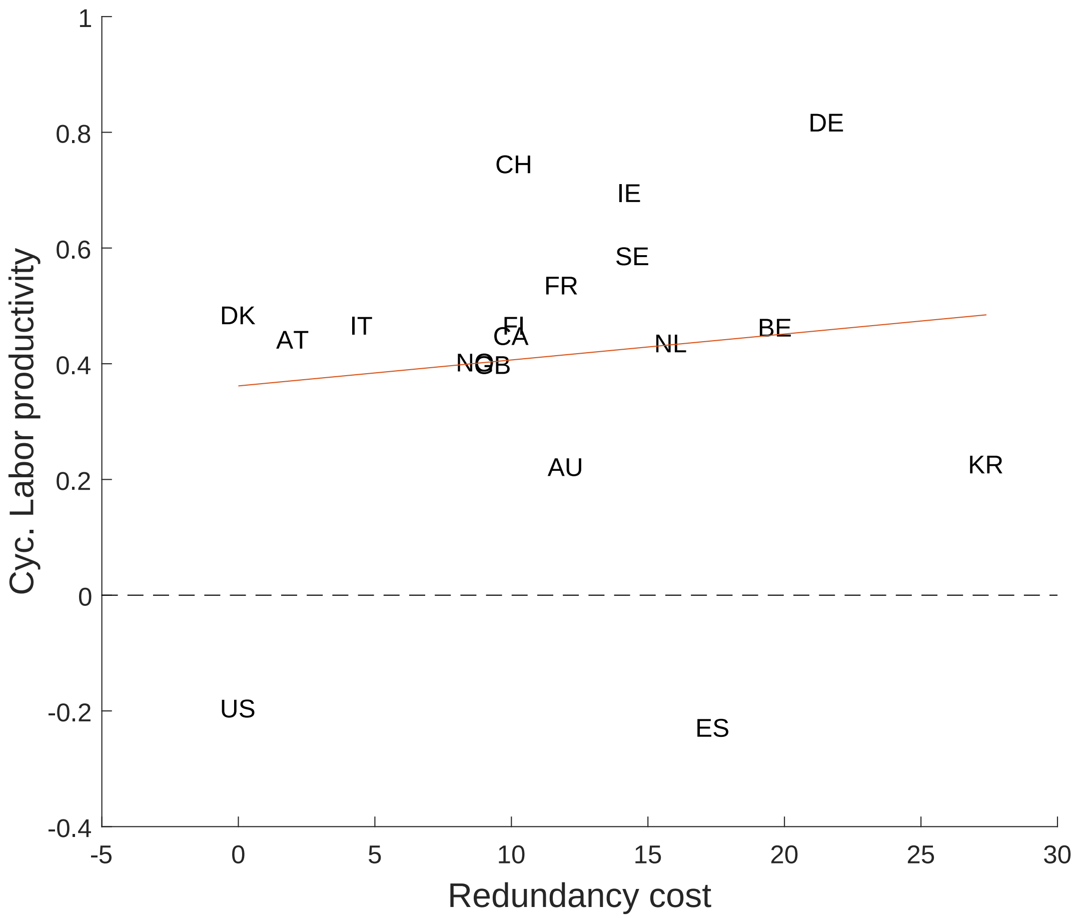

Figure 6: Redundancy pay and cyclicality of labor productivity

Notes. Sample period for labor productivity cyclicality 1984q1 to 2019q4. Countries covered are the same as in Figure 1. Labor productivity measured as quarterly real output per hour. Cyclicality measured as correlation between cyclical components of output and labor productivity. Cyclical component extracted with the HP filter. Redundancy pay measured in weeks of salary; number refers to the value in 2018.

2 Baseline model

Our baseline model has three labor margins, employment, hours and effort, the latter modelled as in Bils and Cho (1994). Employment adjustment costs shift the burden of adjustment away from the extensive margin and towards the two intensive margins.

2.1 Households

There is a representative household with a large number of members, a fraction $n_t \in (0,1)$ of which are employed at time t. Per employed member, h_t denotes hours worked and e_t is effort per hour. All household members consume the average per capita consumption, c_t. There are no unemployment benefits; for simplicity we normalize the value of home production or leisure to zero. The household chooses paths for consumption, capital k_t and employment to maximize expected lifetime utility,

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \frac{A}{1+\eta} n_t^{1+\eta} - \frac{B}{1+\gamma} n_t h_t^{1+\gamma} - \frac{C}{1+\tau} n_t h_t e_t^{1+\tau} \right\}, \tag{1}$$

subject to the budget constraint

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 \leq w_t h_t n_t + r_t k_t + D_t, \tag{2}$$

where $\beta \in (0,1)$ is the household's subjective discount factor, $\delta \in (0,1)$ is the capital depreciation rate, and $\phi_k \geq 0$ scales capital adjustment costs. The elasticities $\eta, \gamma, \tau \geq 0$ capture the curvature of labor disutility in employment, hours and effort, respectively. The parameters A, B and C are non-negative weights on, respectively, employment, hours and effort in labor disutility. Furthermore, w_t is the wage per hour, r_t is the rental rate on capital, and D_t are dividends. All variables are expressed in units of the consumption good.

With the household's Lagrangian problem written as

$$\begin{aligned}\mathcal{L} =& \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \frac{A}{1+\eta} n_t^{1+\eta} - \frac{B}{1+\gamma} n_t h_t^{1+\gamma} - \frac{C}{1+\tau} n_t h_t e_t^{1+\tau} \right| \\ & - \left| \lambda_t \left[c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 - w_t h_t n_t - r_t k_t - D_t \right] \right\},\end{aligned} \tag{3}$$

the first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \left(\frac{1}{c_t} - \lambda_t \right) \stackrel{!}{=} 0, \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\beta^t \lambda_t [1 + \phi_k(k_{t+1} - k_t)] + \beta^{t+1} \mathrm{E}_t \{ \lambda_{t+1} [(1-\delta) + \phi_k(k_{t+2} - k_{t+1}) + r_{t+1}] \} \stackrel{!}{=} 0, \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = -\beta^t \left(A n_t^\eta + \frac{B}{1+\gamma} h_t^{1+\gamma} + \frac{C}{1+\tau} h_t e_t^{1+\tau} \right) + \beta^t \lambda_t w_t h_t \overset{!}{=} 0. \tag{6}$$

Combining (4) with (5) and (6) respectively, the household's equilibrium conditions satisfy:

$$1 + \phi_k (k_{t+1} - k_t) = \mathrm{E}_t \left\{ \beta_{t,t+1} \left[(1-\delta) + \phi_k (k_{t+2} - k_{t+1}) + r_{t+1} \right] \right\}, \tag{7}$$

$$\left(A n_t^\eta + \frac{B}{1+\gamma} h_t^{1+\gamma} + \frac{C}{1+\tau} h_t e_t^{1+\tau} \right) c_t = w_t h_t, \tag{8}$$

where $\beta_{t-1,t} = \beta \lambda_t / \lambda_{t-1}$ is the household's stochastic discount factor, (7) is the optimality condition for capital, and (8) equates the marginal rate of substitution between employment and consumption on the left hand side to the wage per employee, $w_t h_t$, on the right hand side.

2.2 Firms

Firms' current profits, paid out to the household as dividends, can be written as:

$$D_t = y_t - w_t h_t n_t - r_t k_t. \tag{9}$$

From a firm's output - which is net of employment adjustment costs - we subtract wage payments and capital rental costs to obtain profits. Firms choose optimally their inputs capital k_t and employment n_t to maximize the expected discounted stream of profits,

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{0,t} \left\{ y_t - w_t h_t n_t - r_t k_t \right\}, \tag{10}$$

subject to a production function that incorporates employment adjustment costs g_t,

$$y_t = z_t k_t^\alpha (e_t h_t n_t)^{1-\alpha} - g_t, \tag{11}$$

where z_t denotes (exogenous) total factor productivity and $\alpha \in (0,1)$ is the capital share. The production technology is thus a Cobb-Douglas function of capital and total effective hours, the latter defined as

$$H_t = e_t h_t n_t. \tag{12}$$

We use $\beta_{0,t}$ for discounting in the firm's problem, given that the households own the firms.

As in Cacciatore et al. (2017), we regard the employment adjustment cost as a pure loss and not as a transfer to workers. Llosa et al. (2014) and Hopenhayn and Rogerson (1993) model g_t as a firing cost. Instead, our employment adjustment cost is approximated as a quadratic function,

$$g_t = \frac{\phi_n}{2} (n_t - n_{t-1})^2, \tag{13}$$

such that the first derivative with respect to employment is $\partial g_t / \partial n_t = \phi_n(n_t - n_{t-1})$. In steady state, the employment adjustment cost (13) is zero.

The firm's Lagrangian is

$$\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{0,t} \{ z_t k_t^{\alpha} H_t^{1-\alpha} - 0.5 \cdot \phi_n (n_t - n_{t-1})^2 - w_t h_t n_t - r_t k_t \}. \tag{14}$$

The firm's first order conditions with respect to capital and employment are, respectively,

$$\frac{\partial \mathcal{L}}{\partial k_t} = \beta_{0,t} \left(\alpha z_t k_t^{\alpha-1} H_t^{1-\alpha} - r_t \right) \overset{!}{=} 0,$$

$$\frac{\partial \mathcal{L}}{\partial n_t} = \beta_{0,t} [(1-\alpha) z_t k_t^{\alpha} H_t^{-\alpha} e_t h_t - w_t h_t - \phi_n (n_t - n_{t-1})] + \mathrm{E}_t \{ \beta_{0,t+1} \phi_n (n_{t+1} - n_t) \} \overset{!}{=} 0.$$

We can write these equilibrium conditions as follows:

$$\alpha \frac{y_t + g_t}{k_t} = r_t, \tag{15}$$

$$(1-\alpha) \frac{y_t + g_t}{n_t} = w_t h_t + \phi_n (n_t - n_{t-1}) - \mathrm{E}_t \{ \beta_{t,t+1} \phi_n (n_{t+1} - n_t) \}, \tag{16}$$

The left hand side of (15) is the firm's marginal product of capital, $\alpha z_t k_t^{\alpha-1} H_t^{1-\alpha}$. At the optimum, this has to be equated with the marginal cost of capital, i.e. the rental rate r_t. In (16), the marginal product of employment, $(1-\alpha) z_t k_t^{\alpha} H_t^{-\alpha}$, has to equal the marginal cost of employment, which has three components. First, an additional employee who works h_t hours costs the firm $w_t h_t$ in wage payments. Second, the change in the firm's workforce gives rise to an adjustment cost $\phi_n(n_t - n_{t-1})$ in the current period. Third, since next period's employment adjustment cost depends on current employment, $\phi_n(n_{t+1} - n_t)$ also enters the first order condition.

2.3 Effort

The worker chooses the combination of hours and effort that minimizes labor disutility, given a certain number of effective hours demanded by his employer. Each period, the worker thus solves the following intratemporal optimization problem:

$$\min_{e_t, h_t} \frac{A}{1+\eta} n_t^{1+\eta} + \frac{B}{1+\gamma} n_t h_t^{1+\gamma} + \frac{C}{1+\tau} n_t h_t e_t^{1+\tau}, \tag{17}$$

subject to the production technology (11). The Lagrangian can be written as

$$\mathcal{L} = \frac{A}{1+\eta} n_t^{1+\eta} + \frac{B}{1+\gamma} n_t h_t^{1+\gamma} + \frac{C}{1+\tau} n_t h_t e_t^{1+\tau} - \varphi_t [y_t - z_t k_t^{\alpha} (e_t h_t n_t)^{1-\alpha} + g_t].$$

Taking the employment adjustment cost g_t as given, the first order conditions for h_t and e_t are, respectively,

$$\frac{\partial \mathcal{L}}{\partial h_t} = B n_t h_t^{\gamma} + \frac{C}{1+\tau} n_t e_t^{1+\tau} + \varphi_t (1-\alpha) z_t k_t^{\alpha} (e_t h_t n_t)^{-\alpha} e_t n_t \stackrel{!}{=} 0,$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = C n_t h_t e_t^{\tau} + \varphi_t (1-\alpha) z_t k_t^{\alpha} (e_t h_t n_t)^{-\alpha} h_t n_t \stackrel{!}{=} 0,$$

which we can write as

$$B n_t h_t^{\gamma} + \frac{C}{1+\tau} n_t e_t^{1+\tau} + \varphi_t (1-\alpha) \frac{y_t + g_t}{h_t} = 0, \tag{18}$$

$$C n_t h_t e_t^{\tau} + \varphi_t (1-\alpha) \frac{y_t + g_t}{e_t} = 0, \tag{19}$$

We write (19) as $\varphi_t(1-\alpha)(y_t + g_t)/h_t = -C n_t e_t^{1+\tau}$ and plug this into (18) to obtain

$$B n_t h_t^{\gamma} - \frac{\tau}{1+\tau} C n_t e_t^{1+\tau} = 0, \tag{20}$$

Solving (20) for e_t, we obtain equilibrium effort as a function of hours worked,

$$e_t = e_0 h_t^{\frac{\gamma}{1+\tau}}, \tag{21}$$

where the elasticity of effort with respect to hours is given by $\frac{\gamma}{1+\tau} > 0$ and we define

$$e_0 = \left(\frac{1+\tau}{\tau} \frac{B}{C} \right)^{\frac{1}{1+\tau}}. \tag{22}$$

Now, we can substitute the equilibrium effort function (21) back into the production function (11) to obtain

$$y_t = z_t k_t^{\alpha} (e_0 h_t^{\phi} n_t)^{1-\alpha} - g_t. \tag{23}$$

where we define

$$\phi = 1 + \frac{\gamma}{1+\tau}. \tag{24}$$

Also, we can write the household's optimality condition for employment (8) more conveniently as:

$$\left(An_t^\eta + \frac{B}{1+\gamma}h_t^{1+\gamma} + \frac{B}{\tau}h_t^{1+\gamma}\right)c_t = w_t h_t,$$

which simplifies to

$$\left(An_t^\eta + \frac{1+\gamma+\tau}{(1+\gamma)\tau}Bh_t^{1+\gamma}\right)c_t = w_t h_t. \tag{25}$$

Similarly, substituting effort (21) in labor disutility (17), we obtain

$$\frac{A}{1+\eta}n_t^{1+\eta} + \frac{1+\gamma+\tau}{(1+\gamma)\tau}Bn_t h_t^{1+\gamma}. \tag{26}$$

2.4 Hours

Hours are determined jointly by the firm and the worker to maximize the sum of both parties' surpluses. The firm's surplus is given by profits D_t, see (9). The household's surplus is wage income less the disutility from working (26) which we divide by λ_t to convert utils into consumption goods.

$$y_t - w_t h_t n_t - r_t k_t + w_t h_t n_t - \left(\frac{A}{1+\eta}n_t^{1+\eta} + \frac{1+\gamma+\tau}{(1+\gamma)\tau}Bn_t h_t^{1+\gamma}\right)\frac{1}{\lambda_t}.$$

Substituting y_t using the production function (23), cancelling the firm's wage bill with the household's wage income $w_t h_t n_t$, setting $1/\lambda_t = c_t$ from (4), and ignoring terms that are independent of hours, we can write the optimization problem as follows:

$$\max_{h_t}\ z_t k_t^\alpha (e_0 h_t^\phi n_t)^{1-\alpha} - \frac{1+\gamma+\tau}{(1+\gamma)\tau}Bn_t h_t^{1+\gamma}c_t.$$

The first order condition for hours worked satisfies

$$(1-\alpha)\phi e_0 h_t^{\phi-1} n_t z_t k_t^\alpha (e_0 h_t^\phi n_t)^{-\alpha} - \frac{1+\gamma+\tau}{\tau}Bn_t h_t^\gamma c_t = 0,$$

which simplifies to:

$$(1-\alpha)\phi\frac{y_t+g_t}{h_t} = \frac{1+\gamma+\tau}{\tau}Bn_t h_t^\gamma c_t. \tag{27}$$

2.5 Closing the model

Imposing that the household budget constraint (2) hold with equality, and combining it with firm profits (9), we obtain the aggregate accounting relation,

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1}-k_t)^2 + a_t = y_t, \tag{28}$$

where we have introduced a_t as an exogenous component of aggregate demand, such as net exports or government spending. Demand shocks and technology shocks follow first-order autoregressive processes in logarithms with *i.i.d.* normal innovations, that is:

$$\ln a_t = (1-\rho_a)\ln a + \rho_a \ \ln a_{t-1} + \nu_t^a, \tag{29}$$

$$\ln z_t = (1-\rho_z)\ln z + \rho_z \ln z_{t-1} + \nu_t^z, \tag{30}$$

where $\nu_t^i \sim N(0,\sigma_i)$, $i = z, a$. Labor productivity is defined as (measured) output divided by total hours worked,

$$\mathcal{P}_t = \frac{y_t}{n_t h_t}. \tag{31}$$

Measured labor productivity in (31) is distinct from 'true' productivity, which is output per *effective* hour worked, y_t/H_t.

A formal definition of equilibrium, as summarized in Table 2, is as follows.

Definition 1 *A competitive equilibrium in the baseline model is a set of sequences* $\{k_t,\ n_t,\ y_t,\ r_t,\ w_t,\ e_t,\ h_t,\ c_t\}_{t=0}^{\infty}$ *that satisfy the household's first order conditions for capital (7) and employment (8), the firm's production function (11), the firm's demand for capital (15) and employment (16), the optimality conditions for effort (21) and hours (27), and aggregate accounting (28), given exogenous processes for technology* $\{z_t\}_{t=0}^{\infty}$ *and demand shocks* $\{a_t\}_{t=0}^{\infty}$.

Table 2: Equilibrium conditions, baseline model

$1+\phi_k(k_t-k_{t-1}) = \mathrm{E}_t\{\beta_{t,t+1}[1-\delta+\phi_k(k_{t+1}-k_t)+r_{t+1}]\}$	FOC capital
$[An_t^{\eta} + Bh_t^{1+\gamma}/(1+\gamma) + Ce_t^{1+\tau}h_t/(1+\tau)]c_t = w_t h_t$	FOC employment
$y_t = z_t k_{t-1}^{\alpha} H_t^{1-\alpha} - g_t$	Production function
$\alpha(y_t+g_t)/k_{t-1} = r_t$	Demand for capital
$(1-\alpha)(y_t+g_t)/n_t = w_t h_t + \phi_n(n_t-n_{t-1}) - \mathrm{E}_t\{\beta_{t,t+1}\phi_n(n_{t+1}-n_t)\}$	Demand for employment
$e_t = [(1+\tau)B/(\tau C)]^{1/(1+\tau)}\, h_t^{\gamma/(1+\tau)}$	Effort
$(1-\alpha)\phi(y_t+g_t)/h_t = [(1+\gamma+\tau)/\tau]Bn_t h_t^{\gamma} c_t$	Hours
$c_t + k_t - (1-\delta)k_{t-1} + 0.5\cdot\phi_k(k_t-k_{t-1})^2 + a_t = y_t$	Aggregate accounting
$H_t = e_t h_t n_t$	Total effective hours
$g_t = 0.5\cdot\phi_n(n_t-n_{t-1})^2$	Employment adj. cost
$\beta_{t-1,t} = \beta c_{t-1}/c_t$	Stoch discount factor

Note. Endogenous variables: k_t, n_t, y_t, r_t, w_t, e_t, h_t, c_t, H_t, g_t, $\beta_{t-1,t}$.

2.6 Steady state and calibration

Let a variable without a time subscript denote its steady state value. We consider the non-stochastic steady state, where both technology and demand shocks are switched off, $\varepsilon_t^z = \varepsilon_t^a = 0$ for all t.

We normalize gross output (or GDP) at the steady state to unity and set $y + g = 1$. Since employment adjustment costs are zero in steady state, this implies that net output is $y = 1$. Steady state hours and effort are also normalized to 1. From the household's first order condition for capital (7), the rental rate on capital satisfies $r = 1/\beta - (1 - \delta)$. The firm's capital demand (15) is used to derive the capital stock, $k = \alpha(y + g)/r$. Given net output, exogenous demand a and the value of the capital stock, we derive consumption from the aggregate accounting identity (28), $c = y - a - \delta k$. From the production function (11), we find total effective hours at the steady state, $H = [(y + g)/(zk^\alpha)]^{1/(1-\alpha)}$. Then, steady state employment is given by $n = H/(he)$ from the definition of total effective hours. The wage rate can be derived from the firm's employment demand (16). In steady state, the wage rate w is proportional to gross output per hour, $(y + g)/(nh)$, where the factor of proportionality is the labor share in production, $1 - \alpha$. The optimality condition for hours (27) pins down the utility parameter B,

$$B = (1 - \alpha)\phi \frac{y + g}{h} \left(\frac{1 + \gamma + \tau}{\tau} n h^\gamma c \right)^{-1}. \tag{32}$$

Given B, the optimality condition for effort (21) pins down the utility parameter C,

$$C = \left(\frac{1 + \tau}{\tau} B \right) h^\gamma e^{-(1+\tau)}, \tag{33}$$

Finally, we can use the household's first order condition for employment (8), to pin down the remaining utility parameter A as follows,

$$A = \left(\frac{wh}{c} - \frac{B}{1 + \gamma} h^{1+\gamma} + \frac{C}{1 + \tau} h e^{1+\tau} \right) n^{-\eta}. \tag{34}$$

Table 3 summarizes the recursive formulation of the steady state in the baseline model.

