Online-Appendix

for **"When Fiscal Consolidation Meets Private Deleveraging"** by Javier Andrés, Óscar Arce, Dominik Thaler, Carlos Thomas

A. Proof of Lemma 1

The first order conditions of the household problem are

$$1+\lambda_t^c=\lambda_t, \quad \text{with } \lambda_t^c \geq 0 \text{ and } \lambda_t^c c_t=0, \qquad \text{(foc: } c_t\text{)}$$

$$\lambda_t (1+\tau) p_t^h=\vartheta+\beta \lambda_{t+1} (1+\tau) p_{t+1}^h+\xi_t \frac{m_t}{R} p_{t+1}^h 1_{m_t R^{-1} p_{t+1}^h h_t \geq \gamma b_{t-1}}, \qquad \text{(foc: } h_t\text{)}$$

$$\lambda_t=\beta \lambda_{t+1} R+\xi_t 1_{m_t R^{-1} p_{t+1}^h h_t \geq \gamma b_{t-1}}+\mu_t 1_{m_t R^{-1} p_{t+1}^h h_t<\gamma b_{t-1}}-\beta \mu_{t+1} \gamma 1_{m_{t+1} R^{-1} p_{t+2}^h h_{t+1}<\gamma b_t}, \qquad \text{(foc: } b_t\text{)}$$

with $\xi_t \geq 0$ and $\xi_t\left(\frac{m_t}{R} p_{t+1}^h h_t-b_t\right)=0$ whenever $\frac{m_t}{R} p_{t+1}^h h_t \geq \gamma b_{t-1}$, and with $\mu_t \geq 0$ and $\mu_t\left(\gamma b_{t-1}-b_t\right)=0$ whenever $\frac{m_t}{R} p_{t+1}^h h_t<\gamma b_{t-1}$, and where 1_x equals 1 if x holds and 0 otherwise.

We guess (and later verify) that in equilibrium the constraint $c_t \geq 0$ is always slack: $\lambda_t^c=0$ for all t. From (foc: c_t) we then have $\lambda_t=1$. The remaining conditions depend on whether the value of collateral ($\frac{m_t}{R} p_{t+1}^h h_t$) is higher or lower than the contractual amortization path (γb_{t-1}):

- In periods in which $\frac{m_t}{R} p_{t+1}^h h_t \geq \gamma b_{t-1}$, equation (foc: b_t) becomes $\xi_t=1-\beta R>0$, i.e. the collateral constraint binds in such periods. Equation (foc: h_t) then becomes
$$(1+\tau) p_t^h=\vartheta+\beta(1+\tau) p_{t+1}^h+\left(R^{-1}-\beta\right) m_t p_{t+1}^h .$$

- In periods in which $\frac{m_t}{R} p_{t+1}^h h_t<\gamma b_{t-1}$, followed by periods where the same inequality holds, equation (foc: b_t) becomes
$$\mu_t=1-\beta R+\beta \mu_{t+1} \gamma,$$
with terminal condition given by $\mu_{T^*-1}=1-\beta R$, where T^* is the earliest

period in which $\frac{m_t}{R} p_{t+1}^h h_t \geq \gamma b_{t-1}$ holds again. It follows that $\mu_t > 0$, i.e. the constraint $b_t \leq \gamma b_{t-1}$ binds for as long as $\frac{m_t}{R} p_{t+1}^h h_t < \gamma b_{t-1}$. Also, in such periods equation (foc: h_t) becomes

$$(1+\tau) p_t^h = \vartheta + \beta (1+\tau) p_{t+1}^h.$$

Given the equilibrium path $\{b_t\}$, consumption can be determined by the current account identity,

$$c_t = y + b_t - R b_{t-1},$$

which is obtained by imposing $h_t = \bar{h}$ in equation (2) in the main text. We can ensure that consumption is always strictly positive (thus verifying our earlier guess) by setting the fixed endowment y to a sufficiently high value.

B. Equilibrium conditions

Let $\tilde{p}_t \equiv \tilde{P}_{H,t}/P_{H,t}$, $p_{H,t} \equiv P_{H,t}/P_t$, $p_t^* \equiv P_{H,t}/P_{F,t}$. Equilibrium conditions:

- Household budget constraint and first-order conditions (c_t^s, d_t, h_t^s, n_t),

$$(1+\tau_t^c) c_t^s + d_t + b_t^{gs} + p_t^h \left[h_t^s - (1-\delta) h_{t-1}^s\right] = \frac{R_{t-1}}{\pi_t}\left(d_{t-1} + b_{t-1}^{gs}\right) + (1-\tau_t^w) w_t n_t^s - T_t, \tag{17}$$

$$\lambda_t^s = \frac{1}{(1+\tau_t^c) c_t^s}, \tag{18}$$

$$\lambda_t^s = \beta^s E_t \frac{R_t}{\pi_{t+1}} \lambda_{t+1}^s, \tag{19}$$

$$\lambda_t^s p_t^h = \frac{\vartheta}{h_t^s} + \beta^s E_t \lambda_{t+1}^s (1-\delta) p_{t+1}^h, \tag{20}$$

$$\frac{(1+\tau_t^w) w}{(1+\tau_t^c) c} = n^\phi.$$

- Entrepreneur budget constraint, technology, debt constraints, and first-order conditions (c_t^e, b_t, h_t^e, n_t^e),

$$\begin{aligned}(1+\tau_t^c)\, c_t^e &= \left(1\text{-}\tau_t^k\right)(mc_t y_t^e - w_t n_t^e) + b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} - p_t^h \left[h_t^e - (1\text{-}\delta)\, h_{t-1}^e\right] \\ &\quad + \tau_t^k \delta p_t^h h_{t-1}^e + \left(1-\tau_t^k\right)\left(\Pi_t^r + \Pi_t^h\right),\end{aligned} \tag{21}$$

$$y_t^e = \left(h_{t-1}^e\right)^{\alpha} (n_t^e)^{1-\alpha}, \tag{22}$$

$$b_t \leq \max\left\{\overline{m} E_t \frac{\pi_{t+1}}{R_t} p_{t+1}^h h_t^e,\ \gamma \frac{b_{t-1}}{\pi_t}\right\} \tag{23}$$

