COMPANION APPENDIX

Jumping the queue: nepotism and public-sector pay

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A Proofs of propositions

A.1 Lemma 2

We consider that the public-sector unconnected labour market breaks down if the government is not able to hire enough workers to replace the workers that have lost their job. At the limit, it means the government needs to post a wage, defined as $\underline{w}^g_u$, such that it attracts at least $(1-\mu)(s^g+\tau)e^g$ job searches. This means $u^g_u = (1-\mu)(s^g+\tau)e^g$ and the job-finding rate is 1 ($m^g_u = 1$). Applying this to (16) and then setting $U^p_u = U^g_u$ gives

$$b + \frac{1}{r+\tau+s^g+1}[\underline{w}^g_u - b] = (r+\tau)U^p_u$$

Substituting the $(r+\tau)U^p_u$ by equation (15) we get

$$\underline{w}^g_u = \frac{(r+\tau+s^g+1)m(\theta^*)}{r+\tau+s^p+m(\theta^*)}[w^{p,*} - b] + b$$

where θ^* and $w^{p,*}$ are the equilibrium tightness and wages in the private sector.

If $\mu = 0$ then no connections sector exists and all workers hired into the public sector are unconnected. If, on the other hand, a connections sector exists then a share μ of public-sector workers are hired through connections. For the existence of a connections sector, through which the government is able to hire a fraction μ of its employees the government needs to attract at least $\mu(s^g+\tau)e^g$ connected job searchers. This occurs when the government pays a higher wage, $\underline{w}^g_c$, so that queues in the public sector are long enough to induce enough job searchers to use connections to get government jobs.

$$\underline{w}^g_c = \underline{w}^g_u + \Xi^{c,-1}(\mu(s^g+\tau)e^g)(r+\tau+s^g+1)$$

where $\Xi^{c,-1}$ is the inverse of the distribution of connection cost. What it means is that, at the margin, the government has to pay high enough wages such that public-sector queues are long enough and a sufficiently high mass of newborns decide to pay the cost and obtain connections.

Notice that $\underline{w}^g_c$ is increasing in μ, while $\underline{w}^g_u$ is independent of μ. If $\mu = 0$ then we get $\underline{w}^g_c = \underline{w}^g_u$, whereas if $\mu = \bar{\mu}$ then $\underline{w}^g_c = \underline{w}^g_c$ where

$$\underline{w}^g_c = \underline{w}^g_u + \Xi^{c,-1}(\bar{\mu}(s^g+\tau)e^g)(r+\tau+s^g+1)$$

A.2 Proof of Existence and Uniqueness of a Steady-State Equilibrium

It can be easily verified that the free-entry condition in (13) pins down a unique equilibrium value for tightness in the private sector θ^*. To complete the proof of existence and uniqueness we need to show that with θ^* substituted in, the threshold condition in (23) gives a unique equilibrium value for $\tilde{c}$.

Let us write (23) as:

$$\frac{1}{r+\tau}\left[\frac{\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}{r+\tau+s^g+\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}(w^g-b)\right]-\tilde{c}=\frac{1}{r+\tau}\frac{\beta\kappa\theta}{(1-\beta)} \tag{A.1}$$

where $L_c^g = \Xi(\tilde{c})$. The left-hand-side of (A.1) decreases with $\tilde{c}$. This means that with θ^* substituted in we can use (A.1) to solve for the equilibrium value of $\tilde{c}$. The equilibrium conditions (13) and (23) thus give a unique set of equilibrium values $\tilde{c}^*$ and θ^*. This completes the proof of existence and uniqueness.

A.3 Proof of Proposition 1

First, we show that $\frac{dL_u^p}{d\mu} > 0$ (where $\mu = \bar{\mu}$):

Let $L^g = L_u^g + L_c^g$ denote the total number of workers that are either employed or are searching in the public sector. Using conditions (18) and (20) to solve, respectively, for L_u^g and L_c^g, and then adding them up gives:

$$L^g = e^g\left[\lambda+(1-\lambda)(w^g-b)\left[\frac{\mu}{\tilde{c}(r+\tau)+\frac{\beta}{1-\beta}\kappa\theta}+\frac{1-\mu}{\frac{\beta}{1-\beta}\kappa\theta}\right]\right] \tag{A.2}$$

where $\lambda = \frac{r}{r+s^g+\tau}$. Recall that the equilibrium value of θ is given by equation (13) and is independent of μ; thus $\frac{d\theta}{d\mu} = 0$. Given this, we can write:

$$\frac{dL^g}{d\mu} = \frac{\partial L^g}{\partial \mu} + \frac{\partial L^g}{\partial \tilde{c}}\frac{\partial \tilde{c}}{\partial \mu} \tag{A.3}$$

and using (A.1) we can derive that

$$\frac{\partial \tilde{c}}{\partial \mu} > 0 \tag{A.4}$$

Moreover, it can be easily verified from (A.2) that $\frac{\partial L^g}{\partial \mu} < 0$ and $\frac{\partial L^g}{\partial \tilde{c}} < 0$, implying from (A.3) that

$$\frac{dL^g}{d\mu} < 0.$$

Given that $L_u^p = 1 - L^g$, it follows that $\frac{dL_u^p}{d\mu} > 0$.

Next we show that $\frac{du^g}{d\mu} < 0$. The number of workers searching in the public sector with and without connections are given, respectively, by $u_c^g = L_c^g - \mu e^g$ and $u_u^g = L_u^g - (1-\mu)e^g$. By adding them up we get $u^g = u_u^g + u_c^g = L^g - e^g$. The number of workers employed in the public sector, e^g, is exogenous and independent of μ, while, as shown above, $\frac{dL^g}{d\mu} < 0$. It follows that $\frac{du^g}{d\mu} < 0$.

Finally, we show that the employment rate (e) increases. That is, $\frac{de}{d\mu} > 0$. The total employment rate is given by $e = e^g + e^p$, where e^g is exogenously set by the government, while e^p can be derived from (24) and (27): $e^p = \frac{m(\theta)L_u^p}{s^p+\tau+m(\theta)}$. Adding them up gives:

$$e = e^g + \frac{m(\theta)(1-L^g)}{s^p+\tau+m(\theta)} \tag{A.5}$$

Evidently, $\frac{de}{d\mu} > 0$, since θ is independent of μ, while $\frac{dL_u^p}{d\mu} > 0$.

A.4 Proof of Proposition 2

First, let us show that the number of workers searching in the public sector increases as the public-sector wage increases; that is $\frac{dL^g}{dw^g} > 0$, which ultimately implies that $\frac{dL^p}{dw^g} < 0$, since $L_u^p = 1 - L^g$.

Using condition (18) we can solve for L_u^g and obtain:

$$L_u^g = (1-\mu)e^g \left[\lambda + (1-\lambda)\left(\frac{w^g-b}{\frac{\beta}{1-\beta}\kappa\theta}\right)\right] \tag{A.6}$$

where λ is as defined above (in Proposition 1).

The total number of workers attached to the public sector is given by $L^g = L_u^g + L_c^g$ where L_u^g is as derived above and $L_c^g = \Xi(\tilde{c})$. Taking the derivative with respect to w^g gives:

$$\frac{dL^g}{dw^g} = \frac{dL_u^g}{dw^g} + \frac{dL_c^g}{d\tilde{c}}\frac{d\tilde{c}}{dw^g} \tag{A.7}$$

It is straightforward to verify from (A.6) that $\frac{dL_u^g}{dw^g} > 0$. Moreover, $\frac{dL_c^g}{d\tilde{c}} = \xi(\tilde{c}) > 0$ and using (A.1) we can derive that:

$$\frac{d\tilde{c}}{dw^g} = \frac{M}{r+\tau+\frac{M(1-M)(w^g-b)}{L_c^g-\mu e^g}\frac{dL_c^g}{d\tilde{c}}} > 0 \tag{A.8}$$

where $M = \frac{\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}{r+\tau+s^g+\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}$. It follows from (A.7) that:

$$\frac{dL^g}{dw^g} > 0 \tag{A.9}$$

Using (A.1), (A.2) and (A.8) we can further show that:

$$\frac{dL^g}{dw^g} = e^g(1-\lambda)\left[\frac{\mu}{\tilde{c}(r+\tau)+\frac{\beta}{1-\beta}\kappa\theta}\left(1-\frac{r+\tau}{r+\tau+\left(\tilde{c}(r+\tau)+\frac{\beta}{1-\beta}\kappa\theta\right)\frac{(1-M)}{L_c^g-\mu e^g}\frac{dL_c^g}{d\tilde{c}}}\right)+\frac{1-\mu}{\frac{\beta}{1-\beta}\kappa\theta}\right] \tag{A.10}$$

Note that if $\mu = 0$ then

$$\frac{dL^g}{dw^g} = e^g(1-\lambda)\left[\frac{1}{\frac{\beta}{1-\beta}\kappa\theta}\right] \tag{A.11}$$

Comparing (A.10) to (A.11) shows:

$$\left.\frac{dL^g}{dw^g}\right|_{\mu=0} > \left.\frac{dL^g}{dw^g}\right|_{\mu>0} \tag{A.12}$$

and the increase in the number of workers searching in the public sector due to an increase in the public-sector wage is larger when $\mu = 0$ than when $\mu > 0$.

Since $L_u^p = 1 - L^g$ and $u^g = L^g - e^g$, the decrease and increase, respectively, in L_u^p and u^g, is also larger when $\mu = 0$ than when $\mu > 0$.

B Efficiency

As also mentioned in the text, the existence of a connections sector and of queues for public-sector jobs are both inefficient. These two types of inefficiencies can be eliminated by setting $\mu = 0$, which implies $L_c^g = 0$, and $w^g = \underline{w}^g$, which ensures that $u_u^g = (s^g + \tau)e^g$ and the job-finding rate for government jobs is 1. We next compare the central planner's solution to the decentralized one, described in the text, and show that the remaining inefficiency, the congestion externalities can be eliminated with the Hosios condition.

