

Online Appendix

D.1 Equilibrium analysis of spot trade only

The following Proposition characterizes the equilibrium when investors can only trade spot. It also establishes formally the claim made in Section 2.3 that spot trades alone can never allow to support the first best allocation.

Proposition 6. The first best allocation can never be attained when investors trade spot only. There exists a threshold $\bar{a}_{spot}$ such that

1. Low asset quantity: if $a < \bar{a}_{spot}$, then investor 1 sells his entire asset holdings in period 1. The liquidity premium $\mathcal{L}$ is strictly positive.
2. High asset quantity: if $a \geq \bar{a}_{spot}$, then investor 1 sells less $\bar{a}_{spot}$ in period 1. The liquidity premium is $\mathcal{L} = 0$.

Proof. We write down investor 1 consumption in period 3 and 2 and the first order conditions with respect to spot trades in period 1 and 2 for both investors:

$$c_3^1(s) = \omega + as, \tag{D.1}$$

$$c_2^1(s) = \omega + p_2(s)(a_1^1 - a_2^1(s)), \tag{D.2}$$

$$-p_2(s)v'(c_2^1(s)) + s + \gamma_2^1(s) = 0, \tag{D.3}$$

$$-p_2(s)u'(c_2^2(s)) + \beta s + \gamma_2^2(s) = 0, \tag{D.4}$$

$$-p_1 + \mathbb{E} [p_2(s)v'(c_2^1(s))] + \gamma_1^1 = 0, \tag{D.5}$$

$$-p_1 + \mathbb{E} [p_2(s)u'(c_2^2(s))] + \gamma_1^2 = 0, \tag{D.6}$$

where γ_t^i is the Lagrange multiplier on the no-short sale constraint of investor i in period t . In period 2, the state s is an argument of the multiplier.

We first show that investor 1 must always carry at least some asset into period 3, that is $a_2^1(s) > 0$ or $\gamma_2^1(s) = 0$ for all s . Suppose it is not the case, that is $a_2^1(s) = 0$ for some state s . From equation 1, we obtain $c_2^1(s) > \omega > c_2^2(s)$ so that $v'(c_2^1(s)) < u'(c_2^2(s))$ since $u'(\omega) > v'(\omega)$ by assumption. This is a contradiction with equations (D.3)-(D.4) since $\beta < 1$.

We now characterize $p_2(s)$ and $a_2^2(s)$ considering the cases where $\gamma_2^2(s) > 0$ and $\gamma_2^2(s) = 0$ in turn. Let s be such that $\gamma_2^2(s) > 0$ so that $a_2^2(s) = 0$. Then, we have $c_2^1(s) = \omega - p_2(s)a_1^2$ and from equation (D.3), $p_2(s)$ is defined implicitly as a function of a_1^2 by $p_2(s)v'(\omega - p_2(s)a_1^2) = s$. In particular, $p_2(s)$ is strictly increasing in s since v is concave. Consider now values of s for which $\gamma_2^2(s) = 0$. Equations (D.3) and (D.4) imply that

$$\beta v'(c_2^1(s)) = u'(c_2^2(s))$$

Since $c_2^2(s) = 2\omega - c_2^1(s)$ by the resource constraint, it must be that $c_2^1(s)$ does not depend on s . We let $\hat{c}_2^1$ be this constant. From equation (D.3), we obtain that $p_2(s)v'(\hat{c}_2^1) - s = 0$. Hence, $p_2(s)$ is again strictly increasing in s . In addition, equation (D.2) pins down $a_2^2(s)$ since then $\hat{c}_2^1 = \omega - p_2(s)(a_1^2 - a_2^2(s))$.

We now show that there is a threshold $\hat{s}(a_1^2)$ such that $\gamma_2^2(s) > 0$ for $s < \hat{s}(a_1^2)$ and $\gamma_2^2(s) = 0$ for $s \geq \hat{s}(a_1^2)$. If it is not the case, there exists $(s_1, s_2) \in [\underline{s}, \bar{s}]$ such that $s_1 < s_2$ and $\gamma_2^2(s_1) = 0$ while $\gamma_2^2(s_2) > 0$. From the characterization, in the previous paragraph, this implies that $u'(c_2^2(s_2)) > \beta v'(c_2^1(s_2))$ so that $c_2^1(s_2) > \hat{c}_2^1 = c_2^1(s_1)$. To establish a contradiction, observe that

$$c_2^1(s_2) = \omega - p_2(s_2)a_1^2 = \omega - \frac{s_2}{v'(c_2^1(s_2))}a_1^2 < \omega - \frac{s_2}{v'(\hat{c}_2^1)}a_1^2 < \omega - \frac{s_1}{v'(\hat{c}_2^1)}a_1^2 < c_2^1(s_1)$$

The first inequality follows from $c_2^1(s_2) > \hat{c}_2^1$. The last inequality follows from equation (D.2) with $p_2(s_1) = s_1/v'(\hat{c}_2^1)$ by definition of s_1 . Hence the claimed threshold exists. We thus obtain the following pattern for investor 1 consumption in period 2:

$$c_2^1(s) = \begin{cases} \omega - p_2(s)a_1^2 & s \leq \hat{s}(a_1^2) \\ \hat{c}_2^1 & s > \hat{s}(a_1^2) \end{cases} \quad (\text{D.7})$$

It is clear from this expression and the definition of $\hat{c}_2^1$ that the first best allocation can never be implemented from spot trades only. Indeed, when $s \leq \hat{s}(a_1^2)$, investors' consumption varies with s while when $s \geq \hat{s}(a_1^2)$ marginal utilities between periods 1 and 2 are not equalized. This proves our claim in the main text.

