Online Appendix for "Who Works for Whom? Worker Sorting in a Model of Entrepreneurship with Heterogenous Labor Markets"*

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A Algorithm for Solving The Model's Equilibrium

A stationary equilibrium of the model is computed using an algorithm based on Huggett and Ventura (1999). The algorithm finds an equilibrium by iterating over value functions and decision rules over a discretized state space. Labor productivity and entrepreneurial ability processes are discretized on a 21-point support for the distribution implied by the process. The support is bounded below and above the mean by 2.5 times the standard deviation. The asset grid is discretized to 201 points with linear interpolation of all functions between grid points. The spacing between points on the asset grid increases with asset levels. Asset grid points are placed according to $a_1 = 0$, $a_j = \psi j^\chi$ for $j = 2, ..., 201$, where $\chi = 2.0$, $\psi = \bar{a}/(201^\chi)$ and $\bar{a}$ is an upper bound. The algorithm is as follows.

1. Guess a value for the capital-labor ratio in the corporate sector, K/L, and δ_τ,
2. Calculate the values $w_f = (1-\nu)AK^\nu L^{-\nu}$ and $r = \nu AK^{\nu-1}L^{1-\nu} - \delta - \delta_\tau$,
3. Set the initial value for the entrepreneurial sector wage equal to the corporate sector wage: $w_e = w_f$,

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4. Calculate the optimal decision rules $c^i(s)$, $a^{i\prime}(s)$, $h^d(\widetilde{s})$, $m^d(\widetilde{s})$, $k(s), l(s)$, $(i, d \in \{n, f, e, m\})$

5. Calculate K'/L', $\int l(s)d\Psi^m(s)$, $\int p\tau(s)d\Psi^e(s)$, and $\int zd\Psi^e(s)$ implied by the optimal decision rules,

6. If

$$\begin{aligned} |K'/L' - K/L| &< \eta_1, \\ |\int l(s)d\Psi^m(s) - \int zd\Psi^e(s)| &< \eta_2, \\ \left| \int p\tau(s)d\Psi^e(s) - \delta_\tau K \right| &< \eta_3 \end{aligned}$$

for small $\eta_1, \eta_2, \eta_3 > 0$, then a stationary equilibrium has been found. Otherwise, update K/L, $\{w_e, w_f\}$, and δ_τ, and repeat steps 4-6.

The parameter vector $M = \{\alpha, \gamma_e, \gamma_f, \bar{\gamma}, \phi_e, \phi_f, \mu\}$ is recovered using a Nelder-Mead Simplex algorithm, where the objective function is set to minimize the distance between the target and simulation moments. Target moments m_j $(j = 1, ..., J)$ are described in Section 4. The objective function used is

$$\sum_{j=1}^{J} [(\ln(m_j/d_j))^2],$$

where m_j is the moment j calculated using simulated data in the stationary equilibrium and d_j is the corresponding data moment. The Fortran 90 code for the simplex algorithm is taken from the public domain and was written by Alan Miller.[1]

B Estimation of the Parameters $\rho_\theta, \sigma_\theta$, and ξ

The estimation of the decreasing returns parameter, ξ, for entrepreneurial firms, and the parameters for the entrepreneurial productivity process, $\{\rho_\theta, \sigma_\theta\}$, is based on the framework of Abraham and White (2015).[2] The framework allows the estimation of the parameters $\{\rho_\theta, \sigma_\theta, \xi\}$ simultaneously. Consider a production function for a manufacturing firm i in the form of

$$y_{it} = \theta_{it} \left(k_{it}^{a_k} l_{it}^{a_l} x_{it}^{1-a_k-a_l} \right)^\xi, \tag{1}$$

which includes materials and energy, x_{it}, as an input, and a productivity process $\ln \theta_{it} = (1 - \rho_\theta)\mu_i + \delta_t + \rho_\theta \ln \theta_{it-1} + \epsilon_t$, where μ_i is a firm-specific productivity parameter, δ_t is a year effect that captures general changes in productivity that apply to all firms, and $\epsilon_t \sim N(0, \sigma_\theta)$. The

[1] Available at `http://jblevins.org/mirror/amiller/`.

[2] Also see Castiglionesi and Ornaghi (2013) for a similar estimation methodology.

parameters ρ_θ and σ_θ are allowed to vary across industries. The inclusion of the materials and energy in the production function controls for the use of intermediate inputs (materials and energy) in estimating the underlying total factor productivity process. The estimation also allows for a markup, η, common to all firms in an industry, which can be thought of as the average markup across firms that is assumed to be constant over time. Abraham and White (2015) estimate the parameters, ξ, ρ_θ and σ_θ in a GMM framework using the log-linear form of the production function and the Solow residual obtained from the gross output and cost shares of the inputs. See Abraham and White (2015) or Castiglionesi and Ornaghi (2013) for a derivation of the exact model estimated.

