Online Appendix

Fiscal Commitment and Sovereign Default Risk

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A Data Sources

Table A.1: Data Sources

Data	Source	Years Used
Gross Domestic Product	Eurostat	1995-2011
Final consumption expenditure of general government	Eurostat	1995-2011
Final consumption expenditure of households	Eurostat	1995-2011
General government: interest payable	Eurostat	1995-2011
Long-Term Government Bond Yields: 10-year: Main for Greece	FRED, St. Louis Fed	1998-2011
Long-Term Government Bond Yields: 10-year: Main for Germany	FRED, St. Louis Fed	1998-2011
3-Month Rates and Yields: Interbank Rates for Germany	FRED, St. Louis Fed	1995-2011
GDP Implicit Price Deflator in Germany	FRED, St. Louis Fed	1995-2011
Income tax rate for Greece	Vegh and Vuletin (2015)	1995-2011

B Solution Methods

In the quantitative model, we solve for the policy and value functions for the economies under i) a discretionary tax policies (DP), ii) a non-contingent tax commitment (NC), and iii) a contingent tax commitment (CC) using the value function iteration method. We first solve for the equilibrium of a finite-horizon economy with the number of periods large enough so that the deviation of value functions and bond price between the first and second periods is smaller than 10^{-6}. We then use the first-period policy and value functions as the equilibrium functions in the infinite-horizon economy. As we will discuss later, the state variables for models under DP and CC are the current debt b and current income y. For the model under NC, there is an additional state variable: tax rate τ. When solving the value function maximization, we use linear interpolation to find the values of b and τ choices that are not on the grids. We follow Tauchen (1986) and discretize the income process into 51 points spanning $\pm$ 3 standard deviations of the unconditional distribution. We use an equidistant b grid, which has 700 points and is spaced over [0, 0.7]. The τ grid has 40 points and is spaced over [0.2, 0.6].

The solution methods for the models with DP and NC are relatively standard, so we focus on the solution method for the economy under CC. The contingent commitment problem is outlined by equations (15) - (18). In the original form of the problem, the government solves an infinite-dimensional optimization problem since it needs to choose a tax rate for each possible income realization in the next period. After discretizing the income state to 51 points, the problem is still a 52 (51)-dimensional optimization for the repaying (defaulting) government, which is prohibitively time-consuming to solve. Therefore, we reformulate the commitment problem into a *recursive problem of the government with incentive-compatibility (IC) constraints*. The reformulated problem only has two state variables and three choice variables, which can be solved using a standard algorithm.[2] [3]

To illustrate how this reformulation works, we recall the concept of the counterfactual discretionary tax rate defined in section 4.3. Specifically, at each income realization y, the "counterfactual discretionary tax rate" is the tax rate that a government would choose at the end of a period, if it is allowed to conduct a one-time deviation from its committed tax schedule $\tau(y)$, given its default decision d, capital inflow $qb' - b$, and the tax commitment $\tau'(y')$. As shown by equation (19), we can write the counterfactual discretionary tax rate as a function of income and capital flow: $\tau^{CF}(y, qb' - b)$. Therefore, these tax rates are *ex post* optimal for the next-period government, given its capital flow and income.[4] In the original contingent commitment problem, the current government commits to a tax rate for every possible income realization in the next period. For most of the income states, the government will commit to the *ex post* optimal tax rates as long as they deliver the repayment-default decision that the next-period government would like to pursue by itself.

For the other income states, however, the government commits to tax rates that are not *ex post* optimal since it needs to use the tax commitment to influence the next-period government's default decisions. Therefore, the current government's problem is essentially choosing the debt issuance b' and a default threshold $y^{*\prime}$ for the next period, and leaves the taxation decisions to the next-period government as long as those tax rates are compatible with the pre-determined default threshold.[5] As a result, we reformulate the contingent commitment problem into a time-consistent recursive problem with the choice of tax rates restricted by the "incentive-compatibility" constraints. The new problem is characterized by an "incentive-compatible value of repayment" $V^{R,IC}$, an "incentive-

[2]We thank one of the anonymous referees to point out this method to reformulate the government's recursive problem. In the previous version of this paper, we limited the choice set of tax rates to two values to limit the dimension of the optimization. However, such an assumption significantly limited the quantitative implications of our model.

[3]We prove the equivalence of the original CC problem and its reformulation in section C.

[4]In fact, once the capital flow $qb' - b$ is determined, the government's taxation decision becomes a static problem. So, for both the repaying and defaulting government, the optimal discretionary tax rate is always a function of income and net capital flow: $\tau^{CF}(y, qb' - b)$, as defined in equation (19).

[5]Since the utility loss function $U^D(y)$ is an increasing function of income y, there exists a default threshold below which the government chooses to default. Such property of default decision allows us to adopt this reformulation.

compatible value of default" $V^{D,IC}$, and a price function q.[6]

Specifically, if the current government is assigned to repay by the previous-period government, it must chooses a tax rate that makes repayment a more attractive option than default. The incentive-compatible value of repayment is given by

$$V^{R,IC}(b,y) = \tag{B.1}$$
$$\max_{b',y^{*\prime},\tau} \left\{ u(c,g) + \beta \left[\int_{y_{min}}^{y^{*\prime}} V^{D,IC}(b',y')dF(y'|y) + \int_{y^{*\prime}}^{y_{max}} V^{R,IC}(b',y')dF(y'|y) \right] \right\},$$
$$\text{s.t.} \quad c = (1-\tau)y,$$
$$g = \tau y + q(y^{*\prime})b' - b,$$
$$V^{R,IC}(b,y) \geq V^{D,CF}(\tau,y), \qquad \text{(IC – Repayment)} \tag{B.2}$$

where $V^{D,CF}(\tau,y)$ is the counter-factual value of default if the government faces the same income level y and choice of tax rate τ:

$$V^{D,CF}(\tau,y) = \tag{B.3}$$
$$\max_{y^{*\prime}} \left\{ u\left((1-\tau)y, \tau y\right) - U^D(y) + \beta \left[\int_{y_{min}}^{y^{*\prime}} V^{D,IC}(0,y')dF(y'|y) + \int_{y^{*\prime}}^{y_{max}} V^{R,IC}(0,y')dF(y'|y) \right] \right\}$$

The value function $V^{R,IC}(b,y)$ represents the problem of a government that is required to repay its debt in the current period. It needs to choose a contemporaneous tax rate τ that makes the value of repayment higher than the counter-factual value of default $V^{D,CF}(\tau,y)$. The IC constraint (B.2) says that the tax rate τ is chosen so that the government has no incentive to default if the defaulting government also implements this tax rate. This IC constraint is needed to incorporate the tax commitment problem. Because there is no enforcement of debt repayment in our model, the tax rate serves as a commitment device used to induce the current government to repay, given the state of the economy. So, we reformulate the commitment problem into a self-disciplined recursive problem of the government.

If the IC constraint (B.2) does not bind, the repaying government does not need any tax discipline for the debt repayment and would choose an *ex post* optimal tax rate to maximize its contemporaneous utility (shown as the unshaded region in the right panel of figure 6). However, when the IC constraint is binding, the government must deviate from the *ex post* optimal taxes and set a higher tax rate to enforce itself to repay (shown

[6] In the reformulated problem, the bond price function only depends on the choice of default threshold: $q(y^{*\prime})$.

as the shaded region). Lastly, for some of the states of (b, y), there is no possible choice of $(\tau, y^{*\prime}, b')$ that satisfies the IC constraint. These are the states that it is impossible for the government to repay its debt without violating the IC constraint. In this case, we set the value of repayment $V^{R,IC}(b, y)$ to a "death value" (-1e150) in our numerical solution so that the government never wants this state of the economy to realize in equilibrium.

Similarly, the incentive-compatible value of default is given by

$$
\begin{aligned}
& V^{D,IC}(b,y) = && \text{(B.4)} \\
& \max_{y^{*\prime},\tau} \left\{ u(c,g) - U^D(y) + \beta \left[\int_{y_{min}}^{y^{*\prime}} V^{D,IC}(0,y')dF(y'|y) + \int_{y^{*\prime}}^{y_{max}} V^{R,IC}(0,y')dF(y'|y) \right] \right\}, && \\
\text{s.t.} \quad & c = (1-\tau)y, && \\
& g = \tau y, && \\
& V^{D,IC}(b,y) \geq V^{R,CF}(b,\tau,y), \qquad \text{(IC – Default)} && \text{(B.5)}
\end{aligned}
$$

where $V^{R,CF}(b,\tau,y)$ is the counter-factual value of repayment if the government faces the state variables (b, y) and choice of tax rate τ:

$$
\begin{aligned}
& V^{R,CF}(b,\tau,y) = \qquad\qquad \text{(B.6)} \\
& \max_{b',y^{*\prime}} \left\{ u\left((1-\tau)y, \tau y + q(y^{*\prime})b' - b\right) + \beta \left[\int_{y_{min}}^{y^{*\prime}} V^{D,IC}(b',y')dF(y'|y) + \int_{y^{*\prime}}^{y_{max}} V^{R,IC}(b',y')dF(y'|y) \right] \right\}.
\end{aligned}
$$

The defaulting government also follows an IC constraint (B.5) such that the government has no incentive to repay under the same tax rate τ. For the states that the IC constraint cannot be satisfied, we set $V^{D,IC}(b, y)$ to the death value (-1e150) as well.

After reformulating the problem, the current-period government only needs to choose a default threshold $y^{*\prime}$ for the next period and let the next-period government decide the optimal incentive-compatible tax rates. After the next-period income y' realizes, the next-period government has no incentive to deviate from the committed repayment-default decision since the tax rates it implements must also satisfy the IC constraints. In addition, our algorithm ensures that the government will never choose a default threshold that is infeasible to be implemented in the next period since it will yield an extremely low continuation value (due to the "death value" we assign when there is no solution). Therefore, the described reformulated problem yields the same solution to our original problem in equation (15) - (18). Lastly, the bond price in this reformulation only depends on the promised default threshold for the next period:

$$q(y^{*\prime}) = \frac{\int_{y^{*\prime}}^{y_{max}} 1 dF(y'|y)}{1+r^*}. \tag{B.7}$$

The algorithm to solve the government's incentive problem is summarized as the following,

1. We use the value functions from the discretionary fiscal policy as an initial guess for the incentive-compatible value functions $V^{R,IC}$ and $V^{D,IC}$.

2. For each state (b, y):

   (a) We solve the incentive-compatible repayment problem given by (B.1) - (B.2). Specifically, we use discrete choice method to search for a combination of (b', τ, $y^{*\prime}$) that maximizes the value of repayment in equation (B.1) while keeping the IC constraint (B.2) satisfied. If there is no incentive-compatible choice that can deliver positive public and private consumption, set the value of repayment for this state to the death value.

   (b) We solve the incentive-compatible default problem given by (B.4) - (B.5). Specifically, we search for a combination of (τ, $y^{*\prime}$) that optimizes the value of default in equation (B.4) while the IC constraint (B.5) is satisfied. If there is no incentive-compatible choice that can deliver positive public and private consumption, set the default value for this state to the death value.

   (c) Record the equilibrium bond price $q(y^{*\prime})$, which reflects the probability that the government will repay in the next period.

3. Check for the convergence of $V^{R,IC}$, $V^{D,IC}$, q. Stop if the distance between successive iterations is smaller than 10^{-6}; otherwise, fully update and iterate.

Computation of Welfare Gains. Similar to Bianchi et al. (2019), we use the value function iteration method to calculate the conditional welfare gains discussed in section 4.3.3 and table 4. In our quantitative analysis, the welfare gain is calculated as the percentage of permanent private consumption (c) that makes the household indifferent between living in an economy with tax commitment and an economy without it. For a given initial state and the guessed value for the welfare gain, we iterate on the value functions of the government with discretionary tax policies, keeping the policy functions and default strategies fixed. Once the convergence is achieved, we calculate the difference between the newly computed value function and the value function under tax commitment evaluated at the selected initial state. If the absolute value of the difference is lower than 10^{-6}, we stop and report the welfare gain. Otherwise, we update the guessed welfare gain and iterate again. We repeat this procedure for all the initial states of interest. For the average welfare gain, it is calculated as the conditional welfare gains weighted by the ergodic distribution of an economy under the non-contingent or contingent tax commitment.

C Equivalence between the Contingent Commitment Problem and Its Modified Problem

This section shows the equivalence between the contingent commitment problem (CC) in the main text and the government's incentive-compatible problem (IC) discussed in the numerical method in online appendix B. In subsection C.1, we first prove that the feasibility sets of the two problems coincide. Subsection C.2 displays the equivalence in the two-period environment. Lastly, in subsection C.3, we show that the two problems have the same optimality conditions in the quantitative environment.

C.1 A Heuristic Proof for the Equivalence between the IC and CC Problem

We first show that under the CC problem, the committed tax schedule always implies a default threshold such that the government will choose to default when the income is below that threshold. We then argue that the CC government's choices and the corresponding allocations belong to the feasibility set of the IC problem and vice versa. As a result, the two problems deliver the same value functions in the corresponding income states. Our proof proceeds in the following three steps:

(a) First, we prove that in each period, the optimally-committed tax schedule is associated with a default threshold in the next period such that the government will choose to service its debt only when its income is no less than that threshold.

Equivalently, we only need to show that if the government finds it optimal to repay at $y_1 \in \mathcal{Y}'$,[7] it is also optimal to repay at any $y_2 \in \mathcal{Y}'$ such that $y_2 \geq y_1$. For each level of debt issuance b', denote the optimally-committed tax schedule conditional on that government will repay its debt in the next period as $\tau^{R*}(y')$. If the government defaults, the optimal tax rate is τ^{D*}. From the form of the utility function, we know that the optimal tax rate under default does not depend on income realizations. Since the government finds the repayment option more attractive than the default option at y_1, we have

$$V^R(b', \tau^{R*}(y_1), y_1) \geq V^D(\tau^{D*}, y_1). \tag{C.1}$$

Additionally, $\tau^{R*}(y_1)$ is also compatible with the repayment incentive: the govern-

[7]Denote $\mathcal{Y}$ as the space of the current income; $\mathcal{Y}'$ refers to the space of the next-period income.

ment has an incentive to repay if the tax rate is fixed at $\tau^{R*}(y_1)$, that is,

$$V^R(b', \tau^{R*}(y_1), y_1) \geq V^D(\tau^{R*}(y_1), y_1). \tag{C.2}$$

We make the assumption that, for any pairs of τ^R and $\tau^D \in (0,1)$ under repayment and default, $V^R(b', \tau^R, y) - V^D(\tau^D, y)$ is monotonically increasing in y. Implicitly, that means as income y rises, the utility cost of default $U^D(y)$ becomes larger, which is consistent with our choice of the default cost function and parameter values. As a result, based on equation (C.2), at the higher income of y_2, we have $V^R(b', \tau^{R*}(y_1), y_2) \geq V^D(\tau^{R*}(y_1), y_2)$. That means levying a tax rate of $\tau^{R*}(y_1)$ is enough to incentivize the government to repay at y_2.

