Preventing Self-fulfilling Debt Crises: A Global Games Approach

Online Appendix

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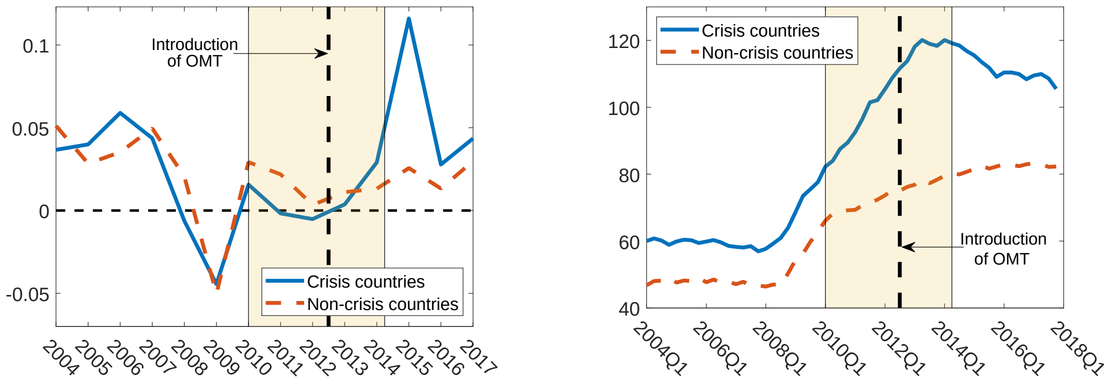
Contents

1 Additional Evidence

In this section, I compare growth of GDP per capita and level of debt-over GDP ratio for Ireland, Italy, Portugal, and Spain, with that for Finland, France, and the UK. 1 further highlights that economic fundamentals of Italy, Spain, Slovenia were similar to the economic fundamentals of Finland, France and UK.



(a) Growth of GDP per capita (b) Debt-to-GDP ratio

Figure 1: Growth of GDP per capita and the debt-to-GDP ratio for Italy, Spain, and Slovenia (countries trapped in a “bad equilibrium”) and for Finland, France, the UK (countries in a “good equilibrium”) during the period 2004Q1–2017Q4. The shaded area marks the period of the European debt crisis. All series depict unweighted averages. Source: ECB Statistical Warehouse.

The left panel of Figure 1 depicts the average growth of GDP per capita for Ireland, Italy, Portugal, and Spain (solid blue curve) and Finland, France, and the UK (red dashed curve). It shows that while the recovery from the Great Recession was slower in Ireland, Italy, Portugal, and Spain than in Finland, France and UK, the latter set of countries also suffered from low GDP growth for a prolonged period of time.

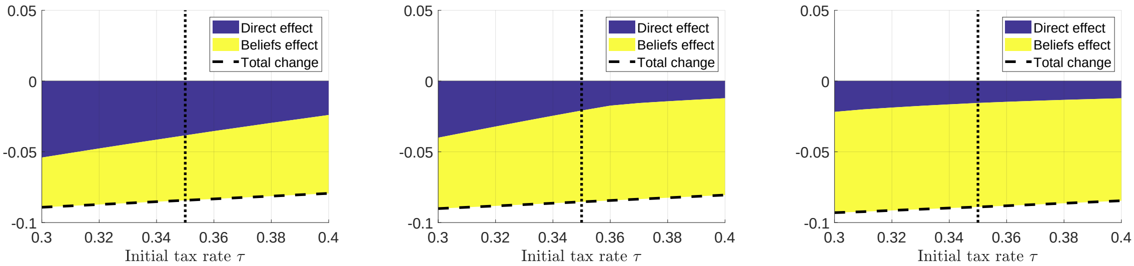
The right panel of Figure 1 depicts the average level of debt-to-GDP for Ireland, Italy, Portugal, and Spain (solid blue line) and Finland, France and UK (red dashed line). It shows that up until 2012, the debt-to-GDP ratios for these two groups of countries were similar. Despite these similarities, Ireland, Italy, Portugal, and Spain experienced a large increase in their cost of borrowing during the period 2010–2012 while the cost of borrowing for Finland, France, and the UK changed very little during that period (as shown in Figure 1 in the paper).

2 Robustness of Numerical Results

2.1 Robustness of Results to Changes in σ_x and ε

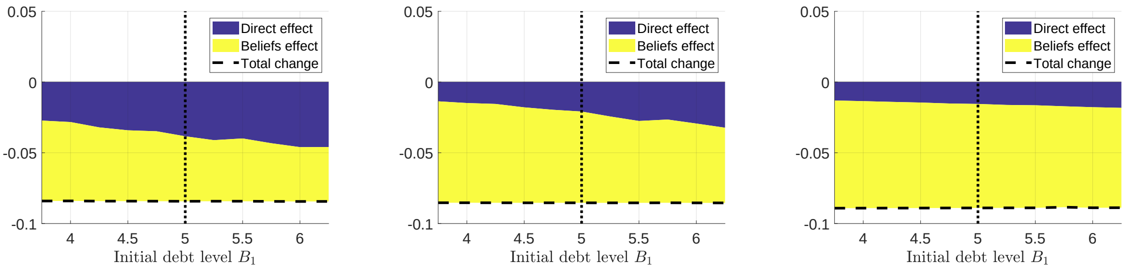
To choose values of σ_x and ε, I followed the approach outlined in Schaal and Taschereau-Dumouchel (2016) and chose these parameters to match the interquartile range of current GDP growth for Europe forecast by professional forecasters. One objection one may have to this approach is that the professional forecasters provide forecasts for Europe as a whole, and the dispersion of their beliefs about GDP growth for Europe may differ from the dispersion of their beliefs about GDP growth for of Italy, Slovenia, and Spain. Therefore, in this section I investigate how my results depend on the choice of σ_x and ε.

Figures 2–5 compare the results of the benchmark calibration with calibrations where σ_x is set to a lower value than in the benchmark calibration ($\sigma_x = 0.000831$) and where σ_x is set to a higher value than in the benchmark calibration ($\sigma_x = 0.0042$). The low σ_x is equal to the benchmark σ_x divided by $\sqrt{5}$, which implies that signals are five times as precise as in the benchmark calibration.[1] Similarly, the high σ_x is equal to the benchmark σ_x multiplied by $\sqrt{5}$, which implies that signals are one-fifth as precise as in the benchmark calibration. As usual, I set $\varepsilon = \sqrt{3}\sigma_x$, so that households and lenders' signals have the same precision.



(a) Low σ_x (b) Benchmark σ_x (c) High σ_x

Figure 2: Change in the probability of default following a 1% increase in the tax rate for different initial values of τ centered around the calibrated value of τ (indicated by the vertical dotted line). All other parameters are chosen as in the benchmark calibration.

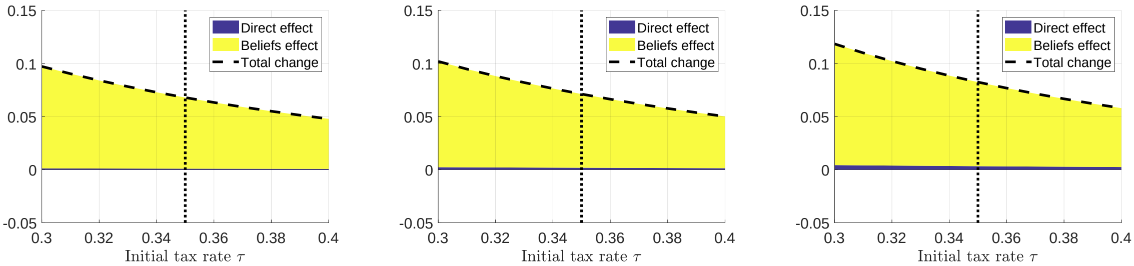


(a) Low σ_x (b) Benchmark σ_x (c) High σ_x

Figure 3: Change in the probability of default following a 1% increase in the tax rate for different initial values of $B1$ centered around the calibrated value of $B1$ (indicated by the vertical dotted line). All other parameters are chosen as in the benchmark calibration.

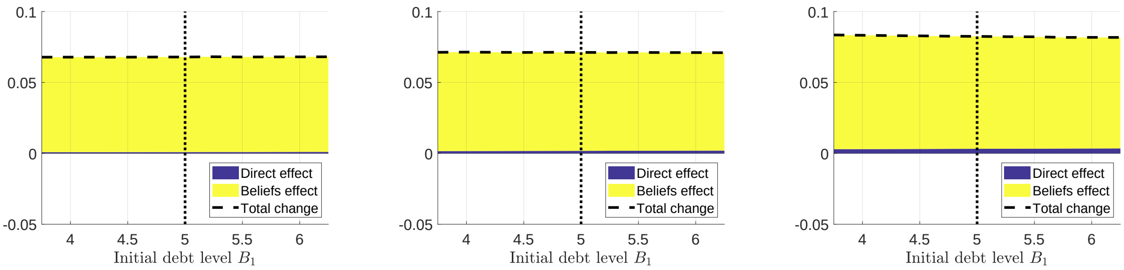
The above figures show that as σ_x increases, a tax hike leads to a to somewhat larger decline in the probability of default, while the opposite is true for a stimulus. Moreover, as σ_x increases, the role of the beliefs effect decreases. The reason for this is that when σ_x is high, the default threshold is more

[1] Recall that precision is defined as the reciprocal of the variance of the signals.



(a) Low σ_x (b) Benchmark σ_x (c) High σ_x

Figure 4: Change in the probability of default following a stimulus equal to 1for different initial values of τ centered around the calibrated value of τ (indicated by the vertical dotted line). All other parameters are chosen as in the benchmark calibration.



(a) Low σ_x (b) Benchmark σ_x (c) High σ_x

Figure 5: Change in the probability of default following a stimulus equal to 1for different initial values of B_1 centered around the calibrated value of B_1 (indicated by the vertical dotted line). All other parameters are chosen as in the benchmark calibration.

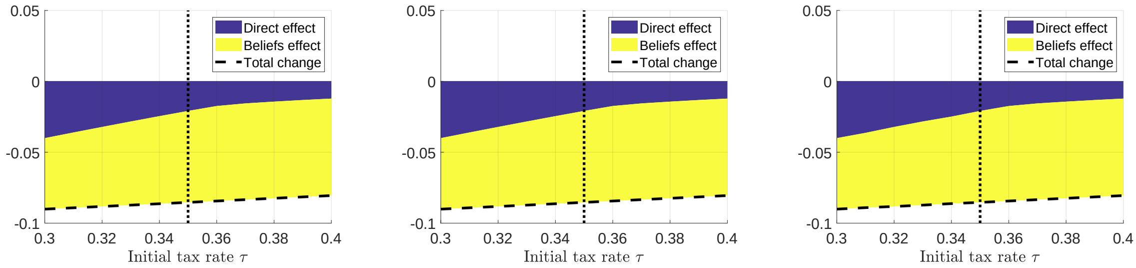
sensitive to changes in the interest rate for relatively high A_{-1}, and a relatively high A_{-1} is needed to ensure a 10% ex-ante probability of default. The high sensitivity of the default threshold to changes in r means that the government chooses a higher r when σ_x is high—and as a consequence it borrows more in equilibrium. But as explained in the paper, the importance of the beliefs effect is high when the government borrows more.

Nevertheless, the above figures show that changes is σ_x and ε do not affect the results dramatically, and the same conclusion holds regardless of the values chosen for σ_x and ε.

2.2 Robustness of Results to Changes in the Initial Debt Level (B_1)

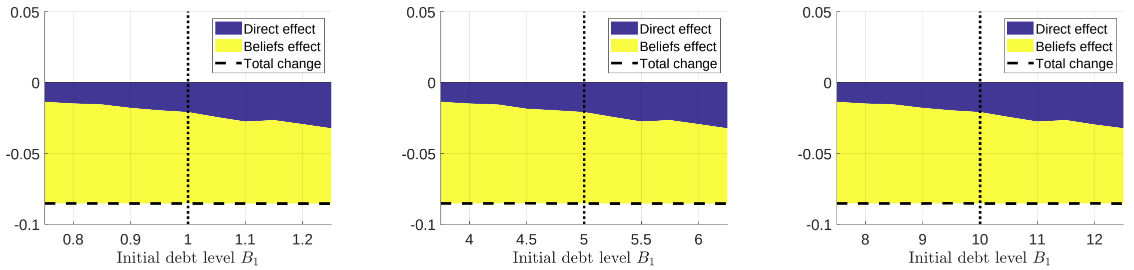
In the benchmark calibration, I set $B_1 = 5$. In this section, I show that the results are not affected when I consider different values of B_1. I then explain why B_1 cannot be chosen to match to natural targets (that is, debt-to-GDP and debt-to-capital ratios). It turns out that once all the other parameters are chosen, the model is invariant to changes in B_1. In other words, the model scales up or down as B_1 increases or decreases, and it does so in a such a way that all the results are left unchanged.

Figures 6–9 compare the results of the benchmark calibration with calibrations where B_1 is set to a lower value than in the benchmark calibration ($B_1 = 1$) and where B_1 is set to a higher value than in the benchmark calibration ($B_1 = 10$). As can be readily judged from these figures, changing the initial value of B_1 does not affect the results: The figures are identical regardless of B_1. This is because when all other parameters of the model are set to match the target values, varying B_1 leaves the debt-to-GDP and debt-to-capital ratios unchanged. In other words, the model scales up and down with the choice of B_1. For more details, see Section 4.2 of the Online Appendix. I explain why this is the case below



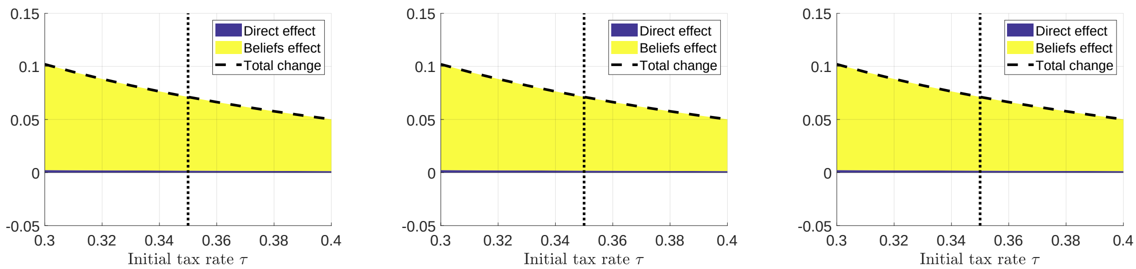
(a) Low B_1 (i.e., $B_1 = 1$) (b) Benchmark B_1 (i.e., $B_1 = 5$) (c) High B_1 (i.e., $B_1 = 10$)

Figure 6: Change in the probability of default following a 1% increase in the tax rate for different initial values of τ centered around the calibrated value of τ (indicated by the vertical dotted line). All other parameters are chosen as in the benchmark calibration.



(a) Low B_1 (i.e., $B_1 = 1$). (b) Benchmark B_1 (i.e., $B_1 = 5$). (c) High B_1 (i.e., $B_1 = 10$).

Figure 7: Change in the probability of default following a 1% increase in the tax rate for different initial values of τ centered around the calibrated value of τ (denotted by the vertical dotted line). The left panel depicts the results for low B_1, the middle panel depicts the results for the benchmark B_1, and the right panel depicts the results for high B_1. All other parameters are chosen as in the benchmark calibration.



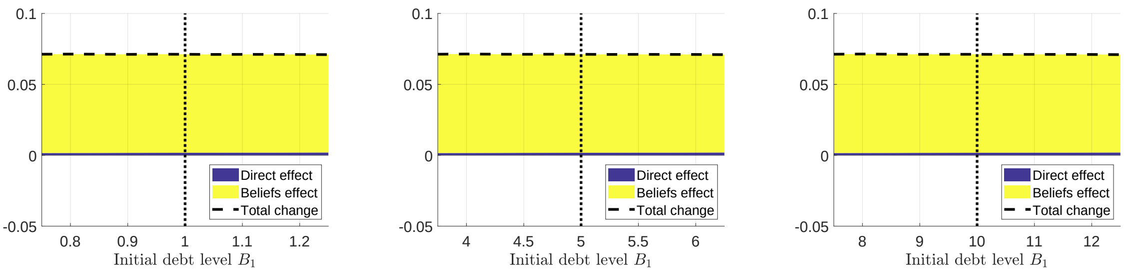
(a) Low B_1 ($B_1 = 1$). (b) Benchmark B_1 ($B_1 = 5$). (c) High B_1 ($B_1 = 10$).

Figure 8: Change in the probability of default following a stimulus equal to 1 for different initial values of τ centered around the calibrated value of τ (indicated by the vertical dotted line). All other parameters are chosen as in the benchmark calibration.

(Sections 2.2.1 and 2.2.2), where I argue that when all other parameters of the model are set to match the target values, varying B_1 leaves both the debt-to-GDP and debt-to-capital ratios unchanged. In other words, the model scales up and down with the choice of B_1.

2.2.1 Invariance of debt-to-GDP ratio to changes in B_1

In this section, I explain why in the model the debt-to-GDP ratio is essentially invariant to changes in B_1 once other parameters are chosen to match their respective targets. In what follows, I will denote by



(a) Low B_1 $(B_1 = 1)$.

(b) Benchmark B_1 $(B_1 = 5)$.

(c) High B_1 $(B_1 = 10)$.

Figure 9: Change in the probability of default following a stimulus equal to 1initial values of B_1 centered around the calibrated value of B_1 (denotted by the vertical dotted line). All other parameters are chosen as in the benchmark calibration.

g_k the target growth rate of capital between periods 1 and 2.

Note that the upper bound of the fragility region is given by[2]

$$\overline{A} = \exp\left(\frac{B_1}{\tau f(k_1)\left(1 - \frac{Z^4}{1+\alpha-\alpha Z}\right)}\right)$$

On the other hand, the lower bound of the fragility region, $\underline{A}(r)$ is bounded from below by

$$\exp\left(\frac{B_1}{2(1+g_k)^{\alpha}\tau f(k_1)\left(1 - \sqrt{Z^3(Z(1+\alpha)-\alpha)}\right)}\right)$$

Thus, once we set the parameters of the model, the debt-to-GDP ratio, B_1/Y_1, satisfies

$$\frac{B_1}{Y_1} \in \left[\tau\left(1 - \frac{Z^4}{1+\alpha-\alpha Z}\right), 2(1+g_k)^{\alpha}\tau\left(1 - \sqrt{Z^3(Z(1+\alpha)-\alpha)}\right)\right]$$

since $Y_1 = e^A f(k_1)$. In that sense, once we fix all the parameters the debt-to-GDP ratio is determined to lie in the above interval. For the calibration used in the current version of the manuscript, this corresponds to

$$\frac{B_1}{Y_1} \in [0.128, 0.142]$$

2.2.2 Invariance of debt-to-capital ratio to changes in B_1

In this section, I explain why in the model he debt-to-capital ratio is essentially invariant to changes B_1 once other parameters are chosen to match their respective targets. As in Section 2.2.1, I denote by g_k the target growth rate of capital between periods 1 and 2. To ease the exposition, I assume that households have perfect information when making their decisions.

[2] To compute $\overline{A}$, I assume that A is common knowledge, and I solve the government's indifference condition for A when the lenders refuse to lend to the government, so that $B_2 = 0$, and households expect default when choosing their investment, so that for all $i \in [0,1]$ we have $k_2^{D,i} = k_2^D, Z$ where $k_2^D = (1-\tau)e^A f(k_1)\alpha/(1+\alpha)$. That is, $\overline{A}$ is the unique solution to

$$V^R\left(A, 0, k_2^D, r\right) - V^D\left(A, 0, k_2^D, r\right) = 0$$

Since $B_2 = 0$, the interest rate does not affect the solution to the above equation.

As noted in Section 2.2.1, the upper bound of the fragility region is given by

$$\overline{A} = \exp\left(\frac{B_1}{\tau f(k_1)\left(1 - \frac{Z^4}{1+\alpha-\alpha Z}\right)}\right),$$

which implies that

$$B_1 = \tau f(k_1)\, e^{\overline{A}} \left(1 - \frac{Z^4}{1+\alpha-\alpha Z}\right)$$

Next, note that the households' investment at $\overline{A}$ is given by

$$k_2 = (1-\tau)\, e^{\overline{A}} f(k_1) \frac{Z\alpha}{1+\alpha}$$

Since $(1+g_k) = k_2/k_1$, it follows that

$$k_1 = (1-\tau)\, e^{\overline{A}} f(k_1) \frac{Z\alpha}{1+\alpha}(1+g_k)$$

Thus at $\overline{A}$ the ratio of debt to capital is equal to

$$\frac{B_1}{k_1} = \frac{\tau}{(1-\tau)}\left(1 - \frac{Z^4}{1+\alpha-\alpha Z}\right)\frac{1}{(1+g_k)}\frac{1+\alpha}{Z\alpha},$$

which is independent of the choice of k_1 and B_1.

A similar argument applies to the lower bound of the fragility region. This shows that once we fix all the parameters and also choose k_1 to match the target growth rate of capital g_k, then the capital-to-debt ratio within the model is uniquely pinned down.

3 Assumptions

In this section, I provide a detailed discussion of a number of special assumptions that I made in the paper.

3.1 Uncertainty

In the baseline model, I assumed that the government's policy is the only source of uncertainty affecting the households' investment decision. In this section, I analyze the model when this assumption is relaxed.

3.1.1 Additional Uncertainty in the Model

There are many ways to introduce additional uncertainty into the households' problem. In this section, I assume that each household's productivity is equal to A, the average level of productivity in the economy. However, unlike the benchmark model, households now do not observe their productivity when they make their investment decisions.[3] Instead, household i observes A_i, a noisy signal about A, where

$$A_i = A + \varepsilon_i,$$

with ε_i *i.i.d.* across households and uniformly distributed on $[-\varepsilon, \varepsilon]$, $\varepsilon > 0$. Thus in contrast to the model in the paper, here households face uncertainty about their income today and tomorrow, in addition to the uncertainty about the government's policies.