The data used for the estimation is the U.S. Census Bureau's Annual Survey of Manufactures (ASM), which provides an unbalanced panel of manufacturing establishments for the period 1972-2009. The data include, for each establishment, annual measures of output (value of shipments) and inputs (employment, materials/energy use, capital). This information is aggregated to the firm level. The age of the firm is also available, which is the age of the oldest establishment of the firm. The establishments included in the ASM sampling frame typically have size 20 employees or more, so the parameter estimates are not representative of very small firms. The model yields estimates of ξ, ρ_θ, and σ_θ for young versus old firms at the 4-digit SIC industry level. The estimated values for young firms are then averaged across industries to be used in the calibration of the baseline model. The analysis is limited to the manufacturing sector because of the unavailability of similar data for other sectors of the economy to calculate the revenue-based productivity of an establishment.

A remark is in order for how the estimated parameters of the three-input production function in (1) are used to calibrate the model's two-factor production function. Recall, the model production function is specified as $y_t = \theta_t(k_t^\nu l_t^{1-\nu})^\xi$. In the production function (1) used for estimation, the decreasing returns parameter, ξ, is the same for each of the three inputs. Because the decreasing returns parameter is common to all inputs, in the model's calibration the estimated decreasing returns parameter $\xi = 0.88$ is applied to the two-factor production function. Similarly, the total factor productivity process is not specific to any input (i.e. Hicks neutral) in (1). Therefore, the estimated productivity process based on the three-factor production function in (1) is assumed to apply to the two-factor production in the model.

C Additional Model Results

C.1 Firm Transitions from Entrepreneurial to Corporate Sector

The purpose of this section is to explore the sensitivity of transition probability p as discussed in the calibration section. The notion of transition from the entrepreneurial sector to the corporate sector was introduced only to generate a simple link between the two sectors and allow some entrepreneurial firms to operate without scale and borrowing constraints. The experiments in Table C.1 show that the exact magnitude of the transition rate does not matter substantially in the model's ability to match the data targets, as long as the rate is not too large. Compare the prices and allocations in the baseline economy, where $p = 0.001$, to two stationary equilibria with much higher transition probability values. In all instances, sorting remains broadly consistent with the baseline economy with relatively small changes in how the model matches targeted moments.

C.2 Persistence of Entrepreneurial Ability

The stochastic process for entrepreneurial ability is estimated based on the methodology in Abraham and White (2006). The estimation yields a smaller persistence than what has been found in other studies, such as Decker et al. (2018), Lee and Mukoyama (2015) and Foster et al. (2008).[3] It is of interest to know how the properties of the baseline economy change if a higher value of persistence is used. For this purpose, the model is re-calibrated with a higher value for the quarterly persistence parameter, $\rho_\theta = 0.6$, that is closer to the quarterly transformations of annual values found in other studies. The results are in Table C.2.

The economy with a higher persistence in entrepreneurial ability is able to broadly capture worker sorting as found in the baseline calibration. The new calibration captures most of the targeted moments with the exception of the corporate earnings premium. While the baseline model implies a 16.6% corporate earnings premium, the economy with higher persistence yields a premium of only 9.0%. Overall, there is also a lower degree of worker sorting based on both productivity and assets.

[3]Lee and Mukoyama (2015) does not allow for heterogeneity in the persistence parameter, and Foster et al. (2008) provides estimates for a sample of 11 narrowly defined manufacturing product categories.

Table C.1. The Role of Transition Probability (p)

	Baseline	No Transitions	Larger Transition Rates	
Variable	($p = 0.001$)	($p = 0.000$)	($p = 0.002$)	($p = 0.004$)
Employment-to-population ratio	82.5%	82.4%	82.7%	83.1%
Share of employment (Entrepreneurial sector)	14.0%	13.2%	13.5%	13.8%
Fraction of entrepreneurs	4.0%	3.5%	5.3%	8.6%
Corporate average earnings premium	16.6%	16.2%	16.6%	14.5%
Wage ratio (w_f/w_e)	1.23	1.23	1.22	1.19
Employment-to-nonemployment (E-to-N) flows	10.3%	10.0%	11.0%	11.0%
Nonemployment-to-employment (N-to-E) flows	39.0%	40.3%	37.3%	32.6%
Job-to-job flows	1.9%	1.9%	1.9%	1.9%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	0.94	0.95	0.96
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.29	1.26	1.19

Notes: Each column represents equilibrium allocations that vary from the baseline by probability of transition to corporate sector. See text for details.

