# Aggregate productivity and the allocation of resources over the business cycle

Online Appendix

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## Appendix A. Derivations

This appendix shows how to derive the decompositions of the sectoral and economy-wide productivity within the general framework presented in Section 3. To find the equations of the simplified framework, given in Section 2, set $\theta_s = 1$, $\alpha_s = 0$, and $\gamma_s = \beta$ in the expressions for sectoral TFP. For simplicity, the time subscripts have been dropped when this creates no ambiguity.

### A.1 Sectoral production function

This section gives the expressions of the sectoral production function and sectoral TFP. We can show that, when firms use a Cobb Douglas production function with identical factor elasticities, there exists a real-valued function $\psi$ such that the sectoral production function can be written

$$Y_s = \Psi(\mathbf{A}_s, \boldsymbol{\tau}_s)\; F(K_s, L_s).$$

To derive the sectoral production function, use $\frac{p_i}{P_s} = (\frac{Y_i}{Y_s/n_s})^{\theta_s - 1}$ to rewrite equations (16) and (17) as

$$\begin{aligned}
\left(\frac{Y_s}{n_s}\right)^{1-\theta_s} \alpha_s A_i^{\theta_s} K_i^{\alpha_s\theta_s - 1} L_i^{\beta_s\theta_s} &= \mu_s^K (1 + \tau_i^K), \\
\left(\frac{Y_s}{n_s}\right)^{1-\theta_s} \beta_s A_i^{\theta_s} K_i^{\alpha_s\theta_s} L_i^{\beta_s\theta_s - 1} &= \mu_s^L (1 + \tau_i^L).
\end{aligned}$$

Solving for $K_i$ and $L_i$ and using the resource constraints to substitute for the Lagrange multipliers, we can then write the allocation rules (i.e., the link between firm-level inputs and aggregate inputs):

$$\begin{aligned}
\frac{K_i}{K_s} &= \frac{g^K(A_i, \tau_i)}{\sum\limits_{i \in \mathcal{N}_s} g^K(A_i, \tau_i)}, \\
\frac{L_i}{L_s} &= \frac{g^L(A_i, \tau_i)}{\sum\limits_{i \in \mathcal{N}_s} g^L(A_i, \tau_i)},
\end{aligned}$$

where $\tau_i \equiv (\tau_i^K, \tau_i^L)$ and the expressions for the $g$ functions are given below. To simplify notations, the dependence of $g$ on $s$ is implied. Substituting for $K_i$ and $L_i$ in the expression for sectoral output gives the sectoral production function

$$Y_s = \frac{1}{n_s^{\frac{1-\theta_s}{\theta_s}}} \left( \sum_{i \in \mathcal{N}_s} A_i^{\theta_s} \left( \frac{g^K(A_i,\tau_i)}{\sum\limits_{i \in \mathcal{N}_s} g^K(A_i,\tau_i)} K_s \right)^{\alpha_s \theta_s} \left( \frac{g^L(A_i,\tau_i)}{\sum\limits_{i \in \mathcal{N}_s} g^L(A_i,\tau_i)} L_s \right)^{\beta_s \theta_s} \right)^{1/\theta_s}.$$

The sectoral production function can then be written $Y_s = \Psi(\mathbf{A}_s, \boldsymbol{\tau}_s) K_s^{\alpha} L_s^{\beta}$ and sectoral TFP is then:

$$\mathrm{TFP}_s \equiv \Psi(\mathbf{A}_s, \boldsymbol{\tau}_s) = \frac{1}{n_s^{\frac{1-\theta_s}{\theta_s}}} \frac{\left( \sum\limits_{i \in \mathcal{N}_s} g^Y(A_i,\tau_i) \right)^{1/\theta_s}}{\left( \sum\limits_{i \in \mathcal{N}_s} g^K(A_i,\tau_i) \right)^{\alpha_s} \left( \sum\limits_{i \in \mathcal{N}_s} g^L(A_i,\tau_i) \right)^{\beta_s}}, \tag{A.1}$$

where the $g$ functions are given by:

$$\begin{aligned}
g^K(A_i,\tau_i) &= A_i^{\frac{\theta_s}{1-\gamma_s\theta_s}} (1+\tau_i^K)^{-\frac{1-\beta_s\theta_s}{1-\gamma_s\theta_s}} (1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma_s\theta_s}}, \\
g^L(A_i,\tau_i) &= A_i^{\frac{\theta_s}{1-\gamma_s\theta_s}} (1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma_s\theta_s}} (1+\tau_i^L)^{-\frac{1-\alpha_s\theta_s}{1-\gamma_s\theta_s}}, \\
g^Y(A_i,\tau_i) &= A_i^{\frac{\theta_s}{1-\gamma_s\theta_s}} (1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma_s\theta_s}} (1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma_s\theta_s}}.
\end{aligned}$$

The sectoral production function is separable in aggregate TFP. This property is key to deriving an exact decomposition of the technical and allocative efficiency components, and it facilitates the measure of the contribution of entry and exit. Both the identical factor elasticities and the Cobb-Douglas specification are crucial to obtain separability. With a more general CES production function, we would need to further assume that the labor and capital distortions are equal or, equivalently, that firms only face output distortions.

## A.2 Taylor approximation of sectoral TFP

In this section, I show how to write sectoral TFP as a function of the cross-sectional moments of firm-level productivity and distortions. Assume that firm-level productivity is given by $A_i = \bar{A}(1+\hat{A}_i)$ and distortions by $1+\tau_i^K = (1+\bar{\tau}^K)(1+\hat{\tau}_i^K)$ and $1+\tau_i^L = (1+\bar{\tau}^L)(1+\hat{\tau}_i^L)$, where $\hat{A}_i$, $\hat{\tau}_i^K$, and $\hat{\tau}_i^L$ are jointly distributed with mean zero, with variance $\sigma_A^2$, $\sigma_K^2$ and $\sigma_L^2$ and with covariance $\sigma_{AK}$, $\sigma_{AL}$ and $\sigma_{KL}$. Taking a first-order Taylor approximation of equation (A.1) around the cross-sectional mean yields

$$\mathrm{TFP}_s \simeq \frac{1}{n_s^{\frac{1-\theta_s}{\theta_s}}} \frac{\left( \sum\limits_i \bar{g}^Y \right)^{\frac{1}{\theta_s}}}{\left( \sum_i \bar{g}^K \right)^{\alpha_s} \left( \sum_i \bar{g}^L \right)^{\beta_s}} \left( 1 + \frac{1}{\theta_s} \frac{\sum\limits_i (g_i^Y - \bar{g}^Y)}{\sum\limits_i \bar{g}^Y} - \alpha_s \frac{\sum\limits_i (g_i^K - \bar{g}^K)}{\sum_i \bar{g}^K} - \beta_s \frac{\sum\limits_i (g_i^L - \bar{g}^L)}{\sum_i \bar{g}^L} \right),$$

where I have used the more compact notations $g_i^X \equiv g^X(A_i, \tau_i)$ and $\overline{g}^X \equiv g^X(\bar{A}, \overline{\tau})$. Using the law of large numbers, sectoral TFP can then be approximated by

$$\mathrm{TFP}_s \simeq n_s^{1-\gamma_s} \frac{\left(\overline{g}^Y\right)^{\frac{1}{\theta_s}}}{\left(\overline{g}^K\right)^{\alpha_s}\left(\overline{g}^L\right)^{\beta_s}} \left(1 + \frac{1}{\theta_s}\frac{E(g_i^Y - \overline{g}^Y)}{\overline{g}^Y} - \alpha_s \frac{E(g_i^K - \overline{g}^K)}{\overline{g}^K} - \beta_s \frac{E(g_i^L - \overline{g}^L)}{\overline{g}^L}\right),$$

where $E$ denotes the expectation operator. Each expectation term can be approximated using a second order Taylor approximation:

$$\begin{aligned} g_i^X - \overline{g}^X \quad = \quad & \frac{\partial \overline{g}^X}{\partial A_i}(A_i - \bar{A}) + \frac{\partial \overline{g}^X}{\partial \tau_i^K}(\tau_i^K - \overline{\tau}^K) + \frac{\partial \overline{g}^X}{\partial \tau_i^L}(\tau_i^L - \overline{\tau}^L) \\ & + \frac{1}{2}\left(\frac{\partial^2 \overline{g}^X}{\partial A_i^2}(A_i - \bar{A})^2 + \frac{\partial^2 \overline{g}^X}{\partial {\tau_i^K}^2}(\tau_i^K - \overline{\tau}^K)^2 + \frac{\partial^2 \overline{g}^X}{\partial {\tau_i^L}^2}(\tau_i^L - \overline{\tau}^L)^2\right) \\ & + \frac{\partial^2 \overline{g}^X}{\partial A_i \partial \tau_i^K}(\tau_i^K - \overline{\tau}^K)(A_i - \bar{A}) + \frac{\partial^2 \overline{g}^X}{\partial A_i \partial \tau_i^L}(\tau_i^L - \overline{\tau}^L)(A_i - \bar{A}) + \frac{\partial^2 \overline{g}^X}{\partial \tau_i^K \partial \tau_i^L}(\tau_i^K - \overline{\tau}^K)(\tau_i^L - \overline{\tau}^L), \end{aligned}$$

where $\frac{\partial \overline{g}^X}{\partial A_i} \equiv \frac{\partial g^X}{\partial A_i}(\bar{A}, \overline{\tau})$, for $X = Y, K, L$.

We then obtain

$$\mathrm{TFP}_s \simeq n_s^{1-\gamma_s} \bar{A} \left(1 + \eta_A \sigma_A^2 - \eta_K \sigma_K^2 - \eta_L \sigma_L^2 - \eta_{KL}\sigma_{KL}\right),$$

with

$$\eta_A = \frac{1}{2}\left(\frac{\theta_s}{1 - \gamma_s \theta_s} - 1\right); \eta_K = \frac{1}{2}\frac{\alpha_s(1 - \beta_s \theta_s)}{1 - \gamma_s \theta_s}; \eta_L = \frac{1}{2}\frac{\beta_s(1 - \alpha_s \theta_s)}{1 - \gamma_s \theta_s}; \eta_{KL} = \frac{\alpha_s \beta_s \theta_s}{1 - \gamma_s \theta_s}.$$

### A.3 Exact decomposition of sectoral TFP

Let us now derive the exact decomposition of sectoral TFP. Note that an exact decomposition is possible only when the aggregate production function is separable, as is the case for the sectoral production function. These are the equations used for the empirical application.

We first separate the change in TFP into an intensive and an extensive margin component. The decomposition is given by

$$\frac{\mathrm{TFP}_{st}}{\mathrm{TFP}_{st-1}} = I_{IN}\ I_{EX},$$

where the intensive margin reads:

$$I_{IN} \equiv \left(\frac{\sum\limits_{i \in \mathcal{C}_{st}} g^Y(A_{it}, \tau_{it})}{\sum\limits_{i \in \mathcal{C}_{st}} g^Y(A_{it-1}, \tau_{it-1})}\right)^{\frac{1}{\theta_s}} \left(\frac{\sum\limits_{i \in \mathcal{C}_{st}} g^K(A_{it}, \tau_{it})}{\sum\limits_{i \in \mathcal{C}_{st}} g^K(A_{it-1}, \tau_{it-1})}\right)^{-\alpha_s} \left(\frac{\sum\limits_{i \in \mathcal{C}_{st}} g^L(A_{it}, \tau_{it})}{\sum\limits_{i \in \mathcal{C}_{st}} g^L(A_{it-1}, \tau_{it-1})}\right)^{-\beta_s},$$

and the extensive margin component is:

$$I_{EX} \equiv \left(\frac{n_{st}}{n_{st-1}}\right)^{\frac{\theta_s-1}{\theta_s}} \left(\frac{\sum\limits_{i\in\mathcal{N}_{st}} g^Y(A_{it},\tau_{it})}{\sum\limits_{i\in\mathcal{N}_{st-1}} g^Y(A_{it-1},\tau_{it-1})} \frac{\sum\limits_{i\in\mathcal{C}_{st}} g^Y(A_{it-1},\tau_{it-1})}{\sum\limits_{i\in\mathcal{C}_{st}} g^Y(A_{it},\tau_{it})}\right)^{\frac{1}{\theta_s}}$$

$$\left(\frac{\sum\limits_{i\in\mathcal{N}_{st}} g^K(A_{it},\tau_{it})}{\sum\limits_{i\in\mathcal{N}_{st-1}} g^K(A_{it-1},\tau_{it-1})} \frac{\sum\limits_{i\in\mathcal{C}_{st}} g^K(A_{it-1},\tau_{it-1})}{\sum\limits_{i\in\mathcal{C}_{st}} g^K(A_{it},\tau_{it})}\right)^{-\alpha_s} \left(\frac{\sum\limits_{i\in\mathcal{N}_{st}} g^L(A_{it},\tau_{it})}{\sum\limits_{i\in\mathcal{N}_{st-1}} g^L(A_{it-1},\tau_{it-1})} \frac{\sum\limits_{i\in\mathcal{C}_{st}} g^L(A_{it-1},\tau_{it-1})}{\sum\limits_{i\in\mathcal{C}_{st}} g^L(A_{it},\tau_{it})}\right)^{-\beta_s},$$