$$\lambda_t^e = \frac{1}{(1+\tau_t^c)\, c_t^e}, \tag{24}$$

$$\lambda_t^e = \beta^e E_t \frac{R_t}{\pi_{t+1}} \lambda_{t+1}^e + \xi_t^e \mathbf{1}_{\vartheta_t^e \geq 0} + \mu_t^e \mathbf{1}_{\vartheta_t^e < 0} - \beta^e \gamma \frac{\mu_{t+1}^e}{\pi_{t+1}} \mathbf{1}_{\vartheta_{t+1}^e < 0}, \tag{25}$$

$$\lambda_t^e p_t^h = \beta^e E_t \frac{\left(1-\tau_{t+1}^k\right) mc_{t+1} \alpha_h y_{t+1}^e / h_t^e + \left(1-\delta+\tau_{t+1}^k \delta\right) p_{t+1}^h}{1/\lambda_{t+1}^e} + \xi_t^e \overline{m} E_t \frac{\pi_{t+1}}{R_t} p_{t+1}^h \mathbf{1}_{\vartheta_t^e \geq 0}, \tag{26}$$

$$w_t = mc_t (1-\alpha) \left(h_{t-1}^e\right)^{\alpha} (n_t^e)^{-\alpha}, \tag{27}$$

where μ_t^e is the Lagrange multiplier on $b_t \leq \gamma \frac{b_{t-1}}{\pi_t}$ and $\vartheta_t^e \equiv \overline{m} E_t \frac{\pi_{t+1}}{R_t} p_{t+1}^h h_t^e - \gamma b_{t-1}/\pi_t$.

- Retailers' optimal price decision, and aggregate profits,

$$E_t \sum_{s=0}^{\infty} (\beta^e \theta_p)^s \frac{\lambda_{t+s}^e \left(1-\tau_{t+s}^k\right)}{\lambda_t^e} \left[\frac{\tilde{p}_t}{\prod_{j=1}^{s} \pi_{H,t+j}} p_{H,t+s} - \frac{\varepsilon_p}{\varepsilon_p - 1} mc_{t+s}\right] \left(\frac{\prod_{j=1}^{s} \pi_{H,t+j}}{\tilde{p}_t}\right)^{\varepsilon_p} y_{t+s} = 0, \tag{28}$$

$$\Pi_t^r = y_t \left(p_{H,t} - mc_t \Delta_t\right), \tag{29}$$

- Dynamics of PPI inflation and price dispersion,

$$1 = (1-\theta)\, \tilde{p}_t^{1-\varepsilon_p} + \theta \pi_{H,t}^{\varepsilon_p - 1}, \tag{30}$$

$$\Delta_t \equiv (1-\theta)\,\tilde{p}_t^{-\varepsilon_p} + \theta \pi_{H,t}^{\varepsilon_p} \Delta_{t-1}. \tag{31}$$

- Construction firm output, first order conditions (n_t^h, i_t), and profits,

$$I_t = \left(n_t^h\right)^{\omega} (i_t)^{1-\omega} \left[1 - \frac{\Phi}{2}\left(\Gamma_t - 1\right)^2\right], \tag{32}$$

$$\begin{aligned} \frac{W_t}{P_t} &= p_t^h \omega \left(n_t^h\right)^{\omega-1} (i_t)^{1-\omega} \left\{1 - \frac{\Phi}{2}\left(\Gamma_t - 1\right)^2 - \Phi\left(\Gamma_t - 1\right)\Gamma_t\right\} \\ &\quad + \beta^e \frac{\lambda_t^e}{\lambda_0^e} \frac{1-\tau_{t+1}^k}{1-\tau_t^k} p_{t+1}^h \omega \left(n_t^h\right)^{\omega} (i_t)^{1-\omega} \Phi\left(\Gamma_t - 1\right)\Gamma_t \end{aligned}$$

$$\begin{aligned} 1 &= p_t^h (1-\omega) \left(n_t^h\right)^{\omega} (i_t)^{-\omega} \left\{1 - \frac{\Phi}{2}\left(\Gamma_t - 1\right)^2 - \Phi\left(\Gamma_t - 1\right)\Gamma_t\right\} \\ &\quad + \beta^e \frac{\lambda_t^e}{\lambda_0^e} \frac{1-\tau_{t+1}^k}{1-\tau_t^k} p_{t+1}^h (1-\omega) \left(n_{t+1}^h\right)^{\omega} (i_{t+1})^{1-\omega} \Phi\left(\Gamma_t - 1\right)\Gamma_t \end{aligned}$$

$$\Pi_t^h = p_t^h I_t - w_t n_t^h - i_t, \tag{33}$$

where $\Gamma_t \equiv \frac{\left(n_t^h\right)^{\omega} (i_t)^{1-\omega}}{\left(n_{t-1}^h\right)^{\omega} (i_{t-1})^{1-\omega}}$.

- Fiscal authority's budget constraint and fiscal rule,

$$\begin{aligned} b_t^g &= \frac{R_{t-1}}{\pi_t} b_{t-1}^g + p_{H,t} g_t - \sum_{x=s,e} T_t - \tau_t^w w_t n_t^s - \tau_t^c \left(c_t^s + c_t^e\right) \\ &\quad - \tau_t^k \left[mc_t y_t^e - w_t n_t^e - \delta p_t^h h_{t-1}^e + \Pi_t^r + \Pi_t^h\right], \end{aligned} \tag{34}$$

$$fi_t = fi_{t-1} + \phi_b \left(b_{t-1}^{gy} - \bar{b}^{gy}\right) + \phi_{\Delta b} \left(b_t^{gy} - b_{t-1}^{gy}\right), \tag{35}$$

with $fi \in \{g, \tau^w, \tau^c, \tau^k, T\}$.