The households' problem can now be written as

$$\begin{aligned} \max_{k_2^i} \; E & \left[\sum_{t=1,2} \left[\log\left(c_t^i\right) + \log\left(g_t\right) \right] \middle| A_i, \boldsymbol{\sigma} \right] \\ s.t. \; c_1^i &= (1-\tau) Z^{d_1} e^A f(k_1) - k_2^i \\ c_2^i &= (1-\tau) Z^{d_1} e^A f\left(k_2^i\right), \end{aligned}$$

where $\boldsymbol{\sigma} = \{\mathbf{k}_2, \boldsymbol{\beta}, r, d_1, g_1, g_2, B_2\}$ is the strategy profile of all players, and the expectations are taken over the government's default decision and productivity A. Household i forms its expectations based on its private signal, A_i, and the prior belief, $A \sim N(A_{-1}, \sigma_A)$.

The main issue that complicates the analysis is that now the households' problem does not have a closed-form solution. In particular, household i's optimal investment is defined only implicitly by the first-order condition for the above problem. In contrast, in the baseline model, household i's FOC can be solved explicitly to obtain a closed-form expression for the optimal investment.

3.1.2 Small uncertainty and relation to the benchmark model

Analytical results are in general impossible to obtain once additional uncertainty is introduced into the household's problem. However, I show below that as the additional uncertainty disappears, that is as $\varepsilon \to 0$, the solution of the above model converges to the equilibrium of the benchmark model when dispersion of households' idiosyncratic productivity becomes vanishingly small. It follows that as long as ε is small, the difference between the two versions of my model will be small.

[3] The assumption that households share the same productivity is made for simplicity. I could assume that, as in the benchmark model, households have different productivities, in which case household i would observe a signal

$$\widehat{A}_i = A_i + \eta_i,$$

where η_i is distributed according to a CDF G, η_i is *i.i.d.* across households, and is independent of A_i.

Lemma 1 *Consider the model where households face uncertainty about their future income and let A^* be the default threshold in that model. Suppose that the noise in households's signal vanishes, that is $\varepsilon \to 0$. Then, households choice of capital for period 2 converges to*

$$k_2(A_i;\kappa) = \begin{cases} \frac{\alpha}{1+\alpha}(1-\tau)e^A f(k_1) & \text{if } A_i > A^* \\ (1-\tau)e^A f(k_1)\widetilde{\Lambda}(\kappa) & \text{if } A_i = A^* \\ \frac{Z\alpha}{1+\alpha}(1-\tau)e^A f(k_1) & \text{if } A_i < A^* \end{cases} \tag{1}$$

where

$$\widetilde{\Lambda}(A_i) = \frac{(1+\kappa)Z + (1-\kappa) + 2\alpha(1+Z) - \sqrt{[(1+\kappa)Z + (1-\kappa) + 2\alpha(1+Z)]^2 - 16\alpha(1+\alpha)Z}}{4(1+\alpha)} \tag{2}$$

and $\kappa = \frac{A_i - A}{\varepsilon}$.

Proof. To establish the above results, it is convenient to make the following change of the variables $\kappa = \frac{\varepsilon_i}{\varepsilon}$, where $\varepsilon_i \in [-\varepsilon, \varepsilon]$ so that $\kappa \in [-1,1]$. It follows that $A_i = A + \varepsilon_i = A + \kappa\varepsilon$. Then the FOC for household i's choice of capital can be written as

$$\int_{A^*}^{A+(1+\kappa)\varepsilon} \frac{1}{(1-\tau)e^A f(k_1) - k_2} p(A|A_i)\, dA + \int_{A-(1-\kappa)\varepsilon}^{A^*} \frac{1}{Z(1-\tau)e^A f(k_1) - k_2} p(A|A_i)\, dA = \frac{\alpha}{k_2},$$

where

$$p(A|A_i) = p(A|A+\kappa\varepsilon) = \frac{\phi\left(\frac{A-\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{(A+\kappa\varepsilon)+\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{(A+\kappa\varepsilon)-\varepsilon-A_{-1}}{\sigma_A}\right)}$$

Note that when $A > A^*$ then there exists sufficiently small $\bar{\varepsilon}(A)$ such that for all $\varepsilon < \bar{\varepsilon}(A)$ all households know that $A > A^*$. In that case the above FOC becomes

$$\int_{A-(1-\kappa)\varepsilon}^{A+(1+\kappa)\varepsilon} \frac{1}{(1-\tau)e^A f(k_1) - k_2} p(A|A_i)\, dA = \frac{\alpha}{k_2}$$

Using the definition of $p(A|A_i)$ we obtain

$$\frac{\int_{A-(1-\kappa)\varepsilon}^{A+(1+\kappa)\varepsilon} \frac{1}{(1-\tau)e^A f(k_1) - k_2} \frac{1}{\sigma_A}\phi\left(\frac{A-\varepsilon-A_{-1}}{\sigma_A}\right) dA}{\Phi\left(\frac{(A+\kappa\varepsilon)+\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{(A+\kappa\varepsilon)-\varepsilon-A_{-1}}{\sigma_A}\right)} = \frac{\alpha}{k_2}$$

Therefore,

$$\lim_{\varepsilon\to 0} \frac{\int_{A-(1-\kappa)\varepsilon}^{A+(1+\kappa)\varepsilon} \frac{1}{(1-\tau)e^A f(k_1) - k_2} \frac{1}{\sigma_A}\phi\left(\frac{A-\varepsilon-A_{-1}}{\sigma_A}\right) dA}{\Phi\left(\frac{(A+\kappa\varepsilon)+\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{(A+\kappa\varepsilon)-\varepsilon-A_{-1}}{\sigma_A}\right)} = \frac{1}{(1-\tau)e^A f(k_1) - k_2}$$

It follows that in the limit, the optimal investment by households is given by

$$k_2 = \frac{\alpha}{1+\alpha}(1-\tau)e^A f(k_1)$$

By the same argument, one can show that if $A < A^*$ then households' optimal investment converges to

$$k_2 = \frac{Z\alpha}{1+\alpha}(1-\tau)e^A f(k_1)$$

It remains to consider the case when $A = A^*$. In that case we have to consider each integral separately

$$= \lim_{\varepsilon \to 0} \frac{\int_{A^*}^{A^*+(1+\kappa)\varepsilon} \frac{1}{(1-\tau)e^A f(k_1) - k_2} \frac{1}{\sigma_A} \phi\left(\frac{A-\varepsilon-A_{-1}}{\sigma_A}\right) dA}{\Phi\left(\frac{(A^*+\kappa\varepsilon)+\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{(A^*+\kappa\varepsilon)-\varepsilon-A_{-1}}{\sigma_A}\right)} = \frac{1}{2} \frac{(1+\kappa)}{(1-\tau)\, e^{A^*} f(k_1) - k_2}$$

Similarly,

$$\lim_{\varepsilon \to 0} \frac{\int_{A^*-(1-\kappa)\varepsilon}^{A^*} \frac{1}{Z(1-\tau)e^A f(k_1) - k_2} \frac{1}{\sigma_A} \phi\left(\frac{A-\varepsilon-A_{-1}}{\sigma_A}\right) dA}{\Phi\left(\frac{(A^*+\kappa\varepsilon)+\varepsilon-A_{-1}}{\sigma_A}\right) - \Phi\left(\frac{(A^*+\kappa\varepsilon)-\varepsilon-A_{-1}}{\sigma_A}\right)} = \frac{1-\kappa}{2} \frac{1}{Z(1-\tau)\, e^{A^*} f(k_1) - k_2}$$

It follows that in the limit the F.O.C. becomes

$$\frac{1}{2} \frac{(1+\kappa)}{(1-\tau)\, e^{A^*} f(k_1) - k_2} + \frac{1}{2} \frac{1-\kappa}{Z(1-\tau)\, e^{A^*} f(k_1) - k_2} = \frac{\alpha}{k_2} \tag{3}$$

Solving Equation (3) for k_2, we obtain

$$k_2 = (1-\tau)\, e^{A^*} f(k_1)\, \widetilde{\Lambda}(\kappa)$$

where

$$\widetilde{\Lambda}(\kappa) \equiv \frac{(1+\kappa)Z + (1-\kappa) + 2\alpha(1+Z) - \sqrt{[(1+\kappa)Z + (1-\kappa) + 2\alpha(1+Z)]^2 - 16\alpha(1+\alpha)Z}}{4(1+\alpha)}$$

This concludes the proof. ■

Next, I consider the baseline model (as described in the paper). I show that as the dispersion of idiosyncratic productivity becomes vanishingly small households' choice of capital converges to the expression (1). In other words, both in the model where households face uncertainty about their income and in the baseline model households make the same choices for capital as $\varepsilon \to 0$.[4]

Lemma 2 *Consider the benchmark model (as described in the paper). Then, as dispersion of idiosyncratic productivity vanishes (i.e., as $\varepsilon \to 0$) households choice of capital for given A period 2 converges to $k_2(A)$ defined in Equation (1).*

Proof. In Section A.1 of the appendix to the paper, I showed that when $\varepsilon > 0$, then household i's optimal choice of capital is given by

$$k_2(A_i;\varepsilon) = \begin{cases} \frac{\alpha}{1+\alpha}(1-\tau)\, e^A f(k_1) & \text{if } A_i > A^* + \varepsilon \\ (1-\tau)\, e^A f(k_1)\, \Lambda(A_i;\varepsilon, A^{**}) & \text{if } A_i \in [A^* - \varepsilon, A^* + \varepsilon] \\ \frac{Z\alpha}{1+\alpha}(1-\tau)\, e^A f(k_1) & \text{if } A_i < A^* - \varepsilon \end{cases}$$

where

$$\begin{aligned} \Lambda(A_i;\varepsilon, A^*) \quad &= \quad \frac{\alpha(1+Z) + P(A^*|A_i) + Z(1 - P(A^*|A_i))}{2(1+\alpha)} \\ &\quad - \frac{\sqrt{[\alpha(1+Z) + P(A^*|A_i) + Z(1 - P(A^*|A_i))]^2 - 4\alpha Z(1+\alpha)}}{2(1+\alpha)} \end{aligned}$$

[4] Note that in the baseline model ε determines the dispersion of households' productivities while in model with income uncertainty (as described in Section 3.1 of this letter) ε determines dispersion of households' beliefs about their productivity.

and

$$P\left(A^*|A_i\right) \equiv \Pr\left(A < A^*|A_i\right) = \frac{\Phi\left(\frac{A^*-A_{-1}}{\sigma}\right) - \Phi\left(\frac{A_i-\varepsilon-A_{-1}}{\sigma}\right)}{\Phi\left(\frac{A_i+\varepsilon-A_{-1}}{\sigma}\right) - \Phi\left(\frac{A_i-\varepsilon-A_{-1}}{\sigma}\right)}$$

Recall that household i's idiosyncratic productivity A_i is given by $A_i = A + \varepsilon_i$ for some $\varepsilon_i \in [-\varepsilon, \varepsilon]$. To establish the limit of households' investment strategy as $\varepsilon \to 0$ I use the same change of variables as in the proof of Lemma 1, that is I let $\kappa = \frac{\varepsilon_i}{\varepsilon}$, where $\varepsilon_i \in [-\varepsilon, \varepsilon]$, so that $\kappa \in [-1, 1]$. It follows that $A_i = A + \varepsilon_i = A + \kappa\varepsilon$ for some $\kappa \in [-1, 1]$. After this change of variables, we have $\Pr\left(A < A^*|A_i\right) = \Pr\left(A < A^*|A + \kappa\varepsilon\right)$.

It is immediate that for if $A > A^*$ then for sufficiently low ε we have

$$P\left(A^*|A_i\right) = P\left(A^*|A + \kappa\varepsilon\right) = 0,$$

and hence

$$\lim_{\varepsilon \to 0} k_2\left(A_i\right) = \frac{\alpha}{1+\alpha}\left(1-\tau\right) e^{A} f\left(k_1\right) \text{ if } A > A^*$$

By an analogous argument it follows that

$$\lim_{\varepsilon \to 0} k_2\left(A_i\right) = \frac{Z\alpha}{1+\alpha}\left(1-\tau\right) e^{A} f\left(k_1\right) \text{ if } A < A^*$$

It remains to consider the case when $A = A^*$. In this case household i's idiosyncratic productivity is given by

$$A_i = A^* + \kappa\varepsilon$$

for some $\kappa \in [-1, 1]$. Therefore,

$$\Pr\left(A < A^*|A_i\right) = \Pr\left(A < A^*|A^* + \kappa\varepsilon\right) = \Pr\left(\frac{\Phi\left(\frac{A^*-A_{-1}}{\sigma}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma}\right)}\right)$$

Using L'Hopital rule we have

$$\lim_{\varepsilon \to 0} \Pr\left(\frac{\Phi\left(\frac{A^*-A_{-1}}{\sigma}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma}\right) - \Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma}\right)}\right) = \frac{1-\kappa}{2}$$

Thus, $\lim_{\varepsilon \to 0} P\left(A^*|A + \kappa\varepsilon\right) = \left(1-\kappa\right)/2$. It follows that

$$\lim_{\varepsilon \to 0} k_2\left(A^* + \kappa\varepsilon\right) = \left(1-\tau\right) e^{A^*} f\left(k_1\right) \widetilde{\Lambda}\left(\kappa\right)$$

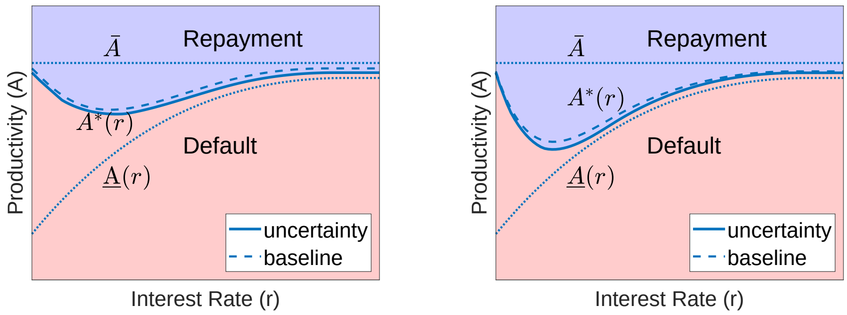
where

$$\begin{aligned} \widetilde{\Lambda}\left(\kappa\right) &= \frac{\alpha\left(1+Z\right) + \left(1-\kappa\right) + Z\left(1+\kappa\right)}{4\left(1+\alpha\right)} \\ &\quad - \frac{\sqrt{\left[\alpha\left(1+Z\right) + \left(1-\kappa\right) + Z\left(1+\kappa\right)\right]^2 - 4\alpha Z\left(1+\alpha\right)}}{4\left(1+\alpha\right)} \end{aligned} \tag{4}$$

Note that expression for $\widetilde{\Lambda}\left(\kappa\right)$ in Equation (4) is the same as expression for $\widetilde{\Lambda}\left(\kappa\right)$ in Equation (2) in Lemma 1. This completes the proof. ■

3.1.3 Robustness (Numerical Analysis)

In this section, I investigate numerically whether the results reported in the paper are robust to introducing an additional source of uncertainty into the households' problem, as described above. In particular, I compare the results of the model described above (Section 3.1.1) to the benchmark model (as described in Section 2 of the manuscript).

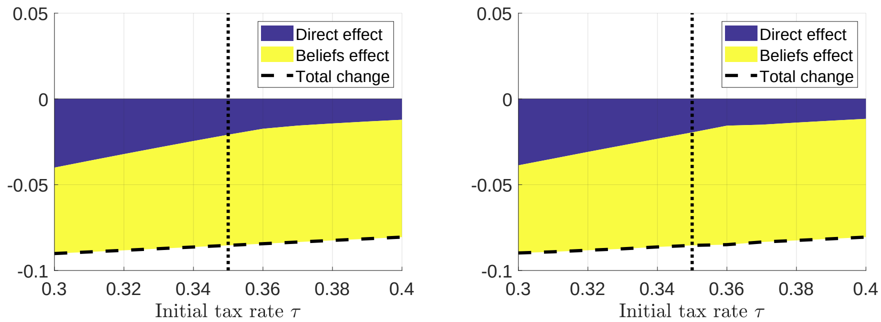


(a) Low A_{-1}

(b) High A_{-1}

Figure 10: Equilibrium default threshold when A_{-1} is low (the left panel) and A_{-1} is high (the right panel). The parameters are set to their calibrated values described in Section 6 of the paper.

Figure 10 depicts the equilibrium default threshold in the baseline model and in the model with an additional source of uncertainty. We see that the default thresholds in the two economies are very similar. Thus I conclude that inclusion of uncertainty about households' productivity in the households' has no significant effect on the equilibrium.

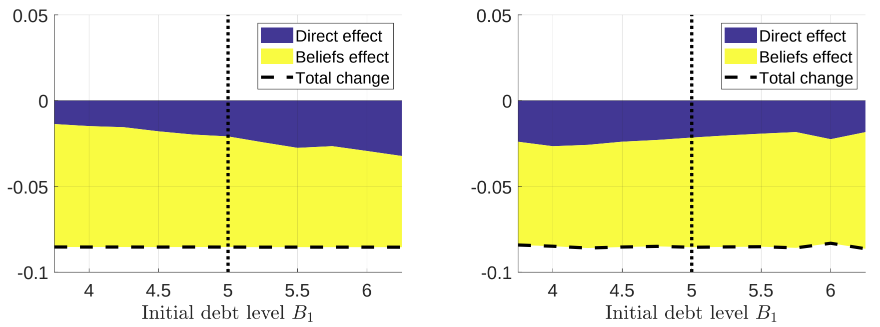


(a) Benchmark

(b) Uncertainty

Figure 11: Change in the probability of default following a 1% increase in the tax rate for different initial values of τ in the benchmark model (the left panel) and the model with an additional source of uncertainty (the right panel). The vertical dotted line indicates the baseline values. All the parameters (except τ) are set to their calibrated values. The initial probability of default is 0.1.

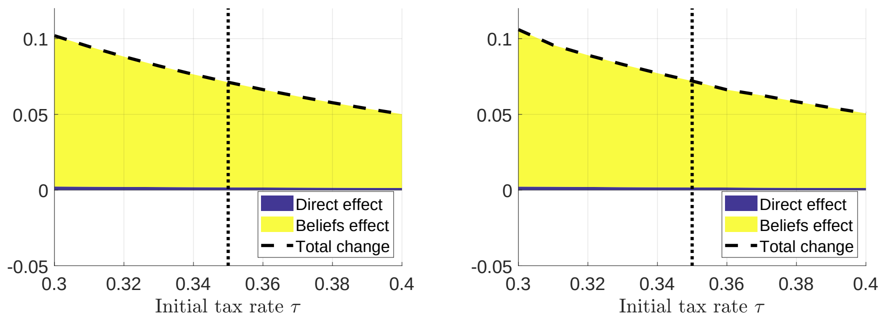
Next, I consider how inclusion of an additional source of uncertainty in the households' problem affects the importance of beliefs in shaping the effectiveness of government policies. Figures 11 and 12 depict the change in the probability of default, and the contributions of the beliefs and direct effects to this change, following a 1% increase in taxes in the benchmark model (left panels in Figures 11 and 12) and the model with the additional source of uncertainty (right panels in Figures 11 and 12). Figures 11 and 12 show that inclusion of the additional source of uncertainty in households' problem does not alter



(a) Benchmark (b) Uncertainty

Figure 12: Change in the probability of default following a 1% increase in the tax rate for different initial values of B_1 in the benchmark model (the left panel) and the model with an additional source of uncertainty (the right panel). The vertical dotted line indicates the baseline values. All parameters (except B_1) are set to their calibrated values. The initial probability of default is 0.1.

the conclusions reached in the baseline model.



(a) Benchmark (b) Uncertainty

Figure 13: Change in the probability of default following a stimulus equal to 1% of the initial capital stock for different initial values of τ in the benchmark model (the left panel) and the model with an additional source of uncertainty (the right panel). The vertical dotted line indicates the baseline values. All parameters (except τ) are set to their calibrated values. The initial probability of default is 0.1.

Finally, Figures 13 and 14 depict the change in the probability of default, and the contributions of the beliefs and direct effects to this change, following a stimulus equal to 1% of the initial capital stock in the benchmark model (left panels in Figures 13 and 14) and the model with the additional source of uncertainty (right panels in Figures 13 and 14). As in the case of an increase in taxes, we see that inclusion of the additional source of uncertainty faced by households does not affect the results.

3.2 Partial Depreciation

In the paper, I assume that capital fully depreciates in each period, that is, I set $\delta = 1$. In this section, I consider the model with $\delta < 1$. I first explain how I adjust the model to incorporate partial depreciation into the model. I then provide numerical results which suggest that the qualitative and quantitative results reported in the paper are unaffected by setting $\delta < 1$. In what follows, I use a subscript h to denote an individual households and i_h to denote investment of household h.

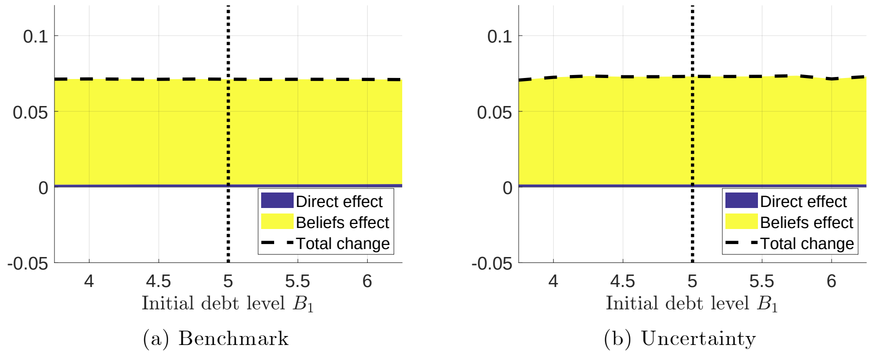


Figure 14: Change in the probability of default following a stimulus equal to 1% of the initial capital stock for different initial values of B_1 in the benchmark model (the left panel) and the model with an additional source of uncertainty (the right panel). The vertical dotted line indicates the baseline values. All parameters (except B_1) are set to their calibrated values. The initial probability of default is 0.1.

