Online Appendix
"Misallocation and Financial Frictions: the Role of Long-Term Financing"*

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A Additional Information on Data and Model Notation

In Table 1, we compare statistics regarding sales, employment and investment between aggregate national income accounts (NIPA), Compustat firms, and Compustat firms that have issued bonds in our data (merged sample). Compustat firms represent a fairly large share of the aggregate economy: 49% of aggregate sales, 58% of aggregate employment, and 30% of aggregate investment. Bond-issuing Compustat firms represent a smaller but still sizable share of the aggregate economy: 19% of aggregate sales, 20% of aggregate employment, and 10%

Table 1: Compustat vs. NIPA

	Compustat/NIPA	Bond-issuing firms/NIPA
Sales Share	0.49	0.19
Employment Share	0.58	0.20
Investment Share	0.30	0.10
Correlation of Sales Growth	0.80	0.88
Correlation of Employment Growth	0.61	0.57
Correlation of Investment Growth	0.91	0.94

Note: This table uses time series from the national income accounts (NIPA), Compustat firms, and a subset of Compustat firms that have issued at least one bond in our sample (denoted "bond-issuing firms"). We focus on two set of statistics: (a) the share of sales, employment, and investment in Compustat or bond-issuing firms in Compustat relative to the aggregate and (b) the correlation between sales, employment, and investment growth between NIPA, Compustat, and bond-issuing firms in Compustat.

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of aggregate investment. The correlation between sales and investment growth across time is high when comparing both Compustat and NIPA and bond-issuing Compustat firms and NIPA. On the other hand, the correlation of employment growth in NIPA and Compustat is low, around 0.6, as most of the employment growth usually comes from small firms. Table 2 describes the construction of variables in our empirical analysis.

Table 2: Definition of Variables: Model vs. Data

Variable	Notation	Definition
Capital	k	Gross value of property, plant, and equipment (PPEGT, data item #7)
Investment	$k' - (1-\delta)k$	Capital expenditures on property, plant, and equipment (CAPXV, #30) minus sales of capital stock (SPPE, #107).
Depreciation	δ	Current depreciation (DP, #14)
Dividends	d	Common dividends (DVC, #268) plus preferred dividends (DVP, #270) minus stock repurchases (PRSTKC, #115)
Employees	n	Stock of employees (EMP, #29)
Leverage	qb'/k'	Debt in current liabilities (DLC, #34) plus long-term debt (DLTT, #9) divided by book value of assets (AT, #6)
Sales	–	Total Sales (SALE, #12)
Value added	–	Sales minus materials
Materials	–	Total Expenses [(SALE, #12) - (OIBDP, #13)] - Labor Expenses [(EMP, #29) × Wages]
Credit spread	$(\theta + c)/q - \theta - r$	Interest paid over a similar maturity Treas. bill
Productivity	z	See main text
Firm age at bond issuance	–	Calendar year minus founding year

Note: We describe the empirical counterpart of model variables. For variables computed using Compustat data we include the definition and data item in parentheses.

In Table 2 we define model variables based on their empirical counterparts. Capital k is defined as the gross value of property, plant, and equipment (PPEGT, data item #7). In our model, capital is beginning-of-the-period capital. To be consistent with this definition, we measure capital in period t as the gross value of property, plant, and equipment reported by each firm in period $t-1$. Investment $i = k' - (1-\delta)k$ is defined as capital expenditures on property, plant, and equipment (data item #30) minus sales of capital stock (data item #107). δ is current depreciation (DP, data item #14) and n is the stock of employees (Emp, data item #29). Debt is the sum of debt in current liabilities (data item #34) and long-term debt (data item #9). The empirical equivalent of leverage qb'/k' is debt divided by book value of assets (data item #6).

Value added is calculated as sales (data item #12) minus materials which is equal to total expenses minus labor expenses. Total expenses is sales minus operating income before

Table 3: Robustness

	Benchmark		Merged Compustat/TR	
	# Obs.	Mean	# Obs.	Mean
Leverage	133,786	0.29	16,526	0.31
Assets/Sales	129,730	2.55	16,504	1.39
Investment rate	119,991	0.16	15,641	0.11
# Employees	126,983	9,309	4,262	53,366
Bond amount ($million)	18,369	365.5	4,326	421.0
Maturity (years)	18,369	11.6	4,326	11.0
Mean credit spread (p.p.)	18,369	2.32	4,326	2.12
St. dev. of credit spread (p.p.)	18,369	2.16	4,326	2.05
Firm Age at Bond Issuance	18,042	61.1	4,309	69.0
	# Obs.	Coeff.	# Obs.	Coeff.
TFP-age elasticity (w/ FE)	18,042	-0.41	4,309	-0.40

Note: This table reports a statistics for Benchmark sample and bond issuing firms in Compustat (denoted "Merged Compustat/TR").

depreciation and amortization (OIBPD, data item #13). Labor expenses is calculated as the number of employees times the average wages which are calculated from the Social Security Administration.

B Compustat and Thomson Reuters Merged Sample

We explore how our main statistics vary between the Benchmark which is a combination of statistics calculated from Compustat and TR and the merged sample (denoted "Merged Compustat/TR") which considers firms that appear in both samples (i.e., Compustat firms that have issued at least one bond). In the merged sample, the number of observations drops significantly as 9.8% of Compustat firms have issued a bond.

The statistics associated with credit spreads do not vary greatly between the samples (see Table 3). In particular, bonds have a similar average maturity while the average credit spread falls by 0.20 percentage points between the benchmark and the merged sample. Similarly, the standard deviation of credit spreads decreases slightly by 0.11 percentage points between the benchmark and the merged sample. Moreover, the relationship between credit spreads and age remains intact. Note that the elasticity of credit spreads to TFP is not reported as we can

Table 4: Credit Spreads by Industry

Industry	# Obs.	Mean	Standard Deviation
Media and Entertainment	1,276	3.45	2.63
Telecommunications	771	3.07	2.48
Healthcare	1,172	2.14	1.88
Retail	1,115	2.22	2.23
Materials	1,459	2.93	3.05
Industrials	2,843	2.19	2.03
Consumer Products and Services	1,117	2.24	2.19
Energy and Power	5,116	2.08	1.75
High Technology	861	2.41	2.37
Real Estate	1,183	2.04	1.33

Note: This table reports credit spreads across industries. Industry classification is based on Thomson Reuters data.

only use the merged sample to compute this statistic. The discrepancies in the two samples appear with respect to the average size of the firms. First, in the merged sample, firms are have a lower assets-to-sales ratio and employ more employees, on average.