We follow Hosios (1990), Charlot and Decreause (2005), among others, and set $r = 0$, so that the central planner maximizes the steady-state surplus. The planner's problem is to choose θ, u^p to maximize total output, plus unemployment income, minus job creation costs. Given that public sector employment is fixed. The planner's objective is to

$$\max(1 - L^g)\left[(1 - u^p)y + u^p b - \theta\kappa u^p\right]$$

s.t

$$u^p = \frac{s^p + \tau}{s^p + \tau + m(\theta)}$$

We set the Langrangian

$$\mathcal{L} = (1 - L^g)\left[(1 - u^p)y + u^p b - \theta\kappa u^p\right] + \phi\left[u^p - \frac{s^p + \tau}{s^p + \tau + m(\theta)}\right] \tag{B.1}$$

The three optimality conditions are

$$\frac{\partial\mathcal{L}}{\partial\theta} = 0 \;\Rightarrow\; \phi\frac{m'(\theta)}{s^p + \tau + m(\theta)} = (1 - L^g)\kappa \tag{B.2}$$

$$\frac{\partial\mathcal{L}}{\partial u^p} = 0 \;\Rightarrow\; \phi = (1 - L^g)\left[y - b + \kappa\theta\right] \tag{B.3}$$

$$\frac{\partial\mathcal{L}}{\partial\phi} = 0 \;\Rightarrow\; u^p = \frac{s^p + \tau}{s^p + \tau + m(\theta)} \tag{B.4}$$

Substituting (B.3) into (B.2) gives:

$$\frac{\kappa}{q(\theta)} = \frac{\eta(y - b)}{s^p + \tau + m(\theta)(1 - \eta)} \tag{B.5}$$

where it may be useful to recall that $m(\theta) = \theta^\eta$ and $m'(\theta) = \eta q(\theta)$. It is easy to verify by comparing (B.5) to (13), that given $r = 0$, if $\beta = (1 - \eta)$, then the decentralized equilibrium is identical to the central planner's solution.

C Random search

C.1 Setup

In this appendix we give the full set of equations of the model with random search and characterize its steady-state equilibrium. Further, we show that in the limiting case where $\mu = 1$ the model with random search becomes identical to the model with segmented markets and we provide proofs of Proposition 4.

The values of being unemployed and employed for connected workers remain as in the Benchmark model; given by (4) and (6). The same holds for the values of being employed in either the private or the public sector for unconnected workers (equations 2 and 5), and the values of a private-sector filled jobs and vacancies (equations 7 and 8). The cutoff connection cost as well as the selection of workers into the two groups, L_c^g, L_u, also remain as given in equations (21) and (22). As discussed in the text, only the value of unemployment for unconnected workers changes. It is now given by equation (39). The Nash bargaining wage of the private sector changes accordingly and is as given in (40).

Both government and private firms that seek to hire workers through regular search in the market meet with workers at rate $q(\theta) = \frac{m(\theta)}{\theta}$, where $\theta = \frac{v_u^p + v_u^g}{u_u}$. The number of vacancies in the private sector is determined endogenously by free entry that drives the value of a private-sector vacancy to zero, $V_u^p = 0$. The government needs to post enough vacancies for workers without connections to ensure that the total number of matches with such workers, $q(\theta)v_u^g$, equals the number of unconnected workers that it needs to hire. Hence, the government posts v_u^g vacancies to ensure $q(\theta)v_u^g = (1-\mu)\,(s^g + \tau)\,e^g$.

Setting $V_u^p = 0$ and using the Nash bargaining conditions in (10), we can write the surplus of a private-sector match as

$$S_u^p = \frac{y - b - D(w^g - b)}{r + \tau + s^p + (1-D)\beta m(\theta)\gamma^p} \tag{C.1}$$

and the zero-profit condition that determines job creation in the private sector becomes:

$$\frac{\kappa}{q(\theta)} = \frac{(1-\beta)(y - b - D(w^g - b))}{r + \tau + s^p + (1-D)\beta m(\theta)\gamma^p} \tag{C.2}$$

We can write the threshold level of connection costs, $\tilde{c} = U_c^g - U_u$, as:

$$\tilde{c} = \frac{1}{r+\tau}\left[\frac{\frac{\mu(s^g+\tau)e^g}{u_c^g}}{r + \tau + s^g + \frac{\mu(s^g+\tau)e^g}{u_c^g}}(w^g - b) - D(w^g - b) - (1-D)\frac{\beta\kappa\theta\gamma^p}{(1-\beta)}\right] \tag{C.3}$$

As in the benchmark model we treat public sector employment as an exogenous policy variable. There are e^g workers employed in the public sector. Among these workers, μ^g are workers who were hired through connections (e_c^g) and the remaining $(1-\mu)e^g$ are workers hired through regular search in the market (e_u^g). The number of workers employed in the private sector is endogenous and depends on job creation in the private sector as well as conditions in the public sector. The labor force of workers without connections consists of those employed in the public sector, those employed in the private sector (e_u^p), and the unemployed (u_u). Hence, $u_u = L_u - (1-\mu)e^g - e_u^p$. By equating the flows in, $m(\theta)\gamma^p u_u$, to the flows out of the state where a worker is employed in the private sector, $e_u^p(s^p+\tau)$ we obtain:

$$e_u^p = \frac{m(\theta)\gamma^p\left[L_u-(1-\mu)e^g\right]}{m(\theta)\gamma^p+\tau+s^p} \tag{C.4}$$

$$u_u = \frac{(\tau+s^p)\left[L_u-(1-\mu)e^g\right]}{m(\theta)\gamma^p+\tau+s^p} \tag{C.5}$$

Given $\theta = \frac{v_u^p+v_u^g}{u_u}$ and $q(\theta)v_u^g = (1-\mu)\left(s^g+\tau\right)e^g$, we can use (C.5) to write:

$$\gamma^p = \frac{s^p+\tau}{m(\theta)}\left[\frac{m(\theta)\left[L_u-(1-\mu)e^g\right]-(1-\mu)(s^g+\tau)e^g}{(s^p+\tau)\left[L_u-(1-\mu)e^g\right]+(1-\mu)e^g(s^g+\tau)}\right] \tag{C.6}$$

Using (C.4) and (C.6) we can write the total employment of workers without connections, $e_u = e_u^p + (1-\mu)e^g$ as:

$$e_u = \frac{m(\theta)L_u+(1-\mu)e^g(s^p-s^g)}{s^p+\tau+m(\theta)} \tag{C.7}$$

C.2 The case $\mu = 1$

If $\mu = 1$, then, as can be seen from (C.6), $\gamma^p = 1$, which implies $D = 0$. Setting $\gamma^p = 1$ and $D = 0$ in (40), (C.2) and (C.3) gives:

$$w^p = b+\beta\left[y-b+\theta\kappa\right] \tag{C.8}$$

$$\frac{\kappa}{q(\theta)} = (1-\beta)\left(\frac{y-b}{r+s^p+\tau+\beta m(\theta)}\right) \tag{C.9}$$

$$\tilde{c} = \frac{1}{r+\tau}\left[\frac{\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}{r+\tau+s^g+\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}(w^g-b)-\frac{\beta\kappa\theta}{(1-\beta)}\right] \tag{C.10}$$

The zero-profit condition in (C.9) gives a unique equilibrium values for θ which is independent of government policy or conditions in the government sector. Moreover, the zero-profit condition in (C.9), private-sector wages in (C.8) and the cut-off cost in (C.10) are identical to those obtained under segmented markets (equations 13, 14 and 23, respectively). Hence, if $\mu = 1$, private-sector job creation and tightness, wages, as well as the composition of the labor force in terms of connections in the model with random search are identical to those obtained under segmented markets.

C.3 Definition of Equilibrium

A steady state equilibrium consists of a cut-off cost $\{\tilde{c}\}$, tightness $\{\theta\}$, and unemployed $\{u_u, u_c^g\}$, such that, given some exogenous government policies $\{w^g, e^g, \mu\}$, the following apply.

1. Private-sector firms satisfy the free-entry condition (C.2).
2. Private-sector wages are the outcome of Nash Bargaining (40).
3. Newborns decide optimally their investments in connections and the population shares are determined by equations (21)-(22).
4. Flows between private employment and unemployment are constant

$$(s^p + \tau)e^p = m(\theta)\gamma^p u_u^p,$$

5. Population add up constraints are satisfied:

$$L_u = e^p + (1 - \mu)e^g + u_u \tag{C.11}$$
$$L_c^g = \mu e^g + u_c^g \tag{C.12}$$
$$L_u + L_c^g = 1 \tag{C.13}$$

C.4 Proof of Existence and Uniqueness

To prove the existence and uniqueness of a steady state equilibrium under random search we show below that the free-entry condition in (C.2) gives a unique equilibrium value for θ. The equilibrium values of the cut-off costs can then be determined by substituting the equilibrium value of θ in equation (C.3). Then using (21) and (22) we can determine L_u and L_c^g, which in turn, together with the equilibrium value of θ can be substituted in equations (40), (C.4), (C.5), (C.11) and (C.12) to determine wages and employment in the private sector.

The job creation condition in (C.2) and the cut-off connection cost in (C.3) can be written as:

$$\frac{\kappa}{q(\theta)} = \frac{(y - b - OO)}{r + \tau + s^p} \tag{C.14}$$
$$\tilde{c} = \frac{1}{r + \tau}\left[A_c - OO\right] \tag{C.15}$$

where $A_c \equiv \frac{\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}{r+\tau+s^g+\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}(w^g - b)$,

$$OO = D(w^g - b) + (1 - D)\frac{\beta}{1 - \beta}\kappa\theta\gamma^p \tag{C.16}$$

is the expression for the outside option of workers, and $D = \frac{(1-\gamma^p)m(\theta)}{r+\tau+s^g+(1-\gamma^p)m(\theta)}$.