3.2.1 Adding $\delta < 1$ into the model

When capital depreciates at the rate $\delta \in (0,1)$, household h's problem becomes

$$\max_{i_h} \; E\left[\sum_{t\in\{1,2\}} \left[\log\left(c_t^h\right) + \log\left(g_t\right)\right] \middle| A_h, \boldsymbol{\sigma}\right]$$
$$s.t. \; c_1^h = (1-\tau)\, Z^{d_1} e^{A_h} f\left(k_1\right) - i_h$$
$$c_2^h = (1-\tau)\, Z^{d_1} e^{A_h} f\left(k_2^h\right)$$
$$k_2^h = (1-\delta)\, k_1 + i_h,$$

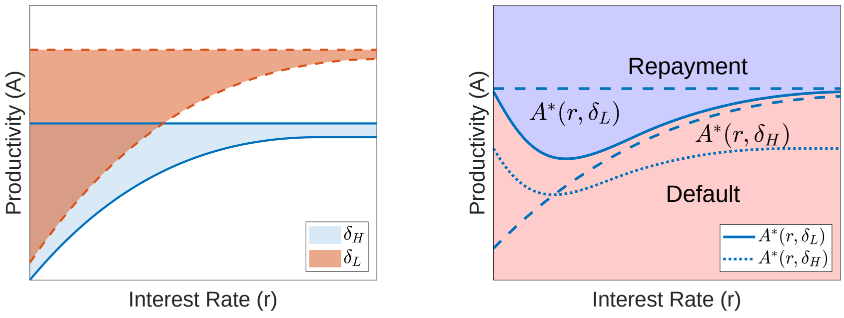
where $\boldsymbol{\sigma}$ is the agents' strategy profile, d_1 is the government's default policy function, and the expectation is taken over the government's default decision as well as over the average level of productivity, A. This is the only change to the model.

3.2.2 Robustness (Numerical analysis)

Below I analyze numerically how assuming that capital depreciates only partially affects the results. In particular, I compare numerically the results of the model with $\delta = 1$ to those with $\delta = 0.04$, which is the average depreciation rate of capital for Ireland, Italy, Portugal, and Spain during the years 2004–2009 (see Penn World Tables). Throughout this section, I refer to $\delta = 1$ as the "high δ" and denote it by δ_H; similarly, I refer to $\delta = 0.04$ as the "low δ" and denote it by δ_L. The other parameters are as in the baseline calibration considered in the paper.[5]

Figure 15 depicts the fragility region and the change in the default threshold for $\delta = \delta_H$ and $\delta = \delta_L$ (i.e., full depreciation and partial depreciation, respectively). The left panel of Figure 15 shows that, compared to the case where $\delta = 1$, a lower depreciation rate shifts the fragility region upwards. This means that when δ decreases, the government is willing to default for a wider range of productivity values. This is because when $\delta = \delta_L$, households invest a smaller fraction of their income in period 1, implying that they consume more in period 1. A higher consumption in period 1 implies a lower costs

[5]The only exception to this is b, the total wealth of lenders. Since the importance of the beliefs effect depends on the equilibrium interest rate, in order to make the numerical results comparable with the benchmark model, I adjusted b so that the equilibrium interest rate in the model with δ_L is the same as in the model with δ_H.



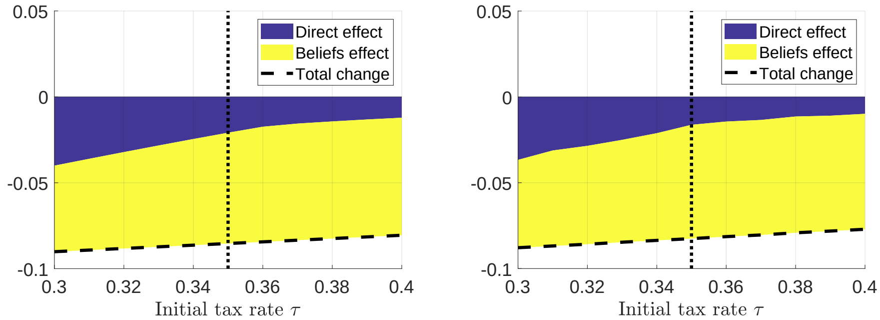
(a) Fragility region (b) Default threshold

Figure 15: Fragility region (left panel) and the equilibrium default threshold (right panel) when $\delta \in \{\delta_L, \delta_H\}$. In the right panel, dashed lines corresponds to the boundries of the fraglity region when $\delta = \delta_L$. All parameters are set to their calibrated values described in Section 6 of the paper.

of default, because by the concavity of the utility function, a drop in consumption following default is less costly when consumption is high than when it is low. This decrease in the cost of default increases the government's incentive to default.

The right panel of Figure 15 depicts the equilibrium default threshold when $\delta \in \{\delta_L, \delta_H\}$. We see that in both models the default threshold has the same shape as a function of the interest rate, r. However, when $\delta = \delta_L$ the default threshold is uniformly higher than in the case where $\delta = \delta_H$. The same force that shifts the fragility region upwards when δ decreases also leads to an increase in the default threshold, $A^*(r)$.

Next, I investigate whether allowing for partial depreciation of capital affects the effectiveness of government policies and the relative importance of the direct and beliefs effects.

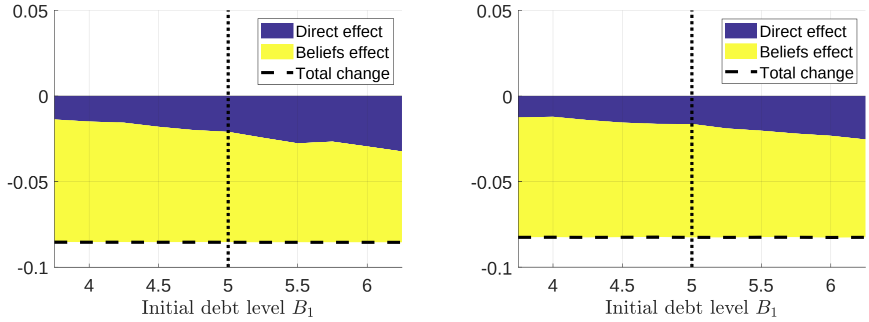


(a) High δ ($\delta = 1$) (b) Low δ ($\delta = 0.04$)

Figure 16: Change in the probability of default following a 1% increase in the tax rate for different initial values of τ centered around the calibrated value of τ (indicated by the vertical dotted line). All parameters (ecept τ) are set to their calibrated values (see Section 6 of the paper). The initial probability of default is 0.1.

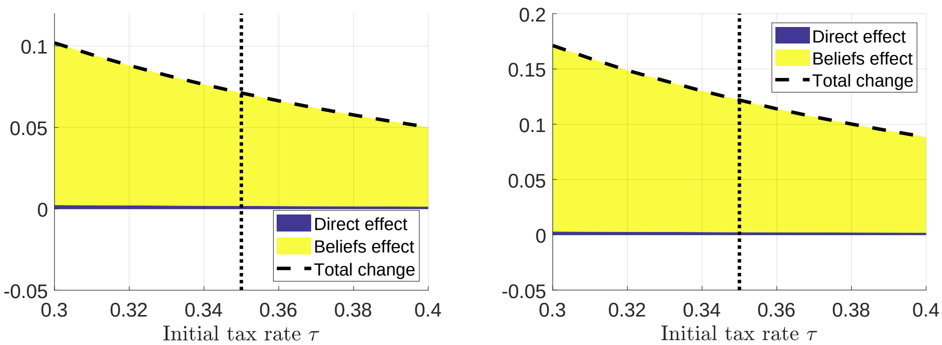
Figures 16 and 17 depict the effect of a 1% increase in taxes for different initial levels of τ (Figure 16) and different initial levels of B_1 (Figure 17) when $\delta = \delta_H$ (left panels) and when $\delta = \delta_L$ (right panels). We see that in both versions of the model an increase in the tax rate leads to a substantial decrease in the probability of default and that the belief effect is the main driver of this decrease.

Figures 18 and 19 depict the effect of a stimulus equal to 1% of the initial capital stock for different



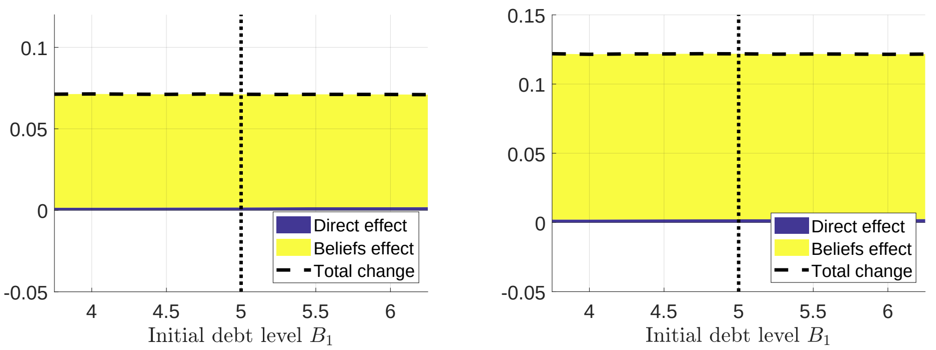
(a) High δ ($\delta = 1$) (b) Low δ ($\delta = 0.04$)

Figure 17: Change in the probability of default following a 1% increase in the tax rate for different values of B_1 centered around the calibrated value of B_1 (indicated by the vertical dotted line). All parameters (except for B_1) are set to their calibrated values (see Section 6 of the paper). The initial probability of default is 0.1.



(a) High δ ($\delta = 1$) (b) Low δ ($\delta = 0.04$)

Figure 18: The change in probability of default following a stimulus equal to 1% the initial capital stock for different values of τ centered around the calibrated value of the tax rate (denotted by the vertical dotted line). Parameters are set to their calibrated values (see Section 6 of the paper).



(a) High δ ($\delta = 1$) (b) Low δ ($\delta = 0.04$)

Figure 19: The change in probability of default following a stimulus equal to 1% the initial capital stock for different values of B_1 centered around the calibrated value of initial debt (denotted by the vertical dotted line). Parameters are set to their calibrated values (see Section 6 of the paper).

initial levels of τ (Figure 18) and different initial levels of B_1 (Figure 19) when when $\delta = \delta_H$ (left panels) and when $\delta = \delta_L$ (right panels). As in the case of an increase in taxes, assuming partial depreciation does not seem to substantially change the results obtained in the benchmark model.

I conclude that the choice of δ affects neither the effectiveness of government policies nor the relative importance of the beliefs effect.

3.3 Model with an Endogenous Interest Rate

In this section, I describe the model where the government posts the amount of bonds it wants to issue, and the price of the bonds is then determined in equilibrium. As in the baseline model, the uncertainty faced by all the agents is about the current level of productivity A (which determines output and hence he government's tax revenue).

Compared to the baseline model, I make four changes:

1. The government posts the amount of bonds it wants to issue, and the price of the bonds is then determined in equilibrium.

2. Households make all their choices (including their investment decisions) at the end of the period and share the same productivity level.

3. The market clearing determines the price at which the government is able to sell its bonds, q. The interest rate is then computed as $r = \frac{1}{q} - 1$.

4. Lender j observes two private signals, x_{1j} and x_{2j}, where

$$\begin{aligned} x_{1j} &= A + \tau_{x_1}^{-1/2}\varepsilon_{1j}, \quad \varepsilon_{1j} \sim N(0,1) \\ x_{2j} &= A + s + \tau_{x_2}^{-1/2}\varepsilon_{2j}, \quad \varepsilon_{2j} \sim N(0,1) \quad \text{and} \quad s \sim N\left(0, \tau_s^{-1}\right) \end{aligned}$$

   Here, s is a correlated noise in the lenders' signals, while ε_{1j} and ε_{2j} both capture idiosyncratic noise in the signals; s, ε_{1j}, and ε_{2j} are assumed to be independent of one another as well as of A.

The first assumption implies that the rate of return on government bonds is determined in equilibrium. The second assumption is made for simplicity. It implies that household expectations do not matter for the outcome of the model. The third assumption is needed to ensure existence of equilibrium. The last assumption is needed to ensure that the endogenous interest rate is a noisy signal about the underlying productivity.[6]

Figure 20 depicts the timing of this version of the model. In particular, we see that at the beginning of period 1 the government chooses the amount of debt it would like to issue. After that, the bond market opens, the lenders simultaneously make their lending decisions, and the market clears. As is standard in the noisy rational expectations literature (see, for example, Grossman and Stiglitz (1980) or Hellwig et al. (2006)), I allow lenders to condition their choices on the price of the government's bonds. After the price of new debt is realized, A is revealed and the government chooses whether to default and how much to spend. As in the baseline model, default is associated with both an output cost and a litigation cost. Once the government makes its decisions, households produce and make their investment and consumption decisions. Period 2 is unchanged.

[6] This approach to ensuring that the price does not fully resolve the underlying uncertainty has been used before, by Zabai (2014). If correlated noise is absent, then the price will perfectly reveal the true value of A and the model will feature multiple equilibria.

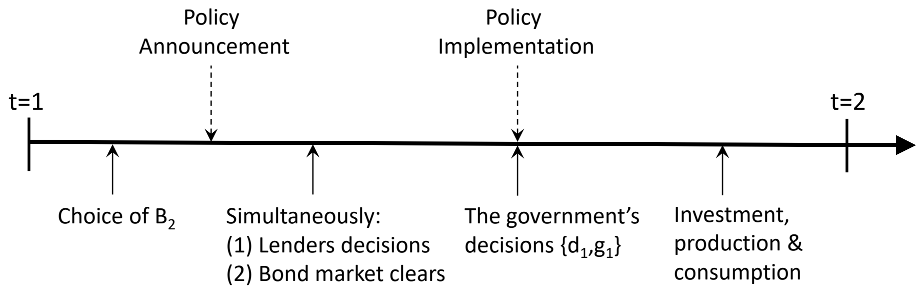


Figure 20: Timeline

3.3.1 Equilibrium

The model described above can be solved by backward induction. Notice that, as in the benchmark model, the outcomes in period 2 are determined by agents' period-1 decisions. Thus we can focus on computing agents' optimal decisions in period 1.

The households' problem is unchanged, except that households now make their choices at the end of period 1, and hence they face no uncertainty. Thus it is easy to show that all households make the same investment choices and household i's optimal investment in repayment and default, respectively, is given by

$$\begin{aligned} k_2^R &= (1-\tau)\, e^A k_1^\alpha \frac{\alpha}{1+\alpha} \\ k_2^D &= Z\,(1-\tau)\, e^A k_1^\alpha \frac{\alpha}{1+\alpha} \end{aligned}$$

The value (to the government) of repaying, given that the government posted B_2 and the price that cleared the market was q, is

$$\begin{aligned} V^R(A, B_2, q) &= \log\left((1-\tau)\, Y_1^R - k_2^R\right) + \log\left(\tau Y_1^R - B_1 + qB_2\right) \\ &\quad \log\left((1-\tau)\, Y_2^R\right) + \log\left(\tau Y_2^R - B_2\right), \end{aligned}$$

where $Y_1^R = e^A f(k_1)$ and $Y_2^R = e^A f\left(k_2^R\right)$. The value of defaulting is given by

$$\begin{aligned} V^D(A) &= \log\left((1-\tau)\, Y_1^D - k_2^D\right) + \log\left(\tau Y_1^D\right) \\ &\quad \log\left((1-\tau)\, Y_2^D\right) + \log\left(\tau Y_2^D\right), \end{aligned}$$

where $Y_1^D = Ze^A f(k_1)$ and $Y_2^D = Ze^A f\left(k_2^D\right)$. Therefore, the default threshold $A^*(q, B_2)$ is the solution to

$$V^R(A, q, B_2) - V^D(A) = 0 \tag{5}$$

Finally, lenders decide the amount of government bonds to buy at a given price q. In other words, lenders choose their demand schedule for bonds. Let $\widehat{x}_j$ denote lender j's compound private signal, where

$$\widehat{x}_j = \frac{\tau_1 x_{1j} + \tau_2 x_{2j}}{\tau_1 + \tau_2}$$

and

$$\tau_1 \equiv \tau_{x_1} \text{ and } \tau_2 = \left(\tau_s^{-1} + \tau_{x_2}^{-1}\right)^{-1}$$

In a monotone equilibrium, lenders' demand schedules take a particularly simple form: Lender j purchases no bonds if $\widehat{x}_j < \widehat{x}^*(q, B_2)$, and uses all his funds to purchase bonds otherwise.

Let $S(q, B_2)$ be the total supply of funds by lenders, given the price q, and given that the government wants to borrow B_2 (i.e., S is the aggregate demand for bonds). Then the market clearing requires that

$$S(q, B_2) = B_2$$

We are now ready to define the equilibrium of this model.

Definition 1 *The equilibrium of the model consists of households' investment decisions in repayment and default* $\{k_2^R, k_2^D\}$*, the government's spending and default decisions* $\{d_1, g_1, g_2\}$*, lenders' monotone strategies characterized by the thresholds* $\widehat{x}^*(q, B_2)$*, the equilibrium price* q*, and the government's borrowing decision* B_2*, such that:*

1. *k_2^R and k_2^D maximize households' utility in repayment and default, respectively.*

2. *d_1, g_1, g_2 maximize the government's payoff, given the price q and borrowing B_2.*

3. *$\widehat{x}^*(q, B_2)$ is the optimal threshold that makes lenders indifferent between buying the bonds and investing in the risk-free asset, given q and B_2.*

4. *The price q clears the bond market.*

5. *The government's choice of B_2 maximizes the government's expected payoff at the beginning of period* 1.

In this version of the model, the price q provides information to lenders. All else equal, a high q implies that a lot of lenders received high signals, and hence A is likely to be high. Similarly, all else equal, a low q implies that a lot of lenders received low signals, and hence A is likely to be low. Thus, the price q reveals information. The key step is to determine exactly what the informational content of q is, and whether q is uniquely determined. As I show below, it turns out that observing a price is equivalent to observing a signal y about A, where

$$y = A + \frac{\tau_2}{\tau_1 + \tau_2} s$$

and s is the correlated noise in lenders' signals. The next proposition follows from the above observations.

Proposition 3 *Given B_2, the equilibrium is determined by the following equations:*

1. *Households' FOCs*

$$\begin{aligned} k_2^R &= (1-\tau) e^A k_1^\alpha \frac{\alpha}{1+\alpha} \\ k_2^D &= Z(1-\tau) e^A k_1^\alpha \frac{\alpha}{1+\alpha} \end{aligned}$$

2. *Government's indifference condition*

$$V^R(A, q, B_2) - V^D(A) = 0$$

3. Lenders' signal threshold $\widehat{x}^*(q, B_2)$ given by

$$\begin{aligned}\widehat{x}^*(q, B_2) &= \frac{\tau_1+\tau_2+\tau_y+\tau_A}{\tau_1+\tau_2+\tau_y} A^*(q, B_2) - \frac{\tau_A}{\tau_1+\tau_2+\tau_y}\mu_A \\ &+ \frac{(\tau_1+\tau_2+\tau_y+\tau_A)^{1/2}}{\tau_1+\tau_2+\tau_y}\Phi^{-1}(q) \\ &- \frac{\tau_y \tau_{\widehat{x}}^{-1/2}}{\tau_1+\tau_2+\tau_y}\Phi^{-1}\left(\frac{B_2}{b}\right)\end{aligned}$$

4. Market clearing condition

$$y = \widehat{x}^*(q, B_2) + \tau_{\widehat{x}}^{-1/2}\Phi^{-1}\left(\frac{B_2}{b}\right)$$

One can establish that if the precision of the private information and the precision of the prior belief are small enough, then the equilibrium is unique.[7]

Finally, the optimal choice of B_2 maximizes the government's expected payoff

$$\int_{y=-\infty}^{\infty}\int_{A^*(q(y),B_2)}^{\infty} V^R(A, q(y), B_2) f(A, y)\, dAdy + \int_{y=-\infty}^{\infty}\int_{-\infty}^{A^*(q(y),B_2)} V^D(A) f(A, y)\, dAdy,$$

where $f(A, y)$ is the joint density of $\{A, y\}$.

3.3.2 Results

Having characterized the equilibrium of the model, I now investigate numerically the effectiveness of government fiscal policy in decreasing the likelihood of a debt crisis. For the purpose of this analysis, I use the calibration described in the paper.

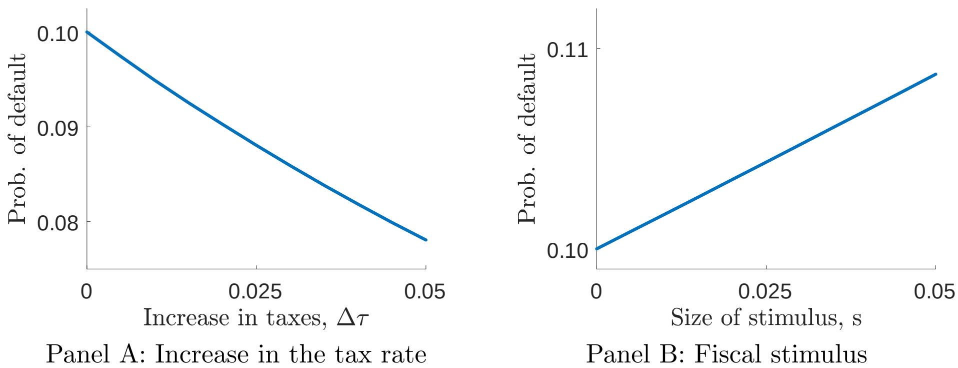


Figure 21: Ex-ante probability of default (conditional on B_2) as the tax rate (the left panel) and the size of the stimulus (the right panel) vary. The parameters are the same as in the baseline calibration (see Section 6 in the paper).