To finish the characterization of the spot only equilibrium, we are left to pin down a_1^2 , the quantity investor 2 initially buys from investor 1. We first show that $\gamma_1^2 = 0$ that is some spot trade occurs in period 1. Indeed, if $\gamma_1^2 > 0$, $a_1^2 = 0$ and from equation

(D.7), $c_2^1(s) \geq \omega$. By assumption, for all s , we thus have $u'(c_2^2(s)) - v'(c_2^1(s)) > 0$ which is incompatible with $\gamma_1^2 > 0$, using equations (D.5) and (D.6) with $\gamma_1^1 = 0$. Hence, we must have $\gamma_1^2 = 0$. Then, from equations (D.3) and (D.4):

$$\begin{aligned} \gamma_1^1 &= \mathbb{E} \{ p_2(s) [u'(c_2^2(s)) - v'(c_2^1(s))] \} \\ &= \int_{\underline{s}}^{\hat{s}(a_1^2)} (p_2(s)(u'(\omega + p_2(s)a_1^2) - s) dF(s) - (1 - \beta)v'(\hat{c}_2^1) \int_{\hat{s}(a_1^2)}^{\bar{s}} p_2(s) dF(s) := K(a_1^2) \end{aligned} \quad (\text{D.8})$$

$$(\text{D.9})$$

The total derivative of K with respect to a_1^2 is equal to

$$K'(a_1^2) = \int_{\underline{s}}^{\hat{s}(a_1^2)} \left[\frac{\partial p_2(s)}{\partial a_1^2} u'(\omega + p_2(s)a_1^2) + \frac{\partial a_1^2 p_2(s)}{\partial a_1^2} u''(\omega + p_2(s)a_1^2) p_2(s) \right] dF(s)$$

When $s \leq \hat{s}(a_1^2)$, $p_2(s)v'(\omega - p_2(s)a_1^2) = s$ so that $p_2(s)$ is strictly decreasing in a_1^2 while $a_1^2 p_2(s)$ is strictly increasing in a_1^2 on $[\underline{s}, \hat{s}(a_1^2)]$. Since u is strictly concave, this proves that $K' < 0$. In addition, observe that given a_1^2 , $\hat{s}(a_1^2)$ is the minimal state where $\omega - p_2(s)a_1^2 = \hat{c}_2^1$ for $p_2(s) = s/v'(\hat{c}_2^1)$ and thus that $\hat{s}(a_1^2)$ is decreasing in a_1^2 . Hence as a_1^2 goes to 0, $\hat{s}(a_1^2)$ becomes larger than $\underline{s}$. As a_1^2 goes to ∞ , $\hat{s}(a_1^2)$ becomes smaller than $\underline{s}$. Then, observe that

$$\begin{aligned} K(0) &= \mathbb{E}[p_2(s)](u'(\omega) - v'(\omega)) > 0 \\ \lim_{x \rightarrow \infty} K(x) &= -(1 - \beta)v'(\hat{c}_2^1)\mathbb{E}[p_2(s)] < 0 \end{aligned}$$

Hence, since K is strictly decreasing and continuous, by the intermediate value theorem, there exists a unique value $\bar{a}_{spot} > 0$ such that $K(\bar{a}_{spot}) = 0$. Two cases are then possible. Either $a \geq \bar{a}_{spot}$ and $\gamma_1^1 = 0$ and $a_1^2 = \bar{a}_{spot}$ or $a < \bar{a}_{spot}$ and $\gamma_1^1 > 0$ that is $a_1^2 = a$. This concludes the proof. $\square$

D.2 Additional results for the Proof of Proposition 1

D.2.1 Proof of Claim 1

We show that investors reach the first best allocation and $s^*(\nu_2) \leq \underline{s}$ when $\gamma_1^1 = 0$. Since $\gamma_1^1 = 0$, condition (C.7) may hold for any positive function $\tilde{f}_{12}$ only if $u'(c_2^2(s)) = v'(c_2^1(s))$ for all s . This implies that $(c_2^1(s), c_2^2(s)) = (c_{2,*}^1, c_{2,*}^2)$ for all s , by definition of $(c_{2,*}^1, c_{2,*}^2)$.

Using equation (C.14), we obtain $p_2(s) = s/v'(c_{2,*}^1)$ for all $s \in [\underline{s}, \bar{s}]$, which is strictly increasing in s . We showed in the main text that the consumption of investor 2 in period 2 satisfies

$$c_2^2(s) \leq \omega + M \frac{as}{(1 - \theta_1)v'(c_{2,*}^1)}$$

Hence, $c_2^2(s) = c_{2,*}^2$ can hold for all s if and only if the right hand side of this inequality is larger than $c_{2,*}^2$ for $s = \underline{s}$. By definition of $s^*(\nu_2)$ in (24), this is equivalent to $s^*(\nu_2) \leq \underline{s}$. We observe that when the stronger condition $s^*(0) \leq \underline{s}$ holds, collateral re-use is not essential. Then, the allocation can be implemented thanks to a repo contract with a fixed repurchase price $\bar{f} \in \left[\frac{p_2(s^*(0))}{1 - \theta_1}, \frac{p_2(\underline{s})}{1 - \theta_1} \right]$ with full segregation of the collateral. Observe that, since $u'(c_2^2(s)) = v'(c_2^1(s))$ for all s , conditions (C.8) and (C.10) hold and the first order condition with respect to spot trades in period 1 hold with

$$p_1 = \mathbb{E}[p_2(s)v'(c_{2,*}^1)] = \mathbb{E}[p_2(s)u'(c_{2,*}^2)] = \mathbb{E}[s]$$

This proves that the first-best allocation can be implemented when $s^*(\nu_2) \leq \underline{s}$. To complete the analysis of this case, we show that the first best allocation can be attained if ν_2 is high enough. Observe from (24) that $s^*(\nu_2)$ is decreasing in ν_2 and that $\lim_{\nu_2 \rightarrow 1} s^*(\nu_2) < 0$. Hence there exists $\nu^* \in (0, 1)$ such that $s^*(\nu^*) = \underline{s}$. To find the expression for ν^* , from equation (24), we have

$$\frac{s^*(\nu_2)}{1 - \nu_2} [1 - \nu_2(1 - \theta_1)] = s^*(0)$$

We get $\nu^* = \frac{s^*(0) - \underline{s}}{s^*(0) - (1 - \theta_1)\underline{s}}$.