Taking the derivative of (C.16) and (C.15) with respect to θ we obtain:

$$\frac{dOO}{d\theta} = \frac{\partial OO}{\partial\theta} + \frac{\partial OO}{\partial\tilde{c}}\frac{d\tilde{c}}{d\theta} \tag{C.17}$$
$$\frac{d\tilde{c}}{d\theta} = \frac{1}{r + \tau}\left[\frac{\partial A_c}{\partial L_c^g}\frac{dL_c^g}{d\tilde{c}}\frac{d\tilde{c}}{d\theta} - \frac{dOO}{d\theta}\right] \tag{C.18}$$

where

$$\frac{\partial OO}{\partial \tilde{c}} = -\frac{(1-D)m(\theta)\left(E_u^g - E_u^p\right)(s^p+\tau)(1-\gamma^p)}{(s^p+\tau)\left[L_u-(1-\mu)e^g\right]+(1-\mu)e^g(s^g+\tau)}\frac{dL_u}{d\tilde{c}} \tag{C.19}$$

$$\frac{\partial OO}{\partial \theta} = (1-D)q(\theta)\left[(1-\gamma^p)\eta\left(\frac{E_u^g m(\theta)+E^P(s^p+\tau)}{s^p+\tau+m(\theta)}-U_u\right)+\gamma^p\left(E_u^p-U_u\right)\right] \tag{C.20}$$

Recall that $L_c^g = \Xi(\tilde{c})$ and $L_u = 1-\Xi(\tilde{c})$ so that $\frac{dL_c^g}{d\tilde{c}} = \xi(\tilde{c}) > 0$ and $\frac{dL_u}{d\tilde{c}} = -\xi(\tilde{c}) < 0$ (where ξ is the pdf of the distribution of connection costs). It can also be easily verified that $\frac{\partial A_c}{\partial L_c^g} < 0$.

Equations (C.17) and (C.18) can be used to solve for $\frac{d\tilde{c}}{d\theta}$:

$$\frac{d\tilde{c}}{d\theta} = \frac{-\frac{\partial OO}{\partial \theta}}{r+\tau-\frac{\partial A_c}{\partial L_c^g}\xi(\tilde{c})+\frac{\partial OO}{\partial \tilde{c}}} < 0 \tag{C.21}$$

Plugging the above into (C.17) we get:

$$\frac{dOO}{d\theta} = \frac{\partial OO}{\partial \theta}\left[\frac{r+\tau-\frac{\partial A_c}{\partial L_c^g}}{r+\tau-\frac{\partial A_c}{\partial L_c^g}\xi(\tilde{c})+\frac{\partial OO}{\partial \tilde{c}}}\right] > 0 \tag{C.22}$$

For private and unconnected public-sector jobs to exist it must be the case that $E_u^p - U_u > 0$ and $E_u^p - U_u > 0$, respectively, which ensures that they generate positive profits. This ensures that $\frac{\partial OO}{\partial \theta} > 0$. As can be seen from (C.19) sufficient (but not necessary) condition to ensure also that $\frac{\partial OO}{\partial \tilde{c}} > 0$ is $E_u^g - E_u^p > 0$ meaning that the value to an unconnected worker is higher when that worker is working for the government than in the private sector. If this condition holds then we know for sure that the term in the bracket of (C.22) is positive and thus, $\frac{dOO}{d\theta} > 0$, while as shown in (C.21) $\frac{d\tilde{c}}{d\theta} < 0$. If $\frac{dOO}{d\theta} > 0$ holds, then the right-hand-side of (C.14) is decreasing while its left-hand-side is increasing in θ. Equation (C.14) thus pins down a unique equilibrium value for θ, which can then be used to solve for $\tilde{c}$ and the rest of the endogenous variables.

C.5 Proof of Proposition 4

From (C.14) and (C.15) we get:

$$\frac{d\theta}{dw^g} = -\left[\frac{\frac{\partial OO}{\partial w^g}\left(r+\tau-\frac{\partial A_c}{\partial L_c^g}\xi(\tilde{c})\right)+\frac{\partial OO}{\partial \tilde{c}}\frac{\partial A_c}{\partial w^g}}{\left(r+\tau-\frac{\partial A_c}{\partial L_c^g}\xi(\tilde{c})\right)\frac{\partial OO}{\partial \theta}-\frac{\kappa q'(\theta)}{q^2(\theta)}(r+s^p+\tau)\left(r+\tau-\frac{\partial A_c}{\partial L_c^g}\xi(\tilde{c})+\frac{\partial OO}{\partial \tilde{c}}\right)}\right] \tag{C.23}$$

$$\frac{d\tilde{c}}{dw^g} = \frac{\frac{\partial A_c}{\partial w^g}-\frac{\partial OO}{\partial w^g}(1-B)}{r+\tau+\frac{\partial OO}{\partial \tilde{c}}(1-B)-\frac{\partial A_c}{\partial L_c^g}\xi(\tilde{c})} \tag{C.24}$$

where $B \equiv \frac{\frac{\partial OO}{\partial \theta}}{\frac{\partial OO}{\partial \theta} - \frac{\kappa q'(\theta)}{q^2(\theta)}(r+s^p+\tau)}$.

As shown above (in Section C.4) $\frac{\partial OO}{\partial \theta} > 0$ and $\frac{\partial OO}{\partial \tilde{c}} > 0$ while $q'(\theta) < 0$. These imply that the denominators in the above expressions are always positive. We know in addition that $\frac{\partial A_c}{\partial w^g} > 0$, $\frac{\partial A_c}{\partial L_c^g} < 0$ and $\frac{\partial OO}{\partial w^g} > 0$, meaning that the numerator in the bracket of (C.23) is positive also. Further, from (C.15) we can verify that for $\tilde{c} > 0$ it must be the case that $\frac{\partial A_c}{\partial w^g} - \frac{\partial OO}{\partial w^g} > 0$, which ensures, also, that the numerator of (C.24) is positive. It follows, then, that

$$\frac{d\theta}{dw^g} < 0 \tag{C.25}$$

$$\frac{d\tilde{c}}{dw^g} > 0 \tag{C.26}$$

Using (C.7) we can easily verify that total employment $e = e_u + \mu e^g$ will decrease with an in increase in w^g since:

$$\frac{de}{dw^g} = \frac{de}{d\tilde{c}}\frac{d\tilde{c}}{dw^g} + \frac{de}{d\theta}\frac{d\theta}{dw^g}$$

and

$$\frac{de}{d\tilde{c}} < 0$$

$$\frac{de}{d\theta} > 0 \tag{C.27}$$

Since $L_c^g = \Xi(\tilde{c})$, $L_u = 1 - \Xi(\tilde{c})$ and $\frac{d\tilde{c}}{dw^g} > 0$, it follows that $\frac{dL_c^g}{dw^g} > 0$ and $\frac{dL_u}{dw^g} < 0$.

D Heterogenous Workers

D.1 Setup

Since the two types differ in terms of productivity, the value of searching for a job as well as the value of being employed in the private sector both differ depending on the worker's ability. Thus,

$$(r+\tau)U^p_{u,i} = b + m(\theta_i)\left[E^p_{u,i} - U^p_{u,i}\right] \tag{D.1}$$

$$(r+\tau)E^p_{u,i} = w^p_i - s^p\left[E^p_{u,i} - U^p_{u,i}\right] \tag{D.2}$$

where $i = [h, l]$ denotes high- and low-ability.

The value of a private-sector vacancy and job, respectively, of type-i is given by

$$rV^p_i = -\kappa + q(\theta_i)\left[J^p_i - V^p_i\right] \tag{D.3}$$

$$rJ^p_i = y_i - (s^p + \tau)\left[J^p_i - V^p_i\right] \tag{D.4}$$

Using the Bellman equations and the Nash bargaining conditions $\beta S^p_{u,i} = E^p_{u,i} - U^p_{u,i}$, $(1-\beta)S^p_{u,i} = J^p_{u,i} - V^p_{u,i}$ we can write the surplus of a private job of type i as:

$$S^p_{u,i} = \frac{y_i - b}{r + \tau + s^p + \beta m(\theta_i)} \tag{D.5}$$

There are now two free-entry conditions one for each private-sector market, $V^p_i = 0$. Imposing the free-entry and Nash bargaining conditions gives the conditions shown in the main text:

$$\frac{\kappa}{q(\theta_i)} = \frac{(y_i - b)(1-\beta)}{r + \tau + s^p + \beta m(\theta_i)}. \tag{D.6}$$

$$w^p_i = b + \beta(y_i - b + \kappa\theta_i). \tag{D.7}$$

It can be easily verified that $\theta_h > \theta_l$ because $y_h > y_l$. It follows also that $w^p_h > w^p_l$ and high-ability workers enjoy a higher value when applying to the private sector than lower ability workers. That is, $E^p_{u,h} > E^p_{u,l}$ and $U^p_{u,h} > U^p_{u,l}$. Moreover, following Lemma 1, θ_h and θ_l are both independent of public-sector employment and wage policies.

In the public sector, despite wages being the same for all workers irrespective of ability, the values of being employed or unemployed for workers that do not have connections differ by ability, because the government screens candidates and gives preference to high-ability workers.

$$(r+\tau)U^g_{u,i} = b + m^g_{u,i}\left[E^g_{u,i} - U^g_{u,i}\right] \tag{D.8}$$

$$(r+\tau)E^g_{u,i} = w^g - s^g\left[E^g_{u,i} - U^g_{u,i}\right]. \tag{D.9}$$

In the connection market, we assume that ability does not affect your probability of being hired, as long as workers have connections. Since in the connections sector not only both types receive the same wage, but have also equal chances of getting a job, given by m^g_c. As such, the values of unemployment and employment for connected workers are the same irrespective of ability and remain as in (4) and (6).