Figure 21 shows how an increase in tax rate (the left panel) and a stimulus of different size (the right panel) affect the ex-ante probability of default. We see that the main message from the baseline model continues to hold: An increase in tax rate tends to decrease the probability of default while stimulus tends to increase it. Thus, assuming that the government posts the amount of bonds rather than the interest rate does not affect the main conclusions of the model.

[7] For the equilibrium to be unique, it is important that the exogenous sources of information be relatively uninformative. Otherwise, the price would reveal too much information, leading to equilibrium multiplicity. See, for example, Hellwig et al. (2006).

3.3.3 Detailed Solution

In this section, I solve the lenders' problem and determine the informational content of the market clearing price.

Lenders' decisions and market clearing Following Hellwig et al. (2006) and Zabai (2014), I use a guess-and-verify technique to compute the lenders' optimal strategies and the market clearing price. Recall that each lender receives two private signals, x_{1j} and x_{2j}. Let τ_1 and τ_2 denote the precisions of these signals, respectively, that is,

$$\tau_1 \equiv \tau_{x_1} \text{ and } \tau_2 = \left(\tau_s^{-1} + \tau_{x_2}^{-1}\right)^{-1}$$

Now define a compound private signal

$$\widehat{x}_j = \frac{\tau_1 x_{1j} + \tau_2 x_{2j}}{\tau_1 + \tau_2},$$

and note that $\widehat{x}_j$ is a sufficient statistic for the information contained in x_{1j} and x_{2j}. I conjecture that lender i will demand bonds when the price of them is q if and only if

$$\widehat{x}_j \geq \widehat{x}^* \left(q, B_2\right) \tag{6}$$

Under the above assumption, the total demand for bonds is

$$b \Pr\left(\widehat{x}_j \geq \widehat{x}^* \left(q, B_2\right) | A, s\right),$$

while the supply is simply B_2. It follows that the equilibrium price q^* has to solve

$$b \Pr\left(\widehat{x}_j \geq \widehat{x}^* \left(q, B_2\right) | A, s\right) = B_2 \tag{7}$$

Since

$$\widehat{x}_i | A, s \sim N\left(A + \frac{\tau_2}{\tau_1 + \tau_2} s, \tau_{\widehat{x}}^{-1}\right),$$

where

$$\tau_{\widehat{x}} = \left[\left(\frac{\tau_1}{\tau_1 + \tau_2}\right)^2 \tau_{x_1}^{-1} + \left(\frac{\tau_2}{\tau_1 + \tau_2}\right)^2 \tau_{x_2}^{-1}\right]^{-1},$$

the market clearing condition can be written as

$$b\left[1 - \Phi\left(\frac{\widehat{x}^* \left(q\right) - A - \frac{\tau_2}{\tau_1 + \tau_2} s}{\tau_{\widehat{x}}^{-1/2}}\right)\right] = B_2$$

Rearranging, we obtain

$$\underbrace{A + \frac{\tau_2}{\tau_1 + \tau_2} s}_{\substack{\equiv y \\ \text{(endogenous signal)}}} = \widehat{x}^* \left(q\right) + \tau_{\widehat{x}}^{-1/2} \Phi^{-1}\left(\frac{B_2}{b}\right) \tag{8}$$

Equation (8) is the equilibrium condition that determines the informational role of prices. It shows that from the lenders' perspective, observing price q in the market is equivalent to observing a public signal y, where

$$y \equiv A + \frac{\tau_2}{\tau_1 + \tau_2} s$$

Having determined the informational content of price q, I now consider the lenders' decisions whether to purchase government debt. Lender j makes his decision based on four pieces of information: his two private signals, the prior belief, and the price. Thus lender j will purchase government bonds if

$$\Pr\left(A \geq A^*\left(q, B_2\right) | x_{1j}, x_{2j}, q\right) \geq q$$

or

$$\left[1 - \Phi\left(\frac{A^*\left(q, B_2\right) - \frac{\tau_1 x_{ij} + \tau_2 x_{2j} + \tau_y y + \tau_A \mu_A}{\tau_1 + \tau_2 + \tau_y + \tau_A}}{\left(\tau_1 + \tau_2 + \tau_y + \tau_A\right)^{-1/2}}\right)\right] \geq q$$

Rearranging, and using the definition of $\widehat{x}_j$, we obtain

$$\frac{\left(\tau_1 + \tau_2\right) \widehat{x}_j + \tau_y y + \tau_A \mu_A}{\tau_1 + \tau_2 + \tau_y + \tau_A} \geq A^*\left(q, B_2\right) + \frac{1}{\left(\tau_1 + \tau_2 + \tau_y + \tau_A\right)^{1/2}} \Phi^{-1}(q)$$

Let $\widehat{x}^*(q)$ denote the threshold compound signal at which lender j is willing to purchase government debt when the price is q. That is

$$\frac{\left(\tau_1 + \tau_2\right) \widehat{x}^*(q) + \tau_y y + \tau_A \mu_A}{\tau_1 + \tau_2 + \tau_y + \tau_A} = A^*\left(q, B_2\right) + \frac{1}{\left(\tau_1 + \tau_2 + \tau_y + \tau_A\right)^{1/2}} \Phi^{-1}(q)$$

Using the fact that $y = \widehat{x}^*\left(q, B_2\right) + \tau_{\widehat{x}}^{-1/2} \Phi^{-1}\left(\frac{B_2}{b}\right)$ and rearranging, we obtain

$$\begin{aligned} \widehat{x}^*\left(q, B_2\right) &= \frac{\tau_1 + \tau_2 + \tau_y + \tau_A}{\tau_1 + \tau_2 + \tau_y} A^*\left(q, B_2\right) - \frac{\tau_A}{\tau_1 + \tau_2 + \tau_y} \mu_A \\ &+ \frac{\left(\tau_1 + \tau_2 + \tau_y + \tau_A\right)^{1/2}}{\tau_1 + \tau_2 + \tau_y} \Phi^{-1}(q) \\ &- \frac{\tau_y \tau_{\widehat{x}}^{-1/2}}{\tau_1 + \tau_2 + \tau_y} \Phi^{-1}\left(\frac{B_2}{b}\right) \end{aligned} \tag{9}$$

Equation (9) determines the lenders' optimal signal threshold given q.

At this point, we have determined $\widehat{x}^*\left(q, B_2\right)$ and $A^*\left(q, B_2\right)$ as functions of q. It remains to determine the equilibrium price q. To do so, we plug the expression for $\widehat{x}^*\left(q, B_2\right)$ into the market clearing condition (8). We obtain

$$\begin{aligned} y &= \frac{\tau_1 + \tau_2 + \tau_y + \tau_A}{\tau_1 + \tau_2 + \tau_y} A^*\left(q, B_2\right) - \frac{\tau_A}{\tau_1 + \tau_2 + \tau_y} \mu_A \\ &+ \frac{\left(\tau_1 + \tau_2 + \tau_y + \tau_A\right)^{1/2}}{\tau_1 + \tau_2 + \tau_y} \Phi^{-1}(q) \\ &- \frac{\tau_y \tau_{\widehat{x}}^{-1/2}}{\tau_1 + \tau_2 + \tau_y} \Phi^{-1}\left(\frac{B_2}{b}\right) \\ &+ \tau_{\widehat{x}}^{-1/2} \Phi^{-1}\left(\frac{B_2}{b}\right) \end{aligned} \tag{10}$$

Equation (10) defines a price correspondence. The exogenous part of it is y, which is determined by the realization of A and the common noise s in lenders' signal. Then for given y, any q that satisfies Equation (10) is an equilibrium price. In particular, if for each q there exists a unique price that satisfies Equation (10), then the equilibrium is unique. If there exist several prices, then there are multiple equilibria.

Finally, if there is no price that satisfy the above equation, then no equilibrium exists. One can establish that if the precision of the private information and the precision of the prior belief is small enough, then the equilibrium is unique.

3.4 Optimal Choice of Taxes

In this section, I consider a version of the model where the government chooses taxes optimally at the same time that it chooses B_2. I make three simplifying assumptions. First, I assume that households make their investment decisions at the end of period 1, after the government makes its default decision. Second, I assume that all households have the same productivity, A. Finally, for simplicity I assume that lenders ignore the "competition effect" when deciding whether to lend to the government or not.[8] This implies that a change in the government tax rates will affect lenders' decisions only via the default threshold. These assumptions simplify the computation of the equilibrium, since otherwise households and lenders would have to predict the entire optimal tax schedule as a function of A based on their information.

3.4.1 The Government's Problem

Suppose that the tax rate τ is chosen by the government optimally simultaneously with the optimal borrowing, after the government observes the underlying aggregate productivity level, A. In this case, the value to the government of repaying its debt when the average productivity is equal to A, the households' investment profile is $\mathbf{k}_2$, and the supply of funds in the bond market is S is given by

$$\begin{aligned} V_1^R\left(A,\mathbf{k}_2,S;r\right) = & \max_{\{\tau\in[0,1],B_2\in[0,S]\}} \sum_{t=1,2}\left\{\int_0^1\left[\log\left(c_t^{i,R}\right)+\log\left(g_t^R\right)\right]di\right\} \\ & s.t.\ g_1^R = \tau Y_1^R - B_1 + B_2 \\ & \quad\ \ g_2^R = \tau Y_2^R - (1+r)B_2, \end{aligned}$$

while the value associated with defaulting is given by[9]

$$\begin{aligned} V_1^D\left(A,\mathbf{k}_2,S\right) = & \max_{\{\tau\in[0,1],B_2\in[0,S]\}} \sum_{t=1,2}\left\{\int_0^1\left[\log\left(c_t^{i,D}\right)+\log\left(g_t^D\right)\right]di\right\}, \\ & s.t.\ g_1^D = \tau\left(ZY_1^R\right) \\ & \quad\ \ g_2^D = \tau\left(ZY_2^R\right) \end{aligned}$$

where

$$\begin{aligned} Y_1^R &= e^A f\left(k_1\right),\ Y_2^R = e^A f\left(k_2^R\right) \\ Y_1^D &= Ze^A f\left(k_1\right),\ Y_2^R = Ze^A f\left(k_2^D\right) \\ k_2^R &= \left(1-\tau\right)e^A f\left(k_1\right)\frac{\alpha}{1+\alpha} \\ k_2^D &= Zk_2^D \end{aligned}$$

The main difference is that now the government has two instruments for smoothing its spending across periods. First, it can borrow from lenders. Second, it can adjust the tax rate. To understand

[8] This is equivalent to Assumption 1, in the introduction to Section 4 of the current manuscript.
[9] I assume here that the litigation costs are given by $\xi\left(B_1,B_2\right)=B_2/\left(B_1+B_2\right)$.

why adjusting the tax rate allows the government to smooth spending, note that by increasing τ the government increases its tax revenue today, but by discouraging investment it decreases its tax revenue tomorrow. Thus by increasing the tax rate the government can effectively trade off spending tomorrow for spending today. Thus we should expect the government to choose a relatively high tax rate when it cannot borrow much, and government spending today would be low relative to government spending tomorrow if the tax rate were low.

3.4.2 Numerical result

While the version of the model where the government chooses taxes optimally at the same time as its borrowing is analytically intractable, it is possible to solve numerically. Below, I explain how this extension of the model affects the equilibrium.

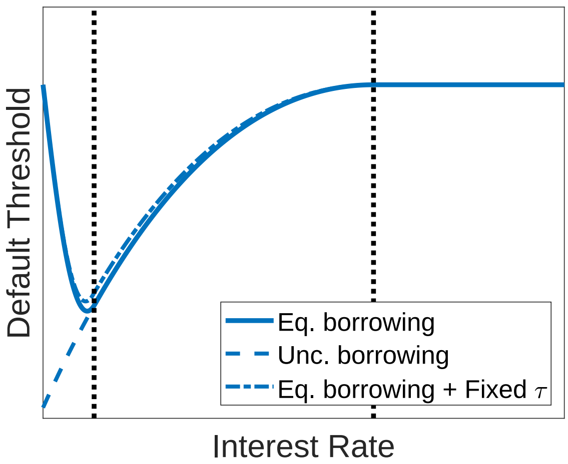
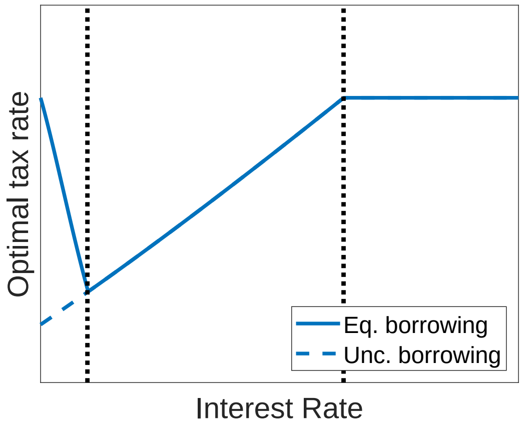


Figure 22: Optimal tax rate and optimal borrowing when the government borrows the equilibrium amount (solid line) and when the government borrows theunconstrained optimal amount (dashed-line). The unconstrained optimal amount is the amount that maximizes the value of repaying the debt. The parameters are set at their baseline values (see Section 6 of the paper.). The left dotted vertical line indicates an interest rate at which the government's borrowing in equilibrium is equal to its desired borrowing. The right dotted vertical line indicates the level of the interest rate above which the government's desired borrowing and the equilibrium borrowing are 0.

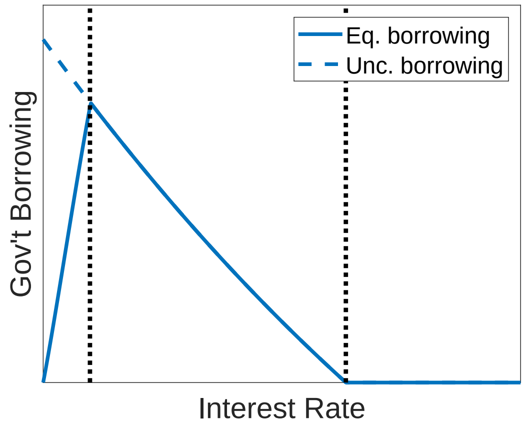
Figure 22 shows that the ability to set taxes optimally does not substantially alter the equilibrium default threshold. In particular, Figure 22 depicts the equilibrium default threshold when the government sets taxes optimally and can borrow only as much as lenders supply to the market (the solid curve referred to as "equilibrium borrowing"), the equilibrium default threshold when the government cannot adjust the tax rate and can borrow only as much as lenders supply to the market (the dot-dashed curve referred to as "equilibrium borrowing + fixed τ") and the equilibrium default threshold when the government sets the tax rate optimally and can borrow its desired amount (the dashed curve referred to as "unconstrained borrowing"). We see that the equilibrium threshold with fixed taxes is in general higher than the equilibrium threshold when taxes are set optimally, but the difference is very small.

Figure 23 depicts the optimal tax rate (the left panel) and the optimal borrowing by the government (the right panel) at the default threshold as a function of the interest rate in the global game equilibrium[10]

[10] By theglobal game equilibrium, I mean the equilibrium in which lenders optimally choose how much funds to supply to their market based on their signal as described in the paper.



(a) Optimal choice of τ



(b) Optimal borrowing B_2

Figure 23: Optimal tax rate and optimal borrowing when the government borrows equilibrium amount (solid line) and when the government borrows unconstrained optimal amount (dashed-line). The unconstrained optimal amount is the amount that maximizes the value of repaying the debt. Parameters are set at their baseline values (see Section 6 of the paper.).

(solid lines) and, for comparison, in the situation where the government could borrow the unconstrained optimal amount (dashed lines). In order to understand the optimal choice of the tax rate, it is useful to start by considering the government's optimal borrowing as depicted in the right panel of Figure 23. This panel shows that when r is low, the government is unable to borrow much in equilibrium (as indicated by the solid line), while it would like to borrow a lot (as indicated by the dashed line). As r increases, the government's desired borrowing decreases, while the supply of funds increases. This continues up to an interest rate at which the government's borrowing in equilibrium is equal to its desired borrowing (the left dotted vertical line). As the interest rate increases further, the government's desired borrowing and the equilibrium borrowing decrease, up to an interest rate at which they both reach 0 (this point is depicted by the right dotted vertical line). Thus Figure 23 indicates that the government chooses both the tax rate and the borrowing to smooth its spending across the two periods.

To sum up, while a government would find it optimal to adjust the tax rate at the time it chooses its borrowing, such choice does not seem to significantly affect the equilibrium outcome. Moreover, as argued earlier, choosing the tax rate and the borrowing separately seems to capture better the behavior of European governments during the recent European debt crisis. Finally, allowing for the endogenous choice of taxes affects tractability of the model substantially. For these reasons, in the current version of the manuscript I continue to consider the choice of the tax rate separately from the choice of borrowing.

3.5 Litigation Cost: Numerical Investigation

In this section, I show that the non-existence issues described in Section A.7 in the paper are not merely theoretical possibilities, but they do arise in my model for reasonable values of parameters.

In the paper, I assumed that a litigation cost are equal to government's borrowing in default, that is, the litigation cost was equal to the government choice of B_2. Below, I parameterize the litigation cost by ξ with $\xi \in [0, 1]$ and assume that the litigation cost is equal to fraction ξ of the government's borrowing in default (i.e., it is equal to ξB_2). This allows me to investigate how large the litigation cost needs to be in order to ensure that the fragility region and a monotone equilibrium exist.

3.5.1 Non-Existence of the Fragility Region

Let $\Delta V^{CI}\left(A, k_2^R(A), B_2; r\right)$ denotes the difference between the value of repaying and the value of defaulting in the complete information version of the model (described in more details in Section 4 below) as a function of productivity A and its borrowing B_2, when the households expect the government to repay its debt (so that they invest $k_2^R(A)$; see Equation 15 below). Moreover, denote by $B_2^{R,u}(A)$ the amount of new debt the government would like to issue if it were planning to repay its legacy debt, B_1.

Figure 24 depicts $\Delta V^{CI}\left(A, k_2^R(A), B_2; r\right)$ when the government can borrow for $B_2^{R,u}(A)$ at each value of productivity A for different values of the litigation cost ξ (red, yellow, and purple curves) and when the government cannot borrow any funds, i.e., when $B_2 = 0$ (in which case ΔV^{CI} is independent of ξ).

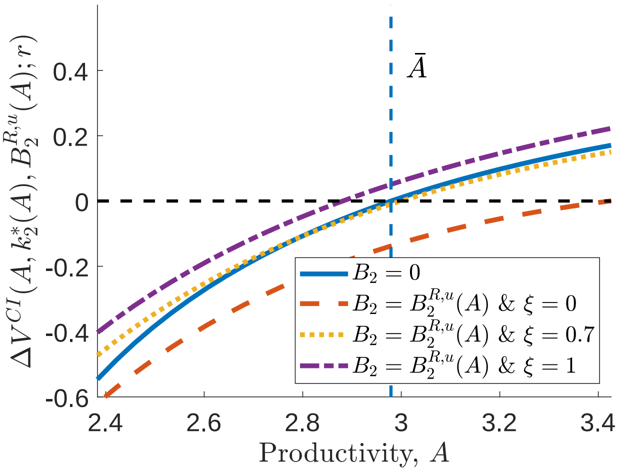
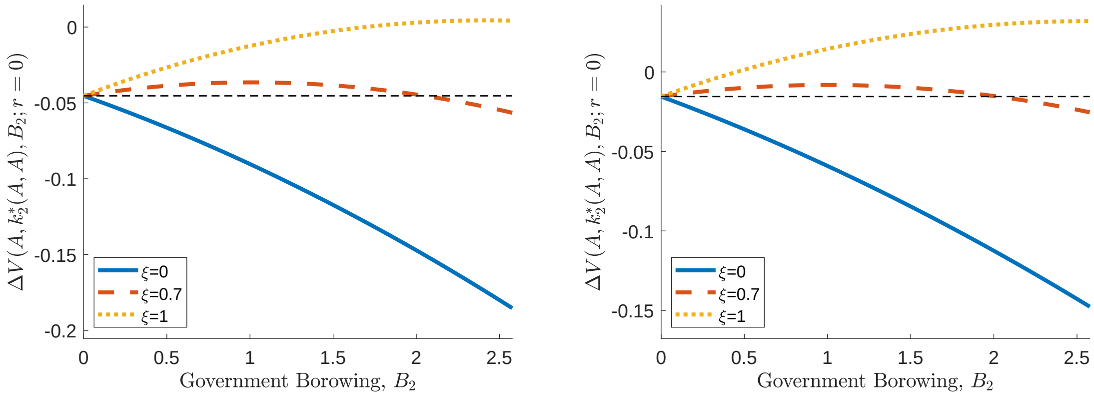


Figure 24: $\Delta V^{CI}(A, k_2^R(A), B_2, \xi)$ when the government is unable to borrow any funds (solid blue curve) and when the government can borrow $B_2^{R,u}(A)$ for different values of ξ (red, yellow, and purple curves). The interest rate is set to $r = 0$ to minimize the government's incentives to default. Households are assumed to not expect default and invest the optimal amount. The parameters are the same as in the baseline calibration (see Section 6 of the paper).