where $\mathcal{N}_{st}$ is the set of active firms, and $\mathcal{C}_{st}$ is the set of continuing firms, and the expressions of the $g$ functions are given in section A.1. To compute the decomposition of the intensive margin component into the contribution of firm-level efficiency $I_{TE}$ and that of allocative efficiency $I_{AE}$,

$$I_{IN} = I_{TE}\ I_{AE},$$

I use a decomposition similar to the statistical indexes used to separate price and volume changes. To avoid any asymmetry induced by the functional form of the indexes, I measure the firm-level and allocative efficiency component with a Fisher-like index, i.e., the geometric mean of the Laspeyres-like and Paasche-like indexes: $I_{TE} \equiv \sqrt{I_{TE0}}\sqrt{I_{TE1}}$, and $I_{AE} \equiv \sqrt{I_{AE0}}\sqrt{I_{AE1}}$; the Laspeyres-like and Paasche-like indexes are given by:

$$I_{TE0} = (I^Y_{TE0})^{1/\theta_s}(I^K_{TE0})^{-\alpha_s}(I^L_{TE0})^{-\beta_s}, \tag{A.2}$$

$$I_{TE1} = (1/I^Y_{TE1})^{-1/\theta_s}(1/I^K_{TE1})^{\alpha_s}(1/I^L_{TE1})^{\beta_s}, \tag{A.3}$$

where the components of the Laspeyres index are given by:

$$I^Y_{TE0} \equiv \frac{\sum\limits_{i\in\mathcal{C}_{st}} g^Y(A_{it},\tau_{it-1})}{\sum\limits_{i\in\mathcal{C}_{st}} g^Y(A_{it-1},\tau_{it-1})},$$

and the components of the Paasche index are given by:

$$1/I^Y_{TE1} \equiv \frac{\sum\limits_{i\in\mathcal{C}_{st}} g^Y(A_{it-1},\tau_{it})}{\sum\limits_{i\in\mathcal{C}_{st}} g^Y(A_{it},\tau_{it})}.$$

The other components $I^K_{TE0}, I^L_{TE0}, I^K_{TE1}, I^L_{TE1}$, and the respective components of the allocative efficiency index are similarly defined.

Noticing that $g^Y(A_{it},\tau_{it})/\left(\sum_i g^Y(A_{it},\tau_{it})\right) = p_{it}Y_{it}/\left(\sum p_{it}Y_{it}\right)$, we can rewrite $I^Y_{TE0}$ and $I^Y_{TE1}$ as:

$$I^Y_{TE0} = \sum_{i\in C_{st}} \left(\frac{A_{it}}{A_{it-1}}\right)^{\frac{\theta_s}{1-\gamma_s\theta_s}} \frac{p_{it-1}Y_{it-1}}{\sum\limits_{i\in C_{st}} p_{it-1}Y_{it-1}},$$

$$1/I^Y_{TE1} = \sum_{i\in C_{st}} \left(\frac{A_{it-1}}{A_{it}}\right)^{\frac{\theta_s}{1-\gamma_s\theta_s}} \frac{p_{it}Y_{it}}{\sum\limits_{i\in C_{st}} p_{it}Y_{it}}.$$

The components $I_{TE0}^K, I_{TE0}^L, I_{TE1}^K, I_{TE1}^L$, can be rewritten similarly as a function of capital and labor shares. The respective components of the allocative efficiency index are computed in a similar way. Finally, the extensive margin component can be rewritten:

$$I_{EX} = \left(\frac{n_{st}}{n_{st-1}}\right)^{1-\alpha_s-\beta_s} (I_{EX}^Y)^{\frac{1}{\theta_s}} \; (I_{EX}^K)^{-\alpha_s} (I_{EX}^L)^{-\beta_s}, \tag{A.4}$$

with

$$I_{EX}^Y = \frac{\frac{1}{n_{st}} \sum\limits_{i \in \mathcal{N}_{st}} p_{it} Y_{it}}{\frac{1}{c_{st}} \sum\limits_{i \in c_{st}} p_{it} Y_{it}} \frac{\frac{1}{c_{st}} \sum\limits_{i \in C_{st}} p_{it-1} Y_{it-1}}{\frac{1}{n_{st-1}} \sum\limits_{i \in \mathcal{N}_{st-1}} p_{it-1} Y_{it-1}},$$

and the other components $I_{EX}^K$ and $I_{EX}^L$ similarly defined.

Aggregate productivity growth can hence be broken down into three components: the change in technical efficiency, the change in allocative efficiency, and the change induced by entry and exit

$$\Delta \ln \mathrm{TFP}_{st} = \ln I_{TE} + \ln I_{AE} + \ln I_{EX}.$$

This gives equation (21) where $\Delta TE_s \equiv \ln I_{TE}$, $\Delta AE_s \equiv \ln I_{AE}$ and $\Delta EX_s \equiv \ln I_{EX}$.

## A.4 Approximate decomposition of sectoral TFP

For simplicity, the main text reports approximations of $\Delta TE$, $\Delta AE$, $\Delta EX$ (equations (22) to (24)) rather than the exact expressions (given in Appendix A.3). In this section, I show how to derive the approximations presented in the main text. The approximation of $\Delta TE$ and $\Delta AE$ is the discrete-time approximation of the continuous-time decomposition applied to continuing firms. Taking the derivative of (the log of) TFP, equation (A.1), with respect to firm-level productivity:

$$\Delta TE_s \simeq \frac{1}{\theta_s} \sum_i \frac{\partial g_i^Y / \partial A_i}{\sum\limits_i g_i^Y} dA_i - \alpha_s \sum_i \frac{\partial g_i^K / \partial dA_i}{\sum_i g_i^K} dA_i - \beta_s \sum_i \frac{\partial g_i^L / \partial A_i}{\sum_i g_i^L} dA_i,$$

where I have used the more compact notation $g_i^X \equiv g^X(A_i, \tau_i)$. Using the expressions for $g^Y$ (given in section A.1) and noticing that $g_i^Y / \left(\sum_i g_i^Y\right) = p_i Y_i / \left(\sum_i p_i Y_i\right)$, and proceeding similarly for the $g_i^K$ and $g_i^L$ terms, yields equations (22) and (23). The approximation of $\Delta AE_s$ is obtained, similarly, by taking the derivative of $\ln \mathrm{TFP}$ with respect to distortions.

If average output is close to the average output of continuing firms, the components of the extensive margin index (equation (A.4)) can be approximated as follows

$$
\begin{aligned}
\ln I_{EX}^{Y} \quad &\simeq \quad \frac{\frac{1}{n_{st}}\left[\sum\limits_{i\in\mathcal{E}_{st}} p_{it}Y_{it} + \sum\limits_{i\in\mathcal{C}_{st}} p_{it}Y_{it}\right] - \frac{1}{c_{st}t}\sum\limits_{i\in\mathcal{C}_{st}} p_{it}Y_{it}}{\frac{1}{c_{st}}\sum\limits_{i\in\mathcal{C}_{st}} p_{it}Y_{it}} \\
&\quad - \frac{\frac{1}{n_{t-1}}\left[\sum\limits_{i\in\mathcal{X}_{st}} p_{it-1}Y_{it-1} + \sum\limits_{i\in\mathcal{C}_{st}} p_{it-1}Y_{it-1}\right] - \frac{1}{c_{st}}\sum\limits_{i\in C_{st}} p_{it-1}Y_{it-1}}{\frac{1}{c_{st}}\sum\limits_{i\in C_{st}} p_{it-1}Y_{it-1}} \\
&\simeq \quad \frac{\frac{e_{st}}{n_{st}}\left[\frac{1}{e_{st}}\sum\limits_{i\in\mathcal{E}_{st}} p_{it}Y_{it} - \frac{1}{c_{st}}\sum\limits_{i\in C_{st}} p_{it}Y_{it}\right]}{\frac{1}{c_{st}}\sum\limits_{i\in\mathcal{C}_{st}} p_{it}Y_{it}} - \frac{\frac{x_{st}}{n_{st-1}}\left[\frac{1}{x_{st}}\sum\limits_{i\in\mathcal{X}_{st}} p_{it-1}Y_{it-1} - \frac{1}{c_{st}}\sum\limits_{i\in\mathcal{C}_{st}} p_{it-1}Y_{it-1}\right]}{\frac{1}{c_{st}}\sum\limits_{i\in\mathcal{C}_{st}} p_{it-1}Y_{it-1}},
\end{aligned}
$$

where $\mathcal{E}_{st}$ is the set of entering firms, and $\mathcal{X}_{st}$ is the set of exiting firms. Computing the same approximation for $\ln I_{EX}^{K}$ and $\ln I_{EX}^{L}$ yields equation (24).

## A.5 Aggregate elasticities and between-sector decomposition

This section describes how to derive the elasticities of the economy-wide production function and the decomposition of the economy-wide TFP.

I derive the economy-wide production function by aggregating the sectoral production functions. Since factor elasticities are heterogeneous across sectors, there is no closed form solution for the economy-wide production function. The economy-wide production function can be characterized by:

$$
F(K, L, \mathbf{TFP}, \boldsymbol{\omega}) = \left(\sum_{s=1}^{S}\left(\mathrm{TFP}_s K_s^{\alpha_s} L_s^{\beta_s}\right)^{\rho}\right)^{\frac{1}{\rho}},
$$

together with the allocation rules:

$$
\begin{aligned}
K_s &= k_s(\mathbf{TFP}, \boldsymbol{\omega}, K, L), \\
L_s &= l_s(\mathbf{TFP}, \boldsymbol{\omega}, K, L),
\end{aligned}
$$

where $\mathbf{TFP} = \{\mathrm{TFP}_1, ..., \mathrm{TFP}_S\}$ is the vector of sectoral level productivity, and $\boldsymbol{\omega}_t = \{\omega_1, ..., \omega_S\}$ is the vector of sectoral level distortions. The allocation rule is defined implicitly by the first order conditions for sectoral allocation, together with the resource constraints (which relate the Lagrange multipliers to aggregate capital and labor). I derive the aggregate elasticities and the between-sector decompositions in continuous time and then use the average of the initial and end period weights to derive the discrete-time approximation.

The derivation of the aggregate elasticities and that of the between-sector decomposition is somewhat tedious. For simplicity, I derive the decomposition under the assumption that firms produce using only labor. Aggregate output is $Y = (\sum_s Y_s^{\rho})^{1/\rho}$, with $Y_s = TFP_s L_s^{\beta_s}$ and $L_s = L_s(TFP_s, \omega_s^L, \lambda^L)$.[1] The resource constraint is $\sum_s L_s(TFP_s, \omega_s^L, \lambda^L) = L$.

Let us first derive the elasticity of the aggregate production function with respect to labor $\varepsilon^L \equiv (dY/dL)(L/Y)$.

Using $dY/dL = \sum_s (dY/dY_s)(dY_s/dL_s)(dL_s/dL)$ and $dY/dY_s = P_s/P$, the elasticity can be computed as follows:

$$\varepsilon^L = \sum_s \frac{P_s Y_s}{PY} \frac{dY_s}{dL_s} \frac{L_s}{Y_s} \frac{dL_s}{dL} \frac{L}{L_s}.$$

By combining the input demand function and the aggregate resource constraint, we have:

$$\begin{aligned} \frac{dL_s}{dL}\frac{L}{L_s} &= \frac{dL_s}{d\lambda^L}\frac{\lambda^L}{L_s} \frac{d\lambda^L}{dL}\frac{L}{\lambda^L}, \\ &= \frac{dL_s}{d\lambda^L}\frac{\lambda^L}{L_s} \left( \sum_s \frac{dL_s}{d\lambda^L}\frac{\lambda^L}{L_s}\frac{L_s}{L} \right)^{-1}, \end{aligned}$$

where the last line is obtained from the aggregate resource constraint, $dL = \sum_s \frac{dL_s}{d\lambda^L} d\lambda^L$, which gives $\frac{dL}{L} = \sum_s \frac{dL_s}{d\lambda^L}\frac{\lambda^L}{L_s}\frac{d\lambda^L}{\lambda^L}\frac{L_s}{L}$. The aggregate elasticity is therefore:

$$\varepsilon^L = \frac{\sum_s \frac{\beta_s}{1-\beta_s\rho}\frac{P_s Y_s}{PY}}{\sum_s \frac{1}{1-\beta_s\rho}\frac{L_s}{L}}.$$

We proceed similarly to derive the between-sector decomposition. By definition, the change in aggregate TFP is equal to the change in aggregate output that is not due to a change in aggregate inputs. In this simplified framework where labor is the only input, this gives (in continuous time):

$$d\ln \text{TFP} = d\ln Y - \varepsilon^L d\ln L.$$

Alternatively, we can compute the change in TFP as:

$$d\ln \text{TFP} = \underbrace{\sum_s \frac{dY}{d\text{TFP}_s}\frac{\text{TFP}_s}{Y}\frac{d\text{TFP}_s}{\text{TFP}_s}}_{\Delta E_{\text{within}}} + \underbrace{\sum_s \frac{dY}{d\omega_s^L}\frac{\omega_s^L}{Y}\frac{d\omega_s^L}{\omega_s^L}}_{\Delta E_{\text{between}}}.$$

The first component captures the effect of changes in sectoral productivity, holding fixed sectoral distortions. The second component captures changes in between-sector allocative efficiency.