- Export demand,

$$x_t = \zeta \left(p_t^*\right)^{-\varepsilon_F} y_{F,t}. \tag{36}$$

- Intermediate good market clearing,

$$y_t \Delta_t = \left(h_{t-1}^e\right)^{\alpha} \left(n_t^e\right)^{1-\alpha}, \tag{37}$$

- Labor market clearing,

$$n_t^s = n_t^e + n_t^h, \tag{38}$$

- Domestic consumption goods market clearing,

$$y_t = c_{H,t}^s + c_{H,t}^e + i_{H,t} + x_t + g_t \tag{39}$$

- Real estate market clearing,

$$h_t^s + h_t^e = I_t + (1-\delta)\left(h_{t-1}^s + h_{t-1}^e\right). \tag{40}$$

- Terms of trade,

$$p_t^* = p_{t-1}^* \frac{\pi_{H,t}}{\pi_{F,t}}. \tag{41}$$

- Relative demand for domestic goods,

$$c_{H,t}^s = \omega_H p_{H,t}^{-\varepsilon_H} c_t^s, \tag{42}$$

$$c_{H,t}^e = \omega_H p_{H,t}^{-\varepsilon_H} c_t^e, \tag{43}$$

$$i_{H,t} = \omega_H p_{H,t}^{-\varepsilon_H} i_t, \tag{44}$$

- Relative domestic producer prices,

$$p_{H,t} = p_{H,t-1} \frac{\pi_{H,t}}{\pi_t}, \tag{45}$$

- CPI inflation,

$$\pi_t^{1-\varepsilon_H} = \frac{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H}}{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H} + 1 - \omega_H} \pi_{H,t}^{1-\varepsilon_H} + \frac{1-\omega_H}{\omega_H \left(p_{t-1}^*\right)^{1-\varepsilon_H} + 1 - \omega_H}, \tag{46}$$

- Real (PPI-deflated) GDP,

$$gdp_t = y_t + \frac{1}{p_{H,t}} \left(p_t^h I_t - i_t\right), \tag{47}$$

- Gross nominal interest rate,

$$R_t = R^* \exp\left(-\psi \frac{(d_t + b_t^{gs}) - b_t - b_t^g}{p_{H,t} gdp_t}\right) \tag{48}$$

C. The size of the fiscal adjustment: tax based consolidations.

Figure 7 displays the GDP effects of fiscal (of size 20pp) consolidations based on the four taxes respectively. For capital and lump sum taxes (panels 2 and 4) we appreciate a similar pattern as for government spending (see Figure 4): a large consolidation produces similar short-run costs (relative to consolidation size), but persistently higher costs in the medium run. This is so, because like spending cuts, these taxes delay the deleveraging process. For consumption taxes, the effect of the fiscal consolidation is pretty much linear in the size of the consolidation, such that the 2 lines in panel 1 are almost identical. This reflects the lack of effect of this kind of consolidation on the duration of deleveraging. For labor income taxes, which – as discussed in the main text – shorten the deleveraging process, the result is reversed and this type of fiscal consolidation affects GDP proportionally while the deleveraging phase lasts but less then proportionally after that.

These dynamic patterns can again be summarized by one number, the present value multiplier. As for spending based consolidations, the PVM of a 1 pp consolidation is smaller for than that of a 20 pp consolidation when capital income tax or lump-sum tax are used. For consumption tax based consolidations they are essentially identical. For labor taxes the PVM is lower for large consolidations than for small.

Column 6 in the next section reports a '1' whenever the PVM patterns across consolidation sizes coincide with the ones described here.

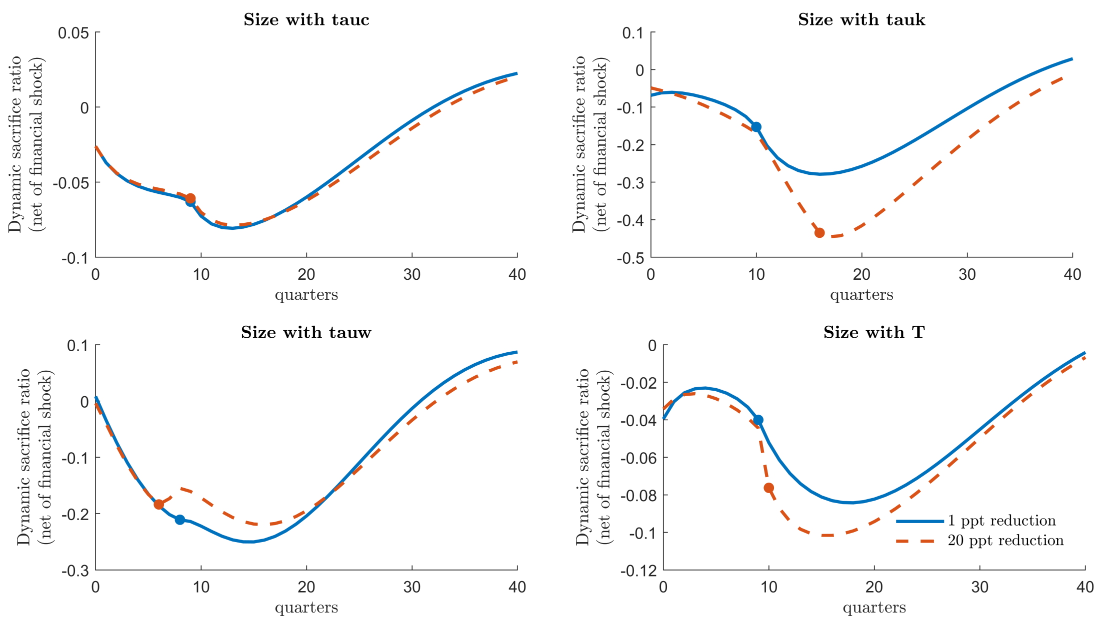

Figure 7: Fiscal sacrifice ratios across instruments and sizes. Triangles and circles mark the no-consolidation and the consolidation T^*.