Calibration. There are two types of parameters in the model, those that affect the steady state, and those that do not. We fix some of the parameters in the first category as follows. Technology at the steady state is normalized to unity, $z = 1$. The share of exogenous demand in GDP is set to the average post-war US government spending share of 20%, that is, we set $a = 0.2$. The model is calibrated to a quarterly frequency. The household discount factor is $\beta = 0.99$ to produce a steady state net interest rate of 4% per annum. Over the post-war period in the US, the average labor share is roughly 65% such that $\alpha = 0.35$. The capital depreciation

Table 3: Recursive steady state, baseline model

$GDP = h = e = 1$	GDP, hours and effort (normalization)
$g = 0$	employment adjustment cost
$y = GDP - g$	net output
$r = 1/\beta - (1 - \delta)$	rental rate on capital
$k = \alpha(y + g)/r$	capital stock
$c = y - a - \delta k$	consumption
$H = [(y + g)/(zk^{\alpha})]^{1/(1-\alpha)}$	total effective hours
$n = H/(eh)$	employment
$w = (1 - \alpha)(y + g)/(nh)$	wage per hour
$B = (1 - \alpha)\phi[(y + g)/h]\,\{[(1 + \gamma + \tau)/\tau]nh^{\gamma}c\}^{-1}$	weight on hours in utility
$C = [(1 + \tau)B/\tau]h^{\gamma}e^{-(1+\tau)}$	weight on effort in utility
$A = [wh/c - Bh^{1+\gamma}/(1 + \gamma) - Ce^{1+\tau}h/(1 + \tau)]n^{-\eta}$	weight on employment in utility

Notes. Calibrated parameters: β, γ, τ, η, α, δ, a, z.

rate, $\delta = 0.011$, implied by an average share of consumption of fixed capital in GDP equal to 14% in US post-war data. The curvature of the utility function with respect to employment η is set to 0.5 as in Cho and Cooley (1994). Llosa et al. (2014) set $\gamma = 0.6$ to match the volatility of employment relative to hours per worker. Finally, without loss of generality we set steady state hours and effort to unity, $e = h = 1$; from this normalization we obtain the implied utility function parameters A, B, and C, as explained above.

Second, there are parameters that are purely dynamic and do not affect the steady state. More specifically, these are the capital adjustment cost parameter, ϕ_k, and the employment adjustment cost ϕ_n. We calibrate ϕ_k to target the volatility of investment relative to output, which in US data is equal to 4.6. The parameter ϕ_n is set to match the volatility of employment relative to output, which equals 0.84 in the US.

Finally, we need to calibrate the size and persistence of the two shock processes. Following the procedure in Batini et al. (2019), we set the shock parameters to match the empirical volatility and persistence of output and labor productivity. We search numerically for those parameters that minimize the quadratic loss function $\sum_{j=1}^{4}(x_j^m - x_j^d)^2$, where x_j^m is the j-th moment in the model and x_j^d is its analogue in the data. Data on real GDP were downloaded from the St. Louis Fed database, series GDPC1, sample period 1947q1 to 2019q4. **Labor productivity was computed as real output divided by total hours, both obtained from Ohanian and Raffo (2012) for the period 1960q1-2010q4.** The time series are detrended using the Hodrick-Prescott filter with smoothing parameter 1600. Volatility is measured as the standard deviation and persistence is measured as the first order autocorrelation coefficient of the cyclical component. We obtain the following data moments: $\sigma_y = 0.0130$, $\rho_y = 0.5526$, $\sigma_{\mathcal{P}} = 0.0095$, $\rho_{\mathcal{P}} = 0.7450$.

Table 4: Parameterization, baseline model

Value	Name	Target/Source
$\beta = 0.99$	household discount factor	4% real return p.a.
$\alpha = 0.4$	capital share in production	Giandrea and Sprague (2017)
$\delta = 0.011$	capital depreciation rate	US data (BEA)
$a = 0.2$	government share in output	US data (BEA)
$\eta = 0.5$	elasticity of disutility to employment	Cho and Cooley (1994)
$\gamma = 0.6$	elasticity of disutility to hours	Llosa et al. (2014)
$\tau = 350.05$	elasticity of disutility to effort	$corr(\mathcal{P}, y) = -0.216$
$\phi_k = 0.5929$	capital adjustment cost	$\sigma_I/\sigma_y = 4.6$
$\phi_n = 3.3633$	employment adjustment cost	$\sigma_n/\sigma_y = 1.012$
$\sigma_z = 0.0059$	size technology shock	*Shock processes chosen to match*
$\rho_z = 0.2151$	persistence technology shock	*standard deviation and persistence*
$\sigma_a = 0.0830$	size demand shock	*of real output and labor productivity*
$\rho_a = 0.7743$	persistence demand shock	*observed in US post-war data(*)*

Notes: BLS: U.S. Bureau of Labor Statistics, BEA: U.S. Bureau of Economic Analysis. σ_x denotes the standard deviation of variable x_t; $corr(x, y)$ denotes the correlation between the variables x_t and y_t. (*)See main text for more details.

— [insert Figures 7 and 8 here] —

3 Firing cost model

Our baseline model features quadratic employment adjustment costs. In the following, we instead assume that firms face firing costs as in Hopenhayn and Rogerson (1993).

Model with firing costs. The representative firm chooses paths for k_t and n_t to maximize the expected discounted stream of profits (10), subject to the following production technology,

$$y_t = z_t k_t^{\alpha} H_t^{1-\alpha} - f \max\{n_{t-1} - n_t, 0\}, \tag{35}$$

where $f \geq 0$ captures the size of the firing cost. Following Llosa et al. (2014), $f \max\{n_{t-1} - n_t, 0\}$, is approximated as a differentiable function,

$$g_t = g(n_t, n_{t-1}) = \frac{1}{2} f[(n_{t-1} - n_t) + \sqrt{(n_{t-1} - n_t)^2 + \iota}]. \tag{36}$$

The first derivatives with respect to current and lagged employment, defined as $g_{1t} \equiv \partial g_t / \partial n_t$ and $g_{2t} \equiv \partial g_t / \partial n_{t-1}$, respectively, are

$$g_{1t} = -f \left[\frac{1}{2} + \frac{n_{t-1} - n_t}{2\sqrt{(n_{t-1} - n_t)^2 + \iota}} \right], \tag{37}$$

and $g_{2t} = -g_{1t}$. As in Llosa et al. (2014), we introduce a correction term to eliminate nonzero firing costs at the steady state. The firm's Lagrangian is thus

$$\mathcal{L} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{0,t} \{ z_t k_t^{\alpha} H_t^{1-\alpha} - [g(n_t, n_{t-1}) - g(n, n)] - w_t h_t n_t - r_t k_t \}.$$

The firm's first order conditions with respect to employment is

$$\frac{\partial \mathcal{L}}{\partial n_t} = \beta_{0,t} [(1-\alpha) z_t k_t^{\alpha} H_t^{-\alpha} e_t h_t - w_t h_t - g_{1t}] - \mathrm{E}_t \{ \beta_{0,t+1} g_{2t+1} \} \stackrel{!}{=} 0.$$

Using the relation $g_{2t} = -g_{1t}$, we can write this equilibrium condition as follows:

$$(1-\alpha) \frac{y_t + g_t^{dev}}{n_t} = w_t h_t + g_{1t} - \mathrm{E}_t \{ \beta_{t,t+1} g_{1t+1} \}, \tag{38}$$

where $g_t^{dev} = g_t - g$ denotes the deviation of the firing cost from its steady state level. In steady state, the firing cost (36) and its first derivative (37) are $g^{dev} = 0$ and $g_1 = -\frac{1}{2} f$.

Table 6 summarizes the recursive formulation of the steady state in the firing cost model.

Calibration. As in Llosa et al. (2014), the approximation parameter ι is set to 10^{-3}. The firing cost parameter f is set to 0.1 so that firing one worker would cost the firm 10% of its quarterly output.

— [insert Figures 11 and 12 here] —

4 Search-and-matching model

In this section, we replace employment adjustment costs of the baseline model with search-and-matching frictions and vacancy posting costs. Wages are determined through (efficient) Nash bargaining.

Table 5: Equilibrium conditions, firing cost model

$1 + \phi_k(k_t - k_{t-1}) = \mathrm{E}_t \left\{\beta_{t,t+1} \left[1 - \delta + \phi_k(k_{t+1} - k_t) + r_{t+1}\right]\right\}$	FOC capital
$[An_t^\eta + Bh_t^{1+\gamma}/(1+\gamma) + Ce_t^{1+\tau}h_t/(1+\tau)]c_t = w_t h_t$	FOC employment
$y_t = z_t k_{t-1}^\alpha H_t^{1-\alpha} - g_t^{dev}$	Production function
$\alpha(y_t + g_t^{dev})/k_{t-1} = r_t$	Demand for capital
$(1-\alpha)(y_t + g_t^{dev})/n_t = w_t h_t + g_{1t} - \mathrm{E}_t\{\beta_{t,t+1} g_{1t+1}\}$	Demand for employment
$e_t = [(1+\tau)B/(\tau C)]^{1/(1+\tau)}\, h_t^{\gamma/(1+\tau)}$	Effort
$(1-\alpha)\phi(y_t + g_t^{dev})/h_t = [(1+\gamma+\tau)/\tau]Bn_t h_t^\gamma c_t$	Hours
$c_t + k_t - (1-\delta)k_{t-1} + 0.5 \cdot \phi_k(k_t - k_{t-1})^2 + a_t = y_t$	Aggregate accounting
$H_t = e_t h_t n_t$	Total effective hours
$g_t^{dev} = 0.5 \cdot f[(n_{t-1} - n_t) + \sqrt{(n_{t-1} - n_t)^2 + \iota}] - 0.5 \cdot f\sqrt{\iota}$	Firing cost
$g_{1t} = -0.5 \cdot f[1 + (n_{t-1} - n_t)/\sqrt{(n_{t-1} - n_t)^2 + \iota}]$	Firing cost derivative
$\beta_{t-1,t} = \beta c_{t-1}/c_t$	Stoch discount factor

Note. Endogenous variables: k_t, n_t, y_t, r_t, w_t, e_t, h_t, c_t, H_t, g_t^{dev}, g_{1t}, $\beta_{t-1,t}$.

Table 6: Recursive steady state, firing cost model

$GDP = h = e = 1$	GDP, hours and effort (normalization)
$g^{dev} = 0$	firing cost
$y = GDP - g^{dev}$	net output
$g_1 = -0.5 \cdot f$	firing cost derivative
$r = 1/\beta - (1-\delta)$	rental rate on capital
$k = \alpha(y + g^{dev})/r$	capital stock
$c = y - a - \delta k$	consumption
$H = [(y + g^{dev})/(zk^\alpha)]^{1/(1-\alpha)}$	total effective hours
$n = H/(eh)$	employment
$w = (1-\alpha)(y + g^{dev})/(nh) - (1-\beta)g_1/h$	wage per hour
$B = (1-\alpha)\phi[(y + g^{dev})/h]\,\{[(1+\gamma+\tau)/\tau]nh^\gamma c\}^{-1}$	weight on hours in utility
$C = [(1+\tau)B/\tau]h^\gamma e^{-(1+\tau)}$	weight on effort in utility
$A = [wh/c - Bh^{1+\gamma}/(1+\gamma) - Ce^{1+\tau}h/(1+\tau)]n^{-\eta}$	weight on employment in utility

Notes. Calibrated parameters: β, γ, τ, η, α, δ, a, z, ι, f.

4.1 Labor market

The labor force is held constant and is normalized to 1, such that the unemployment rate is given by

$$u_t = 1 - n_t. \tag{39}$$

Employment relationships are formed in a search-and-matching framework, where firms post vacancies v_t and unemployed workers search for jobs. New jobs or matches m_t are produced according to a Cobb-Douglas matching function,

$$m_t = \chi u_t^{\xi} v_t^{1-\xi}, \tag{40}$$

where χ denotes matching efficiency. We define labor market tightness, the vacancy filling rate and the job finding rate as follows,

$$\theta_t = v_t/u_t, \tag{41}$$

$$q_t = m_t/v_t = \chi\theta_t^{-\xi}, \tag{42}$$

$$p_t = m_t/u_t = \theta_t q_t. \tag{43}$$

The stock of employed workers evolves according to the law of motion:

$$n_t = (1-\rho)(n_{t-1} + m_{t-1}), \tag{44}$$

where $\rho \in (0,1)$ is the (constant) separation rate.

4.2 Firms

The production function of the representative firm reads

$$y_t = z_t k_t^{\alpha}(e_t h_t n_t)^{1-\alpha}. \tag{45}$$

Current production costs are $w_t h_t n_t + r_t k_t + \Phi n_t$, where as in Christoffel et al. (2009) the term Φ denotes per-period job-related 'overhead costs' that are independent of the number of hours per worker. We do not restrict this term to be positive, such that a more appropriate interpretation would be a hiring tax, with negative values corresponding to a subsidy. The firm posts v_t vacancies at a cost of κ each. Current profits are given by

$$D_t = y_t - w_t h_t n_t - r_t k_t - \Phi n_t - \kappa v_t, \tag{46}$$

and the firm's value is

$$V_t^f = y_t - w_t h_t n_t - r_t k_t - \Phi n_t - \kappa v_t + E_t\{\beta_{t,t+1} V_{t+1}^f\}. \tag{47}$$

Firm's match surplus. Let $\mathcal{J}_t$ denote the firm's match surplus, i.e. the value of filling a vacancy. Technically, this is the derivative of firm value (47) with respect to employment n_t. It is equal to the output of the marginal worker, $\partial y_t/\partial n_t = (1-\alpha) z_t k_t^\alpha H_t^{-\alpha}$, minus per-worker wage payments and overhead costs, plus the continuation value of the match. The latter is the expected future match surplus in case the employment relationship continues, which happens with probability $(1-\rho)$. The firm's value is zero in case the worker and the firm separate, which happens with probability ρ. Thus,

$$\mathcal{J}_t = (1-\alpha) y_t/n_t - w_t h_t - \Phi + (1-\rho) E_t\{\beta_{t,t+1} \mathcal{J}_{t+1}\}. \tag{48}$$

The value of posting a vacancy is given by minus the vacancy posting cost κ, plus the expected future value of the vacancy. The latter is the weighted average of the value of filling the vacancy, i.e. the firm's match value in the next period, which has probability $q_t(1-\rho)$, and the future value of the unfilled vacancy, V_{t+1}, which has probability $1 - q_t(1-\rho)$. Therefore,

$$V_t = -\kappa + E_t\{\beta_{t,t+1}[q_t(1-\rho)\mathcal{J}_{t+1} + (1 - q_t(1-\rho))V_{t+1}]\}.$$

Free entry drives the value of a vacancy to zero at each point in time, such that $V_t = 0$ for all t and thus

$$\kappa/q_t = (1-\rho) E_t\{\beta_{t,t+1} \mathcal{J}_{t+1}\}. \tag{49}$$

Combining the firm's match value (48) and the free entry condition (49), we get the following expression for the firm's match surplus

$$\mathcal{J}_t = (1-\alpha) y_t/n_t - w_t h_t - \Phi + \kappa/q_t. \tag{50}$$

The derivative of the firm's match surplus with respect to the wage is $\partial \mathcal{J}_t/\partial w_t = -h_t$. Finally, using the firm's match surplus (50) to substitute $\mathcal{J}_{t+1}$ in the free entry condition (49), we obtain

$$\kappa/q_t = (1-\rho) E_t\left\{\beta_{t,t+1}[(1-\alpha) y_{t+1}/n_{t+1} - w_{t+1} h_{t+1} - \Phi + \kappa/q_{t+1}]\right\}. \tag{51}$$

4.3 Households

Utility maximization is given by

$$\max_{\{c_t,k_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{\ln c_t - n_t \mathcal{G}(h_t, e_t)\},$$

with individual labor disutility specified as

$$\mathcal{G}(h_t, e_t) = \frac{B}{1+\gamma} h_t^{1+\gamma} + \frac{C}{1+\tau} h_t e_t^{1+\tau}, \tag{52}$$

subject to the household's budget constraint,

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 \leq w_t h_t n_t + r_t k_t + D_t + T_t, \tag{53}$$

where T_t denotes lump-sum government transfers. The labor disutility function in (52) is as in the baseline model with parameter A set to zero, see (1). With the Lagrangian written as

$$\begin{aligned} \mathcal{L} =& E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - n_t \left(\frac{B}{1+\gamma} h_t^{1+\gamma} + \frac{C}{1+\tau} h_t e_t^{1+\tau} \right) \right| \\ & - \lambda_t \left| \left[c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 - w_t h_t n_t - r_t k_t - D_t - T_t \right] \right\}, \end{aligned}$$

the household's equilibrium conditions for consumption c_t and capital k_t satisfy (4) and (7) as in the baseline model.

Worker's match surplus. Denote the value of being employed W_t and the value of being unemployed U_t. In period t, an employed worker receives the wage income $w_t h_t$ and suffers the disutility $\mathcal{G}(h_t, e_t)$. In the next period, he is either still employed with probability $1-\rho$, in which case he has an expected value of $E_t\{\beta_{t,t+1} W_{t+1}\}$, or the employment relation is dissolved with probability ρ, then his expected value is $E_t\{\beta_{t,t+1} U_{t+1}\}$. The worker's asset value of being matched to a firm is

$$W_t = w_t h_t - \mathcal{G}(h_t, e_t) c_t + E_t\{\beta_{t,t+1}[(1-\rho) W_{t+1} + \rho U_{t+1}]\}, \tag{54}$$

where we divide labor disutility $\mathcal{G}(h_t, e_t)$ by the marginal utility of consumption $1/c_t$ to convert utils into consumption. The value of being unemployed U_t is in turn given by

$$U_t = b + E_t\{\beta_{t,t+1}[p_t(1-\rho) W_{t+1} + (1 - p_t(1-\rho)) U_{t+1}]\}. \tag{55}$$

An unemployed worker receives b units of consumption goods in period t. In the next period, he faces a probability $p_t(1-\rho)$ of finding a new job and a probability $1-p_t(1-\rho)$ of remaining unemployed. Defining the worker's surplus as $\mathcal{W}_t = W_t - U_t$, we can subtract (55) from (54) to write the match surplus going to the worker as

$$\mathcal{W}_t = w_t h_t - \mathcal{G}(h_t, e_t)c_t - b + (1-\rho)E_t\left\{\beta_{t,t+1}[(1-p_t)\mathcal{W}_{t+1}]\right\}. \tag{56}$$

The derivative of the worker's match surplus with respect to the wage is $\partial \mathcal{W}_t/\partial w_t = h_t$.

