Online Appendix for "Risk Aversion, Uninsurable Idiosyncratic Risk, and the Financial Accelerator"

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This Appendix provides supplementary material for the article "Risk Aversion, Uninsurable Idiosyncratic Risk, and the Financial Accelerator". Section A collects the proofs of all the Propositions. Section B presents the optimality conditions that are not present in the main text. Section C recapitulates the log-linearized equilibrium conditions of the baseline model. Section D provides an additional analysis of partial and general equilibrium effects. Section E summarizes the equilibrium conditions for the two-sector model and Section F contains additional specifications for the investment regressions.

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A. Proofs

Proof of Lemma 1

The entrepreneur is maximizing

$$\max_K \mathbb{E}\Big[QK\omega Rk - BR\Big]^{1-\rho}, \tag{A.1}$$

using $B = QK - N$, then we can reformulate it

$$\max_K \mathbb{E}\Big[NR + QK(\omega R^k - R)\Big]^{1-\rho} \tag{A.2}$$

Now since NR does not affect the optimal choice, we can divide everything by NR and obtain

$$\max_k \mathbb{E}\Big[1 + k(\omega\frac{R^k}{R} - 1)\Big]^{1-\rho} \tag{A.3}$$

Denote $\log(\omega\frac{R^k}{R}) = x$ and $\omega\frac{R^k}{R} - 1 = y$. Now we can use the relationship

$$y = \omega\frac{R^k}{R} - 1 = e^x - 1 = x + \frac{1}{2}x^2 + o(x^2) \tag{A.4}$$

Let us rewrite this expression as

$$\max_k \mathbb{E}\Big[1 + ky\Big]^{1-\rho} \tag{A.5}$$

Taking a second-order Taylor approximation around $y = 0$, where $y = k(\omega\frac{R^k}{R} - 1)$ we obtain

$$\max_k \mathbb{E}\Big[1 + (1-\rho)ky - \frac{\rho(1-\rho)}{2}k^2y^2 + o(y^2)\Big] \tag{A.6}$$

Now we will assume that variance of ωR^k is small, and the mean $E(\omega R^k)$ is close to R. Taking the first-order condition with respect to k yields

$$\mathbb{E}\Big[(1-\rho)y - \rho(1-\rho)ky^2 + o(y^2)\Big] = 0 \tag{A.7}$$

so we can express k as

$$k = \frac{\mathbb{E}y}{\rho\mathbb{E}y^2} + o(y^2) \tag{A.8}$$

We now substitute the expression $y = x + \frac{x^2}{2} + o(x^2)$ to obtain

$$k = \frac{\mathbb{E}(x + \frac{x^2}{2})}{\rho \mathbb{E} x^2} + o(x^2) \tag{A.9}$$

We would like to get the expression for leverage and omit small terms of higher orders. We will consider $\sigma_\omega^2, \triangle$ to have small values of the first order. On the other hand, variables like $\triangle^2, \sigma^2\triangle, \sigma_\omega^4$ will be considered to be of higher orders and dropped out from the expression or be $o(x)$.

$$\mathbb{E}\log^2\omega = \mathbb{E}(\log\omega)^2 - (\mathbb{E}\log\omega)^2 + \mathbb{E}(\log\omega)^2 = \sigma_\omega^2 + \frac{\sigma_\omega^4}{4} = \sigma_\omega^2 + o(x). \tag{A.10}$$

Then

$$\mathbb{E}x^2 = \mathbb{E}(\log\omega + \triangle)^2 =$$
$$\mathbb{E}(\log\omega)^2 + 2\mathbb{E}(\log\omega)\mathbb{E}\triangle + \mathbb{E}(\triangle^2) = \sigma_\omega^2 + o(x_{t+1}) \tag{A.11}$$

Now we obtain

$$k = \frac{\mathbb{E}\triangle + o(x)}{\rho\sigma_\omega^2 + o(x)} + o(x^2) = \frac{\mathbb{E}\triangle}{\rho\sigma_\omega^2} + o(x) \tag{A.12}$$

If we focus on the expression

$$\kappa = \frac{\mathbb{E}\triangle}{\rho\sigma^2}, \tag{A.13}$$

then the elasticity of leverage with respect to excess returns and risk are given by:

$$\nu_k = \frac{d\log k}{d\mathbb{E}\triangle} = \frac{1}{\triangle} \tag{A.14}$$

$$\nu_\sigma = \frac{d\log k}{d\log\sigma} = -2 \tag{A.15}$$

Note that neither of the elasticities depends on the risk-aversion parameter or the size of risk.