D. Robustness.

Robustness exercise: In this section we analyze the robustness of our numerical results. We repeat all our exercises for all instruments, considering alternative model variants. This may concern the value of a particular parameter, or a modeling choice. For the parameters we usually consider two more extreme values in both directions from the baseline calibration. One of the parameters considered is the elasticity of substitution between housing and consumption. For this exercise we replaced our baseline log preferences $log(c) + \vartheta \log(h)$ by $\frac{c^{1-\sigma}-1}{1-\sigma} + \vartheta \frac{h^{1-\sigma}-1}{1-\sigma}$, with $\sigma \neq 1$. The alternative modeling choices that we consider regard real debt contracts,[47] absence of nominal frictions (i.e. real contracts and no price stickiness), and the borrowing constraint. In the latter case we consider the following two alternatives to the constraint used in the main text: A collateral constraint that depends on the current real estate price $b_t = \max\left\{\overline{m} p_t^h h_t,\ \frac{\gamma}{\pi_t} b_{t-1}\right\}$, and a fixed borrowing limit $b_t = \max\left\{\overline{m},\ \frac{\gamma}{\pi_t} b_{t-1}\right\}$. In each case we adjust $\overline{m}$ such that the initial and the terminal steady state is the same as in the baseline model.

We summarize the results in the table below. The first three columns define each particular model specification: Which parameter or assumption has been modified and which fiscal instrument is used. The adjacent two columns (4 and 5) show how the period in which deleveraging ends (T^*) is affected by the large consolidation (of size 20pp).[48] The next column (6) assesses the robustness of our 'size' result from section 4.3. In particular, we report a '1' whenever the present value multiplier (PVM) of the large consolidation is larger than that of a small consolidation whenever T^* increases, smaller whenever T^* decreases, and approximately the same whenever T^* remains unchanged; for those cases that do not fit any of these three cases, we report a '0'.[49] The next six columns analyze the robustness of the gradualism result from section 4.4. Columns 7-9 show T^* for the baseline (20 pp) consolidation at three different speeds (front-loaded, baseline, gradual). Columns 10-12 report a '1' whenever the gradual consolidation has the lowest PVM and is preferred by households and entrepreneurs, respectively.

Discussion of general results: First consider the result from section 4.2 that g-based consolidations delay the end of deleveraging (T^*). This remains true for all

[47] By real we mean relative to the *domestic* consumer price level, not to the union-wide price level, which is exogenous and assumed to be constant. In other words, we mean debt contracts that are indexed to domestic inflation, not to union-wide inflation.

[48] Notice that the small consolidation never affects T^* by assumption. This required reducing its size (below the 1pp size assumed in the main text) for a handful of the alternative calibrations.

[49] We consider two PVMs to be 'approximately the same' if they differ by less than 10%. Very often the actual difference for these cases is even smaller.

other g-based specifications as columns 4 and 5 show.

Second, consider the result from section 4.5, that tax based consolidations have different effects on T^*. As columns 4 and 5 show, capital taxes tend to delay T^* the most. Furthermore, lump-sum and consumption taxes typically lead to only a short delay, if any. Finally, labor income taxes tend to shorten the deleveraging period (an exception is discussed below).

Third consider the size results from section 4.3 and appendix C. As column 6 shows, the PVM follows the patterns documented before. In basically all cases, the PVM is larger for the larger consolidations whenever they delay T^*, smaller when they shorten T^* (the latter only happens for labor income taxation), and approximately the same when T^* remains unchanged.[50]

Fourth, consider the gradualism result from section 4.4. There we showed that gradual spending-based consolidations deliver the lowest PVM and are preferred by both agents. These results are largely robust across specifications and instruments. However, for consumption taxes the PVM result largely fails to hold. This is consistent with the fact that consumption tax hikes have lower effects on T^*, such that often the gradual and the baseline consolidation result in the same T^*. In that case the 'duration channel' remains muted and it is no longer clear that the gradual consolidation must lead to a lower PVM, instead it is often the baseline specification that does. One reason for this is that, through the 'buffering effect' discussed in the main text (section 2 for the simple model, and 4.3 for the full model), shifting part of the fiscal adjustment from the post-deleveraging to the deleveraging phase may actually be beneficial, since borrowers' net debt flows are unaffected but such front-loading.

Another exception to the gradualism result on PVMs regards the model without adjustment costs and labor income taxes. In that case, as we explain in section 4.6, a fast consolidation delivers the strongest reduction in T^* (remember that τ^w is the only instrument for which a consolidation *shortens* the deleveraging) and hence the lowest PVM. This is an example of how the (inverse) duration effect can even overturn general tax smoothing considerations for certain calibrations of the model.

[50] This pattern is only violated three times. In each case the violation is marginal.

Finally, while for most specifications individual welfare is unanimously the highest with a gradual consolidation, there are a number of exceptions where the two agents disagree. With labor income tax hikes, entrepreneurs, which benefit the most from the fact that labor income taxes speed up deleveraging through the duration channel, sometimes prefer front loaded consolidations. With government spending, households sometimes prefer front-loaded consolidations. This is so because (general equilibrium effects aside) they benefit a lot from front loaded government spending cuts, which imply a shift of government spending from wasteful public expenditures to non-wasteful debt repayment to the government's creditors, i.e. Ricardian households.

Discussion of particular cases: These robustness exercises include two interesting special cases: (i) In the model with real debt there is no debt deflation, just as in the simple model in section 2. Hence inflation does not change the speed with which the debt stock decays during the deleveraging period. (ii) In the model with the fixed borrowing limit, the collateral value is irrelevant in determining the end of deleveraging T^*. Considering these two models hence allows us to assess the role of the two channels that explain how fiscal consolidations delay deleveraging in the full model in isolation: *Fisherian debt deflation* and *collateral value depression.*

As the table shows, the model with real debt, i.e. where T^* depends only the collateral value, behaves very much like the baseline model. The deleveraging process takes similarly long and is similarly sensitive to the consolidation. However there is one exception: For labor income tax based consolidations, real debt implies that such consolidations no longer speed up the deleveraging process. This highlights the fact that debt deflation plays a crucial role in explaining why τ^w-based consolidations tend to shorten private deleveraging.