4.4 Effort

Every period, the worker chooses hours and effort in order to minimize labor disutility (52), subject to the production function (45). The Lagrangian can be written as

$$\mathcal{L} = \frac{B}{1+\gamma}h_t^{1+\gamma} + \frac{C}{1+\tau}h_t e_t^{1+\tau} - \varphi_t[y_t - z_t k_t^\alpha (e_t h_t n_t)^{1-\alpha}].$$

The first order conditions for h_t and e_t are, respectively,

$$\frac{\partial \mathcal{L}}{\partial h_t} = Bh_t^\gamma + \frac{C}{1+\tau}e_t^{1+\tau} + \varphi_t(1-\alpha)z_t k_t^\alpha (e_t h_t n_t)^{-\alpha} e_t n_t \stackrel{!}{=} 0,$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = Ch_t e_t^\tau + \varphi_t(1-\alpha)z_t k_t^\alpha (e_t h_t n_t)^{-\alpha} h_t n_t \stackrel{!}{=} 0.$$

The two FOCs can be written more simply as follows,

$$Bh_t^\gamma + \frac{C}{1+\tau}e_t^{1+\tau} + \varphi_t(1-\alpha)\frac{y_t}{h_t} = 0, \tag{57}$$

$$Ch_t e_t^\tau + \varphi_t(1-\alpha)\frac{y_t}{e_t} = 0. \tag{58}$$

Equilibrium effort is given by (21) as in the baseline model. Using the optimal effort choice, we can rewrite the production function (45) as

$$y_t = z_t k_t^\alpha (e_0 h_t^\phi n_t)^{1-\alpha}, \tag{59}$$

and the disutility of working can be written as a function of hours only,

$$\mathcal{G}(h_t) = B\frac{1+\gamma+\tau}{(1+\gamma)\tau}h_t^{1+\gamma}. \tag{60}$$

To lighten the notation, we will use $\mathcal{G}_t$ as shorthand for $\mathcal{G}(h_t)$ from here onwards. Note that $\mathcal{G}'_t = B\frac{1+\gamma+\tau}{\tau}h_t^{\gamma}$, with $\lim_{\tau\to\infty}(1+\gamma+\tau)/\tau = 1$. Combining the latter two equations, we obtain

$$\mathcal{G}'_t h_t = (1+\gamma)\mathcal{G}_t. \tag{61}$$

4.5 Hours and wages

Hours and wages are set simultaneously. Hours are determined jointly by the firm and the worker to maximize the sum of the firm's surplus and the worker's surplus, (48) and (56),

$$\mathcal{W}_t + \mathcal{J}_t = (1-\alpha)z_t k_t^{\alpha}(e_0 h_t^{\phi} n_t)^{1-\alpha}/n_t - \Phi - \mathcal{G}_t c_t - b + (1-\rho)E_t\left\{\beta_{t,t+1}[(1-p_t)\mathcal{W}_{t+1} + \mathcal{J}_{t+1}]\right\}.$$

The first order condition for hours worked satisfies

$$\phi e_0 h_t^{\phi-1}(1-\alpha)^2 z_t k_t^{\alpha}(e_0 h_t^{\phi} n_t)^{-\alpha} - \mathcal{G}'_t c_t \overset{!}{=} 0,$$

which simplifies to $(1-\alpha)^2\phi y_t/(n_t h_t) = \mathcal{G}'_t c_t$. Using the relation (61), $\mathcal{G}'_t h_t = (1+\gamma)\mathcal{G}_t$, this can also be expressed as

$$(1-\alpha)^2\phi y_t/n_t = (1+\gamma)\mathcal{G}_t c_t. \tag{62}$$

For future reference, substitute $\mathcal{G}_t$ by plugging in the labor disutility function (60),

$$(1-\alpha)^2\phi\frac{y_t}{n_t} = B\frac{1+\gamma+\tau}{\tau}h_t^{1+\gamma}c_t. \tag{63}$$

Under Nash bargaining, the equilibrium wage satisfies

$$\max_{w_t}\ \mathcal{W}_t^{\omega}\mathcal{J}_t^{1-\omega},$$

where $\omega \in (0,1)$ measures the worker's bargaining power. The first order condition to this problem is

$$\omega\mathcal{W}_t^{\omega-1}\frac{\partial\mathcal{W}_t}{\partial w_t}\mathcal{J}_t^{1-\omega} + (1-\omega)\mathcal{J}_t^{-\omega}\frac{\partial\mathcal{J}_t}{\partial w_t}\mathcal{W}_t^{\omega} \overset{!}{=} 0,$$

which can be simplified to yield a surplus sharing rule of the form

$$\omega\frac{\mathcal{J}_t}{\mathcal{W}_t}\frac{\partial\mathcal{W}_t}{\partial w_t} + (1-\omega)\frac{\partial\mathcal{J}_t}{\partial w_t} = 0.$$

Recall that the derivative of the worker's surplus to the wage is h_t, while the derivative of the firm's surplus to the wage is $-h_t$. Plugging these derivatives into the sharing rule, we find

$$\mathcal{W}_t = \frac{\omega}{1-\omega}\mathcal{J}_t. \tag{64}$$

Inserting the firm's and worker's surplus, (48) and (56), for $\mathcal{J}_t$ and $\mathcal{W}_t$, we obtain

$$w_t h_t - \mathcal{G}_t c_t - b + (1-\rho) E_t\{\beta_{t,t+1}(1-p_t)\mathcal{W}_{t+1}\}$$
$$= \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}\right].$$

Using the sharing rule (64) to replace $\mathcal{W}_{t+1}$ with $\frac{\omega}{1-\omega}\mathcal{J}_{t+1}$ yields

$$w_t h_t - \mathcal{G}_t c_t - b + (1-\rho) E_t\{\beta_{t,t+1}(1-p_t)\frac{\omega}{1-\omega}\mathcal{J}_{t+1}\}$$
$$= \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}\right].$$

Cancelling terms in $(1-\rho)\frac{\omega}{1-\omega}E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}$ yields

$$w_t h_t - \mathcal{G}_t c_t - b - \frac{\omega}{1-\omega} p_t (1-\rho) E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\} = \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - \Phi - w_t h_t\right].$$

Using the free entry condition (49) to replace $(1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}$ with κ/q_t yields

$$w_t h_t - \mathcal{G}_t c_t - b = \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + p_t\frac{\kappa}{q_t}\right].$$

Use $p_t/q_t = \theta_t$, multiply by $(1-\omega)$, collect terms in $w_t h_t$ and rearrange to get the per-person wage

$$w_t h_t = (1-\omega)(\mathcal{G}_t c_t + b) + \omega\left[(1-\alpha)y_t/n_t - \Phi + \kappa\theta_t\right]. \tag{65}$$

Let us plug this expression for the wage into the vacancy posting condition (51) to get

$$\frac{\kappa}{q_t} = (1-\rho)E_t\left\{\beta_{t,t+1}\left((1-\alpha)\frac{y_{t+1}}{n_{t+1}} - \left[(1-\omega)(\mathcal{G}_{t+1}c_{t+1} + b) + \omega\left((1-\alpha)\frac{y_{t+1}}{n_{t+1}} - \Phi + \kappa\theta_{t+1}\right)\right] - \Phi + \frac{\kappa}{q_{t+1}}\right)\right.$$

Rearranging, this simplifies to

$$\kappa/q_t = (1-\rho)E_t\left\{\beta_{t,t+1}[(1-\omega)((1-\alpha)y_{t+1}/n_{t+1} - \Phi - \mathcal{G}_{t+1}c_{t+1} - b) - \omega\kappa\theta_{t+1} + \kappa/q_{t+1}]\right\}. \tag{66}$$

4.6 Closing the model

The government budget is balanced each period,

$$T_t = \Phi n_t. \tag{67}$$

Imposing that the household budget constraint (53) hold with equality, and combining it with firm profits (46) and the government budget balance (67), we obtain the aggregate accounting relation,

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 + a_t = y_t - \kappa v_t. \tag{68}$$

A formal definition of equilibrium, as summarized in Table 7, is as follows.

Definition 2 *A competitive equilibrium in the search model is a set of sequences* $\{u_t,\, m_t,\, \theta_t,\, q_t,\, n_t,\, k_t,\, y_t,\, r_t,\, \mathcal{G}_t,\, h_t,\, e_t,\, c_t,\, v_t\}_{t=0}^{\infty}$ *that satisfy the constant labor force assumption (39), the matching function (40), the definitions of labor market tightness (41) and the vacancy filling rate (42), the law of motion for employment (44), the household's first order conditions for capital (7), the vacancy posting condition (51), the firm's production function (59), the firm's demand for capital (15), the optimality condition for effort (21) and aggregate accounting (68), given exogenous processes for technology* $\{z_t\}_{t=0}^{\infty}$ *and demand shocks* $\{a_t\}_{t=0}^{\infty}$*. Hours and the bargaining wage are given by (62) and (65), respectively.*

Table 7: Equilibrium conditions, search-and-matching model

$u_t = 1 - n_t$	Unemployment
$m_t = \chi u_t^{\xi} v_t^{1-\xi}$	Matching function
$\theta_t = v_t / u_t$	Labor market tightness
$q_t = m_t / v_t$	Vacancy filling rate
$n_t = (1-\rho)(n_{t-1} + m_{t-1})$	Employment dynamics
$y_t = z_t k_{t-1}^{\alpha} H_t^{1-\alpha}$	Production function
$H_t = e_t h_t n_t$	Total effective hours
$\kappa / q_t = (1-\rho) E_t \{\beta_{t,t+1}[(1-\alpha)y_{t+1}/n_{t+1} - w_{t+1}h_{t+1} - \Phi + \kappa/q_{t+1}]\}$	Vacancy posting
$\alpha y_t / k_{t-1} = r_t$	Capital demand
$1 + \phi_k(k_t - k_{t-1}) = \mathrm{E}_t\{\beta_{t,t+1}[(1-\delta) + \phi_k(k_{t+1} - k_t)] + r_{t+1}\}$	FOC capital
$e_t = [(1+\tau)B/(\tau C)]^{1/(1+\tau)} h_t^{\gamma/(1+\tau)}$	Effort
$c_t + k_t - (1-\delta)k_{t-1} + 0.5 \cdot \phi_k(k_t - k_{t-1})^2 + a_t = y_t - \kappa v_t$	Aggregate accounting
$\mathcal{G}_t = B(1+\gamma+\tau)h_t^{1+\gamma}/[(1+\gamma)\tau]$	Labor disutility
$\beta_{t-1,t} = \beta c_{t-1}/c_t$	Stochastic discount factor
$(1-\alpha)^2 \phi y_t / n_t = (1+\gamma)\mathcal{G}_t c_t$	Hours
$w_t h_t = (1-\omega)(\mathcal{G}_t c_t + b) + \omega[(1-\alpha)y_t/n_t - \Phi + \kappa\theta_t]$	Wage

Notes. Endogenous variables u_t, m_t, θ_t, q_t, n_t, y_t, v_t, k_t, r_t, e_t, c_t, $\mathcal{G}_t$, $\beta_{t-1,t}$, h_t, w_t.

4.7 Steady state and calibration

We normalize firm output at the steady state to unity and set $y = 1$. Steady state hours and effort are also normalized to 1. From the household's first order condition for capital (7), the rental rate on capital satisfies $r = 1/\beta - (1-\delta)$. The firm's capital demand (15) is used to derive the capital stock, $k = \alpha y/r$. Investment is the product of depreciation rate and the capital stock, δk. From the constant labor force assumption (39), we obtain the following expression for the employment rate, $n = 1 - u$. Rearranging the law of motion for employment (44) yields an expression for the number of matches, $m = \frac{\rho}{1-\rho} n$. We then solve the definition of the vacancy filling probability (42) for the number of vacancies, $v = m/q$. The degree of labor market tightness is obtained as $\theta = v/u$. The probability of a vacancy being filled (42) determines the matching efficiency parameter, $\chi = q\theta^{\xi}$. The share of total vacancy posting costs in output, $\kappa v/y$, is calibrated. Therefore, knowing y and v we can back out the vacancy posting cost κ. Given firm output, employment, vacancy posting costs, exogenous demand a and investment, we derive consumption from the aggregate accounting identity (68), $c = y - \kappa v - a - \delta k$.

The optimality condition for hours (62) pins down steady state labor disutility

$$\mathcal{G} = \frac{(1-\alpha)^2 \phi y/n}{(1+\gamma)c}. \tag{69}$$

and the utility parameter B is computed from (60) as

$$B = \mathcal{G} \left(\frac{1+\gamma+\tau}{(1+\gamma)\tau} h^{1+\gamma} \right)^{-1}. \tag{70}$$

Given B, the optimality condition for effort (21) pins down the utility parameter C,

$$C = \left(\frac{1+\tau}{\tau} B \right) h^{\gamma} e^{-(1+\tau)}, \tag{71}$$

We calibrate the leisure value and solve the steady state vacancy posting condition (66) for overhead costs,[2]

$$\Phi = (1-\alpha)\frac{y}{n} - \mathcal{G}c - b - \frac{1}{1-\omega} \left(\frac{1-(1-\rho)\beta}{(1-\rho)\beta} \frac{\kappa}{q} + \omega\kappa\theta \right). \tag{72}$$

Finally, we find the wage rate at the steady state from (65),

$$wh = (1-\omega)(\mathcal{G}c + b) + \omega((1-\alpha)y/n - \Phi + \kappa\theta). \tag{73}$$

[2]Alternatively, with overhead costs calibrated to a certain value, the vacancy posting condition (66) at the steady state determines the leisure value.

Table 8 summarizes the recursive steady state equations of the search-and-matching model.

Table 8: Recursive steady state, search-and-matching model

$y = h = e = 1$ (normalization)	firm output, hours, effort
$n = 1 - u$	employment
$H = ehn$	total effective hours
$m = \rho n/(1-\rho)$	matches
$v = m/q$	vacancies
$\theta = v/u$	labor market tightness
$\chi = q\theta^{\xi}$	matching efficiency
$r = 1/\beta - (1-\delta)$	rental rate on capital
$k = \alpha y/r$	capital stock
$z = y/(k^{\alpha}H^{1-\alpha})$	production efficiency
$\kappa = (\kappa v/y)y/v$	vacancy posting cost
$c = y - \kappa v - a - \delta k$	consumption
$\mathcal{G} = [(1-\alpha)^2\phi y/n]/[(1+\gamma)c]$	labor disutility
$B = \mathcal{G}\left[(1+\gamma+\tau)h^{1+\gamma}/(1+\gamma)\tau\right]^{-1}$	weight on hours in labor disutility
$C = [(1+\tau)B/\tau]h^{\gamma}e^{-(1+\tau)}$	weight on effort in labor disutility
$\Phi = (1-\alpha)\frac{y}{n} - \mathcal{G}c - b - \frac{1}{1-\omega}\left(\frac{1-(1-\rho)\beta}{(1-\rho)\beta}\frac{\kappa}{q} + \omega\kappa\theta\right)$	overhead cost
$w = [(1-\omega)(\mathcal{G}c+b) + \omega((1-\alpha)y/n - \Phi + \kappa\theta)]/h$	wage

Notes. Calibrated parameters: β, γ, τ, α, δ, a, u, q, ρ, ξ, $\kappa v/y$, b, ω. Composite parameter: $\phi = 1 + \gamma/(1+\tau)$.

— [insert Figures 17 and 18 here] —

5 Hiring cost model

The key difference between the search model and the hiring cost model is that in the latter setup, a firm that wishes to hire a worker always finds one. The firm therefore hires directly, rather than posting vacancies, and there is no search *per se*.

5.1 Labor market

We maintain the assumption of a constant labor force also in this model variant, such that employment and unemployed sum to unity, $u_t = 1 - n_t$. As in the search model, new jobs are determined by the matching function $m_t = \chi u_t^{\xi} v_t^{1-\xi}$, and employment in the current period equals a constant fraction of the sum of last period's workforce and last period's job matches,

Table 9: Parameterization, search-and-matching model

Value	Name	Reference
$u = 0.12$	Steady state unemployment rate	Krause and Lubik (2010)
$\rho = 0.10$	Job separation rate	Shimer (2005)
$q = 0.70$	Steady state vacancy filling rate	den Haan et al. (2000)
$\xi = 0.50$	Match elasticity to unemployment	Petrongolo and Pissarides (2001)
$\kappa v/y = 0.01$	Vacancy posting cost share in output	Andolfatto (1996)
$b = 0.40$	Leisure value	Shimer (2005)
$\omega = 0.50$	Workers' bargaining weight	Hosios (1990)

$n_t = (1-\rho)(n_{t-1}+m_{t-1})$. For notational convenience, we introduce the hiring rate as the ratio of matches to employment,

$$x_t = \frac{m_t}{n_t}. \tag{74}$$

5.2 Firms

The firm's match surplus is given by (48) as in the search model. Meeting a worker entails a cost, κ, which is proportional to the aggregate hiring rate x_t and taken as given by the firm. Free entry into the labor market ensures that the following no-arbitrage condition must hold for all t,

$$\kappa x_t = (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}. \tag{75}$$

Combining the firm's asset value (48) and the free entry condition (75), we get the following expression for the firm's match surplus

$$\mathcal{J}_t = (1-\alpha)y_t/n_t - w_t h_t - \Phi + \kappa x_t. \tag{76}$$

Finally, using the firm's match surplus (76), iterated by one period, to substitute out $\mathcal{J}_{t+1}$ in the free entry condition (75), we obtain the hiring condition

$$\kappa x_t = (1-\rho)E_t\left\{\beta_{t,t+1}\left[(1-\alpha)y_{t+1}/n_{t+1} - w_{t+1}h_{t+1} - \Phi + \kappa x_{t+1}\right]\right\}. \tag{77}$$

5.3 Households

The household's intertemporal choice problem is the same as in the search model, see Section 4. Accordingly, the first order conditions for consumption and capital, and worker's surplus are given by (4), (7) and (56), respectively.

5.4 Effort

Labor effort is determined as in the baseline model, and thus equilibrium effort is given by (21). Also, the production function and the disutility of working are given by (59) and (60), respectively.

5.5 Hours and wages

As in the search model, hours are determined jointly by the firm and the worker to maximize the sum of the firm's surplus and the worker's surplus, (48) and (56). The optimal hours choice is therefore given by (63).

Under Nash bargaining, the surplus sharing rule is identical to the one in the search model, $\mathcal{W}_t = \frac{\omega}{1-\omega}\mathcal{J}_t$. Using the worker's and the firm's surplus, (56) and (48), to replace $\mathcal{W}_t$ and $\mathcal{J}_t$, we obtain

$$w_t h_t - \mathcal{G}_t c_t - b + (1-\rho)E_t\{\beta_{t,t+1}(1-p_t)\mathcal{W}_{t+1}\} \\ = \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}\right].$$

Then, using the sharing rule to replace $\mathcal{W}_{t+1}$ with $\frac{\omega}{1-\omega}\mathcal{J}_{t+1}$ yields

$$w_t h_t - \mathcal{G}_t c_t - b + (1-\rho)E_t\left\{\beta_{t,t+1}(1-p_t)\frac{\omega}{1-\omega}\mathcal{J}_{t+1}\right\} \\ = \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}\right].$$

Collecting terms in $(1-\rho)\frac{\omega}{1-\omega}E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}$ yields

$$w_t h_t - \mathcal{G}_t c_t - b = \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + p_t(1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}\right]$$

Using the free entry condition (75) to replace $(1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}$ with κx_t yields

$$w_t h_t - \mathcal{G}_t c_t - b = \frac{\omega}{1-\omega}\left[(1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + p_t\kappa x_t\right].$$

Finally, we can solve for the wage per person $w_t h_t$ as follows:

$$w_t h_t = (1-\omega)(\mathcal{G}_t c_t + b) + \omega[(1-\alpha)y_t/n_t - \Phi + p_t\kappa x_t]. \tag{78}$$

It will be useful to substitute the wage $w_t h_t$ in the hiring condition (77) to obtain

$$\kappa x_t = (1-\rho)E_t\{\beta_{t,t+1}[(1-\alpha)y_{t+1}/n_{t+1} - (1-\omega)(\mathcal{G}_{t+1}c_{t+1}+b) \\ - \omega[(1-\alpha)y_{t+1}/n_{t+1} - \Phi + p_{t+1}\kappa x_{t+1}] - \Phi + \kappa x_{t+1}]\}.$$

Collecting terms, this becomes

$$\kappa x_t = (1-\rho)E_t\left\{\beta_{t,t+1}\left[(1-\omega)((1-\alpha)y_{t+1}/n_{t+1} - \Phi - \mathcal{G}_{t+1}c_{t+1} - b) + (1-\omega p_{t+1})\kappa x_{t+1}\right]\right\}. \tag{79}$$

5.6 Closing the model

Aggregate accounting is given by

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1}-k_t)^2 + a_t = y_t - \kappa x_t m_t. \tag{80}$$

A formal definition of equilibrium, as summarized in Table 10, is as follows.

Definition 3 *A competitive equilibrium in the hiring cost model is a set of sequences $\{u_t, m_t, x_t, p_t, n_t, y_t, k_t, r_t, e_t, c_t, v_t, h_t, w_t\}_{t=0}^{\infty}$ that satisfy the constant labor force assumption (39), the matching function (40), the definitions of hiring rate (74) and the job finding rate (43), the law of motion for employment (44), the household's first order conditions for capital (7), the hiring condition (77), the firm's production function (59), the firm's demand for capital (15), the optimality condition for effort (21), the bargaining outcomes for hours (62) and the wage (78), and aggregate accounting (80), given exogenous processes for technology $\{z_t\}_{t=0}^{\infty}$ and demand shocks $\{a_t\}_{t=0}^{\infty}$.*

5.7 Steady state and calibration

We normalize firm output at the steady state to unity and set $y = 1$. Steady state hours and effort are also normalized to 1. The constant labor force assumption (39) implies that employment is $n = 1 - u$. From the employment dynamics equation (44) in steady state, we have that hiring rate is given by $x = \rho/(1-\rho)$. Then we can solve the definition of the hiring rate for the steady state number of matches, $m = xn$. The number of matches determines the job finding rate as $p = m/u$. We then solve definition of the vacancy filling probability (42) for the number of vacancies, $v = m/q$. The degree of labor market tightness is obtained as $\theta = v/u$. The probability of a vacancy being filled, $q = \chi\theta^{-\xi}$, determines the matching efficiency parameter, $\chi = q\theta^{\xi}$. From the household's first order condition for capital (7), the

Table 10: Equilibrium conditions, hiring cost model

$u_t = 1 - n_t$	Unemployment
$m_t = \chi u_t^{\xi} v_t^{1-\xi}$	Matching function
$x_t = m_t/n_t$	Hiring rate
$p_t = m_t/u_t$	Job finding rate
$n_t = (1-\rho)(n_{t-1} + m_{t-1})$	Employment dynamics
$y_t = z_t k_{t-1}^{\alpha} H_t^{1-\alpha}$	Production function
$H_t = e_t h_t n_t$	Total effective hours
$\kappa x_t = (1-\rho) E_t \{\beta_{t,t+1} [(1-\alpha) y_{t+1}/n_{t+1} - w_{t+1} h_{t+1} - \Phi + \kappa x_{t+1}]\}$	Vacancy posting
$\alpha y_t / k_{t-1} = r_t$	Capital demand
$1 + \phi_k (k_t - k_{t-1}) = \mathrm{E}_t \{\beta_{t,t+1} [(1-\delta) + \phi_k (k_{t+1} - k_t)] + r_{t+1}\}$	FOC capital
$e_t = [(1+\tau) B/(\tau C)]^{1/(1+\tau)} h_t^{\gamma/(1+\tau)}$	Effort
$c_t + k_t - (1-\delta) k_{t-1} + 0.5 \cdot \phi_k (k_t - k_{t-1})^2 + a_t = y_t - \kappa x_t m_t$	Aggregate accounting
$(1-\alpha)^2 \phi y_t / n_t = (1+\gamma) \mathcal{G}_t c_t$	Hours
$w_t h_t = (1-\omega)(\mathcal{G}_t c_t + b) + \omega[(1-\alpha) y_t/n_t - \Phi + p_t \kappa x_t]$	Wage
$\mathcal{G}_t = B(1+\gamma+\tau) h_t^{1+\gamma} / [(1+\gamma)\tau]$	Labor disutility
$\beta_{t-1,t} = \beta c_{t-1}/c_t$	Stoch. discount factor

Notes. Endogenous variables u_t, m_t, x_t, p_t, n_t, y_t, v_t, k_t, r_t, e_t, c_t, h_t, w_t, $\mathcal{G}_t$, $\beta_{t-1,t}$.

rental rate on capital satisfies $r = 1/\beta - (1-\delta)$. The firm's capital demand (15) is used to derive the capital stock, $k = \alpha y/r$. Investment is the product of depreciation rate and the capital stock, δk. The share of total vacancy posting costs in output, $\kappa v/y$, is calibrated. Therefore, knowing y and v we can back out the vacancy posting cost κ. Given firm output, employment, vacancy posting costs, exogenous demand a and investment, we derive consumption from the aggregate accounting identity (68), $c = y - \kappa x m - a - \delta k$. As in the search model with efficient bargaining, the optimality condition for hours pins down the utility parameter B. Given B, the optimality condition for effort pins down the utility parameter C. See equations (70) and (71), respectively. Steady state labor disutility is given by (69). We calibrate the leisure value b and solve the steady state hiring condition for overhead costs,

$$\Phi = (1-\alpha)\frac{y}{n} - \mathcal{G}c - b - \frac{1 - (1-\rho)\beta(1-\omega p)}{(1-\rho)\beta(1-\omega)}\kappa x. \tag{81}$$

Finally, we find the wage rate at the steady state from (78). Table 11 summarizes the recursive steady state equations of the hiring cost model.