Using the expressions for aggregate output, labor demand and the resource constraint, the elasticity of aggregate output with respect to $\text{TFP}_s$ can be computed as

$$\frac{dY}{d\text{TFP}_s}\frac{\text{TFP}_s}{Y} = \frac{P_s Y_s}{PY} + \frac{\beta_s\rho}{1-\beta_s\rho}\frac{P_s Y_s}{PY} - \sum_j \frac{\beta_j}{1-\beta_j\rho}\frac{P_j Y_j}{PY}\frac{d\lambda^L}{d\text{TFP}_s}\frac{\text{TFP}_s}{\lambda^L}.$$

[1] Sectoral input is given by $L_s = Y^{\frac{1-\rho}{1-\beta_s\rho}} \text{TFP}_s^{\frac{\rho}{1-\beta_s\rho}} (\frac{\lambda^L}{\beta_s}(1+\omega_s^L))^{-\frac{1}{1-\beta_s\rho}}$.

Using the implicit function theorem on the aggregate resource constraint, we have $-\sum_s \frac{1}{1-\beta_s} L_s \frac{d\lambda^L}{\lambda^L} + \sum_s \frac{1}{1-\beta_s} L_s \frac{dTFP_s}{TFP_s} = 0$, and therefore:

$$\frac{d\lambda^L}{d\text{TFP}_s} \frac{\text{TFP}_s}{\lambda^L} = \frac{\frac{\rho}{1-\beta_s\rho} L_s}{\sum_j \frac{1}{1-\beta_j\rho} L_j}.$$

The contribution of within-sector productivity changes can then be written:

$$\begin{aligned}
\Delta E_{\text{within}} &= \sum_s \frac{1}{1-\beta_s\rho} \frac{P_s Y_s}{PY} \frac{d\text{TFP}_s}{\text{TFP}_s} - \left( \frac{\sum_s \frac{\beta_s}{1-\beta_s\rho} \frac{P_s Y_s}{PY}}{\sum_s \frac{1}{1-\beta_s\rho} \frac{L_s}{L}} \right) \frac{\rho}{1-\beta_s\rho} \frac{L_s}{L} \frac{d\text{TFP}_s}{\text{TFP}_s}, \\
&= \sum_s \frac{1}{1-\beta_s\rho} \left( \frac{P_s Y_s}{PY} - \rho \varepsilon^L \frac{L_s}{L} \right) \frac{d\text{TFP}_s}{\text{TFP}_s}.
\end{aligned}$$

The contribution of changes in between-sector allocative efficiency is computed similarly. Applying this method to the general case where firms produce using labor and capital inputs, and approximating the continuous-time decomposition using the average of the initial and end period as weights, gives equations (3.4) and (27), as well as the aggregate elasticities:

$$\varepsilon^K = \frac{a_Y c_L - \rho a_L (a_Y + b_Y)}{c_L c_K - \rho (c_L b_K + a_L c_K)},$$

$$\varepsilon^L = \frac{b_Y c_K - \rho b_K (a_Y + b_Y)}{c_L c_K - \rho (c_L b_K + a_L c_K)},$$

where $a_X = \sum_s \frac{X_s}{X} \frac{\alpha_s}{1-\gamma_s\rho}$ , $b_X = \sum_s \frac{X_s}{X} \frac{\beta_s}{1-\gamma_s\rho}$ and $c_X = \sum_s \frac{X_s}{X} \frac{1}{1-\gamma_s\rho}$.

If $\gamma_s = \gamma$, the elasticities becomes:

$$\varepsilon^K = \frac{\alpha^Y - \rho \alpha^L (\alpha^Y + \beta^Y)}{1 - \rho(\alpha^L + \beta^K)},$$

$$\varepsilon^L = \frac{\beta^Y - \rho \beta^K (\alpha^Y + \beta^Y)}{1 - \rho(\alpha^L + \beta^K)},$$

where $\alpha^X = \sum_s \frac{X_s}{X} \alpha_s$ and $\beta^X = \sum_s \frac{X_s}{X} \beta_s$. Note that if the factor elasticities are identical, the elasticities are simply equal to the sectoral level elasticities $\varepsilon^K = \alpha$ and $\varepsilon^L = \beta$.

## A.6 Alternative decompositions

This section provides the derivation details for the reallocation and shift-share decompositions.

### A.6.1 The reallocation decomposition

To derive equation (8), I use the discrete-time approximation of the continuous-time decomposition. Recall that, within the simplified framework, aggregate productivity is $\ln \mathrm{TFP} = \ln Y - \beta \ln L$, and aggregate output and labor are $Y = \sum_i Y_i$ and $L = \sum_i L_i$. The continuous-time decomposition is:

$$
\begin{aligned}
d \ln \mathrm{TFP} &= \sum_i \frac{Y_i}{Y} \frac{dY_i}{Y_i} - \beta \sum_i \frac{L_i}{L} \frac{dL_i}{L_i}, \\
&= \sum_i \frac{Y_i}{Y} \left( \frac{dA_i}{A_i} + \beta \frac{dL_i}{L_i} \right) - \beta \sum_i \frac{L_i}{L} \frac{dL_i}{L_i}, \\
&= \sum_i \frac{Y_i}{Y} \frac{dA_i}{A_i} + \beta \sum_i \frac{Y_i}{Y} \left( 1 - \frac{L_i/Y_i}{L/Y} \right) \frac{dL_i}{L_i}.
\end{aligned}
$$

Using equation (1), we can write $\frac{L_i/Y_i}{L/Y} = \frac{1+\overline{\tau}^L}{1+\tau_i}$ where $\overline{\tau}^L \equiv \sum_i \tau_i \frac{L_i}{L}$. Substituting in the equation above, and writing the discrete-time version of the equation, gives equation (8).

To go from equation (8) to (9), use the firm's first-order condition to replace $dL_i/L_i$ in equation (8) by

$$
\frac{dL_i}{L_i} = \frac{1}{1-\beta} \left( \frac{dA_i}{A_i} - \frac{d\tau_i}{1+\tau_i} - \frac{d\mu}{\mu} \right).
$$

The term in $d\mu/\mu$ cancels out.

For the empirical application, I use an exact decomposition that is based on the index number approach. After expressing TFP as a function of input shares,

$$
\mathrm{TFP}_{st} = n_{st}^{\frac{\theta_s - 1}{\theta_s}} \left( \sum_{i \in \mathcal{N}_{st}} A_{it}^{\theta_s} \left( \frac{K_{it}}{K_{st}} \right)^{\alpha_s \theta_s} \left( \frac{L_{it}}{L_{st}} \right)^{\beta_s \theta_s} \right)^{1/\theta_s},
$$

we can derive the following decomposition:

$$
\Delta \ln \mathrm{TFP}_{st} = \ln I_A + \ln I_{RE} + \ln I_{EX},
$$

where $I_{EX}$ is the same as in my benchmark decomposition; the within-firm productivity component is $I_A \equiv \sqrt{I_{A0}}\sqrt{I_{A1}}$; and the reallocation component is $I_{RE} \equiv \sqrt{I_{RE0}}\sqrt{I_{RE1}}$.

The Laspeyres-like and Paasche-like indexes used to compute $I_A$ are:

$$
I_{A0} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{A_{it}}{A_{it-1}} \right)^{\theta_s} \frac{p_{it-1} Y_{it-1}}{\sum_{i \in \mathcal{C}_{st}} p_{it-1} Y_{it-1}} \right)^{1/\theta_s},
$$

$$
1/I_{A1} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{A_{it-1}}{A_{it}} \right)^{\theta_s} \frac{p_{it} Y_{it}}{\sum_{i \in \mathcal{C}_{st}} p_{it} Y_{it}} \right)^{1/\theta_s}.
$$

The components of the reallocation index $I_{RE}$ are given by:

$$I_{RE0} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{K_{it} / \sum\limits_{i \in \mathcal{C}_{st}} K_{it}}{K_{it-1} / \sum\limits_{i \in \mathcal{C}_{st}} K_{it-1}} \right)^{\alpha_s \theta_s} \left( \frac{L_{it} / \sum\limits_{i \in \mathcal{C}_{st}} L_{it}}{L_{it-1} / \sum\limits_{i \in \mathcal{C}_{st}} L_{it-1}} \right)^{\beta_s \theta_s} \frac{p_{it-1} Y_{it-1}}{\sum\limits_{i \in \mathcal{C}_{st}} p_{it-1} Y_{it-1}} \right)^{1/\theta_s},$$

$$1/I_{RE1} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{K_{it-1} / \sum\limits_{i \in \mathcal{C}_{st}} K_{it-1}}{K_{it} / \sum\limits_{i \in \mathcal{C}_{st}} K_{it}} \right)^{\alpha_s \theta_s} \left( \frac{L_{it-1} / \sum\limits_{i \in \mathcal{C}_{st}} L_{it-1}}{L_{it} / \sum\limits_{i \in \mathcal{C}_{st}} L_{it}} \right)^{\beta_s \theta_s} \frac{p_{it} Y_{it}}{\sum\limits_{i \in C_{st}} p_{it} Y_{it}} \right)^{1/\theta_s}.$$

The between-sector decomposition is computed using the discrete-time approximation of the continuous-time decomposition, using the average of the initial and end periods as weights:

$$\begin{aligned} d \ln \mathrm{TFP} &= d \ln Y - \varepsilon_L d \ln L - \varepsilon_K d \ln K \\ &= \sum_s \frac{P_s Y_s}{PY} \frac{dY_s}{dY_s} - \varepsilon_L \sum_s \frac{L_s}{L} \frac{dL_s}{L_s} - \varepsilon_K \sum_s \frac{K_s}{K} \frac{dK_s}{K_s}. \end{aligned}$$

Using $d \ln Y_s = d \ln \mathrm{TFP}_s + \alpha_s d \ln dK_s + \beta_s d \ln dL_s$ and rearranging, we obtain:

$$d \ln \mathrm{TFP} = \sum_s \frac{P_s Y_s}{PY} \frac{d\mathrm{TFP}_s}{\mathrm{TFP}_s} + \sum_s \left( \beta_s \frac{P_s Y_s}{PY} - \varepsilon_L \frac{L_s}{L} \right) \frac{dL_s}{L_s} + \sum_s \left( \alpha_s \frac{P_s Y_s}{PY} - \varepsilon_K \frac{K_s}{K} \right) \frac{dK_s}{K_s}.$$

The discrete-time approximation is then obtained by using the average of the time $t-1$ and time $t$ of the output shares, input shares, and elasticities.

### A.6.2 The shift-share decomposition

I show here how to obtain the expression of the shift-share decomposition when inputs are efficiently allocated. When resources are efficiently allocated, $\tau_i = \tau$, and the firm's output share is

$$s_i = \frac{Y_i}{Y} = \frac{A_i^{\frac{1}{1-\beta}}}{\sum_i A_i^{\frac{1}{1-\beta}}}.$$

The change in the firm's share can then be approximated by:

$$\Delta s_i \simeq \frac{1}{1-\beta} s_i \left( \frac{\Delta A_i}{A_i} - \sum_j s_j \frac{\Delta A_j}{A_j} \right).$$

Replacing in equation (13) gives

$$\Delta \ln \bar{A} \simeq \sum_i s_i \, \Delta \ln A_i + \frac{1}{1-\beta} \sum_i s_i \ln A_i \left( \Delta \ln A_i - \sum_j s_j \, \Delta \ln A_j \right).$$

Use $\ln \bar{A} = \sum_i s_i \ln A_i$ and rearrange to obtain the equation of Section 2.4.2

# Appendix B - Extensions and alternative specifications

In this appendix, I describe how to incorporate intermediate inputs, firm-specific demand shocks, overhead labor and firm-specific factor elasticities. Whereas the first two dimensions can be accommodated under the benchmark framework presented in the main text, overhead labor and firm-specific factor elasticities lead to a different measure of firm-level distortions and allocative efficiency.

## B.1 From the gross output production function to the value added production function

I discuss here the assumptions required for the use of a value added production function and derive the link between the wedge, measured using the value added production function, and the underlying distortions faced by the firms.

In the absence of distortions, the existence of a value added production function is guaranteed when the gross output production function is separable in intermediate and primary inputs (see Bruno (1978)). The presence of distortions complicates the analysis. In particular, the value added production function becomes a function of the distortions faced by the firm. To avoid this complication, I follow the traditional approach used in the macro literature (e.g., Rotemberg and Woodford, 1993) and assume that firms use intermediate inputs in fixed proportion to output.