The model with a fixed borrowing limit behaves very differently from the baseline quantitatively. Here T^* is much lower and for 3 of the 5 instruments unaffected by the consolidation. This highlights the fact that the duration channel operates mainly through collateral values and less so through Fisherian debt deflation. However, for consumption taxes and (front-loaded) government spending, fiscal consolidations again delay the duration of deleveraging – if only by one period – because they cause deflation.[51]

Furthermore, the specification without price stickiness highlights the relatively minor role this friction plays in our scenarios. As you can see in Figure 8, which reproduces figure 3 in the text with and without price stickiness, the dynamics of real quantities are mostly unaffected. The main effect of price stickiness is to (plausibly) smooth out the inflation responses.

[51] For larger fiscal consolidations or larger private deleveraging shocks, examples can be found where also the other taxes shift T^* highlighting the fact that, while quantitatively less important, debt deflation alone can in principle affect the length of deleveraging.

Model version			Duration channel Deleveraging		Size Output	Gradualism Deleveraging			Output	Welfare	
Modified param./assumption	Modification	Instr	T_0^*	T_{20}^*	PVM	T_{front}^*	T_{base}^*	T_{gradual}^*	PVM	HH	E
baseline		g	9	12	1	13	12	11	1	1	1
		τ^c	9	9	1	10	9	9	0	1	1
		τ^k	10	16	1	19	16	12	1	1	1
		τ^w	8	6	1	6	6	7	1	1	1
		T	9	10	1	12	10	9	1	1	1
elasticity of sub. housing-cons.	$\sigma = 1.5$	g	10	13	1	13	13	11	1	0	1
		τ^c	9	10	1	11	10	10	1	1	1
		τ^k	11	17	1	19	17	14	1	1	1
		τ^w	9	7	1	6	7	7	1	1	0
		T	10	11	1	14	11	10	1	1	0
elasticity of sub. housing-cons.	$\sigma = 0.5$	g	10	12	1	14	12	11	1	1	1
		τ^c	9	11	1	11	11	10	0	1	1
		τ^k	9	21	1	23	21	13	1	1	1
		τ^w	8	8	1	7	8	8	1	0	1
		T	8	8	1	10	8	8	1	1	1
inverse labor supply elasticity	$\varphi = 2/3$	g	9	12	1	13	12	11	1	1	1
		τ^c	8	9	1	10	9	9	0	1	1
		τ^k	10	17	1	20	17	12	1	1	1
		τ^w	8	5	1	5	5	6	1	1	1
		T	9	10	1	12	10	9	1	1	1
inverse labor supply elasticity	$\varphi = 2$	g	9	12	1	13	12	11	1	1	1
		τ^c	9	10	1	10	10	9	1	1	1
		τ^k	10	16	1	19	16	12	1	1	1
		τ^w	8	8	1	7	8	8	1	1	0
		T	9	10	1	12	10	9	1	1	1
high housing adjustment cost	$\Phi = 10$	g	10	12	1	13	12	11	1	1	1
		τ^c	9	10	1	10	10	9	1	1	1
		τ^k	10	16	1	19	16	13	1	1	1
		τ^w	9	8	1	8	8	8	1	1	1
		T	9	10	1	12	10	9	1	1	1
no housing adjustment cost	$\Phi = 0$	g	9	12	1	12	12	10	1	1	0
		τ^c	8	9	1	10	9	9	0	1	1
		τ^k	10	16	1	19	16	12	1	1	1
		τ^w	8	6	1	5	6	7	0	1	1
		T	9	10	1	12	10	9	1	1	1
price stickiness	$\theta = 0.8$	g	9	11	1	12	11	10	1	1	1
		τ^c	8	9	1	10	9	9	0	1	1
		τ^k	9	16	1	19	16	12	1	1	1
		τ^w	8	6	1	6	6	7	1	1	1
		T	9	9	1	12	9	9	1	1	1
flexible prices	$\theta = 0$	g	9	12	1	13	12	11	1	1	1
		τ^c	9	10	1	10	10	9	1	1	1
		τ^k	10	16	1	19	16	13	1	1	1
		τ^w	8	6	1	6	6	7	1	1	1
		T	9	10	1	12	10	9	1	1	1
debt amortization rate	$\gamma = 0.98$	g	16	20	1	20	20	19	1	1	1
		τ^c	15	17	1	17	17	16	1	1	1
		τ^k[52]	18	23	1	25	23	21	1	1	0
		τ^w	14	12	1	13	12	13	1	1	1
		T	15	19	1	22	19	16	1	1	0
short term debt	$\gamma = 0$	g			1				1	0	1
		τ^c			1				0	1	1
		τ^k			1				1	1	1
		τ^w			0				1	1	0
		T			1				1	1	1
real debt		g	10	12	1	12	12	11	1	0	1
		τ^c	10	10	1	11	10	10	0	1	1
		τ^k	12	21	1	24	21	17	1	1	1
		τ^w	9	9	0	12	9	9	1	1	1
		T	10	11	1	15	11	10	1	1	1
no nominal rigidities (real debt & flexible prices)		g	11	13	1	14	13	12	1	1	1
		τ^c	10	11	1	12	11	11	0	1	1
		τ^k	13	23	1	24	23	18	1	1	1
		τ^w	10	9	0	10	9	9	1	1	0
		T	11	12	1	16	12	11	1	1	0
no nominal rigidities (real debt & flexible prices) and short term debt		g			1				1	0	1
		τ^c			1				0	1	1
		τ^k			1				1	1	1
		τ^w			1				1	1	0
		T			1				1	1	1
contemporaneous collateral constraint		g	10	12	1	13	12	11	1	1	1
		τ^c	9	10	1	10	10	9	1	1	1
		τ^k	10	17	1	20	17	13	1	1	1
		τ^w	9	7	1	7	7	7	1	1	0
		T	9	10	1	13	10	9	1	1	1
fix borrowing limit		g	5	5	1	6	5	5	1	0	1
		τ^c	4	5	1	5	5	4	0	1	1
		τ^k	4	4	1	4	4	4	1	1	1
		τ^w	4	4	1	4	4	4	1	1	1
		T	4	4	1	4	4	4	1	1	1
counterfactual model		g	9	9	1	9	9	9	0	0	1
		τ^c	9	9	1	9	9	9	0	1	1
		τ^k	10	10	1	10	10	10	0	0	1
		τ^w	8	8	1	8	8	8	1	1	0
		T	9	9	1	9	9	9	1	1	1