High- and low-ability workers without connections can search in either the public or the private sector. In an equilibrium where both markets are active, the values of these two options would have to equate:

$$U_{u,i} = U^p_{u,i} = U^g_{u,i} \tag{D.10}$$

These two conditions would determine the numbers of high- and low-ability unconnected searchers in the public sector, $u^g_{u,h}$ and $u^g_{u,l}$. Using the Nash bargaining conditions and (41) we can write

$$(r+\tau)U_{u,i} = b + \frac{\beta\theta_i\kappa}{(1-\beta)} \tag{D.11}$$

Alternatively they can use connections to get into the public sector after paying the cost c drawn from $\Xi(\cdot)$. The threshold level of cost at which a worker of type-i is indifferent between using and not using connections to find a government job is

$$\tilde{c}_i = U_c - U_{u,i} \tag{D.12}$$

These two thresholds determine the allocation of each type of worker into those who use connections to get government jobs and those who search either in the private or in the public sector without connections. We can measure each of these four groups' share in the labor force as:

$$\begin{aligned} L^g_{c,i} &= X_i\Xi(\tilde{c}_i) \\ L_{u,i} &= X_i(1-\Xi(\tilde{c}_i)) \end{aligned} \tag{D.13}$$

where it may be recalled that $L_{u,i} = L^g_{u,i} + L^p_i$ and X_i is the fraction of type-i workers in the labor force. Finally, population add up constraints are satisfied so that

$$\begin{gathered} L^g_{u,i} = (1-\mu)e^g + u^g_{u,i} \\ L^g_{c,i} = \mu e^g + u_{c,i} \\ L^p_{u,h} + L^g_{u,h} + L^g_{c,h} = X_h \\ L^p_{u,l} + L^g_{u,l} + L^g_{c,l} = X_l \end{gathered} \tag{D.14}$$

and by equating the flows in to the flows out of unemployment in the private sector we obtain

$$u^p_i = \left(\frac{s^p+\tau}{s^p+\tau+m(\theta_i)}\right) L^p_{u,i} \tag{D.15}$$

D.2 Wage Cutoffs

The cutoff wage $\underline{w}^g_{u,h}$ is such that a high-ability (unconnected) worker is indifferent between searching for a public or a private job, given $m^g_{u,h} = 1$. Setting $m^g_{u,h} = 1$ in (D.8) and using (D.10), (D.11) gives

$$\underline{w}^g_{u,h} = \frac{\beta\kappa\theta_h}{(1-\beta)}(r + s^g + \tau + 1) + b \tag{D.16}$$

If the public sector wage equals $\underline{w}^g_{c,h}$ then $U^g_{u,h} = U^g_{c,h}$ and a high-ability worker is indifferent between using connections or not. Setting (D.11) equal to (4) and solving for the wage gives:

$$\underline{w}^g_{c,h} = \frac{\beta\kappa\theta_h}{(1-\beta)}\left(\frac{r+s^g+\tau}{m^g_c}+1\right)+b \tag{D.17}$$

Comparing (D.16) to (D.17) reveals that $\underline{w}^g_{c,h} > \underline{w}^g_{u,h}$ since $m^g_c < 1$. We know that at any wage above $\underline{w}^g_c$ connected workers will be queuing up for government jobs, meaning that $m^g_c < 1$.

Note that for $\underline{w}^g_{c,h}$ to exist it must be the case the $\bar{\mu}$ is large enough. Incentives for high ability workers to use connections are at the maximum when queues for government jobs through the regular (unconnected) channel are as long as possible. This occurs when $L^g_{u,h} = X_h$, meaning that all high ability workers search in the public sector. The threshold wage $\underline{w}^g_{c,h}$ will not exist, if, even in this case, where the unconnected public market is as crowded as possible, $U^g_{u,h} > U^g_{c,h}$, meaning that no high type wants to obtain connections. Notice that $U^g_{u,h} > U^g_{c,h}$ implies $m^g_{u,h} > m^g_c$, which gives $\bar{\mu} < \frac{L^g_c}{L^g_c + L^g_{u,h}}$. Setting $L^g_{u,h} = X_h$ gives $\bar{\mu} < \frac{L^g_c}{L^g_c + X_h}$. If $\bar{\mu} \leq \frac{L^g_c}{L^g_c + X_h}$ then no matter how large w^g is, the high-type workers will never opt for connections; in other words, $\underline{w}^g_{c,h}$ does not exist. This threshold wage exists only if $\bar{\mu} > \frac{L^g_c}{L^g_c + X_h}$.

D.3 Proof of Proposition 5

Using (D.10) (with (D.8) and (D.11) substituted in) to solve for $L^g_{u,i}$ we obtain:

$$L^g_{u,i} = (1-\bar{\mu})e^g\left[\lambda + (1-\lambda)\left(\frac{w^g - b}{\frac{\beta}{1-\beta}\kappa\theta_i}\right)\right] \tag{D.18}$$

where $\lambda = \frac{r}{r+s^g+\tau}$. Using (D.12) with either $i = l$ or $i = h$ and (4) and (D.11) substituted in to solve for L^g_c, we get:

$$L^g_c = \bar{\mu}e^g\left[\lambda + (1-\lambda)\left(\frac{w^g - b}{\tilde{c}_l(r+\tau) + \frac{\beta}{1-\beta}\kappa\theta_l}\right)\right] \tag{D.19}$$

$$L^g_c = \bar{\mu}e^g\left[\lambda + (1-\lambda)\left(\frac{w^g - b}{\tilde{c}_h(r+\tau) + \frac{\beta}{1-\beta}\kappa\theta_h}\right)\right] \tag{D.20}$$

which implies:

$$\tilde{c}_l(r+\tau) + \frac{\beta\kappa}{(1-\beta)}\theta_l = \tilde{c}_h(r+\tau) + \frac{\beta\kappa}{(1-\beta)}\theta_h \tag{D.21}$$

Given $w^g > \underline{w}^g_{u,h}$, meaning that all unconnected workers attached to the public sector are of high ability ($L^g_u = L^g_{u,h}$), the total number of workers in the public sector is $L^g = L^g_{u,h} + L^g_c$. Thus, $\frac{dL^g}{d\bar{\mu}} = \frac{dL^g_{u,h}}{d\bar{\mu}} + \frac{dL^g_c}{d\bar{\mu}}$.

Notice that $L^g_c = L^g_{c,l} = X_l\Xi(\tilde{c}_l)$ ($L^g_{c,h} = 0$) if $w^g \leq \underline{w}^g_{c,h}$ ($\tilde{c}_h \leq 0$) and $L^g_c = L^g_{c,l} + L^g_{c,h} =$

$X_l\Xi(\tilde{c}_l) + X_h\Xi(\tilde{c}_h)$ if $w^g > \underline{w}^g_{c,h}$ ($\tilde{c}_h > 0$). As can be verified from (41) $\frac{d\theta_i}{d\tilde{c}_i} = 0$, $i = [h, l]$ so that we know from (D.21) that $\frac{d\tilde{c}_l}{d\bar{\mu}} = \frac{d\tilde{c}_h}{d\bar{\mu}}$. We can then use either (D.20) or (D.19) to get:

$$\begin{aligned}
\frac{dL^g_c}{d\bar{\mu}} &= \frac{L^g_c}{\bar{\mu}}\left[\frac{X_l\xi(\tilde{c}_l) + X_h\xi(\tilde{c}_h)}{X_l\xi(\tilde{c}_l) + X_h\xi(\tilde{c}_h) + \Delta}\right] = \frac{X_l\Xi(\tilde{c}_l) + X_h\Xi(\tilde{c}_h)}{\bar{\mu}}\left[\frac{X_l\xi(\tilde{c}_l) + X_h\xi(\tilde{c}_h)}{X_l\xi(\tilde{c}_l) + X_h\xi(\tilde{c}_h) + \Delta}\right] > 0 \text{ if } w^g > \underline{w}^g_{c,h} \\
\frac{dL^g_c}{d\bar{\mu}} &= \frac{L^g_c}{\bar{\mu}}\left[\frac{X_l\xi(\tilde{c}_l)}{X_l\xi(\tilde{c}_l) + \Delta}\right] = \frac{X_l\Xi(\tilde{c}_l)}{\bar{\mu}}\left[\frac{X_l\xi(\tilde{c}_l)}{X_l\xi(\tilde{c}_l) + \Delta}\right] > 0 \text{ if } w^g \leq \underline{w}^g_{c,h} \qquad \text{(D.22)}
\end{aligned}$$

where $\Delta = \frac{\bar{\mu}e^g(1-\lambda)(w^g-b)(r+\tau)}{(\tilde{c}_l(r+\tau)+\frac{\beta}{1-\beta}\kappa\theta_l)^2} = \frac{\bar{\mu}e^g(1-\lambda)(w^g-b)(r+\tau)}{(\tilde{c}_h(r+\tau)+\frac{\beta}{1-\beta}\kappa\theta_h)^2} > 0$

From (D.18) we get:

$$\frac{dL^g_{u,h}}{d\bar{\mu}} = -\frac{L^g_{u,h}}{1-\bar{\mu}} < 0 \qquad \text{(D.23)}$$

Thus,

$$\begin{aligned}
\frac{dL^g}{d\bar{\mu}} &= -\left[\frac{L^g_{u,h}}{1-\bar{\mu}} - \frac{L^g_c}{\bar{\mu}}\left(\frac{X_l\xi(\tilde{c}_l)}{X_l\xi(\tilde{c}_l) + \Delta}\right)\right] \text{ if } w^g \leq \underline{w}^g_{c,h} \\
\frac{dL^g}{d\bar{\mu}} &= -\left[\frac{L^g_{u,h}}{1-\bar{\mu}} - \frac{L^g_c}{\bar{\mu}}\left(\frac{X_l\xi(\tilde{c}_l) + X_h\xi(\tilde{c}_h)}{X_l\xi(\tilde{c}_l) + X_h\xi(\tilde{c}_h) + \Delta}\right)\right] \text{ if } w^g > \underline{w}^g_{c,h} \qquad \text{(D.24)}
\end{aligned}$$

The terms in the parentheses of the above expressions are less than 1. Further, it can be easily verified from (D.18) (with $i = h$) and (D.20) that $\frac{L^g_{u,h}}{1-\bar{\mu}} \gtreqless \frac{L^g_c}{\bar{\mu}}$ when $\tilde{c}_h \gtreqless 0$ ($w^g \gtreqless \underline{w}^g_{c,h}$). Therefore,

$$\begin{aligned}
&\tfrac{dL^g}{d\bar{\mu}} < 0, \tfrac{du^g}{d\bar{\mu}} < 0, \quad \text{if } w^g > \underline{w}^g_{c,h} \\
&\tfrac{dL^g}{d\bar{\mu}} \lesseqgtr 0, \tfrac{du^g}{d\bar{\mu}} \lesseqgtr 0, \quad \text{if } w^g \leq \underline{w}^g_{c,h}
\end{aligned}$$

where it may be recalled that $u^g = u^g_u + u^g_c = L^g - e^g$.