To illustrate the implications of this assumption, consider, for simplicity, the framework of Section 2 augmented to include intermediate goods. Assume that firms produce using the following gross output production function:

$$Q_i = \min\left[\frac{A_i L_i^{\beta}}{1-c}, \frac{M_i}{c}\right],$$

where $Q_i$ denotes gross output, and $M_i$ denotes the intermediate input used by firm $i$. Firms face distortions on gross output $t_i^Q$ and on labor $t_i^L$. The firm maximizes

$$(1-t_i^Q)Q_i - w(1+t_i^L)L_i - M_i,$$

where $w$ is the real wage. Give the form of the production function, the firm uses intermediate inputs in fixed proportion to output, $M_i = cQ_i$, and the program of the firm can then be rewritten

$$\max_{L_i}(1-t_i^Q-c)Q_i - w(1+t_i^L)L_i.$$

The first order condition is:

$$\frac{1-t_i^Q-c}{1-c}\beta\frac{Y_i}{L_i} = w(1+t_i^L),$$

where the value added is given by $Y_i \equiv (1-c)Q_i = A_i L_i^{\beta}$. Hence, we can rewrite the problem of the firm as having the value added production function $Y_i = A_i L_i^{\beta}$ and facing distortions $1+\tau_i^L \equiv (1-c)(1+t_i^L)/(1-t_i^Q-c)$ in the choice of its primary inputs. This shows that the distortions, computed using the value added production function, are also function of the distortions on gross output, as well as of the intermediate input share $c$.

### B.2 Firm-specific demand shocks

For simplicity, the general framework of Section 3.1 abstracts from firm-specific demand shocks. I show here that the aggregation and the decomposition of productivity are not modified when allowing for firm-specific demand shocks. The interpretation of firm-level productivity, however, is different. In the presence of firm-specific demand shocks, the productivity of the firm becomes a function of both demand and technological shocks.

To show this clearly, let us incorporate firm-specific demand shocks $b_i$ into the general framework of Section 3.1. The sectoral output is then

$$Y_{st} = n_{st}^{\frac{\theta_s-1}{\theta_s}} \left(\sum b_{it} Y_{it}^{\theta_s}\right)^{1/\theta_s}.$$

Denote by $z_i$ the technological productivity of the firm. The firm-level production function is

$$Y_{it} = z_{it} K_{it}^{\alpha_s} L_{it}^{\beta_s}.$$

The first order conditions (equations (16) and (17)) are unchanged[2]

$$\begin{aligned} \alpha_s \frac{p_{it} Y_{it}}{P_{st} K_{it}} &= \mu_{st}^K (1+\tau_{it}^K), \\ \beta_s \frac{p_{it} Y_{it}}{P_{st} L_{it}} &= \mu_{st}^L (1+\tau_{it}^L). \end{aligned}$$

Hence, the measure of distortions is unaffected by the presence of firm-specific demand shocks. The firms' marginal productivity is sufficient to capture the efficiency of input allocation even in the presence of firm-specific demand shocks.

The sectoral TFP is

$$\mathrm{TFP}_{st} = n_{st}^{\frac{\theta_s-1}{\theta_s}} \left(\sum b_{it} z_{it}^{\theta} \left(\frac{K_{it}}{K_{st}}\right)^{\alpha_s\theta_s} \left(\frac{L_{it}}{L_{st}}\right)^{\beta_s\theta_s}\right)^{1/\theta_s},$$

[2] The relative price is now

$$\frac{p_{it}}{P_{st}} = b_i \left(n_{st} \frac{Y_{it}}{Y_{st}}\right)^{\theta_s - 1}.$$

where

$$\frac{K_{it}}{K_{st}} = \frac{g^K(z_{it}b_{it}^{1/\theta_s}, \tau_{it})}{\sum\limits_{i\in\mathcal{N}_s} g^K(z_{it}b_i^{1/\theta_s}, \tau_{it})},$$
$$\frac{L_{it}}{L_{st}} = \frac{g^L(z_{it}b_{it}^{1/\theta_s}, \tau_{it})}{\sum\limits_{i\in\mathcal{N}_s} g^L(z_{it}b_i^{1/\theta_s}, \tau_{it})},$$

and the $g$ functions are given in Appendix A.1. From the expression of TFP given above, it appears clearly that the contribution of each firm to aggregate productivity depends not only on the technological productivity $z_i$ but also on the firm-specific demand shock $b_i$. This, however, does not affect the estimation of the decomposition. In fact, the measured firm-level productivity $A_i$ turns out to be exactly equal to the combination of the technology and demand factors that enters the expression of aggregate TFP.

$$A_{it} \equiv \frac{\left(\frac{Y_{st}}{n_{st}}\right)^{\frac{\theta_s-1}{\theta_s}}\left(\frac{p_{it}Y_{it}}{P_{st}}\right)^{\frac{1}{\theta_s}}}{K_{it}^{\alpha_s}L_{it}^{\beta_s}} = b_i^{\frac{1}{\theta_s}} z_i.$$

The decomposition of aggregate TFP is therefore unchanged.

## B.3 Overhead labor

I now consider how the presence of overhead labor modifies the measure of firm-level distortions and the decomposition of aggregate productivity changes. Let us assume that the production function is

$$Y_{it} = A_{it}K_{it}^{\alpha_s}(L_{it} - f_i)^{\beta_s},$$

where $f_i$ is the amount of overhead labor that is needed for production each period. With this production function, the first order conditions are:

$$\alpha_s \frac{p_{it}Y_{it}}{P_{st}K_{it}} = \mu_{st}^K(1 + \tau_{it}^K),$$
$$\beta_s \frac{p_{it}Y_{it}}{P_{st}(L_{it} - f_i)} = \mu_{st}^L(1 + \tau_{it}^L)$$

Whereas the first order condition for capital is identical to the benchmark case (cf. equation (16)), the condition for labor is now a function of production labor (i.e., total labor minus overhead labor). The sectoral production function with overhead labor is: $Y_{st} = \text{TFP}_{st}K_{st}^{\alpha}(L_{st} - f_s)^{\beta_s}$, with $f_s = \sum f_i$, and sectoral productivity is given, as in the benchmark case, by:

$$\text{TFP}_{st} \equiv \Psi(\mathbf{A}_{st}, \boldsymbol{\tau}_{st}) = \frac{1}{n_{st}^{\frac{1-\theta_s}{\theta_s}}} \frac{\left(\sum\limits_{i\in\mathcal{N}_{st}} g^Y(A_{it}, \tau_{it})\right)^{1/\theta_s}}{\left(\sum\limits_{i\in\mathcal{N}_{st}} g^K(A_{it}, \tau_{it})\right)^{\alpha_s}\left(\sum\limits_{i\in\mathcal{N}_{st}} g^L(A_{it}, \tau_{it})\right)^{\beta_s}},$$

where the $g$ functions are given in Appendix A.1. As in the benchmark framework, we can then compute the change in TFP as a function of the weighted averages of productivity and distortions. The only difference is that the relevant labor weight is the share of production labor

$$\frac{L_{it} - f_i}{L_{st} - f_s} = \frac{g^L(A_{it}, \tau_{it})}{\sum_{i \in \mathcal{N}_{st}} g^L(A_{it}, \tau_{it})}.$$

The decomposition is then identical to the benchmark case after replacing the share of overall labor by the share of production labor.

## B.4 Firm-specific factor elasticities

Let us now consider the case where firms have heterogenous factor elasticities within sectors. That is, the production function is given by:

$$Y_{it} = A_{it} K_{it}^{\alpha_i} L_{it}^{\beta_i}.$$

The first order conditions are then:

$$\begin{aligned} \alpha_i \frac{p_{it} Y_{it}}{P_{st} K_{it}} &= \mu_{st}^K (1 + \tau_{it}^K), \\ \beta_i \frac{p_{it} Y_{it}}{P_{st} L_{it}} &= \mu_{st}^L (1 + \tau_{it}^L). \end{aligned}$$

As factor elasticities are heterogenous, the sectoral production function can no longer be solved in closed form. Similarly to the economy-wide production function, the sectoral production function is now characterized by:

$$F(K_{st}, L_{st}, \mathbf{A}_{st}, \boldsymbol{\tau}_{st}) = \left( \sum_{i \in \mathcal{N}_{st}} \left( A_{it}^{\theta_s} K_{it}^{\alpha_i \theta_s} L_{it}^{\beta_i \theta_s} \right) \right)^{\frac{1}{\theta_s}},$$

together with the allocation rules:

$$\begin{aligned} K_{it} &= k_i(\mathbf{A}_{st}, \boldsymbol{\tau}_{st}, K_{st}, L_{st}), \\ L_{it} &= l_i(\mathbf{A}_{st}, \boldsymbol{\tau}_{st}, K_{st}, L_{st}), \end{aligned}$$

where $\mathbf{A}_{st} = \{A_i, i \in \mathcal{N}_{st}\}$ is the vector of firm-level productivity and $\boldsymbol{\tau}_{st} = \{\tau_i, i \in \mathcal{N}_{st}\}$ the vector of firm-level distortions. The allocation rules are characterized implicitly by both the first order conditions and the aggregate sectoral constraints.[3]

As for the economy-wide production function, sectoral TFP cannot be expressed as a scalar. This implies that the contribution of entry and exit will be more complicated to derive. I focus here on the decomposition of the productivity of continuing firms. The contribution of firm-level productivity can be computed as

[3] See Appendix A.5 for details on the derivation.

$$\Delta TE_{st} \simeq \frac{1}{1-\gamma_i\theta_s}\sum_{i\in\mathcal{C}_{st}}\frac{\Delta A_{it}}{A_{it-1}}\left[\frac{p_{it-1}Y_{it-1}}{\sum\limits_{i\in\mathcal{C}_{st}} p_{it-1}Y_{it-1}} - \varepsilon_s^K\theta_s\frac{K_{it-1}}{\sum\limits_{i\in\mathcal{C}_{st}} K_{it-1}} - \varepsilon_s^L\theta_s\frac{L_{it-1}}{\sum\limits_{i\in\mathcal{C}_{st}} L_{it-1}}\right].$$

The change in allocative efficiency is a combination of weighted averages of the firm-level changes in distortions

$$\begin{aligned}\Delta AE_{st} \simeq\ & \frac{1}{1-\gamma_i\theta_s}\sum_{i\in\mathcal{C}_{st}}\frac{\Delta\tau_{it}^K}{1+\tau_{it-1}^K}\left[\varepsilon_s^K(1-\beta_i\theta_s)\frac{K_{it}}{\sum\limits_{i\in\mathcal{C}_{st}} K_{it}} + \varepsilon_s^L\alpha_i\theta_s\frac{L_{it}}{\sum\limits_{i\in\mathcal{C}_{st}} L_{it}} - \alpha_i\frac{p_{it}Y_{it}}{\sum\limits_{i\in\mathcal{C}_{st}} p_{it}Y_{it}}\right]\\ &+\frac{1}{1-\gamma_i\theta_s}\sum_{i\in\mathcal{C}_{st}}\frac{\Delta\tau_{it}^L}{1+\tau_{it-1}^L}\left[\varepsilon_s^K\beta_i\theta_s\frac{K_{it}}{\sum\limits_{i\in\mathcal{C}_{st}} K_{it}} + \varepsilon_s^L(1-\alpha_i\theta_s)\frac{L_{it}}{\sum\limits_{i\in\mathcal{C}_{st}} L_{it}} - \beta_i\frac{p_{it}Y_{it}}{\sum\limits_{i\in\mathcal{C}_{st}} p_{it}Y_{it}}\right].\end{aligned}$$

The sectoral elasticities are given by:

$$\varepsilon_s^K = \frac{\alpha_s^Y(1-\theta_s\alpha_s^L) - \theta_s\beta_s^Y\alpha^L}{1-\theta_s(\alpha_s^L+\beta_s^K)},\ \text{and}\ \varepsilon_s^L = \frac{\beta_s^Y(1-\theta_s\beta_s^K) - \theta_s\alpha_s^Y\beta_s^K}{1-\theta_s(\alpha_s^L+\beta_s^K)}.$$

where $\alpha_s^X = \sum_s(X_i/X_s)\alpha_i$ and $\beta_s^X = \sum_s(X_i/X_s)\beta_i$.

# Appendix C - Application to French data

In this appendix, I provide more detail on the dataset and report all the equations used in the estimation.

## C.1 Data

I use firm-level data collected annually by the French tax administration and combined with the responses to annual business surveys in the INSEE unified system for business statistics (SUSE). I exclude non-market sectors and the finance sector and consider firms with at least one employee that declare under the "normal" regime (SUSE-BRN) from 1989 to 2007. This tax regime is mandatory for firms with sales above 763 000 euros (230 000 in the service sector), but is also widely chosen by firms that are below the threshold: in 2003, 46% of the firms were below the sales threshold in the manufacturing sector and 32% in the services sector. Moreover, 70% of firms in the sample had less than 20 employees in the manufacturing sector and less than 9 employees in the service sector.

**Trimming procedure**

I keep the observations with non zero and positive output, employment and capital and then make adjustments to the dataset for (a) the introduction of a new industry classification in 1993 (b) spurious entry and exit flows, and (c) outliers.