Proof of Proposition 1

The proof follows Tamayo (2014). First, note that when the report is not verified ($\omega \notin \Omega^V$) the repayment function must only depend on the report $\tilde{\omega}$, i.e. we have $R(\tilde{\omega})$. Therefore, the entrepreneur will choose $\omega^* = \arg\min_{\tilde{\omega}} R(\tilde{\omega})$ so the contract

may as well set $R(\tilde{\omega}) = \bar{R}$. Second, under the optimal contract, in the verification region $R(\omega) \leq \bar{R}$ because otherwise the contract would not be incentive compatible. Specifically, the entrepreneur would prefer to misreport $\omega \notin \Omega^V$ and pay $\bar{R}$. Finally, it can also be shown that Ω^V must be a lower interval (for the proof see Lemma 3 in Tamayo (2014)). These findings can be summarized by saying that the optimal repayment function follows:

$$R(\omega) = \begin{cases} R(\omega) \leq \bar{R}, & \text{if } \omega \leq \bar{\omega} \\ \bar{R}, & \text{if } \omega > \bar{\omega} \end{cases} \tag{A.16}$$

Now let us rewrite the contracting problem using the above results as

$$\max \int_0^{\bar{\omega}} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - R(\omega)]^{1-\rho} d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - \bar{R}]^{1-\rho} d\Phi(\omega) \tag{A.17}$$

$$\text{s.t. } \kappa \frac{R^k}{R} \left(\int_0^{\bar{\omega}} R(\omega) d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu \Phi(\bar{\omega}) \right) \geq (\kappa - 1) \tag{A.18}$$

$$\bar{R} \leq \bar{\omega} \tag{A.19}$$

$$R(\omega) \leq \omega \ \forall \omega \leq \bar{\omega} \tag{A.20}$$

$$R(\omega) \geq 0 \ \forall \omega \leq \bar{\omega} \tag{A.21}$$

where we have plugged in the constraint (6), used the definition of leverage $\kappa = \frac{QK}{N}$ and rescaled the objective function and constraints by the exogenous parameters N and R. Assign the multipliers $\lambda, \xi, \gamma_1(\omega)$ and $\gamma_2(\omega)$ to the constraints. The Lagrangian reads:

$$\begin{aligned} &\max \int_0^{\bar{\omega}} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - R(\omega)]^{1-\rho} d\Phi(\omega) + \int_{\bar{\omega}}^{\infty} \left(\kappa \frac{R^k}{R}\right)^{1-\rho} [\omega - \bar{R}]^{1-\rho} d\Phi(\omega) + \\ &\lambda \left[\kappa \frac{R^k}{R} \left(\int_0^{\bar{\omega}} R(\omega) d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu \Phi(\bar{\omega}) \right) - (\kappa - 1) \right] + \\ &\xi(\bar{\omega} - \bar{R}) + \int_0^{\bar{\omega}} \gamma_1(\omega)(\omega - R(\omega))\phi(\omega) d\omega + \int_0^{\bar{\omega}} \gamma_2(\omega)(R(\omega))\phi(\omega) d\omega \end{aligned}$$

The first order necessary conditions with respect to $R(\omega), \bar{R}, \bar{\omega}$ after appropriate

rescaling of the multipliers can be written as:[1]

$$-\gamma_1(\omega)\phi(\omega) - \left(\kappa\frac{R^K}{R}\right)^{1-\rho}\{[\omega - R(\omega)]^{-\rho}\phi(\omega) + \lambda\kappa\frac{R^K}{R}\phi(\omega) + \gamma_2(\omega)\phi(\omega) = 0 \;\forall\omega \leq \bar{\omega} \tag{A.22}$$

$$-\xi - \left(\kappa\frac{R^K}{R}\right)^{1-\rho}\int_{\bar{\omega}}^{\infty}[\omega - \bar{R}]^{-\rho}d\Phi(\omega) + \lambda\kappa\frac{R^K}{R}[1 - \Phi(\bar{\omega})] = 0 \tag{A.23}$$

$$-\tfrac{\xi}{\phi(\bar{\omega})} - \left(\kappa\tfrac{R^K}{R}\right)^{1-\rho}[\bar{\omega} - R(\bar{\omega})]^{1-\rho} + \left(\kappa\tfrac{R^K}{R}\right)^{1-\rho}[\bar{\omega} - \bar{R}]^{1-\rho} - \lambda\kappa\tfrac{R^K}{R}[R(\bar{\omega}) - \bar{R} - \mu] = 0 \tag{A.24}$$

and the complementary slackness conditions:

$$0 = \lambda\left\{\kappa\frac{R^k}{R}\left(\int_0^{\bar{\omega}}\bar{R}(\omega)d\Phi(\omega) + R[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega})\right) - (\kappa - 1)\right\} \tag{A.25}$$