The blue solid curve depicts ΔV^{CI} when lenders refuse to lend any funds to the government. This value is independent of ξ, since in this case $B_2 = 0$. The level of productivity at which the blue solid curve intersects the $\Delta V^{CI} = 0$ line corresponds to $\overline{A}$, the upper bound of the fragility region, which is marked by the vertical dashed line. The other three curves depict ΔV^{CI} as a function of A when the government can borrow its optimal amount, B_2^{R*}, for corresponding values of ξ. The values of A at which these curves intersect the $\Delta V^{CI} = 0$ line correspond to the lower bounds of the fragility region, $\underline{A}(r)$, for the corresponding values of ξ. Figure 24 shows that when $\xi = 0$ the graph of the function $\Delta V^{CI}\left(A, k_2^R(A), B_2^{R,u}(A); r\right)$ intersects the $\Delta V^{CI} = 0$ at a productivity level $A > \overline{A}$. This implies that when $\xi = 0$ we have $\underline{A}(0) > \overline{A}$, hence the fragility region does not exist. The same is true when $\xi = 0.7$. Thus, we see that in order to ensure that the fragility region exists the litigation cost needs to be high.

3.5.2 Non-Existence of a Monotone Equilibrium

As explained in Section A.6 in order to ensure that a monotone equilibrium exists we need the government incentives to default to be decreasing as the supply of funds in the bond market increases. As I show numerically below, unless the litigation cost, ξ, is high the government incentives to default are either increasing or are non-monotone in B_2 at the calibrated values of the parameters.



(a) Low A (b) High A

Figure 25: $\Delta V(A, k_2^*(A,A), B_2; r)$ as B_2 varies, holding A, $k_2^*(A,A)$, and r constant. The parameters are the same as in the baseline calibration (see Section 6 of the paper). Low productivity is equal to $A = (4/5)\underline{A}(r) + (1/5)\bar{A}$, while high productivity is equal to $A = (1/5)\underline{A}(r) + (4/5)\bar{A}$.

Figure 25 depicts the difference between the value of repaying and the value of defaulting in my benchmark model , denoted by $\Delta V\left(A, k_2^*\left(A, A\right), B_2; r=0\right)$, as a function of B_2 when productivity is low $(A = (4/5)\,\underline{A}\,(0) + (1/5)\overline{A}$, left panel) and when productivity is high $(A = (1/5)\,\underline{A}\,(0) + (4/5)\overline{A}$, right panel) assuming that households expect the default threshold to be A. I set $r = 0$ to minimize the government's incentive to default (with $r > 0$ the results are even starker).[11] All parameters are set to their baseline calibration values as described in Section 6 of the paper. We see that $\Delta V\left(A, k_2^*\left(A\right), B_2; r\right)$ is a decreasing function of B_2 when $\xi = 0$, non-monotone if $\xi = 0.7$ (which is also problematic, for the same reason as when it is decreasing), and monotonically increasing when $\xi = 1$. Thus, unless ξ is large a monotone equilibrium will not exist.

[11] The productivity values are chosen so that both low A and high A belong to the fragility region.

4 Model with Complete Information

In this section, I consider the complete information version of the model and the derive analytically the "fragility zone." As in the paper, I denote by $\underline{A}(r)$ the lower bound of the fragility region and by $\overline{A}$ the upper bound of the fragility region.[12]

In the complete information game all agents observe the productivity parameter A so that A is common knowledge. In addition, I assume that all households have the same productivity level, that is $A_i = A$ for all $i \in [0,1]$. This assumption is for simplicity and is common when constructing "multiplicity region" in standard global games (see, for example, Morris and Shin (1998)).

4.1 Households' and Lenders' Choices and the Government's Borrowing Decision

Lemma 4 *Consider households choice of capital, k_2.*

1. *If households expect the government to repay its debt they invest*

$$k_2^R(A) = \frac{\alpha}{1+\alpha}(1-\tau)e^A f(k_1) \tag{11}$$

2. *If households expect the government to default they invest*

$$k_2^D(A) = \frac{Z\alpha}{1+\alpha}(1-\tau)e^A f(k_1) \tag{12}$$

Proof. Suppose first that households expect the government to repay its debt. In this case, the households' problem is given by

$$\max_{k_2} \log\left((1-\tau)e^A f(k_1) - k_2\right) + \log\left((1-\tau)e^A f(k_2)\right) \tag{13}$$

The F.O.C. associated with the above problem is given by

$$\frac{-1}{(1-\tau)e^A f(k_1) - k_2} + \frac{f'(k_2)}{f(k_2)} = 0 \tag{14}$$

Since $f(k) = k^\alpha$ Equation (14) simplifies to

$$\frac{1}{(1-\tau)e^A f(k_1) - k_2} = \frac{\alpha}{k_2}$$

Denoting by k_2^R the households' optimal choice of capital when households expect the government to repay its debt and solving Equation (14) we obtain

$$k_2^R(A) = \frac{\alpha}{1+\alpha}(1-\tau)e^A f(k_1) \tag{15}$$

Next, suppose that all households expect the government to default. In this case, the households' problem becomes

$$\max_{k_2} \left(Z(1-\tau)e^A f(k_1) - k_2\right) + \log\left(Z(1-\tau)e^A f(k_2)\right) \tag{16}$$

The F.O.C. is given by

$$\frac{-1}{(1-\tau)e^A f(k_1) - k_2} + \frac{f'(k_2)}{f(k_2)} = 0 \tag{17}$$

[12] As I show below, the upper bound of the fragility region does not depend on the interest rate, r.

Denoting by k_2^D the households' optimal choice of capital when households expect the government to default and solving Equation (17) we obtain

$$k_2^D(A) = \frac{Z\alpha}{1+\alpha}(1-\tau)e^A f(k_1) \tag{18}$$

■

In what follows, to simplify the notation I suppress the dependence of k_2^D and k_2^R on the productivity whenever it does not lead to a confusion.

Lemma 5 *Consider lenders choices.*

1. *If lenders expect the government to repay they supply all their funds to the bond market, that is $S = b$.*

2. *If lenders expect the government to default they supply no funds to the bond market, that is $S = 0$.*

Proof. Follows immediately from the fact that lenders are risk-neutral. ■

Finally, I consider the government borrowing decision when the government decides to repay its debt. Let $V^R(A, k_2, S; r)$ is the value to the government of repaying its debt when the productivity is equal to A, households' choose capital k_2, and lenders supply S. Then

$$V^R(A, k_2, S; r) = \max_{B_2 \in [0,S]} \sum_{t=1,2} \log\left(c_t^R\right) + \log\left(g_t^R\right) \tag{19}$$

$$\begin{aligned} \text{s.t. } g_1^R &= \tau e^A f(k_1) - B_1 + B_2 \\ g_2^R &= \tau e^A f(k_2) - (1+r) B_2 \\ c_1^R &= (1-\tau) e^A f(k_1) - k_2 \\ c_2^R &= (1-\tau) e^A f(k_2) \end{aligned}$$

Let $B_2^{R,u}$ denote the government's "desired" borrowing if the government decides to repay its debt. That is, $B_2^{R,u}$ is the amount the government would like to borrow if it did not face the constraint $B_2 \leq S$. Note that if $B_2^{R,u} < 0$ then the government does not want to borrow a positive amount. The alternative for the government is to lend at the risk-free interest rate, $r = 0$. However, in that case lenders' choices do not affect the government's default decision. Since I am interested in self-fulfilling debt crises driven partially by lenders' expectations I make the following assumption.

Assumption 1 $B_2^{R,u}(A, r) \geq 0$ *for all* A *such that* $V^R(A, k_2, S; r)$ *is well-defined*

Once I characterize the fragility region I provide a sufficient condition for the parameters of the model which ensure that Assumption 1 is satisfied (see Lemma 14).

Lemma 6 *Let* $B_2^{R,u}(A, r)$ *be the government's desired choice of borrowing when the government decides to repay its debt. Then for all* $A \geq A$

$$B_2^{R,u}(A, r) = \begin{cases} \frac{(1+r)B_1 + \tau e^A f\left(k_2^R\right) - (1+r)\tau e^A f(k_1)}{2(1+r)} & \text{if } \Theta(A, r) \geq 0 \\ 0 & \text{if } \Theta(A, r) < 0 \end{cases}$$

where

$$\Theta(A) \equiv (1+r) B_1 + \tau e^A f\left(k_2^R\right) - (1+r)\tau e^A f(k_1)$$

Proof. $B_2^{R,u}$ maximizes

$$\max_{B_2} \log\left(\tau e^A f(k_1) - B_1 + B_2\right) + \log\left(\tau e^A f\left(k_2^R\right) - (1+r) B_2\right)$$

The F.O.C. associated with the above problem is given by

$$\frac{1}{\tau e^A f(k_1) - B_1 + B_2} - \frac{1+r}{\tau e^A f\left(k_2^R\right) - (1+r) B_2} = 0$$

Solving the above F.O.C. we obtain

$$B_2^{R,u}(A, r) = \frac{(1+r) B_1 + \tau e^A f(k_2) - (1+r) \tau e^A f(k_1)}{2(1+r)} \tag{20}$$

However, the interest rate, r, applies only to the government borrowing. That is, the above solution applies only to the case when $\Theta(A, r) > 0$ where $\Theta(A, r)$ is defined as

$$\Theta(A, r) \equiv (1+r) B_1 + \tau e^A f(k_2) - (1+r) \tau e^A f(k_1)$$

If $\Theta(A, r) \leq 0$ then the government does not want to borrow a strictly positive amount and its choice is either to lend at the risk-free interest rate $r = 0$ or to choose $B_2^{R,u}(A, r) = 0$. Assumption 1 implies that the government never wants to lend a positive amount at $r = 0$. Therefore, it must be the case that $B_2^{R,u}(A, r) = 0$. ■

In what follows, to simplify the notation, I suppress the dependence of $B_2^{R,u}$ on A and r whenever it does not lead to a confusion. I also assume that b is large enough so that for all A in the fragility region we have $B_2^{R,u}(A, r) < S$.

4.2 The Fragility Region

In this section, I prove the following result, which corresponds to Lemma 1 in the paper.

Lemma 7 *There exist $\underline{A}(r)$ and $\overline{A}$ such that for all $A \in \left[\underline{A}(r), \overline{A}\right]$ the government's default decision depends on households and lenders expectations.*

Proof. I prove this results in steps. First, I establish that government's incentive to repay its debt are increasing in households investment choices and lenders supply decisions (Lemma 8). Then, I derive the upper bound of the fragility region (Lemma 9). Finally, in a series of lemmas I prove existence and characterize $\underline{A}(r)$ and show that for all $r \geq 0$ we have $\underline{A}(r) < \overline{A}$ (Lemma 10-13). Together these results establish Lemma 7. ■

Recall that $V^R(A, k_2, S; r)$ denotes the value to the government of repaying its debt (see (19)). Let $V^D(A, k_2)$ be the value associated with government defaulting on its debt. Then

$$\begin{aligned} V^D(A, k_2) &= \sum_{t=1,2} \log\left(c_t^D\right) + \log\left(g_t^D\right) \\ \text{s.t. } g_1^D &= \tau Z e^A f(k_1) \\ g_2^D &= \tau Z e^A f(k_2) \\ c_1^D &= (1-\tau) Z e^A f(k_1) - k_2 \\ c_2^D &= (1-\tau) Z e^A f(k_2). \end{aligned}$$

The government repays its debt if and only if

$$\Delta V\left(A,k_2,S;r\right)=V^R\left(A,k_2,S;r\right)-V^D\left(A,k_2\right)\geq 0,$$

I start by establishing that the government's incentives to repay the debt are increasing in households' investment choices (for all $k_2\in\left[k_2^D,k_2^R\right]$) and with lenders supply decisions (as long as $B_2^{R,u}>0$ and $S\leq B_2^{R,u}$).

Lemma 8 *$\Delta V\left(A,k_2,S;r\right)$ is strictly increasing in k_2 for $k_2\in\left[k_2^D,k_2^R\right]$ and strictly increasing in S for $S\in\left[0,B_2^{R,u}\right]$ when $B_2^{R,u}>0$.*

Proof. The derivative of $\Delta V\left(A,k_2,S;r\right)$ with respect to k_2 is given by

$$\frac{\partial\Delta V\left(A,k_2,S;r\right)}{\partial k_2}=\frac{\left(1-Z\right)k_2}{\left(1-\tau\right)e^Af\left(k_2\right)-k_2}+\frac{\left(1+r\right)Z\tau e^Af'\left(k_2\right)}{\tau e^Af\left(k_2\right)-\left(1+r\right)B_2}>0.$$

This establishes the first part of the claim.

We show first that $\Delta V\left(A,k_2,S;r\right)$ is concave in B_2:

$$\frac{\partial\Delta V\left(A,k_2,S;r\right)}{\partial B_2^2}=\frac{1}{\tau e^Af\left(k_1\right)-B_1+B_2}-\frac{\left(1+r\right)}{\tau e^Af\left(k_2\right)-\left(1+r\right)B_2}$$

and

$$\frac{\partial^2\Delta V\left(A,k_2,S;r\right)}{\partial B_2^2}=-\frac{1}{\left[\tau e^Af\left(k_1\right)-B_1+B_2\right]^2}-\frac{\left(1+r\right)^2}{\left[\tau e^Af\left(k_2\right)-\left(1+r\right)B_2\right]^2}<0$$

It follows that if $B_2^{R,u}>0$ then the value of repaying is increasing in S for $S\in\left[0,B_2^R\right]$ as $B_2^{R,u}$ is an interior optimal choice of borrowing. ■

Next, I derive the upper bound for the fragility region, $\overline{A}$. I define $\overline{A}$ as a threshold such that for all $A\geq\overline{A}$ the government repays its debt even if lenders decide to supply no funds to the bond market ($S=0$) and households invest expecting default ($k_2=k_2^D$).

Lemma 9 *If households and lenders expect the government to default then the government repays its debt if and only if*

$$A\geq\overline{A},$$

where

$$\overline{A}=\log\left(\frac{B_1}{\tau f\left(k_1\right)\left[1-\frac{Z^4}{1+\alpha-Z\alpha}\right]}\right)\tag{21}$$

Proof. If households and lenders expect the government to default on its debt then the government chooses to repay its debt if and only if

$$V^R\left(A,k_2^D\left(A\right),0;r\right)-V^D\left(A,k_2^D\left(A\right)\right)\geq 0\tag{22}$$

Note that

$$\begin{aligned}V^R\left(A,k_2^D,0;r\right)&=\log\left(\left(1-\tau\right)e^Af\left(k_1\right)-k_2^D\left(A\right)\right)+\log\left(\left(1-\tau\right)e^Af\left(k_2^D\left(A\right)\right)\right)\\&\quad+\log\left(\tau e^Af\left(k_1\right)-B_1\right)+\log\left(\tau e^Af\left(k_2^D\right)\right)\end{aligned}$$

while

$$
\begin{aligned}
V^D\left(A, k_2^D\right) &= \log\left(Z\left(1-\tau\right)e^A f\left(k_1\right) - k_2^D\right) + \log\left(Z\left(1-\tau\right)e^A f\left(k_2^D\right)\right) \\
&\quad + \log\left(Z\tau e^A f\left(k_1\right)\right) + \log\left(Z\tau e^A f\left(k_2^D\right)\right)
\end{aligned}
$$

Using the above observation and the definition of k_2^D, Equation (22) can be simplified to

$$
\log\left(\frac{1+\alpha-Z\alpha}{Z^4}\right) + \log\left(\frac{\tau e^A f\left(k_1\right) - B_1}{Z\tau e^A f\left(k_1\right)}\right) \geq 0 \tag{23}
$$

Note that the LHS of 23 is strictly increasing in A and denote by $\overline{A}$ the value of productivity at which the LHS is equal to zero. Computing $\overline{A}$ establishes the claim. ■

Next, I characterize the lower bound of the fragility region, $\underline{A}\left(r\right)$. Recall that $\underline{A}\left(r\right)$ is the value of productivity such that for all $A < \underline{A}\left(r\right)$ the government defaults on its debt even if lenders decide to supply all their funds to the bond market ($S = b$) and households invest expecting the government to repay its debt. In what follows, I make the following assumption.

Assumption 2 *For all $A \leq \overline{A}$ we have $B_2^{R,u}\left(A, r\right) \leq b$.*

Assumption 2 states that for all $A < \overline{A}$ the government is able to borrow its desired amount if all lenders decide to supply their funds to the bond market. This assumption is satisfied if b is large enough and is made for simplicity.[13] A reader can easily verify that all arguments presented below continue to hold with only slight modifications when Assumption 2 is not imposed.[14] Given Assumption 2, I slightly abuse the notation and write the condition that determines $\underline{A}\left(r\right)$ as

$$
\Delta V\left(A, k_2^R\left(A\right), B_2^{R,u}\left(A, r\right); r\right) = 0 \tag{24}
$$

since under Assumption 2 the government is able to borrow $B_2^{R,u}\left(A, r\right)$ for all $A \leq \overline{A}$.

Lemma 10 *There exists a solution to Equation* (24)*.*

Proof. First, note that

$$
\Delta V\left(A, k_2^R\left(A\right), B_2^{R,u}\left(A, r\right); r\right) \geq \Delta V\left(A, k_2^R\left(A\right), 0; r\right)
$$

It is straightforward to show that

$$
\Delta V\left(A, k_2^R, 0; r\right) = \log\left(\frac{1}{Z^3\left(Z\left(1+\alpha\right)-\alpha\right)}\right) + \log\left(\frac{\tau e^A f\left(k_1\right) - B_1}{\tau e^A f\left(k_1\right)}\right)
$$

Therefore,

$$
\lim_{A\to\infty} \Delta V\left(A, k_2^R, B_2^R\left(A.r\right); r\right) \geq \lim_{A\to\infty} \Delta V\left(A, k_2^R, 0; r\right) = \log\left(\frac{1}{Z^3\left(Z\left(1+\alpha\right)-\alpha\right)}\right) > 0
$$

Thus, for a sufficiently high A we have $\Delta V\left(A, k_2^R\left(A\right), B_2^{R,u}\left(A, r\right); r\right) > 0$.

[13] In the absence of Assumption 2, when deriving $\underline{A}\left(r\right)$ we would need to consider three possibilities: (1) at $\underline{A}\left(r\right)$ the government borrows $B_2^{R,u} > 0$, (2) at $\underline{A}\left(r\right)$ the government borrows S, and (3) at $\underline{A}\left(r\right)$ the government borrows $B_2^{R,u} = 0$. Under Assumption 2 we only need to consider cases (1) and (3).

[14] To be precise, the only part of the argument presented below that needs to be modified is part 1 of the proof of Lemma 12 where one needs to use $\max\left\{B_2^{R,u}\left(A, r\right), S\right\}$ rather than $B_2^{R,u}\left(A, r\right)$ for all $A \leq \overline{A}$.

Next, let $\widehat{B}_2(A,r)$ by the government's optimal choice of borrowing when we allow the government to both borrow and lend at the interest rate $r \geq 0$.[15] Then

$$\Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right) \leq \Delta V\left(A, k_2^R(A), \widehat{B}_2(A,r); r\right)$$

From the definition of $\widehat{B}_2(A,r)$ we have

$$\tau e^A f(k_1) - B_1 + \widehat{B}_2(A,r) = \frac{(1+r)\tau e^A f(k_1) + \tau e^A f(k_2) - (1+r) B_1}{2(1+r)}$$

and

$$\tau e^A f\left(k_2^R\right) - (1+r)\widehat{B}_2(A,r) = \frac{(1+r)\tau e^A f(k_1) + \tau e^A f(k_2) - (1+r) B_1}{2}$$

Therefore,

$$\begin{aligned}
\Delta V\left(A, k_2^R(A), B_2(A,r); r\right) &= \log\left((1-\tau) e^A f(k_1) - k_2^R\right) + \log\left((1-\tau) e^A f\left(k_2^R\right)\right) \\
&\quad + \log\left(\frac{(1+r)\tau e^A f(k_1) + \tau e^A f(k_2) - (1+r) B_1}{2(1+r)}\right) \\
&\quad + \log\left(\frac{(1+r)\tau e^A f(k_1) + \tau e^A f(k_2) - (1+r) B_1}{2}\right) \\
&\quad - \log\left(Z(1-\tau) e^A f(k_1) - k_2^R\right) - \log\left(Z(1-\tau) e^A f\left(k_2^R\right)\right) \\
&\quad - \log\left(Z\tau e^A f(k_1)\right) - \log\left(Z\tau e^A f\left(k_2^R\right)\right)
\end{aligned}$$

Let $\widehat{A}$ be the unique solution to

$$(1+r)\tau e^A f(k_1) + \tau e^A f\left(k_2^R\right) - (1+r) B_1 = 0 \tag{25}$$

We see that $\Delta V\left(A, k_2^R(A), \widehat{B}_2(A,r); r\right)$ is only well-define for $A > \widehat{A}$ and that $\Delta V\left(A, k_2^R(A), \widehat{B}_2(A,r); r\right)$ approaches negative infinity when $A \searrow \widehat{A}$. Since

$$\Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right) \leq \Delta V\left(A, k_2^R(A), \widehat{B}_2(A,r); r\right)$$

it follows that there exists $\underline{\underline{A}} \geq \widehat{A}$ such that: (1) $\Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right)$ is defined only for $A > \underline{\underline{A}}$ and (2) $\lim_{A \searrow \underline{\underline{A}}} \Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right) < 0$ Thus, $\Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right)$ is negative for small enough values of A. We conclude that there exists a solution to $\Delta V\left(A, k_2^R(A), B_2^{R,u}(A); r\right) = 0$.
■

Corollary 11 *Let*

$$\overline{A}^R = \frac{B_1}{\tau f(k_1)\left[1 - Z^3(Z(1+\alpha) - \alpha)\right]}$$

Then for all $A \geq \overline{A}^R$ we have

$$\Delta V\left(A, B_2^R(A,r), k_2^R(A); r\right) \geq 0$$

[15] Recall that the government can borrow at the interest rate r but lend only at the interest rate of zero. Thus, $\widehat{B}$ is a hypothetical borrowing when the government faces the same interest rate when borrowing and when lending and is given by

$$\widehat{B}_2(A,r) = \frac{(1+r) B_1 + \tau e^A f\left(k_2^R\right) - (1+r)\tau e^A f(k_1)}{2(1+r)}$$

Proof. As shown in the proof of Lemma 10 we have

$$\Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right) \geq \Delta V\left(A, k_2^R(A), 0; r\right) = \log\left(\frac{1}{Z^3(Z(1+\alpha)-\alpha)}\right) + \log\left(\frac{\tau e^A f(k_1) - B_1}{\tau e^A f(k_1)}\right)$$

It is immediate to see that the last expression is increasing in A and intersect the zero line at

$$\overline{A}^R = \frac{B_1}{\tau f(k_1)\left[1 - Z^3(Z(1+\alpha)-\alpha)\right]}$$

■

The next proposition characterizes the solution to Equation (24).