[52]For this row, we simulate a 15 pp consoldiation, since for 20 pp the numerical solution algorithm

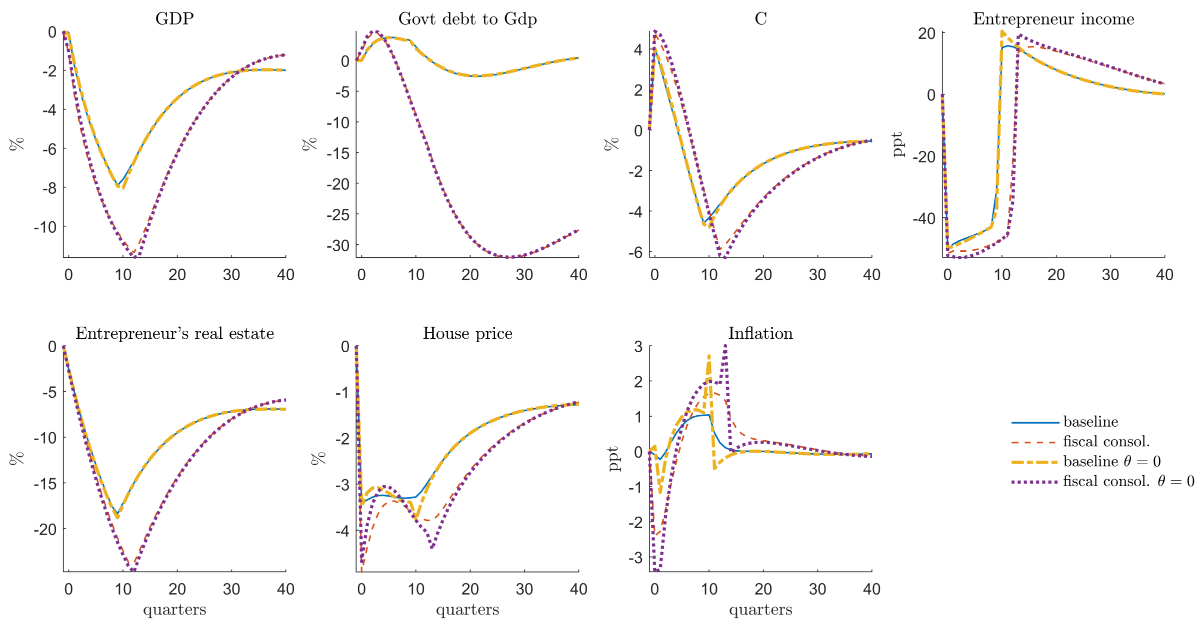

Figure 8: Macroeconomic adjustment with and without price stickiness.

Besides these robustness exercises we also consider a counterfactual model, in which we shut off the effect that the fiscal consolidation has on borrowing. In particular, for each fiscal instrument we first simulate the private deleveraging shock (*without* fiscal consolidation) in the baseline model. From this simulation we record the path of collateral values, $cv_t \equiv \overline{m} E_t \frac{\pi_{t+1}}{R_t} p^h_{t+1} h^e_t$, and debt outstanding (net of current period's amortizations), $ad_t \equiv \gamma \frac{b_{t-1}}{\pi_t}$. Then we construct our "counterfactual" fiscal consolidation scenarios as follows. We replace the asymmetric collateral constraint by

$$b_t \leq \max\left\{ \overline{m} E_t \frac{\pi_{t+1}}{R_t} p^h_{t+1} h^e_t + \epsilon^1_t, \gamma \frac{b_{t-1}}{\pi_t} + \epsilon^2_t \right\},$$

where ϵ^1_t and ϵ^2_t are exogenous anticipated shocks. We then simulate both the deleveraging and the fiscal consolidation shocks, but we set the sequence of shocks ϵ^1_t and ϵ^2_t

did not converge.

such that in the equilibrium of the counterfactual model the paths of the two terms in the max operator coincide with the paths of cv_t and ad_t in the no-consolidation scenario of the baseline model. That is, $cv_t^{\text{counterf consol}} + \epsilon_t^1 = cv_t^{\text{baseline, no consol}}$ and $ad_t^{\text{counterf consol}} + \epsilon_t^2 = ad_t^{\text{baseline, no consol}}$. We thus make sure that the counterfactual "collateral constraint" allows for exactly as much debt as the collateral constraint in the baseline model's no-consolidation scenario: $b_t^{\text{counterf consol}} = b_t^{\text{baseline, no consol}}$. This way we can simulate the effect of a fiscal consolidation on the (rest of the) economy while fully insulating the path of private debt from such consolidation.

Table D.2 compares the effects of the fiscal consolidation across the baseline and the counterfactual economies for each instrument. The second panel ("counterfactual") allows us to understand how a fiscal consolidation would affect the PVM and T^* if there was no interaction between public and private deleveraging. The lower panel computes the difference of PVMs between the baseline and the counterfactual model, which allows us to isolate the effect of the consolidation through its impact on collateral values and private debt.

As the lower panel shows, the interaction of private and public deleveraging makes the fiscal consolidation more costly in terms of PVM whenever the latter triggers a delay in the deleveraging process ($\Delta T^* > 0$) in the baseline model; this is the case for g, τ^k and T. Conversely, it makes the consolidation less costly when the latter reduces T^*, as is the case for τ^w. When T^* is unaffected by the consolidation, as for the baseline and gradual τ^c consolidation, the PVM is essentially equal in both models.

Furthermore, such interaction, and the associated 'duration channel', are crucial in shaping our gradualism results. The costs of front-loading are larger (i.e. the relationship between consolidation speed and PVMs is steeper) in the baseline model than in the counterfactual one whenever speeding up the adjustment delays T^* in the former model (this is the case for g, τ^k and T, and also for τ^c when moving from baseline speed to front-loaded), conversely when it brings forward T^* (this is the case for τ^w). In fact, the second panel shows that, in the absence of the fiscal consolidation-private deleveraging interaction, a fast consolidation can actually lead to lower PVMs (as for g, τ^c, τ^k). That is, the duration channel not only constitutes an argument in favor of gradualism, it can even be decisive in making gradualism the least costly approach.