$$0 = \xi[\bar{\omega} - \bar{R}] \tag{A.26}$$

$$0 = \gamma_1(\omega)[\omega - R(\omega)] \tag{A.27}$$

$$0 = \gamma_2(\omega)R(\omega) \tag{A.28}$$

Suppose that $\gamma_1(\omega) > 0$ for all $\omega < \bar{\omega}$. Then it must be that $\gamma_2(\omega) = 0$, from the complementary slackness conditions. Then equation (A.22) would imply that $\lambda > \left(\kappa\frac{R^K}{R}\right)^{-\rho}(0)^{-\rho}$ which is not possible. Hence it must be true that $\gamma_1(\omega) = 0$ for all $\omega \leq \bar{\omega}$ and a standard debt contract is not optimal. We know from (A.22) that $\gamma_1(\omega) = 0 \iff (\omega - R(\omega))^{-\rho} \geq \lambda$. Now there are two possible cases. Suppose $\gamma_2(\omega) = 0$ for all $\omega \leq \bar{\omega}$. Then the contract specifies that $R(\omega) = \omega - \lambda^{-1/\rho}\left(\frac{R}{R^K\kappa}\right)$. By complementary slackness it should be the case that $R(\omega) > 0$ for all ω, which is not possible because if $\omega = 0$, $R(\omega) > 0$ would not be feasible. Then it must be the case that $\gamma_2(\omega) > 0$ for some ω which implies $R(\omega) = 0$ and $\omega \leq \lambda^{-1/\rho}\left(\frac{R}{R^K\kappa}\right)$ for the same ω. Hence there is a lower interval where $R(\omega) = 0$. Call the upper bound of this interval $\underline{\omega} \equiv \lambda^{-1/\rho}\left(\frac{R}{R^K\kappa}\right)$. Therefore $R(\omega) = 0$ if $\omega \leq \underline{\omega}$ and $R(\omega) = \omega - \underline{\omega}$ if $\underline{\omega} \leq \omega \leq \bar{\omega}$.■

[1] We do not need the first-order condition with respect to κ to prove the proposition.

Proof of Proposition 2

The Lagrangian is

$$\begin{aligned}\mathcal{L} =&\mathbb{E}_t\Bigg\{\frac{(\kappa_t R_{t+1}^k)^{1-\rho} g(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t})\Psi_{t+1}}{1-\rho}+\\ &\lambda_{t+1}\Bigg(\kappa_t R_{t+1}^k h(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) - (\kappa_t - 1)R_t\Bigg)\Bigg\}\end{aligned}$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t\Bigg\{(\kappa_t R_{t+1}^k)^{1-\rho} g_{t+1}\Psi_{t+1} - \lambda_{t+1}R_t\Bigg\} = 0 \tag{A.29}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{\omega}} = \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\bar{\omega},t+1}\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\kappa_t R_{t+1}^k h_{\bar{\omega},t+1} = 0 \tag{A.30}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\underline{\omega},t+1}\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\kappa_t R_{t+1}^k h_{\underline{\omega},t+1} = 0 \tag{A.31}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\bar{R},t+1}\Psi_{t+1}}{1-\rho} + \lambda_{t+1}\kappa_t R_{t+1}^k h_{\bar{R},t+1} = 0 \tag{A.32}$$

Now we can express λ_{t+1} from $\frac{\partial \mathcal{L}}{\partial \bar{\omega}} = 0$

$$\lambda_{t+1} = -\frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\bar{\omega},t+1}\Psi_{t+1}}{1-\rho}\frac{1}{\kappa_t R_{t+1}^k h_{\bar{\omega},t+1}} \tag{A.33}$$

Now we plug this condition into the three other equations and obtain

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t\Bigg\{(\kappa_t R_{t+1}^k)^{1-\rho} g_{t+1}\Psi_{t+1} + \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\bar{\omega},t+1}\Psi_{t+1}}{1-\rho}\frac{1}{\kappa_t R_{t+1}^k h_{\bar{\omega},t+1}}R_t\Bigg\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\underline{\omega},t+1}\Psi_{t+1}}{1-\rho} - \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\bar{\omega},t+1}\Psi_{t+1}}{1-\rho}\frac{1}{\kappa_t R_{t+1}^k h_{\bar{\omega},t+1}}\kappa_t R_{t+1}^k h_{\underline{\omega},t+1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\bar{R},t+1}\Psi_{t+1}}{1-\rho} - \frac{(\kappa_t R_{t+1}^k)^{1-\rho} g_{\bar{\omega},t+1}\Psi_{t+1}}{1-\rho}\frac{1}{\kappa_t R_{t+1}^k h_{\bar{\omega},t+1}}\kappa_t R_{t+1}^k h_{\bar{R},t+1} = 0$$

we can transform this system to

$$\frac{\partial \mathcal{L}}{\partial k_t} = \mathbb{E}_t\left\{(R^k_{t+1})^{1-\rho}\Psi_{t+1}\left(g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t\right)\right\} = 0 \tag{A.34}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{\omega}} = g_{\underline{\omega},t+1} - g_{\bar{\omega},t+1}\frac{h_{\underline{\omega},t+1}}{h_{\bar{\omega},t+1}} = 0 \tag{A.35}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{R}} = g_{\bar{R},t+1} - \frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} h_{\bar{R},t+1} = 0 \tag{A.36}$$