Moreover, equation (C.1) and the assumption of monotonicity imply that

$$V^R(b', \tau^{R*}(y_1), y_2) - V^D(\tau^{D*}, y_2) \geq V^R(b', \tau^{R*}(y_1), y_1) - V^D(\tau^{D*}, y_1) \geq 0.$$

Since $\tau^{R*}(y_2)$ is defined as the optimally-committed tax rate at y_2 conditional on repayment, government's value from choosing $\tau^{R*}(y_2)$ must be no less than the value from choosing $\tau^{R*}(y_1)$, that is

$$V^R(b', \tau^{R*}(y_2), y_2) \geq V^R(b', \tau^{R*}(y_1), y_2).$$

Combining the last two equations gives rise to

$$V^R(b', \tau^{R*}(y_2), y_2) \geq V^D(\tau^{D*}, y_2).$$

It indicates that the government finds the repayment option more attractive than default at income level y_2 as well. Therefore, for the CC government, the repayment-default plan is characterized by an income threshold such that government repays its debt only when the realized income exceeds the threshold.

(b) Second, we prove that the optimal choices made by the CC government and the corresponding allocations are feasible for the IC government. As a result, the values functions delivered by the IC problem must be no less than the value functions from the CC problem when tax rates are set optimally.

As discussed in the previous step, we argued that the CC government chooses a combination of borrowing and tax schedule $\{b', \tau^{*'}(y')\}$ so that the next-period government makes the default decision based on an income-threshold rule (by comparing the realized y' with the threshold $y^{*'}$). The default threshold $y^{*'}$ needs to be compatible with the chosen tax schedule in the sense that for $y' \geq y^{*'}$, the government has an incentive to repay with the tax rate $\tau^{*'}(y')$: $V^R(b', \tau^{*'}(y'), y') \geq V^D(\tau^{*'}(y'), y')$. Similarly, for $y' < y^{*'}$, the government has an incentive to default with the tax rate $\tau^{*'}(y')$: $V^R(b', \tau^{*'}(y'), y') < V^D(\tau^{*'}(y'), y')$.

Denote $\tau^*(y)$ as the optimal tax schedule in the current period (inherited from the last period). For a given (b, y), we define the value of the CC government at the optimally-designed tax rate as: $\tilde{V}^R(b, y) \equiv V^R(b, \tau^*(y), y)$ and $\tilde{V}^D(y) \equiv V^D(\tau^*(y), y)$. Then, based on the above argument, we have

$$\tilde{V}^R(b', y') \geq V^D(\tau^{*'}(y'), y'), \quad \text{for any } \; y' \geq y^{*'}, \tag{C.3}$$
$$\tilde{V}^D(y') > V^R(b', \tau^{*'}(y'), y'), \quad \text{for any } \; y' < y^{*'}. \tag{C.4}$$

The inequalities indicate that the next period tax schedule $\tau^{*'}(y')$ satisfies the "IC-repayment" or "IC-Default" constraint in period $t+1$: for income levels greater than or equal to the threshold $y^{*'}$, the value of repayment under the committed tax rate is higher than the counter-factual value of default with the same tax rate. Reverse is true for income levels below the threshold[8]. Now, let us consider the tax decision in period t. Recall $\tau^*(y)$ is the optimal tax rate schedule designed in period $t-1$ under the CC problem, and y^* is the associated default threshold. Based on the above definition, for any $y \geq y^*$, $\tilde{V}^R(b, y) \geq V^D(\tau^*(y), y)$, while for any $y < y^*$, $\tilde{V}^D(y) \geq V^R(b, \tau^*(y), y)$. It means the current tax schedule under the CC problem $\tau^*(y)$ also satisfies the period-t IC constraints. Therefore, the optimal choices of the CC government (and the corresponding allocations), $\{b', \tau^{*'}(y'), y^{*'}, c'(y'), g'(y'), \tau^*(y)\}$, are feasible for the IC government. As a result, the value functions of the IC problem must be (weakly) higher than the value functions of the CC problem with the current τ set optimally, which implies[9]

$$V^{R,IC}(b, y) \geq \tilde{V}^R(b, y) = V^R(b, \tau^*(y), y), \quad \text{at the repayment states } y \geq y^*, \tag{C.5}$$
$$V^{D,IC}(b, y) \geq \tilde{V}^D(y) = V^D(\tau^*(y), y), \quad \text{at the default states } y < y^*. \tag{C.6}$$

[8] The equilibrium conditions (C.3) and (C.4) derived under the CC framework correspond to the incentive-compatibility constraints (B.2) and (B.5) under the IC framework if their corresponding values equal to each other in relevant income states: $\tilde{V}^R(b', y') = V^{R,IC}(b', y')$ in repayment states and $\tilde{V}^D(y') = V^{D,IC}(b', y')$ in default states.

[9] b is a redundant state in the CC problem under default ($\tilde{V}^D$) since $\tau^*(y)$ is already incentive-compatible by our definition: it incentivizes the government to default in the income levels below the threshold: $y < y^*$. However, we still need b as an additional state in the IC default problem ($V^{D,IC}$) because it affects the counter-factual value of repayment ($V^{R,CF}$) in the IC constraint.

(c) Lastly, we show that the allocations from the IC problem are also feasible for the CC government under a specific current tax rate. As a result, the incentive-compatible value functions are no greater than the value functions of the original CC problem with the current tax rate set optimally.

We use subscript t to denote variables in the current period and $t+1$ as variables in the next period. Consider the period-t problem faced by the IC government under repayment. Given state (b_t, y_t), the IC government makes decisions on $\{b_{t+1}, y^*_{t+1}, \tau_t\}$, with the tax rate τ_t satisfying the incentive-compatibility constraints (B.2) and (B.5). In contrast, for the CC problem under repayment (V^R), we can calculate the optimal current tax rate conditional on that government has an incentive to repay, which is the following problem:

$$\tilde{V}^R(b_t, y_t) = \max_{\tau_t} \left\{ V^R(b_t, \tau_t, y_t), \quad \text{s.t. } V^R(b_t, \tau_t, y_t) \geq V^D(\tau_t, y_t) \right\}.$$

Note that for $y_t \geq y^*_t$, the optimizing tax rate should be exactly the same one as the CC government would commit in period t-1, that is $\tau^*_t(y_t)$. So, the decisions, $\{b_{t+1}, y^*_{t+1}, \tau_t\}$, of the IC problem are all within the feasible choice set of the CC problem, $\tilde{V}^R(b_t, y_t)$.

Similarly, the IC government under default makes decisions on $\{y^*_{t+1}, \tau_t\}$. These choices also belong to the feasibility set of the CC government if the tax rate $\tau_t(y_t)$ incentivizes the government to default in states that satisfy $y_t < y^*_t$. Finally, we have the following inequalities

$$\tilde{V}^R(b_t, y_t) = \max_{\tau_t} \left\{ V^R(b_t, \tau_t, y_t), \text{ s.t. } V^R(b_t, \tau_t, y_t) \geq V^D(\tau_t, y_t) \right\} \geq V^{R,IC}(b_t, y_t), \ \forall y_t \geq y^*_t, \tag{C.7}$$

$$\tilde{V}^D(y_t) = \max_{\tau_t} \left\{ V^D(\tau_t, y_t), \text{ s.t. } V^D(\tau_t, y_t) \geq V^R(b_t, \tau_t, y_t) \right\} \geq V^{D,IC}(b_t, y_t), \ \forall y_t < y^*_t. \tag{C.8}$$

Combining the relationship (C.5)-(C.6) with (C.7)-(C.8), we have

$$\tilde{V}^R(b_t, y_t) = V^{R,IC}(b_t, y_t), \quad \text{at the repayment states } y_t \geq y^*_t, \tag{C.9}$$

$$\tilde{V}^D(y_t) = V^{D,IC}(b_t, y_t), \quad \text{at the default states } y_t < y^*_t. \tag{C.10}$$

Equation (C.9) implies that, at the income levels that the government repays $y_t \geq y^*_t$, the value from the IC problem $V^{R,IC}(b_t, y_t)$ equals to the value from the CC problem, evaluated at the optimally-committed tax rate, $\tilde{V}^R(b_t, y_t)$. Similarly, equation (C.10) implies that, for the income levels that government defaults $y_t < y_t^*$, the value from the IC problem $V^{D,IC}(b_t, y_t)$ also equals to the value from the CC problem evaluated at the optimally-committed tax rate, $\tilde{V}^D(y_t)$. Therefore, the two problems have the same value functions along the equilibrium path. In addition, since the decisions made by the CC government also satisfy the constraints faced by the IC government and vice versa, the two problems yield the same equilibrium paths of allocations: $\{c_t, g_t, \tau_t, b_{t+1}, y_{t+1}^*\}_{t=0}^{\infty}$.

C.2 Comparing the FOCs of the CC and IC Problems: Two-Period Model

In this section, we show the equivalence between the CC problem and the IC problem in the two-period environment as in section 2. We show that the two problems yield the same set of optimality conditions.

The reformulated CC problem. In section C.1 - Part (a), we showed that under the contingent tax commitment (CC), the government's optimal default decision is characterized by a threshold on the income level. This result also holds in the two-period environment since the default cost $(1-\phi)y_2$ is proportional to the income level in the second period[10]. As a result, the government only repays in high-income states to avoid incurring the high default cost. By defining this default threshold, the discretionary default decisions in the second period can be front-loaded to period 1. To compare the FOCs of the CC and IC problems in the two-period environment, we first reformulate the original CC problem by incorporating the choice of this default threshold in the first-period problem.

In the first period, the government chooses bond issuance (b_2), the second-period tax

[10]We can also see the proof of this reformulation from proposition 4.

schedule $\tau_2(y_2)$, and a default threshold $\tilde{y}$. The reformulated two-period CC problem is:

$$W_1 = \max_{\{b_2,\tau_2(y_2),\tilde{y}\}} \left\{ \frac{[q(\tilde{y})b_2]^{1-\sigma}}{1-\sigma} + \beta \int_{y_{min}}^{\tilde{y}} \left[(1-\pi)\frac{[(1-\tau_2(y_2))\phi y_2]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2(y_2)\phi y_2]^{1-\sigma}}{1-\sigma} \right] f(y_2) dy_2 \right.$$
$$\left. + \beta \int_{\tilde{y}}^{y_{max}} \left[(1-\pi)\frac{[(1-\tau_2(y_2))y_2]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2(y_2)y_2 - b_2]^{1-\sigma}}{1-\sigma} \right] f(y_2) dy_2 \right\} \qquad \text{(C.11)}$$

$$\text{s.t. } q(\tilde{y}) = 1 - \frac{\tilde{y} - y_{min}}{y_{max} - y_{min}} \qquad \text{(C.12)}$$

$$(1-\pi)\frac{[(1-\tau_2(y_2))\phi y_2]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2(y_2)\phi y_2]^{1-\sigma}}{1-\sigma}$$
$$> (1-\pi)\frac{[(1-\tau_2(y_2))y_2]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2(y_2)y_2 - b_2]^{1-\sigma}}{1-\sigma}, \quad \forall y_2 < \tilde{y} \qquad \text{(C.13)}$$

$$(1-\pi)\frac{[(1-\tau_2(y_2))\phi y_2]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2(y_2)\phi y_2]^{1-\sigma}}{1-\sigma}$$
$$\leq (1-\pi)\frac{[(1-\tau_2(y_2))y_2]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2(y_2)y_2 - b_2]^{1-\sigma}}{1-\sigma}, \quad \forall y_2 \geq \tilde{y} \qquad \text{(C.14)}$$

Note inequalities (C.13) and (C.14) show that the government's choice of b_2 and $\tau_2(y_2)$ must be compatible with the default threshold $\tilde{y}$. These inequalities impose restrictions on the government's design of tax schedule ($\tau_2(y_2)$) for the second period. In a low income state $y_2 < \tilde{y}$, the second-period utility under default must be higher than the utility under repay, as indicated by inequality (C.13). Similarly, in a high income state $y_2 \geq \tilde{y}$, the utility under repay must be higher than the utility under default. Denote $\mu^{D,CC}(y_2)$ and $\mu^{R,CC}(y_2)$ as the two Lagrange multipliers attached to the constraints (C.13) and (C.14) for a specific value of income y_2. Next, we drive the first-order conditions in the reformulated CC problem.

First, taking derivative with respect to the default threshold $\tilde{y}$ yields

$$[q(\tilde{y})b_2]^{-\sigma} b_2 q'(\tilde{y}) + \beta \left[(1-\pi)\frac{[(1-\tau_2(\tilde{y}))\phi\tilde{y}]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2(\tilde{y})\phi\tilde{y}]^{1-\sigma}}{1-\sigma} \right.$$
$$\left. - (1-\pi)\frac{[(1-\tau_2(\tilde{y}))\tilde{y}]^{1-\sigma}}{1-\sigma} - \pi \frac{[\tau_2(\tilde{y})\tilde{y} - b_2]^{1-\sigma}}{1-\sigma} \right] f(\tilde{y}) = 0. \qquad \text{(C.15)}$$

Second, the first-order condition with respect to debt issuance b_2 is given by

$$[q(\tilde{y})b_2]^{-\sigma} q(\tilde{y}) + \beta \left[\int_{y_{min}}^{\tilde{y}} \mu^{D,CC}(y_2)\pi[\tau_2(y_2)y_2 - b_2]^{-\sigma} f(y_2) dy_2 \right.$$
$$\left. - \int_{\tilde{y}}^{y_{max}} (\mu^{R,CC}(y_2) + 1)\pi[\tau_2(y_2)y_2 - b_2]^{-\sigma} f(y_2) dy_2 \right] = 0. \qquad \text{(C.16)}$$

Lastly, taking derivative with respect to $\tau_2(y_2)$ at a certain repaying income state $y_2 \geq \tilde{y}$ yields the following expression,

$$\begin{aligned}&(1+\mu^{R,CC}(y_2))\{(1-\pi)[(1-\tau_2(y_2))y_2]^{-\sigma}(-y_2)+\pi[\tau_2(y_2)y_2-b_2]^{-\sigma}y_2\}\\&-\mu^{R,CC}(y_2)\{(1-\pi)[(1-\tau_2(y_2))\phi y_2)]^{-\sigma}(-\phi y_2)+\pi[\tau_2(y_2)\phi y_2]^{-\sigma}\phi y_2\}=0, \quad \text{(C.17)}\end{aligned}$$

while at a defaulting income state $y_2 < \tilde{y}$, we have

$$\begin{aligned}&(1+\mu^{D,CC}(y_2))\{(1-\pi)[(1-\tau_2(y_2)\phi y_2)]^{-\sigma}(-\phi y_2)+\pi[\tau_2(y_2)\phi y_2]^{-\sigma}\phi y_2\}\\&-\mu^{D,CC}(y_2)\{(1-\pi)[(1-\tau_2(y_2))y_2]^{-\sigma}(-y_2)+\pi[\tau_2(y_2)y_2-b_2]^{-\sigma}y_2\}=0. \quad \text{(C.18)}\end{aligned}$$

In the end, we can characterize the solution to the reformulated CC problem by equations (C.15), (C.16), (C.17), (C.18).