Our conclusion is that the merged sample includes larger firms on average in terms of sales, employees, and bond issuance. Nonetheless, most of the financial statistics remain in broadly the same range across samples.

We next evaluate the importance of industry heterogeneity by reporting the credit spread distribution across industry. Table 4 reports the results. The mean credit spreads vary from 2.04-3.45 and the standard deviation varies from 1.33-3.05. The industry with the lowest mean credit spreads is Real Estate and the industry with the highest is Media and Entertainment. The industry with the smaller dispersion in credit spreads is Real Estate and the industry with the largest is Materials.

C TFP and TFP Loss

In this section, we derive first derive expressions for TFP and TFP loss. Then we construct an approximation which shows how TFP loss is related to the dispersion of the marginal product of capital and how the level of TFP depends on TFP loss.

TFP We can derive the following aggregate relationship:

$$Y = AM^{1-\nu}\left[K^{\alpha}N^{1-\alpha}\right]^{\nu}$$

where Y is aggregate output, K is aggregate capital, N is aggregate labor, M is the total mass of firms, and A is TFP.[1] It can be shown that TFP in our economy is given by

$$A = \frac{\left(\frac{1}{M}\int_{i\in\mu}\left(z_i/v_i^{\alpha\nu}\right)^{1/(1-\nu)}di\right)^{1-(1-\alpha)\nu}}{\left(\frac{1}{M}\int_{i\in\mu}\left(z_i/v_i^{1-(1-\alpha)\nu}\right)^{1/(1-\nu)}di\right)^{\alpha\nu}} \tag{1}$$

where i indexes the firm, μ is the distribution of firms in the stationary equilibrium, $M = \int d\mu$ is the total mass of firms, z_i is firm i's productivity, and v_i is the gross deviation of firm i's marginal product of capital from the economy-wide average. In our model, there are several reasons why the marginal product of capital will not be equated across firms: (1) capital adjustment costs, (2) taxes, (3) financial frictions, and (4) uncertainty over productivity while capital is chosen one period in advance. As a result, the level of TFP will be lower.

TFP Loss To compute TFP loss, we take as given the aggregate stock of capital $K = \int_{i\in\mu} k_i di$ and the aggregate stock of labor $N = \int_{i\in\mu} n_i di$ from the original economy, and then solve the following problem:

$$\max_{k_i,n_i}\int_{i\in\mu} z_i\left[k_i^{\alpha}n_i^{1-\alpha}\right]^{\nu}di$$

subject to

$$\int_{i\in\mu} k_i di = K$$
$$\int_{i\in\mu} n_i di = N$$

where K and N are the aggregate capital and labor stocks. The solution to this problem requires that the marginal product of labor (MPL) and the marginal product of capital (MPK) are both equated across firms. The quantities of labor and capital, which solve this problem,

[1] Defining $\bar{Y} = Y/M$, $\bar{K} = K/M$, $\bar{N} = N/M$, this then implies $\bar{Y} = A\left[\bar{K}^{\alpha}\bar{N}^{1-\alpha}\right]^{\gamma}$.

are given by

$$n_i = \left(\frac{N}{\Gamma}\right) z_i^{1/(1-\nu)} \tag{2}$$

$$k_i = \left(\frac{K}{\Gamma}\right) z_i^{1/(1-\nu)} \tag{3}$$

where $\Gamma \equiv \int_{i\in\mu} z_i^{1/(1-\nu)} di$. Total output with this allocation is

$$Y_{fb} = A_{fb} M^{1-\nu} K^{\alpha\nu} N^{(1-\alpha)\nu}$$

In this case, (first-best) TFP is $A_{fb} \equiv \left(\frac{1}{M}\int_{i\in\mu} z_i^{1/(1-\nu)}\right)^{1-\nu}$. This is what we get in Equation (1) if we set $v_i = 1$ for all firms.

TFP loss is then defined to be

$$\text{TFP Loss} = \frac{A_{fb}}{A} - 1 \tag{4}$$

In other words, TFP loss is the percentage increase in TFP that would be obtained by a more efficient allocation of capital and labor.

TFP Approximation To build intuition for TFP (defined in Equation 1) and TFP loss (defined in Equation 4), suppose that $\ln z_i$ and $\ln v_i$ are jointly normally distributed. Specifically, assume

$$\ln z_i \sim N(\mu_z, \sigma_z^2) \tag{5}$$

$$\ln v_i \sim N(\mu_v, \sigma_v^2) \tag{6}$$

Under these assumptions,

$$\ln A = \mu_z + \frac{1}{2}\frac{1}{1-\nu}\sigma_z^2 - \frac{\alpha\nu(1-(1-\alpha)\nu)}{1-\nu}\sigma_v^2 \tag{7}$$

In a similar fashion, we can approximate the first-best level of TFP as

$$\ln A_{fb} = \mu_z + \frac{1}{2}\frac{1}{1-\nu}\sigma_z^2 \tag{8}$$

Therefore, using Equations (7) and (8), TFP loss can be approximated as

$$\text{TFP loss} = \ln A_{fb} - \ln A \approx \frac{\alpha\nu(1-(1-\alpha)\nu)}{1-\nu}\sigma_v^2$$

Dispersion in the marginal product of capital generates TFP losses. Summarizing, for overall TFP, we have:

$$\ln A = \underbrace{\mu_z + \frac{1}{2}\frac{1}{1-\nu}\sigma_z^2}_{\text{composition/selection}} - \underbrace{\frac{\hat{\alpha}(1-\hat{\beta})}{1-\nu}\sigma_v^2}_{\text{TFP Loss}}$$

Financial frictions will affect the level of TFP by influencing the productivity distribution of firms, through (μ_z, σ_z^2). Financial frictions will also reduce TFP by increasing the variation in the marginal product of capital across firms. This will generate misallocation and reduce TFP.