Lemma 12 *There exists a unique* $\underline{A}(r)$ *that solves*

$$\Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right) = 0 \tag{26}$$

Moreover,

$$\underline{A}(r) = \begin{cases} \overline{A}^R & \text{if } r \geq \overline{r} \\ \underline{A}^R(r) & \text{if } r \in [0, \overline{r}) \end{cases}, \tag{27}$$

where

$$\begin{aligned} \overline{A}^R &= \frac{B_1}{\tau f(k_1)\left[1 - Z^3(Z(1+\alpha)-\alpha)\right]} \\ \overline{r} &= \frac{1}{Z^3(Z(1+\alpha)-\alpha)} \frac{f\left(k_2^R\left(\overline{A}^R\right)\right)}{f(k_1)} - 1 \end{aligned}$$

and for all $r \in [0, \overline{r})$ *the threshold* $\underline{A}^R(r)$ *is the unique solution to*

$$(1+r)\tau e^A f(k_1) + \tau e^A f\left(k_2^R\right) - (1+r)B_1 - 2\tau e^A \sqrt{Z^3(Z(1+\alpha)-\alpha)(1+r) f(k_1) f\left(k_2^R\right)} = 0$$

Proof. (Part 1: $r < \overline{r}$) Consider first the case where $r < \overline{r}$ and note that by Corollary 11 we have $\Delta V\left(A, k_2^R(A), 0; r\right) \geq 0$ for all $A \geq \overline{A}^R$ with a strict inequality when $A > \overline{A}^R$. Next, note that $r < \overline{r}$ implies

$$(1+r) Z^3(Z(1+\alpha)-\alpha) f(k_1) < f\left(k_2^R\left(\overline{A}^R\right)\right) \tag{28}$$

From the above inequality and the definition of $\overline{A}^R$ we obtain

$$(1+r)B_1 + \tau e^{\overline{A}^R} f\left(k_2^R\left(\overline{A}^R\right)\right) - (1+r)\tau e^{\overline{A}^R} f(k_1) > 0$$

implying that $B_2^{R,u}\left(\overline{A}^R, r\right) > 0$ for all $r < \overline{r}$. It follows that

$$\Delta V\left(\overline{A}^R, k_2^R\left(\overline{A}^R\right), B_2^R\left(\overline{A}^R, r\right); r\right) > \Delta V\left(\overline{A}^R, k_2^R\left(\overline{A}^R\right), 0; r\right) = 0$$

Since $\Delta V\left(A, k_2^R(A), B_2^R(A,r); r\right) \geq \Delta V\left(A, k_2^R(A), 0; r\right) > 0$ for all $A > \overline{A}^R$. Hence, for all $A \geq \overline{A}$ we have $\Delta V\left(A, k_2^R(A), B_2^{R,u}(A,r); r\right) > 0$. We conclude that if $r < \overline{r}$ then (1) $\underline{A}(r) < \overline{A}^R$ and (2) at $\underline{A}(r)$ we must have $B_2^{R,u}(\underline{A}(r), r) > 0$ (as $\Delta V\left(A, k_2^R(A), 0; r\right) < 0$ for all $A < \overline{A}^R$; see Corollary 11).

From the above discussion we know that at any A that satisfies Equation (26) we have $B_2^{R,u}(A,r) > 0$. Therefore, Equation (26) can be simplified to

$$\log\left(\frac{\left[(1+r)\,\tau e^{A}f(k_1)+\tau e^{A}f\left(k_2^R\right)-(1+r)\,B_1\right]^2}{4Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,\tau e^{A}f(k_1)\,\tau e^{A}f\left(k_2^R\right)}\right)=0 \tag{29}$$

Let $\underline{A}^R(r)$ denote the solution to this equation (which exists by Lemma 10). By the earlier argument, we have $B_2^{R,u}\left(\underline{A}^R(r),r\right) > 0$. By the continuity of $B_2^{R,u}(A,r)$, we have that $(1+r)\,\tau e^{A}f(k_1)+\tau e^{A}f\left(k_2^R\right)-(1+r)\,B_1 > 0$ in the neighborhood of $\underline{A}^R(r)$. Thus, we can rewrite Equation (29) as

$$(1+r)\,\tau e^{A}f(k_1)+\tau e^{A}f\left(k_2^R\right)-(1+r)\,B_1-2\tau e^{A}\sqrt{Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,f(k_1)\,f\left(k_2^R\right)}=0 \tag{30}$$

so that $\underline{A}^R(r)$ is also a solution to Equation (30). The derivative of the LHS of Equation (30) is

$$(1+r)\,\tau e^{A}f(k_1)+(1+\alpha)\,\tau e^{A}f\left(k_2^R\right)-(2+\alpha)\,\tau e^{A}\sqrt{Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,f(k_1)\,f\left(k_2^R\right)}$$

Evaluating the above derivative at $A=\underline{A}^R(r)$ and simplifying the resulting equation we obtain

$$(1+r)\,B_1+\alpha\tau e^{A}f\left(k_2^R\right)-\alpha\tau e^{A}\sqrt{Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,f(k_1)\,f\left(k_2^R\right)}$$

Since $\underline{A}^R(r) < \overline{A}^R$ and $B_2^{R,u}\left(\underline{A}^R(r),r\right) > 0$ we have

$$Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,f(k_1) < f\left(k_2^R\right),$$

which implies that the derivative of the LHS of Equation (30) evaluated at $\underline{A}^R(r)$ is positive. It follows that the LHS of (30) and, hence, he LHS of (29), intersects the zero line from below at $\underline{A}^R(r)$. Therefore, we conclude that if $r < \overline{r}$ then $\underline{A}^R(r)$ is unique solution to Equation (26) and is implicitly defined by Equation (30).

(Part 2: $r \geq \overline{r}$) Note that $r \geq \overline{r}$ implies that

$$Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,f(k_1) \geq f\left(k_2^R\left(\overline{A}^R\right)\right) \tag{31}$$

and, hence, $B_2^{R,u}\left(\overline{A}^R,r\right)=0$. Therefore, by Corollary 11, we conclude that

$$\Delta V\left(\overline{A}^R,k_2^R\left(\overline{A}^R\right),B_2^{R,u}\left(\overline{A}^R,r\right);r\right)=0$$

It remains to show that there is no other solution to Equation (26). That there is no $A > \overline{A}^R$ that satisfies Equation (26) follows from Corollary 11. To see that there is no such $A < \overline{A}^R$ suppose to the contrary that such A exists and call this level of productivity A'. Since $A' < \overline{A}^R$ it has to be the case that $B_2^{R,u}(A',r) > 0$. It is straightforward to show that $\Delta V\left(A',k_2^R(A'),B_2^{R,u}(A',r);r\right)=0$ and $B_2^{R,u}(A',r) > 0$ imply that

$$Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,f(k_1) < f\left(k_2^R(A')\right) \tag{32}$$

However, note that since $k_2^R(A)$ is increasing in A, inequality (31) implies that for all $A < \overline{A}^R$ we have

$$Z^3\left(Z(1+\alpha)-\alpha\right)(1+r)\,f(k_1) > f\left(k_2^R(A)\right),$$

which contradicts inequality (32). Thus, we conclude that there is no $A' < \overline{A}^R$ that solves Equation (26). This establishes the claim. ■

The next result shows that $\underline{A}(r) < \overline{A}$ so that the fragility region indeed exists.

Lemma 13 *We have $\overline{A} > \underline{A}(r)$ for all $r \geq 0$.*

Proof. Note that $\underline{A}(r) < \overline{A}^R$, thus it is enough to show that $\overline{A}^R < \overline{A}$. This follows since

$$Z^3 (Z(1+\alpha) - a) - \frac{Z^4}{1+\alpha - Z\alpha} = \frac{\alpha(1+\alpha) Z^3 \left(1 - 2Z + Z^2\right)}{1+\alpha - Z\alpha} < 0$$

■

4.3 Derivations of bounds for B_1 and Z

It remains to find conditions under which Assumption 1 holds.

Lemma 14 *A sufficient condition for $B^R(A, r) \geq 0$ is that the initial debt B_1 is greater than $\underline{B}_1$, where*

$$\underline{B}_1 \equiv \frac{1+\alpha}{\alpha} \frac{\tau}{(1-\tau)} k_1$$

Proof. Note that the government budget constraint in period 1, in the case the government repays its debt, is given by $g_1 = \tau Y_1^R - B_1 + B_2$. Since $g_1 \geq 0$, B_2 can be negative only if $\tau Y_1^R > B_1$ or

$$e^A > \frac{B_1}{\tau f(k_1)}.$$

Since k_2 is increasing in A and since at $r = 0$ the government would choose

$$B_2 = \frac{B_1 + \tau e^A f(k_2) - \tau e^A f(k_1)}{2} \tag{33}$$

it is enough to show that for all $A \geq \log(B_1 / (\tau f(k_1)))$ the government wants to borrow a positive amount.

Recall that $k_2^R(A)$ is increasing in A. Therefore, from Equation (33) we conclude that it is enough to find condition under which

$$f\left(k_2^R(A)\right) > f(k_1) \tag{34}$$

when $A = \log(B_1 / (\tau f(k_1)))$. Using the definition of k_2^R and setting $A = \log(B_1 / (\tau f(k_1)))$ inequality in (34) becomes

$$\frac{\alpha}{1+\alpha} \frac{(1-\tau)}{\tau} B_1 > k_1$$

Setting

$$\underline{B}_1 \equiv \frac{1+\alpha}{\alpha} \frac{\tau}{(1-\tau)} k_1$$

establishes the claim. ■

Finally, I establish a condition which ensures that for all $r \in [0, \overline{r}]$ and $A \geq \underline{A}^R(r)$ the government unconstrained desired borrowing is increasing in A.

Lemma 1 *There exists $\underline{Z} \in (0,1)$ such that $\frac{\partial B_2^{R,u}}{\partial A} \geq 0$ for all $r \in [0, \overline{r}]$ and $A \geq \underline{A}^R(r)$.*

Proof. Fix $r \in [0, \overline{r}]$. We have

$$\frac{\partial B_2^{R,u}(A,r)}{\partial A} = \frac{(1+\alpha)\tau e^A f(k_2(A)) - (1+r)\tau e^A f(k_1)}{2(1+r)}$$

Therefore, $\frac{\partial B_2^{u}(A,r)}{\partial A} > 0$ if and only if

$$(1+\alpha) f(k_2(A)) \geq (1+r) f(k_1)$$

Moreover, we know that $k_2(A) \geq k_2^D(A)$, where $k_2^D(A) = Z^\alpha k_2^R$. Therefore, a sufficient condition for $\frac{\partial B_2^{R,u}(A,r)}{\partial A} > 0$ is that

$$(1+\alpha) Z^\alpha f\left(k_2^R(A)\right) \geq (1+r) f(k_1) \tag{35}$$

Note that the LHS of inequality 35 is increasing in A. Therefore, it is enough to find a condition such that inequality 35 holds at $A = \underline{A}^R(r)$. From the proof of Lemma 12 we know that at $A = \underline{A}^R(r)$ we have

$$f\left(k_2^R(A)\right) \geq Z^3 (Z(1+\alpha) - \alpha)(1+r) f(k_1)$$

Using this observation in inequality 35 we see that the sufficient condition for $\frac{\partial B_2^{R,u}(A,r)}{\partial A}$ to be positive for all $A \geq \underline{A}^R(r)$ is that

$$(1+\alpha) Z^{3+\alpha} (Z(1+\alpha) - \alpha) - 1 \geq 0 \tag{36}$$

Note that the LHS of inequality 36 is strictly increasing in Z, negative at $Z = 0$ and positive at $Z = 1$. Therefore, there exists a unique $\underline{Z} \in (0,1)$ such that for all $Z \geq \underline{Z}$. The claim follows from the observation that the same above argument holds for any $r \in [0, \overline{r}]$. ■

5 Decomposition of $dA^*/d\psi$ under policy uncertainty

In this section I derive the change in the default threshold when households and lenders are uncertain whether the policy change will be implemented and assign probability p to the government implementing the new policy. This requires solving an extended system of equilibrium condition as explained below.

Let A^* be the threshold if the policy parameter takes value is not implemented and $A^{*\prime}$ be the policy threshold when policy is implemented. Similarly, let ψ be the value of policy parameter if the policy is not implemented and ψ' be the policy parameter if the policy is implemented.[16] Then the equilibrium conditions can be written as

$$
\begin{aligned}
(1-p)\,I\left(A^*+\kappa\varepsilon, A^*, \underline{k}_2^*(\kappa), \psi\right) + pI\left(A^*+\kappa\varepsilon, A^{*\prime}, \underline{k}_2^*(\kappa), \psi'\right) &= 0 \quad (37)\\
(1-p)\,I\left(A^{*\prime}+\kappa\varepsilon, A^*, \underline{k}_2^{*\prime}(\kappa), \psi\right) + pI\left(A^{*\prime}+\kappa\varepsilon, A^{*\prime}, \underline{k}_2^{*\prime}(\kappa), \psi'\right) &= 0 \quad (38)\\
(1-p)\,L\left(A^*, x^*, \psi\right) + pL\left(A^{*\prime}, x^*, \psi'\right) &= 0 \quad (39)\\
\Delta V\left(A^*; \mathbf{k}_2^*, x^*, A^*, \psi\right) &= 0 \quad (40)\\
\Delta V\left(A^{*\prime}; \mathbf{k}_2^*, x^*, A^{*\prime}, \psi'\right) &= 0 \quad (41)
\end{aligned}
$$

where $\underline{k}_2^*$ denotes the households' equilibrium investment when the underlying productivity is equal $\underline{A}^*$ while k_2^* denotes the households' equilibrium investment when the underlying productivity is equal A^*.

The above five equations describe the equilibrium when agents' are uncertain whether a policy parameter will take value ψ (with probability $1-p$) or ψ (with probability p). With the uncertainty regarding policy implementation there are five equilibrium conditions instead of three used in model without political uncertainty. This is because we need now to determine the default threshold both when the policy in implemented and when the policy is not implemented (the possibility of a policy change also affects the threshold even if the policy is not implemented in the end). In particular, to compute the equilibrium default threshold when the policy parameter takes value ψ I need the government default conditions and the household investment decision evaluated both evaluated at ψ (equations 37 and 40). Similarly, to compute the equilibrium default threshold when the policy parameter takes value ψ' I need the government default condition and the household condition both evaluated both evaluated at ψ (equations 38 and 41).

I follow the same approach as when deriving the beliefs effect under no uncertainty (Section B of the appendix to the paper) starting by considering the total derivatives of all equilibrium equations with respect to ψ'. Let I_i^{**} denote the partial derivative of $I(\cdot)$ with respect to its ith argument evaluated at the productivity level A^* and when households is $A^{**}=A^*$, $I_i^{**\prime}$ denote the partial derivative of $I(\cdot)$ with respect to its ith argument evaluated at the productivity level A^* and when households is $A^{**}=A^{*\prime}$, $I_i^{*\prime*}$ when productivity is $A^{*\prime}$ and households' belief is $A^{**}=A^*$ and, finally, $I_i^{*\prime*\prime}$ when productivity is $A^{*\prime}$ and households' belief is $A^{**}=A^{*\prime}$.

[16] For example if the relevant policy parameter is a tax rate τ and the government contemplates increasing tax rate to $\tau' > \tau$ then $\psi=\tau$ while $\psi'=\tau'$.

$$
\begin{aligned}
(1-p)\left[I_1^{**}\frac{dA^*}{d\psi'}+I_2^{**}\frac{dA^{**}}{d\psi'}+I_3^{**}\frac{dk_2^*}{d\psi'}\right]+pI\left(I_1^{**\prime}\frac{dA^*}{d\psi'}+I_2^{**\prime}\frac{dA^{**}}{d\psi'}+I_3^{**\prime}\frac{dk_2^*}{d\psi'}+I_4^*\right) &= 0\\
(1-p)\left[I_1^{*\prime*}\frac{dA^*}{d\psi'}+I_2^{*\prime*}\frac{dA^{**}}{d\psi'}+I_3^{*\prime*}\frac{dk_2^{*\prime}}{d\psi'}\right]+p\left(I_1^{*\prime*\prime}\frac{dA^*}{d\psi'}+I_2^{*\prime*\prime}\frac{dA^{**}}{d\psi'}+I_3^{*\prime*\prime}\frac{dk_2^{*\prime}}{d\psi'}+I_4^{*\prime*\prime}\right) &= 0\\
(1-p)\left[L_1\frac{dA^{**}}{d\psi'}+L_2\frac{dx^*}{d\psi'}\right]+p\left[L_1\frac{dA^{**\prime}}{d\psi'}+L_2\frac{dx^*}{d\psi'}+L_3\right] &= 0\\
\Delta V_1^*\frac{dA^*}{d\psi'}+\Delta V_2^*\int_{-1}^{1}\frac{1}{2}\frac{dk_2^*(\kappa)}{d\psi'}d\kappa+\Delta V_3^*\frac{dx^*}{d\psi'} &= 0\\
\Delta V_1^{*\prime}\frac{dA^{*\prime}}{d\psi'}+\Delta V_2^{*\prime}\int_{-1}^{1}\frac{1}{2}\frac{dk_2^{*\prime}(\kappa)}{d\psi'}d\kappa+\Delta V_3^{*\prime}\frac{dx^*}{d\psi'}+\Delta V_4^{*\prime} &= 0
\end{aligned}
$$

The above equations constitutes the system of five linear equations in five unknowns: $dk_2^*/d\psi'$, $dk_2^{*\prime}/d\psi'$, $dx^*/d\psi'$, $dA^*/d\psi'$, $dA^{*\prime}/d\psi'$. Solving the above system, and using the fact that we consider the change in ψ' where initially $\psi'=\psi$ (and hence $A^{*\prime}=A^*$) we obtain

$$
\frac{dA^{*\prime}}{d\psi'}=\frac{\frac{\partial A^*}{\partial\psi'}+p\left[\frac{\partial A^*}{\partial\mathbf{k}_2^{*\prime}}\frac{\partial\mathbf{k}_2^{*\prime}}{\partial\psi'}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial\psi'}\right]\frac{1}{1-(1-p)\frac{\partial A^*}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}}{1-p\left[\frac{\partial A^*}{\partial\mathbf{k}_2^{*\prime}}\frac{\partial\mathbf{k}_2^{*\prime}}{\partial A^{**\prime}}+\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}\right]\frac{1}{1-(1-p)\frac{\partial A^*}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}}
$$

and

$$
\frac{dA^*}{d\psi'}=\frac{p\frac{\partial A^*}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial\psi'}+p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial\psi'}}{1-(1-p)\frac{\partial A^*}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}+\frac{p\frac{\partial A^*}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}+p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}{1-(1-p)\frac{\partial A^*}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}\frac{dA^{**\prime}}{d\psi'},
$$

where each partial effect is defined in an analogous way as in the case of no policy uncertainty. It follows that the change in $dA^{*\prime}/d\psi'$ and $dA^*/d\psi'$ can be compared directly with the change in the default threshold under no uncertainty. Below I provide detailed derivations of the above equations.

5.1 Detailed Derivations

Start with $dx^*/d\psi'$.

$$
(1-p)\left[L_1^{**}\frac{dA^{**}}{d\psi'}+L_2^{**}\frac{dx^*}{d\psi'}\right]+p\left[L_1^{**\prime}\frac{dA^{**\prime}}{d\psi'}+L_2^{**\prime}\frac{dx^*}{d\psi'}+L_3^{**\prime}\right]=0
$$

Rearranging the above equation we obtain

$$
\frac{dx^*}{d\psi'}=-\frac{(1-p)L_1^{**}}{(1-p)L_2^{**}+pL_2^{**\prime}}\frac{dA^{**}}{d\psi'}-\frac{pL_1^{**\prime}}{(1-p)L_2^{**}+pL_2^{**\prime}}\frac{dA^{**\prime}}{d\psi'}-\frac{pL_3^{**\prime}}{(1-p)L_2^{**}+pL_2^{**\prime}}.
$$

Note that I am interested in computing the effect of a change in ψ' from initial level of $\psi'=\psi$. When $\psi'=\psi$ then $A^{*\prime}=A^*$ and hence it follows that $L_2^{**}=L_2^{**\prime}$ and $L_1^{**}=L_1^{**\prime}$. Thus, the above condition simplifies to

$$
\frac{dx^*}{d\psi'}=(1-p)\frac{L_1^{**}}{L_2^{**}}\frac{dA^{**}}{d\psi'}+p\frac{L_1^{**\prime}}{L_2^{**}}\frac{dA^{**\prime}}{d\psi'}+\frac{pL_3^{**\prime}}{L_2^{**}}.
$$

But then it follows that

$$\frac{dx^*}{d\psi'} = (1-p)\left[\frac{\partial x^*}{\partial A^{**}}\right]^c \frac{dA^{**}}{d\psi'} + p\left[\frac{\partial x^*}{\partial A^{**\prime}}\right]^c \frac{dA^{**\prime}}{d\psi'} + p\left[\frac{\partial x^*}{\partial \psi}\right]^c$$

where the superscript c indicates that a given partial derivative is defined as the corresponding partial derivative under no uncertainty.[17] In what follows I define all partial derivatives in the same way they were defined under no policy uncertainty and, to save on notation, I omit the superscript c. Thus, I write

$$\frac{dx^*}{d\psi'} = (1-p)\frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} + p\frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial x^*}{\partial \psi},$$

where it should be implicitly understood that $\partial x^*/\partial A^{**}$ corresponds to the partial effect of a change in belief regarding A^{**} under no policy uncertainty and so on. Writing all the partial effects in this way make it easy to compare both the derivations and the final results under policy uncertainty with the derivations and results under no policy uncertainty. I will use the same convention below.