The IC problem. As described in section B, in the IC problem, the government chooses the default threshold and debt issuance in period 1, and defers the taxation decisions to the second period. In the second period, the government has to implement the tax rate that is incentive-compatible with the default threshold designed in the first period. The IC government's first-period problem is

$$V_1^{IC}=\max_{\tilde{y},b_2}\left\{\frac{[q(\tilde{y})b_2]^{1-\sigma}}{1-\sigma}+\beta\left[\int_{y_{min}}^{\tilde{y}}V^{D,IC}(b_2,y_2)f(y_2)dy_2+\int_{\tilde{y}}^{y_{max}}V^{R,IC}(b_2,y_2)f(y_2)dy_2\right]\right\} \quad \text{(C.19)}$$

In the second period, the government is obligated to repay if the income is higher than the default threshold: $y_2 \geq \tilde{y}$. The incentive-compatible value of repayment is given by

$$V^{R,IC}(b_2,y_2)=\max_{\tau_2}\left\{(1-\pi)\frac{[(1-\tau_2)y_2]^{1-\sigma}}{1-\sigma}+\pi\frac{[\tau_2 y_2-b_2]^{1-\sigma}}{1-\sigma}\right\} \quad \text{(C.20)}$$

$$\text{s.t. } (1-\pi)\frac{[(1-\tau_2)y_2]^{1-\sigma}}{1-\sigma}+\pi\frac{[\tau_2 y_2-b_2]^{1-\sigma}}{1-\sigma}\geq(1-\pi)\frac{[(1-\tau_2)\phi y_2]^{1-\sigma}}{1-\sigma}+\pi\frac{[\tau_2\phi y_2]^{1-\sigma}}{1-\sigma}, \quad \text{(C.21)}$$

where the IC constraint (C.21) indicates that the government's period-2 utility under repayment cannot be lower than the utility under default. Denote $\mu^{R,IC}(y_2)$ as the Lagrange multiplier attached to this incentive constraint at the specific income y_2.

Similarly, if the income is lower than the default threshold: $y_2 < \tilde{y}$, the government is obligated to default on its debt. The incentive-compatible value of default is defined as

$$V^{D,IC}(b_2, y_2) = \max_{\tau_2} \left\{ (1-\pi)\frac{[(1-\tau_2)\phi y_2]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2 \phi y_2]^{1-\sigma}}{1-\sigma} \right\} \tag{C.22}$$

$$\text{s.t. } (1-\pi)\frac{[(1-\tau_2)y_2]^{1-\sigma}}{1-\sigma} + \pi\frac{[\tau_2 y_2 - b_2]^{1-\sigma}}{1-\sigma} < (1-\pi)\frac{[(1-\tau_2)\phi y_2]^{1-\sigma}}{1-\sigma} + \pi\frac{[\tau_2\phi y_2]^{1-\sigma}}{1-\sigma}, \tag{C.23}$$

where inequality (C.23) is the IC constraint. Denote $\mu^{D,IC}(y_2)$ as the Lagrange multiplier attached to the incentive constraint at the income level of y_2.

First, the envelope theorem gives rise to the following expressions,

$$\frac{\partial V^{R,IC}(b_2, y_2)}{\partial b_2} = -(1+\mu^{R,IC}(y_2))\pi[\tau_2 y_2 - b_2]^{-\sigma}, \tag{C.24}$$

$$\frac{\partial V^{D,IC}(b_2, y_2)}{\partial b_2} = \mu^{D,IC}(y_2)\pi[\tau_2 y_2 - b_2]^{-\sigma}. \tag{C.25}$$

In the second-period incentive-compatible problems, taking FOCs with respect to τ_2 yield

$$\begin{aligned}(1+\mu^{R,IC}(y_2))\{(1-\pi)[(1-\tau_2)y_2]^{-\sigma}(-y_2) + \pi[\tau_2 y_2 - b_2]^{-\sigma} y_2\} \\ - \mu^{R,IC}(y_2)\{(1-\pi)[(1-\tau_2)\phi y_2)]^{-\sigma}(-\phi y_2) + \pi[\tau_2\phi y_2]^{-\sigma}\phi y_2\} = 0,\end{aligned} \tag{C.26}$$

$$\begin{aligned}(1+\mu^{D,IC}(y_2))\{(1-\pi)[(1-\tau_2)\phi y_2)]^{-\sigma}(-\phi y_2) + \pi[\tau_2\phi y_2]^{-\sigma}\phi y_2\} \\ - \mu^{D,IC}(y_2)\{(1-\pi)[(1-\tau_2)y_2]^{-\sigma}(-y_2) + \pi[\tau_2 y_2 - b_2]^{-\sigma} y_2\} = 0.\end{aligned} \tag{C.27}$$

for $y_2 \geq \tilde{y}$ and $y_2 < \tilde{y}$, respectively.

In the first-period problem, taking derivative with respect to b_2 gives rise to

$$[q(\tilde{y})b_2]^{-\sigma}q(\tilde{y}) + \beta\left[\int_{y_{min}}^{\tilde{y}} \frac{\partial V^{D,IC}(b_2, y_2)}{\partial b_2} f(y_2)dy_2 + \int_{\tilde{y}}^{y_{max}} \frac{\partial V^{R,IC}(b_2, y_2)}{\partial b_2} f(y_2)dy_2\right] = 0. \tag{C.28}$$

Plugging in equations (C.24) and (C.25), we arrive at the following expression

$$\begin{aligned}[q(\tilde{y})b_2]^{-\sigma}q(\tilde{y}) + \beta\Bigg[\int_{y_{min}}^{\tilde{y}} \mu^{D,IC}(y_2)\pi[\tau_2^{D,IC}(b_2, y_2)y_2 - b_2]^{-\sigma} f(y_2)dy_2 \\ - \int_{\tilde{y}}^{y_{max}} (1+\mu^{R,IC}(y_2))\pi[\tau_2^{R,IC}(b_2, y_2)y_2 - b_2]^{-\sigma} f(y_2)dy_2\Bigg] = 0,\end{aligned} \tag{C.29}$$

where $\tau_2^{R,IC}(b_2, y_2)$ and $\tau_2^{D,IC}(b_2, y_2)$ are the optimal tax rates that the IC-government will choose in period 2, given the states (b_2, y_2).

Lastly, the first-order condition with respect to $\tilde{y}$ is

$$[q(\tilde{y})b_2]^{-\sigma} b_2 q'(\tilde{y}) + \beta \left[V^{D,IC}(b_2, \tilde{y}) - V^{R,IC}(b_2, \tilde{y}) \right] f(\tilde{y}) = 0. \tag{C.30}$$

Combined with the expressions of $V^{D,IC}$ and $V^{R,IC}$, the equation becomes

$$\begin{aligned} &[q(\tilde{y})b_2]^{-\sigma} b_2 q'(\tilde{y}) + \beta \Bigg[(1-\pi) \frac{[(1-\tau_2^{D,IC}(b_2,\tilde{y}))\phi\tilde{y}]^{1-\sigma}}{1-\sigma} + \pi \frac{[\tau_2^{D,IC}(b_2,\tilde{y})\phi\tilde{y}]^{1-\sigma}}{1-\sigma} \\ &\qquad - (1-\pi) \frac{[(1-\tau_2^{R,IC}(b_2,\tilde{y}))\tilde{y}]^{1-\sigma}}{1-\sigma} - \pi \frac{[\tau_2^{R,IC}(b_2,\tilde{y})\tilde{y} - b_2]^{1-\sigma}}{1-\sigma} \Bigg] f(\tilde{y}) = 0. \end{aligned} \tag{C.31}$$

The solution of the IC problem is characterized by equations (C.26), (C.27), (C.29), (C.31).

Equivalence between the CC and IC problems. We can see that the CC and IC problems yield the same set of optimality conditions in the two-period model. Specifically, the first-order conditions with respect to the default threshold $\tilde{y}$ are the same in the CC problem (C.15) and IC problem (C.31). The first-order conditions with respect to bond issuance b_2 are identical as well, as shown in equations (C.16) and (C.29). Lastly, the first-order conditions with respect to the tax rates in the CC problem (equations C.17 - C.18) are identical to the ones from the IC problem (C.26 - C.27).

C.3 Comparing the FOCs of the CC and IC Problems: Infinite-Horizon Model

This section shows that the CC and IC problems have the same optimality conditions in the infinite-horizon setup as in section 3. Similar to the two-period model, the CC problem can be reformulated by allowing the current government to choose the next-period default threshold.

The reformulated CC problem. We first reformulate the original CC problem by incorporating the choice of the next-period default threshold. In the original CC problem, the government chooses the bond issuance (b') and a menu of committed tax rates ($\tau'(y')$), while allowing the next-period government to make its default decision ($d'(y')$) based on the preset tax menu. As shown in section C.1, this problem can be reformulated into a framework that time-t government jointly chooses bond issuance, a menu of tax rates, and a default threshold $(b', \tau'(y'), y^{*'})$ such that $y^{*'}$ and $\tau'(y')$ are compatible. This compatibility is written in terms of a set of inequality restrictions imposed on the next-period problem.

If the government repays, the reformulated CC problem is

$$W^R(b,\tau,y) = \tag{C.32}$$
$$\max_{\tau'(y'),b',y^{*'}} \left\{ u(c,g) + \beta \left[\int_{y_{min}}^{y^{*'}} W^D(\tau'(y'),y')dF(y'|y) + \int_{y^{*'}}^{y_{max}} W^R(b',\tau'(y'),y')dF(y'|y) \right] \right\},$$

$$\text{s.t.} \quad g + b = \tau y + \frac{1}{1+r^*}\mathbb{P}(y' \geq y^{*'})b',$$
$$c = (1-\tau)y,$$
$$W^R(b',\tau'(y'),y') \geq W^D(\tau'(y'),y'), \qquad \forall y' \geq y^{*\prime}, \tag{C.33}$$
$$W^R(b',\tau'(y'),y') < W^D(\tau'(y'),y'), \qquad \forall y' < y^{*\prime}. \tag{C.34}$$

Note that one incentive constraint is imposed on each income level y' in the next period to ensure that the next-period default incentives are compatible with the *ex ante* choice of tax rates. Similarly, if the government defaults on its debt, the reformulated CC problem is

$$W^D(\tau,y) = \tag{C.35}$$
$$\max_{\tau'(y'),y^{*'}} \left\{ u(c,g) - U^D(y) + \beta \left[\int_{y_{min}}^{y^{*'}} W^D(\tau'(y'),y')dF(y'|y) + \int_{y^{*'}}^{y_{max}} W^R(0,\tau'(y'),y')dF(y'|y) \right] \right\}$$

$$\text{s.t.} \quad g = \tau y,$$
$$c = (1-\tau)y,$$
$$W^R(0,\tau'(y'),y') \geq W^D(\tau'(y'),y'), \qquad \forall y' \geq y^{*\prime}, \tag{C.36}$$
$$W^R(0,\tau'(y'),y') < W^D(\tau'(y'),y'), \qquad \forall y' < y^{*\prime}. \tag{C.37}$$

Still, the incentive constraints guarantee that the default threshold $y^{*'}$ is consistent with the menu of tax rates $\tau'(y')$ chosen in *ex ante*. Next, we are ready to derive the optimality conditions in the IC and reformulated CC problems and prove their equivalence. Under the reformulated CC problem, we focus on the government's period-t optimization.

First, in the reformulated CC problem, we take derivative with respect to the tax rate τ_{t+1} on each of the next-period income levels. The FOC depends on whether the next-period government is designated to repay or default in the corresponding income state.

The FOC in the repaying income states ($y_{t+1} \geq y^*_{t+1}$) writes as

$$\frac{\partial W^R(b_{t+1}, \tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} + \mu^{R,CC}_{t+1}(y_{t+1}) \left(\frac{\partial W^R(b_{t+1}, \tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} - \frac{\partial W^D(\tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} \right) = 0, \tag{C.38}$$

where $\mu^{R,CC}_{t+1}(y_{t+1})$ is the Lagrange multiplier on the constraint (C.33). Similarly, at any defaulting state, taking derivative with respect to τ_{t+1} yields

$$\frac{\partial W^D(\tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} + \mu^{D,CC}_{t+1}(y_{t+1}) \left(\frac{\partial W^D(\tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} - \frac{\partial W^R(b_{t+1}, \tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} \right) = 0, \tag{C.39}$$

where $\mu^{D,CC}_{t+1}(y_{t+1})$ is the Lagrange multiplier on the constraint (C.34).

Denote $u^{R,CC}_t$ as the households' period-t utility under repayment. Similarly, denote $u^{D,CC}_t$ as the households' period-t utility under default. The envelope theorem implies

$$\frac{\partial W^R(b_{t+1}, \tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} = \left(\frac{\partial u^{R,CC}_{t+1}}{\partial g_{t+1}} - \frac{\partial u^{R,CC}_{t+1}}{\partial c_{t+1}} \right) y_{t+1}, \tag{C.40}$$

$$\frac{\partial W^D(\tau_{t+1}, y_{t+1})}{\partial \tau_{t+1}} = \left(\frac{\partial u^{D,CC}_{t+1}}{\partial g_{t+1}} - \frac{\partial u^{D,CC}_{t+1}}{\partial c_{t+1}} \right) y_{t+1}. \tag{C.41}$$

Substituting equations (C.40) and (C.41) into (C.38) and (C.39) gives rise to the following expressions for the next-period repayment and default states respectively,

$$(1 + \mu^{R,CC}_{t+1}) \left(\frac{\partial u^{R,CC}_{t+1}}{\partial g_{t+1}} - \frac{\partial u^{R,CC}_{t+1}}{\partial c_{t+1}} \right) y_{t+1} - \mu^{R,CC}_{t+1} \left(\frac{\partial u^{D,CC}_{t+1}}{\partial g_{t+1}} - \frac{\partial u^{D,CC}_{t+1}}{\partial c_{t+1}} \right) y_{t+1} = 0, \quad \forall y_{t+1} \geq y^*_{t+1} \tag{C.42}$$

$$(1 + \mu^{D,CC}_{t+1}) \left(\frac{\partial u^{D,CC}_{t+1}}{\partial g_{t+1}} - \frac{\partial u^{D,CC}_{t+1}}{\partial c_{t+1}} \right) y_{t+1} - \mu^{D,CC}_{t+1} \left(\frac{\partial u^{R,CC}_{t+1}}{\partial g_{t+1}} - \frac{\partial u^{R,CC}_{t+1}}{\partial c_{t+1}} \right) y_{t+1} = 0, \quad \forall y_{t+1} < y^*_{t+1}. \tag{C.43}$$

Next, we take the derivative with respect to the default threshold y^*_{t+1},

$$\frac{\partial u^{R,CC}_t}{\partial g_t} \left[-b_{t+1} \frac{f(y^*_{t+1}|y_t)}{1 + r^*} \right] + \beta \left[W^D(\tau^{CC}_{t+1}(y^*_{t+1}), y^*_{t+1}) - W^R(b_{t+1}, \tau^{CC}_{t+1}(y^*_{t+1}), y^*_{t+1}) \right] f(y^*_{t+1}|y_t) = 0. \tag{C.44}$$

The first term in equation (C.44) implies that increasing default threshold (y^*_{t+1}) dampens bond price in period t and reduces the current public consumption. The second term indicates how the change in default threshold influences the government's continuation values. $\tau^{CC}_{t+1}(y_{t+1})$ is the optimally-committed tax rate at a certain income state in the next period.