D.2.2 Proof of Claim 2

We first show that there exists a threshold $\hat{s}$ such that $c_2^1(s) > c_{2,*}^1$ for $s < \hat{s}$ and $c_2^1(s) = c_{2,*}^1$ for $s \geq \hat{s}$. Suppose this were not true, that is we can find two states $(s_1, s_2) \in [\underline{s}, \bar{s}]^2$ such that $s_1 < s_2$ and $c_2^1(s_1) = c_{2,*}^1$ while $c_2^1(s_2) > c_{2,*}^1$. Then, from the argument in the previous paragraph, we get $f(s_2) = p_2(s_2)/(1 - \theta_1)$, with $p_2(s_2)$ as specified in (25). Since by assumption $c_2^1(s_2) > c_{2,*}^1$, from (25) we obtain

$$p_2(s_2) > \frac{s_2}{v'(c_{2,*}^1)} > \frac{s_1}{v'(c_{2,*}^1)} = p_2(s_1)$$

where the last equality follows from the fact that $c_2^1(s_1) = c_{2,*}^1$. But then

$$c_2^1(s_2) = \omega - aM \frac{p_2(s_2)}{1 - \theta_1} < \omega - aM \frac{p_2(s_1)}{1 - \theta_1} \leq c_2^1(s_1) = c_{2,*}^1$$

a contradiction, which establishes that the claimed threshold $\hat{s}$ exists.

Since $p_2(s) = s/v'(c_{2,*}^1)$ is increasing in s for $s \geq \hat{s}$, the threshold $\hat{s}$ is the minimum state where the first best allocation can be financed given that the spot market price verifies $p_2(s)v'(c_{2,*}^1) = s$. Hence, by the definition of $s^*(\nu_2)$, the threshold $\hat{s}$ coincides with $s^*(\nu_2)$.

D.2.3 Proof of claim 3

We first prove that investor 1 does not want to use another repo contract to sell the asset. For any $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$, we have in fact $f(s) > \tilde{f}_{12}(s)$ for any s such that $u'(c_2^2(s)) > v'(c_2^1(s))$, while in the other states $u'(c_2^2(s)) = v'(c_2^1(s))$. This proves that condition (C.8) holds.

We now show that investor 2 does not sell the asset in a repo, that is (C.10) holds for any $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$. Since $u'(c_2^2(s)) - v'(c_2^1(s)) \geq 0$, this inequality holds for any $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ if it holds for $f_{21} = \nu_1 p_2(s)$, the contract with the minimal repayment in this set. Using the expression for the equilibrium contract $f(s, \nu_2)$ sold by investor 1, we obtain:

$$\begin{aligned} \nu_1 &\geq \frac{(1 - \nu_1 \nu_2)(1 - \theta_1) - (1 - \nu_1)}{(1 - \theta_1)(1 - \nu_2)} \\ \Leftrightarrow \quad \nu_1(1 - \nu_2)(1 - \theta_1) &\geq \nu_1(1 - \nu_2) - \theta_1(1 - \nu_1 \nu_2) \\ \Leftrightarrow \quad \theta_1(1 - \nu_1 \nu_2) &\geq \theta_1 \nu_1(1 - \nu_2) \end{aligned}$$

The last inequality holds since $\nu_1 < 1$.

D.2.4 Proof of Claim 4

We show that the equilibrium contract f we derived is also preferred to any other contract inducing default. Let us denote such a repo contract by f_{12}^d . Recall that in the claimed equilibrium the first best allocation is attained for $s \geq s^*(\nu_2)$. Hence, there is no possible gain in this region from trading a contract with a different payoff. We can thus set $f_{12}^d(s) = f(s)$ for $s \geq s^*(\nu_2)$ without loss of generality. Let $\mathcal{S}_d \subseteq [\underline{s}, s^*(\nu_2)]$ denote the

set of states in the region $s < s^*(\nu_2)$ where $\tilde{f}_{12}^d$ violates (3). Then, $\mathcal{S}_{nd} = [\underline{s}, s^*(\nu_2)] \setminus \mathcal{S}_d$ is the set of states where investor 1 does not default. Building on our argument earlier, investors do not trade contract $\tilde{f}_{12}^d$ in equilibrium if and only if

$$\begin{aligned} & \int_{\mathcal{S}_{nd}} \left(f(s) - \tilde{f}_{12}^d(s) \right) \left(u'(c_2^2(s)) - v'(c_2^1(s)) \right) dG(s) \\ & + \int_{\mathcal{S}_d} \left(f(s) - \alpha \tilde{f}_{12}^d(s) - (1 - \alpha(1 - \kappa))p_2(s) \right) \left(u'(c_2^2(s)) - v'(c_2^1(s)) \right) dG(s) \\ & + \pi \int_{\mathcal{S}_d} \left(\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s) \right) v'(c_2^1(s)) dG(s) \geq 0 \end{aligned}$$