The total employment rate in the model with worker heterogeneity is given by $e = e^g + e^p_h + e^p_l$, where e^g is exogenously set by the government, while e^p_h and e^p_l can be derived from (D.14) and (D.15): $e^p_i = \frac{m(\theta_i)L^p_i}{s^p+\tau+m(\theta_i)}$, $i = [h, l]$ so that

$$e = e^g + \frac{m(\theta_h)(X_h - L^g_h)}{s^p + \tau + m(\theta_h)} + \frac{m(\theta_l)(X_l - L^g_l)}{s^p + \tau + m(\theta_l)} \qquad \text{(D.25)}$$

We know that for $w^g > \underline{w}^g_{u,h}$ all low-ability workers in the public sector are connected. Hence, $L^g_l = L^g_{c,l} = X_l\Xi(\tilde{c}_l)$, and clearly $\frac{dL^g_l}{d\bar{\mu}} > 0$, since $\frac{d\tilde{c}_l}{d\bar{\mu}} > 0$. But the composition of the high-ability workers attached to the public sector (L^g_h) in terms of connections depends on whether wages are above or below $\underline{w}^g_{c,h}$.

If $w^g > \underline{w}^g_{c,h} (> \underline{w}^g_{u,h})$, then $L^g_h = L^g_{c,h} + L^g_{u,h}$, where $L^g_{c,h} = X_h\Xi(\tilde{c}_h)$. Since $\frac{d\tilde{c}_h}{d\bar{\mu}} > 0$ then $\frac{dL^g_{c,h}}{d\bar{\mu}} > 0$, while, as shown above, $\frac{dL^g_{u,h}}{d\bar{\mu}} < 0$. Moreover, as shown above, for $w^g > \underline{w}^g_{c,h}$,

$\frac{dL^g}{d\bar{\mu}} < 0$ which means that the decrease in $L^g_{u,h}$, dominates over increase in $L^g_{c,h}$ and $L^g_{c,l}$. It follows, then, that the decrease in L^g_h dominates over the increase in L^g_l. This, together with the fact that $m(\theta_h) > m(\theta_l)$ implies $\frac{de}{d\bar{\mu}} > 0$. That is, e^p_l decreases and e^p_h increases with the increase in $\bar{\mu}$ but the increase in e^p_h is larger than the decrease in e^p_l and thus total employment increases.

If $w^g \leq \underline{w}^g_{c,h}$ we can not show that $\frac{de}{d\bar{\mu}} > 0$ because the sign of $\frac{dL^g}{d\bar{\mu}}$ is ambiguous. In this case $L^g_l = L^g_{c,l}$ and $L^g_h = L^g_{u,h}$ and we have an increase in $L^g_{c,l}$ and decrease in $L^g_{u,h}$, but we do not know if the decrease in $L^g_{u,h}$ dominates over the increase in $L^g_{c,l}$. To sum up:

$$\frac{de^p_h}{d\bar{\mu}} > 0, \frac{de^p_l}{d\bar{\mu}} < 0, \frac{de}{d\bar{\mu}} \lesseqgtr 0 \text{ if } w^g \leq \underline{w}^g_{c,h}$$
$$\frac{de^p_h}{d\bar{\mu}} > 0, \frac{de^p_l}{d\bar{\mu}} < 0, \frac{de}{d\bar{\mu}} > 0 \text{ if } w^g > \underline{w}^g_{c,h}$$

D.4 Effects of increasing w^g when $w^g > \underline{w}^g_{u,h}$

First, let us show that the number of workers searching in the public sector increases as the public-sector wage increases; that is $\frac{dL^g}{dw^g} > 0$, which ultimately implies that $\frac{dL^p}{dw^g} < 0$, since $L^p = 1 - L^g$. Recall also that for $w^g > \underline{w}^g_{u,h}$ all workers searching for public jobs without connections are of high ability. The total number of workers attached to the public sector is given by $L^g = L^g_u + L^g_c$ where $L^g_u = L^g_{u,h}$ and $L^g_{u,h}$ is as given in (D.18) (with $i = h$), while $L^g_c = L^g_{c,h} + L^g_{c,l} = X_h\Xi(\tilde{c}_h) + X_l\Xi(\tilde{c}_l)$ if $w^g > \underline{w}^g_{c,h} (> \underline{w}^g_{u,h})$ and $L^g_c = L^g_{c,l} = X_l\Xi(\tilde{c}_l)$ if $w^g \leq \underline{w}^g_{c,h}$.

$$\begin{aligned}
\frac{dL^g}{dw^g} &= \frac{dL^g_{u,h}}{dw^g} + X_h\xi(\tilde{c}_h)\frac{d\tilde{c}_h}{dw^g} + X_l\xi(\tilde{c}_l)\frac{d\tilde{c}_l}{dw^g}, \text{ if } w^g > \underline{w}^g_{c,h} \\
\frac{dL^g}{dw^g} &= \frac{dL^g_{u,h}}{dw^g} + X_l\xi(\tilde{c}_l)\frac{d\tilde{c}_l}{dw^g}, \text{ if } w^g \leq \underline{w}^g_{c,h} \qquad \text{(D.26)}
\end{aligned}$$

It is straightforward to verify from (D.18) that $\frac{dL^g_{u,h}}{dw^g} > 0$. To show that $\frac{dL^g}{dw^g} > 0$ we need to show further that $\frac{d\tilde{c}_i}{dw^g} > 0$, $i = [h, l]$.

We can use (D.12), together with (4), (6) and (D.11) to derive:

$$\tilde{c}_i - \frac{1}{r+\tau}\left[\frac{\frac{\bar{\mu}(s^g+\tau)e^g}{L^g_c - \bar{\mu}e^g}}{r+\tau+s^g+\frac{\bar{\mu}(s^g+\tau)e^g}{L^g_c-\bar{\mu}e^g}}(w^g - b)\right] = \frac{1}{r+\tau}\frac{\beta\kappa\theta i}{(1-\beta)}, \quad i = [h, l] \qquad \text{(D.27)}$$

and obtain:

$$\begin{aligned}
\frac{d\tilde{c}_l}{dw^g} &= \frac{M}{r+\tau+\frac{M(1-M)(w^g-b)}{L^g_c-\bar{\mu}e^g}X_l\xi(\tilde{c}_l)} \geq 0, \text{ if } w^g \leq \underline{w}^g_{c,h} \\
\frac{d\tilde{c}_l}{dw^g} &= \frac{d\tilde{c}_h}{dw^g} = \frac{M}{r+\tau+\frac{M(1-M)(w^g-b)}{L^g_c-\bar{\mu}e^g}(X_l\xi(\tilde{c}_h) + X_h\xi(\tilde{c}_l))} \geq 0, \text{ if } w^g > \underline{w}^g_{c,h} \qquad \text{(D.28)}
\end{aligned}$$

where $M = \frac{\frac{\bar{\mu}(s^g+\tau)e^g}{L_c^g-\bar{\mu}e^g}}{r+\tau+s^g+\frac{\bar{\mu}(s^g+\tau)e^g}{L_c^g-\bar{\mu}e^g}}$. It is evident from the above equations that $\frac{d\tilde{c}_i}{dw^g} > 0$, $i = [h, l]$ only if $\bar{\mu} > 0$, while $\frac{d\tilde{c}_i}{dw^g} = 0$ if $\bar{\mu} = 0$.

It follows that:

$$\begin{aligned} \frac{dL^g}{dw^g} &> 0, \frac{dL^g_{u,h}}{dw^g} > 0 \\ \frac{dL^g_c}{dw^g} &> 0, \text{ if } \bar{\mu} > 0 \\ \frac{dL^g_c}{dw^g} &= 0, \text{ if } \bar{\mu} = 0 \end{aligned}$$

Therefore $\frac{dL^g_h}{dw^g} > 0$ and $\frac{dL^g_l}{dw^g} > 0$ if $\bar{\mu} > 0$ and $\frac{dL^g_l}{dw^g} = 0$ if $\bar{\mu} = 0$. From (D.25) and given $u^g = L^g - e^g$ we can also write:

$$\begin{aligned} \frac{de}{dw^g} &< 0, \frac{du^g}{dw^g} > 0, \frac{de^p_h}{dw^g} < 0 \\ \frac{de^p_l}{dw^g} &< 0, \text{ if } \bar{\mu} > 0 \\ \frac{de^p_l}{dw^g} &= 0, \text{ if } \bar{\mu} = 0 \end{aligned} \tag{D.29}$$

D.5 Proof of Proposition 6

Let us firs consider the case $\underline{w}^g_c < w^g \leq \underline{w}^g_{u,h}$ where there are only low-type workers in the public sector. As summarized in Proposition 1, the private-sector labor force and employment increase with an increase in $\bar{\mu}$ due to some workers who would otherwise search without connections for public jobs choosing to search for private jobs instead. Since all workers in the public sector are of low ability the additional workers entering the private sector are of low ability, thereby lowering average ability in the private sector, while all workers in the public sector remain of low ability.

Consider next the case where $\underline{w}^g_{c,h} \geq w^g > \underline{w}^g_{u,h}$ where all low ability workers attached to the public sector are connected while all high ability workers attached to the public sector have no connections. In this case, as shown above (see Section D.3) an increase in $\bar{\mu}$ will attract more low-ability workers into the public sector ($\frac{dL^g_{c,l}}{d\bar{\mu}} > 0$) and will drive high ability workers away from the public and into the private sector ($\frac{dL^g_{u,h}}{d\bar{\mu}} < 0$). This means that the skill composition of employment/labor force in the public sector deteriorates, while those in the private sector improve.

When $w^g > \underline{w}^g_{c,h}$, there are also some high-ability workers in the connections sector. An increase in $\bar{\mu}$ in this case will induce more of the high-ability workers to get connections and move into the public sector. However, as shown above (Section D.3), the decrease in the number of high-ability unconnected workers attached to the public sector dominates over the increase in the number of connected (high or low ability workers) attached to the public sector. Hence, in this case also, the skill composition in the public sector deteriorates while that in the private sector improves.

E Competitive Search in the Private Sector

Suppose now that, as in the benchmark model, the two sectors, private and public, are segmented. However, we depart from the assumptions of Nash bargaining and random search in the private sector. Instead, as in Moen (1997), we introduce a competitive search equilibrium in the private sector. To this end, we assume that the private-sector market consists of submarkets with different posted wages and equilibrium tightness.