Lastly, in the period-t problem, taking FOC with respect to the bond issuance b_{t+1} (inter-temporal Euler equation) gives us the following

$$\frac{\partial u_t^{R,CC}}{\partial g_t} q_t + \beta \int_{y^*_{t+1}}^{y_{max}} \frac{\partial W^R(b_{t+1}, \tau^{CC}_{t+1}(y'), y')}{\partial b_{t+1}} (1 + \mu^{R,CC}_{t+1}(y')) f(y'|y_t) dy' \\ - \beta \int_{y_{min}}^{y^*_{t+1}} \mu^{CC}_{t+1}(y') \frac{\partial W^R(b_{t+1}, \tau^{D,CC}_{t+1}(y'), y')}{\partial b_{t+1}} f(y'|y_t) dy' = 0. \tag{C.45}$$

The first term shows the utility benefit of one more unit of borrowing in the current period, while the second and third terms capture the debt repayment cost in the next period. Since we reformulate the CC problem so that the default threshold in period $t+1$ is chosen in period t, the cost terms incorporate how a higher debt balance tightens (or relaxes) the incentive constraints that make the default threshold consistent with the *ex ante* borrowing and taxation decisions. Furthermore, the envelope theorem implies that

$$\frac{\partial W^R(b_{t+1}, \tau_{t+1}, y_{t+1})}{\partial b_{t+1}} = -\frac{\partial u^{R,CC}_{t+1}}{\partial g_{t+1}}. \tag{C.46}$$

Taking equation (C.46) into (C.45), the inter-temporal Euler equation becomes

$$\frac{\partial u_t^{R,CC}}{\partial g_t} q_t - \beta \int_{y^*_{t+1}}^{y_{max}} \frac{\partial u^{R,CC}_{t+1}}{\partial g_{t+1}} (1 + \mu^{R,CC}_{t+1}(y')) f(y'|y_t) dy' + \beta \int_{y_{min}}^{y^*_{t+1}} \mu^{D,CC}_{t+1}(y') \frac{\partial u^{R,CC}_{t+1}}{\partial g_{t+1}} f(y'|y_t) dy' = 0. \tag{C.47}$$

In the end, the solution of the CC problem can be characterized by equations (C.42), (C.43), (C.44), (C.47).

The IC problem. Next, we derive the optimality conditions of the IC problem. The IC problem in the infinite horizon environment is described in section B and characterized by equations (B.1)-(B.6). We first calculate the first-order condition with respect to the default threshold y^*_{t+1}. In the incentive-compatible problem of repayment ($V^{R,IC}$), taking derivative with respect to the default threshold y^*_{t+1} yields the following FOC,

$$\frac{\partial u_t^{R,IC}}{\partial g_t} \left[-b_{t+1} \frac{f(y^*_{t+1}|y_t)}{1+r^*} \right] + \beta \left[V^{D,IC}(b_{t+1}, y^*_{t+1}) - V^{R,IC}(b_{t+1}, y^*_{t+1}) \right] f(y^*_{t+1}|y_t) = 0, \tag{C.48}$$

where $u_t^{R,IC}$ is the household's period-t utility under repayment in the IC problem, and $u_t^{D,IC}$ is the period-t utility under default. Analogous to the optimality condition of (C.44) under CC, the first term indicates the effect of varying default threshold on bond price and the current utility, while the second term shows how the default threshold affects the government's continuation values.

Then, taking derivative with respect to the bond issuance b_{t+1} generates the intertemporal Euler equation as follows,

$$\frac{\partial u_t^{R,IC}}{\partial g_t} q_t + \beta \int_{y_{t+1}^*}^{y_{max}} \frac{\partial V^{R,IC}(b_{t+1}, y')}{\partial b_{t+1}} f(y'|y_t) dy_{t+1} + \beta \int_{y_{min}}^{y_{t+1}^*} \frac{\partial V^{D,IC}(b_{t+1}, y')}{\partial b_{t+1}} f(y'|y_t) dy' = 0. \quad \text{(C.49)}$$

The envelope theorem implies that in the repayment states of period $t+1$, we have

$$\frac{\partial V^{R,IC}(b_{t+1}, y_{t+1})}{\partial b_{t+1}} = -(1 + \mu_{t+1}^{R,IC}(y_{t+1})) \frac{\partial u_{t+1}^{R,IC}}{\partial g_{t+1}}, \quad \text{(C.50)}$$

where $\mu_{t+1}^{R,IC}(y_{t+1})$ is the Lagrange multiplier attached to the IC constraint (B.2). Similarly, in the income states that the government is obligated to default, we have

$$\frac{\partial V^{D,IC}(b_{t+1}, y_{t+1})}{\partial b_{t+1}} = \mu_{t+1}^{D,IC}(y_{t+1}) \frac{\partial u_{t+1}^{R,IC}}{\partial g_{t+1}}, \quad \text{(C.51)}$$

where $\mu_{t+1}^{D,IC}(y_{t+1})$ is the Lagrange multiplier attached to the IC constraint (B.5).

Substituting equations (C.50), (C.51) into (C.49), the Euler equation for b_{t+1} is simplified to

$$\frac{\partial u_t^{R,IC}}{\partial g_t} q_t - \beta \int_{y_{t+1}^*}^{y_{max}} (1 + \mu_{t+1}^{R,IC}(y')) \frac{\partial u_{t+1}^{R,IC}}{\partial g_{t+1}} f(y'|y_t) dy' + \beta \int_{y_{min}}^{y_{t+1}^*} \mu_{t+1}^{D,IC}(y') \frac{\partial u_{t+1}^{R,IC}}{\partial g_{t+1}} f(y'|y_t) dy' = 0. \quad \text{(C.52)}$$

Lastly, under the problems of $V_t^{R,IC}$ and $V_t^{D,IC}$, taking derivatives with respect to the current tax rate τ_t allows us to have the following FOCs: if the government is obligated to repay in time t, we have

$$(1 + \mu_t^{R,IC}) \left(\frac{\partial u_t^{R,IC}}{\partial g_t} - \frac{\partial u_t^{R,IC}}{\partial c_t} \right) y_t - \mu_t^{R,IC} \left(\frac{\partial u_t^{D,IC}}{\partial g_t} - \frac{\partial u_t^{D,IC}}{\partial c_t} \right) y_t = 0;$$

Otherwise, if the government is obligated to default in time t, we have

$$(1 + \mu_t^{D,IC}) \left(\frac{\partial u_t^{D,IC}}{\partial g_t} - \frac{\partial u_t^{D,IC}}{\partial c_t} \right) y_t - \mu_t^{D,IC} \left(\frac{\partial u_t^{R,IC}}{\partial g_t} - \frac{\partial u_t^{R,IC}}{\partial c_t} \right) y_t = 0.$$

Iterating these FOCs forward by one period gives rise to

$$(1+\mu_{t+1}^{R,IC})\left(\frac{\partial u_{t+1}^{R,IC}}{\partial g_{t+1}}-\frac{\partial u_{t+1}^{R,IC}}{\partial c_{t+1}}\right)y_{t+1}-\mu_{t+1}^{R,IC}\left(\frac{\partial u_{t+1}^{D,IC}}{\partial g_{t+1}}-\frac{\partial u_{t+1}^{D,IC}}{\partial c_{t+1}}\right)y_{t+1}=0,\quad \forall y_{t+1}\geq y_{t+1}^{*}, \tag{C.53}$$

$$(1+\mu_{t+1}^{D,IC})\left(\frac{\partial u_{t+1}^{D,IC}}{\partial g_{t+1}}-\frac{\partial u_{t+1}^{D,IC}}{\partial c_{t+1}}\right)y_{t+1}-\mu_{t+1}^{D,IC}\left(\frac{\partial u_{t+1}^{R,IC}}{\partial g_{t+1}}-\frac{\partial u_{t+1}^{R,IC}}{\partial c_{t+1}}\right)y_{t+1}=0,\quad \forall y_{t+1}< y_{t+1}^{*}. \tag{C.54}$$

Ultimately, the solution of the IC problem is characterized by equations (C.48), (C.52), (C.53), (C.54).

Equivalence between IC and CC problems: Now, we are ready to show the equivalence between IC and CC problems in the infinite-horizon environment. First, a comparison between equations (C.42) - (C.43) and (C.53) - (C.54) indicates that the first-order conditions with respect to the τ_{t+1} are the same for each value of income (y_{t+1}). Second, a comparison between equations (C.44) and (C.48) shows that the first-order conditions with respect to the default threshold y_{t+1}^{*} take the same form in the CC and IC problem. Lastly, the two problems yield the same first-order conditions with respect to the bond issuance b_{t+1}, as shown in equations (C.47) and (C.52).

D Simplified Two-Period Model

In this section, we simplify the two-period model to better illustrate the benefit and cost of the tax commitments. The basic setup and assumptions are the same as in the baseline two-period model in section 2, except that we assume: i) a log utility function; ii) the second-period endowment follows a binary distribution: $y = y^H$ with a probability of $1 - p$ and $y = y^L$ with a probability of p; and iii) the first-period debt issuance is a constant, b. The government can set tax rates based on the income realization y under a discretionary policy or contingent commitment, or otherwise independent of y under a non-contingent commitment. Because the income follows a discrete distribution with two values, the bond price can only take one of the three values $q = 0, 1-p, 1$. The timeline of the model is shown in the following diagram.

$t = 1$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $t = 2$

$u_1 = \log qb$

$$y = \begin{cases} y^H & \text{with prob } 1-p \\ y^L & \text{with prob } p \end{cases}$$

Repay:

$V_2^R(\tau, y) = \pi \log(\tau y - b) + (1-\pi)\log((1-\tau)y)$

Default:

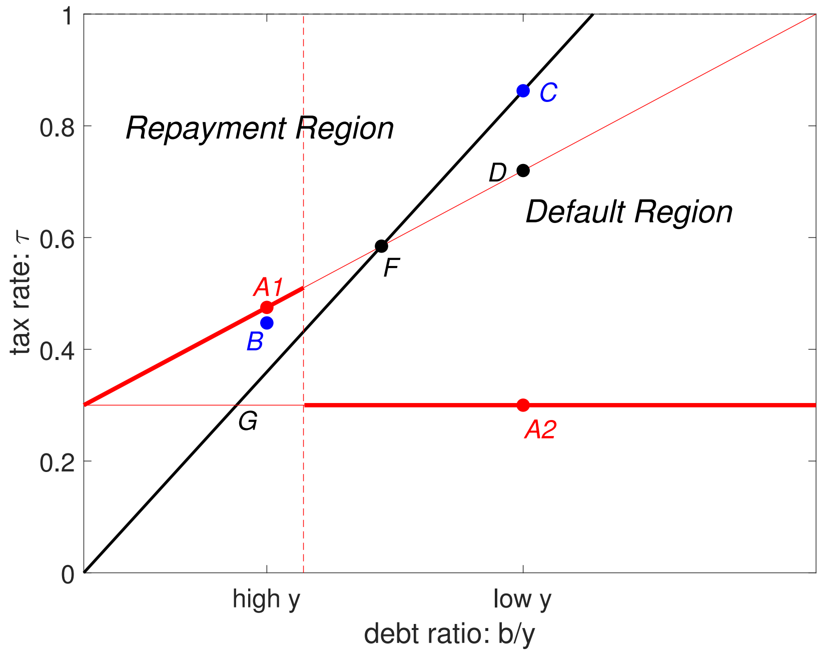
$V_2^D(\tau, y) = \pi \log((\tau\phi y) + (1-\pi)\log((1-\tau)\phi y)$

First, we characterize two tax schedules. They are plotted as the red and black lines in figure D.1. The first one is the "optimal discretionary tax schedule" (red lines), which corresponds to the optimal tax rates chosen by a government under discretionary fiscal policy. Under discretion, the government chooses the tax rates based on income realizations and the default decision,

$$\tau^{*R}\left(\frac{b}{y}\right) = \pi + (1-\pi) \times \frac{b}{y},$$
$$\tau^{*D} = \pi.$$

Note that the tax rate under repayment is an affine function of the debt-to-GDP ratio with an intercept π and slope $1 - \pi$. We also know that, under a discretionary policy, the government opts to default if debt-ratio is higher than the default threshold: $1 - \phi$ (dashed vertical line). Therefore, in equilibrium, the implemented taxes can only lie on the thick segments of the red lines.

The second tax schedule is the "threshold tax rate" that makes the government indifferent between repayment and default (black line). If the government wants to use its



Note: The thick red lines represent the "optimal discretionary tax schedules" (as functions of the debt ratio) under repayment and default. The black line represents the "threshold tax rates" that makes the second-period government indifferent between repaying and defaulting. The vertical red-dashed line indicates the default threshold for a discretionary government.

Figure D.1: Optimal Discretionary Tax Schedule and Threshold Tax Rate

tax commitment to discipline the default incentives in the second period, the tax rate has to be raised up to a point where the value of repayment is no worse than the value of default, $V_2^R(\tau, b, y) \geq V_2^D(\tau, y)$. In other words, the government repays its debt for any combination of $(\frac{b}{y}, \tau)$ above the black line. The "threshold tax rate" can be described by the following function:

$$\tau^*\left(\frac{b}{y}\right) = \frac{1}{1-\phi^{\frac{1}{\pi}}} \times \frac{b}{y},$$

which is also linear in the debt-ratio with a slope bigger than 1 for relevant parameter values.

Figure D.1 shows that the upper phase of red line and black line intersect at point F where the debt-to-GDP ratio equals to $\frac{\pi}{\frac{1}{1-\phi^{\frac{1}{\pi}}}-(1-\pi)}$. If the debt-to-GDP ratio is below this point, the government follows the optimal discretionary tax ($\tau^{*R}\left(\frac{b}{y}\right)$) and chooses to repay its debt. However, if the debt-to-GDP ratio goes above that point, the committed tax rate must deviate from the discretionary schedule to make repayment more attractive than default (point C), which generates a second-period utility loss if the government is forced to repay.