D Additional Model Statistics

Table 5: Summary Statistics

Variable	p(10)	p(90)
Data		
Leverage	0.00	0.60
Investment rate	0.01	0.36
Credit spread (p.p.)	0.60	5.37
Model		
Leverage	0.08	0.55
Investment rate	-0.07	0.47
Credit spread (p.p.)	0.30	5.44

Note: We report model statistics for leverage, investment, and credit spreads at the 10^{th} and the 90^{th} percentile of the distribution.

An additional way to evaluate our model is to compare model statistics along the distribution with empirical counterparts. Table 5 reports the leverage, investment, and credit spread at the 10^{th} and the 90^{th} percentile of the distribution in the model and in the data. Our model is reasonably consistent with the data at these points of the distribution.

E Credit versus Equity Market Constraints

In this section, we distinguish between the effects of credit market constraints (CMC) and equity market constraints (EMC). In our quantitative analysis, we bundled these two sets of frictions together. Table 6 reports the results when we separately eliminate CMC and

Table 6: Effect of Different Frictions on TFP

	Bench. Model	No FF	Diff. (%)	No CMC	Diff. (%)	No EMC	Diff. (%)
Wage	1.00	1.23	22.5%	1.15	15.2%	1.21	21.2%
Capital/firm	1.93	2.57	32.9%	2.46	27.3%	2.59	34.0%
Output/firm	1.09	1.20	10.1%	1.21	11.2%	1.21	11.6%
# firms	1.00	1.11	11.3%	1.04	3.6%	1.09	8.6%
Investment share	0.30	0.08	-74.1%	0.19	-37.3%	-0.03	-111.2%
TFP loss	18.36%	14.81%	-3.5%	15.45%	-2.9%	14.76%	-3.6%
TFP	1.18	1.27	7.3%	1.25	5.6%	1.27	7.0%

Note: Aggregate macroeconomic variables in the Benchmark and three alternative models. In the model without credit market constraints (CMC), we (i) set the borrowing premium r_p to zero, (ii) set the collateral constraint, ψ, to a large number, and (iii) assume public firms cannot default on their debt obligations. In the model without equity market constraints (EMC), we assume that issuing funding is equity markets is costless (parameters ϕ_d and γ are zero). In the model without financial frictions (FF), we shut down both CMC and EMC.

EMC from the economy. In the model without credit market constraints (CMC), we (i) set the borrowing premium r_p to zero, (ii) set the collateral constraint, ψ, to a large number, and (iii) assume public firms cannot default on their debt obligations. In the model without equity market constraints (EMC), we assume that issuing funding in equity markets is costless (parameters ϕ_d and γ are zero). In the model without financial frictions (FF), we shut down both CMC and EMC.

In the economy without CMC, wages increase 15% and average capital per firm increases 27%. TFP loss is reduced by 2.9% while TFP increases by 5.6%. In this economy, while firms no longer face any frictions in credit markets, they still face frictions raising funds from shareholders and also distributing funds to shareholders. In the economy without EMC, the effects are similar to the economy without both frictions (CMC and EMC). Wages increase by 21% and average capital per firm increases by 34%. TFP loss falls by 3.6% and TFP increases by 7%. In this economy, while firms do not face any frictions raising funds from or distributing funds to shareholders, there still are taxes and capital adjustment costs. In particular, taxes still create a small tax advantage for debt issuance.

F Numerical Solution Method

We use value function iteration to jointly solve for the value functions and the equilibrium bond pricing function, where we discretize the productivity shocks using the method of Tauchen (1986). To solve for the distribution of firms, we simulate $N = 100,000$ firms for $T = 1000$ periods, saving the last 25 periods. Because we assume long-duration bonds, the value functions and price functions may fail to converge. As discussed by Chatterjee and Eyigungor (2012), the intuition for why this can happen is that the shape of the bond price schedule will imply that the firm's optimization problem is not globally concave and thus has multiple local maxima. As a result, there may be different combinations of (k', b'), which are far apart, yet deliver similar levels of utility. The convergence failure then occurs as the solution oscillates between two different local maxima.

Therefore, to solve our model, we iterate on the value function and pricing functions for a fixed number of iterations. We use grid search to compute the optimal policies for (k', b') in order to ensure the global maximum is found. To assess the validity of our solution, we check whether our results change when we solve the model with an alternative solution technique which takes care of the convergence problems. In this alternative solution technique, we modify the approach of Dvorkin, Sánchez, Sapriza, and Yurdagul (2019), who introduce a vector of i.i.d. shocks, drawn from a generalized extreme value distribution, in a way that avoids the need for an additional state variable (see also Gordon, 2019). In Table 7, we compare the resulting statistics from the two different solution methods for our benchmark model. We find that the overall conclusions remain intact under both solution methods.

We describe this solution technique as follows. We assume that firms can only choose values of (k', b') on a discrete grid, where $k' \in \mathcal{K} = \{k_1, \ldots, k_{N_k}\}$, $b' \in \mathcal{B} = \{b_1, \ldots, b_{N_b}\}$, and (N_k, N_b) are the number of grid points for capital and debt, respectively. We index every possible choice $(k', b') \in \mathcal{K} \times \mathcal{B}$ by $n = 1, \ldots, N$, where $N = N_k \times N_b$. Let (k'_n, b'_n) denote the $(k', b') \in \mathcal{K} \times \mathcal{B}$ indexed by n.

We then introduce two sets of (low-variance) i.i.d. shocks. First, we introduce a vector of i.i.d. shocks, $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_N)$, which affect the utility of each choice $(k', b') \in \mathcal{K} \times \mathcal{B}$. Second, we introduce two additional i.i.d. shocks, $(\varepsilon_c, \varepsilon_d)$, which affect the utility of continuing and the utility of exiting (private firms) or defaulting (public firms). We assume these shocks arrive to both public and private firms in order to prevent them from affecting the benefit of being public vs. private. We allow the $\boldsymbol{\varepsilon}$ shocks to be correlated with each other, but not with any other shocks. The two shocks $(\varepsilon_c, \varepsilon_d)$ are assumed to be independent of each other and all other shocks.