Since the equilibrium contract is such that $f(s)$ equals the borrowing capacity without default for $s \leq s^*(\nu_2)$, we must have $\tilde{f}_{12}^d(s) \leq f(s)$ for $s \in \mathcal{S}_{nd} \subseteq [\underline{s}, s^*(\nu_2)]$. Since $u'(c_2^2(s)) - v'(c_2^1(s)) > 0$ for $s \leq s^*(\nu_2)$, the term on the first line in the above expression must be positive. It suffices then to show that the expression in the second line is also positive. Rearranging terms, this property is equivalent to:

$$\begin{aligned} \int_{\mathcal{S}_d} (f(s) - p_2(s)) \left(u'(c_2^2(s)) - v'(c_2^1(s)) \right) dG(s) & \geq \int_{\mathcal{S}_d} \alpha \left(\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s) \right) \left(u'(c_2^2(s)) - v'(c_2^1(s)) \right) dG(s) \\ & - \pi \int_{\mathcal{S}_d} \left(\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s) \right) v'(c_2^1(s)) dG(s) \end{aligned} \quad (\text{D.10})$$

Using the fact that $u'(c_2^2(s)) \leq u'(\omega)$ and $v'(c_2^1(s)) \geq v'(\omega)$ for all s and $\tilde{f}_{12}^d(s) \geq p_2(s)$ for $s \in \mathcal{S}_d$, the term on the right hand side of the above inequality is bounded above by:

$$[\alpha(u'(\omega) - v'(\omega)) - \pi v'(\omega)] \int_{\mathcal{S}_d} \left(\tilde{f}_{12}^d(s) - (1 - \kappa)p_2(s) \right) dG(s)$$

which is negative, under assumption (8). Since $f(s) = p_2(s)/(1 - \theta_1) \geq p_2(s)$ for $s \in [\underline{s}, s^*(\nu_2)]$, and thus for all $s \in \mathcal{S}_d$, the term on the left-hand side of inequality (D.10) is positive. This proves that investors do not trade any repo contract inducing default.

D.2.5 Proof of Claim 5

We now prove the uniqueness part of the result in Proposition 1 when $\theta_1 > 0$ and $s^*(\nu_2) > \underline{s}$. We must show that the pattern of trades is unique and in particular that investors do not trade spot or a repo sold by investor 2. We first analyse and dismiss two cases. In Case 1, agents would only trade spot. In Case 2, agents would only trade spot and a repo f_{21} . Then, we show that in any equilibrium, investors only trade a repo

$f_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$. This establishes uniqueness since we showed in Proposition 1 that the equilibrium is unique based on this conjecture.

Case 1: Only spot trades

The first order condition with respect to spot trades are given by equations (C.1), (C.3), (C.5) and (C.6). We find that investors do not want to trade a repo $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ if and only if

$$\mathbb{E} \left[\left(\tilde{f}_{12}(s) - p_2(s) \right) \left(u'(c_2^2(s)) - v'(c_2^1(s)) \right) \right] \geq 0 \quad (\text{D.11})$$

We have shown that it cannot be the case that $u'(c_2^2(s)) - v'(c_2^1(s)) = 0$ for all s . If there exists a positive measure subset $\mathcal{S}_0$ such that $u'(c_2^2(s)) - v'(c_2^1(s)) > 0$, then condition (D.11) is not compatible with contract $\tilde{f}_{12}$ such that $\tilde{f}_{12}(s) = \nu_2 p_2(s)$ on $\mathcal{S}_0$ and $\tilde{f}_{12}(s) = p_2(s)$ otherwise. If there exists a positive measure subset $\mathcal{S}_0$ such that $u'(c_2^2(s)) - v'(c_2^1(s)) < 0$, then condition (D.11) is not compatible with contract $\tilde{f}_{12}$ such that $\tilde{f}_{12}(s) = \frac{p_2(s)}{1-\theta_1}$ on $\mathcal{S}_0$ and $\tilde{f}_{12}(s) = p_2(s)$ otherwise since $\theta_1 > 0$. This proves that there is no equilibrium where agents only trade spot.

Case 2: Only spot trades and a repo contract $f_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$

When investors trade a repo $f_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$, the first-order conditions with respect to l^{12} and b^{21} are respectively:

$$-q_{21} + \mathbb{E} [f_{21}(s)v'(c_2^1(s))] + \nu_1 \gamma_1^1 = 0 \quad (\text{D.12})$$

$$q_{21} - \mathbb{E} [f_{21}(s)u'(c_2^2(s))] - \gamma_1^2 = 0 \quad (\text{D.13})$$

The other first-order conditions with respect to spot trades are given by equations (D.3) to (D.6). The Lagrange multipliers are given by

$$\gamma_1^1 = \frac{1}{1 - \nu_1} \mathbb{E} [(p_2(s) - f_{21}(s)) (u'(c_2^2(s)) - v'(c_2^1(s)))] \quad (\text{D.14})$$

$$\gamma_1^2 = \frac{1}{1 - \nu_1} \mathbb{E} [(\nu_1 p_2(s) - f_{21}(s)) (u'(c_2^2(s)) - v'(c_2^1(s)))] \quad (\text{D.15})$$

By the same argument used in the proof of Proposition 1, the condition that investors do not trade another contract $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ writes:

$$\mathbb{E} \left[\left(\tilde{f}_{21}(s) - f_{21}(s) \right) \left(u'(c_2^2(s)) - v'(c_2^1(s)) \right) \right] \geq 0 \quad (\text{D.16})$$

The condition that investors do not trade a contract $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ writes:

$$\mathbb{E} \left[\left(\frac{1}{1 - \nu_1} [p_2(s) - f_{21}(s) - \nu_2(\nu_1 p_2(s) - f_{21}(s))] - \tilde{f}_{12}(s) \right) (u'(c_2^2(s)) - v'(c_2^1(s))) \right] \geq 0 \quad (\text{D.17})$$

We first show that $u'(c_2^2(s)) \leq v'(c_2^1(s))$ for all s . Suppose it is not the case on a subset $\mathcal{S}_0$ with positive measure. Then, condition (D.16) can hold for all repos $\tilde{f}_{21}$ if and only if $f_{21}(s) = \nu_1 p_2(s)$ for $s \in \mathcal{S}_0$, that is $f_{21}(s)$ must be equal to the minimum repurchase price possible. But then, condition (D.17) is incompatible with repo contract $\tilde{f}_{12}$ where $\tilde{f}_{12}(s) = p_2(s)/(1 - \theta_1) > p_2(s)$ for $s \in \mathcal{S}_0$ and $\tilde{f}_{12}(s) = p_2(s)$ otherwise. This proves that $u'(c_2^2(s)) \leq v'(c_2^1(s))$ for all s .