Since in the equation $(A.34)$ Ψ_{t+1} and $\hat{R}_{k,t+1}$ enter as multiplicative terms and the term $g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t$ is equal to zero in the steady state, Ψ_{t+1} and $\hat{R}_{k,t+1}$ have no effect in the first order approximation. Therefore, to find the approximate solution it is sufficient to consider the following system:

$$\kappa_t R^k_{t+1} h(\bar{\omega}_{t+1}, \underline{\omega}_{t+1}, \bar{R}_{t+1}, \sigma_{\omega,t}) = (\kappa_t - 1) R_t \tag{A.37}$$

$$\mathbb{E}_t\left\{g_{t+1} + \frac{g_{\bar{\omega},t+1}}{(1-\rho)\kappa_t R^k_{t+1} h_{\bar{\omega},t+1}} R_t\right\} = 0 \tag{A.38}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\bar{R},t+1}}{h_{\bar{R},t+1}} \tag{A.39}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\underline{\omega},t+1}}{h_{\underline{\omega},t+1}} \tag{A.40}$$

We can substitute k_t and obtain

$$\frac{\mathbb{E}_t\left[\frac{g_{\bar{\omega}} R_{t+1}}{(1-\rho) R_{k,t+1} h_{\omega,t+1}}\right]}{E_t g_{t+1}} = \frac{1}{1 - \frac{R_{k,t+1}}{R_t} h_{t+1}} \tag{A.41}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\bar{R},t+1}}{h_{\bar{R},t+1}} \tag{A.42}$$

$$\frac{g_{\bar{\omega},t+1}}{h_{\bar{\omega},t+1}} = \frac{g_{\underline{\omega},t+1}}{h_{\underline{\omega},t+1}} \tag{A.43}$$

Whenever the gradient of this system has full rank at the steady state, we will be able to find an approximate solution of $\hat{\bar{\omega}}_{t+1}, \hat{\underline{\omega}}_{t+1}, \hat{\bar{R}}_{t+1}$ as functions of $\mathbb{E}_t \hat{R}_{k,t+1} - \hat{R}_t$, $\hat{R}_{k,t+1} - \hat{R}_t$ and $\hat{\sigma}_{\omega,t}$. Using this fact and log-linearizing equation (A.37) will give us

$$\hat{k}_t = \nu_p(\mathbb{E}_t \hat{R}_{k,t+1} - \hat{R}_t) + \nu_\sigma \hat{\sigma}_{\omega,t} \tag{A.44}$$

B. Optimality Conditions

Households. Households maximize their utility (24) subject to the budget constraint (25) with respect to consumption, labor, bonds, and deposits, yielding the following first order conditions:

$$C_t^{-\sigma_H} = \beta R_t \mathbb{E}_t \left\{ C_{t+1}^{-\sigma_H} \right\}, \tag{B.1}$$

$$C_t^{-\sigma_H} = \beta R_t^n \mathbb{E}_t \left\{ \frac{C_{t+1}^{-\sigma_H}}{\pi_{t+1}} \right\}, \tag{B.2}$$

$$W_t C_t^{-\sigma_H} = \chi H_t^{\eta}. \tag{B.3}$$

Wholesalers. The objective function for wholesalers is given by

$$\max_{H_t, H_t^e, K_{t-1}} \frac{1}{\mathcal{X}_t} A_t K_{t-1}^{\alpha} (H_t)^{(1-\alpha)\Omega} (H_t^e)^{(1-\alpha)(1-\Omega)} - W_t H_t - W_t^e H_t^e - R_t^r K_{t-1}. \tag{B.4}$$

Here wages and the rental price of capital are in real terms. The first order conditions with respect to capital, household labor and entrepreneurial labor are

$$R_t^r = \frac{1}{\mathcal{X}_t} \alpha \frac{Y_t}{K_{t-1}}, \tag{B.5}$$

$$W_t = \frac{\Omega}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t}, \tag{B.6}$$

$$W_t^e = \frac{(1-\Omega)}{\mathcal{X}_t} (1-\alpha) \frac{Y_t}{H_t^e}. \tag{B.7}$$

Given that equilibrium entreprenerial labor in equilibrium is 1, we have

$$W_t^e = \frac{(1-\Omega)}{\mathcal{X}_t} (1-\alpha) Y_t. \tag{B.8}$$

Retailers. The first order condition with respect to the retailer's price p_{it}^* is

$$\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{ \Lambda_{t,s} \left(\frac{p_{it}^*}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \left[\frac{p_{it}^*}{P_{t+s}} - \frac{\varepsilon}{\varepsilon - 1} \frac{P_{t+s}^w}{P_{t+s}} \right] \right\} = 0. \tag{B.9}$$

From this condition, it is clear that all retailers that are able to reset their prices in period t will choose the same price $p_{it}^* = P_t^* \ \ \forall i$. The aggregate price level evolves

according to

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{B.10}$$

Dividing the left and right hand side of (B.10) by the price level gives

$$1 = \left[\theta \pi_{t-1}^{\varepsilon-1} + (1-\theta)(p_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{B.11}$$

where $p_t^* = P_t^*/P_t$ and $\pi_t = P_t/P_{t-1}$. Using the same logic, we can normalize (B.9) and obtain:

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{\Lambda_{t,s} (1/p_{t+s})^{-\varepsilon} Y_{t+s} p_{t+s}^w\right\}}{\sum_{s=0}^{\infty} \theta^s \mathbb{E}_t \left\{\Lambda_{t,s} (1/p_{t+s})^{1-\varepsilon} Y_{t+s}\right\}}, \tag{B.12}$$

where $p_{t+s}^w = \frac{P_{t+s}^w}{P_{t+s}}$ and $p_{t+s} = P_{t+s}/P_t$.

C. Log-linear Model

The log-linear model has 17 equations and 17 variables, because algebraic manipulations with the Calvo model allow to replace (B.11), (B.12) and (28) with (C.4), and drop p^* and p^w, while simplifying the financial contract allows to replace (18), (19), (A.29), (A.30), (A.31), (A.32) with (C.14) and drop $\bar{\omega}, \underline{\omega}, \bar{R}, \Psi$. The equations

are:

$$-\sigma_H\Big(\mathbb{E}_t\hat{C}_{t+1}-\hat{C}_t\Big)+\hat{R}_t=0, \tag{C.1}$$

$$\hat{R}^n_t=\hat{R}_t+\mathbb{E}_t\hat{\pi}_{t+1}, \tag{C.2}$$

$$\hat{Y}_t-\hat{H}_t-\hat{\mathcal{X}}_t-\sigma_H\hat{C}_t=\eta\hat{H}_t, \tag{C.3}$$

$$\hat{\pi}_t=-\frac{(1-\theta)(1-\theta\beta)}{\theta}\hat{\mathcal{X}}_t+\beta\mathbb{E}_t\hat{\pi}_{t+1}, \tag{C.4}$$

$$\hat{Y}_t=\hat{A}_t+\alpha\hat{K}_{t-1}+(1-\alpha)(1-\Omega)\hat{H}_t, \tag{C.5}$$

$$\hat{K}_t=\delta\hat{I}_t+(1-\delta)\hat{K}_{t-1}, \tag{C.6}$$

$$\hat{Q}_t=\delta\phi_K(\hat{I}_t-\hat{K}_{t-1}), \tag{C.7}$$

$$\hat{R}^k_{t+1}=(1-\epsilon)(\hat{Y}_{t+1}-\hat{K}_t-\hat{\mathcal{X}}_{t+1})+\epsilon\hat{Q}_{t+1}-\hat{Q}_t, \tag{C.8}$$

$$\left(1-\frac{G}{Y}\right)\hat{Y}_t=\frac{C}{Y}\hat{C}_t+\frac{I}{Y}\hat{I}_t, \tag{C.9}$$

$$\hat{\phi}_t=\hat{Q}_{t-1}+\hat{K}_{t-1}-\hat{N}_{t-1}+\nu^m_\sigma\hat{\sigma}_{\omega,t-1}+\nu^m_p(\mathbb{E}_{t-1}R_{k,t}-\hat{R}_{t-1}), \tag{C.10}$$

$$\hat{N}_t=\gamma\big(\kappa R_k(\hat{\kappa}_{t-1}+\hat{R}_{k,t})-\kappa R\hat{\kappa}_{t-1}-(\kappa-1)R\hat{R}_{t-1}-\phi\hat{\phi}_t\big)+\tfrac{W^e}{N}\hat{W}^e_t+\tfrac{N-W^e}{N}\hat{N}_{t-1}+\epsilon^\tau_t, \tag{C.11}$$

$$\hat{\kappa}_t=\hat{K}_t+\hat{Q}_t-\hat{N}_t, \tag{C.12}$$

$$\hat{W}^e_t=\hat{Y}_t-\hat{\mathcal{X}}_t, \tag{C.13}$$

$$\hat{\kappa}_t=\nu_p(\mathbb{E}_t\hat{R}^k_{t+1}-\hat{R}_t)+\nu_\sigma\hat{\sigma}_{\omega,t}, \tag{C.14}$$