In each submarket, there is a subset of unemployed workers and firms with vacant jobs that are searching for each other. A matching function determines the number of matches in each submarket. The number of matches in submarket n is $m(v_n, u_n) = (v_n)^{\eta}(u_n^p)^{(1-\eta)}$, $m(\theta_n)$ is the job finding rate and $q(\theta_n)$ the job filling rate. Unemployed workers are free to move between submarkets. They choose to search for a job in the submarket that yields the highest expected income. Since workers are ex-ante identical, and movement across submarkets is free, in equilibrium, the value of search is equal across submarkets. A market maker determines the number of submarkets in each market and the wage in each submarket. The wage is chosen to maximize the value of a vacancy, and since all vacancies in the same submarket are identical, they offer the same wage. There is free entry of vacancies in each submarket, which drives the value of a vacancy to zero, and determines the number of vacancies posted in each submarket.

We present next the full set of Bellman equations describing the optimal behavior of workers and firms, the equilibrium conditions and the model solution. For a worker in submarket n

$$(r+\tau)U_{u,n}^p = b + m(\theta_n)\left[E_{u,n}^p - U_{u,n}^p\right] \tag{E.1}$$

$$(r+\tau)E_{u,n}^p = w_n^p - s^p\left[E_{u,n}^p - U_{u,n}^p\right] \tag{E.2}$$

Unemployed workers are free to move between submarkets. They will choose to search for a job in the submarket that yields the highest expected income. Since workers are ex-ante identical and movement across submarkets is free, this means that $U_{u,n}^p = U_u^p$. Using (E.1) and (E.2) we can write:

$$m(\theta_n) = \left(\frac{(r+\tau)U_u^p - b}{w_n^p - (r+\tau)U_u^p}\right)(r+\tau+s^p) \tag{E.3}$$

The values of vacancies and filled jobs in submarket n satisfy

$$rV_{u,n}^p = -\kappa + q(\theta_n)\left[J_u^p(w_n^p) - V_{u,n}^p\right] \tag{E.4}$$

$$rJ_u^p(w_n^p) = y_n - w_n^p + (s^p+\tau)\left[V_{u,n}^p - J_u^p(w_n^p)\right] \tag{E.5}$$

Using (E.4) and (E.5) to solve for $V_{u,n}^p$ gives

$$rV_{u,n}^p = \frac{-\kappa(r+s^p+\tau) + q(\theta_n)(y_n - w^p)}{r + q(\theta_n) + s^p + \tau} \tag{E.6}$$

In a competitive search equilibrium a market maker determines the number of submarkets in each market and the wage in each submarket. The wage is chosen to maximize the value of a vacancy. All vacancies in the same submarket offer the same wage. Setting the derivative of (E.6) with respect to w_n^p equal to 0 we get the first order condition for optimal wages:

$$-(1-\eta)(r+s^p+\tau)\frac{d\theta_n}{dw_n^p}\left[y_n - w_n^p + \kappa\right] = \theta_n(r+s^p+\tau) + m(\theta_n) \tag{E.7}$$

There is free entry of vacancies in each submarket, which drives the value of a vacancy to zero. Setting $V_{u,n}^p = 0$ in (E.6) gives:

$$\frac{\kappa}{q(\theta_n)} = \frac{y_n - w^p}{r+s^p+\tau} \tag{E.8}$$

Taking the derivative of (E.3) with respect to w_n^p we obtain

$$\frac{d\theta_n}{dw_n^p} = -\left(\frac{\theta_n}{w_n^p - (r+\tau)U_u^p}\right)\frac{1}{\eta} \tag{E.9}$$

Using (E.8) and (E.9) to substitute for κ and $\frac{d\theta_n}{dw_n^p}$, respectively, in (E.7) and then solving for w_n^p we get

$$w_n^p = (1-\eta)y_n + \eta(r+\tau)U_u^p \tag{E.10}$$

Using (E.3) and (E.8) we can substitute for $(r+\tau)U_u^p$ in (E.10) and obtain

$$w_n^p = b + (1-\eta)\left(y_n - b + \theta_n\kappa\right) \tag{E.11}$$

Substituting w_n^p from (E.11) into (E.8) we get the job creation condition in each submarket

$$\frac{\kappa}{q(\theta_n)} = \frac{\eta(y_n - b)}{r+s^p+\tau+(1-\eta)m(\theta_n)} \tag{E.12}$$

Notice that if $y_n = y$, meaning that productivity is the same across all submarkets then $\theta_n = \theta$ and $w_n^p = w^p$. All submarkets offer the same wage and job finding rate. If in addition the Hosios condition holds, i.e. $1-\eta = \beta$, then job creation, market tightness and the Nash bargaining wage in the Benchmark model described in the text (see equations 13 and 14) are identical to those derived under competitive search. Hence, the results discussed in Sections 4 and 5 carry over to this alternative assumption of competitive search in the private sector.

F Connections Premium

In the benchmark model, we consider that connected and unconnected workers enjoy the same benefits of working in the public sector. We also assume that the costs incurred by the newborns to get connections were wasted. We now assume that newborn pay connections costs to current connected public-sector workers so that current workers will help fast-track them into the public sector. These payments are the "connections premium", Υ, which will further raise the value of working in the public sector for connected workers. We describe here the basic set up and in Section H we compare quantitative results in this alternative setup to those obtained in the benchmark model, in which no such connections premium exists.

$$(r+\tau)E_c^g \quad = \quad w^g + \Upsilon - s^g\left[E_c^g - U_c^g\right]. \tag{F.1}$$

In equilibrium, this connections premium depends on the threshold of connections costs, $\Upsilon = \Upsilon(\tilde{c})$. The total connections cost paid by newborns is $\tau \int_0^{\tilde{c}} c\xi(c)dc$, where ξ is the pdf of the distribution of connection costs. To avoid creating further interactions between sectors, we assume that newborns' total connections cost is divided equally among connected workers:

$$\Upsilon(\tilde{c}) = \frac{\tau \int_0^{\tilde{c}} c\xi(c)dc}{\bar{\mu}e^g}. \tag{F.2}$$

In principle, this extension could create multiple equilibria, with people expecting high returns of connections investing in connections (creating a lot of side payments) or people expecting low returns of connections not investing in connections (generating few side payments). We show, below, that provided some regularity conditions on the distribution of connections costs are satisfied, there are no multiple equilibria.

With the introduction of a connection premium all other Bellman equations but the value of being employed in the public sector for a connected worker (E_c^g) remain as in the Benchmark model described in Section 3. It follows that all equilibrium conditions remain the same, but equations (23) that determine the cut-off connection costs. The cut-off connection cost now change to take into account that the existence of a connection premium increases the value of being a connected and employed public employee. In particular, equation (23) becomes:

$$\tilde{c} = \frac{1}{r+\tau}\left[\frac{\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}{r+\tau+s^g+\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}\left(w^g - b + \frac{\tau\int_0^{\tilde{c}} c\xi(c)dc}{\mu e^g}\right) - \frac{\beta\kappa\theta}{(1-\beta)}\right] \tag{F.3}$$

As shown in Appendix A.2, equations (13) gives unique equilibrium value for θ. To guarantee the existence and uniqueness of a steady-state condition we need to show that with the equilibrium value of θ substituted in, equation (F.3) gives a unique equilibrium value for $\tilde{c}$.

Rearranging terms in (F.3) we can write:

$$\tilde{c} - \frac{1}{r+\tau}\left[\frac{\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}{r+\tau+s^g+\frac{\mu(s^g+\tau)e^g}{L_c^g-\mu e^g}}\left(w^g - b + \frac{\tau\int_0^{\tilde{c}} c\xi(c)dc}{\mu e^g}\right)\right] = \frac{1}{r+\tau}\frac{\beta\kappa\theta}{(1-\beta)} \tag{F.4}$$

Since the right-hand-side of the equation above is independent of $\tilde{c}$ a unique equilibrium value of $\tilde{c}$ exists if the left-hand-side of the equation above is increasing in $\tilde{c}$. Sufficient (but not necessary) condition for the left-hand-side of (F.4) to be increasing in $\tilde{c}$ is:

$$1 - \frac{m_c^g}{r+\tau+s^g+m_c^g}\frac{\tau}{r+\tau}\frac{\tilde{c}\xi(\tilde{c})}{\mu e^g} > 0$$

Sufficient but not necessary condition for the above inequality to be always satisfied is

$$\bar{c}\xi(\bar{c}) \leq \mu e^g$$

G Survey data

Table G1: Quality of government survey - European Countries

Country	QoG Indexes			Aggregate public-
	Skills and Merit	Political connections	Personal connections	private wage ratio
Austria	5.00	5.00	4.67	1.72
Belgium	5.71	3.00	2.14	1.40
Bulgaria	3.23	5.38	5.00	1.96
Croatia	4.20	4.10	3.80	
Cyprus	3.20	5.40	5.40	2.41
Czech Republic	5.00	4.11	4.10	1.33
Denmark	6.29	1.57	2.60	1.08
Estonia	4.67	3.33	3.56	
Finland	6.00	3.33	2.50	1.16
France	5.67	2.64	3.00	1.16
Germany	5.89	2.62	2.40	1.38
Greece	4.13	3.87	3.73	2.43
Hungary	3.67	5.07	4.60	1.36
Iceland	4.83	2.83	2.83	1.61
Ireland	6.64	1.82	2.45	2.31
Italy	3.25	4.25	4.20	2.05
Latvia	4.60	3.40	3.60	1.36
Lithuania	4.88	3.56	3.44	1.14
Luxembourg				0.88
Malta	3.75	4.50	3.50	1.48
Netherlands	6.08	2.46	2.68	1.92
Norway	6.40	2.07	1.87	0.98
Poland	5.50	2.80	3.20	
Portugal	4.25	4.63	4.56	2.23
Romania	4.24	4.94	4.33	
Slovakia	2.67	4.78	5.56	1.26
Slovenia	4.38	3.63	4.13	1.51
Spain	5.04	3.17	3.17	2.05
Sweden	5.92	1.92	2.69	0.93
Switzerland	6.20	3.00	2.80	
United Kingdom	5.79	2.82	2.88	1.09