— [insert Figures 19 and 20 here] —

Table 11: Recursive steady state, hiring cost model

$y = h = e = 1$ (normalization)	firm output, hours, effort
$n = 1 - u$	employment
$H = ehn$	total effective hours
$x = \rho/(1-\rho)$	hiring rate
$m = xn$	matches
$p = m/u$	job finding rate
$v = m/q$	vacancies
$\theta = v/u$	labor market tightness
$\chi = q\theta^{\xi}$	matching efficiency
$r = 1/\beta - (1-\delta)$	rental rate on capital
$k = \alpha y/r$	capital stock
$\kappa = (\kappa v/y)y/v$	vacancy posting cost
$c = y - \kappa x m - a - \delta k$	consumption
$z = y/(k^{\alpha}H^{1-\alpha})$	production efficiency
$\mathcal{G} = [(1-\alpha)^2\phi y/n]/[(1+\gamma)c]$	labor disutility
$B = \mathcal{G}/\left[(1+\gamma+\tau)h^{1+\gamma}/(1+\gamma)\tau\right]$	weight on hours in disutility
$C = [(1+\tau)B/\tau]h^{\gamma}e^{-(1+\tau)}$	weight on effort disutility
$\Phi = [(1-\alpha)y/n - \mathcal{G}c - b] - \{[1-(1-\rho)\beta(1-\omega p)]/[(1-\rho)\beta(1-\omega)]\}\kappa x$	overhead cost
$wh = \omega[(1-\alpha)y/n - \Phi + p\kappa x] + (1-\omega)(\mathcal{G}c + b)$	wage

Notes. Calibrated parameters: β, γ, τ, α, δ, a, u, b, q, ρ, ξ, ω. Composite parameter: $\phi = 1 + \gamma/(1+\tau)$.

6 Sticky-price model

We now extend the model to allow for monopolistic competition and price setting power on the part of intermediate goods firms, as well as price adjustment costs. Households have access to a one-period nominal bond. The nominal interest rate is set by the central bank.

6.1 Households

The household chooses paths for consumption, bonds b_t, capital and employment to maximize lifetime utility (1), subject to the budget constraint

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 + b_t \leq w_t h_t n_t + r_t k_t + D_t + T_t + b_{t-1}R_{t-1}/\pi_t, \tag{82}$$

where b_t are holdings of one-period risk-free nominal bonds that yield a gross interest rate R_t and π_t is the gross inflation rate between $t-1$ and t. The household's equilibrium conditions satisfy (7) and (8) as well as the standard Euler equation for bonds,

$$1 = R_t \mathrm{E}_t\{\beta_{t,t+1}/\pi_{t+1}\}. \tag{83}$$

6.2 Firms

The goods market structure follows Dixit and Stiglitz (1977). Final goods producers operate under perfect competition. For each intermediate good i on the unit interval that costs P_{it} currency units, they choose an amount y_{it}, to minimize their production cost, $\int_0^1 P_{it} y_{it} di$, subject to the production function $y_t = (\int_0^1 y_{it}^{(\varepsilon-1)/\varepsilon} di)^{\varepsilon/(\varepsilon-1)}$, where $\varepsilon > 1$ is the elasticity of substitution between inputs. This yields the demand functions

$$y_{it} = (P_{it}/P_t)^{-\varepsilon} y_t, \tag{84}$$

where $P_t = (\int_0^1 P_{it}^{1-\varepsilon} di)^{1/(1-\varepsilon)}$ is the price of the final good, which can be interpreted as the price index.

Intermediate goods firms instead have market power, represented by (the inverse of) ε, which allows them to set prices. They also choose optimally their inputs employment and capital. The representative firm i thus chooses paths for k_{it}, n_{it} and P_{it} to maximize the expected discounted stream of profits,

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \left\{ (P_{it}/P_t)^{1-\varepsilon} y_t - w_{it} h_{it} n_{it} - r_t k_{it} - 0.5 \cdot \phi_p (P_{it}/P_{i,t-1} - 1)^2 y_t \right\}, \tag{85}$$

where $\phi_p \geq 0$ scales the (quadratic) price adjustment costs (Rotemberg, 1982), subject to the constraint

$$(P_{it}/P_t)^{-\varepsilon} y_t = z_t k_{it}^{\alpha} H_{it}^{1-\alpha} - 0.5 \cdot \phi_n (n_{it} - n_{it-1})^2. \tag{86}$$

The firm's Lagrangian can be written as

$$\begin{aligned}\mathcal{L} =& \mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \{ (P_{it}/P_t)^{1-\varepsilon} y_t - w_{it} h_{it} n_{it} - r_t k_{it} - 0.5 \cdot \phi_p \left(P_{it}/P_{i,t-1} - 1 \right)^2 y_t \\ & - \Psi_t \left[(P_{it}/P_t)^{-\varepsilon} y_t - z_t k_{it}^{\alpha} H_{it}^{1-\alpha} + 0.5 \cdot \phi_n (n_{it} - n_{it-1})^2 \right] \},\end{aligned} \tag{87}$$

where the Lagrange multiplier Ψ_{it} on the constraint (86) represents the firm's real marginal cost, or the cost of increasing firm output by one unit. The firm's first order conditions with respect to capital and employment are, respectively,

$$\frac{\partial \mathcal{L}}{\partial k_{it}} = -r_t + \Psi_{it} \alpha z_t k_{it}^{\alpha-1} H_{it}^{1-\alpha} \stackrel{!}{=} 0,$$

$$\frac{\partial \mathcal{L}}{\partial n_{it}} = -w_{it} h_{it} + \Psi_{it} \left[(1-\alpha) z_t k_{it}^{\alpha} H_{it}^{-\alpha} e_{it} h_{it} - \phi_n (n_{it} - n_{it-1}) + \mathrm{E}_t \{ \beta_{t,t+1} \phi_n (n_{it+1} - n_{it}) \} \right] \stackrel{!}{=} 0.$$

In a symmetric equilibrium, the optimality conditions become

$$\alpha \frac{y_t + g_t}{k_t} = \frac{r_t}{\Psi_t}, \tag{88}$$

$$(1-\alpha) \frac{y_t + g_t}{n_t} - \phi_n (n_t - n_{t-1}) + \mathrm{E}_t \left\{ \beta_{t,t+1} \phi_n (n_{t+1} - n_t) \right\} = \frac{w_t h_t}{\Psi_t}. \tag{89}$$

The left hand side of (88) is the firm's marginal product of capital, $\alpha z_t k_t^{\alpha-1} H_t^{1-\alpha}$. At the optimum, this equals the marginal cost of capital, i.e. the rental rate r_t, divided by the real marginal cost Ψ_t. In (89), the marginal product of employment on the left hand side equals the marginal cost of employment, i.e. the wage per worker, $w_t h_t$, divided by the real marginal cost Ψ_t.

The first order condition with respect to prices is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial P_{it}} =& (1-\varepsilon) \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} \frac{y_t}{P_t} - \phi_p \left(\frac{P_{it}}{P_{i,t-1}} - 1 \right) \frac{y_t}{P_{i,t-1}} + \Psi_t \varepsilon \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon-1} \frac{y_t}{P_t} \\ & + \mathrm{E}_t \left\{ \beta_{t,t+1} \phi_p \left(\frac{P_{it+1}}{P_{i,t}} - 1 \right) \frac{P_{it+1}}{P_{i,t}} \frac{y_{t+1}}{P_{i,t}} \right\} \stackrel{!}{=} 0,\end{aligned}$$

Multiplying the whole equation by $P_{i,t}/y_t$, this becomes

$$
\begin{aligned}
(1-\varepsilon)\left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} - \phi_p\left(\frac{P_{it}}{P_{i,t-1}}-1\right)\frac{P_{i,t}}{P_{i,t-1}} + \Psi_t\varepsilon\left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} \\
+ \phi_p \mathrm{E}_t\left\{\beta_{t,t+1}\left(\frac{P_{it+1}}{P_{i,t}}-1\right)\frac{P_{it+1}}{P_{i,t}}\frac{y_{t+1}}{y_t}\right\} = 0.
\end{aligned}
$$

If all firms' real marginal costs are the same, individual goods prices are identical and equal to the price index, $P_{it} = P_t$. This gives rise to the standard New Keynesian Phillips Curve,

$$(1-\varepsilon) - \phi_p(\pi_t - 1)\pi_t + \varepsilon\Psi_t + \phi_p \mathrm{E}_t\left\{\beta_{t,t+1}(\pi_{t+1}-1)\pi_{t+1}y_{t+1}/y_t\right\} = 0. \tag{90}$$

In the sticky-price model, firm profits are given by:

$$D_t = y_t - w_t h_t n_t - r_t k_t - 0.5 \cdot \phi_p(\pi_t - 1)^2 y_t. \tag{91}$$

From a firm's net output y_t we subtract wage payments, capital rental costs and price adjustment costs to obtain profits.

6.3 Effort and hours

For each employer i on the unit interval, the household chooses the combination of hours and effort that minimizes labor disutility, i.e. he solves the problem (17), subject to the firm's demand constraint (86). The Lagrangian can be written as

$$
\begin{aligned}
\mathcal{L} =& \mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1}\left\{\frac{A}{1+\eta}n_{it}^{1+\eta} + \frac{B}{1+\gamma}n_{it}h_{it}^{1+\gamma} + \frac{C}{1+\tau}n_{it}h_{it}e_{it}^{1+\tau}\right| \\
&- \left|\varphi_t\left[(P_{it}/P_t)^{-\varepsilon}y_t - z_t k_{it}^{\alpha}(e_{it}h_{it}n_{it})^{1-\alpha} + g_{it}\right]\right\},
\end{aligned}
$$

The first order conditions for h_{it} and e_{it} are, respectively,

$$\frac{\partial\mathcal{L}}{\partial h_{it}} = Bn_{it}h_{it}^{\gamma} + \frac{C}{1+\tau}n_{it}e_{it}^{1+\tau} + \varphi_t(1-\alpha)z_t k_{it}^{\alpha}(e_{it}h_{it}n_{it})^{-\alpha}e_{it}n_{it} \stackrel{!}{=} 0,$$

$$\frac{\partial\mathcal{L}}{\partial e_{it}} = Cn_{it}h_{it}e_{it}^{\tau} + \varphi_t(1-\alpha)z_t k_{it}^{\alpha}(e_{it}h_{it}n_{it})^{-\alpha}h_{it}n_{it} \stackrel{!}{=} 0,$$

In a symmetric equilibrium, we obtain the same equilibrium conditions for hours and effort as in the baseline model.

6.4 Closing the model

Market clearing in the bond market implies that $b_t = 0$ for all t as bonds are in zero net supply in equilibrium. Imposing this condition in the household budget constraint (82) holding with equality, and combining with symmetric firm profits (91) and government budget balance $T_t = \Phi n_t$, we obtain the aggregate accounting relation,

$$c_t + k_{t+1} - (1-\delta)k_t + 0.5 \cdot \phi_k (k_{t+1} - k_t)^2 + a_t = [1 - 0.5 \cdot \phi_p (\pi_t - 1)^2] y_t. \tag{92}$$

The model is closed with a feedback rule that determines the interest rate R_t in response to deviations of inflation from steady state,

$$\frac{R_t}{R} = \left(\frac{\pi_t}{\pi}\right)^{\Omega_\pi}, \tag{93}$$

where $\Omega_\pi > 0$. A formal definition of equilibrium in the sticky-price model, as summarized in Table 12, is as follows.

Definition 4 *A competitive equilibrium in the sticky-price model is a set of sequences* $\{\pi_t, k_t, n_t, y_t, r_t, w_t, \Psi_t, e_t, h_t, c_t, R_t\}_{t=0}^{\infty}$ *that satisfy the household's first order conditions for bonds (83), capital (7) and employment (8), the firm's production function (23), the firm's demand for capital (88) and employment (89), the price setting decision (90), the optimality conditions for effort (21) and hours (27), aggregate accounting (92), and the interest rate rule (93), given exogenous processes for technology* $\{z_t\}_{t=0}^{\infty}$ *and demand shocks* $\{a_t\}_{t=0}^{\infty}$.

We now turn to the zero-inflation steady state.

6.5 Steady state and calibration

The recursive steady state is computed in the same way as in the baseline model. Table 13 summarizes the steady state in the sticky-price model.

There are three additional parameters in the sticky-price model as compared with the baseline model. The elasticity of substitution between intermediate goods varieties ε is calibrated to 7.5, which corresponds to the median elasticity of substitution across brand modules estimated in Broda and Weinstein (2010). This value implies a steady state net price markup of 15%, $\Psi = 1.15$. The parameter scaling the price adjustment cost ϕ_p is set to 13, which corresponds to a Phillips curve coefficient of $(\varepsilon - 1)/\phi_p$ equal to 0.5 as in Lubik and Schorfheide (2004). Finally, Ω_π is set to 1.5 in line with the work of Taylor (1993).

— [insert Figures 13 and 14 here] —

Table 12: Equilibrium conditions, sticky-price model

$1 = R_t \mathrm{E}_t\{\beta_{t,t+1}/\pi_{t+1}\}$	FOC bonds
$1 + \phi_k(k_t - k_{t-1}) = \mathrm{E}_t\{\beta_{t,t+1}[1 - \delta + \phi_k(k_{t+1} - k_t) + r_{t+1}]\}$	FOC capital
$[An_t^\eta + Bh_t^{1+\gamma}/(1+\gamma) + Ce_t^{1+\tau}h_t/(1+\tau)]c_t = w_t h_t$	FOC employment
$y_t = z_t k_{t-1}^\alpha H_t^{1-\alpha} - g_t$	Production function
$\alpha(y_t + g_t)/k_{t-1} = r_t/\Psi_t$	Capital demand
$(1-\alpha)(y_t + g_t)/n_t - \phi_n(n_t - n_{t-1}) + \phi_n \mathrm{E}_t\{\beta_{t,t+1}(n_{t+1} - n_t)\} = w_t h_t/\Psi_t$	Employment demand
$(1-\varepsilon) + \varepsilon\Psi_t = \phi_p(\pi_t - 1)\pi_t - \phi_p \mathrm{E}_t\{\beta_{t,t+1}(\pi_{t+1} - 1)\pi_{t+1}y_{t+1}/y_t\}$	Price setting
$e_t = [(1+\tau)B/(\tau C)]^{1/(1+\tau)}\, h_t^{\gamma/(1+\tau)}$	Effort
$(1-\alpha)\phi(y_t + g_t)/h_t = [(1+\gamma+\tau)/\tau]Bn_t h_t^\gamma c_t$	Hours
$c_t + k_t - (1-\delta)k_{t-1} + 0.5 \cdot \phi_k(k_t - k_{t-1})^2 + a_t = [1 - 0.5 \cdot \phi_p(\pi_t - 1)^2]y_t$	Aggregate accounting
$R_t/R = (\pi_t/\pi)^{\Omega_\pi}$	Monetary policy
$H_t = e_t h_t n_t$	Total effective hours
$g_t = 0.5 \cdot \phi_n(n_t - n_{t-1})^2$	Empl. adj. cost
$\beta_{t-1,t} = \beta c_{t-1}/c_t$	Stoch discount factor

Notes. Endogenous variables: π_t, k_t, n_t, H_t, y_t, r_t, w_t, Ψ_t, e_t, h_t, c_t, R_t, g_t, $\beta_{t-1,t}$.

Table 13: Recursive steady state, sticky-price model

$GDP = h = e = 1$ (normalization)	GDP, hours and effort
$g = 0$	employment adjustment cost
$y = GDP - g$	net output
$\Psi = (\varepsilon - 1)/\varepsilon$	inverse gross price markup
$\pi = 1$	gross inflation
$R = \pi/\beta$	policy rate
$r = 1/\beta - (1-\delta)$	rental rate on capital
$k = \Psi\alpha(y+g)/r$	capital stock
$H = [(y+g)/(zk^\alpha)]^{1/(1-\alpha)}$	total effective hours
$n = H/(eh)$	employment
$c = y - a - \delta k$	consumption
$w = \Psi(1-\alpha)(y+g)/(nh)$	real wage per hour
$B = (1-\alpha)\phi[(y+g)/h]\,\{[(1+\gamma+\tau)/\tau]nh^\gamma c\}^{-1}$	weight on hours in utility
$C = [(1+\tau)B/\tau]h^\gamma e^{-(1+\tau)}$	weight on effort in utility
$A = [wh/c - Bh^{1+\gamma}/(1+\gamma) - Ce^{1+\tau}h/(1+\tau)]n^{-\eta}$	weight on employment in utility

Notes. Calibrated parameters: β, γ, τ, η, α, δ, a, z as in the baseline model, as well as ε, ϕ_p, Ω_π.

7 Right-to-manage bargaining model

In this section, we consider search-and-matching frictions as in Section 4, together with sticky prices as in Section 6 and 'right-to-manage' bargaining. In the right-to-manage (RTM) model of Trigari (2006), wages are set first in a Nash bargaining framework. Once wages are determined, the firm sets the number of hours per worker in a profit-maximizing fashion. Thus, firms take the bargained wage as given when choosing hours.

7.1 Labor market

The specification of the labor market is the same as in the search-and-matching model of Section 4.

7.2 Firms

The production function of the representative intermediate firm i reads

$$y_{it} = z_t k_{it}^{\alpha} H_{it}^{1-\alpha}, \tag{94}$$

where $H_{it} = e_{it} h_{it} n_{it}$ are total effective hours worked at firm i. Period-t profits, expressed in terms of the final consumption good, are

$$D_{it} = \frac{P_{it} y_{it}}{P_t} - w_{it} h_{it} n_{it} - r_t k_{it} - \Phi n_{it} - \frac{\phi_p}{2} \left(\frac{P_{it}}{P_{i,t-1}} - 1 \right)^2 y_t.$$

Firm i chooses paths for k_{it} and P_{it} to maximize the expected discounted stream of profits,

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \{ (P_{it}/P_t)^{1-\varepsilon} y_t - w_{it} h_{it} n_{it} - r_t k_{it} - \Phi n_{it} - 0.5 \cdot \phi_p (P_{it}/P_{i,t-1} - 1)^2 y_t \}, \tag{95}$$

subject to the demand constraint $(P_{it}/P_t)^{-\varepsilon} y_t = z_t k_{it}^{\alpha} H_{it}^{1-\alpha}$. With the Lagrangian written as

$$\begin{aligned} \mathcal{L} = & \mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \{ (P_{it}/P_t)^{1-\varepsilon} y_t - w_{it} h_{it} n_{it} - r_t k_{it} - \Phi n_{it} - 0.5 \cdot \phi_p \left(P_{it}/P_{i,t-1} - 1 \right)^2 y_t \\ & - \Psi_t \left[(P_{it}/P_t)^{-\varepsilon} y_t - z_t k_{it}^{\alpha} H_{it}^{1-\alpha} \right] \}, \end{aligned} \tag{96}$$

the first order condition with respect to capital is $\partial \mathcal{L} / \partial k_{it} = -r_t + \Psi_{it} \alpha z_t k_{it}^{\alpha-1} H_{it}^{1-\alpha} \stackrel{!}{=} 0$, which under symmetry simplifies to

$$\Psi_t \alpha y_t / k_t = r_t. \tag{97}$$

The firm's first order condition with respect to the price P_{it} is derived in the sticky-price model of Section 6 and leads to the New Keynesian Phillips Curve (90). In a symmetric equilibrium, firm profits are given by:

$$D_t = [1 - 0.5 \cdot \phi_p(\pi_t - 1)^2]y_t - w_t h_t n_t - r_t k_t - \Phi n_t - \kappa v_t. \tag{98}$$

Firm's match surplus. The firm's match surplus and the vacancy posting condition are derived as in the search model of Section 4. In a symmetric equilibrium with identical firms, the firm's surplus and the vacancy posting condition are given by (48) and (51), respectively.