(a) New industry classification

In 1993, a new industry classification system is introduced: the "Nomenclature d'Activités Franąise" replaced the "Nomenclature d'Activités et de Produits". To construct a conversion table, I use firms present in the dataset both before and after 1993 and build a contingency table linking each pre-1993 4-digit sector to a post-1993 2-digit sector (NES36). I then use the pairs with the highest frequencies to assign all firms to a post-1993 2-digit sector.

(b) Spurious entry and exit flows

To avoid spurious entry and exit flows caused by missing observations, I use a linear interpolation to recover the employment, fixed assets, and value added of firms that temporarily disappear from the dataset.[4] Because of those temporary exits, the entry rate is likely to be overestimated in the first years of the sample and the exit rate in the last years of the sample. I therefore remove the first two years and the last year of the sample and undertake the analysis over the period 1991-2006.[5] I also exclude firms that appear or disappear from the dataset with employment, capital, or valued added above the 99th percentile to limit spurious entry and exit flows caused by takeover or restructuring.

(c) Outliers

Finally, I also trim the dataset to exclude from the sample firms whose productivity changes are in the bottom and top 2 percentiles.

Figure D.1 and D.2 compares real output growth in the manufacturing and service sectors to the corresponding national accounts statistics (INSEE). The figure indicates that the dataset captures quite well the cyclical dynamics of aggregate output, although the movements are exacerbated. Table D.1 gives some descriptive statistics on each sector.

**Description of variables**

Nominal output: gross value added, less operating subsidies, plus taxes.

Sectoral and aggregate price indexes: deflators published by the national accounts (INSEE).

[4] The alternative, which consists in excluding firms that temporarily disappear from the dataset, leads to overestimating the entry and exit rates because those firms tend to survive longer than firms with no missing observations. Moreover, as the number of missing observations varies over time, excluding temporary exits may also affect the cyclicality of entry and exit rates.

[5] Since about 75% of the firms that temporarily disappear from the database are absent only one year, dropping the first and the last observations of the sample considerably reduces the scope for spurious entry and exit flows. The first two years are removed because entry can only be identified from the second year of the sample.

Labour: average number of employees over the year.

Capital: real capital stock, computed using the perpetual inventory method from the previous period undepreciated stock and the period's real investment in tangible and intangible assets. I use a linear sector-specific depreciation rate, based on Sylvain (2003) estimates of equipments' life span for the manufacturing sector, and use a depreciation rate of 6% for the service sector. Investment is computed as the change in the firm's fixed asset, divided by the 2-digit (NES36) sectoral investment deflator published by the national accounts (INSEE, base 1995 and 2000). For negative investment values, the change in the book value does not reflect the actual change in the capital stock and must be adjusted for depreciation. More specifically, real capital $k_t$ is computed as follows: $K_{it} = (1-\delta)K_{it-1} + I_t$, with $I_{it} = inv/p_{inv}$ if $inv \geq 0$ and $I_{it} = inv * (1-\delta)^{age}$ if $inv < 0$. Where $inv$ is the book value of investment, $\delta$ the sector specific depreciation rate, $p_{inv}$ the sectoral investment deflator, and $age$ is the age of the firm. The first year capital stock is computed using the book value of assets over the sectoral NES36 deflator for fixed assets.

Sectoral labor share: computed, from the firm-level data aggregated at the 2-digit (NES36) level, as one minus the ratio of the sectoral gross operating surplus over value added. The labor share includes social contribution costs.

Debt-to-asset ratio: the debt is computed as the sum of bonds and bank loans (does not include accounts payable), and assets are computed as the book value of tangible and intangible fixed assets.

Interest rate: computed as the ratio of interest payments over debt.

## C.2 The decompositions used in the empirical application

I report here all the equations used in the empirical application. All the derivation details are in Appendix A.

### C.2.1 Benchmark decomposition

Within sectors

The decomposition of sectoral productivity is given by:

$$\Delta \ln \mathrm{TFP}_{st} = \ln I_{TE} + \ln I_{AE} + \ln I_{EX},$$

with

$$I_{EX} = \left(\frac{n_{st}}{n_{st-1}}\right)^{1-\alpha_s-\beta_s} (I_{EX}^{Y})^{\frac{1}{\theta_s}} \, (I_{EX}^{K})^{-\alpha_s} (I_{EX}^{L})^{-\beta_s},$$

$$I_{TE} \equiv \sqrt{I_{TE0}}\sqrt{I_{TE1}},$$

$$I_{AE} \equiv \sqrt{I_{AE0}}\sqrt{I_{AE1}}.$$

The Laspeyres-like and Paasche-like indexes for firm-level efficiency are

$$I_{TE0} = (I^Y_{TE0})^{1/\theta}(I^K_{TE0})^{-\alpha_s}(I^L_{TE0})^{-\beta_s},$$

$$I_{TE1} = (1/I^Y_{TE1})^{-1/\theta}(1/I^K_{TE1})^{\alpha_s}(1/I^L_{TE1})^{\beta_s}.$$

The indexes $I^Y_{EX}$, $I^Y_{TE0}$ and $I^Y_{TE1}$ are given by:

$$I^Y_{EX} = \frac{\frac{1}{n_{st}}\sum\limits_{i\in\mathcal{N}_{st}} p_{it}Y_{it}}{\frac{1}{c_{st}}\sum\limits_{i\in c_{st}} p_{it}Y_{it}} \frac{\frac{1}{c_{st}}\sum\limits_{i\in C_{st}} p_{it-1}Y_{it-1}}{\frac{1}{n_{st-1}}\sum\limits_{i\in\mathcal{N}_{st-1}} p_{it-1}Y_{it-1}}$$

$$I^Y_{TE0} = \sum_{i\in C_{st}} \left(\frac{A_{it}}{A_{it-1}}\right)^{\frac{\theta_s}{1-\gamma_s\theta_s}} \frac{p_{it-1}Y_{it-1}}{\sum\limits_{i\in C_{st}} p_{it-1}Y_{it-1}}.$$

$$1/I^Y_{TE1} = \sum_{i\in C_{st}} \left(\frac{A_{it-1}}{A_{it}}\right)^{\frac{\theta_s}{1-\gamma_s\theta_s}} \frac{p_{it}Y_{it}}{\sum\limits_{i\in C_{st}} p_{it}Y_{it}}.$$

The other indexes are similarly defined.

Between sectors

The decomposition of aggregate productivity between sector is

$$\Delta \ln \mathrm{TFP}_t \simeq \Delta E_{\mathrm{within}} + \Delta E_{\mathrm{between}},$$

with

$$\Delta E_{\mathrm{within}} = \sum_{s=1}^{S} \frac{1}{1-\gamma_s\rho}\left(v^Y_{st,t-1} - \rho\varepsilon^K v^K_{st,t-1} - \rho\varepsilon^L v^L_{st,t-1}\right)\frac{\Delta\mathrm{TFP}_{st}}{\mathrm{TFP}_{st-1}},$$

and

$$\begin{aligned}\Delta E_{\mathrm{between}} \quad = \quad & \sum_{s=1}^{S} \frac{1}{1-\gamma_s\rho}\left(\varepsilon^K(1-\beta_s\rho)v^K_{st,t-1} + \varepsilon^L\alpha_s\rho v^L_{st,t-1} - \alpha_s v^Y_{st,t-1}\right)\frac{\Delta\omega^K_{st}}{1+\omega^K_{st-1}} \\ & + \sum_{s=1}^{S} \frac{1}{1-\gamma_s\rho}\left(\varepsilon^K\beta_s\rho v^K_{st,t-1} + \varepsilon^L(1-\alpha_s\rho)v^L_{st,t-1} - \beta_s v^Y_{st,t-1}\right)\frac{\Delta\omega^L_{st}}{1+\omega^L_{st-1}}.\end{aligned}$$

where $v^Y_{st,t-1} \equiv \left(\frac{P_{st-1}Y_{st-1}}{P_{t-1}Y_{t-1}} + \frac{P_{st}Y_{st}}{P_tY_t}\right)/2$, and $v^K_{st,t-1}$ and $v^K_{st,t-1}$ are similarly defined. The elasticities $\varepsilon^K$ and $\varepsilon^L$ are computed as the average of time $t-1$ and $t$ elasticities.

### C.2.2 Alternative decompositions

The shift-share decomposition

I compare the results of my decomposition to Foster et al. (2001)'s decomposition (denoted Method 2 in their paper)[6]. Within sectors, the change in the output-weighted average of firm-level productivity, $\ln \bar{A}_{st} \equiv \sum_{i \in \mathcal{N}_{st}} v_{it} \ln A_{it}$, is decomposed as follows:

$$
\begin{aligned}
\Delta \ln \bar{A}_{st} \;=\; & \sum_{i \in \mathcal{C}_{st}} v_{it-1,t}\, \Delta \ln A_{it} + \sum_{i \in \mathcal{C}_{st}} \Delta v_{it}\, (\ln A_{it-1,t} - \ln \bar{A}_{st-1,t}) \\
& + \sum_{i \in \mathcal{E}_{st}} v_{it}\, (\ln A_{it} - \ln \bar{A}_{st-1,t}) - \sum_{i \in \mathcal{X}_{st}} v_{it-1}\, (\ln A_{it-1} - \ln \bar{A}_{st-1,t}),
\end{aligned}
$$

where $v_{it-1,t} \equiv \left(\frac{p_{it-1}Y_{it-1}}{P_{st-1}Y_{st-1}} + \frac{p_{it}Y_{it}}{P_{st}Y_{st}}\right)/2$, $\ln A_{it-1,t} = (\ln A_{it-1} + \ln A_{it})/2$, and $\ln \bar{A}_{st-1,t} = (\ln \bar{A}_{st-1} + \ln \bar{A}_{st})/2$.

The same decomposition (without the entry and exit component) applies between sectors.

$$
\Delta \ln \bar{A}_t = \sum_s v_{st-1,t}\, \Delta \ln \bar{A}_{st} + \sum_s \Delta v_{st}\, (\ln \bar{A}_{st-1,t} - \ln \bar{A}_{t-1,t}),
$$

where $\ln \bar{A}_t \equiv \sum_s v_{st} \ln A_{st}$, and $v_{st,t-1}$ and $\ln \bar{A}_{t-1,t}$ are defined similarly to $v_{it,t-1}$, and $\ln \bar{A}_{st-1,t}$.

The reallocation decomposition

The reallocation decomposition is given by:

$$
\Delta \ln \mathrm{TFP}_{st} = \ln I_A + \ln I_{RE} + \ln I_{EX},
$$

where $I_{EX}$ is the same as in my benchmark decomposition, $I_A \equiv \sqrt{I_{A0}}\sqrt{I_{A1}}$ and $I_{RE} \equiv \sqrt{I_{RE0}}\sqrt{I_{RE1}}$. The Laspeyres-like and Paasche-like indexes used to compute $I_A$ are:

$$
I_{A0} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{A_{it}}{A_{it-1}} \right)^{\theta_s} \frac{p_{it-1}Y_{it-1}}{\sum\limits_{i \in \mathcal{C}_{st}} p_{it-1}Y_{it-1}} \right)^{1/\theta_s},
$$

$$
1/I_{A1} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{A_{it-1}}{A_{it}} \right)^{\theta_s} \frac{p_{it}Y_{it}}{\sum\limits_{i \in \mathcal{C}_{st}} p_{it}Y_{it}} \right)^{1/\theta_s}.
$$

The components of the reallocation index $I_{RE}$ are given by:

$$
I_{RE0} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{K_{it}/\sum\limits_{i \in \mathcal{C}_{st}} K_{it}}{K_{it-1}/\sum\limits_{i \in \mathcal{C}_{st}} K_{it-1}} \right)^{\alpha_s\theta_s} \left( \frac{L_{it}/\sum\limits_{i \in \mathcal{C}_{st}} L_{it}}{L_{it-1}/\sum\limits_{i \in \mathcal{C}_{st}} L_{it-1}} \right)^{\beta_s\theta_s} \frac{p_{it-1}Y_{it-1}}{\sum\limits_{i \in \mathcal{C}_{st}} p_{it-1}Y_{it-1}} \right)^{1/\theta_s},
$$

[6] Method 2 is closely related to the decomposition of Griliches and Regev (1995). Similar to the method used for my decomposition, Method 2 does not introduce any exogenous asymmetry between the within and between components.

$$1/I_{RE1} = \left( \sum_{i \in \mathcal{C}_{st}} \left( \frac{K_{it-1}/\sum_{i\in\mathcal{C}_{st}} K_{it-1}}{K_{it}/\sum_{i\in\mathcal{C}_{st}} K_{it}} \right)^{\alpha_s \theta_s} \left( \frac{L_{it-1}/\sum_{i\in\mathcal{C}_{st}} L_{it-1}}{L_{it}/\sum_{i\in\mathcal{C}_{st}} L_{it}} \right)^{\beta_s \theta_s} \frac{p_{it}Y_{it}}{\sum_{i\in C_{st}} p_{it}Y_{it}} \right)^{1/\theta_s}.$$

The between-sector decomposition reads

$$\Delta \ln \mathrm{TFP}_t = \sum_s v^Y_{st,t-1} \Delta \ln \mathrm{TFP}_{st} + \sum_s \left( \beta_s v^Y_{st,t-1} - \varepsilon_L v^L_{st,t-1} \right) \Delta \ln L_{st} + \sum_s \left( \alpha_s v^Y_{st,t-1} - \varepsilon_K v^K_{st,t-1} \right) \Delta \ln K_{st}$$

where the weights and the elasticities are computed as the average of time $t-1$ and time $t$, as defined above.