Table C.2. The properties of the model with high persistence of entrepreneurial ideas

Variable	**Baseline** ($\rho_\theta = 0.30$)	**Higher Persistence for θ** ($\rho_\theta = 0.6$)
Employment-to-population ratio	82.5%	83.3%
Share of employment (Entrepreneurial sector)	14.0%	14.0%
Fraction of entrepreneurs	4.0%	4.6%
Corporate average earnings premium	16.6%	9.0%
Wage ratio (w_f/w_e)	1.23	1.10
Employment-to-nonemployment (E-to-N) flows	10.3%	9.0%
Nonemployment-to-employment (N-to-E) flows	39.0%	35.6%
Job-to-job flows	1.9%	2.4%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	0.99
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.08
Disutility from labor, α	0.37	0.37
Entrepreneurial sector job offer rate, γ_e	0.07	0.06
Corporate sector job offer rate, γ_f	0.46	0.43
Job-to-job transition rate parameter, $\bar{\gamma}$	0.10	0.11
Entrepreneurial sector job separation rate, ϕ_e	0.05	0.06
Corporate sector job separation rate, ϕ_f	0.10	0.09
Entrepreneurial ability (Mean), $e^{-\mu}$	0.38	0.36

Notes: In the model with a higher persistence for θ, all model parameters are re-estimated to match the data targets.

D Additional Empirical Results

Table D.1. Household net worth by firm size

	Mean		Quasi-median	
Firm Size (employees):	**0-49**	**50+**	**0-49**	**50+**
Net worth (All)	$117,500	$125,400	$17,060	$23,590
s.e.	(1,195)	(652)	(287)	(218)
N	53,000	175,000	53,000	175,000
Net worth (Recent hire)	$97,560	$94,440	$10,640	$11,160
s.e.	(1,842)	(1,148)	(310)	(200)
N	19,000	46,000	19,000	46,000
Earnings (All)	$9,856	$13,520	$7,640	$10,380
s.e.	(68)	(339)	(35)	(23)
N	53,000	175,000	53,000	175,000
Earnings (Recent hire)	$7,070	$10,660	$5,133	$6,586
s.e.	(102)	(1,252)	(45)	(40)
N	19,000	46,000	19,000	46,000

Notes: Standard errors in parentheses. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. For quasi-median, the standard errors are calculated using bootstrap.

Table D.2. Regression analysis of household net worth and earnings by firm size

	Estimated Small Firm Coefficient	
Dependent Variable	**OLS**	**Median**
Net worth (All workers)	-0.313^{***}	-0.157^{***}
s.e.	(0.071)	(0.017)
N	$228,000$	$228,000$
Net worth (Recent hire)	-0.129	-0.064^{**}
s.e.	(0.105)	(0.034)
N	$66,000$	$66,000$
Earnings (All workers)	-0.299^{***}	-0.209^{***}
s.e.	(0.015)	(0.004)
N	$228,000$	$228,000$
Earnings (Recent hire)	-0.307^{***}	-0.214^{***}
s.e.	(0.034)	(0.008)
N	$66,000$	$66,000$

Notes: Standard errors in parentheses (clustered by time and industry for OLS). All regressions include gender, race, marital status, education level dummies, age, and age-squared, as well as state, industry (2-digit NAICS) and year fixed effects. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. (*), (**), and (***) indicate statistical significance at 10, 5, and 1% levels, respectively.

Table D.3. Regression analysis by firm age with 4-digit NAICS controls

	Estimated Young Firm Coefficient	
Dependent Variable	**OLS**	**Median**
Net worth (All workers)	-0.395^{***}	-0.186^{***}
s.e.	(0.078)	(0.022)
N	$228,000$	$228,000$
Net worth (Recent hire)	-0.150^{*}	-0.057^{**}
s.e.	(0.118)	(0.043)
N	$66,000$	$66,000$
Earnings (All workers)	-0.257^{***}	-0.092^{***}
s.e.	(0.022)	(0.006)
N	$228,000$	$228,000$
Earnings (Recent hire)	-0.197^{***}	-0.080^{***}
s.e.	(0.041)	(0.011)
N	$66,000$	$66,000$

Notes: Standard errors in parentheses (clustered by time and industry for OLS). All regressions include gender, race, marital status, education level dummies, age, and age-squared, as well as state, industry (4-digit NAICS) and year fixed effects. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. (*), (**), and (***) indicate statistical significance at 10, 5, and 1% levels, respectively.

Table D.4. Regression analysis by firm size with 4-digit NAICS controls

	Estimated Small Firm Coefficient	
Dependent Variable	**OLS**	**Median**
Net worth (All workers)	-0.289^{***}	-0.146^{***}
s.e.	(0.065)	(0.018)
N	228,000	228,000
Net worth (Recent hire)	-0.105	-0.037
s.e.	(0.106)	(0.041)
N	66,000	66,000
Earnings (All workers)	-0.288^{***}	-0.190^{***}
s.e.	(0.014)	(0.004)
N	228,000	228,000
Earnings (Recent hire)	-0.311^{***}	-0.337^{***}
s.e.	(0.035)	(0.008)
N	66,000	66,000

Notes: Standard errors in parentheses (clustered by time and industry for OLS). All regressions include gender, race, marital status, education level dummies, age, and age-squared, as well as state, industry (4-digit NAICS) and year fixed effects. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. (*), (**), and (***) indicate statistical significance at 10, 5, and 1% levels, respectively.

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