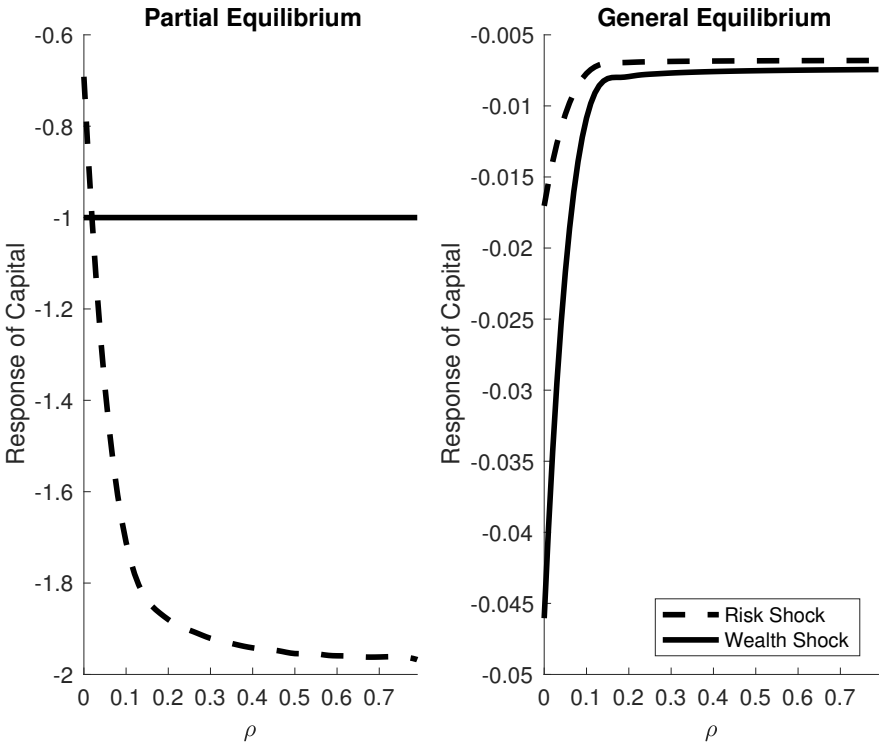
$$\hat{A}_t=\rho^A\hat{A}_{t-1}+\epsilon^A_t, \tag{C.15}$$

$$\hat{R}^n_t=\rho^{R^n}\hat{R}^n_{t-1}+\xi\hat{\pi}_t+\rho^Y\hat{Y}_t+\epsilon^{R^n}_t, \tag{C.16}$$

$$\hat{\sigma}_{\omega,t}=\rho^{\sigma_\omega}\hat{\sigma}_{\omega,t-1}+\epsilon^{\sigma_\omega}_t. \tag{C.17}$$

D. Partial versus General Equilibrium Effect

Figure 1: Partial versus General Effect



Note: The figure shows the effects of changing risk aversion (x-axis) on the partial and general equilibrium response of capital to financial shocks. The rest of the model parameters follow Risk Averse 2 and the steady-state is being kept fixed by adjusting σ_ω as in Figure 5.

Figure 10 demonstrates the partial and general equilibrium effects of risk and wealth shocks on purchases of physical capital for different coefficients of risk-aversion. The results from general equilibrium were obtained from the impulse responses of the model, while partial equilibrium effects of shocks were constructed under the assumption of excess returns to capital being fixed at their steady state values. Each point on the general equilibrium dashed line requires running a DSGE model for the corresponding value of the risk aversion, computing the impulse response for purchases of physical capital on impact. Every point of the partial equilibrium response requires recomputing the steady state derivatives for each value of the risk aversion.

The response of capital purchases under partial equilibrium is more volatile than in general equilibrium by about two orders of magnitude. This is a consequence of the fact that the demand for physical capital in general equilibrium is equal to the supply of physical capital stock, which is almost vertical in the short run. Thus, the

endogenous movement of prices plays a major role in capital dynamics by dampening partial equilibrium effects.

Recall that entrepreneurs with the same preferences choose the same leverage, as the entrepreneurial problem is homothetic with respect to net worth. Therefore, the decrease of net worth by one percent in partial equilibrium leads to a decline of physical capital purchases by one percent as well.

In general equilibrium, there are two additional channels in addition to the partial equilibrium response. First, excess returns to capital increase, R^k/R, because the lower capital purchases reduces temporarily the price of capital. Higher expected returns to capital lead to the rise in leverage and higher purchases of physical capital, having a stabilizing effect. We refer to this mechanism as a leverage channel. Second, a decrease in the price of physical capital leads to lower net worth, especially since the entrepreneurs have a leverage greater than one. This tends to have an amplifying effect on business cycles. We refer to this mechanism as a net worth channel. For low coefficients of risk aversion, the general equilibrium effect is relatively large because of the net worth channel. As the coefficient of risk-aversion increases, the general equilibrium effect becomes smaller because of the higher sensitivity or risk averse entrepreneurs to high excess returns to capital.

Under the partial equilibrium, the effect of a risk shock for risk-neutral agents leads to an increase in defaults, which translate into higher interest rate on loans, and, consequently, reducing leverage. For the agents with higher risk-aversion, we observe a larger reduction of leverage in partial equilibrium, because in addition to higher defaults, the risk shock directly increases the volatility of returns, from which the risk averse borrower tries to back away. In general equilibrium, precautionary motives allow entrepreneurs with higher risk-aversion to increase their leverage more when they face higher excess returns to capital. In other words, for risk-averse agents leverage channel is strong enough to support the price of capital and prevent the fall of net worth, which instead dominates in the case of risk neutral agents.