## C.3 Robustness checks

I consider how sensitive the results are to both the specification of the production function and to the estimation strategy. I estimate the decomposition with overhead labor, as well as with firm-specific factor elasticities, and I use an alternative estimation strategy for factor elasticities to allow for average distortions. This appendix provides details on how to estimate the decomposition under these alternative specifications.

### C.3.1 Overhead labor

The framework with overhead labor is given in Appendix B.3. In the presence of overhead labor, the decomposition is given by the same equations as the benchmark framework except that the decomposition is a function of production labor instead of overall labor. In particular,

(i) the employment share $L_i/L$ is replaced by the share of production labor $(L_i - f_i)/(L - \sum_i f_i)$ in the indexes $I^L_{EX}$, $I^L_{TE0}$,$I^L_{TE1}$, $I^L_{AE0}$ and $I^L_{AE1}$.

(ii) the labor distortions are computed as

$$1 + \tau_{it} = \beta_s \frac{p_{it}Y_{it}}{L_{it} - f_i}.$$

(iii) the firm-level productivity is computed as

$$A_{it} = \frac{Y_{it}^{1/\theta_s}}{(L_{it} - f_i)^{\beta_s} K_{it}^{\alpha_s}} \left( \frac{n_{st}}{Y_{st}} \right)^{(1-\theta_s)/\theta_s}.$$

To estimate the decomposition, I calibrate the overhead labor $f_i$ at 20 percent of each firm's average employment level.[7] As noted by Bartelsman et al. (2013), non-production workers account

[7] I have bounded the number of production workers at $\max(L_{it} - f_i, 1)$ to avoid negative values and outliers in the marginal product of labor.

for about 30 percent of total employment in the US manufacturing industry, and managers account for about 10 percent of total employment. In light of those numbers, 20 percent of overhead labor seems to be a reasonable assumption.

### C.3.2 Firm-specific factor elasticities

The specification is given in Appendix B.4. To compute the decomposition, I compute the labor elasticity for each firm as the average of its labor share

$$\beta_i = \frac{1}{T} \sum_t \frac{w_{it} L_{it}}{p_{it} Y_{it}},$$

and adjust accordingly the measures of the distortions and firm-level productivity. To avoid obtaining factor elasticities that are negative or greater than one, I have restricted the factor elasticities to vary between one standard deviation above and below the average sectoral labor share.[8] With this correction, the labor elasticity varies between 0.37 and 1.00.

The within-sector decomposition is then given by the equations of Appendix B.4, where the shares and elasticities are computed as the average of the initial and end period values, similarly to the economy-wide decomposition.

### C.3.3 Average sectoral distortions

To compute the factor elasticities in the presence of average unobserved distortion, I assume that distortions on capital and labor inputs are equal and use the cost share of labor instead of the income share, as suggested in Hall (1991):

$$\beta_s = \frac{1}{T} \sum_t \frac{w_{st} L_{st}}{w_{st} L_{st} + r_{st} K_{st}}.$$

The rental cost of capital $r_{st}$ is computed as $q + \delta$, where $q$ is the market interest rate, and $\delta$ is the depreciation rate. I use the bond yield series published by the Banque de France for $q$ and the average depreciation rate of 6.5% for $\delta$.

[8] The standard deviation has been computed after getting rid of negative and larger-than-one values. I have taken an upper bound of one for the few sectors in which it was greater than one.

# Appendix D - Additional tables and figures

Table D.1: Descriptive statistics

| | Number of firms | Median size | Value added share (%) | labor elasticity $\beta$ |
|---|---|---|---|---|
| MANUFACTURING | | | | |
| Food products | 10686 | 8 | 12.4 | 0.64 |
| Wearing apparel and leather products | 2961 | 13 | 3.1 | 0.76 |
| Printing and reproduction | 6855 | 7 | 6.7 | 0.77 |
| Pharmaceutical products and perfumes | 659 | 25 | 6.3 | 0.59 |
| Furniture | 4405 | 8 | 4.8 | 0.78 |
| Motor vehicle | 975 | 15 | 6.7 | 0.74 |
| Other transportation equipment | 644 | 10 | 2.0 | 0.84 |
| Mechanical equipment | 9386 | 11 | 12.2 | 0.80 |
| Electric equipment | 3730 | 8 | 6.8 | 0.80 |
| Mineral products | 3441 | 11 | 6.0 | 0.66 |
| Textiles | 1988 | 17 | 2.8 | 0.77 |
| Wood and paper products | 3863 | 12 | 4.5 | 0.75 |
| Rubber and plastics products | 3384 | 17 | 9.9 | 0.71 |
| Fabricated metal products | 10049 | 11 | 11.2 | 0.78 |
| Electronic products | 1446 | 16 | 4.5 | 0.75 |
| SERVICES | | | | |
| Trade and repair of motor vehicles | 24784 | 5 | 6.1 | 0.76 |
| Wholesale trade | 42753 | 6 | 19.9 | 0.73 |
| Retail trade | 71566 | 4 | 14.6 | 0.72 |
| Transportation | 18869 | 8 | 18.3 | 0.77 |
| Real estate | 14834 | 4 | 4.0 | 0.67 |
| Telecommunications and postal services | 413 | 8 | 0.2 | 0.74 |
| Professional and technical services | 39136 | 5 | 17.3 | 0.83 |
| Administrative and support services | 13926 | 7 | 9.8 | 0.84 |
| Accommodation and food services | 23632 | 6 | 5.2 | 0.78 |
| Arts, entertainment and recreation | 5498 | 5 | 3.6 | 0.59 |
| Personal and domestic services | 8653 | 5 | 1.0 | 0.82 |

Note: This table presents the average number of firms, their median size, and the value added of each sector as a fraction of manufacturing or services total value added, as well as the labor elasticity for each sector. The labor elasticity is computed as the average income share of labor. The median size refers to the number of employees. Averages are computed over the 1991-2006 period.

Table D.2: Adjustment costs and distortions

| | capital distortions | labor distortions |
|---|---|---|
| $\sigma_{\text{inaction}}$ | 1.11 | 0.57 |
| $\sigma_{\text{inaction}}/\sigma_{\text{adjust}}$ | 1.22 | 1.22 |
| $\text{corr}(d\ln A_i, d\ln\tau_i)$ | 0.68 | 0.89 |

Note: This table gives the standard deviation of the log of capital and labor distortions for firms that do not adjust their investment or their employment, $\sigma_{\text{inaction}}$, and their standard deviation relative to the firms that do adjust their inputs, $\sigma_{\text{inaction}}/\sigma_{\text{adjust}}$. The inaction group refers to firms that changed their investment-to-asset ratio or their employment by less than 1 percent. The adjust group refers to the rest of the firms after trimming the top and bottom 1 percent of employment and capital and changes. The last line of the table reports the correlation between the log changes of distortions and the log changes of firm-level productivity. The standard deviation and the correlation are computed within sectors. The table reports the median value for 2005.

Figure D.1: Growth of real value added in the manufacturing industry: national accounts (INSEE) vs SUSE-BRN

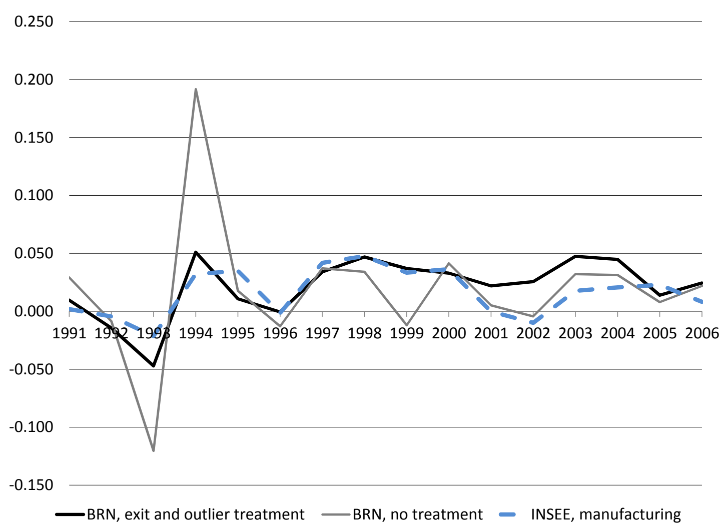


Table D.3: Average components of productivity changes, 1991-2006

| | $\Delta\text{TFP}_s$ | $\Delta\text{TE}_s$ | $\Delta\text{AE}_s$ | $\Delta\text{EX}_s$ |
|---|---|---|---|---|
| MANUFACTURING | | | | |
| Food products | 0.003 | 0.012 | -0.012 | 0.004 |
| Wearing apparel and leather products | 0.033 | 0.020 | -0.020 | 0.034 |
| Printing and reproduction | 0.007 | 0.013 | -0.010 | 0.003 |
| Pharmaceutical products and perfumes | 0.027 | 0.060 | -0.032 | -0.002 |
| Furniture | 0.032 | 0.037 | -0.012 | 0.007 |
| Motor vehicle | 0.022 | 0.031 | -0.011 | 0.001 |
| Other transportation equipment | -0.018 | 0.000 | -0.018 | 0.000 |
| Mechanical equipment | 0.032 | 0.043 | -0.009 | -0.003 |
| Electric equipment | 0.083 | 0.091 | -0.007 | -0.001 |
| Mineral products | 0.010 | 0.015 | -0.009 | 0.004 |
| Textiles | 0.032 | 0.032 | -0.019 | 0.018 |
| Wood and paper products | 0.021 | 0.023 | -0.006 | 0.003 |
| Rubber and plastics products | 0.039 | 0.044 | -0.007 | 0.001 |
| Fabricated metal products | 0.000 | 0.005 | -0.008 | 0.003 |
| Electronic products | 0.057 | 0.069 | -0.009 | -0.003 |
| SERVICES | | | | |
| Trade and repair of motor vehicles | -0.017 | -0.015 | -0.005 | 0.002 |
| Wholesale trade | 0.020 | 0.033 | -0.013 | 0.000 |
| Retail trade | -0.002 | -0.006 | 0.004 | 0.000 |
| Transportation | 0.022 | 0.030 | -0.004 | -0.004 |
| Real estate | -0.004 | -0.002 | 0.001 | -0.003 |
| Telecommunications and postal services | 0.045 | 0.067 | 0.010 | -0.032 |
| Professional and technical services | -0.004 | 0.020 | -0.005 | -0.019 |
| Administrative and support services | -0.011 | -0.001 | 0.003 | -0.013 |
| Accommodation and food services | -0.014 | -0.020 | 0.011 | -0.006 |
| Arts, entertainment and recreation | -0.011 | 0.005 | 0.003 | -0.019 |
| Personal and domestic services | -0.014 | -0.012 | -0.001 | -0.001 |

Note: This table presents the 1991-2006 average of annual sectoral productivity growth and its components. The expression of the decomposition is given in Appendix C. The first column gives the average annual sectoral productivity growth. The second column gives the average firm-level efficiency growth. The third column gives the average change in allocative efficiency, and the last column the contribution of the extensive margin. The components may not sum to average productivity growth due to rounding.

Table D.4: FHK decomposition, Average components of productivity changes, 1991-2006

| | $\Delta \ln \bar{A}_s$ | Within | Reallocation | Extensive margin |
|---|---|---|---|---|
| MANUFACTURING | | | | |
| Food products | 0.013 | -0.000 | 0.009 | 0.004 |
| Wearing apparel and leather products | 0.071 | -0.010 | 0.048 | 0.032 |
| Printing and reproduction | 0.018 | 0.000 | 0.011 | 0.007 |
| Pharmaceutical products and perfumes | 0.041 | 0.029 | 0.010 | 0.002 |
| Furniture | 0.038 | 0.020 | 0.010 | 0.008 |
| Motor vehicle | 0.002 | 0.023 | -0.026 | 0.004 |
| Other transportation equipment | -0.0088 | -0.023 | 0.008 | 0.007 |
| Mechanical equipment | 0.041 | 0.029 | 0.008 | 0.004 |
| Electric equipment | 0.043 | 0.083 | -0.050 | 0.009 |
| Mineral products | 0.012 | 0.004 | -0.000 | 0.008 |
| Textiles | 0.037 | 0.005 | 0.015 | 0.017 |
| Wood and paper products | 0.020 | 0.014 | 0.002 | 0.004 |
| Rubber and plastics products | 0.040 | 0.035 | -0.001 | 0.006 |
| Fabricated metal products | 0.003 | -0.006 | 0.004 | 0.004 |
| Electronic products | 0.070 | 0.055 | 0.010 | 0.006 |
| SERVICES | | | | |
| Trade and repair of motor vehicles | -0.013 | -0.023 | 0.009 | 0.002 |
| Wholesale trade | 0.034 | 0.013 | 0.016 | 0.005 |
| Retail trade | 0.011 | -0.008 | 0.019 | 0.000 |
| Transportation | 0.008 | 0.024 | -0.017 | 0.001 |
| Real estate | 0.023 | -0.001 | 0.016 | 0.008 |
| Telecommunications and postal services | 0.053 | 0.068 | -0.000 | -0.014 |
| Professional and technical services | 0.010 | 0.008 | 0.016 | -0.014 |
| Administrative and support services | 0.008 | 0.002 | 0.015 | -0.009 |
| Accommodation and food services | -0.003 | -0.014 | 0.021 | -0.010 |
| Arts, entertainment and recreation | -0.036 | 0.002 | -0.015 | -0.023 |
| Personal and domestic services | -0.011 | -0.015 | 0.009 | -0.006 |

Note: This table presents the 1991-2006 average of the Foster et al. (2001) decomposition. The first column gives the average growth rate of the weighted average of productivity. The components may not sum to average TFP growth due to rounding. The expression of the decomposition is given in Appendix C.