These shocks are all assumed to be drawn from a generalized extreme value (type-I) distri-

Table 7: Comparing Solution Methods

	Benchmark Solution Method	Solution with Extreme Value Shocks
Selected Statistics:		
Mean credit spreads	2.34%	2.34%
SD of credit spreads	2.12%	2.02%
Credit spread-TFP elast.	-0.35	-0.40
Credit spread-Age elast.	-0.49	-0.52
Mean lev. [0-90]	0.34	0.39
Mean lev., public firms	0.28	0.27
SD of inv. rates, public firms	0.24	0.26
Inv. share of public firms	0.30	0.29
Sales autocorr.	0.76	0.77
Sales std. dev.	0.50	0.53
Rel. emp. of firms, age 0	0.34	0.35
Rel. emp. of firms, age 1-5	0.64	0.66
Rel. emp. of firms, age 6-10	0.93	0.94
Rel. emp. of firms, age 11-15	1.03	1.04
Rel. emp. of firms, age 16-20	1.07	1.08
Rel. emp. of firms, age 21-25	1.11	1.11
Mean capital/sales ratio	2.92	2.90
Frequency of equity issuance	0.34	0.33
Mean default rate	3.19%	3.37%
SD of leverage	0.20	0.20
Effect of Removing Financial Frictions:		
TFP Loss	-3.55%	-3.58%
TFP	7.33%	7.04%
Wage	22.5%	22.0%

bution. Specifically, we assume the following cumulative distribution functions for the shocks:

$$F(\varepsilon_1, \ldots, \varepsilon_N) = \exp\left[-\left(\sum_{n=1}^{N} \exp\left(-\frac{\varepsilon_n - \mu}{\rho\sigma}\right)\right)^{\rho}\right]$$

$$F(\varepsilon_c) = \exp\left[-\exp\left(-\frac{\varepsilon_c - \mu}{\sigma}\right)\right]$$

$$F(\varepsilon_d) = \exp\left[-\exp\left(-\frac{\varepsilon_d - \mu}{\sigma}\right)\right].$$

The parameter μ is a location parameter and σ is a scale parameter which controls the variance of the shocks. We set σ to a small value (0.05) and we set μ so that the mean value of the shocks is zero.[2] The parameter ρ controls the correlation of the shocks $(\varepsilon_1, \ldots, \varepsilon_N)$, where $1 - \rho$ is approximately equal to the correlation. When $\rho = 1$, the shocks are all independent. We set $\rho = 0.02$ so that $\rho\sigma = 0.001$. Essentially, these shocks enable convergence by allowing

[2]Specifically, we set $\mu = -\sigma\gamma$, where γ is Euler's constant.

firms to randomize over their choices of capital and debt.[3] However, to ensure convergence, it was also sometimes necessary to utilize a relaxation parameter when updating the bond price schedule $q_j(z, k', b')$.[4]

We then introduce these shocks to the model as follows. Consider the problem of an incumbent firm of type j, with state (z, k, b). We define the utility this firm obtains from choosing (k'_n, b'_n) as follows (before it learns the shocks $\boldsymbol{\varepsilon}$):

$$\bar{V}_j^n(z, k, b) = d - \Lambda_j(d) + \beta E_{z'|z}\left[\tilde{V}_j(z', k'_n, b'_n)\right]$$

where

$$d = e_j(z, k, b) - k'_n + q_j(z, k'_n, b'_n)\left[b'_n - (1 - \theta_j)b\right] - g(k, k'_n)$$

The firm's optimization problem is then:

$$V_j(z, k, b) = E_{\boldsymbol{\varepsilon}}\left[\max_n\left\{\bar{V}_j^n(z, k, b) + \varepsilon_n\right\}\right] \tag{9}$$

Because of the extreme value shocks, it is now random which choice $(k'_n, b'_n) \in \mathcal{K} \times \mathcal{B}$ will be chosen by the firm. We can compute the probability that choice $n = m$ will be chosen as follows:

$$p_j^m(z, k, b) = \frac{\exp\left(\bar{V}_j^m(z, k, b)/(\rho\sigma)\right)}{\sum_{n=1}^N \exp\left(\bar{V}_j^n(z, k, b)/(\rho\sigma)\right)} \tag{10}$$

Moreover, the expected utility can be computed as

$$V_j(z, k, b) = \mu + \gamma\sigma + \rho\sigma \ln\left(\sum_{n=1}^N \exp\left(\bar{V}_j^n(z, k, b)/(\rho\sigma)\right)\right) \tag{11}$$

where $\gamma \approx 0.577$ is Euler's constant.[5] However, in our solution, we assume $N_k = 101$ and $N_b = 100$, which would yield $N = 10,100$ possible choices for n. This limits the tractability of this approach. Therefore, we make the following approximation. We assume that firms only randomize among the top $\tilde{N} < N$ choices. Without loss of generality, for each (j, z, k, b), we order the indices n such that $\bar{V}_j^1(z, k, b) \geq \bar{V}_j^2(z, k, b) \geq \cdots \geq \bar{V}_j^N(z, k, b)$. We then artificially assume that $p_j^n(z, k, b) = 0$ for all $n > \tilde{N}$. This leads to modified probabilities and expected

[3] There is a trade off between the variance of the shocks and the speed of convergence, where convergence occurs more quickly when the variance of the shocks is larger.

[4] The relaxation parameter ω is the weight placed on the old guess when updating the bond price schedule. Specifically, let q^k be the guess for the bond price schedule at iteration k and let $\hat{q}^k$ denote the new bond price schedule computed given q^k. The guess for the bond price schedule on the next iteration was then set to be $q^{k+1} = (1 - \omega)\hat{q}^k + \omega q^k$. Even though this was not always necessary for convergence, we set $\omega = 0.75$.

[5] To derive Equations (10) and (11), we use Theorem 1 in McFadden (1978).

utility:

$$p_j^m(z,k,b) = \begin{cases} \frac{\exp\left(\bar{V}_j^m(z,k,b)/(\rho\sigma)\right)}{\sum_{n=1}^{\tilde{N}} \exp\left(\bar{V}_j^n(z,k,b)/(\rho\sigma)\right)} & \text{for } m = 1,\ldots,\tilde{N} \\ 0 & \text{for } m = \tilde{N}+1,\ldots,N \end{cases} \tag{12}$$

$$V_j(z,k,b) = \mu + \gamma\sigma + \rho\sigma \ln\left(\sum_{n=1}^{\tilde{N}} \exp\left(\bar{V}_j^n(z,k,b)/(\rho\sigma)\right)\right) \tag{13}$$

We set $\tilde{N} = 200$, so that firms are restricted to randomize among the 200 best choices for (k', b').[6] In a similar fashion, we update $V_R^e(z)$ and $V_B^e(z,k,b)$.