We now show that this inequality must hold as an equality. Suppose it is not the case on a subset $\mathcal{S}_0$ with positive measure. Then, condition (D.16) imposes that $f_{21}(s) = p_2(s)/(1 - \theta_2)$ for $s \in \mathcal{S}_0$. It then follows that $\gamma_1^2 > 0$ since $f_{21}(s) \geq \nu_1 p_2(s)$ for all s and thus that $b^{21} = a_1^2$. We now establish a contradiction. Using the budget constraint (12), we have

$$c_2^1(s) = \omega + (a_1^1 - a_2^1(s))p_2(s) + a_1^2 \frac{p_2(s)}{1 - \theta_2} = \omega - (a_1^2 - a_2^2(s))p_2(s) + a_1^2 \frac{p_2(s)}{1 - \theta_2} \geq \omega$$

where the second equality follows from spot market clearing in periods 1 and 2. By assumption $v'(\omega) < u'(\omega)$. Hence, $c_2^1(s) \geq \omega$ is a contradiction with $u'(c_2^2(s)) - v'(c_2^1(s)) < 0$. The only possibility is that $u'(c_2^2(s)) = v'(c_2^1(s))$, for all s . However, we have that

$$\begin{aligned} c_2^2(s) &= \omega + (a_1^2 - a_2^2(s))p_2(s) - b^{21} f_{21}(s) \leq \omega + a_1^2 p_2(s) - b^{21} f_{21}(s) \\ &\leq \omega + \frac{a}{1 - \nu_1} p_2(s) - \frac{a}{1 - \nu_1} \nu_1 p_2(s) \end{aligned}$$

where to derive the last inequality, we used the collateral constraint (14) of both investors and the fact that $f_{21}(s) \geq \nu_1 p_2(s)$. Then, $c_2^2(s) = c_{2,*}^2$ may hold only if $s^*(0) > \underline{s}$ which contradicts our initial assumption.

Case 3: Spot trades, repo contracts $f_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ and $f_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$

We now analyze the last possibility where in addition to a repo contract f_{12} , agents trade a repo contract f_{21} . We show that this second trade does not arise in equilibrium. The first order conditions with respect to spot trades repo trades of contract f_{12} are given in the proof of Proposition 1 by equation (C.1) to (C.6). The first order conditions with

respect to repo trades of contract f_{21} are given by equations (D.12) and (D.13). Investor 1 consumption in period 2 and state s is given by:

$$c_2^1(s) = \omega + (a_1^1 - a_2^1(s))p_2(s) - b^{12}f_{12}(s) + b^{21}f_{21}(s) \quad (\text{D.18})$$

The Langrange multipliers on the collateral constraint of investor 1 and 2 are given by

$$\begin{aligned} \gamma_1^1 &= \frac{1}{1 - \nu_1} \mathbb{E}[(p_2(s) - f_{21}(s)) (u'(c_2^2(s)) - v'(c_2^1(s)))] \\ \gamma_1^2 &= \frac{1}{1 - \nu_2} \mathbb{E}[(f_{12}(s) - p_2(s)) (u'(c_2^2(s)) - v'(c_2^1(s)))] \end{aligned}$$

The conditions for investors not to trade another contract $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ or another contract $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ are given by equations (C.8) and (D.16) respectively.

We first prove that if (C.8) and (D.16) hold, then we must have $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all s . Observe that (C.8) and (D.16) hold for any $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ and any $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$, if and only if $f_{12}(s) = \nu_2 p_2(s)$ and $f_{21}(s) = p_2(s)/(1 - \theta_2)$ whenever $u'(c_2^2(s)) < v'(c_2^1(s))$ and $f_{12}(s) = p_2(s)/(1 - \theta_1)$ and $f_{21}(s) = \nu_1 p_2(s)$ whenever $u'(c_2^2(s)) > v'(c_2^1(s))$. Suppose then that there exists a subset $\mathcal{S}_0$ of positive measure such that $u'(c_2^2(s)) < v'(c_2^1(s))$ for all $s \in \mathcal{S}_0$. Given our previous observations, this implies that $\gamma_1^2 > 0$ and $\gamma_1^1 > 0$ and thus that the collateral constraints bind:

$$b^{21} = a_1^2 + \nu_2 b^{12} \quad (\text{D.19})$$

$$b^{12} = a_1^1 + \nu_1 b^{21} \quad (\text{D.20})$$

Using equation (D.18), we have that for all $s \in \mathcal{S}_0$:

$$c_2^1(s) = \omega + (a_1^1 - a)p_2(s) - b^{12}\nu_2 p_2(s) + b^{21} \frac{p_2(s)}{1 - \theta_2}$$

where we used the fact that investor 2 does not hold the asset after period 2. To show that $c_2^1(s) > c_{2,*}^1$, we only need to consider the two extreme cases $b^{12} = 0$ and $b^{21} = 0$. We then obtain

$$c_2^1(s) = \begin{cases} \omega + \frac{\nu_2}{1 - \nu_2} a p_2(s) - \frac{\nu_2 a}{1 - \nu_2} p_2(s) & \text{if } b^{21} = 0 \\ \omega - \frac{a}{1 - \nu_1} p_2(s) + \frac{a}{1 - \nu_1} \frac{p_2(s)}{1 - \theta_2} & \text{if } b^{12} = 0 \end{cases}$$

where we used equations (D.19) and (D.20) to substitute for a_1^1 , b^{12} and b^{21} . This proves that $c_2^1(s) \geq \omega > c_{2,*}^1$, which is a contradiction with $u'(c_2^2(s)) < v'(c_2^1(s))$ for $s \in \mathcal{S}_0$. Hence, we showed that $u'(c_2^2(s)) \geq v'(c_2^1(s))$ for all $s \in [\underline{s}, \bar{s}]$.