Table D.5: FHK decomposition, correlation with sectoral value added growth, 1991-2006

| | $\Delta \ln \bar{A}_s$ | Within | Reallocation | Extensive margin |
|---|---|---|---|---|
| MANUFACTURING | | | | |
| Food products | 0.67 | 0.88 | 0.19 | -0.09 |
| Wearing apparel and leather products | 0.88 | 0.95 | 0.58 | 0.30 |
| Printing and reproduction | 0.76 | 0.89 | 0.12 | 0.31 |
| Pharmaceutical products and perfumes | 0.95 | 0.90 | 0.78 | 0.12 |
| Household equipment | 0.74 | 0.89 | -0.05 | -0.12 |
| Motor vehicle | 0.92 | 0.70 | 0.93 | 0.05 |
| Other transportation equipment | 0.96 | 0.96 | 0.38 | 0.22 |
| Mechanical equipment | 0.68 | 0.86 | 0.19 | -0.22 |
| Electric equipment | 0.88 | 0.67 | 0.90 | 0.05 |
| Mineral products | 0.90 | 0.94 | 0.68 | 0.04 |
| Textiles | 0.41 | 0.79 | -0.30 | -0.26 |
| Wood and paper products | 0.92 | 0.94 | 0.44 | 0.18 |
| Chemical, rubber and plastics products | 0.90 | 0.93 | 0.66 | 0.25 |
| Fabricated metal products | 0.61 | 0.88 | 0.01 | -0.84 |
| Electronic products | 0.88 | 0.67 | 0.62 | 0.17 |
| SERVICES | | | | |
| Trade and repair of motor vehicles | 0.86 | 0.90 | 0.68 | 0.39 |
| Wholesale trade | 0.87 | 0.91 | 0.30 | -0.34 |
| Retail trade | 0.71 | 0.77 | 0.32 | -0.38 |
| Transportation | 0.93 | 0.90 | 0.80 | 0.15 |
| Real estate | -0.13 | 0.76 | -0.38 | -0.78 |
| Telecommunications and postal services | 0.74 | 0.73 | 0.61 | 0.01 |
| Professional and technical services | 0.69 | 0.34 | 0.53 | 0.17 |
| Administrative and support services | 0.81 | 0.50 | 0.85 | 0.13 |
| Accommodation and food services | 0.65 | 0.90 | 0.26 | -0.20 |
| Arts, entertainment and recreation | 0.60 | 0.65 | 0.62 | -0.67 |
| Personal and domestic services | 0.30 | 0.37 | 0.10 | -0.15 |

Note: This table presents the correlations between real value added growth and the components of the Foster et al. (2001) decomposition. The expression of the decomposition is given in Appendix C.

Table D.6: FHK decomposition, contribution to sectoral volatility, 1991-2006

| | Within | Reallocation | Extensive margin |
|---|---|---|---|
| MANUFACTURING | | | |
| Food products | 0.54 | 0.34 | 0.11 |
| Wearing apparel and leather products | 0.59 | 0.31 | 0.11 |
| Printing and reproduction | 0.61 | 0.27 | 0.12 |
| Pharmaceutical products and perfumes | 0.57 | 0.41 | 0.02 |
| Household equipment | 0.75 | 0.24 | 0.02 |
| Motor vehicles | 0.50 | 0.49 | 0.00 |
| Other transportation equipment | 0.89 | 0.09 | 0.02 |
| Mechanical equipment | 0.68 | 0.28 | 0.03 |
| Electric equipment | 0.32 | 0.64 | 0.03 |
| Mineral products | 0.68 | 0.30 | 0.02 |
| Textiles | 0.61 | 0.28 | 0.12 |
| Wood and paper products | 0.82 | 0.16 | 0.02 |
| Chemical, rubber and plastics products | 0.70 | 0.27 | 0.03 |
| Fabricated metal products | 0.74 | 0.38 | -0.12 |
| Electronic products | 0.35 | 0.64 | 0.00 |
| SERVICES | | | |
| Trade and repair of motor vehicles | 0.55 | 0.39 | 0.06 |
| Wholesale trade | 0.80 | 0.22 | -0.02 |
| Retail trade | 0.58 | 0.42 | 0.00 |
| Transportation | 0.35 | 0.63 | 0.02 |
| Real estate | 0.31 | 0.16 | 0.52 |
| Telecommunications and postal services | 0.35 | 0.60 | 0.05 |
| Professional and technical services | 0.63 | 0.28 | 0.10 |
| Administrative and support services | 0.22 | 0.74 | 0.04 |
| Accommodation and food services | 0.51 | 0.46 | 0.03 |
| Arts, entertainment and recreation | 0.26 | 0.73 | 0.00 |
| Personal and domestic services | 0.64 | 0.26 | 0.10 |

Note: This table presents the contribution of each components of the Foster et al. (2001) decomposition to the volatility of the growth rate of average productivity. The expression of the decomposition is given in Appendix C, and the contributions are computed as described in section 4.3.2. The sum of the contribution of the within, reallocation, and the extensive margin component equals 1.

Table D.7: Reallocation decomposition, Average components of productivity changes, 1991-2006

| | $\Delta\text{TFP}_s$ | Within | Reallocation | Extensive margin |
|---|---|---|---|---|
| MANUFACTURING | | | | |
| Food products | 0.003 | -0.002 | 0.001 | 0.004 |
| Wearing apparel and leather products | 0.033 | -0.015 | 0.014 | 0.034 |
| Printing and reproduction | 0.007 | -0.002 | 0.005 | 0.003 |
| Pharmaceutical products and perfumes | 0.027 | 0.028 | 0.001 | -0.002 |
| Furniture | 0.032 | 0.018 | 0.008 | 0.007 |
| Motor vehicle | 0.022 | 0.023 | -0.003 | 0.001 |
| Other transportation equipment | -0.018 | -0.024 | 0.005 | 0.000 |
| Mechanical equipment | 0.032 | 0.028 | 0.006 | -0.003 |
| Electric equipment | 0.083 | 0.088 | -0.004 | -0.001 |
| Mineral products | 0.010 | 0.003 | 0.003 | 0.004 |
| Textiles | 0.032 | 0.004 | 0.009 | 0.018 |
| Wood and paper products | 0.021 | 0.013 | 0.004 | 0.003 |
| Rubber and plastics products | 0.039 | 0.034 | 0.003 | 0.001 |
| Fabricated metal products | 0.000 | -0.007 | 0.005 | 0.003 |
| Electronic products | 0.057 | 0.054 | 0.006 | -0.003 |
| SERVICES | | | | |
| Trade and repair of motor vehicles | -0.017 | -0.024 | 0.005 | 0.002 |
| Wholesale trade | 0.020 | 0.013 | 0.006 | 0.000 |
| Retail trade | -0.002 | -0.009 | 0.007 | 0.000 |
| Transportation | 0.022 | 0.025 | 0.001 | -0.004 |
| Real estate | -0.004 | -0.002 | 0.001 | -0.003 |
| Telecommunications and postal services | 0.045 | 0.072 | 0.005 | -0.032 |
| Professional and technical services | -0.004 | 0.009 | 0.006 | -0.019 |
| Administrative and support services | -0.011 | 0.002 | 0.000 | -0.013 |
| Accommodation and food services | -0.014 | -0.015 | 0.006 | -0.006 |
| Arts, entertainment and recreation | -0.011 | 0.003 | 0.006 | -0.019 |
| Personal and domestic services | -0.014 | -0.015 | 0.003 | -0.001 |

Note: This table presents the 1991-2006 average of the components of the reallocation decomposition. The expression of the decomposition is given in Appendix C. The components may not sum to average TFP growth due to rounding.

Table D.8: Reallocation decomposition, correlation with sectoral value added growth, 1991-2006

| | $\Delta \ln \text{TFP}_s$ | Within | Reallocation | Extensive margin |
|---|---|---|---|---|
| MANUFACTURING | | | | |
| Food products | 0.88 | 0.88 | 0.22 | 0.04 |
| Wearing apparel and leather products | 0.94 | 0.95 | 0.64 | 0.05 |
| Printing and reproduction | 0.90 | 0.89 | -0.05 | 0.31 |
| Pharmaceutical products and perfumes | 0.94 | 0.88 | 0.34 | 0.19 |
| Household equipment | 0.88 | 0.89 | -0.38 | -0.29 |
| Motor vehicle | 0.84 | 0.71 | 0.33 | -0.62 |
| Other transportation equipment | 0.96 | 0.96 | -0.13 | 0.38 |
| Mechanical equipment | 0.82 | 0.86 | -0.20 | -0.18 |
| Electric equipment | 0.81 | 0.66 | 0.18 | 0.24 |
| Mineral products | 0.94 | 0.94 | -0.02 | 0.11 |
| Textiles | 0.66 | 0.79 | 0.46 | -0.34 |
| Wood and paper products | 0.93 | 0.94 | 0.12 | -0.12 |
| Chemical, rubber and plastics products | 0.92 | 0.93 | -0.18 | -0.01 |
| Fabricated metal products | 0.76 | 0.88 | -0.42 | -0.86 |
| Electronic products | 0.69 | 0.67 | 0.50 | -0.09 |
| SERVICES | | | | |
| Trade and repair of motor vehicles | 0.88 | 0.89 | -0.07 | 0.30 |
| Wholesale trade | 0.88 | 0.91 | 0.02 | -0.39 |
| Retail trade | 0.66 | 0.78 | -0.08 | -0.50 |
| Transportation | 0.90 | 0.89 | -0.30 | -0.02 |
| Real estate | 0.77 | 0.77 | 0.03 | -0.22 |
| Telecommunications and postal services | 0.77 | 0.74 | 0.45 | -0.12 |
| Professional and technical services | 0.31 | 0.31 | -0.48 | 0.26 |
| Administrative and support services | 0.48 | 0.51 | -0.15 | 0.06 |
| Accommodation and food services | 0.79 | 0.89 | 0.09 | -0.19 |
| Arts, entertainment and recreation | 0.57 | 0.65 | 0.15 | -0.69 |
| Personal and domestic services | 0.40 | 0.38 | 0.10 | -0.25 |

Note: This table presents the correlation of the components of the reallocation decomposition with real value added growth. The expression of the decomposition is given in Appendix C.

Table D.9: Reallocation decomposition, contribution to sectoral volatility, 1991-2006

| | Within | Reallocation | Extensive margin |
|---|---|---|---|
| MANUFACTURING | | | |
| Food products | 0.89 | 0.05 | 0.06 |
| Wearing apparel and leather products | 0.87 | 0.08 | 0.05 |
| Printing and reproduction | 0.89 | -0.00 | 0.11 |
| Pharmaceutical products and perfumes | 0.94 | 0.01 | 0.04 |
| Household equipment | 1.08 | -0.03 | -0.06 |
| Motor vehicles | 1.05 | -0.03 | -0.02 |
| Other transportation equipment | 0.97 | -0.00 | 0.03 |
| Mechanical equipment | 0.97 | -0.00 | 0.03 |
| Electric equipment | 1.04 | -0.09 | 0.04 |
| Mineral products | 0.98 | 0.00 | 0.01 |
| Textiles | 0.89 | 0.04 | 0.07 |
| Wood and paper products | 0.99 | 0.00 | 0.01 |
| Chemical, rubber and plastics products | 0.99 | -0.01 | 0.01 |
| Fabricated metal products | 1.23 | -0.01 | -0.21 |
| Electronic products | 0.91 | 0.06 | 0.03 |
| SERVICES | | | |
| Trade and repair of motor vehicles | 0.90 | 0.01 | 0.10 |
| Wholesale trade | 1.02 | 0.05 | -0.06 |
| Retail trade | 1.03 | 0.00 | -0.03 |
| Transportation | 1.02 | -0.01 | -0.00 |
| Real estate | 0.99 | 0.02 | -0.02 |
| Telecommunications and postal services | 0.75 | 0.09 | 0.16 |
| Professional and technical services | 1.02 | -0.07 | 0.05 |
| Administrative and support services | 0.89 | 0.11 | 0.00 |
| Accommodation and food services | 0.86 | 0.06 | 0.08 |
| Arts, entertainment and recreation | 0.86 | 0.19 | -0.05 |
| Personal and domestic services | 1.03 | -0.08 | 0.05 |

Note: This table presents the contribution of each components of the reallocation decomposition to the volatility of sectoral productivity. The expression of the decomposition is given in Appendix C and the contributions are computed as described in section 4.3.2. The sum of the contribution of the within, reallocation, and the extensive margin component equals 1.