7.3 Households

Utility maximization is given by

$$\max_{\{c_t,k_t,b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - n_t \left(\frac{B}{1+\gamma} h_t^{1+\gamma} + \frac{C}{1+\tau} h_t e_t^{1+\tau} \right) \right\},$$

subject to the household's budget constraint,

$$c_t + k_{t+1} - (1-\delta)k_t + 0.5 \cdot \phi_k (k_{t+1} - k_t)^2 + b_t \leq w_t h_t n_t + r_t k_t + D_t + T_t + b_{t-1} R_{t-1}/\pi_t. \tag{99}$$

With the Lagrangian written as

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \ln c_t - n_t \left(\frac{B}{1+\gamma} h_t^{1+\gamma} + \frac{C}{1+\tau} h_t e_t^{1+\tau} \right) \Bigg| \\
& - \lambda_t \Bigg| \left[c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 + b_t - w_t h_t n_t - r_t k_t - D_t - T_t - b_{t-1} R_{t-1}/\pi_t \right] \Bigg\},
\end{aligned}$$

the household's equilibrium conditions for consumption c_t, capital k_t and bonds b_t satisfy (4), (7) and (83) as in the sticky-price model.

Worker's match surplus. The worker's match surplus is given by (56) as in the search-and-matching model, see Section 4.

7.4 Effort

Labor effort is determined as in the sticky-price model of Section 6.

7.5 Hours and wages

Hours are determined unilaterally by the firm to maximize its surplus (48), taking the wage rate as given:

$$\max_{h_{it}} \; (1-\alpha) z_t k_{it}^{\alpha} (e_0 h_{it}^{\phi} n_{it})^{1-\alpha} / n_{it} - w_{it} h_{it} - \Phi + (1-\rho) E_t \{ \beta_{t,t+1} \mathcal{J}_{it+1} \}. \tag{100}$$

The first order condition for hours worked satisfies

$$\partial \mathcal{J}_{it} / \partial h_{it} = \phi e_0 h_{it}^{\phi-1} (1-\alpha)^2 z_t k_{it}^{\alpha} (e_0 h_{it}^{\phi} n_{it})^{-\alpha} - w_{it} \overset{!}{=} 0,$$

which can be written as follows:

$$\phi (1-\alpha)^2 y_{it} / (h_{it} n_{it}) - w_{it} = 0.$$

In a symmetric equilibrium, this simplifies to

$$(1-\alpha)^2 \phi y_t / (n_t h_t) = w_t. \tag{101}$$

In equilibrium, hours worked are a function of the wage rate, $h_t = h(w_t)$; more specifically, $h_t = (1-\alpha)^2 \phi y_t / (w_t n_t)$, and the first derivative satisfies

$$h_{wt} w_t = -(1-\alpha)^2 \phi y_t / (w_t n_t) = -h_t. \tag{102}$$

Using the hours equation in the vacancy posting condition (51) to replace $w_{t+1} h_{t+1}$ yields:

$$\kappa / q_t = (1-\rho) E_t \left\{ \beta_{t,t+1} ([1-(1-\alpha)\phi](1-\alpha) y_{t+1} / n_{t+1} - \Phi + \kappa / q_{t+1}) \right\}. \tag{103}$$

The surplus sharing rule is given by (64) as before, but now the marginal benefits to the worker and to the firm of raising the wage no longer cancel out. This is because the bargaining wage has an effect on hours through (102). Denoting the effect on the firm's and worker's surplus of raising the wage as $\delta_t^f = \partial \mathcal{J}_t / \partial w_t$ and $\delta_t^w = \partial \mathcal{W}_t / \partial w_t$, respectively, we have

$$\delta_t^f = \phi e_0 h_t^{\phi-1} (1-\alpha)^2 z_t k_t^{\alpha} (e_0 h_t^{\phi} n_t)^{-\alpha} h_{wt} - h_t - h_{wt} w_t,$$

$$\delta_t^w = h_t + h_{wt} w_t - \mathcal{G}_t' c_t h_{wt}.$$

Using $h_t + h_{wt} w_t = 0$ from (102), we obtain

$$\delta_t^f = -\phi (1-\alpha)^2 \frac{y_t}{h_t n_t} \frac{h_t}{w_t},$$

$$\delta_t^w = \mathcal{G}_t' c_t h_t / w_t.$$

These expressions can be rewritten once more using $(1-\alpha)^2 \phi y_t/(n_t h_t) = w_t$ from the choice of hours worked (101) and $\mathcal{G}_t' h_t = (1+\gamma)\mathcal{G}_t$ from (61),

$$\delta_t^f = -h_t, \tag{104}$$

$$\delta_t^w = (1+\gamma)\mathcal{G}_t c_t / w_t. \tag{105}$$

Inserting the worker's and firm's surplus, (56) and (48), into the sharing rule (64) we obtain

$$\begin{aligned}(1-\omega)\delta_t^f [w_t h_t - \mathcal{G}_t c_t - b + &(1-\rho)E_t\{\beta_{t,t+1}(1-p_t)\mathcal{W}_{t+1}\}] \\ &= \omega\delta_t^w \left[(1-\alpha)y_t/n_t - w_t h_t - \Phi + (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}\right].\end{aligned}$$

Dividing by $(1-\omega)\delta_t^f$ and using the sharing rule (64) to replace $\mathcal{W}_{t+1}$ with $\frac{\omega\delta_t^w}{(1-\omega)\delta_t^f}\mathcal{J}_{t+1}$ yields

$$\begin{aligned}w_t h_t - \mathcal{G}_t c_t - b + &(1-\rho)E_t\left\{\beta_{t,t+1}(1-p_t)\frac{\omega\delta_t^w}{(1-\omega)\delta_t^f}\mathcal{J}_{t+1}\right\} \\ &= \frac{\omega\delta_t^w}{(1-\omega)\delta_t^f}\left[(1-\alpha)y_t/n_t - w_t h_t - \Phi + (1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}\right].\end{aligned}$$

Cancelling terms in $(1-\rho)\frac{\omega\delta_t^w}{(1-\omega)\delta_t^f}E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}$, we find

$$w_t h_t - \mathcal{G}_t c_t - b - \frac{\omega\delta_t^w}{(1-\omega)\delta_t^f}p_t(1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\} = \frac{\omega\delta_t^w}{(1-\omega)\delta_t^f}[(1-\alpha)y_t/n_t - w_t h_t - \Phi].$$

Using the free entry condition (49) to replace $(1-\rho)E_t\{\beta_{t,t+1}\mathcal{J}_{t+1}\}$ with κ/q_t yields

$$w_t h_t - \mathcal{G}_t c_t - b = \frac{\omega\delta_t^w}{(1-\omega)\delta_t^f}\left((1-\alpha)\frac{y_t}{n_t} - w_t h_t - \Phi + p_t\frac{\kappa}{q_t}\right).$$

Use $p_t/q_t = \theta_t$, multiply by $(1-\omega)\delta_t^f$, and collect terms in $w_t h_t$, such that

$$[(1-\omega)\delta_t^f + \omega\delta_t^w]w_t h_t = (1-\omega)\delta_t^f(\mathcal{G}_t c_t + b) + \omega\delta_t^w[(1-\alpha)y_t/n_t - \Phi + \kappa\theta_t].$$

Then rearrange to get the per-person wage,

$$w_t h_t = (1-\varphi_t)(\mathcal{G}_t c_t + b) + \varphi_t[(1-\alpha)y_t/n_t - \Phi + \kappa\theta_t], \tag{106}$$

where the effective bargaining weight is given by

$$\varphi_t = \frac{\omega\delta_t^w}{(1-\omega)\delta_t^f + \omega\delta_t^w}. \tag{107}$$

7.6 Closing the model

As in Section 6, bond market clearing implies $b_t = 0$ for all t. Imposing this condition in the household budget constraint (99) holding with equality, and combining with symmetric firm profits (98), yields the aggregate accounting relation,

$$c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 + a_t = [1 - 0.5 \cdot \phi_p (\pi_t - 1)^2]y_t - \kappa v_t. \tag{108}$$

The monetary policy rule is given by (93). A formal definition of equilibrium, as summarized in Table 14, is as follows.

Definition 5 *A competitive equilibrium in the right-to-manage model is a set of sequences* $\{u_t,$ $m_t,$ $\theta_t,$ $q_t,$ $n_t,$ $k_t,$ $y_t,$ $r_t,$ $\mathcal{G}_t,$ $h_t,$ $e_t,$ $c_t,$ $v_t,$ $\pi_t,$ $\Psi_t,$ $R_t\}_{t=0}^{\infty}$ *that satisfy the constant labor force assumption (39), the matching function (40), the definitions of labor market tightness (41) and the vacancy filling rate (42), the law of motion for employment (44), the household's first order conditions for capital (7) and bonds (83), the vacancy posting condition (51), the firm's production function (45), the firm's demand for capital (97), the price setting condition (90), the optimality condition for effort (21), monetary policy (93) and aggregate accounting (108), given exogenous processes for technology* $\{z_t\}_{t=0}^{\infty}$ *and demand shocks* $\{a_t\}_{t=0}^{\infty}$*. Hours and the bargaining wage are given by (101) and (106), respectively.*

Steady state and calibration. Up to the computation of steady state consumption, we follow the same steps as in the search-and-matching model. Then, we derive the steady state wage using the hours choice (101),

$$w = (1-\alpha)^2\phi y/(hn). \tag{109}$$

The vacancy posting condition (51) determines overhead costs

$$\Phi = (1-\alpha)\frac{y}{n} - wh - \frac{1-(1-\rho)\beta}{(1-\rho)\beta}\frac{\kappa}{q}. \tag{110}$$

Table 14: Equilibrium conditions, right-to-manage model

$u_t = 1 - n_t$	Unemployment
$m_t = \chi u_t^{\xi} v_t^{1-\xi}$	Matching function
$\theta_t = v_t/u_t$	Labor market tightness
$q_t = m_t/v_t$	Vacancy filling rate
$n_t = (1-\rho)(n_{t-1} + m_{t-1})$	Employment dynamics
$y_t = z_t k_{t-1}^{\alpha} H_t^{1-\alpha}$	Production function
$\kappa/q_t = (1-\rho)E_t\left\{\beta_{t,t+1}[(1-\alpha)y_{t+1}/n_{t+1} - w_{t+1}h_{t+1} - \Phi + \kappa/q_{t+1}]\right\}$	Vacancy posting
$\alpha y_t/k_{t-1} = r_t/\Psi_t$	Capital demand
$1 + \phi_k(k_t - k_{t-1}) = \mathrm{E}_t\left\{\beta_{t,t+1}\left[(1-\delta) + \phi_k(k_{t+1} - k_t)\right] + r_{t+1}\right\}$	FOC capital
$e_t = [(1+\tau)B/(\tau C)]^{1/(1+\tau)} h_t^{\gamma/(1+\tau)}$	Effort
$c_t + I_t + 0.5\phi_k(k_t - k_{t-1})^2 + a_t = [1 - 0.5\phi_p\left(\pi_t - 1\right)^2]y_t - \kappa v_t$	Aggregate accounting
$I_t = k_t - (1-\delta)k_{t-1}$	Investment
$\mathcal{G}_t = B(1+\gamma+\tau)h_t^{1+\gamma}/[(1+\gamma)\tau]$	Labor disutility
$\beta_{t-1,t} = \beta c_{t-1}/c_t$	Stoch. discount factor
$(1-\alpha)^2\phi y_t/(n_t h_t) = w_t$	Hours
$w_t h_t = (1-\varphi_t)(\mathcal{G}_t c_t + b) + \varphi_t[(1-\alpha)y_t/n_t - \Phi + \kappa\theta_t]$	Wage
$\varphi_t = \omega\delta_t^w/[(1-\omega)\delta_t^f + \omega\delta_t^w]$	Effective bargaining weight
$\delta_t^f = -h_t$	Effect of wage on firm surplus
$\delta_t^w = (1+\gamma)\mathcal{G}_t c_t/w_t$	Effect of wage on worker surplus
$1 = R_t \mathrm{E}_t\{\beta_{t,t+1}/\pi_{t+1}\}$	FOC bonds
$(1-\varepsilon) + \varepsilon\Psi_t = \phi_p(\pi_t - 1)\pi_t - \phi_p \mathrm{E}_t\{\beta_{t,t+1}(\pi_{t+1} - 1)\pi_{t+1} y_{t+1}/y_t\}$	Price setting
$R_t/R = (\pi_t/\pi)^{\Omega_\pi}$	Monetary policy

Notes. Endogenous variables u_t, m_t, θ_t, q_t, n_t, y_t, v_t, k_t, r_t, e_t, c_t, I_t, $\mathcal{G}_t$, $\beta_{t-1,t}$, h_t, w_t, φ_t, δ_t^f, δ_t^w, π_t, Ψ_t, R_t.

We calibrate the effective bargaining weight φ and compute the disutility of labor $\mathcal{G}$ from the wage equation (106),

$$\mathcal{G} = \frac{1}{c}\left(\frac{1}{1-\varphi}\{wh - \varphi[(1-\alpha)y/n - \Phi + \kappa\theta]\} - b\right). \tag{111}$$

We then find the effect of a wage rise on the firm's and worker's surplus, $\delta^f = -h$ and $\delta^w = (1+\gamma)\mathcal{G}c/w$, respectively. The definition of the effective bargaining power (107) determines ω,

$$\omega = \frac{\delta^f}{(1/\varphi - 1)\delta^w + \delta^f}. \tag{112}$$

Then, we solve the expression for steady state labor disutility (60) for B,

$$B = \mathcal{G}\left[\frac{1+\gamma+\tau}{(1+\gamma)\tau}h^{1+\gamma}\right]^{-1}. \tag{113}$$

Given B, the optimality condition for effort (21) pins down the utility parameter C,

$$C = \left(\frac{1+\tau}{\tau}B\right)h^{\gamma}e^{-(1+\tau)}. \tag{114}$$

Table 15 summarizes the recursive steady state equations of the right-to-manage model.

— [insert Figures 21 and 22 here] —

8 Gift exchange model

Based on the pioneering work by Akerlof (1982), various papers (Danthine and Kurmann, 2004; de la Croix et al., 2009) propose a model with efficiency wages of the gift exchange variety. Workers dislike effort. They work harder than some required minimum (the worker's gift to the firm) in exchange for a real wage above some reference compensation level that is considered 'fair' (the firm's gift to the worker). By setting the wage, a firm can influence the amount of effort that the worker exerts on the job. An individual firm setting a high wage imposes an externality on other firms: as the aggregate wage rises, other employers will be forced to raise their wage as well in order to maintain their workers' effort level.

The gift exchange (or fair-wage model) predicts a low correlation between wages and employment, consistent with the data. The New Keynesian version we use here also predicts a dampening of marginal cost dynamics in response to demand shocks, due to a real wage rigidity that arises endogenously through the efficiency wage mechanism.

Table 15: Recursive steady state, right-to-manage model

$y = h = e = 1$ (normalization)	firm output, hours, effort
$n = 1 - u$	employment
$H = ehn$	total effective hours
$m = \rho n/(1-\rho)$	matches
$v = m/q$	vacancies
$\theta = v/u$	labor market tightness
$\chi = q\theta^{\xi}$	matching efficiency
$\Psi = (\varepsilon - 1)/\varepsilon$	inverse gross price markup
$\pi = 1$	gross inflation
$R = \pi/\beta$	policy rate
$r = 1/\beta - (1-\delta)$	rental rate on capital
$k = \alpha y \Psi / r$	capital stock
$\kappa = (\kappa v/y)y/v$	vacancy posting cost
$c = y - \kappa v - a - \delta k$	consumption
$z = y/(k^{\alpha} H^{1-\alpha})$	production efficiency
$w = (1-\alpha)^2 \phi y/(hn)$	wage
$\Phi = (1-\alpha)y/n - wh - (\kappa/q)[1-(1-\rho)\beta]/[(1-\rho)\beta]$	overhead cost
$\mathcal{G} = (\{wh - \varphi[(1-\alpha)y/n - \Phi + \kappa\theta]\}/(1-\varphi) - b)/c$	labor disutility
$\omega = \delta^f/[(1/\varphi - 1)\delta^w + \delta^f]$	effective bargaining weight
$\delta^f = -h$	effect of wage on firm surplus
$\delta^w = (1+\gamma)\mathcal{G}c/w$	effect of wage on worker surplus
$B = \mathcal{G}/\left[(1+\gamma+\tau)h^{1+\gamma}/(1+\gamma)\tau\right]$	weight on hours in labor disutility
$C = [(1+\tau)B/\tau]h^{\gamma}e^{-(1+\tau)}$	weight on effort in labor disutility

Notes. Calibrated parameters: β, γ, τ, α, δ, a, u, b, q, ρ, ξ, φ, ε, ϕ_p, Ω_π. Composite parameter: $\phi = 1 + \gamma/(1+\tau)$.

Table 16: Parameterization, right-to-manage model

Value	Name	Reference
$u = 0.12$	Steady state unemployment rate	Krause and Lubik (2010)
$\rho = 0.10$	Job separation rate	Shimer (2005)
$q = 0.70$	Steady state vacancy filling rate	den Haan et al. (2000)
$\xi = 0.50$	Match elasticity to unemployment	Petrongolo and Pissarides (2001)
$\kappa v/y = 0.01$	Vacancy posting cost share in output	Andolfatto (1996)
$b = 0.25$	Leisure value	$b = 0.40$ violates rank condition
$\varphi = 0.50$	Workers' effective bargaining weight	Hosios (1990)

To facilitate the introduction of this alternative effort specification, we work with constant hours and thus remove h_t from the model.

8.1 Households

Households have preferences over consumption and effort. Their optimization problem is the following,

$$\max_{\{c_t,k_t,b_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \{\ln c_t - n_t \mathcal{G}(e_t)\}, \tag{115}$$

where $\mathcal{G}(e_t)$ is the individual effort disutility function of employed household members, subject to the household budget constraint,

$$c_t + k_{t+1} - (1-\delta)k_t + 0.5 \cdot \phi_k (k_{t+1} - k_t)^2 + b_t \leq w_t n_t + r_t k_t + D_t + b_{t-1} R_{t-1}/\pi_t. \tag{116}$$

With the household's Lagrangian problem written as

$$\begin{aligned} \mathcal{L} =& \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - n_t \mathcal{G}(e_t) \right| \\ & - \left| \lambda_t \left[c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 + b_t - w_t n_t - r_t k_t - D_t - \frac{b_{t-1} R_{t-1}}{\pi_t} \right] \right\}, \end{aligned}$$

the household's equilibrium conditions for consumption c_t, capital k_t and bonds b_t satisfy (7) and (83) as in the sticky-price model.

In a Walrasian labor market, the household would choose labor supply to maximize utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - \frac{A}{1+\eta} n_t^{1+\eta} \right\}, \tag{117}$$

such that the first order condition for labor supply is $\partial \mathcal{L} / \partial n_t = \beta^t \left(-A n_t^{\eta} + \lambda_t w_t \right) \stackrel{!}{=} 0$. Replacing the Lagrange multiplier λ_t with $1/c_t$ from (4), we can write the first order condition as

$$A n_t^{\eta} = \frac{w_t}{c_t}. \tag{118}$$

8.2 Effort

Worker j's disutility of providing effort is specified as the squared deviation of e_{jt} from a minimum level e_{jt}^*, to be defined below,

$$\min_{e_{jt}} \ \mathcal{G}(e_{jt}) = (e_{jt} - e_{jt}^*)^2.$$

The minimum effort level e^*_{jt} depends on the firm's gift to the worker, i.e. the amount by which the individual wage w_{jt} exceeds the fair wage. The latter, in turn, depends on the current aggregate wage w_t and on labor market conditions n_t, and a constant τ_0,

$$e^*_{jt} = \tau_1 \frac{w^\mu_{jt} - (\tau_0 + \tau_2 n^\mu_t + \tau_3 w^\mu_t)}{\mu}, \tag{119}$$

The functional form in (119) loosely follows de la Croix et al. (2009), which is a generalization of the logarithmic function in Danthine and Kurmann (2004). More precisely, we obtain a logarithmic effort function for $\mu \to 0$.

The wage w_{jt} is chosen by the firm and is considered exogenous by the worker. Optimal effort therefore satisfies

$$e_{jt} = \tau_1 \frac{w^\mu_{jt} - (\tau_0 + \tau_2 n^\mu_t + \tau_3 w^\mu_t)}{\mu}. \tag{120}$$

We impose the following parameter restrictions, $\tau_0 \in \mathbb{R}$, $\tau_1 > 0, \tau_2 > 0$, $\tau_3 \in [0, 1)$, $\mu \in [0, 1)$. Consistent with the gift exchange theory, a worker will respond to a higher individual wage w_{jt}, ceteris paribus, by exerting greater effort, such that $\tau_1 > 0$. The reference compensation level depends positively on labor market tightness n_t and on the comparison wage w_t, such that $\tau_2 > 0$ and $\tau_3 > 0$. Furthermore, we impose an upper bound of 1 on the response of effort to the aggregate wage, τ_3, reflecting the assumption that the wage externality alone cannot overwhelm the direct effect of the individual wage on effort. Finally, the elasticity of the effort function, μ, is bounded below by 0, which represents the logarithmic case. We impose an upper bound of 1 to ensure that the firm faces decreasing marginal returns – in terms of worker effort – from raising wages.

8.3 Firms

The final goods sector is as in the sticky-price model of section 6. Intermediate firm i's current profits are

$$D_{it} = P_{it} y_{it}/P_t - w_{it} n_{it} - r_t k_{it} - 0.5 \cdot \phi_p (P_{it}/P_{i,t-1} - 1)^2 y_t. \tag{121}$$

Notice in the profit function (121) that the wage is a choice variable of the firm, while the rental rate on capital is common to all firms. The firm's production function is given by $y_{it} = z_t k^\alpha_{it} H^{1-\alpha}_{it} - g_{it}$, where effective labor input is $H_{it} = e_{it} n_{it}$.