To better illustrate the trade-off underlying a fiscal commitment, we make the following simplifying assumption:

Assumption 1.

$$\frac{b}{y^H} < 1 - \phi < \frac{\pi}{\frac{1}{1-\phi^{\frac{1}{\pi}}} - (1-\pi)} < \frac{b}{y^L}.$$

The second inequality in assumption 1 guarantees that the debt ratio space in figure D.1 can be divided into three regions. In the middle region (between the vertical line and point F), committing to the optimal discretionary tax would induce the government to repay. The first and third inequalities imply that at the good state y^H (to the left of the vertical line), the debt-to-GDP ratio is low enough to fall into the repayment region even for the discretionary government; and at the bad state (above F), the low income y^L makes the debt-to-GDP ratio high enough so that government must deviate from the discretionary tax to avoid a default. We have the following results for the discretionary fiscal policy (DP).

Corollary 1 (Discretionary Policy). *Under assumption 1, the solution for the economy with a discretionary fiscal policy is:*

1. *The government defaults if the second-period income is low ($y = y^L$) and repays if the income is high ($y = y^H$).*

2. *The optimal tax when the second-period income is high equals $\tau^{*R}(\frac{b}{y^H})$, and the optimal tax when the income is low equals τ^{*D}.*

3. *The first-period bond price is $q = 1 - p$.*

4. *The second-period values are given by*

$$V_2^{DP}(y^H) = \log(y^H - b) + \pi \log(\pi) + (1-\pi)\log(1-\pi),$$
$$V_2^{DP}(y^L) = \log(\phi y^L) + \pi \log(\pi) + (1-\pi)\log(1-\pi).$$

Proof. The corollary goes by construction. Since $1-\phi$ is the default threshold under a discretionary policy, the government repays the debt if $\frac{b}{y} < 1-\phi$; and government defaults if $\frac{b}{y} > 1-\phi$. The bond price incorporates the default probability. □

We cannot analytically characterize the optimal policy under a non-contingent commitment because the government has to consider the two states jointly. The government can either choose a relatively low tax so that it only repays at y^H or chooses a relatively high tax rate and repay in both states. In the former case, the tax is set to solve the following optimization problem:

$$\tilde{\tau}^* = \underset{\tau}{\operatorname{argmax}} \Big\{ (1-p)\pi \log(\tau y^H - b) + p\pi \log(\tau) + (1-\pi)\log(1-\tau) \Big\}.$$

The optimal choice is shown as point B in figure D.1, which lies between the optimal discretionary tax rates under repay (A_1) and default(A_2). On the other hand, if the government chooses the latter and intends to repay in both states, the tax rate must be increased to the "threshold tax rate" at y^L state (point C) to induce the government to repay.

The government compares the lifetime utilities associated with these two options (point B and point C). Committing to a higher tax rate $\tau^*(\frac{b}{y^L})$ creates utility losses in both states of income, which is expressed as,

$$(1-p)\left[V_2^R(\tilde{\tau}^*, y^H) - V_2^R(\tau^*(\frac{b}{y^L}), y^H)\right] + p\left[V_2^D(\tilde{\tau}^*, y^L) - V_2^D(\tau^*(\frac{b}{y^L}), y^L)\right],$$

while the smaller default risk indicates a utility gain of $\log(\frac{1}{1-p})$ in the first period.

Lastly, under a contingent tax commitment (CC), the government can set two tax rates, one for y^H and the other for y^L. The government has two options: it may choose to repay only in the good state by committing to the discretionary tax menu,

$$\tau = \begin{cases} \tau^{*R}(\frac{b}{y^H}), & \textit{for } y = y^H \\ \tau^{*D}, & \textit{for } y = y^L, \end{cases}$$

or choose to repay in both states by committing to an "incentive-compatible" tax menu,

$$\tau = \begin{cases} \tau^{*R}(\frac{b}{y^H}), & \textit{for } y = y^H \\ \tau^{*}(\frac{b}{y^L}), & \textit{for } y = y^L. \end{cases}$$

In choosing between these two options, the government compares the *ex post* cost of a higher tax rate at y^L (from point A2 to point C) and the *ex ante* benefit of obtaining a better bond price in the first period. The next corollary summarizes the results.

Corollary 2 (Contingent Commitment). *Under assumption 1, if the following condition holds:*

$$[\pi\log(\pi) + (1-\pi)\log(1-\pi)] - \left[\pi\log(\frac{\frac{b}{y^L}}{1-\phi^{\frac{1}{\pi}}}) + (1-\pi)\log(1-\frac{\frac{b}{y^L}}{1-\phi^{\frac{1}{\pi}}})\right] < \log(\frac{1}{1-p}), \tag{D.1}$$

then the cost of disciplining default at y^L is smaller than its benefit. Then, the government under a contingent tax commitment makes the following decision:

1. *The government repays in both the good and bad states in the second period.*
2. *The optimal tax at the high income equals $\tau(y^H) = \tau^{*R}(\frac{b}{y^H})$, and the optimal tax at the low income equals $\tau(y^L) = \tau^{*}(\frac{b}{y^L})$.*
3. *The first-period bond price is $q = 1$.*
4. *The second-period values are given by,*

$$V_2^{CC}(y^H) = V_2^R(\tau^{*R}(\frac{b}{y^H}), y^H) = V_2^{DP}(y^H),$$
$$V_2^{CC}(y^L) = V_2^D(\tau^{*}(\frac{b}{y^L}), y^L) < V_2^{DP}(y^L).$$

Since the choice of tax rate at y^L is independent of the choice at y^H, a contingent commitment policy delivers a lifetime utility that is no worse than a discretionary fiscal policy.[11]

Proof. Since $\frac{b}{y^H} < 1 - \phi$ holds, it is always desirable for the government to repay at y^H when the tax rate is set equal to the tax rate under a discretionary policy. At y^L, if the government defaults, the optimal tax rate equals to the one that is optimal under discretion τ^{*D}, and the associated value in the second period becomes $V_2^D(\tau^{*D}, y^L)$. Such a tax menu coincides with the one implemented under discretionary fiscal policy.

On the other hand, if the government wants to repay at y^L, the tax rate must be increased to $\tau^*(\frac{b}{y^L})$ where the value of repayment equals to the value of default and the associated second-period value becomes $V_2^D(\tau^*(\frac{b}{y^L}), y^L)$. Specifically, the government would choose to avoid defaulting at y^L if the utility loss in the second period is smaller than the gain in the first period:

$$V_2^D(\tau^{*D}, y^L) - V_2^D(\tau^*(\frac{b}{y^L}), y^L) < \log(b) - \log((1-p)b),$$

which translates into inequality (D.1) in the corollary.

[11] If the debt-ratio at bad state satisfies $1 - \phi < \frac{b}{y^L} < \pi/[\frac{1}{1-\phi^{\frac{1}{\pi}}} - (1-\pi)]$, and given everything else unchanged, the condition that makes a government willing to repay at y^L becomes: $\frac{b}{y^L} < [1 - \phi(1-p)]$. The same intuition applies here: a contingent commitment policy is effective in mitigating default risk when the probability of the bad state is high and the bad state debt-ratio is not too large.

Since the discretionary tax schedule,

$$\tau = \begin{cases} \tau^{*R}(\frac{b}{y^H}), & \textit{for } y = y^H \\ \tau^{*}(\frac{b}{y^L}), & \textit{for } y = y^L, \end{cases}$$

belongs to the choice set of the contingent commitment policy, we can conclude that a contingent commitment delivers a lifetime utility that is no worse than the discretionary policy. □

The left side of the inequality (D.1) equals 0 if the debt-to-GDP ratio falls to point G in figure D.1, where it satisfies $\tau^*(\frac{b}{y^L}) = \tau^{*D}$. However, this condition violates assumption 1. At higher debt-to-GDP ratios, this expression is always positive and decreases as $\frac{b}{y^L}$ increases. Condition (D.1) implies that in order to make repaying at y^L attractive to the government (benefit larger than cost), the probability of the bad state p must be high enough and the debt-to-GDP ratio at bad state cannot be too high.

E Extensions to the Two-Period Model

E.1 Ramsey Tax Plan vs. Contingent Commitment

In the case of contingent commitment in section 2, the government can commit to a menu of tax rates contingent on future income realizations. For each income realization, the committed tax rate has to be implemented, regardless of the government's default decision. In this section, we assume that the government can commit to two different tax rate schedules, one implemented under repayment and the other implemented when the government defaults. Specifically, suppose there is a **Ramsey planner** who makes a function of tax rule in the form of $\mathcal{T}(b_2, y_2, d)$, which depends on the second-period state vector (b_2, y_2) and government's default decision: $d \in \{0, 1\}$. As a result, the government in this economy takes the tax policies set by the Ramsey planner as given and only makes the sequential decisions of borrowing and default.

Formally, there are three types of agents in the economy – a Ramsey planner, a local government, and a continuum of risk-neutral international investors. The Ramsey planner is benevolent and has the same utility function as the government. The Ramsey planner chooses the tax function, $\mathcal{T}(b_2, y_2, d)$, to maximize her lifetime utility:

$$\max_{\mathcal{T}} \left\{ u\left(g_1^*(\mathcal{T})\right) + \beta \mathbb{E} u\left(c_2^*(\mathcal{T}, y_2), g_2^*(\mathcal{T}, y_2)\right) \right\},$$

where g^* and c^* are the policy functions that are consistent with the government's decisions and investor's bond pricing equation under the tax rule. Taking the function $\mathcal{T}(b_2, y_2, d)$ as given, the government makes the borrowing and default decisions:

$$\max_{b_2} \left\{ u\left(q(b_2)b_2\right) + \beta \mathbb{E} \max_{d} \left[(1-d)u(c_2^R, g_2^R) + du(c_2^D, g_2^D) \right] \right\},$$

where $c_2^R = [1 - \mathcal{T}(b_2, y_2, d = 0)]y_2$, $g_2^R = \mathcal{T}(b_2, y_2, d = 0)y_2 - b_2^*$, $c_2^D = [1 - \mathcal{T}(b_2, y_2, d = 1)]\phi y_2$, and $g_2^D = \mathcal{T}(b_2, y_2, d = 1)\phi y_2$. The risk-neutral investors determine the bond price competitively:

$$q(b_2) = \mathbb{E}[1 - d(b_2, y_2)].$$

The Ramsey planner internalizes the effect of its tax rule on the government's default decisions and bond price. The infinite dimensionality of the problem makes it infeasible to solve in its original form. However, the Ramsey problem can be reformulated into a time-consistent Markov problem of the government subject to an incentive-compatibility constraint. Suppose in the first period, the government chooses a combination of borrowing and default threshold. In the second period, after the income shock is realized, the government decides the two tax rates under repayment and default $\{\tau^R(y_2), \tau^D(y_2)\}$, respectively, such that the government's default incentive is compatible with the repayment choice made in the first period. Formally, we use the following algorithm to solve the Ramsey planner's problem. In the first period, the government chooses a bond issuance b_2 and a second-period default threshold $\bar{y}$ to solve the problem:

$$\max_{b_2, \bar{y}} \left\{ \frac{(q(\bar{y})b_2)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E} V_2(b_2, \bar{y}, y_2) \right\}, \tag{E.1}$$

where the bond price is determined as $q(\bar{y}) = \frac{y_{max}-\bar{y}}{y_{max}-y_{min}}$. In the second period, for each realization of y_2, the government decides the tax rates $\{\tau^R, \tau^D\}$ to maximize the utility subject to an "incentive-compatibility" (IC) constraint under either repayment or default:

Ramsey Planner Problem:

$$V_2^{RP}(b_2, \bar{y}, y_2) = \begin{cases} \textit{if } y_2 \geq \bar{y} & \max_{\tau^R, \tau^D} \; V_2^R(b_2, \tau^R, y_2) \\ & s.t. \quad V_2^R(b_2, \tau^R, y_2) \geq V_2^D(\tau^D, y_2) \\ & \\ \textit{if } y_2 < \bar{y} & \max_{\tau^R, \tau^D} \; V_2^D(\tau^D, y_2) \\ & s.t. \quad V_2^R(b_2, \tau^R, y_2) \leq V_2^D(\tau^D, y_2) \end{cases}$$

where $V_2^R(b_2, \tau, y_2) = (1-\pi)\frac{[(1-\tau)y_2]^{1-\sigma}}{1-\sigma} + \pi\frac{[\tau y_2 - b_2]^{1-\sigma}}{1-\sigma}$, and $V_2^D(\tau, y_2) = (1-\pi)\frac{[(1-\tau)\phi y_2]^{1-\sigma}}{1-\sigma} + \pi\frac{[\tau\phi y_2]^{1-\sigma}}{1-\sigma}$. The IC constraint says that if y_2 is above the default threshold $\bar{y}$, government chooses tax rates $\{\tau^R, \tau^D\}$ such that it is optimal for the government to repay. And among all the possible combinations of tax rates satisfying this condition, the government chooses the one that maximizes the value of repayment. The similar logic applies to the problem of default if y_2 is below the threshold.

The government's problem can be further simplified. Because the government can choose $\{\tau^R, \tau^D\}$ independently, the optimal tax rates can be set at the optimal discretionary values (shown as the red lines in figure D.1), and the IC constraint never binds. For example, if the government is induced by the Ramsey planner to repay the debt, the tax rate that maximizes V_2^R in the unconstrained problem is: $\tau^{*R} = (\pi^{1/\sigma} + ((1-\pi)^{1/\sigma})\frac{b_2}{y_2})/(\pi^{1/\sigma} + (1-\pi)^{1/\sigma})$.[12] In this case, there always exists a τ^D that the Ramsey planner can choose such that $V_2^R(b_2, \tau^{*R}, y_2) > V_2^D(\tau^D, y_2)$. Similarly for the default problem, the tax rate that maximizes the value of default V_2^D is given as, $\tau^{*D} = \pi^{1/\sigma}/(\pi^{1/\sigma} +$

[12]The repayment problem is well-defined only when $b_2 < y_2$.

$(1-\pi)^{1/\sigma})$, and there always exists a τ^R such that $V_2^R(b_2,\tau^R,y_2) < V_2^D(\tau^{*D},y_2)$. Thus, the Ramsey problem can be transformed as the first-period government chooses the income states in which it will default in the second period. In the second period, since the repayment-default problem was already planned, the government simply adopts the optimal discretionary tax for each income realization.