We also introduce extreme value shocks which affect the decision of firms to continue or exit/default. For a firm of type j, with state (z', k', b'), we define the continuation value:

$$\tilde{V}_j(z',k',b') = E_{(\varepsilon_c,\varepsilon_d)}\left[\max\left(V_j^c(z',k',b') + \varepsilon_c, V_j^d(z',k',b') + \varepsilon_d\right)\right]$$

where $V_R^d(z',k',b') = V_R^x(z',k',b')$ and $V_B^d(z',k',b') = 0$. In this case, conditional on (z',k',b'), the probability a firm of type j will continue or exit/default is given by

$$p_j^c(z',k',b') = \frac{\exp\left(V_j^c(z',k',b')/\sigma\right)}{\exp\left(V_j^c(z',k',b')/\sigma\right) + \exp\left(V_j^d(z',k',b')/\sigma\right)}$$

$$p_j^d(z',k',b') = \frac{\exp\left(V_j^d(z',k',b')/\sigma\right)}{\exp\left(V_j^c(z',k',b')/\sigma\right) + \exp\left(V_j^d(z',k',b')/\sigma\right)}$$

The corresponding continuation value can then be written as

$$\tilde{V}_j(z',k',b') = \mu + \gamma\sigma + \sigma\ln\left(\exp\left(V_j^c(z',k',b')/\sigma\right) + \exp\left(V_j^d(z',k',b')/\sigma\right)\right)$$

where $\gamma \approx 0.577$ is Euler's constant.

Given the extreme value shocks, the bond pricing function for public firms can now be written as

$$q_B(z,k',b') = \frac{1}{1+r} E_{z'|z}\left[R^d(z',k',b') p_B^d(z',k',b') + R^{nd}(z',k',b') p_B^c(z',k',b')\right] \tag{14}$$

[6] Note that using Equations (12) and (13) directly would cause numerical overflow. Without loss of generality, assume $\bar{V}_j^1(z,k,b)$ is the largest value. Then we can re-write Equation (12) so that $p_j^m = \exp((\bar{V}_j^m - \bar{V}_j^1)/(\rho\sigma))/\sum_{n=1}^{\tilde{N}} \exp((\bar{V}_j^n - \bar{V}_j^1)/(\rho\sigma))$. A similar normalization can be used to compute $V_j(z,k,b)$ in a way that does not cause numerical overflow.

where

$$R^d(z',k',b') \equiv \frac{1}{b'}\left[\pi(z',k') - T_c\left[\pi(z',k') - \delta k'\right] + (1-\xi)(1-\delta)k' - g(k',0)\right]$$

$$R^{nd}(z',k',b') \equiv \theta_B + c + (1-\theta_B)\left[(1-\pi)\sum_{n=1}^{N} p_B^n(z',k',b')q_B(z',k'_n,b'_n) + \pi\right]$$

Conditional on tomorrow's productivity z', a firm will default with probability $p_B^d(z',k',b')$ and the lender will recover $R^d(z',k',b')$. Meanwhile, with probability $p_B^c(z',k',b')$, the firm will continue and not default, and the lender will recover $R^{nd}(z',k',b')$. In non-default states, the amount the lender recovers tomorrow is dependent on the firm's choices for (k',b') tomorrow, which is now random and depends on tomorrow's extreme value shocks.

G Structural Estimation Method

We used SMM to estimate a total of $p = 9$ parameters by matching a set of $m = 11$ moments. The $p = 9$ parameters can be presented by the vector

$$\Theta = [\rho_z, \sigma_{\varepsilon z}, c_o, \mu_\chi, \sigma_\chi, \psi, \phi_d, \gamma, \phi_k].$$

Let $M^m(\Theta)$ denote the $m \times 1$ vector of model-generated moments and M^d their empirical counterparts. We define a loss function

$$L_W(\Theta) = \left[M^d - M^m(\Theta)\right]' W \left[M^d - M^m(\Theta)\right]$$

where W is a $m \times m$ positive-definite weighting matrix. The estimator, $\hat{\Theta}$, minimizes the loss function:

$$\hat{\Theta} = \arg\min_{\Theta} L_W(\Theta). \tag{15}$$

We used a total of $m = 11$ moments as part of the estimation: persistence of log sales, standard deviation of the innovation to log sales, average and standard deviation of credit spreads, the elasticity of credit spreads with respect to TFP and age, average leverage for small firms (bottom 90% in terms of assets), average leverage of Compustat firms, the standard deviation of investment rates in Compustat, the investment share of Compustat firms relative to aggregate investment from national accounts, and finally the relative employment of new firms as computed from the Business Dynamics Statistics (BDS).

We freely picked the weighting matrix W. In particular, we assume the off-diagonal elements are all zero. For the diagonal elements, we assume $W_{ii} = w_i/(m_i^d)^2$, where m_i^d is data moment i and w_i is an additional weight we place on moment i in the estimation. We use the inverse of the data target squared to correct for scaling issues. Furthermore we choose the weights $\{w_i\}_{i=1}^{11}$ to place extra weight on some specific dimensions of the data we view as more important to match. Specifically, for mean credit spreads we chose a weight of 20. For the elasticities of credit spreads with respect to TFP and age, we chose a weight of 5. For the standard deviation of credit spreads, we chose a weight of 5. For the investment share of public firms, we chose a weight of 15. For all other moments, we set the weight to 1.[7]

To compute the standard errors, we construct an estimate of $S = \lim_{N\to\infty} \text{Var}\left(\sqrt{N} M^d\right)$, an $m \times m$ matrix, which is the asymptotic variance-covariance matrix of the data moments. We estimated S using simulated model data. Then, we constructed a $m \times p$ gradient matrix, G, where $G_{ij} = \partial M_i^m(\Theta)/\partial\Theta_j$ is an estimate of the derivative of moment i in the model with respect to parameter j. We then construct the following variance-covariance matrix:

$$V = \left(1 + \frac{1}{\tau}\right) (G'WG)^{-1} G'WSWG (G'WG)^{-1}$$

where $\tau = N_s/N$, N_s is the number of observations in the simulation and N is the number of data observations. We then computed the standard error for parameter Θ_j as $\sqrt{V_{jj}/N}$. Since we used a selection of moments from different sources, to be conservative, we used $N = 4151$, the number of observations in the merged Compustat-Thomson Reuters sample used in the calculation of the credit-spread TFP elasticity.