Table D.2. Baseline vs counterfactual model (with debt path equal to that in no-consolidation scenario)

		PVM			T^*			
model	instr.	front.	base.	grad.	T_0^*	T_{front}^*	T_{base}^*	T_{gradual}^*
	g	4.60	4.04	3.67	9	13	12	11
	τ^c	1.92	1.53	1.77	9	10	9	9
baseline	τ^k	13.05	10.15	7.54	10	19	16	12
	τ^w	5.60	4.58	4.18	8	6	6	7
	T	4.62	2.9	1.95	9	12	10	9
	g	2.45	2.48	2.65	9	9	9	9
	τ^c	1.49	1.57	1.79	9	9	9	9
counterfactual	τ^k	5.20	5.40	6.64	10	10	10	10
	τ^w	*	5.07	4.39	8	*	8	8
	T	2.51	2.13	1.78	9	9	9	9
	g	2.16	1.57	1.02				
	τ^c	0.43	-0.04	-0.02				
baseline vs	τ^k	7.85	4.75	0.89				
counterfactual	τ^w	*	-0.49	-0.21				
	T	2.1	0.77	0.17				

* in this case the solution algorithm failed to converge

E. A medium scale DSGE

The lessons we derived from the model in the main text apply qualitatively also to more sophisticated environments. To show this we repeated our analysis in the medium scale DSGE model of Andrés et al. (2017). Their model largely nests the full model in the main text. Relative to the model in the main text, their model features a richer productive sector the additionally features equipment capital, production of equipment capital and wage stickiness. Furthermore, the household sector consists of two households with different degrees of patience, as in the large model in Iacoviello (2005). The patient household is as in the model in the main text, the impatient household also borrows subject a collateral constraint analogous to the one of the entrepreneur. For a full description of the model and the calibration we refer the reader to their paper. We augmented their model by a fiscal block which is a straightforward extension of the one in the main text.

The mechanisms we described in the main text for the full model play out similarly in this full model. To illustrate this we reproduce the corresponding figures to Figures 2 to 5 from the main text here.

Figure 9 shows that the dynamics of debt and collateral, triggered by a private deleveraging scenario with or without spending based consolidation, exhibit a similar pattern as in Figure 2. This is true for both the entrepreneurs and the borrowing (inpatient) households, though we only show the latter here.

Comparing Figures 10 and 3 we see that macroeconomic variables again behave similarly. This includes real estate prices and inflation, the drivers of the collateral valuation and debt deflation channels. Note that now there are two kinks visible in the dynamics because impatient households' and entrepreneurs' deleveraging ends in different periods.

Comparing Figures 11 and 4 we see that the buffering effect and the duration channel have similar effects on the fiscal sacrifice ratio.[53] Figures 12 and 5 both show how fast consolidations delay the end of deleveraging. Finally, the degree to which deleveraging is delayed depends on the fiscal instrument in a similar fashion as in the model in the main text (compare 12 and 6)

[53]Note that since each path for GDP now has 2 kinks, the sacrifice ratio has up to 4 kinks.