The firm chooses paths for P_{it}, k_{it}, n_{it} and w_{it} to maximize the expected discounted stream of profits,

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \left\{ (P_{it}/P_t)^{1-\varepsilon} y_t - w_{it} n_{it} - r_t k_{it} - 0.5 \cdot \phi_p (P_{it}/P_{i,t-1} - 1)^2 y_t \right\}, \tag{122}$$

subject to the demand constraint (86) and firm i's effort function,

$$e_{it} = \tau_1 \frac{\tau_0 + w_{it}^{\mu} - \tau_2 n_t^{\mu} - \tau_3 w_t^{\mu}}{\mu}. \tag{123}$$

When setting the wage, firm i needs to consider the effect that this decision has on worker effort via (123). The firm maximizes

$$\begin{aligned} \mathcal{L} =& \mathrm{E}_0 \sum_{t=0}^{\infty} \beta_{t,t+1} \{ (P_{it}/P_t)^{1-\varepsilon} y_t - w_{it} n_{it} - r_t k_{it} - 0.5 \cdot \phi_p \left(P_{it}/P_{i,t-1} - 1 \right)^2 y_t \\ & - \Psi_{it} \left[(P_{it}/P_t)^{-\varepsilon} y_t - z_t k_{it}^{\alpha} H_{it}^{1-\alpha} + 0.5 \cdot \phi_n (n_{it} - n_{it-1})^2 \right] \}. \end{aligned} \tag{124}$$

The firm's first order condition with respect to the price P_{it} is derived as in the sticky-price model of Section 6 and leads to the New Keynesian Phillips Curve (90). In a symmetric equilibrium, firm profits are given by:

$$D_t = [1 - 0.5 \cdot \phi_p (\pi_t - 1)^2] y_t - w_t n_t - r_t k_t. \tag{125}$$

The firm's first order conditions with respect to capital, employment and wages are, respectively,

$$\frac{\partial \mathcal{L}}{\partial k_{it}} = -r_t + \Psi_{it} \alpha z_t k_{it}^{\alpha-1} H_{it}^{1-\alpha} \stackrel{!}{=} 0,$$

$$\frac{\partial \mathcal{L}}{\partial n_{it}} = -w_{it} + \Psi_{it} \left[(1-\alpha) z_t k_{it}^{\alpha} H_{it}^{-\alpha} e_{it} - \phi_n (n_{it} - n_{it-1}) + \mathrm{E}_t \{ \beta_{t,t+1} \phi_n (n_{it+1} - n_{it}) \} \right] \stackrel{!}{=} 0.$$

$$\frac{\partial \mathcal{L}}{\partial w_{it}} = -n_{it} + \Psi_{it} (1-\alpha) z_t k_{it}^{\alpha} (e_{it} n_{it})^{-\alpha} n_{it} \frac{\partial e_{it}}{\partial w_{it}} \stackrel{!}{=} 0.$$

Noting that the derivative of the effort function with respect to the individual wage w_{it} is $\tau_1 w_{it}^{\mu-1}$, we can write these equilibrium conditions as follows:

$$\Psi_{it} \alpha \frac{y_{it} + g_{it}}{k_{it}} = r_t, \tag{126}$$

$$(1-\alpha) \frac{y_{it} + g_{it}}{n_{it}} = \frac{w_{it}}{\Psi_{it}} + \phi_n (n_{it} - n_{it-1}) - \mathrm{E}_t \{ \beta_{t,t+1} \phi_n (n_{it+1} - n_{it}) \}, \tag{127}$$

$$\Psi_{it} (1-\alpha) \frac{y_{it} + g_{it}}{e_{it}} \tau_1 w_{it}^{\mu-1} = n_{it}. \tag{128}$$

Writing (128) as $(1-\alpha)(y_{it} + g_{it})/n_{it} = w_{it}^{1-\mu} e_{it}/(\tau_1 \Psi_{it})$, and plugging this result into (127) yields,

$$e_{it} = \tau_1 w_{it}^{\mu} + \tau_1 \Psi_{it} w_{it}^{\mu-1} [\phi_n (n_{it} - n_{it-1}) - \mathrm{E}_t \{ \beta_{t,t+1} \phi_n (n_{it+1} - n_{it}) \}]. \tag{129}$$

Notice from (129) that in the absence of employment adjustment costs, the effort function would be $e_{it} = \tau_1 w_{it}^{\mu}$ as in de la Croix et al. (2009). If, in addition, $\mu = 0$, effort would be constant and equal to τ_1 (Solow, 1979). Now, combining (129) with (123) yields,

$$\tau_1 w_{it}^{\mu} + \tau_1 \Psi_{it} w_{it}^{\mu-1}[\phi_n(n_{it} - n_{it-1}) - \mathrm{E}_t\{\beta_{t,t+1}\phi_n(n_{it+1} - n_{it})\}] = \tau_1 \frac{\tau_0 + w_{it}^{\mu} - \tau_2 n_t^{\mu} - \tau_3 w_t^{\mu}}{\mu}.$$

Imposing symmetry, this becomes

$$\left((\mu + \tau_3 - 1) + \mu \frac{\Psi_{it}}{w_t}[\phi_n(n_t - n_{t-1}) - \mathrm{E}_t\{\beta_{t,t+1}\phi_n(n_{t+1} - n_t)\}]\right) w_t^{\mu} = \tau_0 - \tau_2 n_t^{\mu}.$$

At the steady state, this is

$$w = \left(\frac{\tau_2}{1 - \mu - \tau_3} n^{\mu} - \frac{\tau_0}{1 - \mu - \tau_3}\right)^{\frac{1}{\mu}}.$$

In our calibration, we will follow de la Croix et al. (2009) in assuming that $1 - \mu - \tau_3 > 0$.

In a symmetric equilibrium, the firm's production function and the FOCs for capital and employment are as follows,

$$y_t = z_t k_t^{\alpha} H_t^{1-\alpha} - 0.5 \cdot \phi_n (n_t - n_{t-1})^2, \tag{130}$$

$$\Psi_t \alpha \frac{y_t + g_t}{k_t} = r_t, \tag{131}$$

$$(1 - \alpha)\frac{y_t + g_t}{n_t} = \frac{w_t}{\Psi_t} + \phi_n(n_t - n_{t-1}) - \mathrm{E}_t\{\beta_{t,t+1}\phi_n(n_{t+1} - n_t)\}. \tag{132}$$

Wages are determined by the relation

$$\Psi_t(1 - \alpha)\frac{y_t + g_t}{e_t}\tau_1 w_t^{\mu-1} = n_t. \tag{133}$$

Effort is given by

$$e_t = \tau_1 \frac{\tau_0 + (1 - \tau_3) w_t^{\mu} - \tau_2 n_t^{\mu}}{\mu}. \tag{134}$$

8.4 Closing the model

Market clearing in the bond market implies that $b_t = 0$ for all t as bonds are in zero net supply in equilibrium. Imposing that the household budget constraint (116) hold with equality, and combining it with firm profits (125), we obtain the aggregate accounting relation,

$$c_t + k_{t+1} - (1 - \delta)k_t + \frac{\phi_k}{2}(k_{t+1} - k_t)^2 + a_t = \left[1 - \frac{\phi_p}{2}(\pi_t - 1)^2\right] y_t. \tag{135}$$

A formal definition of equilibrium in the gift exchange model, as summarized in Table 17, is as follows.

Definition 6 *A competitive equilibrium in the gift exchange model is a set of sequences* $\{k_t, \pi_t, y_t, r_t, n_t, w_t, \Psi_t, e_t, R_t, c_t\}_{t=0}^{\infty}$ *that satisfy the household's first order condition for capital (7) and bonds (83), the firm's production function (130), the firm's demand for capital (131) and employment (132), the firm's wage choice (128), the price setting condition (90), the optimality condition for effort (134), monetary policy (93) and aggregate accounting (135), given exogenous processes for technology* $\{z_t\}_{t=0}^{\infty}$ *and demand shocks* $\{a_t\}_{t=0}^{\infty}$.

Table 17: Equilibrium conditions, gift exchange model

$1 + \phi_k(k_t - k_{t-1}) = \mathrm{E}_t\left\{\beta_{t,t+1}\left[1 - \delta + \phi_k(k_{t+1} - k_t) + r_{t+1}\right]\right\}$	FOC capital
$y_t = z_t k_{t-1}^{\alpha} H_t^{1-\alpha} - g_t$	Production function
$\Psi_t \alpha (y_t + g_t)/k_{t-1} = r_t$	Demand for capital
$(1-\alpha)(y_t + g_t)/n_t = w_t/\Psi_t + \phi_n(n_t - n_{t-1}) - \mathrm{E}_t\{\beta_{t,t+1}\phi_n(n_{t+1} - n_t)\}$	Demand for employment
$c_t + k_t - (1-\delta)k_{t-1} + 0.5\phi_k(k_t - k_{t-1})^2 + a_t = [1 - 0.5\phi_p(\pi_t - 1)^2]y_t$	Aggregate accounting
$H_t = e_t n_t$	Effective labor
$g_t = 0.5 \cdot \phi_n(n_t - n_{t-1})^2$	Employment adj. cost
$\beta_{t-1,t} = \beta c_{t-1}/c_t$	Stoch discount factor
$1 = R_t \mathrm{E}_t\{\beta_{t,t+1}/\pi_{t+1}\}$	FOC bonds
$(1-\varepsilon) + \varepsilon\Psi_t = \phi_p(\pi_t - 1)\pi_t - \phi_p \mathrm{E}_t\{\beta_{t,t+1}(\pi_{t+1} - 1)\pi_{t+1} y_{t+1}/y_t\}$	Price setting
$R_t/R = (\pi_t/\pi)^{\Omega_\pi}$	Monetary policy
Gift exchange model	
$e_t = \tau_1[\tau_0 + (1-\tau_3)w_t^{\mu} - \tau_2 n_t^{\mu}]/\mu$	Effort
$\Psi_t(1-\alpha)[(y_t + g_t)/e_t]\tau_1 w_t^{\mu-1} = n_t$	Wage
Walrasian labor market model	
$e_t = e$	Effort
$A n_t^{\eta} = w_t/c_t$	Wage

Note. Endogenous variables: k_t, y_t, r_t, n_t, w_t, e_t, c_t, H_t, g_t, $\beta_{t-1,t}$, π_t, Ψ_t, R_t.

8.5 Steady state and calibration

In both the gift exchange model and the Walrasian labor market model, the computation of the steady state is as in the sticky-price model up to the determination of the wage rate w, see Table 13, with hours normalized to 1. In the *gift exchange model*, the wage setting equation (128) can be solved for the parameter τ_1 as follows, $\tau_1 = w^{1-\mu} n[\Psi(1-\alpha)(y+g)/e]^{-1}$. We plug in the wage to obtain

$$\tau_1 = w^{-\mu} e. \tag{136}$$

Solving the effort function (134) at steady state for τ_0, and plugging in $e/\tau_1 = w^\mu$, we obtain

$$\tau_0 = [\mu - (1 - \tau_3)]w^\mu + \tau_2 n^\mu. \tag{137}$$

In a *Walrasian labor market*, we compute the weight on employment in utility, A, from the labor supply equation (118),

$$A = \frac{w}{n^\eta c} \tag{138}$$

Table 18 summarizes the recursive formulation of the steady state in the gift exchange model and in the model with a Walrasian labor market.

Table 18: Recursive steady state, gift exchange model

$GDP = e = 1$ (normalization)	GDP and effort
$\pi = 1$ (normalization)	gross inflation
$g = 0$	employment adjustment cost
$y = GDP - g$	net output
$\Psi = (\varepsilon - 1)/\varepsilon$	inverse gross price markup
$R = \pi/\beta$	policy rate
$r = 1/\beta - (1 - \delta)$	rental rate on capital
$k = \Psi\alpha(y + g)/r$	capital stock
$c = y - a - \delta k$	consumption
$H = [(y + g)/(zk^\alpha)]^{1/(1-\alpha)}$	effective labor
$n = H/e$	employment
$w = \Psi(1 - \alpha)(y + g)/n$	wage
Gift exchange model	
$\tau_1 = w^{-\mu}e$	coefficient in effort function
$\tau_0 = [\mu - (1 - \tau_3)]w^\mu + \tau_2 n^\mu$	coefficient in effort function
Walrasian labor market model	
$A = w/(n^\eta c)$	weight on employment in utility

Notes. Calibrated parameters: β, α, δ, ϵ, a, z, τ_2, τ_3.

Calibration. When calibrating the coefficients in the effort function, we recall the following parameter restrictions: effort depends negatively on the comparison wage, $\tau_3 > 0$, and it is more sensitive to the individual wage than to the aggregate wage, $\tau_3 < 1$. Economic theory (e.g. Shapiro and Stiglitz, 1984) suggests that the response of effort to labor market tightness is negative, i.e. $\tau_2 > 0$. From the empirical results in Danthine and Kurmann (2004), we know that the current wage is more responsive to the lagged wage than to current employment. To account for this, we impose that τ_2 is smaller than τ_3. In our choice for μ, τ_3 and τ_2 shown in Table 19, we follow de la Croix et al. (2009).

Table 19: Parameterization, gift exchange model

Value	Name	Restriction
$\eta = 0.6$	Labor supply elasticity (Walras model)	Llosa et al. (2014)
$\mu = 0.1$	Elasticity of the effort function	$\mu \in [0, 1)$
$\tau_3 = 0.4$	Response of effort to comparison wage	$\tau_3 \in [0, 1)$, $1 - \mu - \tau_3 > 0$
$\tau_2 = 0.2$	Response of effort to employment	$\tau_2 > 0$ and $\tau_2 < \tau_3$

— [insert Figures 23 and 24 here] —

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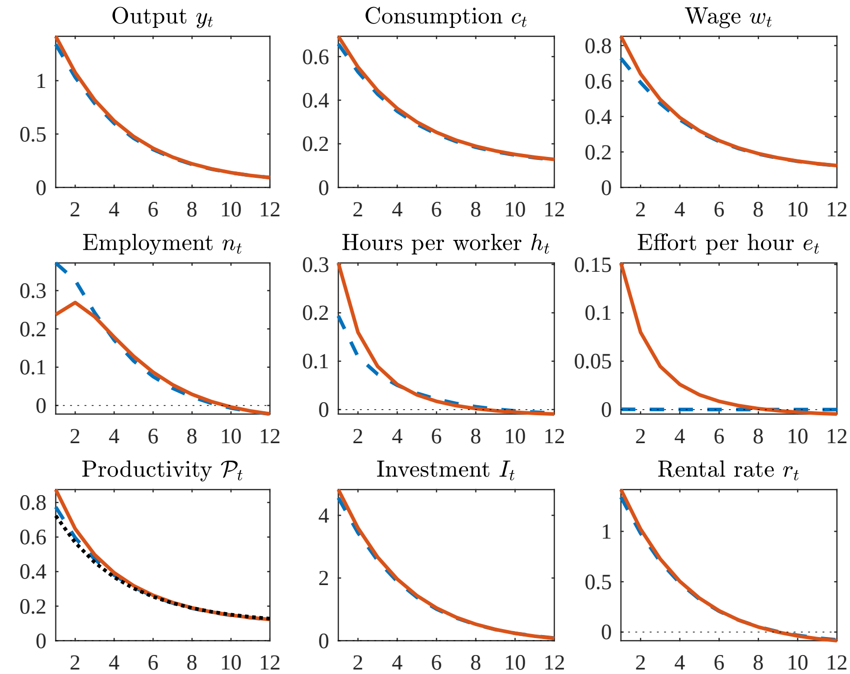

Figure 7: Responses to one-percent technology shock, **baseline model**

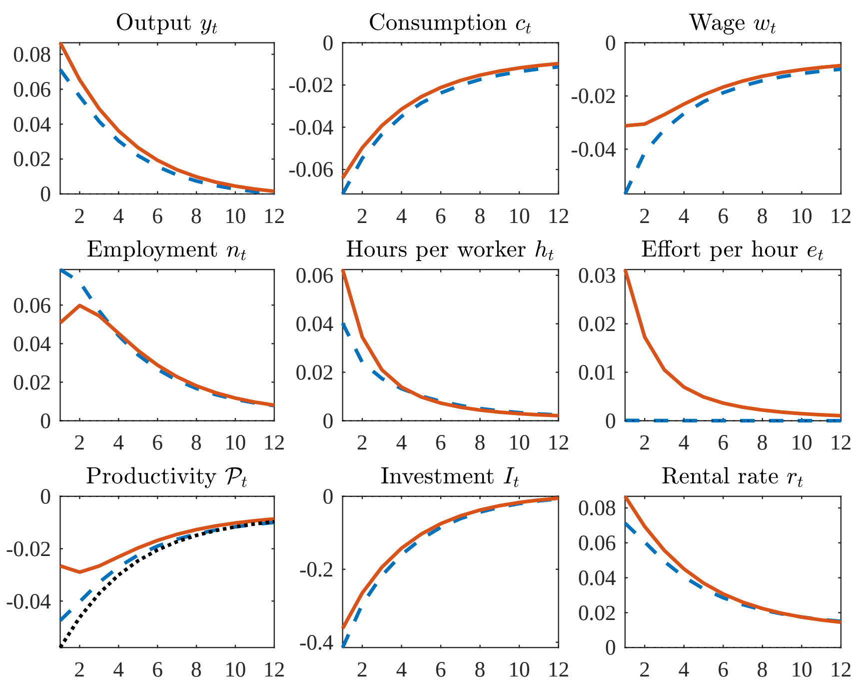

Figure 8: Responses to one-percent demand shock, **baseline model**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.2$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

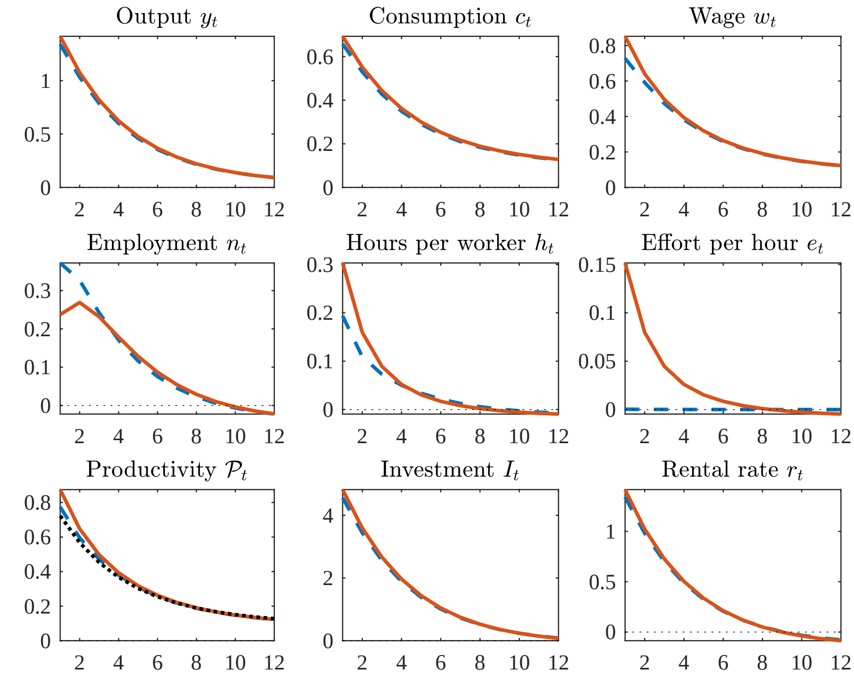

Figure 9: Responses to one-percent technology shock, **baseline model** & **rigid labor market**

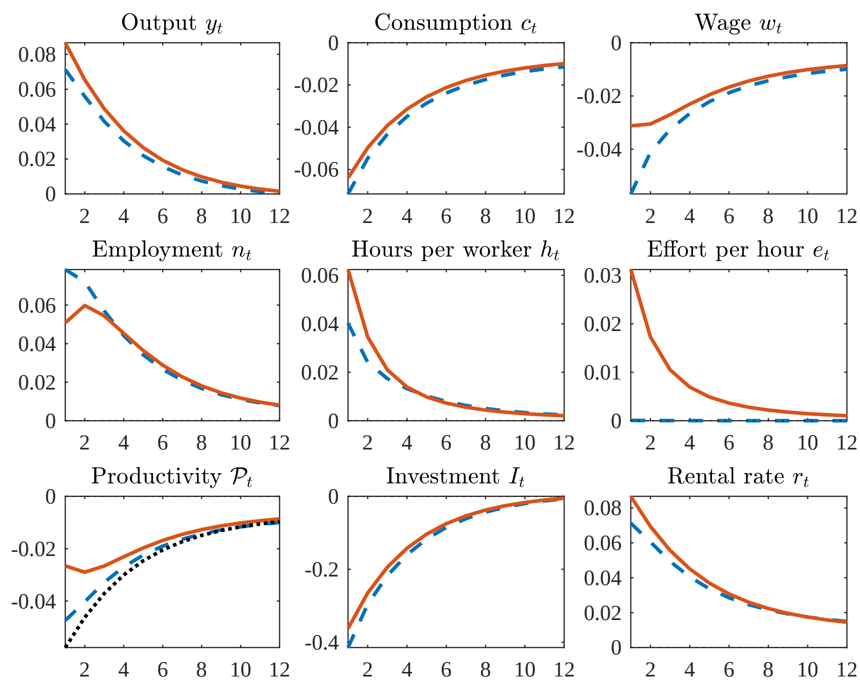

Figure 10: Responses to one-percent demand shock, **baseline model** & **rigid labor market**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.2$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