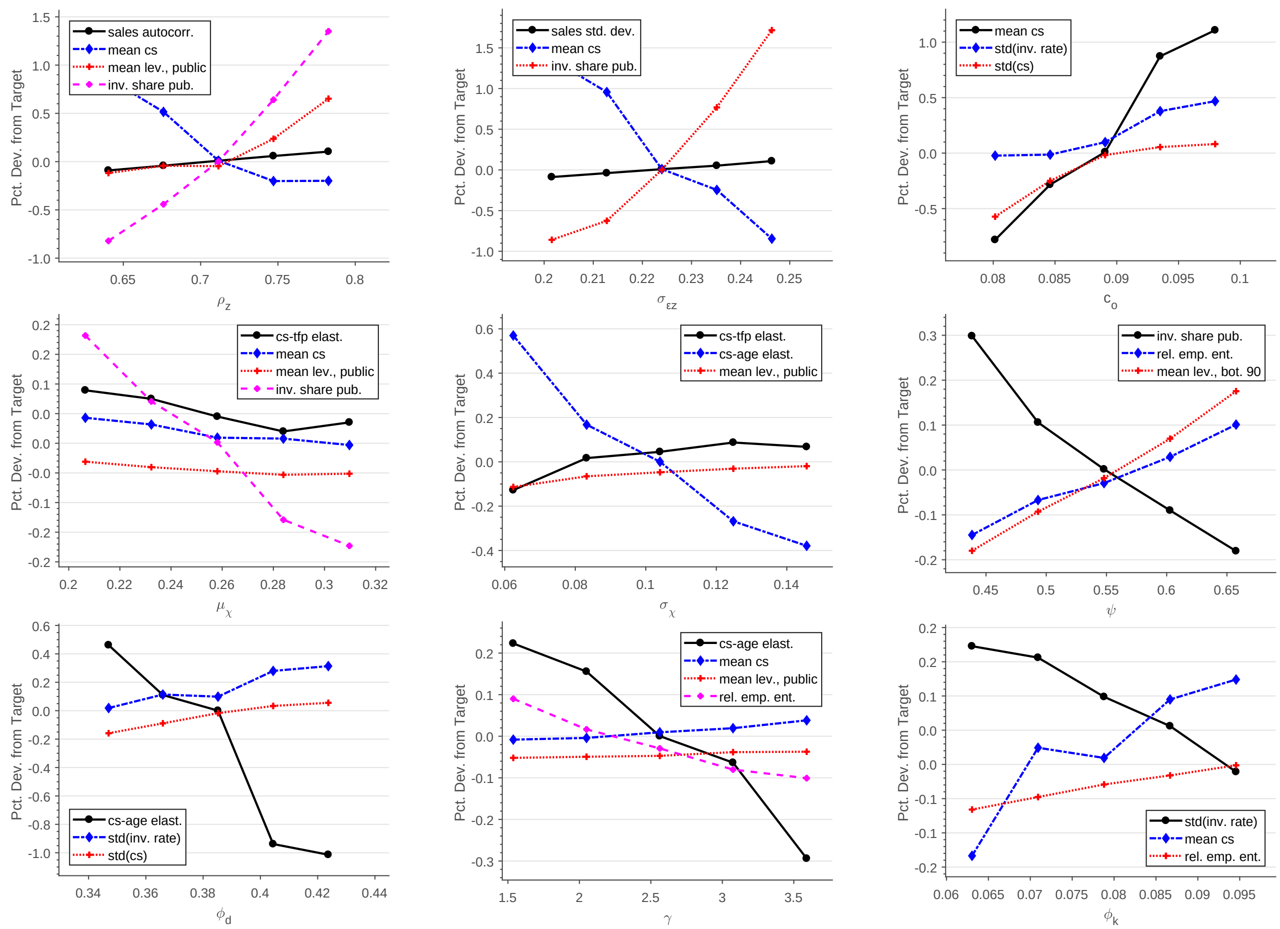
To make identification more transparent, it is useful to determine the moments which are the most responsive to each parameter. For this purpose, Figure 1 plots how a selected subset of moments vary with each of the estimated parameters, $(\rho_z, \sigma_{\varepsilon z}, c_o, \mu_\chi, \sigma_\chi, \psi, \phi_d, \gamma, \phi_k)$. While all the parameters will jointly affect all the moments, we highlight a few moments for each parameter. Meanwhile, in Figure 2, we plot how the total error, $L_W(\Theta)$, varies with each of the estimated parameters.

H Robustness

We re-estimate the model, but make alternative assumptions about the calibrated parameters. Table 8 reports the results. The top two panel reports resulting estimated parameters and the chosen calibrated parameters. The middle panel shows the effect on targeted and untargeted moments. Finally, the bottom panel reports the effect on TFP and TFP loss from removing financial frictions from these alternative economies. We consider six alternative calibrations: (i) public firms issue one-period bonds (i.e., $\theta_B = 1$), (ii) smaller bond premium