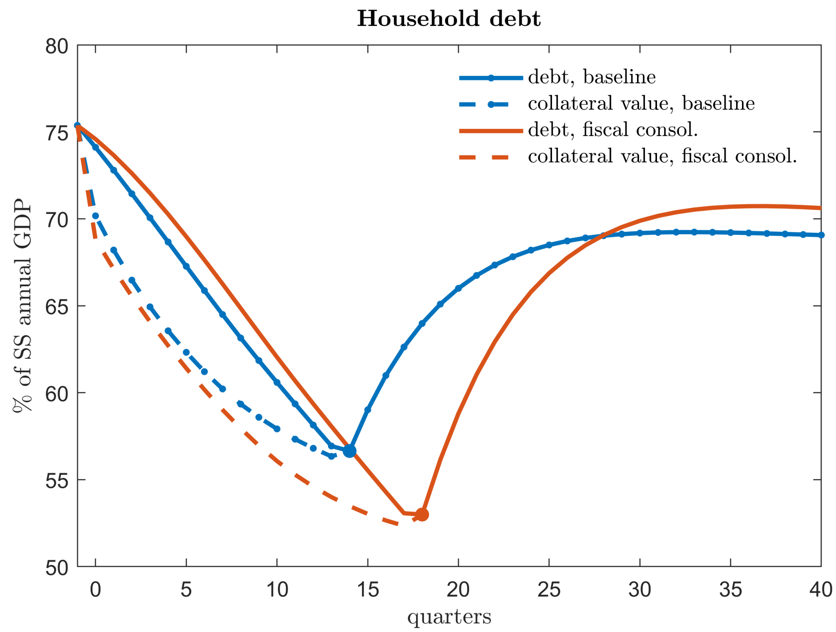

Figure 9: Household debt and collateral dynamics

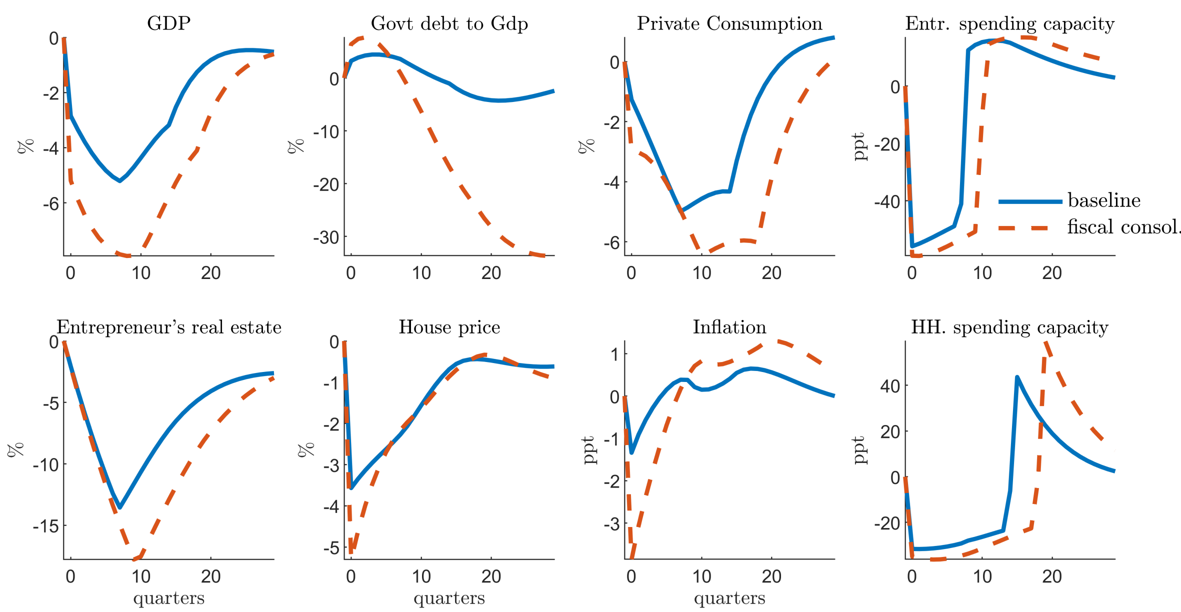

Figure 10: Macroeconomic adjustment

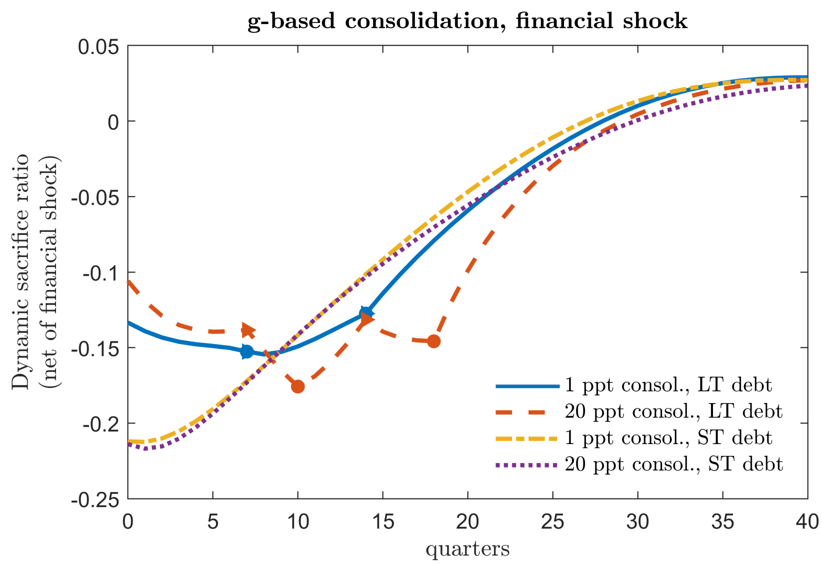

Figure 11: Dynamic sacifice ratios

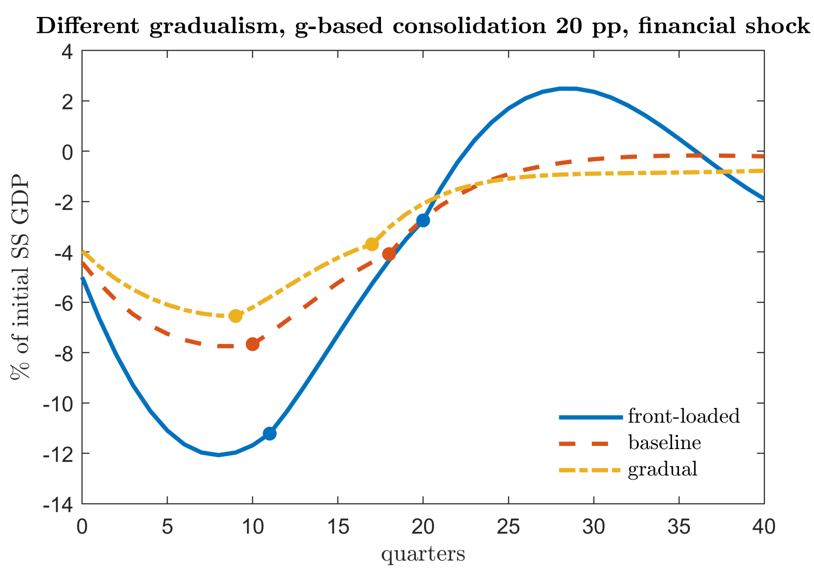

Figure 12: Dynamic sacifice ratios

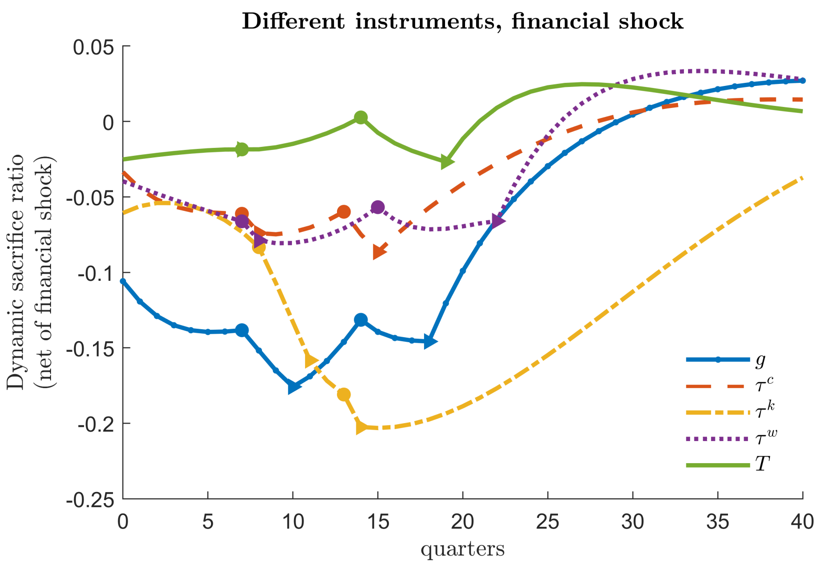

Figure 13: Output costs of consolidation across fiscal instruments. Triangles and circles mark the no-consolidation and the consolidation T^*.