Note: Indexes of recruitment practices are takes from the Quality of Government Survey*. Data on government and private sector employment is from EUROSTAT and OECD. Data on government wage bill and private sector wage bill is from AMECO.*

Table G2: Quality of government survey - World regions

Region	**QoG Indexes**		
(no of countries)	**Skills and Merit**	**Political connections**	**Personal connections**
Eastern Europe and post Soviet Union (25)	3.71	4.54	4.28
Latin America (16)	3.44	4.86	4.51
North Africa and the Middle East (11)	3.32	4.71	4.14
Sub-Saharan Africa (25)	3.60	4.92	5.11
Western Europe and North America (22)	5.51	2.92	1.99
East Asia (4)	5.32	2.87	3.08
South-East Asia (7)	4.35	4.44	4.57
South Asia (6)	3.78	4.78	5.67
The Pacific (1)	3.66	5.00	4.83
The Caribbean (3)	4.00	4.08	3.75

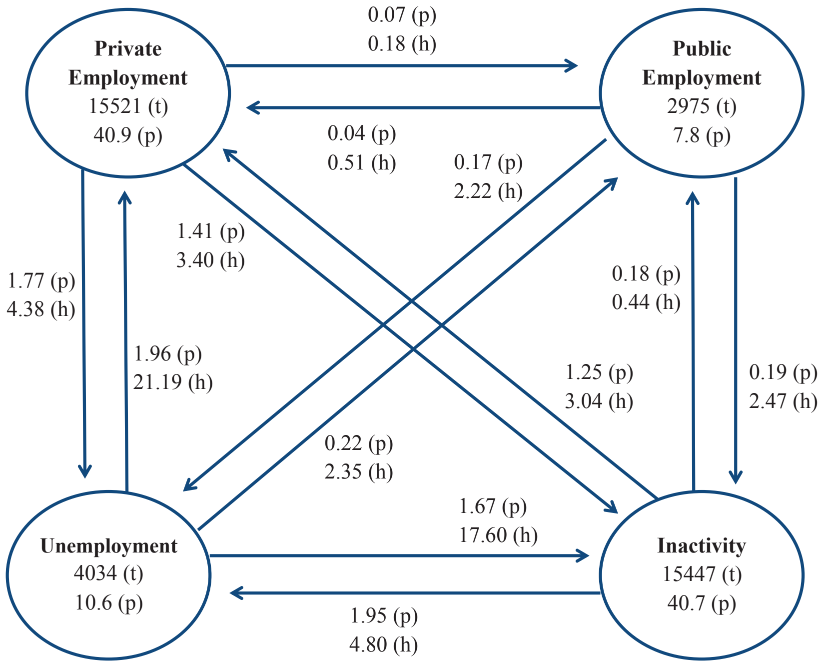
Note: Indexes of recruitment practices are takes from the Quality of Government Survey. *Average for different world regions.*

Table G3: Regression of the ratio of indexes of non-meritocracy

	Baseline variables				**Alternative variables**	
	(1)	(2)	(3)	(4)	(5)	(6)
Public-sector wage premium	0.265**		0.384***	0.723***	3.484***	0.459**
	(2.62)		(3.74)	(4.48)	(3.81)	(2.22)
Unemployment rate		-0.003*	-0.006***			
		(-1.71)	(-3.12)			
× High public wage				-0.009***	-0.038***	-0.010***
				(-4.16)	(-2.98)	(-3.51)
× Low public wage				-0.002	0.000	-0.006*
				(-0.95)	(0.02)	(-1.86)
Observations	70	70	70	70	70	70
R-squared	0.09	0.041	0.207	0.283	0.194	0.169

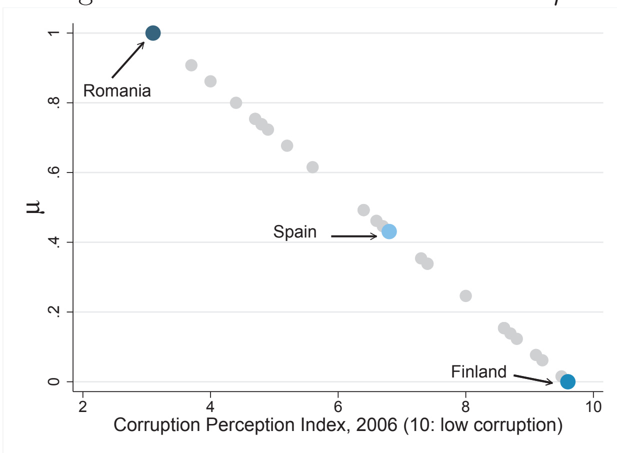
*Notes: The t-statistics are shown in brackets.*** indicates significance at the 1% level, ** at 5% level, and * at the 10% level. The dependent variable is the ratio of the non-meritocracy index for the public sector over the index for the private sector. It increases when the public sector is perceived to be less meritocratic than the private sector. The index is constructed with data taken from European Quality of Government Index dataset. The public-sector wage premium is estimated with microdata from the 2010* Structure of Earnings Survey. *Unemployment rate is taken from Eurostat. In column (5) we use an alternative index which is the difference between the index for the public over the index for the private. In column (6) we use an alternative index which is the ratio between the index for the public sector (answer by only public-sector workers) over the index for the private sector (answered by only private-sector workers).*

Figure G4: 4-state stocks and flows, Spain



Source: Spanish Labour Force Survey, average 2005-2015. The worker stocks and flows are expressed as total number of people in thousands (t), as a percentage of the working-age population (p) or as a hazard rate (h). See Fontaine et al. (2020) for details. For the calibration, we excluded the flows from and to inactivity.

Figure G5: Alternative calculation of $\bar{\mu}$



Source: Corruption Perception Index, 2006, Own calculations.

H Quantitative exercise

The purpose of this appendix is twofold. First, we want to extent our quantitative analysis in Section 5.3 by using our benchmark model with segmented markets calibrated to the Spanish economy to simulating also the effects of wage and employment policies. Given the endogenous limits that government policies place on μ discussed in Section 4.2, and given a set of parameters, we might be in a region where: (i) μ in not constrained and is equal to $\bar{\mu}$ or (ii) μ is constrained. Changes in government policy may switch the economy from one region to the other, making it difficult to solve for their effect in the full model analytically. In our quantitative exercise we account for such switches and are able to characterize the full effect of policy changes. Second, we want to compare the benchmark model with the alternative models proposed in Section 7 – in particular, to compare the transmission mechanisms under the assumptions of segmented markets and random search. We have also done simulations changing the deep parameters of the model is Section 6, but the effects simply boil down to combinations of different policies.

H.1 Effects of policies

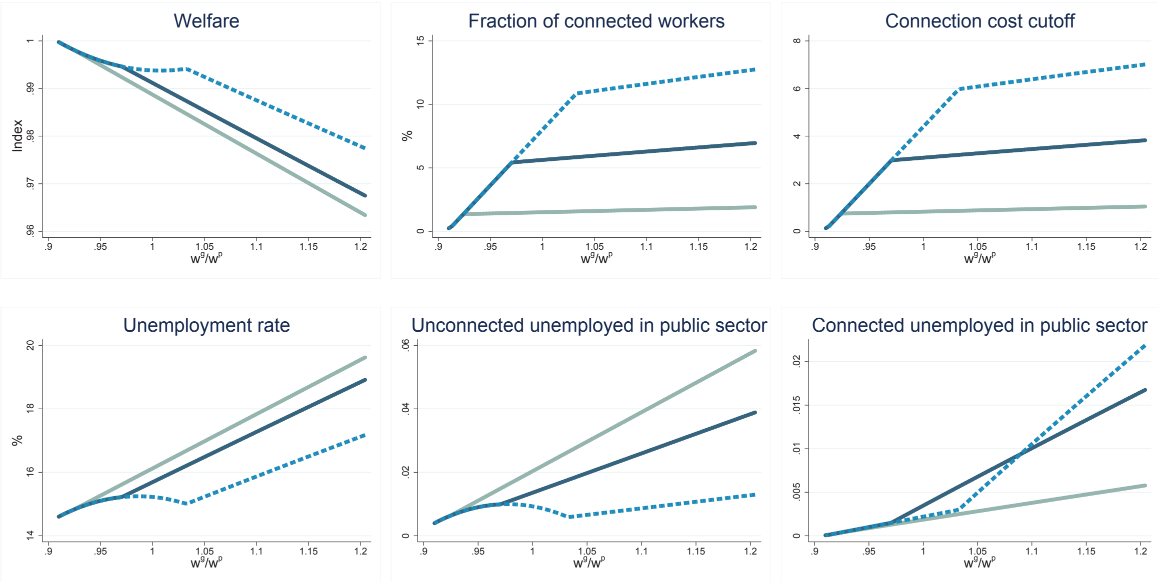
Figure H1 shows the effect of public wages, for three levels of target $\bar{\mu}$: 0.2, 0.40 and 0.8. In general, decreasing public wages raises welfare, since, as outlined in Proposition 2, cutting them has a positive effect on the employment rate. A 10 percent cut in the wages of public-sector workers lowers the unemployment rate by 1.5 percentage points. However, for some combination of parameters (high $\bar{\mu}$), there is a region in which μ becomes constrained. In that region welfare declines with wage cuts. This happens because, as shown in Proposition 3, in the constrained case μ decreases with wage cuts. Decreasing μ means freeing up public jobs for job searchers that do not have connections. This pushes more unemployed workers to queue for public-sector jobs, and increases the unemployment rate.

Figure H2 shows the effects of increasing public-sector employment. The effect of increasing public-sector employment on the selection of workers into the two sectors resembles those of increasing public-sector wages. In both cases the value of searching in the public sector goes up and this drains workers from the private to the public sector. The fact that higher public-sector employment lowers welfare follows trivially from the lack of assumption on the value of public-sector production. What is interesting to notice is that it can increase or decrease unemployment, depending on the level of nepotism. In line with the results outlined in Proposition 2 for the case of increasing w^g, high nepotism prevents large increases in the queues for public-sector jobs, which helps reduce unemployment. Conversely, when most public-sector jobs are available to unconnected workers, more job openings at high wages, attract a disproportionate number of searchers raising unemployment.