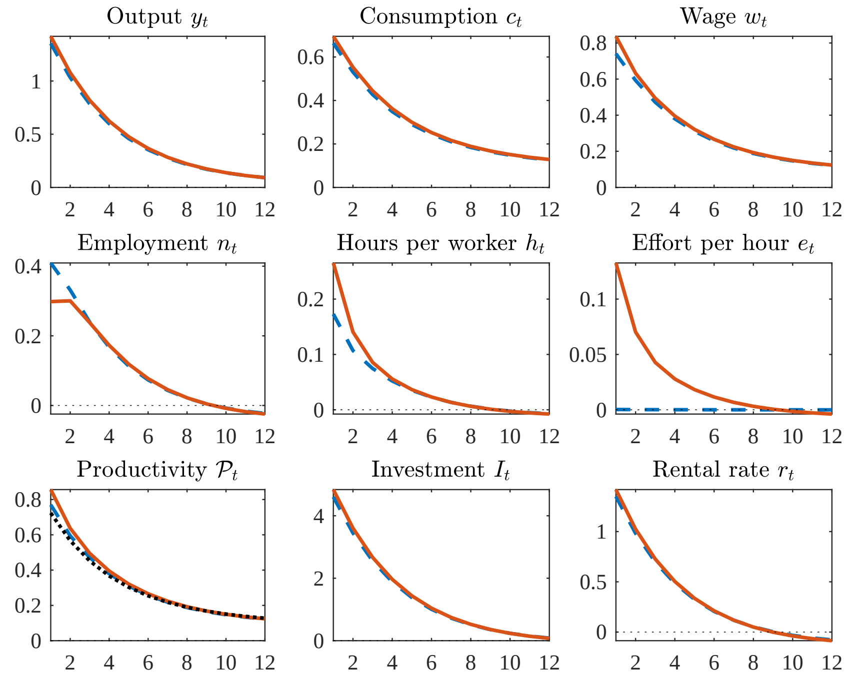

Figure 11: Responses to one-percent technology shock, **firing cost model**

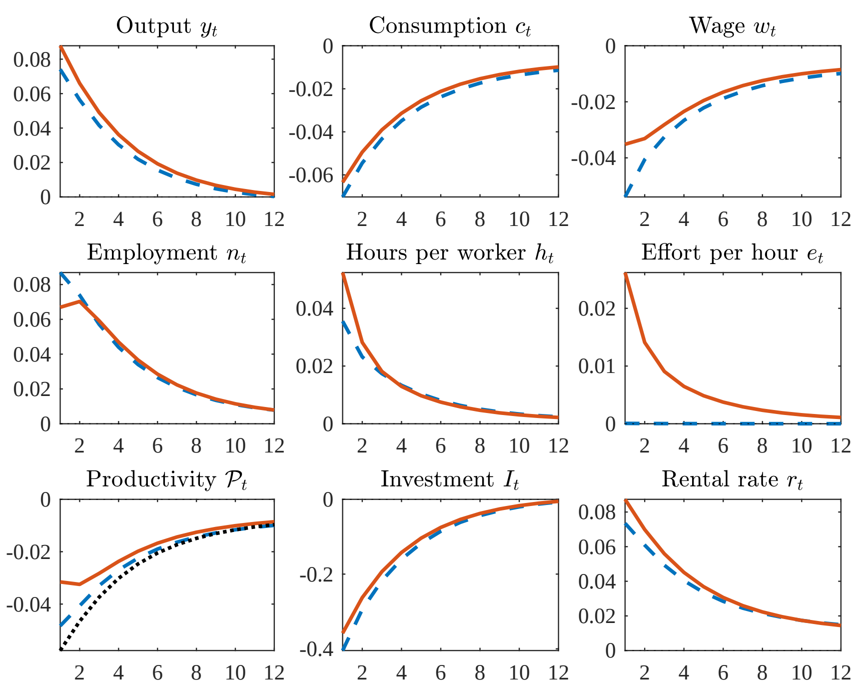

Figure 12: Responses to one-percent demand shock, **firing cost model**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.2$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

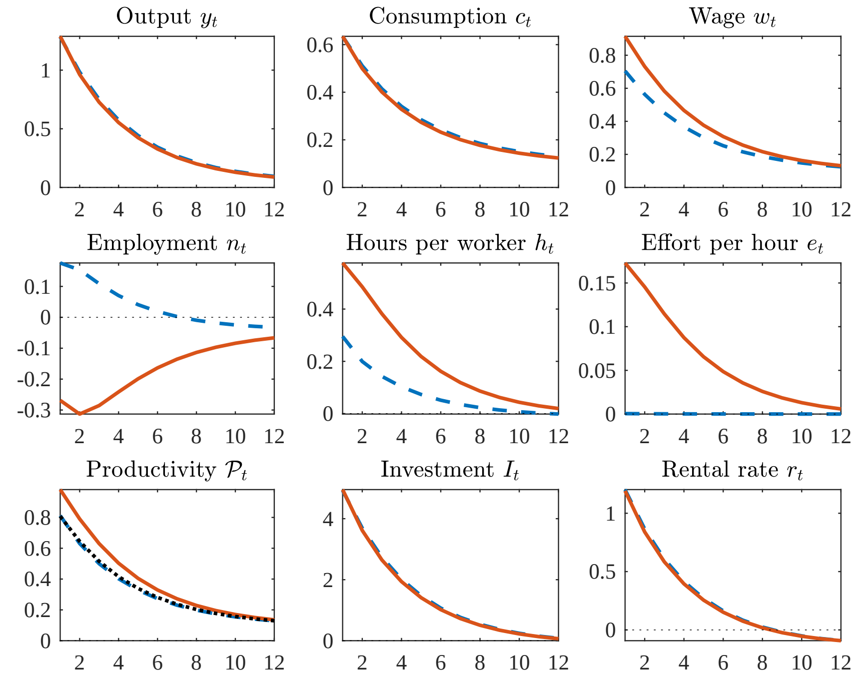

Figure 13: Responses to one-percent technology shock, **sticky-price model**

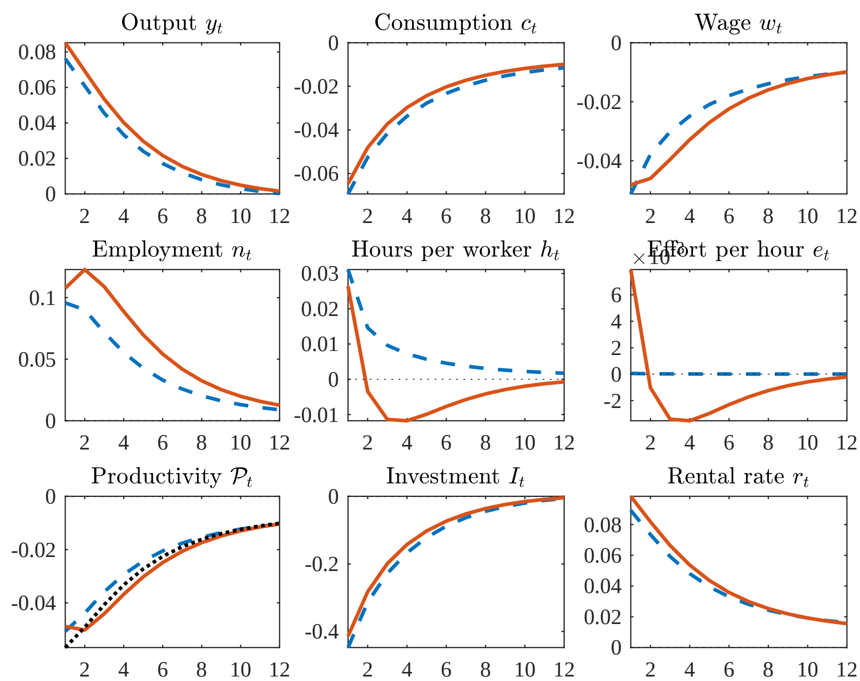

Figure 14: Responses to one-percent demand shock, **sticky-price model**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.2$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

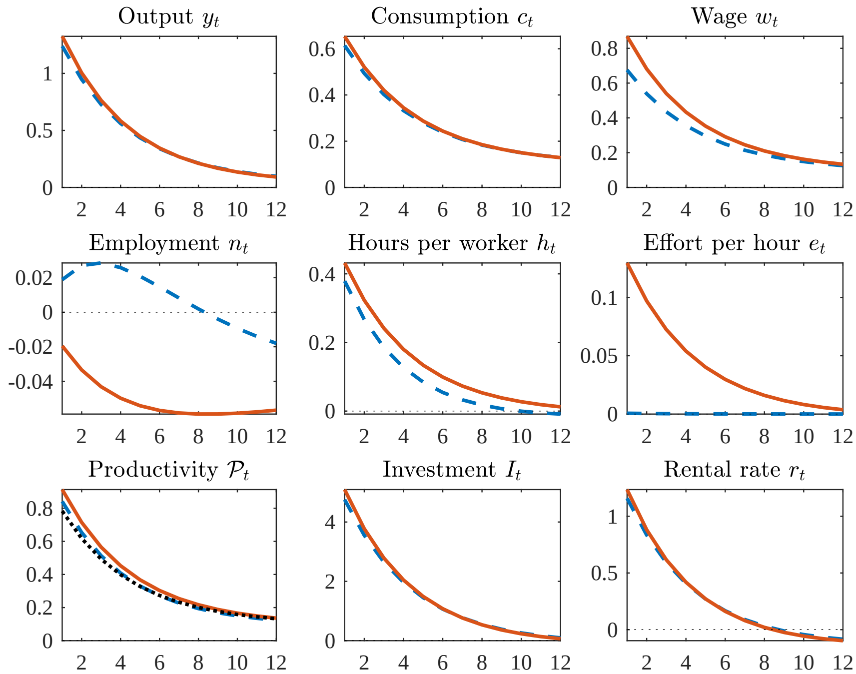

Figure 15: Responses to one-percent technology shock, **sticky prices & rigid labor market**

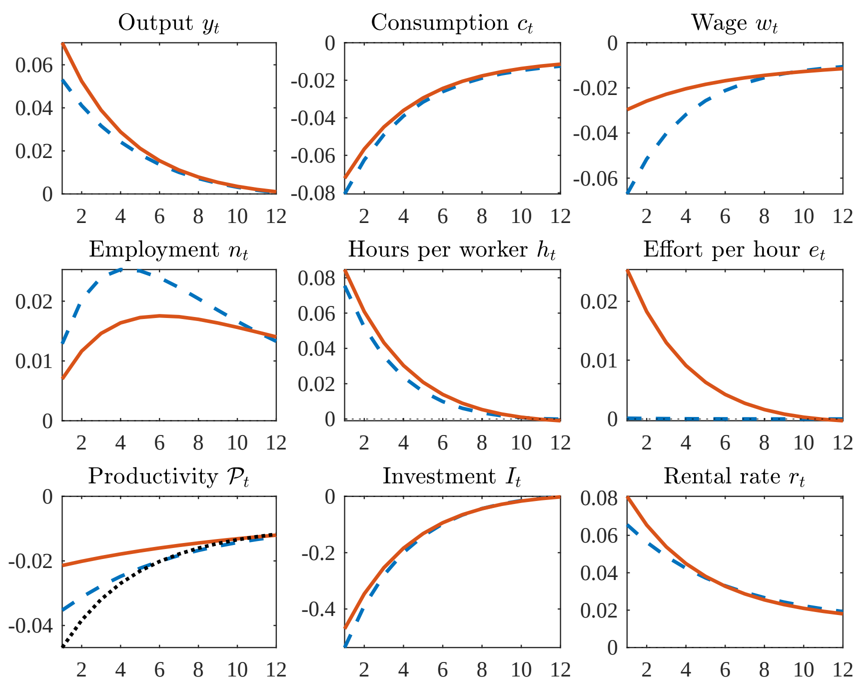

Figure 16: Responses to one-percent demand shock, **sticky prices & rigid labor market**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.2$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

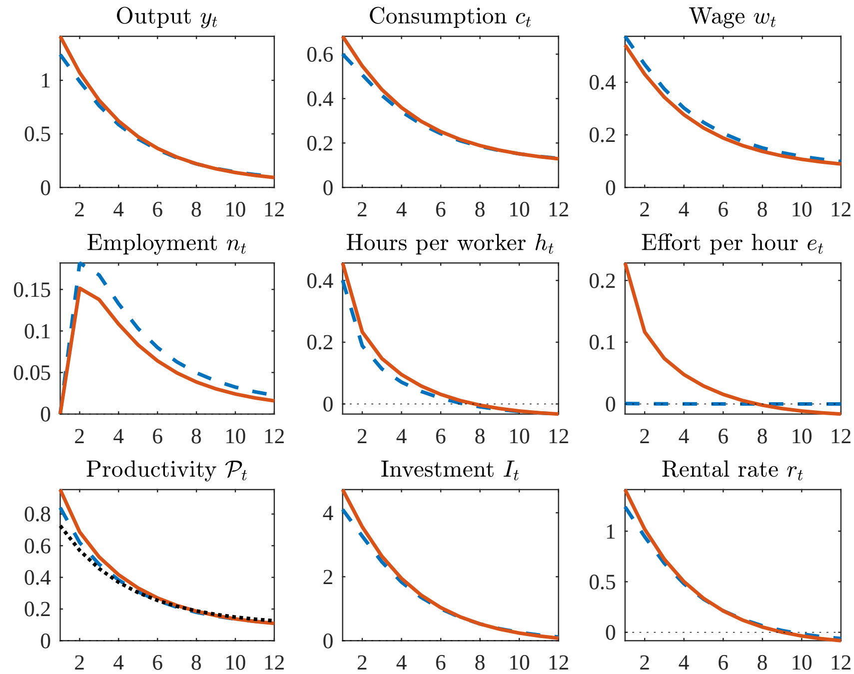

Figure 17: Responses to one-percent technology shock, **search model**

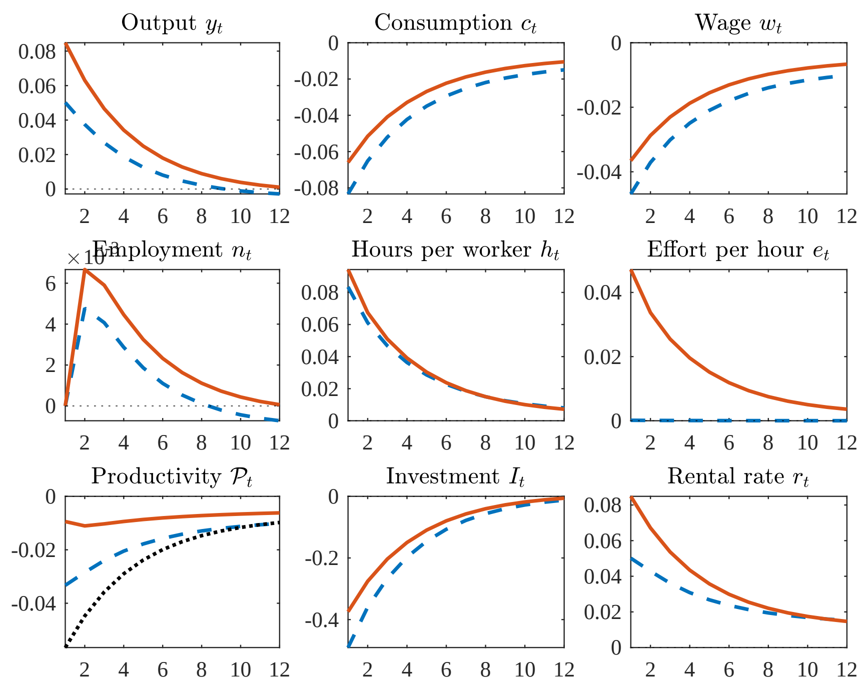

Figure 18: Responses to one-percent demand shock, **search model**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.2$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

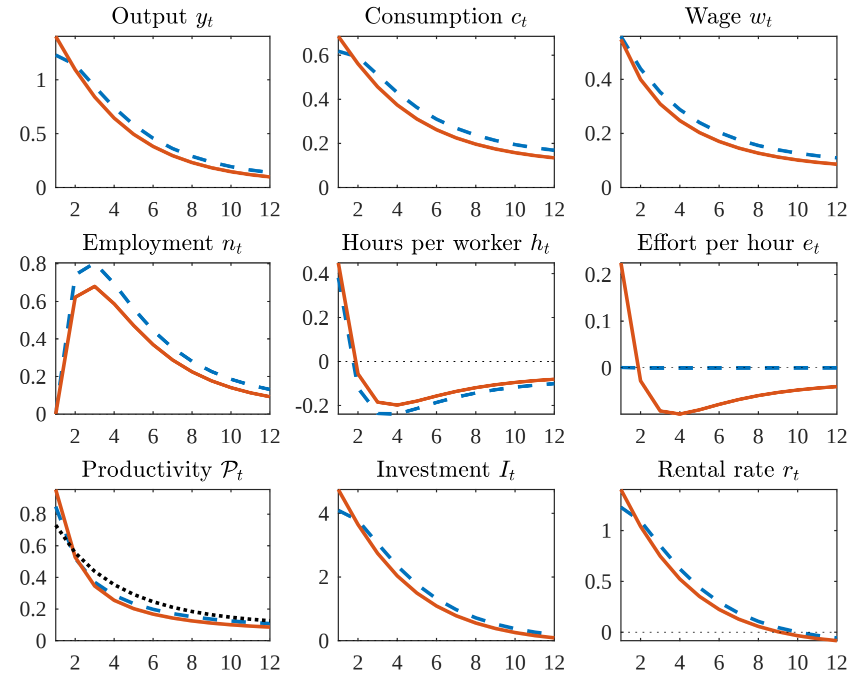

Figure 19: Responses to one-percent technology shock, **hiring cost model**

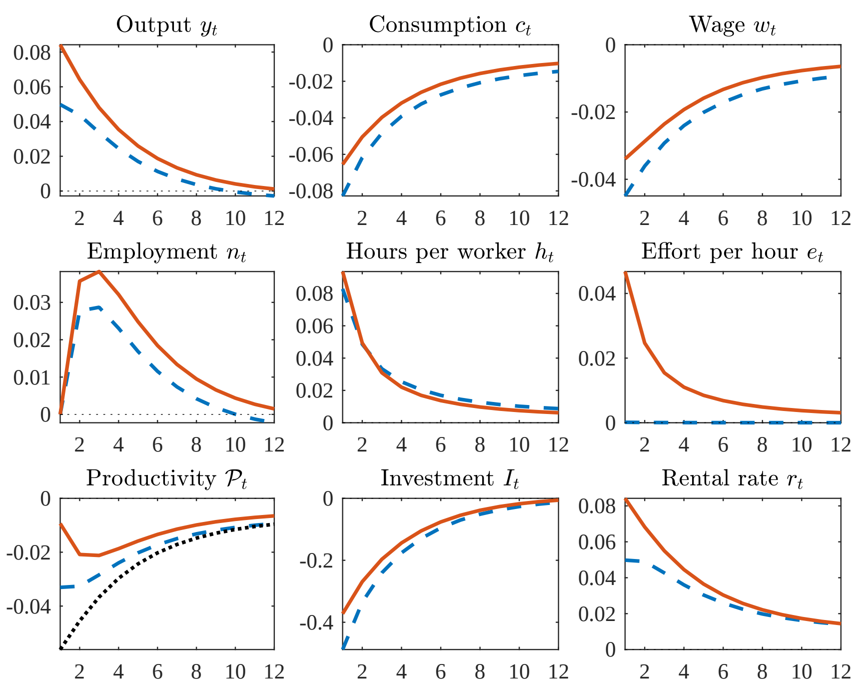

Figure 20: Responses to one-percent demand shock, **hiring cost model**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.2$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