We now prove that no contract $f_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$ is traded. Suppose first that $u'(c_2^2(s)) = v'(c_2^1(s))$ so that the first-best allocation is attained in all states. However, using equation (D.18), we obtain that

$$c_2^1(\underline{s}) \geq \omega - aM \frac{p_2(\underline{s})}{1 - \theta_1}$$

so that $c_2^1(\underline{s}) = c_{2,*}^1$ is possible only if $\underline{s} > s^*(\nu_2)$. Since we assumed $s^*(\nu_2) > \underline{s}$ here, there exists a subset $\mathcal{S}_0$ of $[\underline{s}, \bar{s}]$ of positive measure where $u'(c_2^2(\cdot)) > v'(c_2^1(\cdot))$. Using our previous results, this proves that $\gamma_1^1 > 0$ and $\gamma_1^2 > 0$. Suppose now that a contract f_{21} is traded. We can also express γ_1^2 using the first order conditions with respect to the repo trades of f_{21}

$$\gamma_1^2 = \frac{1}{1 - \nu_1} \mathbb{E}[(\nu_1 p_2(s) - f_{21}(s)) (u'(c_2^2(s)) - v'(c_2^1(s)))]$$

Since either $u'(c_2^2(s)) = v'(c_2^1(s))$ holds or $u'(c_2^2(s)) > v'(c_2^1(s))$ and $f_{21}(s) = \nu_1 p_2(s)$ hold, we find that $\gamma_1^2 = 0$, a contradiction. This proves that no contract f_{21} is traded and hence that $b^{21} = l^{12} = 0$. We proved that the equilibrium is unique and as characterized in Proposition 1.

D.3 Proof of Proposition 4

The proposition states that in equilibrium, investor B acquires at least part of the asset in the spot market and sells it in a repo with 2. Using the results derived in Proposition 1, the repo contracted traded by investors B and 2 is given by:

$$f_{B2}(s) = \begin{cases} \frac{p_2(s)}{1 - \theta_B} & s \leq s_B^*(\nu_2) \\ \frac{p_2(s_B^*(\nu_2))}{1 - \theta_B} + \nu_2(p_2(s) - p_2(s_B^*(\nu_2))) & s > s_B^*(\nu_2) \end{cases}$$

with $s_B^*(\nu_2)$ determined by the following equation, analogous to (24):

$$c_{2,*}^2 = \omega + aM_{B2} \frac{p_2(s_B^*(\nu_2))}{1 - \theta_B} = \omega + aM_{B2} \frac{s_B^*(\nu_2)}{(1 - \theta_B)\delta}$$

where M_{B2} is the collateral multiplier between B and 2 specified as in (20), replacing θ_1 with θ_B . Observe in particular that $s_B^*(\nu_2) < s^*(\nu_2)$ where $s^*(\nu_2)$ is defined by (24) since $\theta_B > \theta_1$. The spot market price is determined by $p_2(s)v'(c_2^1(s)) = s$ for all s . By a similar argument as in the proof of Proposition 1 we can show that only investor 1, who has the greatest marginal utility for consumption in period 3, carries the asset into period 3. Since $v(x) = \delta x$ here, we obtain $p_2(s) = \frac{s}{\delta}$.

We need to prove that investor 1 sells at least part of his asset spot to investor B . Observe that, under the assumption $s^*(\nu_2) > \underline{s}$, it is still possible to attain the first-best allocation in every state with intermediation whenever $s_B^*(\nu_2) \leq \underline{s}$. We consider first the case where this is not possible or $s_B^*(\nu_2) > \underline{s}$. In this case, we actually show that investor 1 sells all his asset spot to investor B and does not sell any repo to investor 2. The first order conditions for the period 1 spot trades of the three types of investors and for the repo trades are:

$$-p_1 + \delta \mathbb{E}[p_2(s)] + \gamma_1^1 = 0 \quad (\text{D.21})$$

$$-p_1 + \delta \mathbb{E}[p_2(s)] + \gamma_1^B = 0 \quad (\text{D.22})$$

$$q_{B2} - \delta \mathbb{E}[f_{B2}(s)] - \gamma_1^B = 0 \quad (\text{D.23})$$

$$-p_1 + \mathbb{E}[p_2(s)u'(c_2^2(s))] + \gamma_1^2 = 0 \quad (\text{D.24})$$

$$-q_{B2} + \mathbb{E}[f_{B2}(s)u'(c_2^2(s))] + \nu_2 \gamma_1^2 = 0 \quad (\text{D.25})$$

From equation (D.21) to (D.25), we obtain:

$$\gamma_1^1 = \gamma_1^B = \frac{1}{1 - \nu_2} \mathbb{E}[(f_{B2}(s) - p_2(s))(u'(c_2^2(s)) - \delta)] > 0$$

where the sign follows from the fact that $u'(c_2^2(s)) > \delta$ for $s \in [\underline{s}, s_B^*(\nu_2)]$. Using (C.7), we obtain that investor 1 does not sell the asset in a repo to investor B if, for all $\tilde{f}_{1B} \in \mathcal{F}_{1B}(\mathbf{p}_2)$,

$$\delta \mathbb{E}[\tilde{f}_{1B}(s)] + \gamma_1^1 \geq \delta \mathbb{E}[\tilde{f}_{1B}(s)] + \nu_B \gamma_B^1$$