H.2 Comparing different models

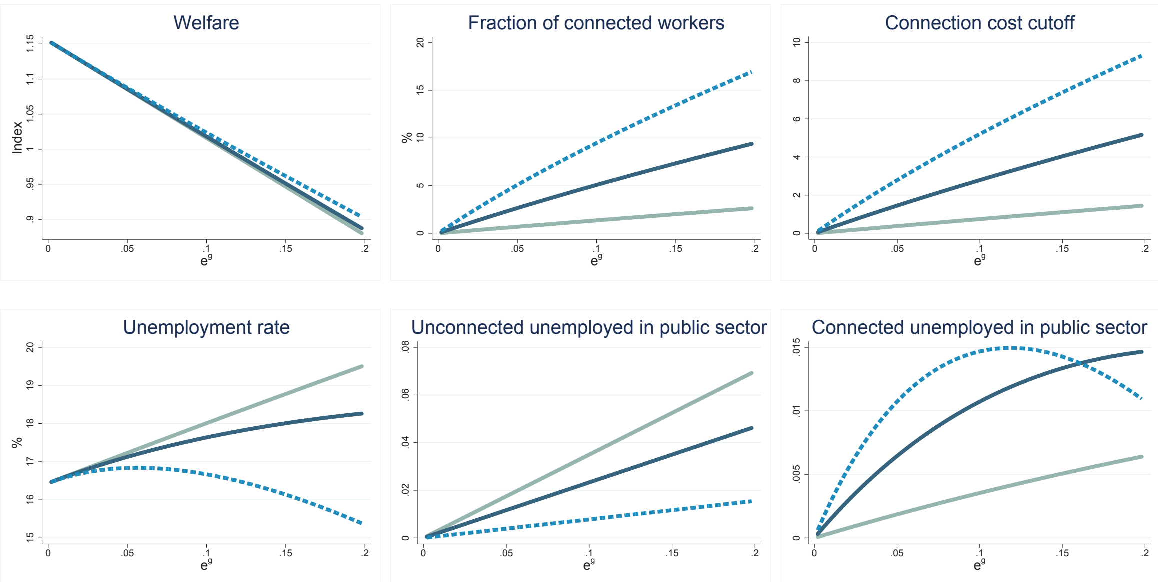
We now compare the results from the baseline segmented market model with those from the alternative models discussed in Section 7. For the model in which search in the public and private sectors is random, we reparameterize the cost of posting vacancies to target the steady-state unemployment rate ($\kappa = 7.31$). We follow the same procedure for the model

Figure H1: Effects of public-sector wages



Note: The ***dark blue line*** *is the benchmark calibration ($\bar{\mu} = 0.4$). The* ***light green line*** *is the scenario with low nepotism ($\bar{\mu} = 0.2$). The* ***bright blue dashed line*** *is the scenario with high nepotism ($\bar{\mu} = 0.8$). Welfare is expressed as a fraction of the efficient steady state. In all scenarios, when skilled public-sector wages are low, μ becomes constrained. Tightness and wages in the private sector are constant and independent of public-sector wages or nepotism ($\theta = 0.06$, $w^p = 0.901$).*

Figure H2: Effects of public-sector employment



Note: The ***dark blue line*** *is the benchmark calibration ($\bar{\mu} = 0.4$). The* ***light green line*** *is the scenario with low nepotism ($\bar{\mu} = 0.2$). The* ***bright blue dashed line*** *is the scenario with high nepotism ($\bar{\mu} = 0.8$). Welfare is expressed as a fraction of the efficient steady state. In all scenarios, μ is never constrained. Tightness and wages in the private sector are constant and independent of public-sector wages or nepotism ($\theta = 0.06$, $w^p = 0.901$).*

with a connections premium ($\kappa = 6.29$). Once recalibrated, the steady state of the remaining variables is very close to that of the benchmark model.

Table H1 shows the effects of three different policies: i) a decrease in $\bar{\mu}$ from 0.4 to 0.2; ii) an increase in $\bar{\mu}$ from 0.4 to 0.6; and iii) a ten-percent decrease in public-sector wages.

We start by comparing the model with segmented markets with the model of random search. Graphs with a more detailed comparison are shown in Figure H3. We can see in the table that random search in the labor market weakens the effects of policies on unemployment. Although the effects go in the same direction, the mechanisms at work are different. Under random search, nepotism affects tightness (θ) positively and private wages negatively. By having fewer unconnected vacancies, the outside option of an unemployed worker bargaining with a firm is weaker, pushing wages down and raising job creation. This effect on private wages raises the public-sector wage premium endogenously.

As discussed above, the effect of $\bar{\mu}$ on welfare are ambiguous. As Figure H3 shows, under this parametrization, and in contrast with segmented markets, the effect is negative. When we move from $\bar{\mu} = 0.40$ to $\bar{\mu} = 0.20$, welfare increases by 0.22 percent. When moving to $\bar{\mu} = 0.60$, welfare also increases but marginally.

Turning, now, to the model with connections premium, it tends to amplify the effects of policies on the number of connected workers, but because the premium represents only 1.3 percent of public-sector wages, the effects are quantitatively similar to those in the benchmark model.

To sum up, we can draw three main conclusions from this section. First, under the baseline model, parameterized to a country with a large public-sector wage premia, welfare is increasing in $\bar{\mu}$, but this is not always true in the random search model. Second, public-sector wage cuts have a large quantitative effect on reducing the unemployment rate. Third, in the random search model, the effects of policies on unemployment are qualitatively similar

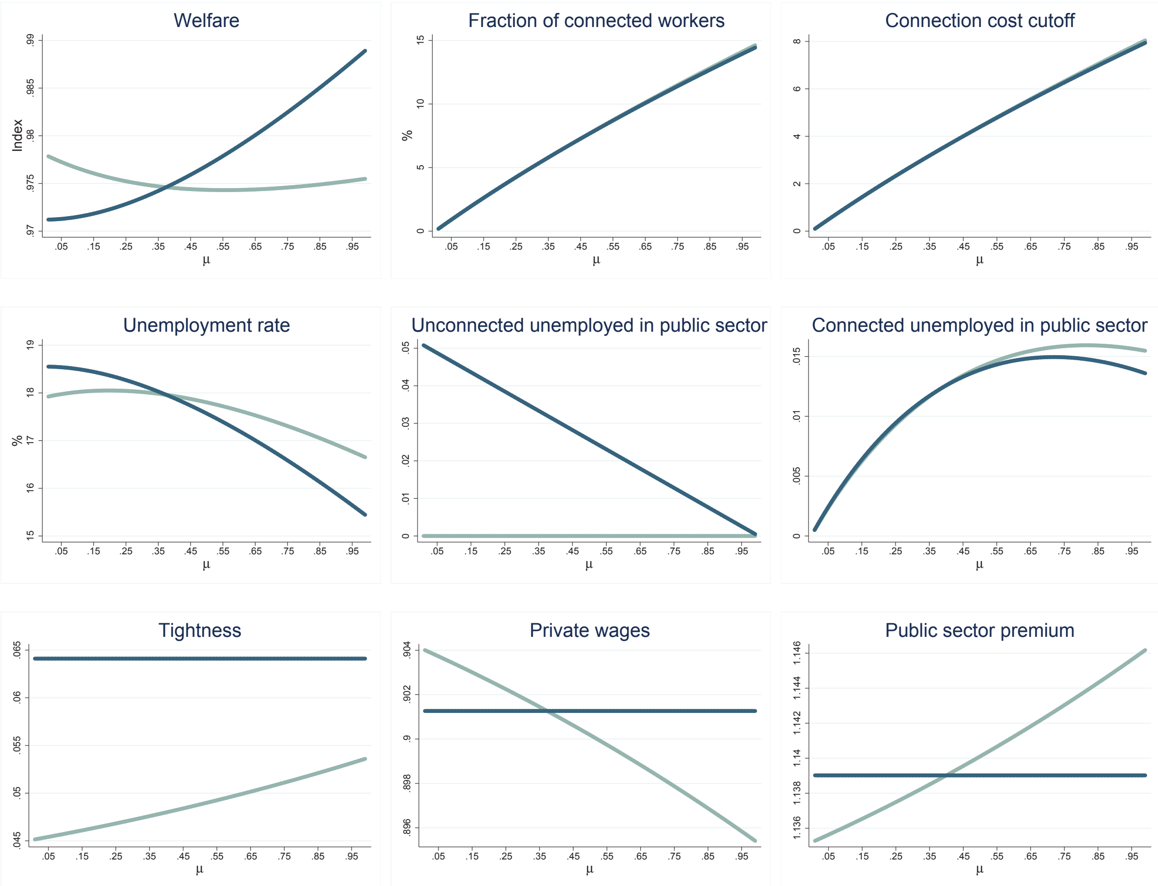
Table H1: Effects of policies under different models

Policy **Policy**	**Segmented** **markets**	**Random** **search**	**Connections** **premium**
Reduction of nepotism to $\bar{\mu} = 0.20$			
%Δ welfare	-0.28%	0.22%	-0.31%
Δ fraction of connected	-3.10 p.p.	-3.12 p.p.	-3.16 p.p.
Δ unemployment rate	0.48 p.p.	0.12 p.p.	0.42 p.p.
Increase of nepotism to $\bar{\mu} = 0.60$			
%Δ welfare	0.40%	0.00%	0.45%
Δ fraction of connected	2.85 p.p.	2.89 p.p.	2.94 p.p.
Δ unemployment rate	-0.68 p.p.	-0.30 p.p.	-0.61 p.p.
Reduction of public-sector wages by 10 percent			
%Δ welfare	1.36%	0.88%	1.35%
Δ fraction of connected	-0.74 p.p.	-0.68 p.p.	-0.76 p.p.
Δ unemployment rate	-1.80 p.p.	-1.03 p.p.	-1.82 p.p.

Note: The random search and connections premium models are recalibrated ($\kappa = 7.31$) and ($\kappa = 6.29$).

but quantitatively smaller than in the model with segmented markets. The same holds for the "connections premium" model.

Figure H3: Effects of nepotism



*Note: The **dark blue line** is the economy with segmented markets. The **light green line** is the economy with random search.*