It is helpful to compare this Ramsey problem with the contingent commitment policy in our paper. The difference is that under contingent commitment, government can only choose one tax rate for each level of income: $\tau(b_2,y_2)$, and the same tax rate has to be implemented no matter the government repays or not. We can formulate a similar incentive-compatible problem for the contingent commitment policy as follows,

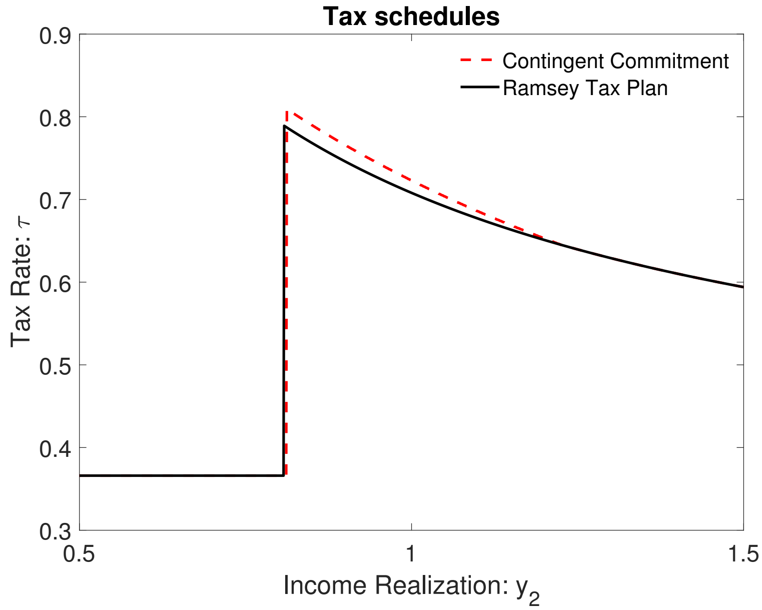
Contingent Commitment Problem:

$$V_2^{CC}(b_2,\bar{y},y_2) = \begin{cases} \textit{if } y_2 \geq \bar{y} & \max_\tau \ V_2^R(b_2,\tau,y_2) \\ & s.t. \quad V_2^R(b_2,\tau,y_2) \geq V_2^D(\tau,y_2) \\ \\ \textit{if } y_2 < \bar{y} & \max_\tau \ V_2^D(\tau,y_2) \\ & s.t. \quad V_2^R(b_2,\tau,y_2) \leq V_2^D(\tau,y_2) \end{cases}$$

The government repays the debt if $y_2 \geq \bar{y}$. In this case, the government chooses a tax rate to maximize V_2^R conditional on the fact that the same tax rate would not make the value of repay smaller than the value of default. The problem under default is constructed in a similar way. Different from the Ramsey problem, there exists some states (b_2,y_2) that the incentive constraint is binding: when the discretionary tax cannot incentivize the government to repay its debt, the committed tax rate need to be raised up to a point where the value of repay is higher than the value of default (as the black line in figure D.1).[13]

The incentive-compatibility (IC) constraints take different forms in the Ramsey problem and the contingent commitment. Notice that a Ramsey policy is more flexible in adjusting tax policies and using the fiscal adjustment to discipline default incentives than the contingent commitment. For each level of b_2, denote $\bar{y}^{RP}(b_2)$ ($\bar{y}^{CC}(b_2)$) as the default threshold below which the government defaults in the Ramsey planner's problem (contingent commitment). Denote V_1^{RP} and V_1^{CC} as the lifetime utilities under the Ramsey tax plan and contingent commitment. Then we have the following proposition to differentiate the two problems.

[13]The region where IC binds expands when the parameter π or ϕ increases. In the numerical example ($\pi = 0.185$ and $\phi = 0.65$) that we provide in section 2, the IC constraint never binds. To better illustrate the comparison between Ramsey tax planner's problem and the contingent commitment problem, we use the parameter values $\pi = 0.25$ and $\phi = 0.75$ in this section.



Note: Model parameters are the same as in the baseline model, except that $\pi = 0.25$ and $\phi = 0.75$. Debt issuance is set at the Ramsey planner's optimal decision.

Figure E.1: Ramsey Tax Plan and Contingent Commitment

Proposition 3 (Ramsey Tax Plan). *The Ramsey tax planner (RP) can always achieve a bond price schedule and a lifetime utility that are no worse than the contingent tax commitment (CC). That means, for any level of debt issuance b_2, we have*

$$\bar{y}^{RP}(b_2) \leq \bar{y}^{CC}(b_2),$$

and,

$$V_1^{RP} \geq V_1^{CC}.$$

Proof. For each y_2, we define $\tau^{*R}(y_2)$ and τ^{*D} as the "optimal discretionary tax schedules" under repayment and default (as the red lines in figure D.1). Let $\tau^*(y_2)$ be the "threshold tax rates" that make the government indifferent between repayment and default (black line in figure D.1). First, we prove that for any b_2, we have $\bar{y}^{RP}(b_2) \leq \bar{y}^{CC}(b_2), \;\; \forall b_2$. Our proof consists of two steps.

1) We prove that for any y_2 that belongs to the repayment set of CC, it also belongs to the repayment set of RP. We only need to prove that for any y_2 in the repayment set of CC, the second-period utility loss of making commitment is smaller under RP than under CC.

   First, if y_2 belongs to the repayment set of discretionary policy, we have $V_2^R(\tau^{*R}, y_2) > V_2^D(\tau^{*D}, y_2)$. In this case, there is no utility loss from making commitment (relative to discretionary tax policy), and both the CC and RP government would like to repay at y_2 by committing to τ^{*R}. Second, if the government under discretionary fiscal policy defaults at y_2 but the government under CC repays at y_2, then the utility loss from making commitment at state y_2 equals to one the following two expressions,

$$\text{CC's utility loss from making commitment at } y_2 = \begin{cases} V_2^D(\tau^{*D}, y_2) - V_2^R(\tau^{*R}(y_2), y_2) > 0 \\ V_2^D(\tau^{*D}, y_2) - V_2^R(\tau^{*}(y_2), y_2) > 0. \end{cases}$$

In the first case, although the CC government repays at y_2, the IC constraint is not binding and the CC government still chooses the optimal discretionary tax $\tau^{*R}(y_2)$. As a result, the RP government would incur the same utility loss if it commits to the same the tax rate $\tau^{*R}(y_2)$ and repays. In the second case, the CC government needs to increase the tax rate at least up to $\tau^{*}(y_2)$ to avoid a default, and the IC constraint binds. However, because the RP can choose the two tax rates τ^R and τ^D separately to circumvent the constraint, τ^R can be still set at the optimal discretionary tax $\tau^{*R}(y_2)$ if the government is supposed to repay. The *ex post* optimal tax rate $\tau^{*R}(y_2)$ will make the value of repayment higher under RP than CC. Therefore, the utility loss under RP when it repays at y_2 is smaller than the utility loss of the CC government, that is,

$$V_2^D(\tau^{*D}, y_2) - V_2^R(\tau^{*R}(y_2), y_2) < V_2^D(\tau^{*D}, y_2) - V_2^R(\tau^{*}(y_2), y_2).$$

For any y_2 in the repayment set of the CC government, the RP always incur smaller utility losses if it commits to repay at the same state. In the meantime, the benefit of making commitment is the same (reduced default risk) between the two problems. As a result, y_2 also belongs to the repayment set of RP.

2) We show that for any y_2 in the repayment set of RP, the RP also repays at any y' such that $y_2' > y_2$.

   This can be easily shown by proving that RP's utility loss from making commitment to repay: $V_2^D(\tau^{*D}, y_2) - V_2^R(\tau^{*R}(y_2), y_2)$, is a decreasing function of y_2. As a result, for higher income states, the government has stronger incentives to repay under the Ramsey tax plan. The RP can accomplish this by committing to the optimal discretionary tax rate.

Hence, we have proved that $\bar{y}^{RP}(b_2) \leq \bar{y}^{CC}(b_2)$ holds, which implies that the Ramsey tax policy delivers a bond price schedule that is no worse than the contingent commitment.

Table E.1: Ramsey Tax Plan in the Two-Period Model

	Discretionary	Non-contingent	Contingent	**Ramsey**
Debt issuance	0.18	0.33	0.53	**0.54**
Bond price	0.77	0.84	0.7	**0.69**
Expected second-period utility	-2.58	-3.67	-3.56	**-3.57**
Lifetime value	-9.70	-7.27	-6.26	**-6.25**

Note: Model parameters are the same as in baseline model, except that $\pi = 0.25$ and $\phi = 0.75$.

Now, we prove that $V_1^{RP} \geq V_1^{CC}$ holds. We assume that $\hat{b}_2$ is the optimal borrowing and $\hat{\tau}(y_2)$ is the committed tax schedule chosen by a CC government. The RP can always restore the equilibrium by choosing the same borrowing and tax schedules under both the repayment and default: $b_2 = \hat{b}_2$, $\tau^R(y_2) = \hat{\tau}(y_2)$, $\tau^D(y_2) = \hat{\tau}(y_2)$. This commitment plan results in the same default decisions in the second period (with the same default threshold $\bar{y}^{CC}(b_2)$) and the same lifetime utility. Therefore, the government under a Ramsey tax plan can always achieve a welfare that is no worse than the contingent commitment. □

Numerical results. Figure E.1 compares the optimal tax schedules under a contingent commitment and a Ramsey tax plan. The debt issuance is set at the optimal value under the Ramsey problem. Table E.1 contrasts the three economies with Ramsey tax policy in the two-period model environment.

As shown in figure E.1, the Ramsey taxes are identical to the ones under contingent commitment in the regions where the IC constraint does not bind (high income states). In these regions, both of them choose the optimal discretionary taxes. However, as income moves closer to the default threshold (intermediate income levels), the discretionary tax is not high enough to incentivize the government to repay if the same tax rate is implemented under default. As a result, under contingent commitment, the tax rates rise above the discretionary policy to induce the second-period government to repay. In contrast, the Ramsey planner can always choose a combination of tax rates (τ^R and τ^D) to ensure the IC constraints strictly hold.[14] Therefore, the tax rates set by the Ramsey planner always coincide with the ones under a discretionary policy until it reaches the preset default threshold. Another insight from figure E.1 is that, for the same level of debt issuance, Ramsey policy has a larger debt repayment region, which is consistent with the result in

[14]By setting τ^D close to zero (one), the Ramsey tax planner can make the second-period public (private) consumption arbitrarily close to zero. Since the utility function satisfies the Inada conditions, the Ramsey planner can make the default value an arbitrarily large negative number and induce the government to repay.

proposition 3.

Table E.1 shows the numerical results in the two-period model. The equilibrium results of the Ramsey problem are very close to those under the contingent commitment. That is because the cost of commitment is similar.[15] Compared to the contingent commitment, the Ramsey tax policy generates a slightly larger probability of default, a slightly lower second-period utility, and a higher welfare.

E.2 Alternative Commitment Measures

In this paper, we only consider commitments to the tax rate. In this section, we discuss how other commitment measures affect the government's default decisions. One possibility is a commitment to future debt issuance, and the other is a commitment to government spending. We show that, unlike tax commitment, these two commitment measures do not necessarily affect the government's default decisions in the one-period bond environment.[16]

Commitment to future debt issuance. To explore the effects of a commitment to future debt issuance, we extend the two-period model to a three-period model. There is no default risk in the third period and the default option is only available in the second period. If the government defaults in the second period, it still has access to the international finance market and can issue one-period debt, which is always repaid in the ending period. The government's lifetime utility is given by

$$\log(g_1) + \mathbb{E}\left\{\beta\left[(1-\pi)\log(c_2) + \pi\log(g_2)\right] + \beta^2\log(g_3)\right\}.$$

In the first period, the government borrows by issuing a defaultable one-period debt and commits to the second-period fiscal policies. The fiscal instruments available for commitment include τ_2 and b_3. Then, the first-period government's problem becomes

$$V_1 = \max_{b_2,\tau_2,b_3} \log\left(q(b_2,\tau_2,b_3)b_2\right) + \beta\mathbb{E}\max\left\{V_2^R(b_2,y_2,\tau_2,b_3), V_2^D(y_2,\tau_2,b_3)\right\}.$$

Notice that the first-period bond price and second-period value functions depend on the committed fiscal policies: $\{\tau_2, b_3\}$. We assume that both τ_2 and b_3 are committed by the

[15]The cost of commitment is smaller under the Ramsey problem, which encourages the first-period government to issue more debt in equilibrium.

[16]Commitment on the future debt issuance may have a different role in the long-term bond environment due to the debt dilution effect. The latter channel is explored in detail in Hatchondo et al. (2015, 2016).

first-period government. We relax this assumption later and let the first-period government have access to one commitment measure at a time, leaving the other one chosen in the second period in a discretionary manner.

If the first-period government commits to both future tax rate and bond issuance, the only decision left for the second period is whether to default. The value of repayment and the value of default are

$$\begin{aligned} V_2^R(b_2, y_2; \tau_2, b_3) &= (1-\pi)\log(c_2^r) + \pi\log(g_2^r) + \beta\log(g_3^r) \\ &= (1-\pi)\log((1-\tau_2)y_2) + \pi\log(\tau_2 y_2 - b_2 + b_3) + \beta\log(y_3 - b_3), \end{aligned} \tag{E.4}$$

$$\begin{aligned} V_2^D(y_2; \tau_2, b_3) &= (1-\pi)\log(c_2^d) + \pi\log(g_2^d) + \beta\log(g_3^d) \\ &= (1-\pi)\log((1-\tau_2)\phi y_2) + \pi\log(\tau_2\phi y_2 + b_3) + \beta\log(y_3 - b_3). \end{aligned} \tag{E.5}$$

We find that the government will repay its debt if y_2 is higher than a threshold:

$$y_2 \geq y_2^*(b_2; b_3, \tau_2) = \frac{b_2 - b_3(1-\phi^{\frac{1-\pi}{\pi}})}{\tau_2(1-\phi^{\frac{1}{\pi}})}.$$

It is clear that under relevant parameter values, either committing to a higher tax rate or committing to a higher debt issuance can lower the default threshold. Next, we assume that the government has only one commitment device at a time, leaving the other one chosen in the second period after observing the income realizations. We analyze the tax commitment and commitment to debt issuance in turn and discuss their difference in affecting the default incentives.

1. *Commitment to b_3; discretionary choice of τ_2.* Now, we suppose that the first-period government can only commit to the future debt issuance: b_3. Conditional on the preset b_3 and the state variables (b_2, y_2), the second-period government chooses an optimal τ_2 for each income realization. Specifically, the government optimizes the value functions (E.4) and (E.5) by choosing τ_2 for each state. The discretionary tax rates are given by

$$\tau_2^R = \pi + (1-\pi)\frac{b_2 - b_3}{y_2}, \tag{E.6}$$

$$\tau_2^D = \pi + (1-\pi)\frac{-b_3}{\phi y_2}. \tag{E.7}$$

We have the following corollary about the contingency of tax policies in this environment.