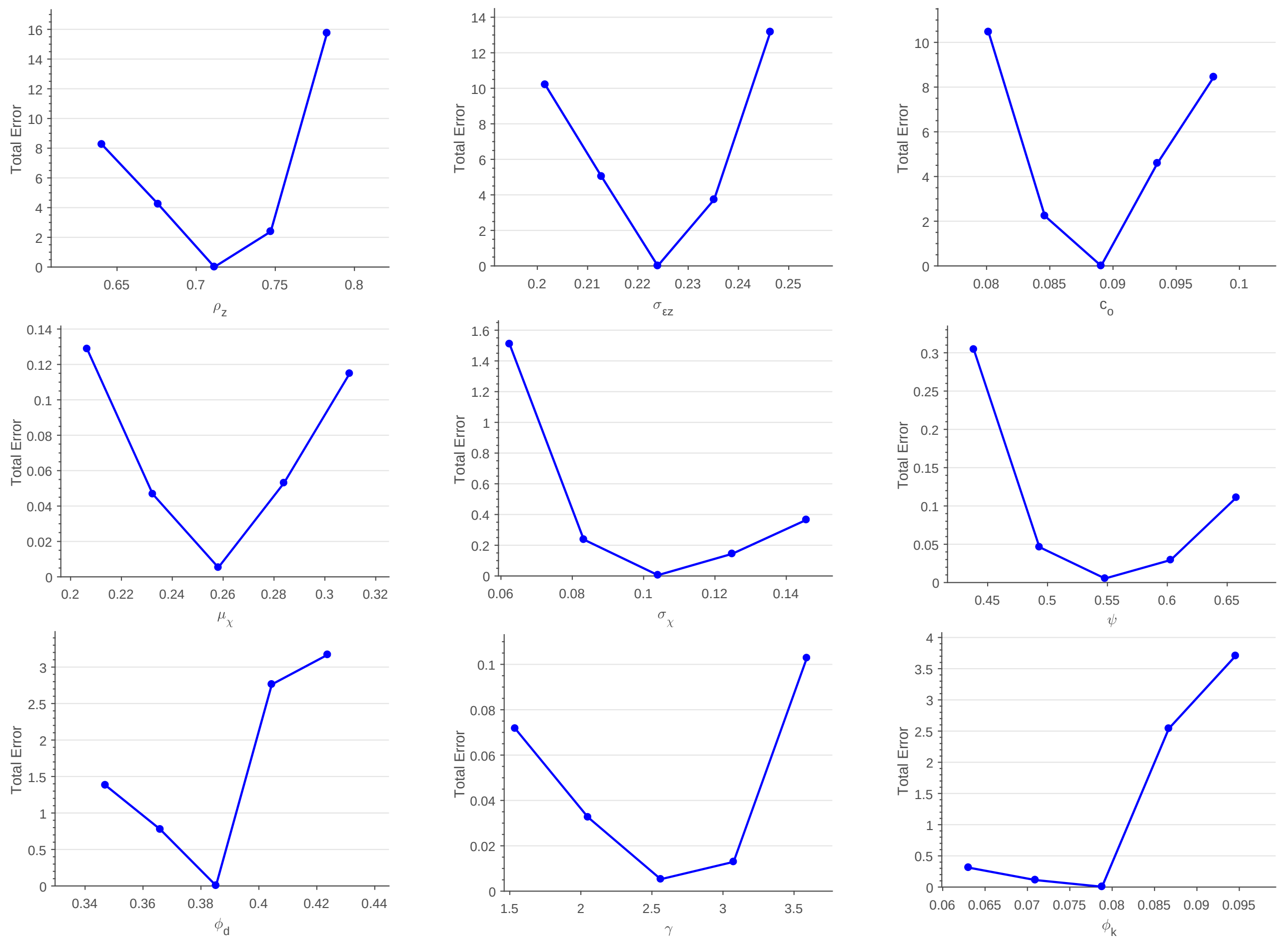
[7] We then also normalized the weights so that $\sum_{i=1}^{11} w_i = 1$.

Figure 1: Effect of Parameters on Selected Model Moments



Note: We plot how a selected subset of targeted moments varies with each of the 9 estimated parameters, $(\rho_z, \sigma_{\varepsilon z}, c_o, \mu_\chi, \sigma_\chi, \psi, \phi_d, \gamma, \phi_k)$. For each moment, we plot the percentage deviation from the target. A total of 11 moments were used: (1) the autocorrelation of log sales ("sales autocorr."), (2) the standard deviation of the innovation to log sales ("sales std. dev."), (3) mean credit spreads ("cs mean"), (4) standard deviation of credit spreads ("std(cs)"), (5) the elasticity of credit spreads with respect to TFP ("cs-tfp elast."), (6) the elasticity of credit spreads with respect to age ("cs-age elast."), (7) mean leverage of firms in the bottom 90% of the asset distribution ("mean lev., bot. 90"), (8) mean leverage for public firms ("mean lev., pub."), (9) standard deviation of investment rates for public firms ("std(inv. rate)"), (10) investment share of public firms ("inv. share pub."), and (11) the relative employment of entrants ("rel. emp. ent.").

Figure 2: Effect of Parameters on Loss Function



Note: The estimated parameters $\Theta = (\rho_z, \sigma_{\varepsilon z}, c_o, \mu_\chi, \sigma_\chi, \psi, \phi_d, \gamma, \phi_k)$ were chosen to minimize the total error or loss, defined to be $L_W(\Theta) = \left[M^d - M^m(\Theta)\right]' W \left[M^d - M^m(\Theta)\right]$. $M^m(\Theta)$ denotes the vector of model-generated moments, M^d denotes their empirical counterparts, and W is a weighting matrix. This figure shows how this loss function varies with each of the estimated parameters.

for private firms ($r_p = 0.75\%$), (iii) lower returns to scale ($\nu = 0.75$), (iv) lower capital share ($\alpha = 0.30$), (v) lower exogenous exit rate ($\pi = 1\%$), and (vi) no taxes ($\tau_i = \tau_c^- = \tau_c^+ = 0$).

In addition, Table 9 reports the results when we re-estimate the model, but constrain some of the estimated parameters to be lower. Specifically, we consider five alternative models: (i) lower capital adjustment costs ($\phi_k = 0.04$), (ii) homogeneous bond entry costs ($\sigma_\chi = 0$), (iii) tighter collateral constraint for private firms ($\psi = 0.45$), (iv) cheaper equity for private firms ($\gamma = 0$), and (v) no private firms, where all new entrants are born as public firms.