Next, consider $dk_2^*/d\psi$ and note that when $\psi' = \psi$ then $A^{*\prime} = A^*$ and hence it follows that $I_1^{**} = I_1^{**\prime}$, $I_2^{**} = I_2^{**\prime}$, $I_3^{**} = I_3^{**\prime}$ and $I_4^{**} = I_4^{**\prime}$. Therefore, one can write $dk_2^*/d\psi'$ as

$$\frac{dk_2^*}{d\psi'} = -(1-p)\frac{\partial k_2^*}{\partial A^*}\frac{dA^*}{d\psi'} - (1-p)\frac{\partial k_2}{\partial A^{**}}\frac{dA^{**}}{d\psi'} - p\frac{\partial k_2^*}{\partial A^*}\frac{dA^*}{d\psi'} - p\frac{\partial k_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} - p\frac{\partial k_2^*}{\partial \psi'}$$

where the partial effects are defined in the same way as under no uncertainty (see the discussion above and footnote).

Now, substitute these expressions into the equation for $\frac{dA^*}{d\psi'}$:

$$\Delta V_1^* \frac{dA}{d\psi'} = -\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{dk_2^*(\kappa)}{d\psi'}d\kappa - \Delta V_3^* \frac{dx^*}{d\psi'}$$

[17] To be precise, let superscript u denote the partial effect of a given derivative under policy uncertainty and c denote the partial effect under no uncertainty (or certainty) about policy change. In addition, let $\psi' = \psi$. Then, the partial effect of an increase in ψ' on x^* under policy uncertainty is

$$\begin{aligned}\frac{\partial x^*}{\partial \psi'}^u &= -p\frac{L_3^{**\prime}}{(1-p)L_2^{**} + pL_2^{**\prime}} = -p\frac{\frac{\partial}{\partial \psi'}L(A^{**\prime}, x^*, \psi')\Big|_{\{A^{**\prime}=A^*, \psi'=\psi\}}}{(1-p)\frac{\partial}{\partial x^*}L(A^{**}, x^*, \psi)\Big|_{A^{**}=A^*} + \frac{\partial}{\partial x^*}pL(A^{**\prime}, x^*, \psi')\Big|_{\{A^{**\prime}=A^*, \psi'=\psi\}}} \\ &= -p\frac{L_1(A^*, x^*, \psi)}{L_2(A^*, x^*, \psi)}\end{aligned}$$

since in equilibrium, when $\psi = \psi'$, we have $A^{**\prime} = A^{**} = A^*$. On the other hand

$$\frac{\partial x^*}{\partial \psi'}^c = -\frac{\frac{\partial}{\partial \psi}L(A^{**}, x^*, \psi)\Big|_{\{A^{**\prime}=A^*\}}}{\frac{\partial}{\partial x^*}L(A^{**}, x^*, \psi)\Big|_{\{A^{**\prime}=A^*\}}} = -\frac{L_1(A^*, x^*, \psi)}{L_2(A^*, x^*, \psi)}.$$

Comparing the above expressions we note that

$$\frac{\partial x^*}{\partial \psi'}^u = p\frac{\partial x^*}{\partial \psi'}^c.$$

In a similar fashion, one can show that

$$\frac{\partial x^*}{\partial A^{**}}^u = (1-p)\frac{\partial x^*}{\partial A^{**}}^c$$

and so on.

so that

$$\Delta V_1^* \frac{dA^*}{d\psi'} = -(1-p)\,\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\left[\frac{\partial k_2^*}{\partial A^*}\frac{dA^*}{d\psi'} + \frac{\partial k_2}{\partial A^{**}}\frac{dA^{**}}{d\psi'}\right] d\kappa$$
$$-p\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\left[\frac{\partial k_2^*}{\partial A^*}\frac{dA^*}{d\psi'} + \frac{\partial k_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + \frac{\partial k_2^*}{\partial \psi'}\right] d\kappa$$
$$-(1-p)\,\Delta V_3^* \frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} - p\Delta V_3^* \frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} - p\Delta V_3^* \frac{\partial x^*}{\partial \psi}$$

Rearranging, so that all the terms that include $dA^*/d\psi'$ are on the left, I get

$$\left[\Delta V_1^* + (1-p)\,\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa + p\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa\right] \frac{dA^*}{d\psi'}$$
$$= -(1-p)\,\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} d\kappa$$
$$-p\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\left[\frac{\partial k_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + \frac{\partial k_2^*}{\partial \psi'}\right] d\kappa$$
$$-(1-p)\,\Delta V_3^* \frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} - p\Delta V_3^* \frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} - p\Delta V_3^* \frac{\partial x^*}{\partial \psi}$$

Dividing both sides by $\left[\Delta V_1^* + \Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa\right]$ we get

$$\frac{dA^*}{d\psi'} = \frac{-(1-p)\,\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} d\kappa - p\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\left[\frac{\partial k_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + \frac{\partial k_2^*}{\partial \psi'}\right] d\kappa}{\left[\Delta V_1^* + \Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa\right]}$$
$$+\frac{-(1-p)\,\Delta V_3^* \frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} - p\Delta V_3^* \frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} - p\Delta V_3^* \frac{\partial x^*}{\partial \psi}}{\left[\Delta V_1^* + \Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa\right]}$$

Note that the second term on the right hand side in the above expression has a natural interpretation as

$$(1-p)\,\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi'},$$

i.e., it captures the effect of a change in lenders' strategy (due to change in lenders' beliefs and the change in the parameter ψ). Similarly, the first term, captures the effect of a change in the households' strategies.

As in Section B of the appendix to the paper, and slightly abusing notation, I define

$$\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} \equiv -\frac{\Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^{**}} d\kappa}{\left[\Delta V_1^* + \Delta V_2^* \int_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa\right]}\frac{dA^{**}}{d\psi'}$$

$$\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} \equiv -\frac{\Delta V_2^* \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^{**\prime}} d\kappa}{\left[\Delta V_1^* + \Delta V_2^* \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa\right]}\frac{dA^{**\prime}}{d\psi'}$$

and

$$\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial \psi'} \equiv -\frac{\Delta V_2^* \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial \psi'} d\kappa}{\left[\Delta V_1^* + \Delta V_2^* \int\limits_{-1}^{1} \frac{1}{2}\frac{\partial k_2^*}{\partial A^*} d\kappa\right]}.$$

With the above definitions, one can write the expression for $dA^*/d\psi'$ as

$$\begin{aligned}\frac{dA^*}{d\psi'} &= (1-p)\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} + p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial \psi'} \\ &\quad + (1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\frac{dA^{**}}{d\psi'} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi'}\end{aligned}$$

In equilibrium it always has to be the case that $A^{**} = A^*$ and, thus, $dA^*/d\psi' = dA^{**}/d\psi'$. Therefore, the above expression for $dA^*/d\psi'$ can be written as

$$\begin{aligned}\left[1-(1-p)\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}} - (1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\right]\frac{dA^*}{d\psi'} &= p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial \psi'} \\ &\quad + p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial \psi'}\end{aligned}$$

or

$$\frac{dA^*}{d\psi'} = \frac{p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial \psi'}}{1-(1-p)\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}} - (1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}} + \frac{p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}\frac{dA^{**\prime}}{d\psi'} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi'}}{1-(1-p)\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}} - (1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}$$

Rearranging, we note that

$$\frac{dA^*}{d\psi'} = \frac{p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial \psi'} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi'}}{1-(1-p)\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}} - (1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}} + \frac{p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**\prime}} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}{1-(1-p)\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}} - (1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}\frac{dA^{**\prime}}{d\psi'}.$$

Thus, we see that even if the policy is not implemented, there will be change in the default threshold. The total change in the default threshold when the policy is not implemented in the end consists of two terms. The first term captures the direct and the beliefs effect of a policy change. In particular, a change in ψ with probability p affects directly households' and lenders' strategy choices leading. This is captured by $p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial \psi'}$ and $p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi'}$. Note that because of the uncertainty, this effect is a fraction p of the effect a policy change has when the change is certain. This initial change in the default thresholds makes households and lenders change their beliefs leading to the already familiar multiplier effect which is equal to $1-(1-p)\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**}} - (1-p)\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}$. The second term captures the new effect that was not present earlier. In particular, a change in ψ will leads to a different threshold in the default threshold when the policy is implemented. In equilibrium, this leads to a change in households' and lenders' beliefs regarding $A^{**\prime}$, the resulting default threshold when the policy is implemented, leading to an adjustment in households' and lenders' strategies. This is captured by $\left[p\frac{\partial A^*}{\partial \mathbf{k}_2^*}\frac{\partial \mathbf{k}_2^*}{\partial A^{**\prime}} + p\frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}\right]\frac{dA^{**\prime}}{d\psi'}$. This initial adjustment in strategies sets off a new round of adjustments in government default threshold when the policy is not implemented driven by the change agents' beliefs regarding the default (i.e., to a multiplier effect for A^*).

Following the same steps as above, we obtain the expression for $dA^{*\prime}/d\psi'$ which is given by

$$\frac{dA^{*\prime}}{d\psi'}=\frac{\frac{\partial A^{*\prime}}{\partial\psi}+p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial\psi'}+p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial\psi'}}{1-p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}+\frac{(1-p)\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}+(1-p)\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}{1-p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}\frac{dA^{**}}{d\psi}$$

Above I have obtained a system of two equations in two unknown ($dA^*/d\psi'$ and $dA^{*\prime}/d\psi'$). Solving the above system for $dA^{*\prime}/d\psi'$ I obtain

$$\frac{dA^{*\prime}}{d\psi'}=\frac{\frac{\partial A^{*\prime}}{\partial\psi}\left[1-(1-p)\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\right]+p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial\psi'}+p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial\psi'}}{1-p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}-(1-p)\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}$$

Let $\frac{dA^{*\prime}}{d\psi'}(p)$ denote the change in the default threshold when the policy is implemented with probability p so that $\frac{dA^{*\prime}}{d\psi'}(1)$ corresponds to the change in the default threshold when the policy is always implemented and $\frac{dA^{*\prime}}{d\psi'}(0)$ to the case when it is never enacted. I now show that $\frac{dA^{*\prime}}{d\psi'}(p)$ is a linear combination of $\frac{dA^{*\prime}}{d\psi'}(1)$ and $\frac{dA^{*\prime}}{d\psi'}(0)$.

Proposition 1 *The change in the default threshold $A^{*\prime}$ is equal to*

$$\frac{dA^{*\prime}}{d\psi'}=p\frac{dA^{*\prime}}{d\psi'}(1)+\frac{dA^{*\prime}}{d\psi'}(0)$$

Proof. First, note that since all the partial derivatives are calculated at $A^{*\prime}=A^*$ and $\psi'=\psi$ it follows that

$$\frac{\partial x^*}{\partial A^{**\prime}}=\frac{\partial x^*}{\partial A^{**}},\ \frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}=\frac{\partial\mathbf{k}_2^*}{\partial A^{**}},\ \frac{\partial A^*}{\partial\mathbf{k}_2^*}=\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\text{ and }\frac{\partial A^{*\prime}}{\partial x^*}=\frac{\partial A^*}{\partial x^*}.$$

Thus, it follows that

$$\frac{dA^{*\prime}}{d\psi'}(0)=\frac{\partial A^*}{\partial\psi}$$

and

$$\frac{dA^{*\prime}}{d\psi'}(1)=\frac{\frac{\partial A^{*\prime}}{\partial\psi}+\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial\psi'}+\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial\psi'}}{1-\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}.$$

Now, using the above observations,

$$\begin{aligned}
&\frac{\frac{\partial A^{*\prime}}{\partial\psi}\left[1-(1-p)\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\right]+p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial\psi'}+p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial\psi'}}{1-p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}-(1-p)\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}}\\
=&\ \frac{p\frac{\partial A^{*\prime}}{\partial\psi}+p\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial\psi'}+p\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial\psi'}}{1-\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}+\frac{\frac{\partial A^{*\prime}}{\partial\psi}\left[1-p-(1-p)\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**}}-(1-p)\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**}}\right]}{1-\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}\\
=&\ p\frac{\frac{\partial A^{*\prime}}{\partial\psi}+\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial\psi'}+\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial\psi'}}{1-\frac{\partial A^{*\prime}}{\partial\mathbf{k}_2^*}\frac{\partial\mathbf{k}_2^*}{\partial A^{**\prime}}-\frac{\partial A^{*\prime}}{\partial x^*}\frac{\partial x^*}{\partial A^{**\prime}}}+(1-p)\frac{\partial A^{*\prime}}{\partial\psi}\\
=&\ p\frac{dA^{*\prime}}{d\psi'}(1)+(1-p)\frac{\partial A^{*\prime}}{\partial\psi}
\end{aligned}$$

as required. ■

6 Model with Fundamental Crises Only and Policy Uncertainty

In this section I compare how the effects of political uncertainty differ between my benchmark model and the model with fundamental crises only. As in Section 5 in the paper, let $p \in [0,1]$ denote the probability with which a policy ψ is implemented. Then,

$$\frac{d\underline{A}}{d\psi}(p) = p\frac{d\underline{A}}{d\psi}(1) + (1-p)\frac{\partial \underline{A}}{\partial \psi} \tag{42}$$

The intuition underlying this result is similar to that underlying Equation (6) in the paper. As agents expect the policy to be implemented with probability p, they respond to the announced change in ψ proportionally less.[18] As a result, as in the model with dispersed information, the presence of political uncertainty decreases the distortionary effects of higher taxes on households' investment but also decreases the expansionary effect of a stimulus.

However, policy uncertainty tends to dampen the effectiveness of policies more in the model with dispersed information, as Proposition 2 establishes. This is because, in the model with dispersed information, political uncertainty affects negatively the belief effect. Since the beliefs effect is absent in the model with only fundamental crises, this negative effect of political uncertainty is absent in this model.

Proposition 2 *Suppose that in the model with dispersed information both ε and σ are small. Then, as long as the desirability of ψ is unaffected by policy uncertainty, we have*

$$\left|\frac{d\underline{A}/d\psi\,(p)}{d\underline{A}/d\psi\,(1)}\right| > \left|\frac{dA^*/d\psi\,(p)}{dA^*/d\psi\,(1)}\right|$$

I now prove the above claim.

Derivations of Equation (42)**.** Suppose that households are uncertain whether the government will implement the announced policy. Let $k_2^{R,i}(\underline{A},\psi;p)$ be the households' investment when agents believe that a new policy will be implemented with probability p. Then, using the households' problem and following the same approach as in the model with dispersed information, it is straightforward to show that

$$\frac{\partial k_2^{R,i}(\underline{A},\psi;p)}{\partial\psi} = p\frac{\partial k_2^{R,i}(\underline{A},\psi;1)}{\partial\psi}$$

The decomposition in Equation (42) follows immediately from this observation. ■

Proof of Proposition 2. This follows immediately from Proposition 5 and Corollary 3 in the paper, and Equation (42) above. ■

[18] With probability $1-p$, households and lenders do not expect the adjustment, in which case a change in $\underline{A}$ is driven by the direct change in the government's default incentive.

7 Policy Analysis: Robustness

7.1 Higher tax rate in repayment only

In this section, I consider the case in which the government implements an increase in taxes only if it repays the debt. Let τ^R denote the tax rate in repayment and τ^D the tax rate in default where, initially, $\tau^R = \tau^D = \tau$. An increase in the tax rate only in repayment is captured by an increase in τ^R holding τ^D constant.

An increase in τ^R can be analyzed the same way as an increase in τ considered above. A higher τ^R leads to a change in the government's incentives to repay the debt equal to

$$\underbrace{Y_1^R u_{g_1}^R + Y_2^R u_{g_2}^R}_{\substack{\text{Increase in the government's} \\ \text{revenue in repayment}}} \quad - \underbrace{\int_{i=0}^{1} \left(u_{c_1}^R \frac{\partial c_1^R}{\partial \tau^R} + u_{c_2}^R \frac{\partial c_2^R}{\partial \tau^R} - u_{c_1}^D \frac{\partial c_1^D}{\partial \tau^R} - u_{c_2}^D \frac{\partial c_2^D}{\partial \tau^R} \right) di}_{\substack{\text{Differential decrease} \\ \text{in private consumption}}} \tag{43}$$
$$\underbrace{-\tau^R \frac{\partial Y_2^R}{\partial \tau^R} \left(u_{g_2}^R - Z u_{g_2}^D \right)}_{\text{Investment distortion}},$$

where, $u_{g_t}^R$ and $u_{g_t}^D$ are the marginal utilities from the government spending in period t in repayment and default, respectively, $u_{c_t}^R$ and c_t^R are household i's marginal utility from the private consumption and private consumption at time t in repayment, and Y_t^R is the total output of the economy in period t in repayment, all evaluated at the threshold A^*. If the expression in (43) is positive, then the government's incentives to repay its debt increase following an increase in τ^R.

There are three noticeable differences compared to the case when the tax rate is increased in both repayment and default. First, a higher τ^R increases government tax revenues only in repayment, which tends to increase the government's incentives to repay the debt more than in the earlier case. On the other hand, a higher τ^R decreases the government's incentives to repay by decreasing private consumption in repayment by more than in default (private consumption in default is affected indirectly through the change in households' investment strategies). Finally, the investment distortion effect, while still present, is now smaller since, at the time that they make their investment decisions, the households are uncertain whether the announced tax increase will be implemented.

While a choice whether to increase the tax rate only in repayment or both in repayment and in default is most likely determined by the political constraints, it is of interest to compare the effect of increasing τ^R against increasing the tax rate in both repayment and default. The following proposition establishes that when the initial tax rate is low, an increase only in τ^R leads to a larger increase in the government's incentives to repay then does an increase in both τ^R and τ^D, while the opposite is true when the initial tax rate is high.[19]

Proposition 15 *Let $\frac{\partial \Delta V}{\partial \tau^R}$ and $\frac{\partial \Delta V}{\partial \tau}$ denote the effect on the government incentives of increasing the tax rate only in repayment and both in repayment and in default, respectively. Then, there exists $\underline{\underline{\tau}}$ and $\overline{\overline{\tau}}$, with $0 < \underline{\underline{\tau}} < \overline{\overline{\tau}} < 1$ such that*

1. *If $\tau > \overline{\overline{\tau}}$ then $\frac{\partial \Delta V}{\partial \tau^R} < \frac{\partial \Delta V}{\partial \tau}$.*

[19] The Proof of Proposition 15 can be found in the "Additional Results" document available on the author's website `https://econ.sites.olt.ubc.ca/files/2016/01/pdf_szkup_debt_crises_additional.pdf`.

2. If $\tau < \underline{\underline{\tau}}$ then $\frac{\partial \Delta V}{\partial \tau^R} > \frac{\partial \Delta V}{\partial \tau}$.

To understand this result, note that when the tax rate is initially low, households' private consumption is relatively high while government spending is relatively low. Thus, the negative effect of higher τ^R on the utility from the private consumption in repayment is small, while the the positive effect of higher τ^D on the utility from the government spending would be high. It follows that at if, initially, both $\tau^R = \tau^D = \tau$ where τ is low, then increasing only τ^R has a larger effect on government's incentives to repay than does increasing both τ^R and τ^D at the same time; the opposite is true when the initial tax level is low.

7.1.1 Proof of Proposition 15

Proof of Proposition 15. Let τ^R denote the tax rate in repayment and τ^D denote the tax rate in default. When $\tau^R \neq \tau^D$ then solving problem of a household with productivity $A_i = A^* + \kappa\varepsilon$ we get

$$k_2\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right) = e^{A+\kappa\varepsilon} f\left(k_1\right) \Lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right)$$

where

$$\begin{aligned}
\Lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right) &\equiv \frac{\lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon\right) - \sqrt{\lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon\right)^2 - 4\alpha Z\left(1+\alpha\right)\left(1-\tau^R\right)\left(1-\tau^D\right)}}{2\left(1+\alpha\right)} \\
\lambda\left(A^* + \kappa\varepsilon, \kappa, \varepsilon\right) &\equiv \left(P\left(\varepsilon\right) + \alpha\right)\left(1-\tau^R\right) + Z\left(1 - P\left(\varepsilon\right) + \alpha\right)\left(1-\tau^D\right)
\end{aligned}$$

Note that if $\tau^R = \tau^D$ the expression for k_2 becomes identical to the expression reported in Section A.1 of the appendix to the paper. Moreover, it can be shown that

$$\frac{\partial k_2\left(A^* + \kappa\varepsilon, \kappa, \varepsilon; \tau^R, \tau^D\right)}{\partial \tau^R} \in \left[-\frac{1}{1-\tau} k_1, 0\right]$$

as κ varies from -1 to 1.[20] The above discussion implies that

$$\frac{\partial Y_2}{\partial \tau^R} = \frac{\partial}{\partial \tau^R} \int_{\kappa=-1}^{1} k_2\left(A^* + \kappa\varepsilon, \kappa, A^*\right)^\alpha d\kappa < \frac{\alpha}{1-\tau} \int_{\kappa=-1}^{1} k_2\left(A^* + \kappa\varepsilon, \kappa, A^*\right)^\alpha d\kappa$$

and hence, as remarked in the text, the distortionary effect of higher taxes is lower when the higher tax is implemented only in repayment.