Finally, it is worthwhile noticing that the responses of capital purchases for both shocks in general equilibrium flatten out as soon as risk aversion reaches 0.1, which shows that our results are not sensitive to a particular value of this parameter.

E. Two-Sector Model

The complete system with a two-sector model is described by the following system of equations:

$$Y_{1,t} = A_t K_{1,t-1}^{\alpha} H_{1,t}^{(1-\alpha)\Omega}, \tag{E.1}$$

$$Y_{2,t} = A_t K_{2,t-1}^{\alpha} H_{2,t}^{(1-\alpha)\Omega}, \tag{E.2}$$

$$K_{1,t} = I_{1,t} + (1-\delta)K_{1,t-1}, \tag{E.3}$$

$$K_{2,t} = I_{2,t} + (1-\delta)K_{2,t-1}, \tag{E.4}$$

$$\frac{1}{Q_{1,t}} = 1 - \phi_K \left(\frac{I_{1,t}}{K_{1,t}} - \delta \right), \tag{E.5}$$

$$\frac{1}{Q_{2,t}} = 1 - \phi_K \left(\frac{I_{2,t}}{K_{2,t}} - \delta \right), \tag{E.6}$$

$$R_{1,t+1}^k = \frac{\frac{1}{X_{t+1}} a \left(\frac{Y_{1,t+1}}{Y_{t+1}} \right)^{\zeta-1} \frac{\alpha Y_{1,t+1}}{K_{1,t+1}} + Q_{1,t+1}(1-\delta)}{Q_{1t}}, \tag{E.7}$$

$$R_{2,t+1}^k = \frac{\frac{1}{X_{t+1}} a \left(\frac{Y_{2,t+1}}{Y_{t+1}} \right)^{\zeta-1} \frac{\alpha Y_{2,t+1}}{K_{2,t+1}} + Q_{2,t+1}(1-\delta)}{Q_{2t}}, \tag{E.8}$$

$$\phi_{1,t} = Q_{1,t-1} K_{1,t-1} g(E_t R_{1k,t}, R_t, R_{1,kt}, \sigma_{1\omega,t}, \rho_1), \tag{E.9}$$

$$\phi_{2,t} = Q_{2,t-1} K_{2,t-1} g(E_t R_{2k,t}, R_t, R_{2,kt}, \sigma_{2\omega,t}, \rho_2), \tag{E.10}$$

$$N_{1,t} = \gamma(K_{1,t}(R_{k,1t} - R_t - \phi_{1,t}) + N_{1,t}R) + W_{e,1t}, \tag{E.11}$$

$$N_{2,t} = \gamma(K_{2,t}(R_{k,2t} - R_t - \phi_{2,t}) + N_{2,t}R) + W_{e,2t}, \tag{E.12}$$

$$\kappa_{1,t} = \frac{K_{1,t}Q_{1,t}}{N_{1,t}}, \tag{E.13}$$

$$\kappa_{2,t} = \frac{K_{2,t}Q_{2,t}}{N_{2,t}}, \tag{E.14}$$

$$W_{e,1t} = a(1-\Omega)(1-\alpha)\frac{Y_{1,t}}{\mathcal{X}_t}\left(\frac{Y_{1,t}}{Y_t} \right)^{\zeta-1}, \tag{E.15}$$

$$W_{e,2t} = (1-a)(1-\Omega)(1-\alpha)\frac{Y_{2,t}}{\mathcal{X}_t}\left(\frac{Y_{2,t}}{Y_t} \right)^{\zeta-1}, \tag{E.16}$$

$$\kappa_{1,t} = f(E_t R_{1k,t+1}, R_t, \sigma_{1\omega,t}, \rho_1), \tag{E.17}$$

$$\kappa_{2,t} = f(E_t R_{2k,t+1}, R_t, \sigma_{2\omega,t}, \rho_2), \tag{E.18}$$

$$\log(\sigma_{1\omega,t}) = (1-\rho^{\sigma_\omega})\log(\sigma_{1\omega,ss}) + \rho^{\sigma_\omega}\log(\sigma_{1\omega,t-1}) + \epsilon_t^{\sigma_{1\omega}} \tag{E.19}$$

$$\log(\sigma_{2\omega,t}) = (1-\rho^{\sigma_\omega})\log(\sigma_{2\omega,ss}) + \rho^{\sigma_\omega}\log(\sigma_{2\omega,t-1}) + \epsilon_t^{\sigma_{2\omega}} \tag{E.20}$$

$$C_t^{-\sigma_H} = \beta R_t E_t C_{t+1}^{-\sigma_H}, \tag{E.21}$$

$$C_t^{-\sigma_H} = \beta R_t^n E_t \left\{ \frac{C_{t+1}^{-\sigma_H}}{\pi_{t+1}} \right\}, \tag{E.22}$$

$$H_t^\eta = (1-\alpha)\Omega \frac{Y_{1,t}}{H_{1,t}\mathcal{X}_t} a \left(\frac{Y_{1,t}}{Y_t} \right)^{\zeta-1} C_t^{-\sigma_H} \tag{E.23}$$

$$H_t^\eta = (1-\alpha)\Omega \frac{Y_{2,t}}{H_{2,t}\mathcal{X}_t} (1-a) \left(\frac{Y_{2,t}}{Y_t} \right)^{\zeta-1} C_t^{-\sigma_H} \tag{E.24}$$

$$(1-g_y)\, Y_t = C_t + I_t, \tag{E.25}$$

$$1 = \left[\theta \pi_{t-1}^{\varepsilon-1} + (1-\theta)(p_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{E.26}$$