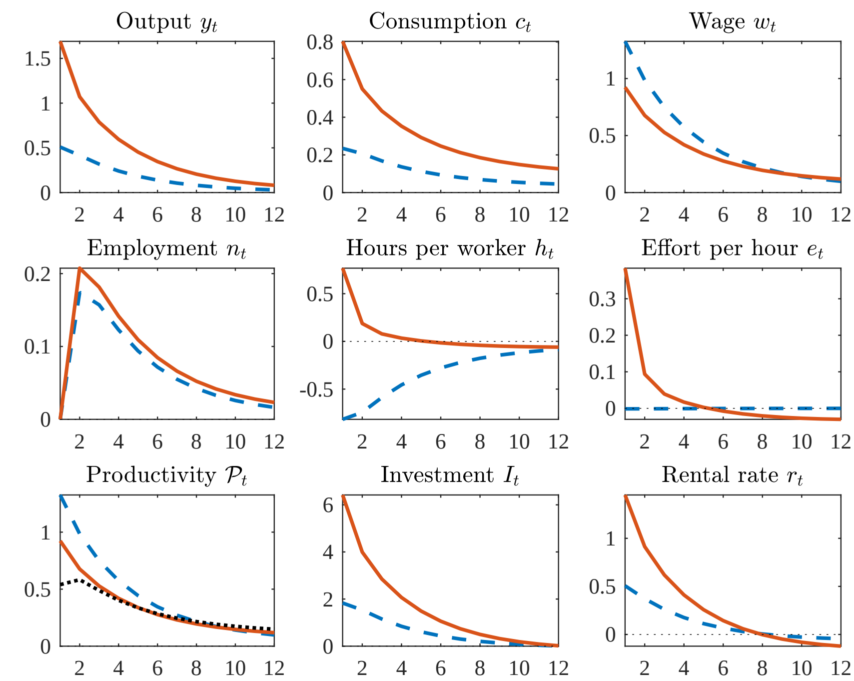

Figure 21: Responses to one-percent technology shock, **right-to-manage model**

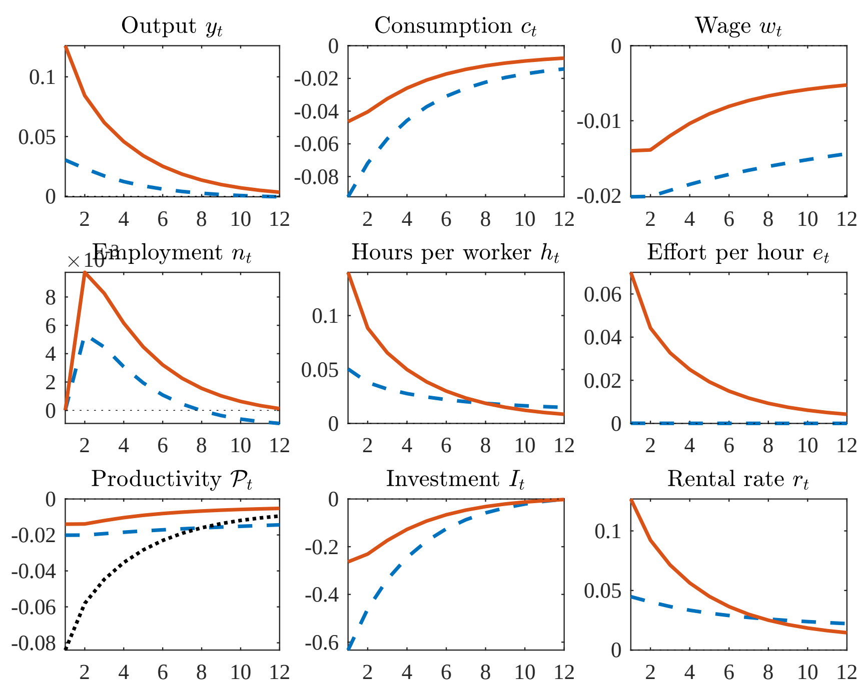

Figure 22: Responses to one-percent demand shock, **right-to-manage model**

Notes. Blue dashed lines show constant-effort model ($\tau = 350$), red solid lines show model with variable labor effort ($\tau = 0.5$). Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

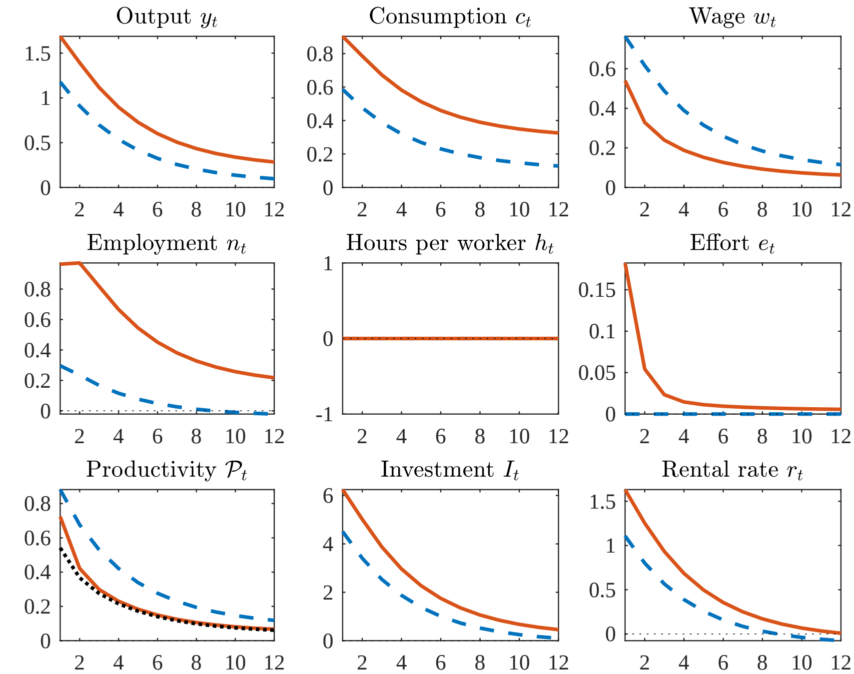

Figure 23: Responses to one-percent technology shock, **gift exchange model**

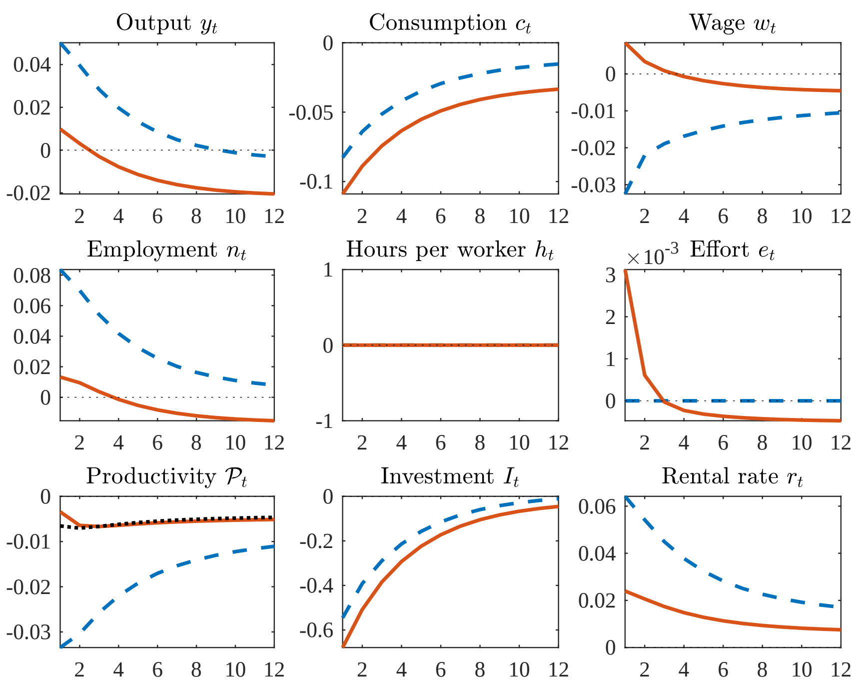

Figure 24: Responses to one-percent demand shock, **gift exchange model**

Notes. Blue dashed lines show model with Walrasian labor market, red solid lines show gift exchange model. Black dotted line shows true productivity y/H. Impulse responses measured as percentage deviation from steady state. Horizontal axis shows time horizon in quarters.

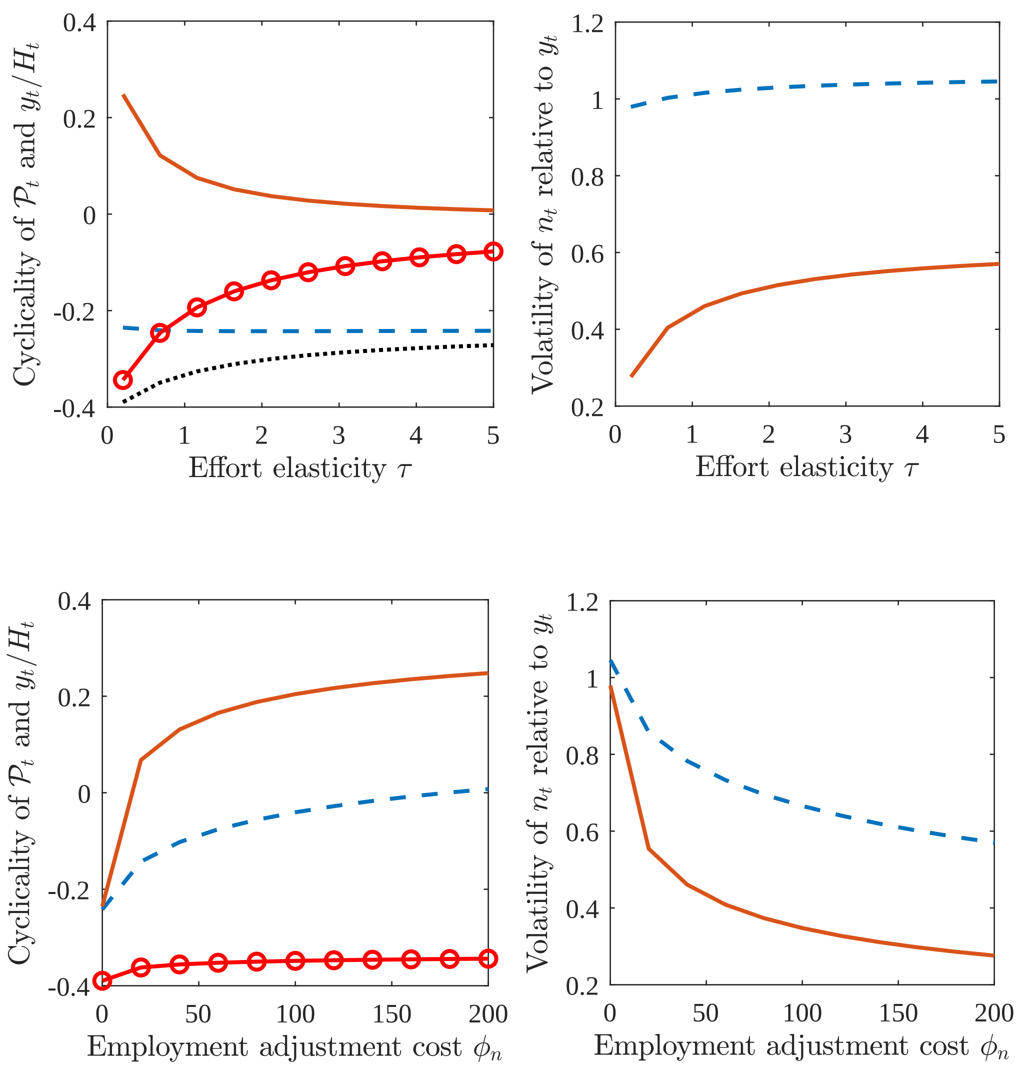

Figure 25: Labor adjustment and second moments, **baseline model**

Notes. In the upper panel, blue dashed lines show model with fluid labor market ($\phi_n = 0.01$), red solid lines show model with rigid labor market ($\phi_n = 200$). Black dotted line shows true productivity, y/H, in model with fluid labor market, red line with circles depicts true productivity in model with rigid labor market. In the lower panel, blue dashed lines show constant-effort model ($\tau = 5$), red solid lines show model with variable labor effort ($\tau = 0.2$). Red line with circles depicts true productivity in model with variable labor effort.

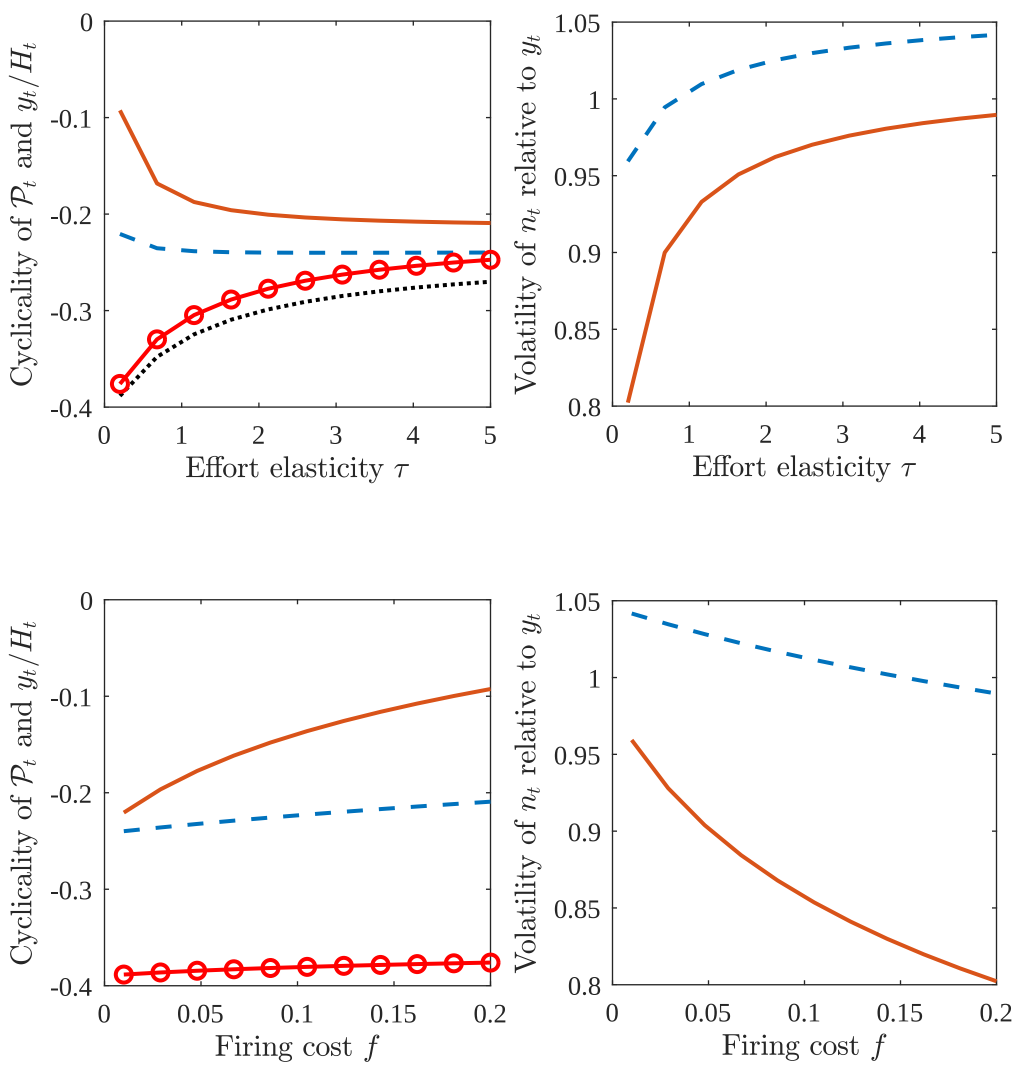

Figure 26: Labor adjustment and second moments, **firing cost model**

Notes. In the upper panel, blue dashed lines show model with fluid labor market ($f = 0.01$), red solid lines show model with rigid labor market ($f = 0.2$). Black dotted line shows true productivity, y/H, in model with fluid labor market, red line with circles depicts true productivity in model with rigid labor market. In the lower panel, blue dashed lines show constant-effort model ($\tau = 5$), red solid lines show model with variable labor effort ($\tau = 0.2$). Red line with circles depicts true productivity in model with variable labor effort.

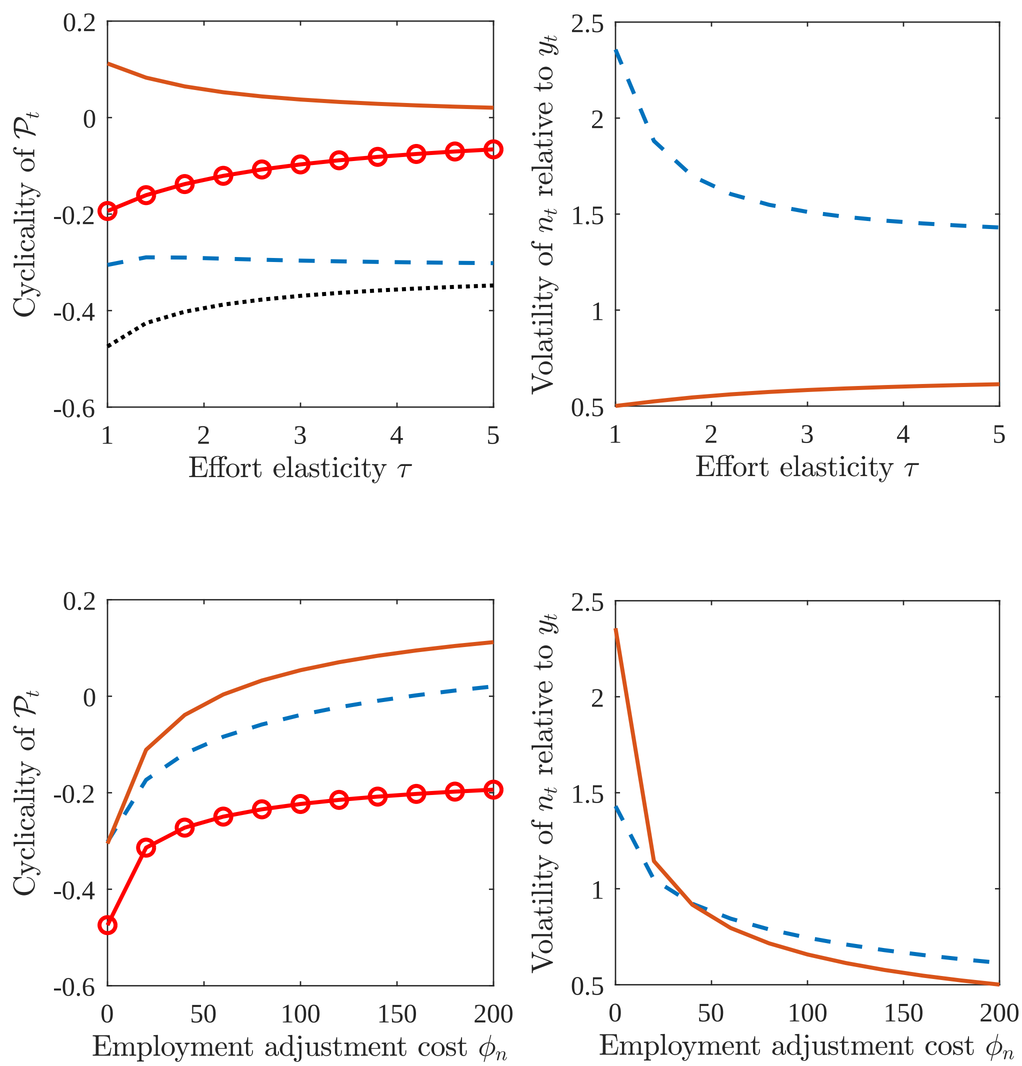

Figure 27: Labor adjustment and second moments, **sticky-price model**

Notes. In the upper panel, blue dashed lines show model with fluid labor market ($\phi_n = 0.01$), red solid lines show model with rigid labor market ($\phi_n = 200$). Black dotted line shows true productivity, y/H, in model with fluid labor market, red line with circles depicts true productivity in model with rigid labor market. In the lower panel, blue dashed lines show constant-effort model ($\tau = 5$), red solid lines show model with variable labor effort ($\tau = 0.2$). Red line with circles depicts true productivity in model with variable labor effort.

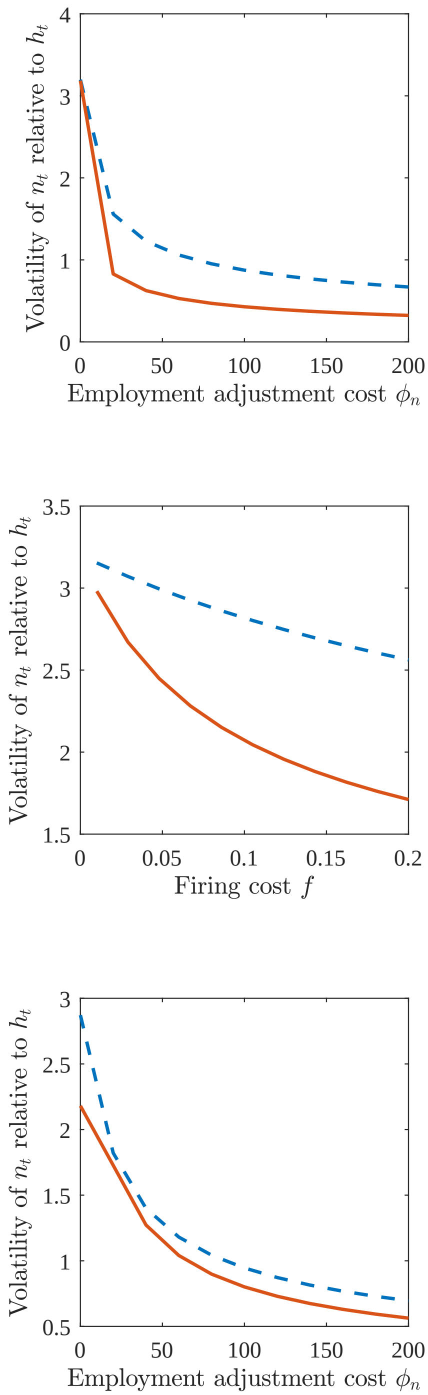

Figure 28: Labor adjustment and employment vs hours volatility

Notes. On the left, blue dashed lines show constant-effort model ($\tau = 5$), red solid lines show model with variable labor effort ($\tau = 0.2$). On the right, blue dashed lines show model with fluid labor market ($\phi_n = 0.01$ or $f = 0.01$), red solid lines show model with rigid labor market ($\phi_n = 200$ or $f = 0.2$). Upper panel: baseline model, middle panel: firing cost model, lower panel: sticky-price model.