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Table 8: Alternative Models 1: Estimated Parameters and Model Fit

	One Period Bonds	Smaller Bond Premium	Lower Returns to Scale	Lower Capital Share	Lower Exog. Exit	No Taxes
Estimated Parameters:						
ρ_z	0.71	0.73	0.72	0.71	0.71	0.71
$\sigma_{\varepsilon z}$	0.22	0.22	0.27	0.22	0.22	0.22
c_o	0.10	0.09	0.11	0.07	0.11	0.14
μ_χ	0.82	0.26	0.33	0.28	0.35	0.33
σ_χ	0.11	0.13	0.10	0.10	0.10	0.09
ψ	0.54	0.58	0.50	0.57	0.56	0.55
γ	2.49	2.44	2.39	2.53	2.26	3.86
ϕ_d	0.42	0.39	0.40	0.38	0.42	0.27
ϕ_k	0.11	0.08	0.08	0.08	0.08	0.08
Calibrated Parameters:						
θ_B	**1.00**	0.086	0.086	0.086	0.086	0.086
r_p	1.5%	**0.75%**	1.5%	1.5%	1.5%	1.5%
ν	0.85	0.85	**0.75**	0.85	0.85	0.85
α	0.35	0.35	0.35	**0.30**	0.35	0.35
π	2%	2%	2%	2%	**1%**	2%
τ_i	0.296	0.296	0.296	0.296	0.296	**0**
τ_c^-	0.2	0.2	0.2	0.2	0.2	**0**
τ_c^+	0.35	0.35	0.35	0.35	0.35	**0**
Selected Statistics:						
Mean credit spreads	0.06%	2.28%	2.42%	2.30%	2.17%	2.33%
SD of credit spreads	0.38%	1.99%	1.83%	1.87%	2.35%	1.98%
Credit spread-TFP elast.	-3.96	-0.39	-0.71	-0.38	-0.80	-0.54
Credit spread-Age elast.	-0.75	-0.51	-0.15	-0.28	-0.63	-0.51
Mean lev. [0-90]	0.51	0.42	0.37	0.35	0.33	0.36
Mean lev., public firms	0.70	0.36	0.39	0.32	0.24	0.33
SD of inv. rates, public firms	0.27	0.25	0.22	0.22	0.21	0.25
Inv. share of public firms	0.28	0.30	0.31	0.30	0.27	0.28
Sales autocorr.	0.76	0.78	0.73	0.74	0.76	0.77
Sales std. dev.	0.50	0.49	0.52	0.55	0.50	0.50
Rel. emp. of firms, age 0	0.24	0.35	0.47	0.42	0.28	0.31
Rel. emp. of firms, age 1-5	0.79	0.63	0.75	0.70	0.67	0.62
Rel. emp. of firms, age 6-10	1.10	0.90	0.95	0.93	0.95	0.90
Rel. emp. of firms, age 11-15	1.16	1.01	1.01	1.02	1.02	1.01
Rel. emp. of firms, age 16-20	1.18	1.06	1.04	1.06	1.06	1.06
Rel. emp. of firms, age 21-25	1.19	1.11	1.07	1.09	1.08	1.11
Mean capital/sales ratio	3.08	2.95	2.66	2.77	3.24	3.22
Frequency of equity issuance	0.37	0.37	0.32	0.38	0.21	0.31
Mean default rate	0.99%	3.10%	2.78%	2.73%	2.86%	2.67%
Credit spread-leverage elast.	5.76	0.31	0.43	0.53	0.05	0.35
SD of leverage	0.20	0.19	0.18	0.16	0.20	0.19
Autocorr. of inv. rates	0.38	0.26	0.28	0.26	0.25	0.27
Effect of Removing Financial Frictions:						
TFP Loss	-5.92%	-3.11%	-2.16%	-2.27%	-5.11%	-4.44%
TFP	11.08%	4.59%	4.66%	3.51%	11.25%	7.69%

Note: We re-estimate the model, making alternative assumptions about calibrated parameters. We highlight in bold the calibrated parameters which we fix to values different from the benchmark model.

Table 9: Alternative Models 2: Estimated Parameters and Model Fit

	Lower Cap. Adj. Cost	Homog. Bond Entry Cost	Tighter Coll. Const.	Cheaper Equity Priv. Firms	No Private Firms
Estimated Parameters:					
ρ_z	0.70	0.70	0.71	0.71	0.71
$\sigma_{\varepsilon z}$	0.22	0.22	0.22	0.22	0.21
c_o	0.09	0.09	0.09	0.09	0.08
μ_χ	0.27	0.04	0.27	0.15	-
σ_χ	0.10	**0.00**	0.11	0.11	-
ψ	0.58	0.52	**0.45**	0.60	-
γ	2.55	1.70	2.68	**0.00**	-
ϕ_d	0.38	0.53	0.37	0.40	0.39
ϕ_k	**0.04**	0.08	0.08	0.09	0.10
Selected Statistics:					
Mean credit spreads	2.10%	2.42%	2.42%	2.31%	4.02%
SD of credit spreads	1.88%	2.51%	2.19%	2.19%	1.91%
Credit spread-TFP elast.	-0.31	-0.81	-0.29	-0.40	-1.27
Credit spread-Age elast.	-0.35	-0.51	-0.57	-0.49	-0.92
Mean lev. [0-90]	0.36	0.40	0.30	0.34	0.35
Mean lev., public firms	0.32	0.40	0.28	0.28	0.36
SD of inv. rates, public firms	0.29	0.36	0.24	0.23	0.26
Inv. share of public firms	0.28	0.29	0.33	0.31	1.00
Sales autocorr.	0.76	0.82	0.75	0.75	0.76
Sales std. dev.	0.50	0.51	0.50	0.51	0.46
Rel. emp. of firms, age 0	0.33	0.30	0.31	0.51	0.63
Rel. emp. of firms, age 1-5	0.65	0.67	0.63	0.76	0.87
Rel. emp. of firms, age 6-10	0.92	0.96	0.93	0.94	1.02
Rel. emp. of firms, age 11-15	1.02	1.05	1.04	1.00	1.12
Rel. emp. of firms, age 16-20	1.06	1.09	1.10	1.05	1.20
Rel. emp. of firms, age 21-25	1.09	1.11	1.15	1.09	1.24
Mean capital/sales ratio	2.92	3.16	2.85	2.93	2.29
Frequency of equity issuance	0.35	0.02	0.34	0.32	0.44
Mean default rate	2.82%	3.38%	3.39%	3.45%	7.80%
Credit spread-leverage elast.	0.26	0.14	0.25	0.21	0.03
SD of leverage	0.20	0.26	0.10	0.20	0.21
Autocorr. of inv. rates	0.20	0.36	0.25	0.23	0.29
Effect of Removing Financial Frictions:					
TFP Loss	-3.31%	-4.49%	-4.07%	-2.45%	-1.97%
TFP	6.43%	10.01%	8.82%	4.04%	3.39%

Note: We re-estimate the model under alternative assumptions. We highlight in bold any estimated parameters which we fix to values different from the benchmark model. In the “No Private Firms” scenario, we assume all new firms are born as public firms.