Corollary 4. *In the economy with a commitment to debt issuance, the government always has an incentive to levy a higher tax rate under repayment than under default.*

Proof. The conclusion can be obtained from comparing equations (E.6) - (E.7). □

Plugging equations (E.6)-(E.7) into (E.4)-(E.5) yields the value functions in the second period as follows,

$$V_2^{R,[1]}(b_2, y_2; b_3) = \log(y_2 - b_2 + b_3) + \beta \log(y_3 - b_3) + C^{[1]}, \tag{E.8}$$

$$V_2^{D,[1]}(y_2; b_3) = \log(\phi y_2 + b_3) + \beta \log(y_3 - b_3) + C^{[1]}, \tag{E.9}$$

where $C^{[1]}$ is a constant that does not contain b_2, y_2, or b_3. The government repays if and only if $V_2^{R,[1]} \geq V_2^{D,[1]}$, which implies a default threshold as follows,

$$y_2 \geq y_2^{*,[1]}(b_2) = \frac{b_2}{1-\phi}. \tag{E.10}$$

Equation (E.10) reveals that the default threshold does not depend on the committed debt issuance b_3. This is because the discretionary tax in the second period is flexible to absorb the fluctuations of income shock and mutes the effects of the commitment to b_3. In fact, we will have the same equilibrium outcomes even if we allow the government to choose the optimal b_3 in the second period, which can be easily derived from equations (E.8) and (E.9). Consequently, a commitment to b_3 does not affect the government's default incentives as long as τ_2 is chosen in a discretionary manner.

2. *Commitment to τ_2; discretionary choice of b_3.* Now, we assume the first-period government only makes commitment to τ_2, and b_3 is chosen in the second period conditional on the preset τ_2 and the state vector. From equations (E.4)-(E.5), we can derive the bond issuance decisions in the second period under repay and default,

$$b_3^R = \frac{\pi y_3 + \beta b_2 - \beta \tau_2 y_2}{\pi + \beta}, \tag{E.11}$$

$$b_3^D = \frac{\pi y_3 - \beta \tau_2 \phi y_2}{\pi + \beta}. \tag{E.12}$$

Notice that $b_3^R > b_3^D$ when $\tau_2 < \frac{1}{1-\phi}\frac{b_2}{y_2}$, and we have the following corollary to characterize the government's incentive to issue debt.

Corollary 5. *In the economy with a commitment to tax, the government has an incentive to issue more debt under repayment than under default if the committed tax rate is not high enough to generate sufficient tax revenue to repay the maturing debt.*

Proof. The conclusion can be obtained from comparing equations (E.11)-(E.12). □

The corollary says that when τ_2 is not large enough to generate enough tax revenues to repay the existing debt balance, the government relies on the debt issuance to do so. The state-contingent debt issuance only helps the government to smooth public consumption across time periods, but it cannot re-optimize the resource allocation between public and private consumption. The indirect value functions under the tax commitment can be derived as,

$$V_2^{R,[2]}(b_2, y_2; \tau_2) = (1-\pi)\log((1-\tau_2)y_2) + \log(\tau_2 y_2 + y_3 - b_2) + C^{[2]}, \quad \text{(E.13)}$$

$$V_2^{D,[2]}(y_2; \tau_2) = (1-\pi)\log((1-\tau_2)\phi y_2) + \log(\tau_2 \phi y_2 + y_3) + C^{[2]}. \quad \text{(E.14)}$$

A comparison of $V_2^{R,[2]}$ and $V_2^{D,[2]}$ implies that the government defaults if income exceeds the default threshold as follows,

$$y_2 \geq y_2^{*,[2]}(b_2; \tau_2) = \frac{b_2 - y_3(1-\phi^{1-\pi})}{\tau_2(1-\phi^{2-\pi})}. \quad \text{(E.15)}$$

As shown in equation (E.15), a higher committed tax rate τ_2 lowers the default threshold and induces the government to repay with a higher probability, which is similar to the results we find in our baseline two-period model.

Our analysis is summarized in the following proposition.

Proposition 6. *Based on the expression of default thresholds in equations* (E.10) *and* (E.15), *we reach the following policy implications:*

1. *A commitment to future debt issuance does not have any effect on the government's default incentive if the tax policies are conducted in a discretionary manner.*

2. *A commitment to tax policy always affects default risk even if we allow the government to choose the debt issuance in a discretionary manner.*

Proof. The proposition can be easily derived from equations (E.10) and (E.15). □

Commitment to government spending. This section shows that, similar to the commitment to future debt issuance, a commitment to the second-period government spending does not influence the government's default decision either. Consider the baseline two-period model in section 2, the discretionary government's spending policy takes the form of

$$g_2^R = \pi(y_2 - b_2), \tag{E.16}$$
$$g_2^D = \pi\phi y_2. \tag{E.17}$$

Solving for the discretionary policy implies that default occurs when $y_2 < \frac{b_2}{1-\phi}$. Therefore, a comparison of equations (E.16) and (E.17) reveals that the g_2 under default is lower than that under repayment. Suppose the first-period government commits to a constant government spending policy, g_2, in a similar way as the non-contingent tax commitment, then the second-period value functions become

$$V_2^R(g_2, b_2, y_2) = (1-\pi)\log(y_2 - b_2 - g_2) + \pi\log(g_2), \tag{E.18}$$
$$V_2^D(g_2, y_2) = (1-\pi)\log(\phi y_2 - g_2) + \pi\log(g_2). \tag{E.19}$$

Notice that the committed g_2 can be implemented by setting the tax rates $\tau_2^R = \frac{b_2+g_2}{y_2}$ and $\tau_2^D = \frac{g_2}{\phi y_2}$, which are different from the optimal discretionary ones. A comparison of the second-period value functions (E.18)-(E.19) reveals that the default threshold under the "g_2" commitment is the same as the one under discretionary policy: $y_2 < \frac{b_2}{1-\phi}$. As a result, a precommitment in g_2 does not have any effect on the government's repayment decision.

F Alternative Calibration: Value-Added Tax

In our baseline setup in section 4.1, we calibrate our model to match the cyclicality of the Greek income tax rate calculated by Vegh and Vuletin (2015). Due to the progressive income tax system in Greece, one drawback of our calibration is that not all residents pay the highest marginal income tax rate. In this section, we show that the results of our model are quantitatively similar if we calibrate the model to match the cyclicality of the value-added tax, which is faced by all Greek residents. The VAT data also come from Vegh and Vuletin (2015), and the correlation between the value-added tax rate and detrended GDP is -0.29. Table F.1 shows parameter values under the alternative calibration and table F.2 displays some basic statistics from the model.

Table F.1: Alternative Calibration – Parameter Values

Description	Parameter	Value
From literature or data:		
Relative risk aversion on c	σ	2
Risk-free interest rate	r^*	0.0214
Std. dev. of innovation to y	σ_e	0.0459
Autocorrelation of income y	ρ	0.7213
Calibrated to fit targets:		
Discount factor	β	0.9395
Weight on public consumption	π	0.1
Relative risk aversion on g	σ_g	2.0
Default cost parameter	α_0	0.42
Default cost parameter	α_1	2.5

Table F.2: Alternative Calibration – Basic Statistics

	Greece 1995-2011	Alternative Calibration Discretionary Policy
Targeted Moments:		
Debt service/ income	6.45%	6.58%
Average spread	2.2%	2.27%
Average $c/(c+g)$	0.76	0.75
$std(c)/std(g)$	0.99	1.00
$corr(\tau, y)$	-0.29	-0.28
Nontargeted Moments:		
Default probability	2.80%	1.93%
Std. dev. of spread	3.6%	2.89%
$std(c)/std(y)$	0.98	1.08
$corr(c, y)$	0.95	0.99
$corr(g, y)$	0.92	0.99
$corr(tb/y, y)$	-0.46	-0.25
$corr(\text{spread}, y)$	-0.58	-0.84

Table F.3 shows the simulation results and welfare implications in the economies with and without tax commitment under the alternative calibration. The results are quantitatively similar to those under the baseline calibration. Similar to the baseline results, the non-contingent tax commitment is not effective in reducing the probability of default. The lack of tax contingency prevents the government from freely adjusting tax rates based on future income realizations and forces the government to rely on bond issuance to smooth consumption. As a result, the economy with a non-contingent tax commitment has a lower level of debt and a higher spread relative to the discretionary policy, which leads to a net welfare loss across the ergodic distribution. On the other hand, a contingent tax commitment enables the government to implement austerity tax rates to induce the government to repay on the states where default risk is relevant. Consequently, a contingent commitment is more effective in reducing the probability of default and suppressing the cost of borrowing. In equilibrium, the government with a contingent commitment can sustain a higher level of debt and enjoys a positive welfare gain.

Table F.3: Alternative Calibration – Simulation Results and Welfare Implications

	Discretionary Tax Policy	Non-contingent Tax Commitment	Contingent Tax Commitment
Default probability	1.93%	1.87%	1.65%
Mean b/y	6.58%	6.04%	10.18%
Mean τ	25.2%	25.2%	25.5%
Mean spread	2.28%	2.32%	1.97%
$std(c)/std(y)$	1.08	1.03	1.27
$std(g)/std(y)$	1.08	1.16	0.99
$std(\tau)$	0.77%	0.35%	1.81%
$corr(tb/y, y)$	-0.25	-0.32	-0.22
$corr(\tau, y)$	-0.28	-0.3	-0.2
Std. dev. of spread	2.89%	3.40%	2.58%
Avg. welfare gain	-	-0.02%	0.11%

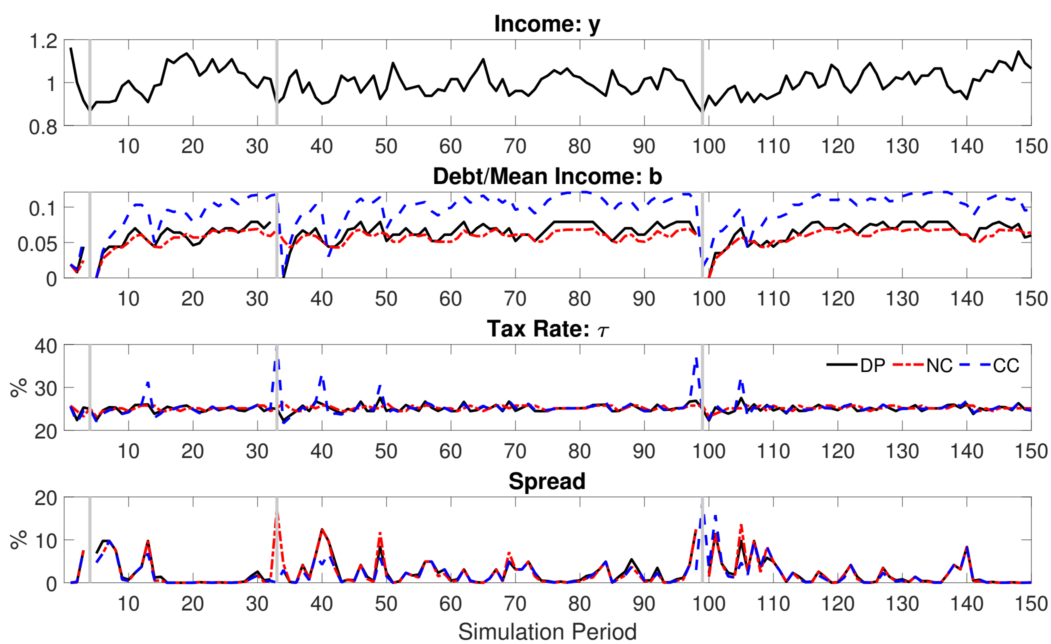
G Additional Numerical Results

This section includes some additional numerical results in our baseline quantitative analysis.

G.1 Simulation Paths

Figure G.1 shows a snapshot of the long-run simulations when the same sequence of income is fed into the three economies with discretionary policy, non-contingent, and contingent tax commitment. The grey vertical lines indicate the three default episodes experienced by the discretionary government, and these episodes occur in periods 4, 33, and 99. After default, the government is excluded from the international financial market for one period, so the simulated bond spread and debt-to-mean income ratios display breaks in these periods.

The dotted red lines in figure G.1 represent the simulated economy under a non-



Note: This figure shows a snapshot of our long-run simulations when the same sequence of income is fed into the three economies. The grey vertical lines indicate the three default episodes experienced by the government with a discretionary tax policy, and these episodes occur in periods 4, 33, and 99. In the legend, "DP," "NC," and "CC" stand for "discretionary policy," "non-contingent commitment," and "contingent commitment," respectively.

Figure G.1: Simulation Paths under Discretionary Policy, Non-contingent, and Contingent Tax Commitment contingent tax commitment. As shown in the third panel, the fluctuation of tax rates is milder under the non-contingent commitment since the government is unable to adjust tax rates flexibly. The reluctance to make fiscal adjustments, in turn, makes it difficult for the government to smooth public consumption and increases its default incentives, as discussed in section 4.3.2. As a result, the government sustains a lower level of debt on average (the second panel) compared to the discretionary policy. And when unfavorable income shocks are realized, the interest rate spreads rise more sharply (the fourth panel).

The dashed blue lines in figure G.1 represent the simulated economy under a contingent tax commitment. First, as shown in the third panel, the tax rates under the contingent commitment closely mimics the discretionary tax policy since the committed tax schedule is also *ex post* optimal for most of the income realizations, as shown in figure 6. More importantly, under a negative income shock, the government with the contingent commitment implements austerity tax rates in order to discipline its future default incentives. In figure G.1, these austerity taxes are displayed as the spikes in periods 13, 33, 40, 49, 98, and 105. Over the long run, the commitment to these austerity tax rates reduces the government's incentive to default and increases its debt sustainability.[17] As a result, the second panel of figure G.1 shows that the debt levels under the contingent commitment are almost always higher than the debt levels in the other two economies.

Lastly, we turn our analysis to the default events. For the discretionary tax policy, the first default episode occurs in period 4 after three consecutive adverse shocks. The adverse shocks are severe enough that neither of the tax commitments can prevent the default. The second default episode happens in period 33, and both the governments with a non-contingent and contingent commitment successfully avoid the default. Notice that the government with a contingent commitment implements an austerity tax in this period that is close to 40%. The high committed tax rate lowers the government's demand for borrowing and reduces its benefit of default. However, for the non-contingent commitment, the government only increases the tax rate slightly due to the fiscal rigidity, which results in a spike in the interest rate in period 33. Finally, only the contingent commitment policy can prevent the government from defaulting its debt in period 99. The default is avoided because the government levies a high tax rate in the previous period, and the high tax rate reduces the debt accumulation and mitigates the default incentive.

[17] When unfavorable income shocks are realized, the austerity tax rates reduce the government's demand for borrowing because the government has collected enough tax revenue to repay its debt. Therefore, the periods with high tax rates are always followed by periods of low debt.

G.2 Comparative Statics

In this section, we conduct comparative statics of the baseline calibration by varying the values of the subjective discount factor and income volatility.

Subjective discount factor. Figure G.2 compares some basic moments of the three economies for different values of subjective discount factor β. When the household becomes less patient (lower β), the governments have stronger incentive to borrow to front-load consumption, resulting in higher debt-to-income ratios. The governments also default more frequently when they are less patient, which is transmitted into higher average spreads.