Next differentiating ΔV (as defined in Section A.3 in the appendix to the paper) with respect to τ^R,

[20] When $\kappa = -1$ then $P\left(\varepsilon\right) = 1$ which means that these households expect default with probability 1 and as a consequence they assign probability 0 to taxes being increased and leave their investment decisions unchanged. On the other end of the spectrum lie households which received $\kappa = 1$.

imposing that initially $\tau^R = \tau^R = \tau$, and simplifying, we obtain

$$\frac{\partial \Delta V}{\partial \tau^R} = \underbrace{-\int_{\kappa=-1}^{1} \frac{1}{(1-\tau)(1-\Lambda(\kappa))} d\kappa - \int_{\kappa=-1}^{1} \frac{\frac{\partial k_2}{\partial \tau^R}}{(1-\tau) e^A f(k_1)(1-\Lambda(\kappa))} \left[1 - \frac{1}{Z}\right] d\kappa - \frac{1}{1-\tau}}_{\text{The effect of a higher } \tau^R \text{ on the private consumption in repayment versus default}}$$
$$+ \underbrace{\frac{Y_1}{\tau Y_1 - B_1 + B_2} + \frac{Y_2}{Y_2 - (1+r) B_2}}_{\text{An increase in government tax revenues in repayment}}$$
$$+ \underbrace{\frac{\tau \frac{\partial Y_2}{\partial \tau^R}}{\tau Y_2 - (1+r) B_2} - \frac{\frac{\partial Y_2}{\partial \tau^R}}{Y_2}}_{\text{Investment distortion in repayment versus default}}$$

Now,

$$\Lambda(\kappa) \in \left[\frac{\alpha Z}{1+\alpha}, \frac{\alpha}{1+a}\right] \text{ and } \frac{\partial Y_2}{\partial \tau^R} \in \left(0, \frac{\alpha}{1-\tau}\right),$$

and therefore,

$$\begin{aligned} \frac{\partial \Delta V}{\partial \tau^R} < \; & \frac{Y_1}{\tau Y_1 - B_1 + B_2} - \frac{1}{1-\tau} + \frac{Y_2}{\tau Y_2 - (1+r) B_2} - \frac{1}{1-\tau} \\ & -\tau \frac{\partial Y_2}{\partial \tau^R} \left[\frac{1}{\tau Y_2 - (1+r) B_2} - \frac{1}{\tau Y_2}\right] - \frac{1}{1-\tau} \frac{\alpha Z}{1+\alpha-\alpha Z} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \Delta V}{\partial \tau^R} > \; & \frac{Y_1}{\tau Y_1 - B_1 + B_2} - \frac{1}{1-\tau} + \frac{Y_2}{Y_2 - (1+r) B_2} - \frac{1}{1-\tau} \\ & -\tau \frac{\partial Y_2}{\partial \tau^R} \left[\frac{1}{\tau Y_2 - (1+r) B_2} - \frac{1}{\tau Y_2}\right] - \frac{\alpha}{1-\tau} \frac{1}{Z} \end{aligned}$$

We obtained the upper bound and lower bound for the effect of an increase in τ^R The result then follows from comparing the upper bound and the lower bound for $\partial \Delta V / \partial \tau^R$ with the expression for $\partial \Delta V / \partial \tau$ derived in the proof of Proposition 3. In particular, one can show that for all low enough τ the lower bound for $\partial \Delta V / \partial \tau^R$ is greater than $\partial \Delta V / \partial \tau$. Similarly, for high enough τ, the upper bound for $\partial \Delta V / \partial \tau^R$ is smaller than $\partial \Delta V / \partial \tau$. ■

7.2 The Effect of the Interest Rate on Policy Adjustments

Above, I analyzed the case in which the policy change takes place after the interest rate has been set, and, thus, the change in the policy and the resulting change in the default threshold A^* do not affect the interest rate r. In this section, I analyze what happens when the policy change is announced before the government chooses the interest rate, in which case I have to take into account how a policy change affects the choice of interest rate and how this change in the interest rate affects the default threshold.

Recall that the government chooses the interest rate to maximize the ex-ante welfare. The optimal interest rate is, then, the solution to the first-order condition associated with this problem, which can be written as

$$R(A^*, k_2, x^*, \psi, r^*) = 0$$

Here, we recognize that r^* depends on the government's future decisions, households' investment choices, and lenders' supply decisions. The choice of r^* is also affected by the policy parameters, since ψ affects the gains and costs associated with a higher r.

Following the same approach as in Section B of this appendix, I find that the total effect of a change in policy ψ on the default threshold is given by

$$\frac{dA^*}{d\psi} = M_{Total}\left[1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right]\left[\frac{\frac{\partial A^*}{\partial \psi} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2^i}{\partial \psi}di}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right]$$
$$+M_{Total}\left[1 - \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right]\left[\frac{\left(\frac{\partial A^*}{\partial r} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi} + \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)}{1 - \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di}\right],$$

where M_{Total} is the (total) beliefs effect that is present in the model when r can adjust; it is given by

$$M_{Total} = \frac{1}{1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di - \frac{\left(\frac{\partial A^*}{\partial r} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)}{1-\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial r}}\left(\frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} + \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}di\right)}.$$ [21]

To understand the above expression, note first that $\left[1 - \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial A^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the beliefs effect in the case when we hold the interest rate constant, and $\left[1 - \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial A^{**}} - \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial A^{**}}\right]^{-1}$ is the beliefs effect when the government's default decision is affected by the change in households' beliefs only through an implied adjustment in the interest rate. Then, the first term in the expression for $dA^*/d\psi$ captures the change in the default threshold implied by a change in the policy, holding the interest rate constant (the expression in the square brackets), weighted by the relative importance of the "partial" beliefs effect (i.e., beliefs effect when r is kept constant, as in Section 4 of the paper) compared to the total beliefs effect, M_{Total}. This effect is familiar from the earlier analysis. The second term captures the total change in the default threshold implied by the adjustment in r^*. Here, $\left(\frac{\partial A^*}{\partial r} + \frac{\partial A^*}{\partial x^*}\frac{\partial x^*}{\partial r}\right)\left(\frac{\partial r^*}{\partial \psi} + \frac{\partial r^*}{\partial x^*}\frac{\partial x^*}{\partial \psi} + \int_{i=0}^{1}\frac{\partial r^*}{\partial k_2}\frac{\partial k_2}{\partial \psi}di\right)$ captures the effect that an adjustment in ψ has on r^* (and, hence, on A^*), holding households' and lenders' expectations constant: A change in ψ leads to a change in r^*, which then affects A^*. This effect is then reinforced by the associated beliefs effect that results from the initial adjustment in r^* and is adjusted by the relative importance of its partial beliefs effect.

How does an adjustment in r^* alter the effectiveness of various government policies compared to the case when r^* is constant? While it is difficult to answer this question analytically, intuition suggests that an adjustment in r^* tends to decrease the magnitude of the change in A^* implied by ψ as long as the default threshold A^* is lower than A_{-1}, the prior of the mean belief about A. To understand this, note that a decrease in A^* decreases the benefit of a higher r (since a lower A^* means that a further decrease in A^* due to the choice of higher r translates into a lower decrease in the probability of default) and increases the cost of a higher r (since a fall in A^* implies that the government has to incur the cost of a higher r for a larger set of productivity values). The opposite is true when A^* increases. This suggests that a policy change that leads to a decrease in A^* is accompanied by a decrease in r^*, which decreases the positive effect of the policy adjustment. On the other hand, a policy change that leads to an increase in A^* is accompanied by an increase in r^*, which tends to partially offset the negative effect that such a policy has on the probability of default.

[21] The above expression can be derived by following the same steps as in Section B.1.

8 Intermediate and Technical Claims

In this section, I provide proofs of several results that have been invoked throughout this appendix. First, I show that $\partial x^*/\partial A^* < \frac{p_x+p_A}{p_x}$. Then, I compute limits of several expressions as $\varepsilon, \sigma_x \to 0$ and which where used in the proof of Proposition 2.

Lemma 16 *The derivative of x^* with respect to A^* is bounded from above by $\frac{\sigma_x^2+\sigma_A^2}{\sigma_A^2}$.*

Proof. Applying the implicit function theorem to the lenders' indifference condition, we get

$$\frac{dx^*}{dA^*} = -\frac{(-1)\left(1+r\min\left\{1, \frac{B_2^u(A^*)}{S(A^*,x^*)}\right\}\right)f(A^*|x^*)}{\frac{\partial}{\partial x^*}\left[\int\limits_{A^*}^{\infty}\left(1+r\min\left\{1,\frac{B_2^u(A)}{S(A.x^*)}\right\}\right)f(A|x^*)\,dA\right]},$$

where $f(A|x)$ is the conditional density of A given lender j observed signal $x_j = x^*$. Define $\mathcal{A}^u = \left\{A \geq A^* | B_2^{R,u}(A) < S(A)\right\}$ and $\mathcal{A}^c = \left\{A \geq A^* | B_2^{R,u}(A) \geq S(A)\right\}$, and note that $B_2^{R,u}(A)$ and $S(A)$ intersect at most finitely many times. Without loss of generality, I assume that $B_2^{R,u}(A)$ and $S(A)$ intersect at least once (otherwise, the result follows immediately). Then we can write $\mathcal{A}^u$ and $\mathcal{A}^c$ as $\mathcal{A}^u = \cup_{i=1}^{N_u}\left[A_{i_0}^u, A_{i_1}^u\right]$ and $\mathcal{A}^c = \cup_{i=1}^{N_c}\left(A_{i_0}^c, A_{i_1}^c\right)$, where $N_u, N_c \in \mathbb{N}$, $\left\{A_{i_0}^u\right\}_{i=1}^{N_u}$ are the values of the productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from above and $\left\{A_{i_1}^u\right\}_{i=1}^{N_u}$ are the values of productivity at which $B_2^{R,u}(A)$ intersects $S(A)$ from below.[22] With these definitions, we can write the above derivative as

$$\frac{dx^*}{dA^*} = \frac{\left(1+r\min\left\{1,\frac{BB_2^{R,u}(A)}{S(A^*,x^*)}\right\}\right)f(A^*|x^*)}{\sum\frac{\partial}{\partial x^*}\int\limits_{A_{i_0}^c}^{A_{i_1}^c}(1+r)f(A|x^*)\,dA + \sum\frac{\partial}{\partial x^*}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\left\{1+r\frac{B_2^{R,u}(A)}{S(A,x^*)}f(A|x^*)\right\}dA}$$

Consider the case where at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. Then the denominator becomes:

$$\sum_{i=1}^{N_u}\int\limits_{A_{i_0}^c}^{A_{i_1}^c}\frac{\partial}{\partial x^*}(1+r)f(A|x^*)\,dA + \sum_{i=1}^{N_c}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\frac{\partial}{\partial x^*}\left\{1+r\frac{B_2^{R,u}(A)}{S(A,x^*)}f(A|x^*)\right\}dA$$

$$= \int\limits_{A^*}^{\infty}\frac{\partial}{\partial x^*}(1+r)f(A|x^*)\,dA + \sum_{i=1}^{N_c}\int\limits_{A_{i_0}^u}^{A_{i_1}^u}\frac{\partial}{\partial x^*}\left\{r\left(\frac{B_2^{R,u}(A)}{S(A,x^*)}-1\right)f(A|x^*)\right\}dA$$

It remains to show that the second of the above terms is positive. Intuitively, that is what we expect, since a higher x^* makes high values of A more likely and $B_2^u(A)$ is increasing in A. The remainder of this proof is devoted to establishing it analytically.

The idea of the next few steps is to change differentiation with respect to x^* with the differentiation with respect to A. First, note that, since $f(A|x^*) = (p_A+p_x)^{1/2}\phi\left(\frac{A-\frac{p_x x^* + p_A A_{-1}}{p_x+p_A}}{(p_A+p_x)^{-1/2}}\right)$, we have

$$\int\limits_{A^*}^{\infty}\frac{\partial}{\partial x^*}(1+r)f(A|x^*)\,dA = -\frac{p_x}{p_x+p_A}\int\limits_{A^*}^{\infty}\frac{\partial}{\partial A}(1+r)f(A|x^*)\,dA = \frac{p_x}{p_x+p_A}(1+r)f(A^*|x^*)$$

[22] If at A^* we have $S(A,x^*) > B_2^{R,u}(A)$, then $A_{1_0}^u = A^*$, $A_{1_1}^u = A_{1_0}^c$, $A_{i_1}^c = A_{2_0}^u$, and so on. If at A^* we have $S(A,x^*) < B_2^{R,u}(A)$ then $A_{1_0}^c = A^*$, $A_{1_1}^c = A_{1_0}^u$, $A_{i_1}^u = A_{2_0}^c$, and so on.

Next, let $H(A,x^*) = \left(\frac{B_2^u(A)}{S(A,x^*)} - 1\right) f(A|x^*)$. Then,

$$\frac{\partial}{\partial x^*} H(A,x^*) = -\frac{p_x}{p_x+p_A}\frac{\partial}{\partial A}H(A,x^*) + \frac{\partial B_2^{R,u}(A)}{\partial A}\frac{1}{S(A,x^*)}f(A|x^*) - \frac{p_A}{p_x+p_A}\frac{B_2^{R,u}(A)}{S(A,x^*)}\frac{\frac{\partial}{\partial x^*}S(A,x^*)}{S(A,x^*)},$$

where, since $\partial B_2^{R,u}(A)/\partial A > 0$ and $\frac{\partial}{\partial x^*}S(A,x^*) < 0$, the last two terms are strictly positive. Moreover, note that for $i = 1, ..., N_c$ we have $H\left(A_{i_1}^u, x^*\right) = H\left(A_{i_0}^u, x^*\right) = 0$.

Therefore,

$$\begin{aligned} & \sum_{i=1}^{N_c}\int_{A_{i_0}^u}^{A_{i_1}^u}\frac{\partial}{\partial x^*}\left\{r\left(\frac{B_2^{R,u}(A)}{S(A,x^*)}-1\right)f(A|x^*)\right\}dA \\ > & \sum_{i=1}^{N_c}\int_{A_{i_0}^u}^{A_{i_1}^u} -\frac{p_x}{p_x+p_A}\frac{\partial}{\partial A}H(A,x^*)\,dA \\ = & -\frac{p_x}{p_x+p_A}\sum_{i=1}^{N_c}\left[H\left(A_{i_1}^u,x^*\right)-H\left(A_{i_0}^u,x^*\right)\right] = 0 \end{aligned}$$

This establishes the claim for the conclusion of the Lemma when at $A = A^*$ we have $B_2^u(A^*) \geq S(A^*, x^*)$. The case when $B_2^u(A^*) < S(A^*, x^*)$ is established in an analogous way. ■

Claim 1 $\lim_{\varepsilon\to 0}\frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} = 0$

Proof. Note that

$$\begin{aligned} & \frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} \\ = & \frac{\frac{1}{\sigma_A}\phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)} + \\ & + \frac{\left[\frac{1}{\sigma_A}\phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right) - \frac{1}{\sigma_A}\phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]\left[\Phi\left(\frac{A^*-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)\right]^2} \end{aligned}$$

Taking the limit as $\varepsilon \to 0$ and using l'Hôpital's rule one can show that the first term converges to $\frac{A^*-A_{-1}}{\sigma_A}\frac{(1-\kappa)}{2}$ while the second term converges to $-\frac{A^*-A_{-1}}{\sigma_A}\frac{(1-\kappa)}{2}$. It follows that $\lim_{\varepsilon\to 0}\frac{\partial \Pr(A^*|A^*+\kappa\varepsilon)}{\partial A^*} = 0$. ■

The next two claims have been used in the proof of Proposition 2.

Claim 2 $\lim_{\varepsilon\to 0}\left.\frac{\partial \Pr(A^{**}|A^*+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^*} = \infty$ *and* $\lim_{\varepsilon\to 0}\frac{\left.\frac{\partial \Pr\left(A^{**}|A^*+\hat{\kappa}\varepsilon\right)}{\partial A^*}\right|_{A^{**}=A^*}}{\left.\frac{\partial \Pr\left(A^{**}|A^*+\kappa\varepsilon\right)}{\partial A^{**}}\right|_{A^{**}=A^*}} = -1$

Proof. If $A^{**} \in (A^* - (1-\kappa)\varepsilon, A^* + (1+\kappa)\varepsilon)$, then

$$\Pr(A^{**}|A^*+\kappa\varepsilon) = \frac{\Phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^*+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^*-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}$$

Differentiating with respect to A^{**}, we get

$$\frac{\partial \Pr\left(A^{**}|A^{*}+\kappa\varepsilon\right)}{\partial A^{**}}=\frac{\phi\left(\frac{A^{**}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}$$

Taking the limit as $\varepsilon\to 0$ at $A^{*}=A^{**}$, we get

$$\lim_{\varepsilon\to 0}\frac{\partial \Pr\left(A^{**}|A^{**}+\kappa\varepsilon\right)}{\partial A^{**}}=\infty$$

Next, consider

$$\frac{\left.\frac{\partial \Pr(A^{**}|A^{*}+\widehat{\kappa}\varepsilon)}{\partial A^{*}}\right|_{A^{**}=A^{*}}}{\left.\frac{\partial \Pr(A^{**}|A^{*}+\kappa\varepsilon)}{\partial A^{**}}\right|_{A^{**}=A^{*}}}=-\frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}}-\frac{\frac{\left[\Phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]\left[\frac{1}{\sigma_A}\phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]}{\left[\Phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)\right]^{2}}}{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}}$$

Using l'Hôpital's rule, one can establish that

$$\lim_{\varepsilon\to 0}-\frac{\frac{\frac{1}{\sigma_A}\phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\widehat{\kappa})\varepsilon-A_{-1}}{\sigma_A}\right)}}{\frac{\phi\left(\frac{A^{*}-A_{-1}}{\sigma_A}\right)}{\Phi\left(\frac{A^{*}+(1+\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)-\Phi\left(\frac{A^{*}-(1-\kappa)\varepsilon-A_{-1}}{\sigma_A}\right)}}=-1$$

Similarly, using l'Hôpital's rule, one can show that the second term converges to 0. ■

Claim 3 $\lim_{\sigma_x\to 0}\frac{\partial}{\partial A}\left.S\left(A,x^{*}\right)\right|_{A=A^{*}}=\infty$

Proof. Note that

$$S\left(A,x^{*}\right)=b\left[1-\Phi\left(\frac{x^{*}-A}{\sigma_x}\right)\right]$$

Taking the derivative with respect to A, we get

$$\frac{\partial S\left(A,x^{*}\right)}{\partial A}=\frac{1}{\sigma_x}\phi\left(\frac{x^{*}-A}{\sigma_x}\right).$$

Under Assumption 1 in the paper, we have $x^{*}=\frac{\sigma_x^2+\sigma_A^2}{\sigma_A^2}A^{*}-\frac{\sigma_x^2}{\sigma_A^2}A_{-1}+\sigma_x^2\sqrt{\frac{1}{\sigma_x^2}+\frac{1}{\sigma_A^2}}\Phi^{-1}\left(\frac{1}{1+r}\right)$, and thus $\lim_{\sigma_x\to 0}\frac{x^{*}-A}{\sigma_x}=\phi\left(\Phi^{-1}\left(\frac{1}{1+r}\right)\right)$. The Claim follows immediately from this observation. ■

Claim 4 *We have $dA^{*}/dA_{-1}<0$.*

Proof. In light of Proposition 2, it is enough to consider the direct effect of increasing A_{-1}. By inspection, we see that A_{-1} does not directly affect the government incentives to default. Next, note that A_{-1} affects household i's investment choice only through its impact on $P\left(A^{**}|A_i\right)\equiv\Pr\left(A<A^{**}|A_{-1},A_i\right)$, and hence on $\Lambda\left(A_i;\varepsilon,A^{**}\right)$. Since $\Lambda\left(A_i;\varepsilon,A^{**}\right)$ is decreasing in $P\left(A^{**}|A_i\right)$ and $P\left(A^{**}|A_i\right)$ is decreasing in A_{-1} it follows that $\partial\Lambda\left(A_i;\varepsilon,A^{**}\right)/\partial A_{-1}$ is increasing in A_{-1}. Thus, an increase in A_{-1} leads to a

higher investment by households. Since a higher investment strictly decreases governments' incentives to default we have

$$\int\limits_{i=0}^{1} \frac{\partial A^*}{\partial k_2} \frac{\partial k_2}{\partial A_{-1}} di < 0$$

Next, consider lenders. Recall that the signal threshold above which lenders supply their funds to the bond market is defined implicitly by

$$\int_{A^{**}}^{\infty} \Delta u \left(A; x^*, A^{**}\right) \left(p_x + p_A\right)^{1/2} \phi \left(\frac{A - \frac{p_x x^* + p_A A_{-1}}{p_x + p_A}}{\left(p_x + p_A\right)^{-1/2}} \right) dA = 0$$

where A^{**} denote expected default threshold by the lenders. Then $\partial x^* / \partial A_{-1} = -p_x / p_A < 0$. Thus, an increase in A_{-1} leads to a decrease in x^* implying that lenders supply more funds to the bond market for any given A. Since a higher supply of funds weakly decreases the government's default incentives we have

$$\frac{\partial A^*}{\partial x^*} \frac{\partial x^*}{\partial A_{-1}} \leq 0.^{23}.$$

The result follows then from Proposition 2. ■

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