This inequality is actually strict since $\gamma_1^1 = \gamma_B^1 > 0$. Also, investor 1 does not wish to sell a repo $\tilde{f}_{12} \in \mathcal{F}(\mathbf{p}_2)$ to investor 2 if

$$\delta \mathbb{E}[\tilde{f}_{12}(s)] + \gamma_1^1 \geq \mathbb{E}[\tilde{f}_{12}(s)u'(c_2^2(s))] + \nu_2 \gamma_1^2$$

Using equations (D.23) and (D.25), we can replace the Lagrange multipliers to obtain:

$$\mathbb{E} [f_{B2}(s) (u'(c_2^2(s)) - \delta)] \geq \mathbb{E} [\tilde{f}_{12}(s) (u'(c_2^2(s)) - \delta)] \quad (\text{D.26})$$

Observe that for all $\tilde{f}_{12} \in \mathcal{F}(\mathbf{p}_2)$ and $s \leq s_B^*(\nu_2)$,

$$\tilde{f}_{12}(s) \leq \frac{p_2(s)}{1 - \theta_1} < \frac{p_2(s)}{1 - \theta_B} = f_{B2}(s)$$

since $\theta_B > \theta_1$. When $s > s_B^*(\nu_2)$, $u'(c_2^2(s)) = \delta$ so that inequality (D.26) holds for any $\tilde{f}_{12} \in \mathcal{F}(\mathbf{p}_2)$.

In the alternative case where $s_B^*(\nu_2) \leq \underline{s}$, that is the first-best allocation can be attained in every state, the pattern of trades described above is still an equilibrium. However, as discussed in the main text after Proposition 1, other patterns of trade can also implement this allocation.

D.4 Chain of Repos with $\nu_2 > 0$.

We prove that an equilibrium with a chain of repos, similar to that of Proposition 5, also exists when ν_2 is positive but not too large.

Proposition 7. *There exists $\bar{\delta}_B(\nu_2) > \underline{\delta}_B(\nu_2) > \delta$ such that the equilibrium features intermediation with a chain of repos if and only if $\delta_B \in [\underline{\delta}_B(\nu_2), \bar{\delta}_B(\nu_2)]$ and*

$$\nu_B \geq \frac{1}{1 - \nu_2(1 - \theta_B)} \left[\frac{(1 - \nu_2)(1 - \theta_B)}{1 - \theta_1} + \nu_2 \theta_B \right] \quad (\text{D.27})$$

Investors 1 sells all the asset in a repo f_{1B} to B with

$$f_{1B}(s) = \frac{s}{1 - \theta_1} \quad \forall s \in [\underline{s}, \bar{s}] \quad (\text{D.28})$$

Investor B sells part of the asset in a repo f_{B2} to 2 with

$$f_{B2}(s) = \begin{cases} \frac{p_2(s)}{1 - \theta_B} & \text{if } s < s_{B2}^*(\nu_2) \\ \frac{p_2(s_{B2}^*)}{1 - \theta_B} + \nu_2(p_2(s) - p_2(s_{B2}^*)) & \text{if } s \geq s_{B2}^*(\nu_2) \end{cases}$$

for some $s_{B2}^(\nu_2) \in [\underline{s}, \bar{s}]$ and the remaining part in a spot sale to investor 1.*

Proof. The first order conditions are those we derived in the proof of Proposition (5) except for that of investor 2 with respect to repo trades of contract f_{B2} which becomes

$$-q_{B2} + \mathbb{E}[f_{B2}(s)u'(c_2^2(s))] + \nu_2\gamma_1^2 = 0 \quad (\text{D.29})$$

With $\nu_2 > 0$, the equivalent of condition (C.40) becomes

$$a = (1 - \nu_B)b^{1B} + (1 - \nu_2)b^{B2}$$

Let us denote $b := b^{B2}(1 - \nu_2)$ which can be described as the amount of asset used in the transaction between investors B and 2. This defines again a range $[0, \nu_B a]$ of possible values for b . We showed in the poof of Proposition 5 that $p_2(s) = s/\delta$ since investor 1 holds the asset into period 3. Let us now define $s_{B2}^*(b)$ implicitly as:

$$u' \left(\omega + \frac{bs_{B2}^*(b)}{(1 - \nu_2)\delta} \left[\frac{1}{1 - \theta_B} - \nu_2 \right] \right) = \delta_B,$$

Using the results from Proposition 1, we know that investors B and 2 will trade repo contract $f_{B2}(b)$ defined implicitly as a function of the amount b used in the transaction:

$$f_{B2}(b, \nu_2, s) = \begin{cases} \frac{p_2(s)}{1 - \theta_B} & \text{if } s < s_{B2}^*(b) \\ \frac{s_{B2}^*(b)}{\delta(1 - \theta_B)} + \frac{\nu_2(s - s_{B2}^*(b))}{\delta} & \text{if } s \geq s_{B2}^*(b) \end{cases} \quad (\text{D.30})$$

We may also define investor 2 consumption as a function of b

$$c_2^2(b, s) = \omega + \frac{b}{1 - \nu_2} (f_{B2}(b, \nu_2, s) - \nu_2 p_2(s))$$

where $f(b, \nu_2, s)$ is defined in (D.30). Since, investors 1 and B are risk-neutral, the same argument used in Proposition 5 applies here to derive the repo contract sold by investor 1 to investor B :

$$f_{1B}(s) = \frac{p_2(s)}{1 - \theta_1} \quad \forall s \in [\underline{s}, \bar{s}] \quad (\text{D.31})$$

We now pin down the equilibrium value of b . From equations (D.21) and (C.32) to (C.34), we obtain:

$$\gamma_1^B = \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \mathbb{E}[p_2(s)]$$

From equations (C.35) and (D.29), we obtain:

$$\gamma_1^B = \frac{1 - (1 - \theta_B)\nu_2}{(1 - \nu_2)} \int_{\underline{s}}^{s_{B2}^*(b)} [u'(c_2^2(b, s)) - \delta_B] \frac{p_2(s)}{1 - \theta_B} dG(s)$$