Figure G.2 also shows that, consistent with the results under our baseline calibration, the government with a discretionary fiscal policy has the highest default probability and the government with a contingent commitment defaults the least frequently for all values of β. Meanwhile, the non-contingent tax commitment delivers the lowest level of debt and the highest average spread. Due to the greater volatility and dampened borrowing ability, the non-contingent tax policy always yields a welfare loss. The welfare loss decreases in β because the government's ability to use bond issuance to smooth consumption is more severely hampered when the average spread is higher.

On the other hand, the contingent tax commitment enables the government to sustain

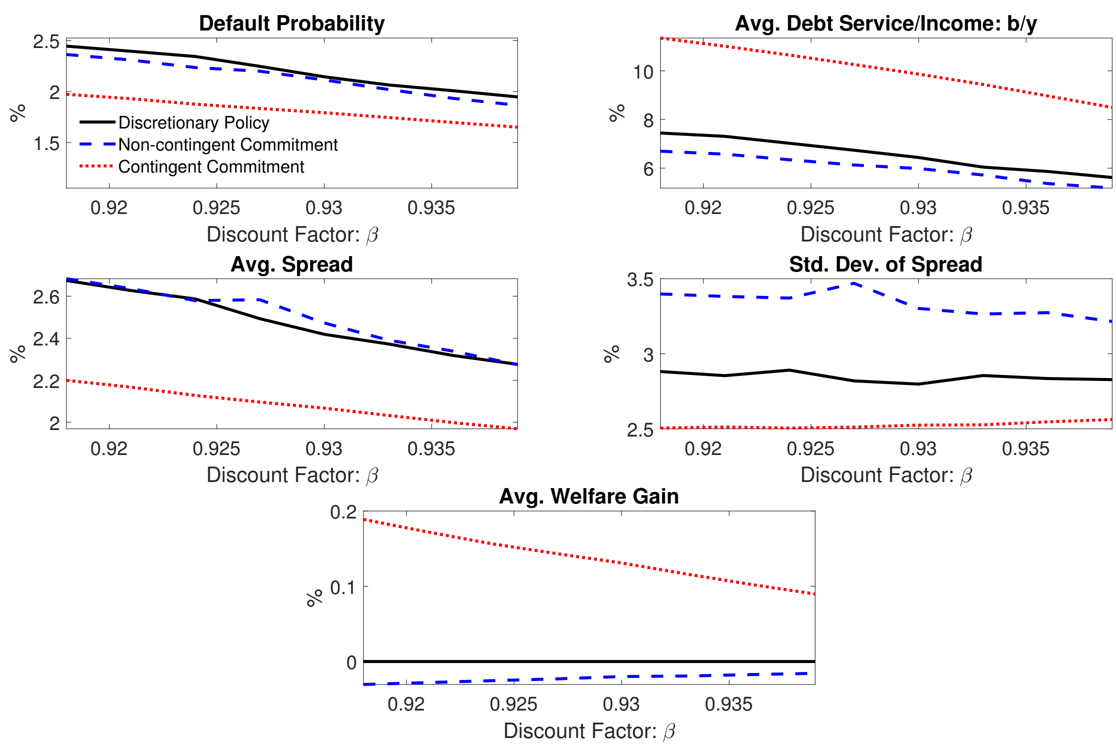


Figure G.2: Comparative Statics – Subjective Discount Factor β.

the highest average debt level at the lowest average spread. Similar to the baseline calibration, the government under a contingent commitment can use a flexible tax schedule to discipline its future default incentives, which reduces the default probability and creates the lowest spreads. Due to the lower volatility and enhanced borrowing ability, the contingent commitment always generates a welfare gain to the economy, and the welfare gain is larger when households are more impatient and the default probabilities are more relevant.

Standard deviation of income innovation. Figure G.3 shows the comparative statics for different values of the income shock standard deviation σ_e. When the income volatility is low, the governments in all three economies are less likely to be impacted by severely unfavorable shocks. As a result, both the default frequencies and average spreads become lower. The lower income volatility also reduces the precautionary saving motive and induces the governments to borrow more, as shown in the second graph of figure G.3.

Compared to the discretionary tax policy, the government under a non-contingent tax commitment can achieve slightly smaller default probabilities, but the lack of contingency reduces the government's debt sustainability and produces higher interest rate spreads for most values of σ_e. Therefore, non-contingent commitment produces a welfare

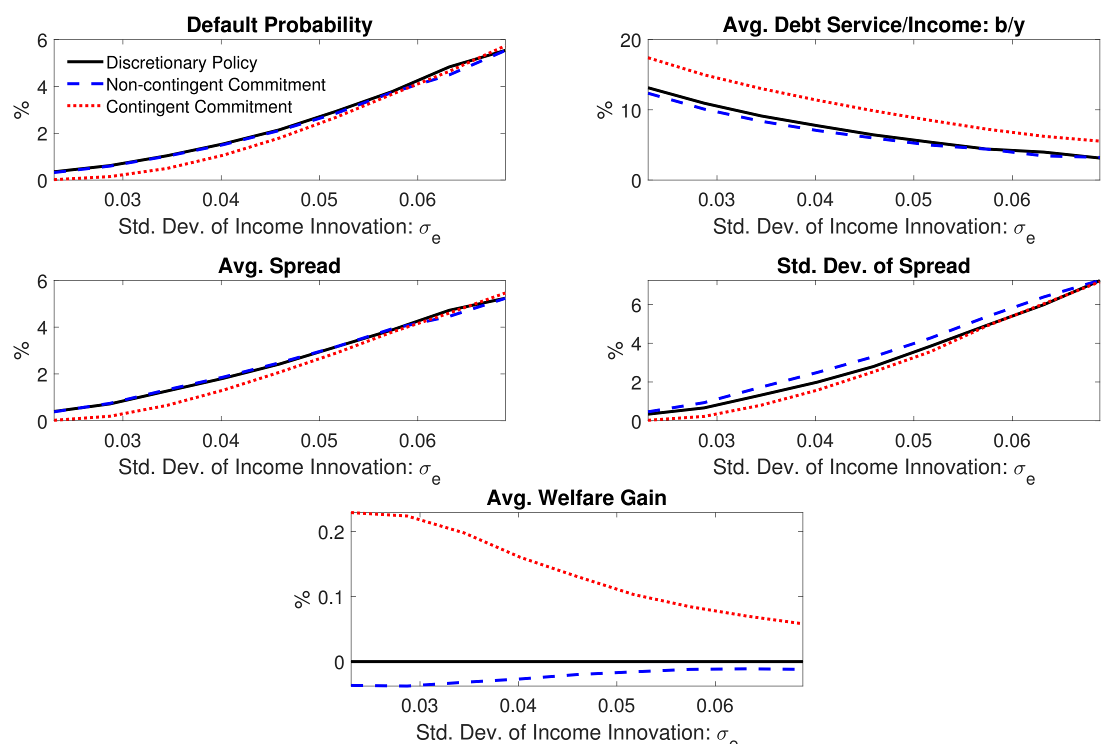


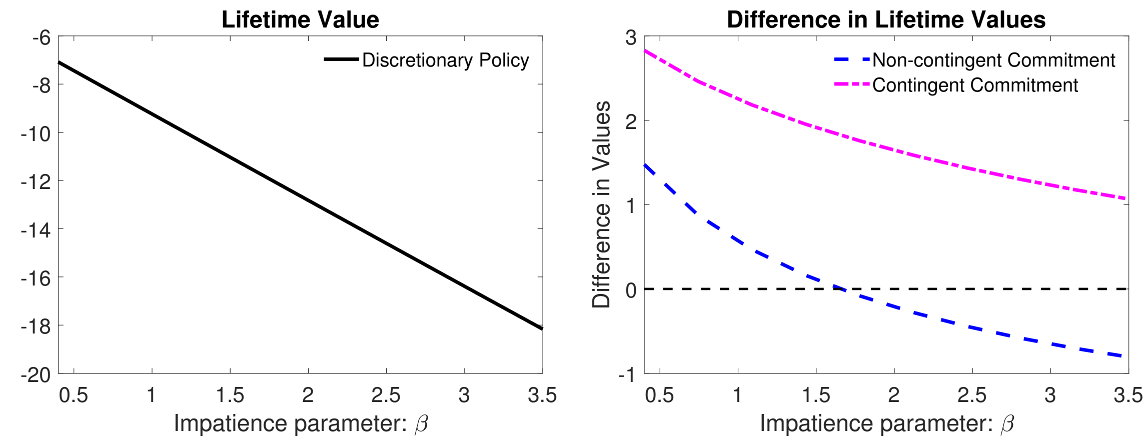
Figure G.3: Comparative Statics – Standard Deviation of Income Innovation σ_e.

loss. The welfare loss is especially large when the income shock is less volatile and the default probability is low because the benefit of tax commitment on default risk becomes marginal.

Figure G.3 also shows that except for extremely high values of σ_e, the government with a contingent tax commitment can always achieve the lowest default frequency and the lowest interest rate spread. As shown in figure 6, the government implements the austerity tax rates in the "intermediate income levels" at which it would otherwise choose to default. When the income volatility is low and the distribution of next-period income is more concentrated, the austerity tax rates are more likely to be implemented in the next period, and they effectively discipline the default risk. For example, when σ_e equals 0.023, the government with a contingent tax commitment implements the austerity tax rates 4.5% of the time, and the default probability is only 0.017%, compared to a default probability of 0.35% under the discretionary policy. On the other hand, when the income volatility is high and the next-period income distribution is more dispersed, the austerity tax rates are implemented less frequently. As a result, the commitment becomes less useful in reducing the probability of default in the next period.[18] Besides, the contingent commitment enables the government to borrow more aggressively relative to the other two polices. The stronger incentive to borrow and the reduced commitment efficiency increase the default probability and the average spread as income volatility rises. In the end, contingent commitment delivers a welfare gain relative to discretionary policy, but the welfare gain is smaller when the income process is more volatile.

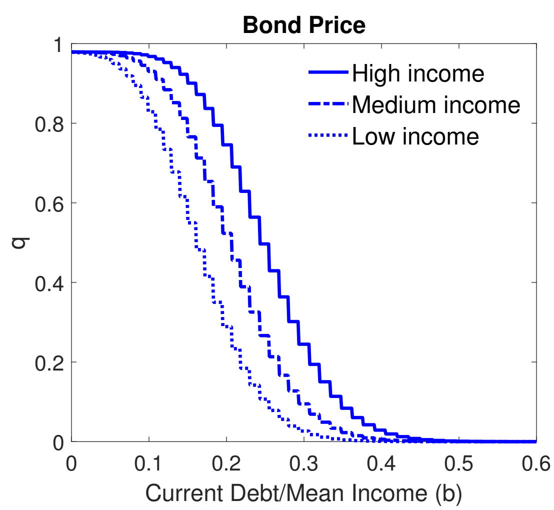
[18] When σ_e equals 0.0689, austerity tax rates are implemented only 2.6% of the time, leading to a default probability of 5.73%. This is higher than the default probability of 5.54% under a discretionary tax policy.

H Additional Figures



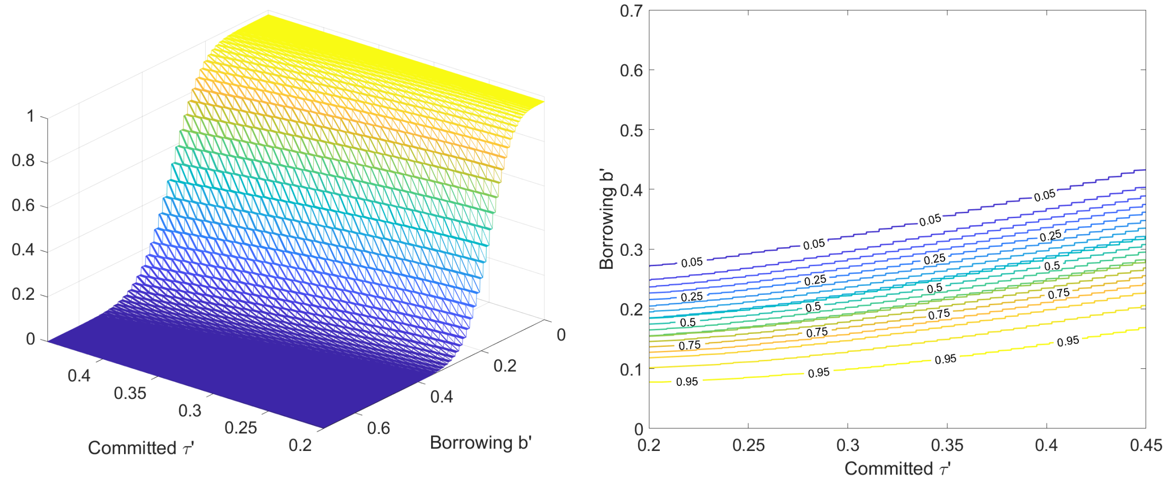
Note: To increase the default risk, we set $\zeta = 0.8$. Other parameters are the same as the baseline model.

Figure H.1: Two-Period Model – Welfare Gain of Tax Commitment for Different β



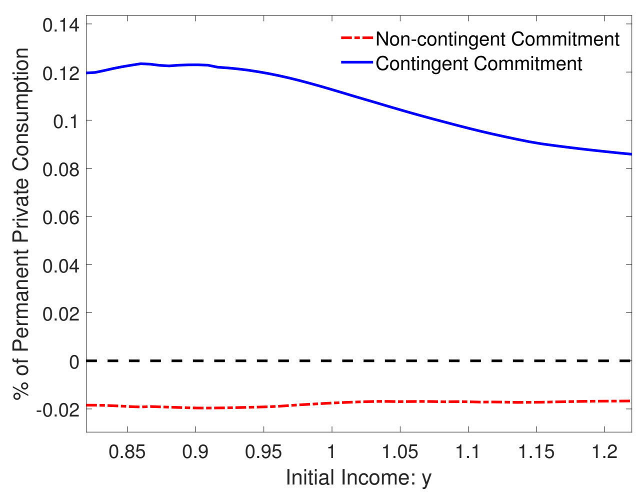
Note: This figure shows the bond price schedules under a discretionary tax policy for three different income levels. The high, medium, low income corresponds to the state that is 4%, 0%, -4% from its unconditional mean.

Figure H.2: Bond Price Schedules Under Discretionary Fiscal Policy



Note: This figure shows the bond prices under the non-contingent commitment for different bond issuance b' and committed tax rate τ'. The right panel plots the pair of (b', τ') delivering a particular price q for different values of q. The current income is at its unconditional mean.

Figure H.3: Bond Prices Under Non-contingent Tax Commitment



Note: This figure shows the state-contingent welfare gains of tax commitments at different initial income states. Debt level is fixed at 0. The welfare gain is defined as the permanent private consumption compensation (in percentage) that households would like to move from an economy with discretionary tax policy to another economy with non-contingent (contingent) tax commitment. A positive number means that the household prefers the economy with a tax commitment.

Figure H.4: State-Contingent Welfare Gains for Different Initial Incomes

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