Figure D.2: Growth of real value added in the service sector: national accounts (INSEE) vs SUSE-BRN

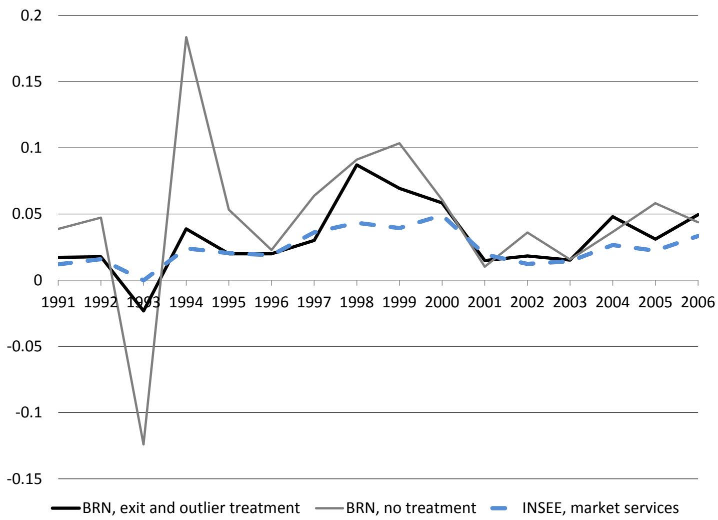
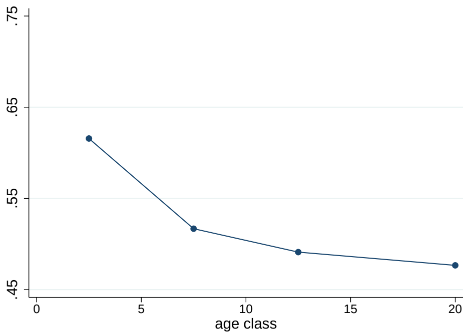
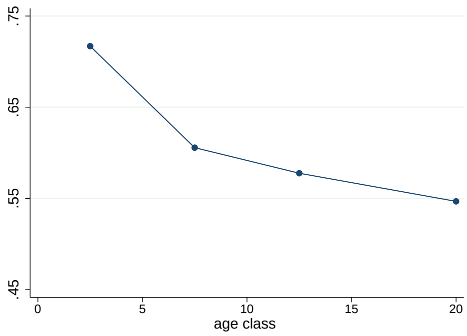


Figure D.3: Dispersion in capital and labor distortions, by firm age (2005)

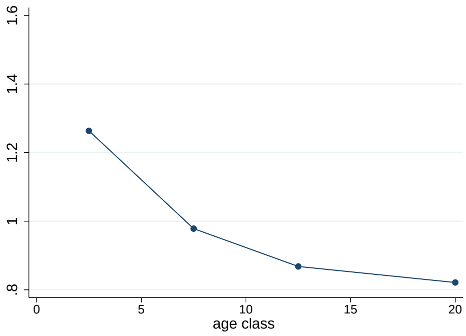
(a) Labor distortions, manufacturing



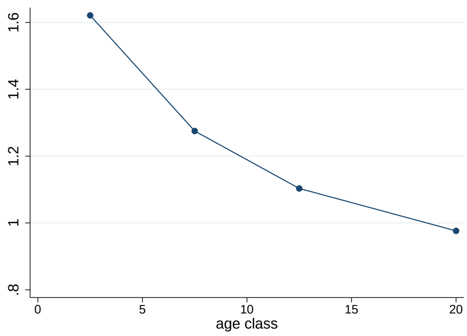

(b) Labor distortions, services




(c) Capital distortions, manufacturing

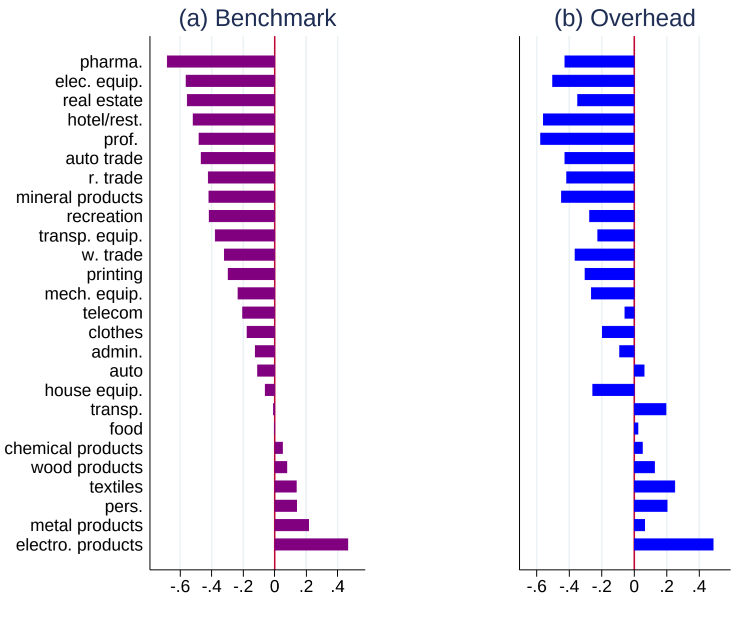



(d) Capital distortions, services



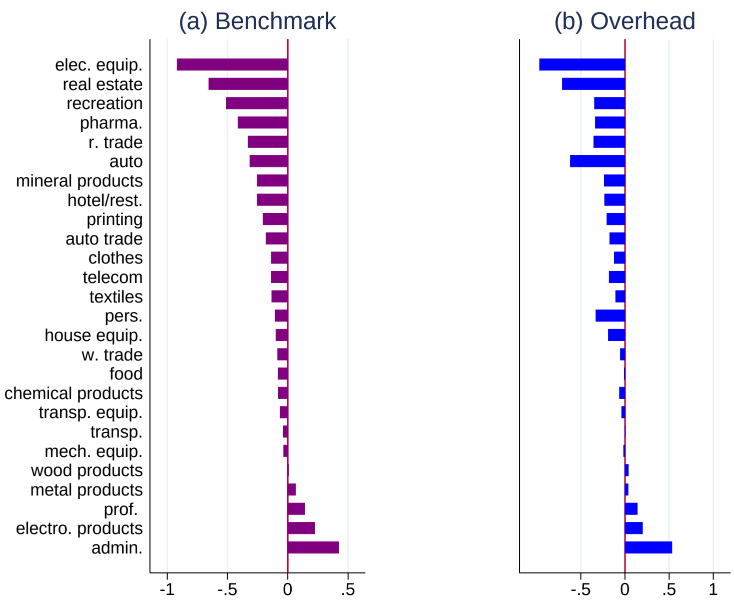

Note: This figure gives the standard deviation of the log of labor and capital distortions across all sectors in 2005 by firm age. The age classes are the following: 0, 1-4, 5-9, 10-14, 15+ years. The “midpoint” 20 refers to the group of firms aged 15 years and older.

Figure D.4: Overhead labor - correlation of $\Delta AE$ with output changes



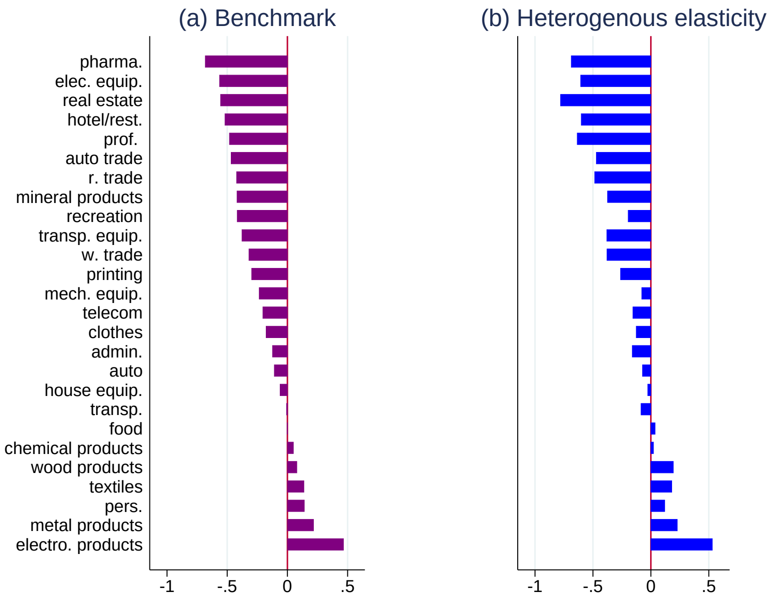

Note: This Figure shows the correlation of changes in allocative efficiency with changes in output for the overhead labor specification. The correlation for the benchmark specification is reported in panel (a).

Figure D.5: Overhead labor - contribution of $\Delta AE$ to the volatility of sectoral TFP



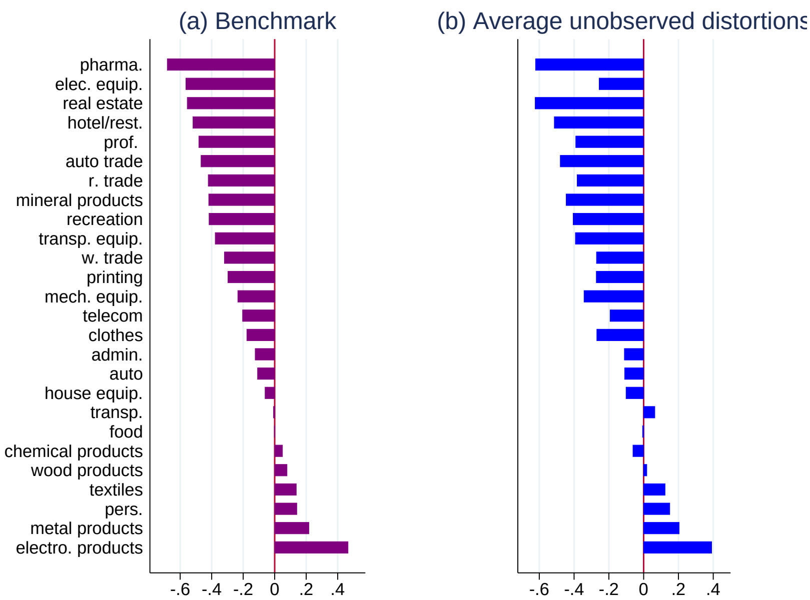

Note: This Figure shows the contribution of changes in allocative efficiency to the volatility of sectoral TFP for the overhead labor specification. The contribution for the benchmark specification is reported in panel (a).

Figure D.6: Firm-specific factor elasticities - correlation of $\Delta AE$ with output changes



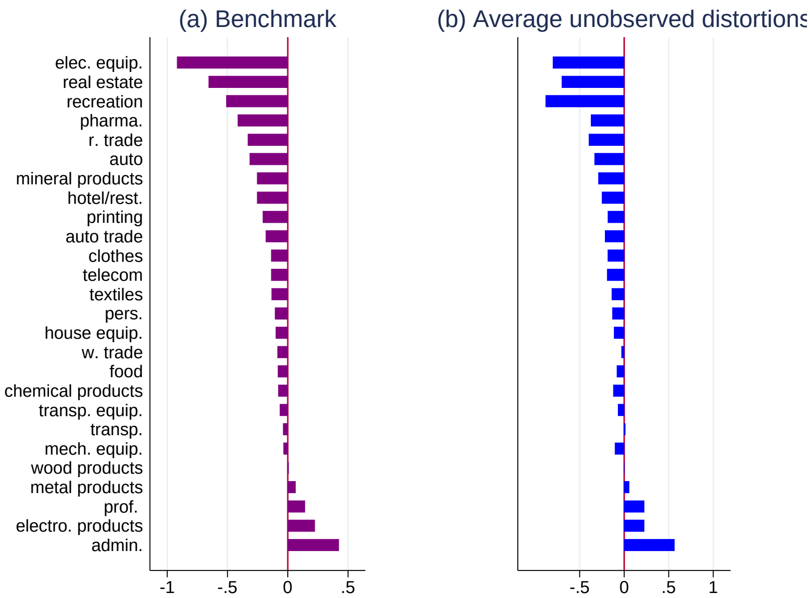

Note: This Figure shows the correlation of changes in allocative efficiency with changes in output for the firm-specific factor elasticities specification. The correlation for the benchmark specification is reported in panel (a).

Figure D.7: Average unobserved distortions - correlation of $\Delta AE$ with output changes




Note: This Figure shows the correlation of changes in allocative efficiency with changes in output under the assumption of average unobserved distortions. The correlation for the benchmark estimation strategy is reported in panel (a).

Figure D.8: Average unobserved distortions - contribution of $\Delta AE$ to the volatility of sectoral TFP




Note: This Figure shows the contribution of changes in allocative efficiency to the volatility of sectoral TFP under the assumption of average unobserved distortions. The contribution for the benchmark estimation strategy is reported in panel (a).

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