Equalizing these two expressions for γ_1^B derived above, we obtain:

$$\frac{1 - (1 - \theta_B)\nu_2}{(1 - \theta_B)(1 - \nu_2)} \int_{\underline{s}}^{s_{B2}^*(b)} [u'(c_2^2(b, s)) - \delta_B] p_2(s) dG(s) = \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \mathbb{E}[p_2(s)] \quad (\text{D.32})$$

Since the mapping

$$b \rightarrow \int_{\underline{s}}^{s_{B2}^*(b)} [u'(c_2^2(b, s)) - \delta_B] p_2(s) dG(s)$$

is strictly decreasing in b , the equality above pins down a unique value for b . Our equilibrium conjecture can only be true if $b \in [0, \nu_B a]$. Plugging the lower bound for b into expression (D.32) yields the following condition:

$$\frac{1 - (1 - \theta_B)\nu_2}{1 - \nu_2} \frac{u'(\omega) - \delta_B}{1 - \theta_B} \geq \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \quad (\text{D.33})$$

This inequality is equivalent to

$$\delta_B \leq \bar{\delta}_B(\nu_2) := \frac{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} \delta + \frac{[1 - (1 - \theta_B)\nu_2] u'(\omega)}{(1 - \theta_B)(1 - \nu_2)}}{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} + \frac{[1 - (1 - \theta_B)\nu_2] u'(\omega)}{(1 - \theta_B)(1 - \nu_2)}}$$

Observe in particular that $\bar{\delta}_B(\nu_2) \leq u(\omega)$. Plugging now the upper bound for b into expression (D.32), we obtain:

$$\frac{1 - (1 - \theta_B)\nu_2}{1 - \nu_2} \int_{\underline{s}}^{s_{B2}^*(\nu_B a)} \frac{u'(c_2^2(\nu_B a, s)) - \delta_B}{1 - \theta_B} s dF(s) \leq \frac{(\delta_B - \delta)\theta_1}{(1 - \nu_B)(1 - \theta_1)} \quad (\text{D.34})$$

which is equivalent to

$$\delta \geq \underline{\delta}_B(\nu_2) = \frac{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} \delta + \frac{1 - (1 - \theta_B)\nu_2}{1 - \nu_2} \int_{\underline{s}}^{s_{B2}^*(\nu_B a)} \frac{u'(c_2^2(\nu_B a, s))}{1 - \theta_B} s dF(s)}{\frac{\theta_1}{(1 - \nu_B)(1 - \theta_1)} + \frac{1 - (1 - \theta_B)\nu_2}{1 - \nu_2} \int_{\underline{s}}^{s_{B2}^*(\nu_B a)} \frac{1}{1 - \theta_B} s dF(s)}$$

with $\underline{\delta}_B(\nu_2) \geq \delta$. Since $\delta_B \leq \bar{\delta}_B(\nu_2)$ and $\delta_B \geq \underline{\delta}_B(\nu_2)$ are respectively equivalent to (D.33)

and (D.34), it is easy to see from these expressions that $\underline{\delta}_B(\nu_2) \leq \bar{\delta}_B(\nu_2)$.

We are left to show that there is no contract in $\mathcal{F}_{12}(\mathbf{p}_2)$ that investor 1 desires to sell to investor 2. Letting $\tilde{f}_{12}$ be a generic contract in $\mathcal{F}_{12}(\mathbf{p}_2)$, the first condition writes:

$$\delta \mathbb{E}[\tilde{f}_{12}(s)] + \gamma_1^1 \geq \mathbb{E} \left[\tilde{f}_{12}(s) u'(c_2^2(s)) \right] + \nu_2 \gamma_1^2$$

which, substituting for γ_1^1 and γ_1^2 thanks to equations (C.38) and (D.29), becomes:

$$\mathbb{E} \left[\left(\tilde{f}_{12}(s) - p_2(s) \right) \left(u'(c_2^2(s)) - \delta \right) \right] \leq \mathbb{E} \left[\left(f_{B2}(s) - p_2(s) \right) \left(u'(c_2^2(s)) - \delta_B \right) \right]$$

This inequality holds for all $\tilde{f}_{12} \in \mathcal{F}_{12}(\mathbf{p}_2)$ if it holds for the repo contract f_{12} with payoff $f_{12}(s) = \frac{p_2(s)}{1-\theta_1}$. Plugging this expression in the inequality above and rearranging terms we obtain:

$$\begin{aligned} 0 &\leq \mathbb{E} \left[\left(f_{B2}(s) - f_{12}(s) \right) \left(u'(c_2^2(s)) - \delta_B \right) \right] - \mathbb{E} \left[\left(f_{12}(s) - p_2(s) \right) \left(\delta_B - \delta \right) \right] \\ \Leftrightarrow \quad 0 &\leq \left[\frac{1}{1-\theta_B} - \frac{1}{1-\theta_1} \right] \int_{\underline{s}}^{s_{B2}^*(b^{B2})} \left[u'(c_2^2(b^{B2}, s)) - \delta_B \right] p_2(s) dG(s) - \frac{\theta_1(\delta_B - \delta)}{1-\theta_1} \mathbb{E}[p_2(s)] \\ \Leftrightarrow \quad 0 &\leq \frac{1-\nu_2}{1-(1-\theta_B)\nu_2} \left[1 - \frac{1-\theta_B}{1-\theta_1} \right] \gamma_1^B - \gamma_1^B (1-\nu_B) \\ \Leftrightarrow \quad 0 &\leq \nu_B - \frac{1}{1-\nu_2(1-\theta_B)} \left[\frac{(1-\nu_2)(1-\theta_B)}{1-\theta_1} \right] \end{aligned}$$

where the last inequality is Condition (D.27). This concludes the proof. $\square$