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{V_t}{\tilde{V}_t} \tag{E.27}$$

$$V_t = C_t^{-\sigma_H} Y_t \mathcal{X}_t^{-1} + \beta\theta E_t (V_{t+1} \pi_{t+1}^{\epsilon}) \tag{E.28}$$

$$\tilde{V}_t = C_t^{-\sigma_H} Y_t + \beta\theta E_t (\tilde{V}_{t+1} \pi_{t+1}^{\epsilon-1}) \tag{E.29}$$

$$\log(A_t) = \rho^A \log(A_{t-1}) + \epsilon_t^A, \tag{E.30}$$

$$\log(R_t^n) - \log(R^n) = \rho^{R^n} (\log(R_{t-1}^n) - \log R) + \xi \pi_{t-1} + \epsilon_t^{R^n}, \tag{E.31}$$

$$Y_t^\zeta = a Y_{1,t}^\zeta + (1-a) Y_{2,t}^\zeta \tag{E.32}$$

$$I_t = I_{1,t} + I_{2,t} \tag{E.33}$$

Equations (E.32) and (E.33) reflect how sector-specific output and investment are combined to produce corresponding aggregates. The parameter ζ characterizes the elasticity of substitution between sectors, and the parameter a corresponds to the weight of the sector 1 in the production of aggregate output. Equations (E.21)-(E.33) describe the behavior of aggregate variables, while equations (E.1)-(E.20) correspond to sector-specific relationships. All other parameters, except a and ζ, are identical to the one-sector model. Overall, we have 20 sector-specific variables $Y_i, K_i, I_i, H_i, Q_i, R_i^k, N_i, \phi_i, \kappa_i, \sigma_{i\omega}$ with $i = 1, 2$ and 13 aggregate variables $C, R, R_n, \pi, p^*, V, \tilde{V}, \mathcal{X}, H, W^e, Y, I, A$.

F. Additional Regressions

Table 1: Additional Regressions

Dependent variable: $(I/K)_{i,t}$	(1)	(2)	(3)	(4)	(5)	(6)
$\log(Y/K)_{i,t}$	0.083	0.084	0.148	0.143	0.102	0.075
	(6.71)	(6.73)	(9.89)	(9.89)	(7.55)	(6.14)
$\log(Y/K)_{i,t} \times INSD_2$	0.018	0.018	0.018	0.017	0.016	0.014
	(0.95)	(0.95)	(0.95)	(0.96)	(0.95)	(0.90)
$\log(Y/K)_{i,t} \times INSD_3$	0.024	0.023	0.102	0.092	0.079	0.052
	(1.01)	(0.98)	(4.07)	(3.75)	(3.22)	(2.33)
$\log(Y/K)_{i,t} \times INSD_4$	0.077	0.078	0.125	0.112	0.091	0.072
	(2.79)	(2.83)	(4.77)	(4.24)	(3.42)	(2.95)
$\log(Y/K)_{i,t} \times INSD_H$	0.048	0.047	0.139	0.122	0.103	0.066
	(2.08)	(2.01)	(6.04)	(5.38)	(4.57)	(3.26)
Observations	32,441	32,441	32,444	32,444	32,444	32,444
R^2	0.79	0.79	0.64	0.65	0.66	0.68
Fixed effects	F, T	F, T, A	F	F, T	F, T	F, T
Controls	Q, K, A	Q, K	No	No	Q	Q, K

$Notes$: F and T, A stand for firm, time and firm-age fixed effects, respectively. Age here means years since IPO, which is collected from Jay Ritter's website. Q and K denote a firm's Tobin's Q ($\log Q_{i,t-1}$) and size ($\log \hat{K}_{i,t-1} = \log(K_{i,t-1}/\frac{1}{N}\sum_{i=1}^{N} K_{i,t-1})$, respectively. Specification (1) controls for the square of the firm's age. Specifications (2)-(5) mimic specification (2)-(5) in Table 4 of the main text, but assigns firms to the insider ownership quintile they belonged to when first observed in the sample. Time and firm fixed effects are interacted with the quintile dummies. Standard errors are clustered at the firm level, and t-statistics are reported in parenthesis.

References

Tamayo, C. (2014). Investor protection and optimal contracts under risk aversion and costly state verification. *Economic Theory*, 1–31.